Strategic Bargaining with Destructive Power

by Paola Manzini,[*]

Department of Economics, University of Exeter, Exeter EX4 4RJ United Kingdom

[*]This paper is based on a chapter of my Ph.D. dissertation. I would like to thank Abhinay Muthoo and my supervisor, Dennis Snower, for encouragement and helpful comments and suggestions on a previous version of this paper. I owe special thanks to Marco Mariotti for extremely constructive insights. Of course the responsibility of any errors is only mine. Financial support from a Royal Economic Society Junior Fellowship is gratefully acknowledged.

Short Running title: Strategic Bargaining with Destructive Power

Address for Proofs:
Paola Manzini
Department of Economics
University of Exeter
Amory Building
Rennes Drive
Exeter EX4 4RJ
United Kingdom
Phone:+44-1392-263212
Fax: +44-1392-263242
e-mail: p.manzini@exeter.ac.uk

Abstract: This paper studies a two-player alternating offers bargaining model in which one of the agents has the ability to damage permanently the "pie" bargained over. I show how this feature can result in an increase of the cost of rejecting an offer for the "non-harming player". Beside the "Rubinstenian" bilateral monopoly outcome, I show that it is possible to select a "harming" equilibrium in which the sequence of damages to the pie is endogenously determined and payoffs do not vary monotonically with the discount factor.

JEL Classification: C78.
Keywords: bargaining, commitment.

1 Introduction

This paper studies a simple two-player alternating offers bargaining model in which one of the players is allowed to destroy part of the surplus bargained over. This is a feature which can arise in a number of circumstances, and that has been addressed to some extent by the existing literature. Wage negotiations between a firm and a union can trigger strikes, which disrupt the productive process, thereby imposing per period costs on the firm (see [Fernandez and Glazer(1991)] and [Haller and Holden(1991)]). Alternatively, one player could increase his opponent's (opportunity) cost of rejecting an offer either by choosing who is going to be the next proposer (as in [Kambe(1994)]), or by appropriately timing offers (as in [Perry and Reny(1993)] and [Sakovics(1993)]). However, these models focus on each player's ability to alter his opponent cost of rejecting an offer, by performing actions which can be detrimental only in terms of one's opponent's payoff; thus, such models consider a "money burning" activity for one or both players[1], thereby focusing on their ability to inflict a *side cost* on their opponent, without actually damaging the object bargained over.

In contrast, there are instances in which - beside time preference considerations - disagreement can result in the reduction of the size of the surplus to be shared. For instance, a worker whose bid for a higher wage has been frustrated can be less effective in his job, and thus reduce the firm's profitability.

Such different approach is adopted in [Dasgupta and Maskin(1989)], where both bargainers have "destructive power", in the sense that they can actually destroy part of the surplus to be shared. More recently, [Bush et al.(1996)] have modelled the bargaining with such endogenous reduction of the pie in a strategic setting: here in case of disagreement one of the players can perform a damaging action which increases the rate at which the pie shrinks. In this paper I model destructive power in an alternative manner, allowing the harming player to destroy a part of the cake. In so doing, I build on [Manzini(1996)]. There I studied how one player's commitment to a specific destructive action affects the equilibrium outcome, and showed that (not surprisingly) destructive power increases the equilibrium payoff of the "harming" player. Besides this quite intuitive result, there is the less obvious one that, contrary to the standard

[1] For a more general treatment of this class of models, see [Bush and Wen(1995)], [Avery and Zemsky(1994)] and references therein.

[Rubinstein(1982)] model, the equilibrium shares do *not* vary monotonically with the discount factor, which therefore has non-trivial effects on equilibrium payoffs. As I shall discuss more in detail later on, variations of the discount factor have two conflicting effects on the equilibrium partition, and in the limit as players become infinitely patient, the "harming" player manages to get the whole pie, *regardless of how small his destructive power*.

However, in many real life negotiations exogenous commitment to destructive activities can be either unlikely or unfeasible.In this paper I relax the commitment assumption, and model a bargaining game with destructive power in which the "harming" player is allowed to choose both whether or not to exercise his destructive power, and of how much. As it turns out, this corresponds to a more flexible way of modelling commitment. Notice that the timing of offers as modelled in [Rubinstein(1982)] can be interpreted as player's commitment to an offer: since no player can change his own offer before a certain amount of time has elapsed, he is *de facto* bound to his proposal over that spell of time. [Perry and Reny(1993)] and [Sakovics(1993)] have modified the basic setting by considering a "destructured" bargaining, in which players can choose the timing of offers and reactions to those offers. This affects players' cost of rejecting an offer, so that different timing structures correspond to a different degree of commitment to a specific proposal. As an alternative to this approach, I propose to model a player's commitment as his ability to choose the level of destructive power he exherts. I show that in equilibrium he can obtain a share whose size depends positively on his destructive power; furthermore, equilibrium shares are non monotonic in the discount factor, and as the discount factor approaches unity the harming player obtains the whole profits. Thus, as long as the player with destructive power is free to choose a suitable "harming structure" (that is, a feasible sequence of credible destructive actions), it is possible to retrieve a result of the same nature of the model with exogenous commitment, this time relying on a more realistic structure.

The paper is organised as follows: in the next section the model is introduced and solved in Markov strategies. Section 3 contains a specific example of a harming structure. Section 4 solves for the equilibria in non-stationary strategies, and section 5 concludes.

2 Bargaining with destructive power

Consider the following simple extension of the two-player alternating offers bargaining game. For the sake of argument, let our two players be a union (indexed with u) and a firm (f), engaged in negotiations over the wage, determined as a share of the firm's profits, normalised to unity. At time $t = 0$ (and at all even stages of the game) player 1 (the union) proposes a partition of the available surplus. The firm can either accept, ending the game, or reject the offer. If the latter, before the firm can propose a split, the union can *choose* whether or not to harm the firm, i.e. the union can reduce the available profits of an amount $c_i \in [0,1]$ (which we will refer to as "harm"). The game then moves to the following round, in which the firm makes a counterproposal; the union can either accept it, ending the game, or reject the offer. In case of disagreement, the game enters the following round of negotiations, in which it is the union's turn to make a proposal, and so on[2]. The union's action space is $X \times C \equiv [0,1] \times [0,1]$; clearly, $c_i = 0$ implies that the union does not harm. Because of the non-stationary structure of this framework, for the moment I focus on Markov strategies, that is strategies which are history independent, and that depend only on the *state* of the play: hence past actions influence current play only through their effect on a state variable which summarises all the (payoff relevant) information. For our purposes, a state is defined simply by π_n, the size of the "pie" after it has been damaged n times. Let c_n be the amount of damage which the union selects the n^{th} time it harms; then, $\pi_n = 1 - \sum_{i=0}^{n-1} c_{i+1}$, (with $\pi_0 = 1$) which implies that $\pi_{n+1} = \pi_n - c_{n+1}$. In other words, a state is characterized by the actual size of the pie, which reveals how many times in the past the union has "hit".

Recalling that a Markov perfect equilibrium (m.p.e.) is a profile of Markov strategies which yields a Nash equilibrium in every proper subgame (see [Bernheim and Ray(1989)], [Maskin and Tirole(1988)]), I can then claim the following:

Proposition 1 *In a bargaining game of alternating offers in which the worker can harm after every rejection of the firm, if $c_n \leq \delta \sum_{i=0}^{\infty} c_{n+i+1} \delta^{2i}$ $\forall n$, there exist two m.p.e., which I will call*

[2] In this model the player with destructive ability (the union) is allowed to destroy part of the cake only after his own offer has been rejected, not after *any* rejection. This is assumed just for ease of exposition, since it is possible to prove that harming after rejecting an opponent's proposal - and before making one' own - would never be credible. See for instance [Avery and Zemsky(1994)], [Bush et al.(1996)], [Fernandez and Glazer(1991)], [Haller and Holden(1991)].

"harming equilibrium" and "Rubinstenian equilibrium", respectively, in which agreement is reached immediately either on the "harming" equilibrium partition:

$$x^* = \left(x_w^*, x_f^*\right) = \left(\frac{1}{1+\delta}\left(1+\delta\sum_{i=0}^{\infty} c_{i+1}\delta^{2i}\right), \frac{\delta}{1+\delta}\left(1-\sum_{i=0}^{\infty} c_{i+1}\delta^{2i}\right)\right)$$

or on the "Rubinstenian" equilibrium partition:

$$\hat{x}^* = (\hat{x}_w^*, \hat{x}_f^*) = \left(\frac{1}{1+\delta}; \frac{\delta}{1+\delta}\right)$$

where in each partition the first entry is the share received by the union, and the second entry is the share received by the firm.

The above states that in a bargaining game of alternating offers in which one of the players can choose the level of damage to the pie, two Markov perfect equilibria exists: in one, no harm is ever committed, so that the Rubinstenian equilibrium results; in the other one, however, the harming player achieves his maximum payoff, which is greater than the Rubinstenian and is positively related to his harming ability.

Proof. The strategies that support the Rubinstenian equilibrium prescribe to the union always to propose a partition $\hat{x}^*$, accept any proposal $\hat{y}^*$ and any $x \geq \hat{y}_u^*$, reject any proposal $x < \hat{y}_u^*$, and never harm the firm when rejecting; to the firm, always to propose a partition $\hat{y}^*$, accept any proposal $\hat{x}^*$ and any $x \geq \hat{x}_f^*$, and reject any proposal $x < \hat{x}_f^*$, where $\hat{y}^*$ is defined as:

$$\hat{y}^* = \left(\hat{y}_u^*; \hat{y}_f^*\right) = \left(\frac{\delta}{1+\delta}; \frac{1}{1+\delta}\right)$$

To specify the strategies that support the harming equilibrium, let $x^*(\pi_n) = (x_u^*(\pi_n), x_f^*(\pi_n))$ be the m.p.e. partition in subgames starting with an offer by the union, defined as:

$$\begin{aligned} x^*(\pi_n) &= \left(x_u^*(\pi_n); x_f^*(\pi_n)\right) = \\ &= \left(\frac{1}{1+\delta}\left(\pi_n + \delta\sum_{i=0}^{\infty} c_{n+i+1}\delta^{2i}\right), \frac{\delta}{1+\delta}\left(\pi_n - \sum_{i=0}^{\infty} c_{n+i+1}\delta^{2i}\right)\right) \end{aligned}$$

Then the m.p.e. strategies for this equilibrium are for the union always to propose a partition $x^*(\pi_n)$, accept any proposal $y^*(\pi_n)$ and any $x \geq y_u^*(\pi_n)$, reject any proposal $x < y_u^*(\pi_n)$, and to harm the firm after a rejection if $c_n \leq \delta\sum_{i=0}^{\infty} c_{n+i+1}\delta^{2i}$; the equilibrium strategy for the firm is always to propose a partition $y^*(\pi_n)$, accept any proposal $x^*(\pi_n)$ and any $x \geq x_f^*(\pi_n)$, and reject any proposal $x < x_f^*(\pi_n)$, where $y^*(\pi_n)$ is defined as:

$$
\begin{aligned}
y^*(\pi_n) &= \left(y_u^*(\pi_n); y_f^*(\pi_n)\right) = \\
&= \left(\frac{\delta}{1+\delta}\left(\pi_n + \delta\sum_{i=0}^{\infty} c_{n+i+1}\delta^{2i}\right), \frac{1}{1+\delta}\left(\pi_n - \delta^2\sum_{i=0}^{\infty} c_{n+i+1}\delta^{2i}\right)\right)
\end{aligned}
$$

Then the proof is easily obtained using standard techniques. Let $x_u(\pi_n)$ and $X_u(\pi_n)$ ($y_f(\pi_n)$ and $Y_f(\pi_n)$) denote the infimum and supremum equilibrium payoff to the union (firm) in subgames in which it is the first to make an offer when bargaining over a pie of size π_n. Then, the unique m.p.e. partition can be retrieved by solving recursively the following system of equations:

$$
x_u(\pi_n) \geq \begin{cases} \pi_n - \delta Y_f(\pi_{n+1}) & if \quad c_{n+1} \neq 0 \\ \pi_n - \delta Y_f(\pi_n) & if \quad c_{n+1} = 0 \end{cases} \tag{1}
$$

$$
X_u(\pi_n) \leq \begin{cases} \pi_n - \delta y_f(\pi_{n+1}) & if \quad c_{n+1} \neq 0 \\ \pi_n - \delta y_f(\pi_n) & if \quad c_{n+1} = 0 \end{cases} \tag{2}
$$

$$
y_f(\pi_n) \geq \pi_n - \delta X_u(\pi_n) \tag{3}
$$

$$
Y_f(\pi_n) \leq \pi_n - \delta x_u(\pi_n) \tag{4}
$$

with

$$
\begin{aligned}
&x_u(\pi_n) + x_f(\pi_n) \leq \pi_n \\
&y_u(\pi_n) + y_f(\pi_n) \leq \pi_n \\
&X_u(\pi_n) + X_f(\pi_n) \leq \pi_n \\
&Y_u(\pi_n) + Y_f(\pi_n) \leq \pi_n
\end{aligned} \tag{5}
$$

Equations 1 and 2 state that whenever the pie is of size π_n, the equilibrium share to the union in subgames in which it is the first mover depends on whether or not the union harms the firm if confronted with a rejection. If it does, the union can claim at least (at most) the actual pie net of the present discounted value of the supremum (infimum) equilibrium share to the firm in the following subgame, in which the two parties would be bargaining on a pie of size now reduced to $\pi_{n+1} = \pi_n - c_{n+1}$; alternatively, if the union decides not to harm, it will have to concede to the firm at most (at least) the supremum (infimum) equilibrium share it could get in the following period, in present discounted value terms, out of a pie of same size. Analogous interpretation is to be given to equations 3 and 4, where however this time the pie does not change from one period to the next. It is worth noticing that in subgames starting with an offer by the firm there can never be a change in state. Finally, equation 5 describes the usual constraints on the sum of the shares to each player not to exceed the size of the pie bargained over.

If the union decides not to harm, then it is easy to check that the above system of equations yields $X_u(\pi_n) = x_u(\pi_n) = \frac{1}{1+\delta}\pi_n$ and $Y_u(\pi_n) = y_u(\pi_n) = \frac{\delta}{1+\delta}\pi_n$. Thus, the equilibrium payoff pair coincides with the Rubinstenian solution out of a "pie" of size π_n; hence never harming carries immediate agreement on the partition $\hat{x}^*$ (since $\pi_0 = 1$). Notice that this also implies that in general, as soon as the union decides to stop harming, the game reverts to a "standard" alternating offers bargain (from that period onwards), which will yield the Rubinstenian solution out of a pie of reduced size. In fact, whenever the union chooses *nh* a subsequent change of the state is no longer possible, since equations 1 and 2 above reduce to:

$$x_u(\pi_n) \geq \pi_n - \delta Y_f(\pi_n) \;\text{ and }\; X_u(\pi_n) \leq \pi_n - \delta y_f(\pi_n)$$

which, coupled with 3 and 4, determine the equilibrium payoffs uniquely for a cake of size π_n.

Consider now the case when the union harms if the firm rejects his proposal. Again, equations 1 and 2 above can be reduced, so that the harming payoffs can be retrieved inductively from:

$$x_u(\pi_n) \geq \pi_n - \delta\pi_{n+1} + \delta^2 x_u(\pi_{n+1})$$

and

$$X_u(\pi_n) \leq \pi_n - \delta\pi_{n+1} + \delta^2 X_u(\pi_{n+1})$$

Thus, a strategy which prescribed to the union to harm z times would yield:

$$x_u(\pi_n) \geq \sum_{i=0}^{z-1}(\pi_{n+i} - \delta\pi_{n+1+i})\delta^{2i} - \frac{\delta^{2z-1}}{1+\delta}\pi_{n+z} \geq X_u(\pi_n)$$

Since $\pi_{n+i} > \delta\pi_{n+1+i} \forall i$ (being $\delta < 1$ and $\pi_{n+i} \geq \pi_{n+1+i} \forall i$), the above increases with z. Thus, the union can attain its highest payoff by following a strategy which prescribes to maximize the number of times the union damages the pie. Besides, notice that a strategy which prescribed the union to harm for a *finite* number of times could never be supported in equilibrium, since it would introduce a "deadline effect" which would make the union's harming threat not credible. In fact, suppose the union were to follow a strategy equal to the one which supports the harming m.p.e. described by Proposition 1 except for the fact that the union harms when confronted with a rejection only for the first Z times, and after that it does not do it any more. Consider now a node off the equilibrium path when the pie is of size $\pi_{Z-1}$and the firm has rejected the union's offer. Then, if the union followed its strategy and harmed, the union could obtain the Rubinstenian share out of $\pi_Z < \pi_{Z-1}$, whereas if it did not harm the union could obtain the same share out of the greater pie π_{Z-1}.Then harming for a finite number of times is never credible. More precisely, harming is never credible as long as the chosen harming structure (that is, a sequence of destructive actions) is such that the finite sum of the damages exhausts the size of the pie bargained over, that is if there exists a finite N such that:

$$1-\sum_{i=1}^{N} c_i = 0$$

where c_i is the amount of damage chosen by the union the i^{th} time it harms. If the above condition is satisfied, then there will be a "last" round, at a time T^*, when harming will no longer be credible. The same is true whenever the union's strategy prescribes $c_i = c_j = c \; \forall i, j$: if harm is held fixed at a constant level throughout the play, there will be a limit to the extension of the branch of the game tree corresponding to the "always harm" strategy of the union, since at some point the pie will vanish completely. So there will be a "last" harming round, after which there will be nothing left to bargain over. But then the same arguments as above apply to show that harming is never credible (see [Manzini(1996)]). However, when the structure of damages is such that only their *infinite* sum converges to the size of the pie, then it is in principle possible to avoid such problem.

Consequently, from the above system one obtains

$$x_u(\pi_n) = X_u(\pi_n) = \sum_{i=0}^{\infty} (\pi_{n+i} - \delta\pi_{n+1+i})\delta^{2i}$$

and recalling that $\pi_{n+1+i} = \pi_{n+i} - c_{n+i+1}$, the expression for $x_u^*(\pi_n)$ follows; one can then proceed similarly to retrieve the other m.p.e. partitions stated above.

To conclude the proof, it is now needed to show that if $c_n \leq \delta \sum_{i=0}^{\infty} c_{n+i+1} \delta^{2i}$ $\forall n$, harming is indeed credible. First of all notice that as long as the harming structure is such that there is no finite number of harms which exhaust the pie, the credibility problem implied by the deadline effect discussed above is not relevant. Next, consider the union when confronted with an unexpected rejection from the firm when the pie is of size π_{n-1}. By harming the union can obtain $y_u^*(\pi_n)$ in the following period, whereas if it does not harm, play reverts to the Rubinstenian equilibrium, yielding the union a payoff $\frac{\delta}{1+\delta}\pi_{n-1}$, delayed one period. Thus, harming is credible as long as $\delta y_u^*(\pi_n) \geq \frac{\delta^2}{1+\delta}\pi_{n-1}$, or

$$\frac{\delta}{1+\delta}\left(\pi_n + \delta \sum_{i=0}^{\infty} c_{n+i+1} \delta^{2i}\right) \geq \frac{\delta}{1+\delta}\pi_{n-1}$$

which can be rearranged as

$$\frac{\delta}{1+\delta}\left(\pi_{n-1} - c_n + \delta \sum_{i=0}^{\infty} c_{n+i+1} \delta^{2i}\right) \geq \frac{\delta}{1+\delta}\pi_{n-1}$$

from which the credibility condition stated in the proposition obtains. ■

Proposition 1 establishes a class of harming equilibria, which exist if the harming structure satisfies two conditions, namely *(a)* it rules out deadline effects, and *(b)* the credibility constraint is satisfied. The first requirement actually does not put any restriction on the behaviour of the sequence $\{c_i\}_{i=1}^{\infty}$, so that the union could alternate harms of any size, as long as they are strictly positive. What is important is that there is no finite N such that the sum of the first N terms of the sequence exhausts the pie. In contrast, the second requirement does indeed restrict somehow the harming structure, since at any round harming is credible only if it is less than the sum, in present discounted value, of all future damages. As it stands, our problem does not allow us to restrict further the family of sequences which satisfy both conditions. In the next section I identify one obvious candidate, which I use to exemplify the basic characteristics of the harming equilibrium. Furthermore, notice that because of the deadline effect explained above, only the two "extreme" payoff pairs can be sustained in equilibrium: in contrast with other results in the literature, I do not derive a range of payoffs which can be sustained in equilibrium, but just two equilibria.

Proposition 1 has some interesting implications[3]:

Corollary 1 *In the limit as players become infinitely patient, the union in equilibrium obtains the whole profits, regardless of the size of the elements of* $\{c_i\}_{i=0}^{\infty}$*, provided that they are positive:*

$$\lim_{\delta\to 1} x_w^* = 1 \tag{6}$$

Notice that in the limit the credibility condition reduces to $c_n \leq \pi_n \; \forall n$, so that one needs not qualify the above statement any further. Moreover, I can state the following

Corollary 2 *The m.p.e. shares do not vary monotonically with the discount factor*

Proof. The result is proved for x_u^*; then, one can proceed similarly to prove the result for the other shares. From Proposition 1, $x_u^* = \frac{1}{1+\delta}\left(1 + \sum_{i=0}^{\infty} c_{i+1}\delta^{2i}\right)$. Differentiating with respect to the discount factor:

$$\frac{\partial x_u^*}{\partial \delta} = \frac{1}{1+\delta}\left[2\sum_{i=0}^{\infty} i c_{i+1}\delta^{2i} + \frac{1}{1+\delta}\left(\sum_{i=0}^{\infty} c_{i+1}\delta^{2i} - 1\right)\right]$$

It is straightforward to verify that

$$\lim_{\delta\to 0} \frac{\partial x_u^*}{\partial \delta} < 0$$

and

$$\lim_{\delta\to 1} \frac{\partial x_u^*}{\partial \delta} > 0$$

The first limit comes from the fact that as $\delta \to 1$ the term first term in square brackets is positive, whereas the second term approaches zero (since by hypothesis $\lim_{z\to\infty} \sum_{i=0}^{z} c_{i+1} = 1$); conversely, as $\delta \to 0$, the first term vanishes, whereas the second term is surely negative. Since $\frac{\partial x_u^*}{\partial \delta}$ is continuous in δ, there must exist at least one δ^* such that $x_u^*(\delta)$ is increasing in δ if $\delta > \delta^*$, and decreasing in δ if $\delta < \delta^*$. Thus, the equilibrium share does not vary monotonically with the discount factor. ∎

A variation of the discount factor brings about two contrasting effects on the cost of rejection: as δ increases, the “Rubinstenian” cost of rejecting an offer decreases, with negative

[3] In [Manzini(1996)] I derived analogous results for a model in which the union is committed to destroy a fixed amount c of the surplus bargained over.

comparative statics effects on the payoff of the proposing player; on the other hand, an increase in the discount factor raises the present discounted value of the sum of the sequences of harms, which reduces the equilibrium payoff to the firm. Furthermore, there is no such "harming effect" for the union, who is the only player empowered with destructive ability in this model. From the above discussion one can conclude that the "Rubinstenian effect" prevails for lower values of the discount factor, whereas the "harming effect" is outweighing for higher values of δ; however, it is not possible to draw any inference as to the uniqueness of a threshold value of the discount factor to determine which of the two effects prevails.

So far I showed how a player who is free to choose the degree of harm to inflict on the object bargained over can exploit this ability to his advantage, without the need to commit to destructive activities before-hand. This result holds provided the harming structure conforms to two requirements (credibility and deadline effect), but the above discussion is still rather abstract: the example of the next section aims at clarifying the results presented above.

3 Proportional harm

I now turn to a specific harming structure. Let the infinite sequence of harms planned by the union in case of disagreement be $\{\alpha^{i-1}(1-\alpha)\}_{i=1}^{\infty}$, where $\alpha \in (0,1]$. This implies that the union damages each time the pie in such a way that it decreases at a constant rate, α, harm after harm. In fact, recall that $\pi_n = 1 - \sum_{i=1}^{n} c_i$; thus, in our example:

$$\pi_n = 1 - \sum_{i=1}^{n} \alpha^i (1-\alpha) = 1 - (1-\alpha)\frac{1-\alpha^n}{1-\alpha} = \alpha^n$$

so that $\pi_0 = 1$. Clearly, the greater is α, the smaller the effect of harm on the pie[4]. Since in this case $\sum_{i=0}^{\infty} c_{n+i+1}\delta^{2i} = \sum_{i=0}^{\infty} \alpha^{n+i}(1-\alpha)\delta^{2i} = \frac{\alpha^n(1-\alpha)}{1-\alpha\delta^2}$, proposition 1 predicts that in the harming

[4]Notice that the succession of harms need not be decreasing: one can readily check that whether or not c_i increases with α depends on whether α is smaller or greater than $\frac{i}{i+1}$.

equilibrium an agreement is reached immediately on the partition[5]

$$x^* = \left(x_u^*, x_f^*\right) = \left(\frac{1-\alpha\delta}{1-\alpha\delta^2}, \frac{\alpha\delta(1-\delta)}{1-\alpha\delta^2}\right)$$

Notice that, not surprisingly, as α approaches unity, the equilibrium partition converges to the Rubinstenian equilibrium, since harm becomes less and less effective. The m.p.e. strategies which support this equilibrium are now defined with respect to

$$x^*(\alpha^n) = \left(x_u^*(\alpha^n); x_f^*(\alpha^n)\right) = \left(\frac{\alpha^n(1-\alpha\delta)}{1-\alpha\delta^2}, \frac{\alpha^{n+1}\delta(1-\delta)}{1-\alpha\delta^2}\right)$$

and

$$y^*(\alpha^n) = \left(y_u^*(\alpha^n); y_f^*(\alpha^n)\right) = \left(\frac{\alpha^n\delta(1-\alpha\delta)}{1-\alpha\delta^2}, \frac{\alpha^n(1-\delta)}{1-\alpha\delta^2}\right)$$

Such strategies amount to sharing the surplus available, α^n, always according to the same proportions in each of the two types of subgames (those starting with an offer by the firm and those starting with an offer by the union). The condition for harming to be credible is

$$\alpha^{n-1}(1-\alpha) \le \delta \sum_{i=0}^{\infty} \alpha^{n+i}(1-\alpha)\delta^{2i} \Rightarrow \alpha \ge \frac{1}{\delta(1+\delta)}$$

which holds for sufficiently large values of the discount factor[6].

This harming structure shows very clearly how the union can exploit its harming ability to increase the firm's cost of rejection. The effect of imposing the sort of "proportional" harm described above is actually equivalent to making the firm more impatient, since in effect it lowers the firm's rate of time preference from δ to $\delta\alpha$. In fact, notice that if we define $\delta_1 = \delta$ and $\delta_2 = \alpha\delta$, the partition which obtains in equilibrium can be expressed as

$$x^{*R} = \left(x_u^{*R}, x_f^{*R}\right) = \left(\frac{1-\delta_1}{1-\delta_1\delta_2}, \frac{\delta_2(1-\delta_1)}{1-\delta_1\delta_2}\right)$$

which is exactly the unique equilibrium partition of a "standard" alternating offers bargaining game in which players have different discount factors! Moreover, since $\frac{\partial x_u^{*R}}{\partial\delta_2} = -\frac{1-\delta_1}{(1-\delta_1\delta_2)^2} < 0$,

[5] Although developed independently, this result corresponds in essence to that stated in [Bush et al.(1996)], Proposition 2. This is not surprising, since in that paper destructive power is modelled as an increase in the pie's shrinking rate; although in my model destructive power consists in "chopping off" part of the cake, the example is designed as to decrease proportionally the size of the pie harm after harm.

[6] Since by definition $\alpha \in (0,1]$, in order for the above to be satisfied it has to be true that $\frac{1}{\delta(1+\delta)} \le 1$, which is satisfied for $\delta \ge \frac{-1+\sqrt{5}}{2}$.

for any given value of δ the equilibrium share to the union increases as δ_2 (and thus α) decreases: the greater the (credible) rate at which the union can reduce the pie, the greater his payoff. Thus, in order to maximize his equilibrium payoff, the union has to choose the smallest α which satisfies the credibility constraint, so that it will set $\alpha = \frac{1}{\delta(1+\delta)}$, yielding the equilibrium payoff pair will be $x^*(\frac{1}{\delta(1+\delta)}) = (\delta, 1-\delta)$[7].

Regardless of the size of α, it is straightforward to check that, as predicted by Corollary 1, in the limit as the discount factor approaches unity, the union obtains the whole of the profits. Moreover, Corollary 2 is also confirmed, since one can easily check that the equilibrium shares do not vary monotonically with the discount factor, as shown in Figure **??**[8]. In this case shares are represented on the $A \times \Delta$ domain, *not* the $C \times \Delta$ domain, where $A = \{\alpha | \alpha \in (0,1]\}$ and $\Delta\{\delta | \delta \in (0,1)\}$[9]. In fact, consider the union's equilibrium share. Then:

$$\frac{\partial(x_u^*)}{\partial\delta} = -\frac{\alpha(\alpha\delta^2 - 2\delta + 1)}{(1-\alpha\delta^2)^2}$$

The expression in brackets at the numerator has two distinct real roots, $\delta_{1,2} = \frac{1 \pm \sqrt{1-\alpha}}{\alpha}$, of which only one is admissible. Consequently, $\frac{\partial(x_u^*)}{\partial\delta} < 0$ if $\delta < \frac{1-\sqrt{1-\alpha}}{\alpha}$, and $\frac{\partial(x_u^*)}{\partial\delta} > 0$ if $\delta > \frac{1-\sqrt{1-\alpha}}{\alpha}$.

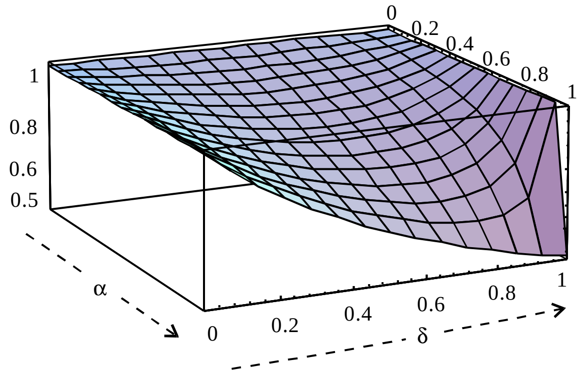
Notice that the function $\frac{\partial(x_u^*)}{\partial\delta}$ is continuous over the domain, since it admits a discontinuity for $\delta = \frac{1}{\sqrt{\alpha}} > 1$.

Then, the above represents an easy "harming rule" which enables the harming player to maximize his payoff.

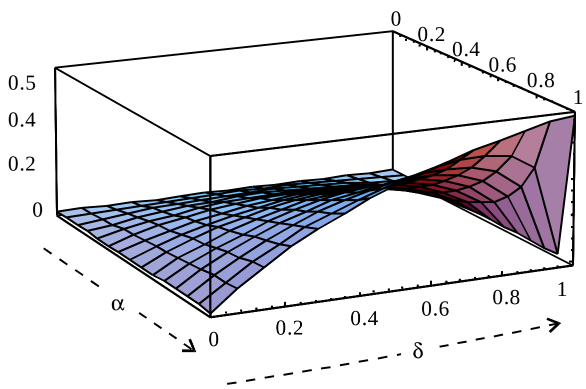
[7][Bush et al.(1996)] obtain a similar result in their Corollary 1.

[8]Notice however that in the degenerate case when $\alpha \to 0$, the non-monotonicity result does not hold any longer. This case is anyway of no practical interest, since for such low values of α harming is not credible.

[9]Thus $\alpha = 1$ corresponds to $c = 0$.



worker's share



firm's share

4 Non-stationary equilibria

If we now turn to non-stationary strategies, we can retrieve for this model results analogous to those that are now standard in the literature on "money burning" bargaining models, which predict the existence of a range of subgame perfect equilibrium payoffs. The type of strategies which can sustain multiple equilibria depend in fact on the existence of just the two "extreme" equilibrium payoffs; they can be used to define two different regimes of play - supported by the strategies defined in Proposition 1 -, one in which the harming player never harms, and in which the bargainers obtain the bilateral monopoly payoff (so that the union receives its "worst" wage); and another one in which the harming player always harms when faced with a rejection, which sustains the "harming" equilibrium payoff. Then, consider the following strategies: both the union and the firm conform to offering a wage within the "worst-best" range, and punish the opponent who proposes a different wage by reverting to playing according to the regime that yields the extreme equilibrium which is less favourable to the deviator. Then, one can verify that such strategies support the following[10]:

Proposition 2 *In a bargaining game of alternating offers in which the worker can harm after every rejection of the firm, if* $c_n \leq \delta \sum_{i=0}^{\infty} c_{n+i+1} \delta^{2i}$ $\forall n$*, then any partition which yields the union a wage* $w^* \in [\widehat{x}_u^*, x_u^*]$ *can be sustained as a subgame perfect equilibrium of the bargaining game.*

Further, as in other models belonging to the money burning literature, in this model the existence of the two extreme equilibria is enough to guarantee also inefficient equilibria, as established in the following:

Proposition 3 *For any finite integer N, if* $c_n \leq \delta \sum_{i=0}^{\infty} c_{n+i+1} \delta^{2i}$ $\forall n$*, then there exists a discount factor* $\widehat{\delta}$ *such that for* $\delta \in \left[\widehat{\delta}, 1\right)$ *every partition which yields the union a wage* $\widetilde{w} \in \left[\frac{\widehat{x}_u^*}{\delta^N}, 1 - \frac{1-\delta x_u^*}{\delta^{N-1}}\right]$ *can be sustained as a subgame perfect equilibrium of the bargaining game in which agreement is reached after N periods.*

Proof. Strategies that can support this equilibrium are for instance: for the firm, for the first N periods, the union (firm) asks for the whole profits and rejects any offer which yields

[10] See [Avery and Zemsky(1994)] or [Bush and Wen(1995)].

less than the whole pie, and the union never harms after a rejection; after N periods, the union (firm) offers a wage $\widetilde{w}$, rejects all wages below (above) $\widetilde{w}$, accepts all wages above (below) $\widetilde{w}$, and the union harms after its own offer has been rejected. If one of the players deviates, then in the following round play reverts to the strategies which support the worst equilibrium payoff for the deviator.

The proof follows standard arguments (e.g., see [Holden(1989)]). To simplify the notation, let us re-define the partitions $x^*(\pi_0) = \left(x_u^*(1); x_f^*(1)\right)$ and $y^*(\pi_0) = \left(y_u^*(1); y_f^*(1)\right)$as $(w_u, 1 - w_u)$ and $(w_f, 1 - w_f)$, respectively, where $w_f = \delta w_u \Rightarrow 1 - w_f = 1 - \delta w_u$. Now, consider a deviation by the firm. If it were to deviate in the first period, then play would revert to the harming equilibrium, yielding the firm a payoff of $1 - w_f$. Thus, it will not be profitable for the firm to deviate as long as $\delta^N(1 - \widetilde{w}) \geq \delta(1 - w_f)$, where the term on the l.h.s. is the firm's payoff from sticking to its equilibrium strategy. The above can be rearranged as $\widetilde{w} \leq 1 - \delta^{1-N}(1 - w_f) = 1 - \delta^{1-N}(1 - \delta w_u)$, which establishes the upper bound of the range in which $\widetilde{w}$ is allowed to vary. Further, notice that the condition required for deviations in periods other than the first not to be profitable are less stringent. For instance, by deviating in the second period the firm would obtain $1 - w_u$ in the following round, so that such deviations is not profitable if $\delta^N(1 - \widetilde{w}) \geq \delta^2(1 - w_u)$, or $\widetilde{w} \leq 1 - \delta^{2-N}(1 - w_u)$. But $1 - \delta^{2-N}(1 - w_u) \geq 1 - \delta^{1-N}(1 - \delta w_u)$ if $\delta(1 - w_u) \leq (1 - \delta w_u)$, or $\delta \leq 1$. Then, one can proceed analogously to verify that the wage level which makes deviations not profitable is greater the further down we move along the game tree.Consider now the union. A deviation in the first round of negotiations would trigger play to revert to the Rubinstenian equilibrium in the following period, yielding the union a payoff of $\frac{\delta^2}{(1+\delta)}$, so that a deviation will not be profitable as long as $\frac{\delta^2}{(1+\delta)} \leq \delta^N \widetilde{w}$, or $\widetilde{w} \geq \frac{1}{\delta^N(1+\delta)} = \frac{\widehat{x}_u^*}{\delta^N}$, which is also the condition for a deviation in the second period not to be profitable. Consider now a deviation in round 3: this will not be profitable as long as $\delta^N \widetilde{w} \geq \frac{\delta^4}{(1+\delta)} \Rightarrow \widetilde{w} \geq \frac{\delta^4}{\delta^N(1+\delta)}$, where $\frac{\delta^4}{\delta^N(1+\delta)} \leq \frac{1}{\delta^N(1+\delta)}$; then one can proceed in a similar fashion to verify that, as for the firm, the condition required for deviations in periods other than the first not to be profitable are less stringent the later the deviation.

Next, I show that $\left[\frac{\widehat{x}_u^*}{\delta^N}, 1 - \frac{(1-\delta x_u^*)}{\delta^{N-1}}\right]$ is non empty. Indeed, $\frac{\widehat{x}_u^*}{\delta^N} = \frac{1}{\delta^N(1+\delta)} \leq 1 - \delta^{1-N}(1 - \delta x_u^*)$ can be rearranged as

$$1 - \frac{1}{\delta^N(1+\delta)} - \frac{1 - \delta w_u}{\delta^{N-1}} = 1 - \frac{1}{\delta^N(1+\delta)} - \frac{w_f}{\delta^{N-1}} \geq 0$$

$$1 - \frac{1}{\delta^N(1+\delta)} - \frac{\frac{1}{1+\delta}\left(1-\delta^2\sum_{i=0}^{\infty} c_{i+1}\delta^{2i}\right)}{\delta^{N-1}} \geq 0$$

$$\frac{\delta^N(1+\delta)-1-\delta}{\delta^N(1+\delta)} + \frac{\delta^3\sum_{i=0}^{\infty} c_{i+1}\delta^{2i}}{\delta^N(1+\delta)} \geq 0$$

$$(1+\delta)\left(1-\delta^N\right) - \delta^3\sum_{i=0}^{\infty} c_{i+1}\delta^{2i} \leq 0 \Rightarrow$$

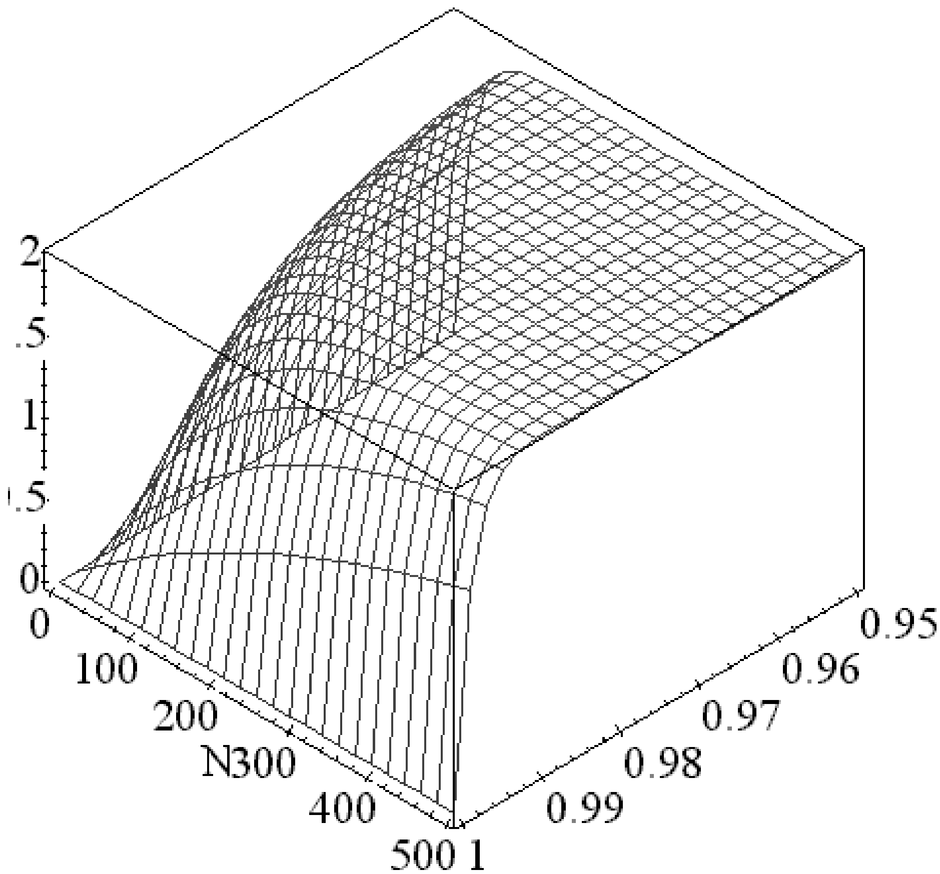
$$(1+\delta)\left(1-\delta^N\right) \leq \delta^3\sum_{i=0}^{\infty} c_{i+1}\delta^{2i}$$

Now, note that $0 < \delta^3\sum_{i=0}^{\infty} c_{i+1}\delta^{2i} \leq 1$, and the higher the value of the discount factor, the closer the r.h.s. of the above inequality gets to unity (recall that $\sum_{i=0}^{\infty} {c_{i+1}}^{2i} = 1$). Then, for any given finite N, as the discount factor approaches unity:

$$\lim_{\delta\to 1}(1+\delta)\left(1-\delta^N\right) - \delta^3\sum_{i=0}^{\infty} c_{i+1}\delta^{2i} = -1$$

so that the above inequality is always verified. ■

Figure 2 reports simulated values for $(1+\delta)\left(1-\delta^N\right)$: for any value of N one can correspondingly find a value of the discount factor high enough as to set $(1+\delta)\left(1-\delta^N\right)$ as close as chosen to unity.



2
.5
1
.5
0
0
100
200
N300
400
500
1
0.99
0.98
0.97
0.96
0.95

5 Concluding remarks

In this paper I have shown how the ability to destroy the surplus bargained over can be successfully exploited by the harming player to increase his equilibrium payoff, since destructive actions in effect increase one opponent's cost of rejecting an offer, which grows with the discount factor. In order for this result to obtain, there is no need to resort neither to "forcing" a player to a specific action in case of disagreement, nor to a repeated game structure, as long as the harming player is free to choose a sequence of harms. I argued that the latter is indeed an alternative form of commitment. Recent literature has modelled commitment either as a time frame for offers and counteroffers, or as a cost on one's payoff for conceding on different agreement from the one previously put forward. In my approach, the harming ability of one player, which modifies the pie bargained over, is the mechanism which increases one opponent's cost of rejecting an offer. As shown in the example, the net effect amounts to increasing the opponent's rate of time preference (by reducing his discount factor). The same effect would be produced by increasing the time span between subsequent offers by the harming player. However such "Rubinstenian" commitment works in favour of the committed player only if he happens to be the first mover in the bargaining game: for the responding player, the "slower" the bargaining (or the smaller the discount factor), the greater the degree of commitment, the smaller the payoff. My approach to modelling commitment brings about an obvious though different result: regardless of whether the harming player starts negotiations as a proposer or as a responder, the greater the commitment achieved through destructive power, the greater the payoff to the committed player.

The model introduced in this chapter possesses at least two appealing features. On the one hand it justifies the intuition that bargaining power increases with the ability to inflict a cost on one's opponent even in a simple "one pie" bargaining game. On the other hand, compared to repeated bargaining games - which generally predict a continuum of equilibria even in stationary strategies (see [Muthoo(1995)]) - it has the advantage of singling out just two m.p.e.. These are supported by an "all or nothing" strategy from the harming player which seems to us simpler and consequently more compelling for applications, since it requires the harming player to carry out his threat either never or every time.

Still further work is needed. The nature of some kinds of negotiations (most notably wage bargaining) calls for the modelling of repeated interactions; the analysis of a repeated play of my model seems therefore an extension worth pursuing.

References

[Avery and Zemsky(1994)] C. Avery and P. B. Zemsky. "Money Burning and Multiple Equilibria in Bargaining". *Games and Economic Behaviour*, 7:154–168, 1994.

[Bernheim and Ray(1989)] D. Bernheim and D. Ray. "Markov-perfect Equilibria in Altruistic Growth Economies with Production Uncertainty". *Journal of Economic Theory*, 47:195–202, 1989.

[Bush and Wen(1995)] L-A. Bush and Q. Wen. "Perfect Equilibria in a Negotiation Model". *Econometrica*, 63:545–565, 1995.

[Bush et al.(1996)] L-A. Bush, S. Shi, and Q. Wen. "Bargaining with Surplus Destruction". mimeo, University Of Waterloo, 1996.

[Dasgupta and Maskin(1989)] P. Dasgupta and E. S. Maskin. "Bargaining and Destructive Power". Discussion Paper 1432, Harvard Institute of Economic Research, Harvard University, 1989.

[Fernandez and Glazer(1991)] R. Fernandez and J. Glazer. "Striking for a Bargain Between Two Completely Informed Agents". *American Economic Review*, 81:240–252, 1991.

[Haller and Holden(1991)] H. Haller and S. Holden. "A Letter to the Editor on Wage Bargaining". *Journal of Economic Theory*, 52:232–236, 1991.

[Holden(1989)] S. Holden. "Non-cooperative Wage Bargaining". Discussion Paper 349, Centre for Labour Economics, LSE, 1989.

[Kambe(1994)] S. Kambe. "When is there a Unique Equilibrium in Less Structured Bargaining?". Discussion Paper n.90, (Economics Discussion Papers), Nuffield College Oxford, August 1994.

[Manzini(1996)] P. Manzini. "Strategic wage bargaining with destructive power: the role of commitment". W.P. 96/17, University of Exeter, 1996.

[Maskin and Tirole(1988)] E. Maskin and J. Tirole. "A Theory of Dynamic Oligopoly. I. Overview and Quantity Competition with Large Fixed Costs". *Econometrica*, 56:549–570, 1988.

[Muthoo(1995)] A. Muthoo. "Bargaining in a Long-Term Relationship with Endogenous Termination". *Journal of Economic Theory*, 66:590–598, 1995.

[Perry and Reny(1993)] M. Perry and P.J. Reny. "A Non-Cooperative Bargaining Model with Strategically Timed Offers". *Journal of Economic Theory*, 59:50–77, 1993.

[Rubinstein(1982)] A. Rubinstein. "Perfect Equilibrium in a Bargaining Model". *Econometrica*, 50:97–109, 1982.

[Sakovics(1993)] J. Sakovics. "Delay in Bargaining Games with Complete Information". *Journal of economic Theory*, 59:78–95, 1993.