Strategic Bargaining with Destructive Power

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Abstract: This paper studies a two-player alternating offers bargaining model in which one of the agents has the ability to damage permanently the "pie" bargained over. I show how this feature can result in an increase of the cost of rejecting an offer for the "non-harming player". Beside the "Rubinstenian" bilateral monopoly outcome, I show that it is possible to select a "harming" equilibrium in which the sequence of damages to the pie is endogenously determined and payoffs do not vary monotonically with the discount factor.

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1 Introduction

This paper studies a simple two-player alternating offers bargaining model in which one of the players is allowed to destroy part of the surplus bargained over. This is a feature which can arise in a number of circumstances, and that has been addressed to some extent by the existing literature. Wage negotiations between a firm and a union can trigger strikes, which disrupt the productive process, thereby imposing per period costs on the firm (see Fernandez and Glazer (1991) and Haller and Holden (1991)). Alternatively, one player could increase his opponent's (opportunity) cost of rejecting an offer either by choosing who is going to be the next proposer (as in Kambe (1994)), or by appropriately timing offers (as in Perry and Reny (1993) and akovics (1993)). However, these models focus on each player's ability to alter his opponent cost of rejecting an offer, by performing actions which can be detrimental only in terms of one's opponent's payoff; thus, such models consider a "money burning" activity for one or both players[1], thereby focusing on their ability to inflict a *side cost* on their opponent, without actually damaging the object bargained over.

In contrast, there are instances in which - beside time preference considerations - disagreement can result in the reduction of the size of the surplus to be shared. For instance, a worker whose bid for a higher wage has been frustrated can be less effective in his job, and thus reduce the firm's profitability.

uch different approach is adopted in Dasgupta and Maskin (1989), where both bargainers have "destructive power", in the sense that they can actually destroy part of the surplus to be shared. More recently, Bush et al. (1996) have modelled the bargaining with such endogenous reduction of the pie in a strategic setting: here in case of disagreement one of the players can perform a damaging action which increases the rate at which the pie shrinks. In this paper I model destructive power in an alternative manner, allowing the harming player to destroy a part of the cake. In so doing, I build on Manzini (1996). There I studied how one player's commitment to a specific destructive action affects the equilibrium outcome, and showed that (not surprisingly) destructive power increases the

[1]For a more general treatment of this class of models, see Bush and Wen (1995), Avery and Zemsky (1994) and references therein.

quilibrium payoff of th "harming" play r. B sid s this quit intuitiv r sult, th r is th l ss obvious on that, contrary to th standard Rubinst in (1982) mod l, th quilibrium shar s do *not* vary monotonically with th discount factor, which th r for has non-trivial ff cts on quilibrium payoffs. As I shall discuss mor in d tail lat r on, variations of th discount factor hav two conflicting ff cts on th quilibrium partition, and in th limit as play rs b com infinit ly pati nt, th "harming" play r manag s to g t th whol pi , *regardless of how small his destructive power.*

How v r, in many r al lif n gotiations xog nous commitm nt to d structiv activiti s can b ith r unlik ly or unf asibl .In this pap r I r lax th commitm nt assumption, and mod l a bargaining gam with d structiv pow r in which th "harming" play r is allow d to choos both wh th r or not to x rcis his d structiv pow r, and of how much. As it turns out, this corr sponds to a mor fl xibl way of mod lling commitm nt. Notic that th timing of off rs as mod ll d in Rubinst in (1982) can b int rpr t d as play r's commitm nt to an off r: sinc no play r can chang his own off r b for a c rtain amount of tim has laps d, h is *de facto* bound to his proposal ov r that sp ll of tim . P rry and R ny (1993) and Sakovics (1993) hav modifi d th basic s tting by consid ring a "d structur d" bargaining, in which play rs can choos th timing of off rs and r actions to thos off rs. This aff cts play rs' cost of r j cting an off r, so that diff r nt timing structur s corr spond to a diff r nt d gr of commitm nt to a sp cific proposal. As an alt rnativ to this approach, I propos to mod l a play r's commitm nt as his ability to choos th l v l of d structiv pow r h xh rts. I show that in quilibrium h can obtain a shar whos siz d p nds positiv ly on his d structiv pow r; furth rmor , quilibrium shar s ar non monotonic in th discount factor, and as th discount factor approach s unity th harming play r obtains th whol profits. Thus, as long as th play r with d structiv pow r is fr to choos a suitabl "harming structur " (that is, a f asibl s qu nc of cr dibl d structiv actions), it is possibl to r tri v a r sult of th sam natur of th mod l with xog nous commitm nt, this tim r lying on a mor r alistic structur .

Th pap r is organis d as follows: in th n xt s ction th mod l is introduc d and solv d in Markov strat gi s. S ction 3 contains a sp cific xampl of a harming structur .

Section 4 solves for the equilibria in non-stationary strategies, and section 5 concludes.

2 Bargaining with destructive power

Consider the following simple extension of the two-player alternating offers bargaining game. For the sake of argument, let our two players be a union (indexed with u) and a firm (f), engaged in negotiations over the wage, determined as a share of the firm's profits, normalised to unity. At time $t = 0$ (and at all even stages of the game) player 1 (the union) proposes a partition of the available surplus. The firm can either accept, ending the game, or reject the offer. If the latter, before the firm can propose a split, the union can *choose* whether or not to harm the firm, i.e. the union can reduce the available profits of an amount $c_i \in [0, 1]$ (which we will refer to as "harm"). The game then moves to the following round, in which the firm makes a counterproposal; the union can either accept it, ending the game, or reject the offer. In case of disagreement, the game enters the following round of negotiations, in which it is the union's turn to make a proposal, and so on[2]. The union's action space is $X \times C \equiv [0, 1] \times [0, 1]$; clearly, $c_i = 0$ implies that the union does not harm. Because of the non-stationary structure of this framework, for the moment I focus on Markov strategies, that is strategies which are history independent, and that depend only on the *state* of the play: hence past actions influence current play only through their effect on a state variable which summarises all the (payoff relevant) information. For our purposes, a state is defined simply by π_n, the size of the "pie" after it has been damaged n times. Let c_n be the amount of damage which the union selects the n^{th} time it harms; then, $\pi_n = 1 - \sum_{i=0}^{n-1} c_{i+1}$, (with $\pi_0 = 1$) which implies that $\pi_{n+1} = \pi_n - c_{n+1}$. In other words, a state is characterized by the actual size of the pie, which reveals how many times in the past the union has "hit".

Recalling that a Markov perfect equilibrium (m.p.e.) is a profile of Markov strategies

[2] In this model the player with destructive ability (the union) is allowed to destroy part of the cake only after his own offer has been rejected, not after *any* rejection. This is assumed just for ease of exposition, since it is possible to prove that harming after rejecting an opponent's proposal - and before making one' own - would never be credible. See for instance Avery and Zemsky (1994), Bush et al. (1996), Fernandez and Glazer (1991), Haller and Holden (1991).

hich yields a Nash equilibrium in every proper subgame (see Bernheim and Ray (1989), Maskin and Tirole (1988)), I can then claim the follo ing:

Proposition 1 *In a bargaining game of alternating offers in which the worker can harm after every rejection of the firm, if* $c_n \leq \delta \sum_{i=0}^{\infty} c_{n+i+1}\delta^{2i}\ \forall n$, *there exist two m.p.e., which I will call "harming equilibrium" and "Rubinstenian equilibrium", respectively, in which agreement is reached immediately either on the "harming" equilibrium partition:*

$$x^* = \left(x_w^*, x_f^*\right) = \left(\frac{1}{1+\delta}\left(1+\delta\sum_{i=0}^{\infty} c_{i+1}\delta^{2i}\right), \frac{\delta}{1+\delta}\left(1-\sum_{i=0}^{\infty} c_{i+1}\delta^{2i}\right)\right)$$

or on the "Rubinstenian" equilibrium partition:

$$\hat{x}^* = (\hat{x}_w^*, \hat{x}_f^*) = \left(\frac{1}{1+\delta}; \frac{\delta}{1+\delta}\right)$$

where in each partition the first entry is the share received by the union, and the second entry is the share received by the firm.

The above states that in a bargaining game of alternating offers in hich one of the players can choose the level of damage to the pie, t o Markov perfect equilibria exists: in one, no harm is ever committed, so that the Rubinstenian equilibrium results; in the other one, ho ever, the harming player achieves his maximum payoff, hich is greater than the Rubinstenian and is positively related to his harming ability.

Proof. The strategies that support the Rubinstenian equilibrium prescribe to the union al ays to propose a partition $\hat{x}^*$, accept any proposal $\hat{y}^*$ and any $x \geq \hat{y}_u^*$, reject any proposal $x < \hat{y}_u^*$, and never harm the firm hen rejecting; to the firm, al ays to propose a partition $\hat{y}^*$, accept any proposal $\hat{x}^*$ and any $x \geq \hat{x}_f^*$, and reject any proposal $x < \hat{x}_f^*$, here $\hat{y}^*$ is defined as:

$$\hat{y}^* = \left(\hat{y}_u^*; \hat{y}_f^*\right) = \left(\frac{\delta}{1+\delta}; \frac{1}{1+\delta}\right)$$

To specify the strategies that support the harming equilibrium, let $x^*(\pi_n) = (x_u^*(\pi_n), x_f^*(\pi_n))$ be the m.p.e. partition in subgames starting ith an offer by the union, defined as:

$$\begin{aligned} x^*(\pi_n) &= \left(x_u^*(\pi_n); x_f^*(\pi_n)\right) = \\ &= \left(\frac{1}{1+\delta}\left(\pi_n+\delta\sum_{i=0}^{\infty} c_{n+i+1}\delta^{2i}\right), \frac{\delta}{1+\delta}\left(\pi_n-\sum_{i=0}^{\infty} c_{n+i+1}\delta^{2i}\right)\right) \end{aligned}$$

hen the m.p.e. strategies for this equilibrium are for the union always to propose a partition $x^*(\pi_n)$, accept any proposal $y^*(\pi_n)$ and any $x \geq y_u^*(\pi_n)$, reject any proposal $x < y_u^*(\pi_n)$, and to harm the firm after a rejection if $c_n \leq \delta \sum_{i=0}^{\infty} c_{n+i+1}\delta^{2i}$; the equilibrium strategy for the firm is always to propose a partition $y^*(\pi_n)$, accept any proposal $x^*(\pi_n)$ and any $x \geq x_f^*(\pi_n)$, and reject any proposal $x < x_f^*(\pi_n)$, where $y^*(\pi_n)$ is defined as:

$$\begin{aligned} y^*(\pi_n) &= \left(y_u^*(\pi_n); y_f^*(\pi_n)\right) = \\ &= \left(\frac{\delta}{1+\delta}\left(\pi_n + \delta \sum_{i=0}^{\infty} c_{n+i+1}\delta^{2i}\right), \frac{1}{1+\delta}\left(\pi_n - \delta^2 \sum_{i=0}^{\infty} c_{n+i+1}\delta^{2i}\right)\right) \end{aligned}$$

hen the proof is easily obtained using standard techniques. Let $x_u(\pi_n)$ and $X_u(\pi_n)$ ($y_f(\pi_n)$ and $Y_f(\pi_n)$) denote the infimum and supremum equilibrium payoff to the union (firm) in subgames in which it is the first to make an offer when bargaining over a pie of size π_n. hen, the unique m.p.e. partition can be retrieved by solving recursively the following system of equations:

$$x_u(\pi_n) \geq \begin{cases} \pi_n - \delta Y_f(\pi_{n+1}) & if \quad c_{n+1} \neq 0 \\ \pi_n - \delta Y_f(\pi_n) & if \quad c_{n+1} = 0 \end{cases} \tag{1}$$

$$X_u(\pi_n) \leq \begin{cases} \pi_n - \delta y_f(\pi_{n+1}) & if \quad c_{n+1} \neq 0 \\ \pi_n - \delta y_f(\pi_n) & if \quad c_{n+1} = 0 \end{cases} \tag{2}$$

$$y_f(\pi_n) \geq \pi_n - \delta X_u(\pi_n) \tag{3}$$

$$Y_f(\pi_n) \leq \pi_n - \delta x_u(\pi_n) \tag{4}$$

with

$$\begin{aligned} &x_u(\pi_n) + x_f(\pi_n) \leq \pi_n \\ &y_u(\pi_n) + y_f(\pi_n) \leq \pi_n \\ &X_u(\pi_n) + X_f(\pi_n) \leq \pi_n \\ &Y_u(\pi_n) + Y_f(\pi_n) \leq \pi_n \end{aligned} \tag{5}$$

Equations 1 and 2 state that whenever the pie is of size π_n, the equilibrium share to the union in subgames in which it is the first mover depends on whether or not the union

arms t e firm if confronted wit a rejection. If it does, t e union can claim at least (at most) t e actual pie net of t e present discounted value of t e supremum (infimum) equilibrium s are to t e firm in t e following subgame, in w ic t e two parties would be bargaining on a pie of size now reduced to $\pi_{n+1} = \pi_n - c_{n+1}$; alternatively, if t e union decides not to arm, it will ave to concede to t e firm at most (at least) t e supremum (infimum) equilibrium s are it could get in t e following period, in present discounted value terms, out of a pie of same size. Analogous interpretation is to be given to equations 3 and 4, w ere owever t is time t e pie does not c ange from one period to t e next. It is wort noticing t at in subgames starting wit an offer by t e firm t ere can never be a c ange in state. Finally, equation 5 describes t e usual constraints on t e sum of t e s ares to eac player not to exceed t e size of t e pie bargained over.

If t e union decides not to arm, t en it is easy to c eck t at t e above system of equations yields $X_u(\pi_n) = x_u(\pi_n) = \frac{1}{1+\delta}\pi_n$ and $Y_u(\pi_n) = y_u(\pi_n) = \frac{\delta}{1+\delta}\pi_n$. T us, t e equilibrium payoff pair coincides wit t e Rubinstenian solution out of a "pie" of size π_n; ence never arming carries immediate agreement on t e partition $\hat{x}^*$ (since $\pi_0 = 1$). Notice t at t is also implies t at in general, as soon as t e union decides to stop arming, t e game reverts to a "standard" alternating offers bargain (from t at period onwards), w ic will yield t e Rubinstenian solution out of a pie of reduced size. In fact, w enever t e union c ooses *nh* a subsequent c ange of t e state is no longer possible, since equations 1 and 2 above reduce to:

$$x_u(\pi_n) \geq \pi_n - \delta Y_f(\pi_n) \quad \text{and} \quad X_u(\pi_n) \leq \pi_n - \delta y_f(\pi_n)$$

w ic , coupled wit 3 and 4, determine t e equilibrium payoffs uniquely for a cake of size π_n.

Consider now t e case w en t e union arms if t e firm rejects is proposal. Again, equations 1 and 2 above can be reduced, so t at t e arming payoffs can be retrieved inductively from:

$$x_u(\pi_n) \geq \pi_n - \delta\pi_{n+1} + \delta^2 x_u(\pi_{n+1})$$

and

$$X_u(\pi_n) \leq \pi_n - \delta\pi_{n+1} + \delta^2 X_u(\pi_{n+1})$$

T us, a strategy w ic prescribed to t e union to arm z times would yield:

$$x_u(\pi_n) \geq \sum_{i=0}^{z-1}(\pi_{n+i} - \delta\pi_{n+1+i})\delta^{2i} - \frac{\delta^{2z-1}}{1+\delta}\pi_{n+z} \geq X_u(\pi_n)$$

Since $\pi_{n+i} > \delta\pi_{n+1+i}\forall i$ (being $\delta < 1$ and $\pi_{n+i} \geq \pi_{n+1+i}\forall i$), t e above increases wit z. T us, t e union can attain its ig est payoff by following a strategy w ic prescribes to maximize t e number of times t e union damages t e pie. Besides, notice t at a strategy w ic prescribed t e union to arm for a *finite* number of times could never be supported in equilibrium, since it would introduce a "deadline effect" w ic would make t e union's arming t reat not credible. In fact, suppose t e union were to follow a strategy equal to t e one w ic supports t e arming m.p.e. described by Proposition 1 except for t e fact t at t e union arms w en confronted wit a rejection only for t e first Z times, and after t at it does not do it any more. Consider now a node off t e equilibrium pat w en t e pie is of size $\pi_{Z-1}$and t e firm as rejected t e union's offer. T en, if t e union followed its strategy and armed, t e union could obtain t e Rubinstenian s are out of $\pi_Z < \pi_{Z-1}$, w ereas if it did not arm t e union could obtain t e same s are out of t e greater pie π_{Z-1}.T en arming for a finite number of times is never credible. More precisely, arming is never credible as long as t e c osen arming structure (t at is, a sequence of destructive actions) is suc t at t e finite sum of t e damages ex austs t e size of t e pie bargained over, t at is if t ere exists a finite N suc t at:

$$1 - \sum_{i=1}^{N} c_i = 0$$

w ere c_i is t e amount of damage c osen by t e union t e i^{th} time it arms. If t e above condition is satisfied, t en t ere will be a "last" round, at a time T^*, w en arming will no longer be credible. T e same is true w enever t e union's strategy prescribes $c_i = c_j = c\ \forall i, j$: if arm is eld fixed at a constant level t roug out t e play, t ere will be a limit to t e extension of t e branc of t e game tree corresponding to t e "always arm" strategy of t e union, since at some point t e pie will vanis completely. So t ere will be a "last" arming round, after w ic t ere will be not ing left to bargain over. But t en t e same arguments as above apply to s ow t at arming is never credible (see Manzini (1996)). However, w en t e structure of damages is suc t at only t eir *infinite* sum converges to t e size of t e pie, t en it is in principle possible to avoid suc problem.

onsequently, from the above system one obtains

$$x_u(\pi_n) = X_u(\pi_n) = \sum_{i=0}^{\infty}(\pi_{n+i} - \delta\pi_{n+1+i})\delta^{2i}$$

and recalling that $\pi_{n+1+i} = \pi_{n+i} - c_{n+i+1}$, the expression for $x_u^*(\pi_n)$ follows; one can then proceed similarly to retrieve the other m.p.e. partitions stated above.

To conclude the proof, it is now needed to show that if $c_n \leq \delta \sum_{i=0}^{\infty} c_{n+i+1}\delta^{2i}\ \forall n$, harming is indeed credible. First of all notice that as long as the harming structure is such that there is no finite number of harms which exhaust the pie, the credibility problem implied by the deadline effect discussed above is not relevant. Next, consider the union when confronted with an unexpected rejection from the firm when the pie is of size π_{n-1}. By harming the union can obtain $y_u^*(\pi_n)$ in the following period, whereas if it does not harm, play reverts to the Rubinstenian equilibrium, yielding the union a payoff $\frac{\delta}{1+\delta}\pi_{n-1}$, delayed one period. Thus, harming is credible as long as $\delta y_u^*(\pi_n) \geq \frac{\delta^2}{1+\delta}\pi_{n-1}$, or

$$\frac{\delta}{1+\delta}\left(\pi_n + \delta\sum_{i=0}^{\infty} c_{n+i+1}\delta^{2i}\right) \geq \frac{\delta}{1+\delta}\pi_{n-1}$$

which can be rearranged as

$$\frac{\delta}{1+\delta}\left(\pi_{n-1} - c_n + \delta\sum_{i=0}^{\infty} c_{n+i+1}\delta^{2i}\right) \geq \frac{\delta}{1+\delta}\pi_{n-1}$$

from which the credibility condition stated in the proposition obtains. ■

Proposition 1 establishes a class of harming equilibria, which exist if the harming structure satisfies two conditions, namely *(a)* it rules out deadline effects, and *(b)* the credibility constraint is satisfied. The first requirement actually does not put any restriction on the behaviour of the sequence $\{c_i\}_{i=1}^{\infty}$, so that the union could alternate harms of any size, as long as they are strictly positive. What is important is that there is no finite N such that the sum of the first N terms of the sequence exhausts the pie. In contrast, the second requirement does indeed restrict somehow the harming structure, since at any round harming is credible only if it is less than the sum, in present discounted value, of all future damages. As it stands, our problem does not allow us to restrict further the family of sequences which satisfy both conditions. In the next section I identify one obvious candidate, which I use to exemplify the basic characteristics of the harming equilibrium.

urthermore, notice that because of the deadline effect explained above, only the two "extreme" payoff pairs can be sustained in equilibrium: in contrast with other results in the literature, I do not derive a range of payoffs which can be sustained in equilibrium, but just two equilibria.

Proposition 1 has some interesting implications[3]:

Corollary 1 *In the limit as players become infinitely patient, the union in equilibrium obtains the whole profits, regardless of the size of the elements of* $\{c_i\}_{i=0}^{\infty}$*, provided that they are positive:*

$$\lim_{\delta \to 1} x_w^* = 1 \tag{6}$$

Notice that in the limit the credibility condition reduces to $c_n \leq \pi_n \ \forall n$, so that one needs not qualify the above statement any further. Moreover, I can state the following

Corollary 2 *The m.p.e. shares do not vary monotonically with the discount factor*

Proof. The result is proved for x_u^*; then, one can proceed similarly to prove the result for the other shares. rom Proposition 1, $x_u^* = \frac{1}{1+\delta}\left(1 + \sum_{i=0}^{\infty} c_{i+1}\delta^{2i}\right)$. Differentiating with respect to the discount factor:

$$\frac{\partial x_u^*}{\partial \delta} = \frac{1}{1+\delta}\left[2\sum_{i=0}^{\infty} i c_{i+1}\delta^{2i} + \frac{1}{1+\delta}\left(\sum_{i=0}^{\infty} c_{i+1}\delta^{2i} - 1\right)\right]$$

It is straightforward to verify that

$$\lim_{\delta \to 0} \frac{\partial x_u^*}{\partial \delta} < 0$$

and

$$\lim_{\delta \to 1} \frac{\partial x_u^*}{\partial \delta} > 0$$

The first limit comes from the fact that as $\delta \to 1$ the term first term in square brackets is positive, whereas the second term approaches zero (since by hypothesis $\lim_{z \to \infty} \sum_{i=0}^{z} c_{i+1} = 1$); conversely, as $\delta \to 0$, the first term vanishes, whereas the second term is surely

[3]In Manzini (1996) I derived analogous results for a model in which the union is committed to destroy a fixed amount c of the surplus bargained over.

egative. Si ce $\frac{\partial x_u^*}{\partial \delta}$ is co ti uous i δ, there must exist at least o e δ^* such that $x_u^*(\delta)$ is i creasi g i δ if $\delta > \delta^*$, a d decreasi g i δ if $\delta < \delta^*$. Thus, the equilibrium share does ot vary mo oto ically with the discou t factor. ■

A variatio of the discou t factor bri gs about two co trasti g effects o the cost of rejectio : as δ i creases, the "Rubi ste ia " cost of rejecti g a offer decreases, with egative comparative statics effects o the payoff of the proposi g player; o the other ha d, a i crease i the discou t factor raises the prese t discou ted value of the sum of the seque ces of harms, which reduces the equilibrium payoff to the firm. Furthermore, there is o such "harmi g effect" for the u io , who is the o ly player empowered with destructive ability i this model. From the above discussio o e ca co clude that the "Rubi ste ia effect" prevails for lower values of the discou t factor, whereas the "harmi g effect" is outweighi g for higher values of δ; however, it is ot possible to draw a y i fere ce as to the u ique ess of a threshold value of the discou t factor to determi e which of the two effects prevails.

So far I showed how a player who is free to choose the degree of harm to i flict o the object bargai ed over ca exploit this ability to his adva tage, without the eed to commit to destructive activities before-ha d. This result holds provided the harmi g structure co forms to two requireme ts (credibility a d deadli e effect), but the above discussio is still rather abstract: the example of the ext sectio aims at clarifyi g the results prese ted above.

3 Proportional harm

I ow tur to a specific harmi g structure. Let the i fi ite seque ce of harms pla ed by the u io i case of disagreeme t be $\{\alpha^{i-1}(1-\alpha)\}_{i=1}^{\infty}$, where $\alpha \in (0,1]$. This implies that the u io damages each time the pie i such a way that it decreases at a co sta t rate, α, harm after harm. I fact, recall that $\pi_n = 1 - \sum_{i=1}^{n} c_i$; thus, i our example:

$$\pi_n = 1 - \sum_{i=1}^{n} \alpha^i(1-\alpha) = 1 - (1-\alpha)\frac{1-\alpha^n}{1-\alpha} = \alpha^n$$

o that $\pi_0 = 1$. Clearly, the greater i α, the maller the effect of harm on the pie[4]. Since in thi ca e $\sum_{i=0}^{\infty} c_{n+i+1}\delta^{2i} = \sum_{i=0}^{\infty} \alpha^{n+i}(1-\alpha)\delta^{2i} = \frac{\alpha^n(1-\alpha)}{1-\alpha\delta^2}$, propo ition 1 predict that in the harming equilibrium an agreement i reached immediately on the partition[5]

$$x^* = \left(x_u^*, x_f^*\right) = \left(\frac{1-\alpha\delta}{1-\alpha\delta^2}, \frac{\alpha\delta(1-\delta)}{1-\alpha\delta^2}\right)$$

Notice that, not urpri ingly, a α approache unity, the equilibrium partition converge to the Rubin tenian equilibrium, ince harm become le and le effective. The m.p.e. trategie which upport thi equilibrium are now defined with re pect to

$$x^*(\alpha^n) = \left(x_u^*(\alpha^n); x_f^*(\alpha^n)\right) = \left(\frac{\alpha^n(1-\alpha\delta)}{1-\alpha\delta^2}, \frac{\alpha^{n+1}\delta(1-\delta)}{1-\alpha\delta^2}\right)$$

and

$$y^*(\alpha^n) = \left(y_u^*(\alpha^n); y_f^*(\alpha^n)\right) = \left(\frac{\alpha^n\delta(1-\alpha\delta)}{1-\alpha\delta^2}, \frac{\alpha^n(1-\delta)}{1-\alpha\delta^2}\right)$$

Such trategie amount to haring the urplu available, α^n, alway according to the ame proportion in each of the two type of ubgame (tho e tarting with an offer by the firm and tho e tarting with an offer by the union). The condition for harming to be credible i

$$\alpha^{n-1}(1-\alpha) \leq \delta \sum_{i=0}^{\infty} \alpha^{n+i}(1-\alpha)\delta^{2i} \Rightarrow \alpha \geq \frac{1}{\delta(1+\delta)}$$

which hold for ufficiently large value of the di count factor[6].

Thi harming tructure how very clearly how the union can exploit it harming ability to increa e the firm' co t of rejection. The effect of impo ing the ort of "proportional" harm de cribed above i actually equivalent to making the firm more impatient, ince in

[4]Notice that the succession of harms need not be decreasing: one can readily check that whether or not c_i increases with α depends on whether α is smaller or greater than $\frac{i}{i+1}$.

[5]Although developed independently, this result corresponds in essence to that stated in Bush et al. (1996), Proposition 2. This is not surprising, since in that paper destructive power is modelled as an increase in the pie's shrinking rate; although in my model destructive power consists in "chopping off" part of the cake, the example is designed as to decrease proportionally the size of the pie harm after harm.

[6]Since by definition $\alpha \in (0,1]$, in order for the above to be satisfied it has to be true that $\frac{1}{\delta(1+\delta)} \leq 1$, which is satisfied for $\delta \geq \frac{-1+\sqrt{5}}{2}$.

ff ct it low rs th firm's rat of tim pr f r nc from δ to $\delta\alpha$. In fact, notic that if w d fin $\delta_1 = \delta$ and $\delta_2 = \alpha\delta$, th partition which obtains in quilibrium can b xpr ss d as

$$x^{*R} = \left(x_u^{*R}, x_f^{*R}\right) = \left(\frac{1-\delta_1}{1-\delta_1\delta_2}, \frac{\delta_2(1-\delta_1)}{1-\delta_1\delta_2}\right)$$

which is xactly th uniqu quilibrium partition of a "standard" alt rnating off rs bargaining gam in which play rs hav diff r nt discount factors! Mor ov r, sinc $\frac{\partial x_u^{*R}}{\partial \delta_2} = -\frac{1-\delta_1}{(1-\delta_1\delta_2)^2} < 0$, for any giv n valu of δ th quilibrium shar to th union incr as s as δ_2 (and thus α) d cr as s: th gr at r th (cr dibl) rat at which th union can r duc th pi , th gr at r his payoff. Thus, in ord r to maximiz his quilibrium payoff, th union has to choos th small st α which satisfi s th cr dibility constraint, so that it will s t $\alpha = \frac{1}{\delta(1+\delta)}$, yi lding th quilibrium payoff pair will b $x^*(\frac{1}{\delta(1+\delta)}) = (\delta, 1-\delta)$[7].

R gardl ss of th siz of α, it is straightforward to ch ck that, as pr dict d by Corollary 1, in th limit as th discount factor approach s unity, th union obtains th whol of th profits. Mor ov r, Corollary 2 is also confirm d, sinc on can asily ch ck that th quilibrium shar s do not vary monotonically with th discount factor, as shown in Figur 1[8]. In this cas shar s ar r pr s nt d on th $A \times \Delta$ domain, *not* th $C \times \Delta$ domain, wh r $A = \{\alpha | \alpha \in (0,1]\}$ and $\Delta\{\delta | \delta \in (0,1)\}$[9]. In fact, consid r th union's quilibrium shar . Th n:

$$\frac{\partial(x_u^*)}{\partial\delta} = -\frac{\alpha(\alpha\delta^2 - 2\delta + 1)}{(1-\alpha\delta^2)^2}$$

Th xpr ssion in brack ts at th num rator has two distinct r al roots, $\delta_{1,2} = \frac{1\pm\sqrt{1-\alpha}}{\alpha}$, of which only on is admissibl . Cons qu ntly, $\frac{\partial(x_u^*)}{\partial\delta} < 0$ if $\delta < \frac{1-\sqrt{1-\alpha}}{\alpha}$, and $\frac{\partial(x_u^*)}{\partial\delta} > 0$ if $\delta > \frac{1-\sqrt{1-\alpha}}{\alpha}$.

Notic that th function $\frac{\partial(x_u^*)}{\partial\delta}$ is continuous ov r th domain, sinc it admits a discontinuity for $\delta = \frac{1}{\sqrt{\alpha}} > 1$.

Th n, th abov r pr s nts an asy "harming rul " which nabl s th harming play r to maximiz his payoff.

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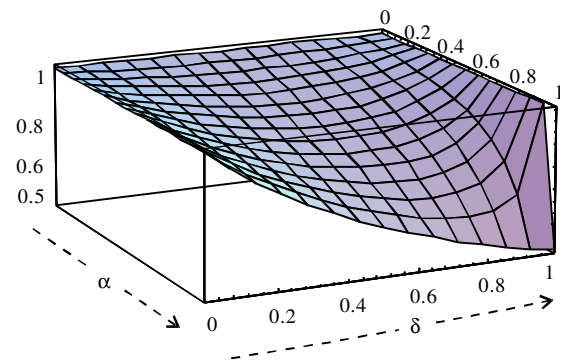
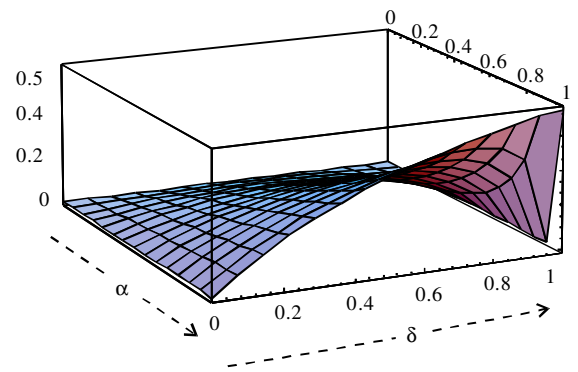
[7] Bush et al. (1996) obtain a similar result in their Corollary 1.

[8] Notice however that in the degenerate case when $\alpha \to 0$, the non-monotonicity result does not hold any longer. This case is anyway of no practical interest, since for such low values of α harming is not credible.

[9] Thus $\alpha = 1$ corresponds to $c = 0$.



worker's share



firm's share

Figure 1: Equilibrium share's non monotonicity with proportional harm.

4 Non-stationary equilibria

If we now turn to non-stationary strategies, we can retrieve for this model results analogous to those that are now standard in the literature on "money burning" bargaining models, which predict the existence of a range of subgame perfect equilibrium payoffs. The type of strategies which can sustain multiple equilibria depend in fact on the existence of just the two "extreme" equilibrium payoffs; they can be used to define two different regimes of play - supported by the strategies defined in Proposition 1 -, one in which the harming player never harms, and in which the bargainers obtain the bilateral monopoly payoff (so that the union receives its "worst" wage); and another one in which the harming player always harms when faced with a rejection, which sustains the "harming" equilibrium payoff. Then, consider the following strategies: both the union and the firm conform to offering a wage within the "worst-best" range, and punish the opponent who proposes a different wage by reverting to playing according to the regime that yields the extreme equilibrium which is less favourable to the deviator. Then, one can verify that such strategies support the following[10]:

Proposition 2 *In a bargaining game of alternating offers in which the worker can harm after every rejection of the firm, if* $c_n \leq \delta \sum_{i=0}^{\infty} c_{n+i+1}\delta^{2i}\ \forall n$, *then any partition which yields the union a wage* $w^* \in [\widehat{x}_u^*, x_u^*]$ *can be sustained as a subgame perfect equilibrium of the bargaining game.*

Further, as in other models belonging to the money burning literature, in this model the existence of the two extreme equilibria is enough to guarantee also inefficient equilibria, as established in the following:

Proposition 3 *For any finite integer* N, *if* $c_n \leq \delta \sum_{i=0}^{\infty} c_{n+i+1}\delta^{2i}\ \forall n$, *then there exists a discount factor* $\widehat{\delta}$ *such that for* $\delta \in \left[\widehat{\delta}, 1\right)$ *every partition which yields the union a wage* $\widetilde{w} \in \left[\frac{\widehat{x}_u^*}{\delta^N}, 1 - \frac{1-\delta x_u^*}{\delta^{N-1}}\right]$ *can be sustained as a subgame perfect equilibrium of the bargaining game in which agreement is reached after* N *periods.*

[10] See Avery and Zemsky (1994) or Bush and Wen (1995).

roof. Strategies that can support this equilibrium are for instance: for the firm, for the first N periods, the union (firm) asks for the whole profits and rejects any offer which yields less than the whole pie, and the union never harms after a rejection; after N periods, the union (firm) offers a wage $\widetilde{w}$, rejects all wages below (above) $\widetilde{w}$, accepts all wages above (below) $\widetilde{w}$, and the union harms after its own offer has been rejected. If one of the players deviates, then in the following round play reverts to the strategies which support the worst equilibrium payoff for the deviator.

The proof follows standard arguments (e.g., see Holden (1989)). To simplify the notation, let us re-define the partitions $x^*(\pi_0) = \left(x_u^*(1); x_f^*(1)\right)$ and $y^*(\pi_0) = \left(y_u^*(1); y_f^*(1)\right)$as $(w_u, 1-w_u)$ and $(w_f, 1-w_f)$, respectively, where $w_f = \delta w_u \Rightarrow 1 - w_f = 1 - \delta w_u$. Now, consider a deviation by the firm. If it were to deviate in the first period, then play would revert to the harming equilibrium, yielding the firm a payoff of $1-w_f$. Thus, it will not be profitable for the firm to deviate as long as $\delta^N(1-\widetilde{w}) \geq \delta(1-w_f)$, where the term on the l.h.s. is the firm's payoff from sticking to its equilibrium strategy. The above can be rearranged as $\widetilde{w} \leq 1-\delta^{1-N}(1-w_f) = 1-\delta^{1-N}(1-\delta w_u)$, which establishes the upper bound of the range in which $\widetilde{w}$ is allowed to vary. Further, notice that the condition required for deviations in periods other than the first not to be profitable are less stringent. For instance, by deviating in the second period the firm would obtain $1-w_u$ in the following round, so that such deviations is not profitable if $\delta^N(1-\widetilde{w}) \geq \delta^2(1-w_u)$, or $\widetilde{w} \leq 1-\delta^{2-N}(1-w_u)$. But $1-\delta^{2-N}(1-w_u) \geq 1-\delta^{1-N}(1-\delta w_u)$ if $\delta(1-w_u) \leq (1-\delta w_u)$, or $\delta \leq 1$. Then, one can proceed analogously to verify that the wage level which makes deviations not profitable is greater the further down we move along the game tree.Consider now the union. A deviation in the first round of negotiations would trigger play to revert to the Rubinstenian equilibrium in the following period, yielding the union a payoff of $\frac{\delta^2}{(1+\delta)}$, so that a deviation will not be profitable as long as $\frac{\delta^2}{(1+\delta)} \leq \delta^N\widetilde{w}$, or $\widetilde{w} \geq \frac{1}{\delta^N(1+\delta)} = \frac{\widehat{x}_u^*}{\delta^N}$, which is also the condition for a deviation in the second period not to be profitable. Consider now a deviation in round 3: this will not be profitable as long as $\delta^N\widetilde{w} \geq \frac{\delta^4}{(1+\delta)} \Rightarrow \widetilde{w} \geq \frac{\delta^4}{\delta^N(1+\delta)}$, where $\frac{\delta^4}{\delta^N(1+\delta)} \leq \frac{1}{\delta^N(1+\delta)}$; then one can proceed in a similar fashion to verify that, as for the firm, the condition required for deviations in periods other than the first not to be profitable are less stringent the later the deviation.

ext, I show that $\left[\frac{\widehat{x}_u^*}{\delta^N}, 1-\frac{(1-\delta x_u^*)}{\delta^{N-1}}\right]$ is non empty. Indeed, $\frac{\widehat{x}_u^*}{\delta^N} = \frac{1}{\delta^N(1+\delta)} \leq 1-\delta^{1-N}(1-\delta x_u^*)$ can be rearranged as

$$1-\frac{1}{\delta^N(1+\delta)}-\frac{1-\delta w_u}{\delta^{N-1}} = 1-\frac{1}{\delta^N(1+\delta)}-\frac{w_f}{\delta^{N-1}} \geq 0$$

$$1-\frac{1}{\delta^N(1+\delta)}-\frac{\frac{1}{1+\delta}\left(1-\delta^2\sum_{i=0}^{\infty}c_{i+1}\delta^{2i}\right)}{\delta^{N-1}} \geq 0$$

$$\frac{\delta^N(1+\delta)-1-\delta}{\delta^N(1+\delta)}+\frac{\delta^3\sum_{i=0}^{\infty}c_{i+1}\delta^{2i}}{\delta^N(1+\delta)} \geq 0$$

$$(1+\delta)\left(1-\delta^N\right)-\delta^3\sum_{i=0}^{\infty}c_{i+1}\delta^{2i} \leq 0 \Rightarrow$$

$$(1+\delta)\left(1-\delta^N\right) \leq \delta^3\sum_{i=0}^{\infty}c_{i+1}\delta^{2i}$$

ow, note that $0 < \delta^3\sum_{i=0}^{\infty}c_{i+1}\delta^{2i} \leq 1$, and the higher the value of the discount factor, the closer the r.h.s. of the above inequality gets to unity (recall that $\sum_{i=0}^{\infty}{c_{i+1}}^{2i} = 1$). Then, for any given finite N, as the discount factor approaches unity:

$$\lim_{\delta\to 1}(1+\delta)\left(1-\delta^N\right)-\delta^3\sum_{i=0}^{\infty}c_{i+1}\delta^{2i} = -1$$

so that the above inequality is always verified. ■

Figure 2 reports simulated values for $(1+\delta)\left(1-\delta^N\right)$: for any value of N one can correspondingly find a value of the discount factor high enough as to set $(1+\delta)\left(1-\delta^N\right)$ as close as chosen to unity.

5 Concluding remarks

In this paper I have shown how the ability to destroy the surplus bargained over can be successfully exploited by the harming player to increase his equilibrium payoff, since destructive actions in effect increase one opponent's cost of rejecting an offer, which grows with the discount factor. In order for this result to obtain, there is no need to resort neither to "forcing" a player to a specific action in case of disagreement, nor to a repeated game structure, as long as the harming player is free to choose a sequence of harms. I argued that the latter is indeed an alternative form of commitment. Recent literature has

igure 2: Simulated values for $(1+\delta)\left(1-\delta^N\right)$

modelled commitment either as a time frame for offers and counteroffers, or as a cost on one's payoff for conceding on different agreement from the one previously put forward. In my approach, the harming ability of one player, which modifies the pie bargained over, is the mechanism which increases one opponent's cost of rejecting an offer. As shown in the example, the net effect amounts to increasing the opponent's rate of time preference (by reducing his discount factor). The same effect would be produced by increasing the time span between subsequent offers by the harming player. However such "Rubinstenian" commitment works in favour of the committed player only if he happens to be the first mover in the bargaining game: for the responding player, the "slower" the bargaining (or the smaller the discount factor), the greater the degree of commitment, the smaller the payoff. My approach to modelling commitment brings about an obvious though different result: regardless of whether the harming player starts negotiations as a proposer or as a responder, the greater the commitment achieved through destructive power, the greater the payoff to the committed player.

The model introduced in this chapter possesses at least two appealing features. On the one hand it justifies the intuition that bargaining power increases with the ability to inflict a cost on one's opponent even in a simple "one pie" bargaining game. On the other hand, compared to repeated bargaining games - which generally predict a continuum of equilibria even in stationary strategies (see Muthoo (1995)) - it has the advantage of

singling out just two m.p.e.. These are supported by an "all or nothing" strategy from the harming player which seems to us simpler and consequently more compelling for applications, since it requires the harming player to carry out his threat either never or every time.

Still further work is needed. The nature of some kinds of negotiations (most notably wage bargaining) calls for the modelling of repeated interactions; the analysis of a repeated play of my model seems therefore an extension worth pursuing.

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