COSTLY COASIAN CONTRACTS*

LUCA ANDERLINI
(St. John's College, Cambridge)

LEONARDO FELLI
(London School of Economics)

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ABSTRACT. We identify and investigate the basic 'hold-up' problem which arises whenever each party to a contingent contract has to pay some *ex-ante cost* for the contract to become feasible. We then proceed to show that, under plausible circumstances, a 'contractual solution' to this hold-up problem is not available. This is because a contractual solution to the hold-up problem typically entails writing a 'contract over a contract' which generates a fresh set of ex-ante costs, and hence is associated with a new hold-up problem. We conclude the paper investigating two applications of our results to a static and to a dynamic principal-agent model.

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KEYWORDS: Hold-Up Problem, Ex-Ante Contractual Costs, Contracts Over Contracts, Incomplete Contracts, Principal-Agent Problems.

ADDRESS FOR CORRESPONDENCE: Luca Anderlini, Faculty of Economics and Politics, University of Cambridge, Austin Robinson Building, Sidgwick Avenue, Cambridge CB3 9DD, United Kingdom. E-mail LA13@ECON.CAM.AC.UK.

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1. INTRODUCTION

1.1. Motivation

Virtually all economic transactions are regulated by a contract of one type or another. Even when a supermarket posts a price on a can of food, it is offering its customers a contract for the purchase of the can at a given price with certain clauses attached. For instance that the food should be of "satisfactory quality"[1] and that the buyer must be "ready and willing to pay the price in exchange for possession"[2] of the food. The parties accept to execute such contract when the item is keyed in at the cash register.

Most larger economic transactions are explicitly regulated by contracts which specify, very often in writing, details of the transaction(s) to be carried out.

It is readily apparent to any observer that most contracts which regulate economic transactions are *costly*: some of the resources available are used up in the process of specifying and/or enforcing the contract between agents. If nothing else, each party to a contract has to invest a certain amount of time in drafting or simply reading the contract proposed by the other party.[3] In this paper, we study the impact of these costs on whether a contract will or will not be drawn-up, and to some extent on the 'shape' of those contracts which agents decide to enter, given their costs.

Clearly, the *size* of contractual costs, relative to the possible surplus which the contract will generate, will be a contributing factor to the impact which contractual costs have: if contractual costs are large in relative terms, then their impact will be accordingly larger.[4] In this paper, though, we choose to concentrate on a further issue — namely, is it the case that contractual costs will have an impact (on both the availability and the shape of possible contracts) which goes beyond what is due to their relative size? We conclude

[1] In the U.K., according Section 14 of the *Sale of Goods* Act of 1979, the definition of "satisfactory quality" includes (among other aspects of the goods in question): "(a) fitness for all the purposes for which goods of the kind in question are commonly supplied, (b) appearance and finish, (c) freedom from minor defects, (d) safety, and (e) durability" (Rose 1987, p. 107).

[2] Cf. Rose (1987) p. 112.

[3] Of course, if legal experts of some kind are involved in the process, drafting and other costs can be substantial.

[4] Cf. Anderlini and Felli (1996b) and the discussion of this paper in the following subsection.

that contractual costs may matter, regardless of their relative size, for *strategic* reasons whenever it is the case that at least some of the costs associated with a contract must be paid *ex-ante* — before the contract itself is negotiated by the agents.

The primary effect of contractual costs is to generate contracts which are *incomplete* in a well defined sense. In the most basic version of our model the agents may end up with no contract at all. When the choice of ex-ante costs is 'gradual', and higher costs paid correspond to a more 'detailed' contract, the agents will, in general, end up with a contract which is less detailed than would be optimal, after the ex-ante costs are taken into account. In its simplest form, the strategic effect which drives our results below is not hard to outline.

Consider any 'Coasian' contractual situation with the following features.[5] Two agents contemplate entering a contract which yields a surplus of an arbitrary given size. Moreover, the two agents' shares of the surplus generated by the contract are exogenously given, say because the extensive form which they must use to negotiate the contract is itself exogenously given.

Suppose now that there are ex-ante costs associated with the contract which the agents contemplate entering into. In particular, suppose that the agents must each pay some cost before the contract-negotiating phase begins. Then, if the *distribution* of ex-ante costs is such that one (or both) agents will not be able to recoup the ex-ante cost given the distribution of surplus, the contract will not be drawn-up. This is possible even when the total of ex-ante costs across the two agents is *less* than the surplus which the contract generates so that it would be *socially efficient* for the agents to pay the ex-ante costs and enter the contract.

We model this situation both taking the distribution of surplus as exogenously given, and considering a variety of extensive forms in which the agents are allowed to bargain over

[5] By this we mean a situation in which the property rights of the agents are sufficiently well defined to allow them to enter into a negotiating phase, and that there are some un-exploited gains from trade (Coase 1960). This obviously covers an extremely wide variety of possible situations, ranging from text-book like externalities to complex contingent contracts. In a previous version of this paper (Anderlini and Felli 1996a), we concentrated on the case of two agents who negotiate a simple contingent contract — namely a *risk-sharing* agreement. All the results which we report here apply to such situation.

the distribution of surplus, provided of course that the ex-ante costs have been paid. We find that the problem we have described is 'pervasive' in the sense that in a whole variety of extensive forms, the agents will not draw-up a contract even though it would be socially efficient to do so.

What we have just described is a version of a source of inefficiencies well known in Contract Theory as the 'hold-up problem' (Grossman and Hart 1986, Hart and Moore 1988, Hart and Moore 1990, among many others). The problem is particularly acute in our setting since it may be impossible to find a 'contractual solution' to this hold-up problem for the following reasons.[6]

Imagine that the two agents in the contractual setting we have described attempt to resolve the inefficiency in the following way. Before the ex-ante costs are paid, they negotiate a transfer of money which will compensate the agent who is unable to recoup the ex-ante cost for his loss, of course contingent on his paying the ex-ante cost. Provided the sum of ex-ante costs does not exceed the surplus generated by the contract, such transfer can always be arranged so that both agents now benefit from paying the ex-ante costs and entering the contract. However, the problem which arises now is that the contingent compensating transfer can itself be viewed as a contract, which will involve a new set of ex-ante costs.

Suppose that the 'second tier' contract we have described does indeed involve a new set of strictly positive ex-ante costs. Suppose moreover that the second tier contract and ex-ante costs must be paid for and negotiated *before* the first order costs and contract are paid for and negotiated respectively.[7]

Then, relying on simple sub-game perfection, it is possible to show that the ex-ante costs associated with the second tier contract will not be paid. Consequently, the contingent

[6] Aghion, Dewatripont, and Rey (1994) and Nöldeke and Schmidt (1995), among others, analyse models of contractual solutions to the hold-up problem.

[7] This is obviously an *assumption* as such. However, we believe it to be plausible in a wide variety of cases. In Sections 7 and 8 below we examine two specific models (a static and a dynamic principal-agent set-up) in which any re-distribution of surplus would have to be negotiated *before* the actual contract-negotiating phase begins because the contract-negotiating phase involves a take-it-or-leave-it offer by the principal to the agent, and the *size* of the surplus depends on its distribution between the principal and the agent via the incentive-compatibility and limited liability constraints.

compensating transfer cannot be negotiated and therefore will not take place. Hence, the ex-ante costs associated with the first tier contract will, in turn, not be paid, and the actual surplus-generating contract will not be drawn-up. As we show in Subsection 6.2, a whole hierarchy of 'higher order' contracts will not resolve the problem either.

In Sections 7 and 8, we show that our results apply naturally to a class of models in which the interplay of incentive constraints and limited liability 'pins down' the distribution of surplus between the agents as the result of exogenously given economic parameters.

We first focus on a familiar principal-agent set-up in the presence of ex-ante contractual costs, and find that the resulting contract may induce the agent to exert a low level of effort although an incentive contract that induces a high level of effort would be socially efficient, after the ex-ante costs are taken into account.

Our last concern is to apply our results to a multi-period model. Here, the choice of ex-ante costs is 'gradual' in the sense that a principal and an agent can choose to pay more ex-ante in order to enter a contract with a longer horizon. We find that, in general, the contract which is drawn-up is *shorter* than would be socially efficient, after the ex-ante costs are taken into account.

1.2. Related Literature

The 'Coase Theorem' (Coase 1960) has had a pervasive influence on Economics in general, and Contract Theory in particular, for over three decades. In the absence of any contractual costs, when ownership rights are well defined, a Pareto-efficient outcome is guaranteed. In this paper, we focus on the impact of *ex-ante* contractual costs on this benchmark result. Ex-ante contractual costs play a crucial strategic role (as opposed, for instance, to those contractual costs which are payable ex-post) and may lead to an outcome which is *constrained inefficient*. In the models which we analyze below, such inefficient outcomes can be interpreted as contracts which are incomplete in a well defined sense.

Starting from Williamson (1985) and Grossman and Hart (1986) a number of papers have focused on the effects of contract incompleteness. Most of these papers assume that contracts are incomplete and concentrate on the role of available mechanisms, and in particular institutions, to mitigate the inefficiencies generated by contract incompleteness.

Some of the mechanisms considered are: vertical and lateral integration (Grossman and Hart 1986), the optimal allocation of ownership rights on physical capital (Hart and Moore 1990), and the delegation of authority within organizations (Aghion and Tirole 1997). We differ from these papers since we do not assume contractual incompleteness but rather derive it *endogenously* from the ex-ante costs associated with a contract.

A second strand of literature has concentrated on some of the possible causes of contractual incompleteness. Hart and Moore (1988) have asked whether contractual incompleteness might be due to the fact that the outcome that the parties want to implement may be, at least in part, un-observable to the enforcing agency (the court). They conclude that this un-observability might lead the parties to write a contract that will leave out some details that the court cannot observe. This will result in a basic 'hold-up problem'. Each party's final allocation of resources will be determined on the basis of an ex-post re-negotiation of the contract that cannot be specified, at least not fully, by means of the ex-ante contract. A number of subsequent papers have explored whether this basic hold-up problem might have a contractual solution. Chung (1991) Rogerson (1992) Aghion, Dewatripont, and Rey (1994) Nöldeke and Schmidt (1995) analyse possible contractual solutions to the hold-up problem under a variety of different assumptions about the nature of informational asymmetries. By contrast, Maskin and Tirole (1995) show that it is *always* possible to devise an ex-ante contract (a mechanism) that implements the same outcome that would be implemented in the absence of the un-observability. This is achieved by asking the contracting parties—once the conditioning event has been realized—to report the event or, equivalently, to report the utility levels or payoffs accruing to each party, while truthful revelation is ensured through appropriate incentives.

The basic hold-up problem we are concerned with in this paper is of the same variety as the problem identified by Holmström (1982) and analyzed in the context of the incomplete contracts literature by Hart and Moore (1988). However, causality is essentially reversed in this paper. In Hart and Moore (1988) the hold-up problem is induced by contractual incompleteness. Instead, in this paper it is the hold-up problem in the negotiation of a contract that yields contractual incompleteness. We also differ from the previous literature since we argue that in our setting a contractual solution to the hold-up problem is unlikely to be available for the reasons we outlined above.

Some recent papers have addressed the impact that *complexity* considerations may have on the form and shape of optimal contracts (Dye 1985, Segal 1995, Anderlini and Felli 1996b, among others). Dye (1985) considers a model in which the contracting parties face fixed cost of 'adding more contingencies' to a contract. Segal (1995) focuses on a contracting problem in which the relevant event is not observable to the enforcing agency (the court). In such an environment he analyzes the parties' welfare gains from using ex-ante message contingent mechanisms as in Maskin and Tirole (1995). He shows that such gains become negligible as the number of events on which the contract is contingent increases without bound. The implication is that, in a 'complex' environment, message contingent mechanisms will not be used, even if they entail an arbitrarily small complexity cost. Aside from the basic differences in the formulation of the models, our analysis in this paper differs from Segal (1995) and Dye (1985) since the incompleteness they derive is constrained *efficient*; a central planner facing the same complex environment will choose the same incomplete contract. In our setting (ex-ante) contractual costs play a strategic role, and the incomplete contracts we derive are constrained *inefficient*.

In Anderlini and Felli (1996b) we model a contract as an algorithmic procedure (a Turing Machine) and we explore whether any complexity measure in a general class associated with such a contract might induce the contracting parties to choose an incomplete contract. We conclude that this is indeed the case. However, in Anderlini and Felli (1996b), while we model explicitly the complexity costs associated to a contract, we also assume away any strategic role they might play. As a consequence, we find that the impact of the complexity costs is directly related to their size, relative to the size of the surplus which the contract itself generates. This is not the case in the models we analyse in this paper.

Finally, starting with Rubinstein (1986) a whole strand of literature (Abreu and Rubinstein 1988, Piccione 1992, Piccione and Rubinstein 1993, among others) has concentrated on the impact of complexity costs on the equilibrium set of a repeated game. Although our model describes a radically different environment, in our view, one similarity can be drawn between the two. As in the repeated games literature, the impact of contractual costs in our model is directly attributable to their *strategic* role.

1.3. Overview

We begin with an informal discussion of the possible interpretations of the ex-ante contractual costs in Section 2. In Section 3 we present the simplest possible model of the basic hold-up problem associated with the writing of a risk-sharing contract. This problem is analysed in the case in which the ex-ante costs associated with the contract are discrete and are either complements or substitutes. In Section 4 we turn to the analysis of a simple model in which the agents' choice of ex-ante costs is continuous. Section 5 generalizes the results of Section 3 to the case in which the distribution of surplus can be negotiated by the agents in a variety of extensive forms. In Section 6 we address the question of whether a contractual solution to our basic hold-up problem is plausible. We do this by analysing the possibility of a 'contract over a contract' and a whole hierarchy of 'contracts over contracts' in two possible extensive forms. In Sections 7 and 8 we introduce two examples of economic environments to which our previous results apply. The model in Section 7 is a simple, static principal-agent model. In Section 8 we analyse a dynamic principal-agent problem. Section 9 offers some concluding remarks. To ease the exposition, we have relegated all proofs to the Appendix.

2. EX-ANTE CONTRACTUAL COSTS

We are concerned with contractual situations in which the parties have to incur some costs ex-ante, before they reach the stage in which the actual contract is negotiated.

The interpretation of these ex-ante contractual costs which we favour is that of time spent 'preparing' for the negotiation of the contract. Typically, a variety of tasks need to be carried out by the contracting parties before the actual negotiation begins.

In those cases in which a contract contingent on a state of nature is concerned, both parties need to conceive of, and agree upon, a suitable contractual language to describe precisely the possible realizations of the state of nature. The contracting parties also need to collect and analyse information about the 'legal environment' in which the contract will be embedded. For instance, in different countries the same contract will need to be drawn-up differently to make it legally binding (enforceable).

In virtually all contractual settings the parties need to spend time arranging a way to 'meet', and they need to 'earmark' some of their time schedules for the actual meeting.

In many cases, before a meaningful negotiation can start, the parties will need to collect and analyse background information which may be relevant to their understanding of the actual trading opportunities. These activities may range from collecting information about (for instance the credit-worthiness of) the other party, to actual 'thinking' or 'complexity' costs incurred to understand the contractual problem. We view this type of ex-ante contractual costs as both relevant and important for the type of effects which we identify in our analysis below. However, it should be emphasized that our model does not *directly* apply to this type of costs. This is because in our model the size of the surplus is fixed and known to the parties. On the other hand, the lack of information and/or understanding of the contractual setting which we have just described, would clearly make the size of the surplus uncertain for the contracting parties. We have not considered the case of uncertain surplus for reasons of space and analytical convenience. However, we conjecture that the general 'flavour' of our results generalizes to this case.

We conclude this section with an observation. In many cases the parties to a contract will have the opportunity to delegate to outsiders many of the tasks which we have mentioned as sources of ex-ante contractual costs. The most common example of this is the hiring of lawyers. In these cases, the time costs which we have just discussed will be *monetized* at an appropriate rate. Abstracting from agency problems (between the contracting party/principal and the lawyer/agent), which are likely to increase the ex-ante costs anyway, our analysis applies, unchanged, to the case in which the ex-ante contractual costs are payable in money.

3. A 'REDUCED FORM' MODEL: DISCRETE EX-ANTE COSTS

3.1. Complementary Ex-Ante Costs

Our model consists of two agents, called A and B, who face a 'Coasian' opportunity to realize some gains from trade. With little loss of generality we normalize the size of the surplus which is realized if an agreement is reached to be one and we set the parties' payoffs in the case disagreement to be equal to zero.[8]

In the simple model we analyse in this section, once the contract-negotiating phase is reached the division of the surplus between the two agents is exogenously given and cannot be changed by the agents. This should be thought of as the result of the agents having exogenously given bargaining power in the contract-negotiating phase (the *extensive form* of the bargaining game they play to divide the surplus is exogenously given), and/or the possibility that the *size* of the surplus may depend on its distribution across the agents as is the case in the two principal-agent models which we analyse in Sections 7 and 8 below.

For the time being we simply let $\lambda \in [0,1]$ be the share of the surplus which accrues to agent A if a contract is drawn-up, and $1-\lambda$ the share of the surplus which accrues to B.

For the contract to become feasible, each agent has to pay a given *ex-ante cost*. In other words, the agents reach the contract-negotiating phase only if they both pay a certain amount before the negotiation of the contract begins.[9] These costs should be thought of as representing a combination of the activities necessary for a contract to become feasible which we discussed in some detail in Section 2 above.

Let $c_A > 0$ and $c_B > 0$ be the two agents' ex-ante costs. Clearly, if $c_A + c_B > 1$ then the two agents will never draw-up a contract yielding the unit surplus, but then neither would a social planner since the total cost of the contract exceeds the surplus which it yields. We are interested in the case in which it would be socially efficient for the two agents to draw-up a contract. Our first assumption guarantees that this is the case.

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[8] If the agreement is interpreted as a *risk-sharing* contract there is some loss of generality in working with a surplus of fixed size as we do here. In Anderlini and Felli (1996a) we spell out in detail the assumptions which are needed to ensure that the model we use here can be though of as a model of a risk-sharing contract. In short, we would need to assume that the agents have constant absolute risk-aversion and that there is no *aggregate* risk.

[9] Notice therefore that we are implicitly assuming that the agents have some endowments of resources out of which the ex-ante costs can be paid. In Anderlini and Felli (1996a) we model explicitly the agents's endowments from which the ex-ante costs are paid, and examine the bounds which they have to satisfy. This becomes inessential in the simplified model which we use in this version of the paper. For our purposes, it is enough to assume that the agents both start off with a unit endowment of resources which is available to them when the ex-ante costs are payable.

ASSUMPTION 1: *The surplus which the contract yields exceeds the total ex-ante costs which are payable for the contract to become feasible. In other words* $c_A + c_B < 1$.

Our two agents play a two-stage game. In period $t = 0$ they both simultaneously and independently decide whether to pay their ex-ante cost. Only if both agents pay their ex-ante cost at $t = 0$, do they have the possibility of negotiating a contract yielding a surplus of size one at $t = 1$.[10] In this case the game at $t = 1$ is, for the time being, a 'black box' yielding pay-offs of λ to A and $1 - \lambda$ to B. If one or both agents do not pay their ex-ante costs at $t = 0$, the game at $t = 1$ is trivial: the contract which yields the unit surplus is not feasible; the agents have no actions to take and they both receive a pay-off of zero at $t = 1$.

Throughout the paper by equilibrium we mean a *subgame perfect* equilibrium of the game at hand.

The normal form which corresponds to the two-stage game we have just described is depicted in Figure 1. From this it is immediate to derive our first two propositions, which therefore are stated without proof.

PROPOSITION 1: *Let a pair of ex-ante costs* $c_A > 0$ *and* $c_B > 0$ *satisfying Assumption 1 be given. Then there exists a range of values — namely* $\Lambda = [0, c_A) \cup (1 - c_B, 1]$ *— for the distribution parameter* λ *such that the only equilibrium of the two-stage game represented in Figure 1 has neither agent paying the ex-ante cost, and therefore yields the no-contract outcome.*

PROPOSITION 2: *Let any value of the distribution parameter* $\lambda \in [0, 1]$ *be given. Then there exists a set* $C = \{c_A, c_B \mid \text{either } c_A > \lambda \text{ or } c_B > 1 - \lambda \text{ and } c_A + c_B < 1\}$ *of pairs of ex-ante costs which satisfy Assumption 1, and such that the only equilibrium of the two-stage game represented in Figure 1 has neither agent paying the ex-ante cost, and therefore yields the no-contract outcome.*

[10] Notice that we are therefore assuming that the two agents' ex-ante costs are *perfect complements* in the 'technology' which decides whether the surplus-generating contract is feasible or not. We examine the cases of perfect and partial substitutes, and of strategic complements in Subsections 3.2, 3.3 and 3.4 below respectively.

	pay c_B	not pay c_B
pay c_A	$\lambda - c_A,\ 1 - \lambda - c_B$	$-c_A,\ 0$
not pay c_A	$0,\ -c_B$	$0,\ 0$

Figure 1: Normal form of the two-stage game with ex-ante costs.

We view Propositions 1 and 2 together as implying that in the presence of ex-ante contractual costs, if the *distribution* of ex-ante costs across the parties is sufficiently 'mis-matched' with the given distribution of surplus, then the ex-ante costs will generate a version of the hold-up problem which will induce the agents not to draw-up a contract even though it would be socially efficient to write one. In this case the agents will end up in a situation which can be interpreted as an 'incomplete' contract in a very strong sense: no contract at all.

The intuition behind our results above is simple enough. If entering a contract involves some costs which are payable ex-ante, the share of the surplus accruing to each party will not depend, in equilibrium, on whether the ex-ante costs are paid. Therefore, the parties will pay the costs only if the distribution of the surplus generated by the contract will allow them to recoup the cost ex-post. If the distribution of surplus and that of ex-ante costs are sufficiently 'mis-matched', then one of the agents will not be able to recoup the ex-ante cost. In this case, a contract will not be drawn-up, even though it would generate a total surplus large enough to cover the ex-ante costs of both agents.

3.2. Perfect Substitutes

So far, we have assumed that the agents' ex-ante costs are 'perfect complements' in determining whether a contract is feasible or not. The next proposition tells us that when the agents' ex-ante costs are *perfect substitutes* in determining whether a contract is possible or not, our simple constrained inefficiency results of Subsection 3.1 no longer hold.[11]

[11] However, this failure of the inefficiency results in the case of perfectly substitutable ex-ante costs is a special feature of the simple model with a *discrete* choice of ex-ante costs which we have analyzed in Subsection 3.1. This is demonstrated by Remark 3 below.

	pay c_B	not pay c_B
pay c_A	$\lambda - c_A,\ 1-\lambda-c_B$	$\lambda - c_A,\ 1-\lambda$
not pay c_A	$\lambda,\ 1-\lambda-c_B$	$0,\ 0$

Figure 2: Normal form when the ex-ante costs are perfect substitutes.

The intuition behind Proposition 3 below is not difficult to outline. If it is socially efficient for the agents to draw-up a contract (Assumption 1), then it must be that for at least one of the agents the share of the surplus generated by the contract 'covers' the ex-ante cost. It then follows that, if one agent alone paying the ex-ante cost is sufficient to enable the parties to draw-up a contract, there always will be an equilibrium of the two stage game in which a contract is drawn-up, whenever it is socially efficient to do so.

The normal form of the two-stage game when one agent alone paying the ex-ante cost is sufficient to make the contract feasible is depicted in Figure 2. Given the pay-offs in Figure 2 Proposition 3 below is immediate, and therefore it is stated without proof.

PROPOSITION 3: *Consider the reduced form model with ex-ante costs which are perfect substitutes, so that a contract is feasible whenever at least one of the agents has paid the ex-ante cost. Suppose that it is socially efficient for the agents to draw-up a contract (Assumption 1 holds). Then in any (pure strategy) equilibrium of the game depicted in Figure 2 one agent pays the ex-ante cost, and hence a contract is drawn-up between the agents.*

3.3. Partial Substitutes

The case of ex-ante costs which are perfect substitutes turns out to be a rather special one. This will be immediately apparent when we turn to the analysis of a model with continuous ex-ante contractual costs in Section 4. However, this is also the case in a simple modification of our model with discrete ex-ante contractual costs.

The easiest way to model the possibility of partial substitution across the ex-ante costs paid by the two agents is to assume that the costs are 'partially transferable' between A and B. We modify the model as follows. At $t = 0$ each agent $i \in \{A, B\}$ chooses a number

$c_i \in [0, \overline{c}_i]$. Since $\overline{c}_A$ and $\overline{c}_B$ are the maximum ex-ante costs which each agent can decide to pay, the analogue of Assumption 1 now reads $\overline{c}_A + \overline{c}_B < 1$.

We also let $\underline{c}_i$ be an arbitrarily given number in the interval $(0, \overline{c}_i)$ for $i \in \{A, B\}$. The number $\underline{c}_i$ represents the minimum ex-ante cost which agent i must pay for the contract yielding one unit of surplus to become feasible.

Finally, we assume that a contract is possible only if another condition is satisfied as well, namely that the *sum* of the ex-ante costs paid by the two agents is at least equal to $\overline{c}$, with $\overline{c}$ an arbitrarily given number in the interval $(\underline{c}_A + \underline{c}_B, \overline{c}_A + \overline{c}_B]$. Note that the assumption of partial transferability of ex-ante costs across the two agents is embodied in the fact that we are imposing $\overline{c} > \underline{c}_A + \underline{c}_B$.

To summarize, we are now assuming that a contract between the two agents is feasible if and only if the ex-ante costs paid satisfy

$$c_i \geq \underline{c}_i \;\; \forall i \in \{A, B\} \quad \text{and} \quad c_A + c_B \geq \overline{c} \tag{1}$$

Propositions 4 and 5 below are the analogs of Propositions 1 and 2 above, when the agents' ex-ante costs are partial substitutes. We view the two of them together as implying that our previous inefficiency result is robust to the possibility of partially transferable ex-ante costs: if the distribution of *minimum* ex-ante costs does not 'match' the parties' shares of the surplus yielded by the contract-negotiating phase, the parties will not draw-up a contract, even though it would be socially efficient to do so.

Once again, we distinguish between the case in which the ex-ante costs $\underline{c}_A$, $\underline{c}_B$ are given (Proposition 4 below) and the case in which the distribution parameter λ is given (Proposition 5 below).

PROPOSITION 4: *Consider the 'reduced form' model described above, modified to allow for partially transferable ex-ante costs. Let a pair $\underline{c}_A$ and $\underline{c}_B$, and a total ex-ante cost necessary for a contingent contract, $\overline{c} \in (\underline{c}_A + \underline{c}_B, \overline{c}_A + \overline{c}_B)$, be given.*[12] *Then, there exists a range*

[12] *Recall that we are assuming that $\overline{c}_A + \overline{c}_B < 1$.*

of values $\widehat{\Lambda} = [0, \underline{c}_A) \cup (1 - \underline{c}_B, 1]$ *for the distribution parameter* λ *such that the unique equilibrium of the reduced form model with partially transferable ex-ante costs has both agents paying zero ex-ante costs, and therefore yields the no-contract outcome.*

PROPOSITION 5: *Consider the 'reduced form' model described above, modified to allow for partially transferable ex-ante costs. Let any* $\lambda \in (0,1)$ *be given. Then there exists a set* $\widehat{C} = \{\underline{c}_A, \underline{c}_B \mid \text{either } \underline{c}_A > \lambda \text{ or } \underline{c}_B > 1 - \lambda\}$ *of pairs of minimum ex-ante costs* $\underline{c}_A$ *and* $\underline{c}_B$ *such that, for any* $\overline{c} \in (\underline{c}_A + \underline{c}_B, 1]$*, the unique equilibrium of the reduced form model with partially transferable ex-ante costs involves both agents paying zero ex-ante costs, and hence yields the no contract outcome.*

3.4. Strategic Complements

A generalization of our results to the case in which the ex-ante costs are perfect substitutes is possible, provided that the model is modified so as to make them *strategic* complements. We conjecture that this is true 'in general', but of course this cannot be shown in a general result since we would need to consider formally all the extensive forms which guarantee strategic complementarity of the agents' ex-ante costs.

We limit our formal analysis to an example which is a modification of the model in which the ex-ante costs are 'technologically' perfect substitutes, but at the same time are strategic complements.

The description of our next model is as follows. At $t = 0$ both agents decide simultaneously and independently whether to pay their ex-ante costs. If both agents decide not to pay the ex-ante costs, then a contract is not feasible and both receive a pay-off of zero. If either agent $i \in \{A, B\}$ pays the ex-ante cost c_i at $t = 0$ the contract which yields one unit of surplus becomes feasible. If both pay their ex-ante costs at $t = 0$ the distribution parameter λ determines the contract which is drawn-up and the A's and B's pay-offs are $\lambda - c_A$ and $1 - \lambda - c_B$ respectively.

However, if only one agent, say A, pays the ex-ante cost at $t = 0$, he is allowed to make a take-it-or-leave-it offer ℓ to B at $t = 1$ The value of ℓ is interpreted as an offer to make A's and B's pay-offs *net* of any costs paid ℓ and $1 - \ell$ respectively. This can be thought of as a crude way to say that if only one agent pays the ex-ante cost then the bargaining power shifts dramatically in his favour.

Moreover, we assume that A, if he alone has paid the ex-ante cost, can, in principle, make some offers which would push agent B below his individual rationality constraint. In other words we assume that the take-it-or-leave-it offer ℓ must be in the interval $[-\varepsilon, 1+\xi]$ with ε and ξ some (possibly small) positive numbers.

At $t = 2$, B has two choices. He can either pay an ex-ante cost $c'_B > 0$ or pay nothing.[13] If he does not pay he does not observe A's offer, but is still allowed to accept or reject it blind. If B decides to pay his ex-ante cost, he can then observe A's offer and subsequently decide to accept or reject it.

The description of the extensive form which is played if it is B alone who pays the ex-ante cost at $t = 0$ is exactly symmetric to the case we have just described.

Notice that the strategic complementarity of the two agents' ex-ante costs is built into the extensive form game we have described precisely via the shift in bargaining power which obtains when one agent alone pays the ex-ante costs at $t = 0$.

Suppose now that the parameters λ, c_A and c_B are such that the agents would not draw-up a contract in the model described in Subsection 3.1. Our next proposition then tells us that, in the model with strategic complementarities we have just described, they will not draw-up a contract either.

PROPOSITION 6: *Consider the model with ex-ante costs which are strategic complements described above in this subsection. Assume that either $\lambda < c_A$ or $1-\lambda < c_B$. Then the unique equilibrium outcome of the model has neither agent paying the ex-ante cost at $t = 0$ and hence the no contract outcome obtains.*

4. A 'REDUCED FORM' MODEL: CONTINUOUS COSTS

So far, we have assumed that the agents' decision regarding the ex-ante costs is 'lumpy'; a contingent contract is not possible unless both agents sink a minimum, strictly positive,

[13] Notice that while Proposition 6 below restricts the values of c_A and c_B to be in an appropriate range, c'_B can take any positive (small) value.

ex-ante cost. This is the reason why Propositions 1, 2, 4 and 5 above refer to a *range* of the distribution parameter λ for any given ex-ante costs, and to a *range* of ex-ante costs given any value of the distribution parameter λ.

In this section we consider a model in which the agents have a continuous choice of ex-ante costs. Our model is still a 'reduced form' one in that we do not model explicitly the effects of increased ex-ante costs paid by the agents.[14] We simply postulate that the *size of the surplus* yielded by the contract which the agents draw-up is an increasing function of the magnitude of the ex-ante costs paid by the two agents.

The interpretation of our reduced form increasing relationship between the ex-ante costs and the surplus generated by the contract, we believe, is a natural one. We imagine a situation in which, as the agents pay larger amounts of ex-ante costs, more *detailed contracts* become feasible between them. The meaning of the word detail here can range from a more accurate description of the 'contractual variables', to a more detailed description of the possible states of nature (and therefore, in a dynamic model, to contracts with a longer time horizon), to a contract which is better specified in legal terms, which as a consequence is more easily enforced, and therefore yields a higher level of surplus 'net of enforcement costs'.

The results which we derive in this section are the analogs in our set-up of the general under-investment results stemming from a hold-up problem (Hart and Moore 1988). Formally, the model which we analyze is close to Holmström (1982), and can be described as follows.

The two agents, A and B, play a two stage game. At $t = 0$, both agents decide, simultaneously and independently, how much ex-ante contractual cost to pay. Agent $i \in \{A, B\}$ chooses a number $c_i \in [0, \overline{c}_i]$ with $\overline{c}_i \in (0, 1)$.[15] At $t = 1$ the agents do not in fact have any choices to make; the pair of ex-ante costs (c_A, c_B) paid at $t = 0$, determines the size

[14] As we mentioned above, in Section 8 we consider a model in which the agents have a 'gradual' choice of ex-ante costs, and the effects of increased ex-ante costs paid is modelled explicitly as affording the agents a more detailed contracts which extends further into the future.

[15] Recall that we are assuming throughout that the agents pay their ex-ante costs out of unit endowments which are available to them at $t = 0$. The constraint that c_i should be in $[0, \overline{c}_i]$ with $\overline{c}_i \in (0, 1)$ is simply designed to ensure that any choice of ex-ante cost is 'affordable' for both agents.

of the surplus that the contract yields to the agents, which is then divided among them according to the exogenously given distribution parameter λ. The surplus corresponding to the pair (c_A, c_B) is denoted by $x(c_A, c_B)$, with $x_A(c_A, c_B)$ and $x_B(c_A, c_B)$ its partial derivatives with respect to the first and second argument respectively. We assume that x is a (differentiable) strictly increasing and concave function[16] which satisfies the Inada conditions $\lim_{c_A \to 0} x_A(c_A, c_B) = \infty$, $\lim_{c_B \to 0} x_B(c_A, c_B) = \infty$, $x_A(c_A, c_B) = 0$ for all $c_A \geq \overline{c}_A$ and $x_B(c_A, c_B) = 0$ for all $c_B \geq \overline{c}_B$.

Given a pair of ex-ante costs (c_A, c_B), the pay-offs accruing to A and B are given by $\lambda x(c_A, c_B) - c_A$ and $(1-\lambda)x(c_A, c_B) - c_B$ respectively. We denote by c_A^* and c_B^* the (unique) equilibrium ex-ante costs which the agents pay in the game we have just described. Given that our assumptions on the function x guarantee an interior solution, the equilibrium is easy to characterize.

REMARK 1: *The model with continuous ex-ante costs we have described above yields a unique pair of equilibrium ex-ante costs (c_A^*, c_B^*), which can be characterized as follows by the corresponding first order conditions.*

$$x_A(c_A^*, c_B^*) = \frac{1}{\lambda} \quad \text{and} \quad x_B(c_A^*, c_B^*) = \frac{1}{1-\lambda} \tag{2}$$

The efficiency benchmark with which to compare the equilibrium identified in Remark 1 is straightforward to define and to characterize.

DEFINITION 1: *The socially efficient levels of ex-ante costs in the model described above are denoted by c_A^E and c_B^E, and are defined as the pair of ex-ante costs which maximize the difference between the surplus given by the contract and the sum of ex-ante costs $x(c_A, c_B) - c_A - c_B$.*

REMARK 2: *In the model with continuous choice of ex-ante costs described above there is a unique socially efficient pair of ex-ante costs (c_A^E, c_B^E), which can be characterized as*

[16]Therefore, we are assuming that there are decreasing returns to scale in the relationship between the hex-ante costs paid and the size of the surplus which the contract generates.

follows using the first order condition for maximization of the surplus from the contract net of ex-ante costs.

$$x_A(c_A^E, c_B^E) = 1 \quad \text{and} \quad x_B(c_A^E, c_B^E) = 1 \tag{3}$$

Using the concavity of x, it is immediate to see that (2) together with (3) imply that $c_i^* < c_i^E$ for all $i \in \{A, B\}$. Therefore, we have already proved the following proposition.[17]

PROPOSITION 7: *Let any value of the distribution parameter $\lambda \in [0,1]$ be given. In equilibrium, in the model with a continuous choice of ex-ante costs the agents pay an inefficiently low level of ex-ante contractual costs in the sense that $c_i^* < c_i^E$ for all $i \in \{A, B\}$. Therefore, the equilibrium of this model involves the agents drawing-up a contract which is incomplete in the sense that it is less 'detailed' than would be socially efficient, even after contractual costs are taken into account.*

The intuitive reason why the agents under-invest in their ex-ante costs according to Proposition 7 is simple to outline. Each party's share of the surplus generated is fixed, and total surplus is an increasing concave function of both ex-ante costs. Each agent invests in his ex-ante costs only up to the point at which *his own* marginal net return is zero. Such point is therefore below the point at which the marginal social (across both agents) net return on his investment in the ex-ante cost is equated to zero.

We conclude this section with two observations. The first is that Proposition 7 is a stronger inefficiency result than Propositions 1 and 2 above since it yields an inefficiently low level of investment in the ex-ante costs *regardless* of the value of λ. Notice that our model of Subsection 3.1 can be viewed as a 'special case' (in which the assumption of concavity of x is violated) of our model with a continuous choice of ex-ante costs in which the size of the surplus yielded by the contract is a *discontinuous* function of the ex-ante costs

[17]Notice that in Proposition 7 below we allow for λ to take the values 0 and 1 as well as all the values in the interval $[0,1]$. For $\lambda = 0$ and $\lambda = 1$, Proposition 7 below is trivially true since for $\lambda = 0$ A will choose to set $c_A = 0$, and for $\lambda = 1$ B will choose to set $c_B = 0$.

paid by the two agents. Intuitively, the 'marginal' conditions for efficiency are therefore easier to satisfy in our model of Subsection 3.1 than in our present set-up.

Our second observation concerns the case of ex-ante costs which are perfect substitutes. Recall that Proposition 3 above says that in our model with discrete ex-ante costs, the agents will always draw-up a contract whenever it is socially efficient to do so if their ex-ante costs are perfect substitutes. This is no longer true in our present set-up: in this sense Proposition 3 is misleadingly strong in the way it amplifies the effects of substitutability of ex-ante costs. We deal formally with the case of ex-ante costs which are perfect substitutes in our model with continuous ex-ante costs in the following remark.

REMARK 3: *Consider our model with a continuous choice of ex-ante costs described above and let any* $\lambda \in (0,1)$ *be given. Suppose that the two agents' ex-ante costs are perfect substitutes in their effect on the size of the surplus generated by the contract. Formally, assume that there exists some strictly increasing and concave function* $f : \Re_+ \to \Re_+$ *such that* $x(c_A, c_B) = f(c_A + c_B)$ *for all pairs* (c_A, c_B). *Then Proposition 7 still applies since all its hypotheses are satisfied. Therefore in this case as well, in equilibrium, the agents will both pay an inefficiently low level of ex-ante costs.*

5. NEGOTIATION OF THE DISTRIBUTION PARAMETER

5.1. Preamble

In our analysis so far, we have assumed that the value of the distribution parameter λ is exogenously given and cannot be changed by the agents.

Notice that since Proposition 7 tells us that the agents will pay inefficiently low levels of ex-ante costs *whatever* the valued of the distribution parameter, this is not a reason for concern in the case of continuous ex-ante costs. Whatever the extensive form which decides the value of the distribution parameter, given the equilibrium value of λ, the agents will pay inefficiently low levels of ex-ante costs.

In this section, we relax the assumption of a given value of λ for the case of discrete ex-ante costs. The hold-up problem which we have identified in Section 3 turns out to be 'pervasive' even when the agents, negotiate the value of the distribution parameter λ.

5.2. Discrete Costs

Propositions 1 and 2 of Section 3 can be paraphrased as saying that, for any given pair c_A and c_B there exists a range of values of λ (Proposition 1), and for any value of λ there exists a range of values of c_A and c_B (Proposition 2), such that the agents will not draw-up a contract, even though it would be socially efficient to do so.

As we mentioned above, it is natural to think of the value of λ as determined by an extensive form game in which the agents negotiate the distribution of surplus if a contract is drawn-up. Therefore, another way to paraphrase Propositions 1 and 2 is as follows.

Proposition 1 tells us that, for any given pair c_A and c_B, there exist a 'set of extensive forms' such that the agents will not draw-up a contract if the distribution parameter is determined as an equilibrium of the given extensive form, even though it would be socially efficient to do so.[18]

Proposition 2, on the other hand, says that, for any given extensive form which determines the value of the distribution parameter λ, there exists a pair of ex-ante costs c_A and c_B such that, it would be socially efficient for the agents to pay the ex-ante costs, but in equilibrium this will not be the case and a contract will not be drawn-up.

The question which we pursue in this subsection is the following. Fix *any positive values* for c_A and c_B. Is it then the case that for any extensive form in some interesting set the agents will not draw-up a contract in equilibrium? The interpretation of the word 'interesting' is of course open to disagreement, but we believe the answer to our question to be yes.

The set of extensive forms described in Cases 1 through to 7 below can be described intuitively as follows. We start with a set of extensive form bargaining games which can be described as 'canonical' in that it includes a full range of bargaining games ranging form

[18]Here and in the next paragraph we refer to 'sets of extensive forms' without making this notion precise. This is simply to save space and new notation. Intuitively, we think of the set of all extensive forms as a set of extensive form games the equilibria of which determine the distribution parameter λ and which is sufficiently rich so as to ensure that given any value of $\lambda \in [0,1]$, there exists some extensive form game in the set considered which yields precisely λ as the equilibrium value of the distribution parameter. Examples of what such extensive form games might be are given in the descriptions of Cases 1 through to 7 below.

take-it-or-leave-it offers to infinite alternating offer games à la Rubinstein (1982). We then modify these models assuming that they generate ex-ante costs for the agents as they are played. In particular, the key assumption is that a set of ex-ante costs must be payable by the agents *immediately before* they choose the actions prescribed by the extensive form (for example, before making an offer or a counter-offer, or before deciding whether to accept or reject a given offer).[19] Relying on sub-game perfection, this allows us to conclude that, given any ex-ante costs, if the distribution parameter is determined by one of the extensive form we consider the agents will not draw-up a contract.

We start by describing the set of extensive forms as Cases 1 to 7. Proposition 8 below summarizes our findings for all these extensive form games. All ex-ante costs in all the extensive forms considered below are arbitrary strictly positive numbers. In all cases we assume that the sum of all ex-ante costs (over all stages of the game and across the two agents) is strictly less than one.

We begin with the simple case in which A makes a take-it-or-leave-it offer to B.

CASE 1: At time $t = 0$ both parties decide simultaneously and independently whether to sink the ex-ante costs (c_A, c_B). If either party decides not to sink this cost then the game moves directly to period 2 and both parties receive a pay-off of zero, minus any costs paid since a contract is not feasible in this case.

If instead both agents pay the ex-ante costs at $t = 0$, the contract which yields one unit of surplus becomes feasible, and A makes an offer to B at $t = 1$. This offer specifies a value of $\lambda \in [0,1]$. Agent B then has the possibility to accept or reject such offer.

In either case the parties move to period $t = 2$. If B accepted A's offer at $t = 1$ the agents' pay-offs are $\lambda - c_A$ and $1 - \lambda - c_B$ respectively. If B rejected A's offer at $t = 1$, then the agents have not reached an agreement on how to draw-up the contract which yields one

[19]This of course does not preclude the consideration of extensive forms in which some contractual costs are paid at other points in the game as well. What matters is that *some* of the contractual costs must be payable immediately before making offers and counter-offers, and accept/reject decisions. However, to keep matters relatively simple, we abstract from all ex-ante costs which are not payable immediately before any such actions are decided.

unit of surplus. Therefore they do not enter such contract and they receive pay-offs of $-c_A$ and $-c_B$ respectively.

The next extensive form we consider is simply the symmetric case in which party B makes a take-it-or-leave-it offer to party A.

CASE 2: The extensive form pertaining to periods $t=0$, $t=1$ and $t=2$ can be described by swapping the names of agents A and B in the description of the extensive form specified in Case 1 above.

The next two extensive forms we consider are a modification of the ones described in Cases 1 and 2 above. They are designed to show that simultaneity of the agents' decisions to pay the ex-ante costs can be abandoned, provided that the party which receives the offer is unable to observe it, unless he pays his ex-ante cost. This observation generalizes to extensive forms considered in Cases 5, 6 and 7 below, although we do not provide the details for reasons of space.

CASE 3: Consider the extensive form described in Case 1 modified as follows. At $t=0$ A chooses whether to sink the ex-ante cost c_A. If A does not pay c_A at $t=0$ the surplus-generating contract is not feasible. Therefore, in this case the game ends and both agents receive a pay-off of zero. If A sinks c_A, he can then make an offer to B specifying a value $\lambda \in [0,1]$ for the distribution parameter if a contract is drawn-up. The game then moves on to period $t=1$. At $t=1$, agent B can observe whether A has decided to make an offer of λ, but he cannot observe the value of λ, unless he pays his own ex-ante cost c_B. If B does not pay c_B the surplus-generating contract is not feasible.[20] Therefore, in this case the game ends and the agents' pay-offs are $-c_A$ and zero respectively. If B decides to pay c_B at $t=1$, then he observes λ, and can subsequently decide to accept or reject A's offer. If B rejects A's offer the game ends and the agents receive pay-offs of $-c_A$ and $-c_B$ respectively.

[20]As in Subsection 3.4 above the complementarity of the ex-ante costs (c_A, c_B) is not crucial. It is enough that the two costs are strategic complements. This is the case if, for example, B can accept or reject the offer even if he has not paid his ex-ante cost provided that in this case B has to decide without seeing the offer and the offer λ can exceed 1 thus pushing agent B below his individual rationality constraint.

If, on the other hand, B accepts A's offer, the game moves on to $t = 2$, when the surplus generating contract is drawn-up. In this case the agents's pay-offs are $\lambda - c_A$ and $1 - \lambda - c_B$ respectively.

The next extensive form is the symmetric one in which B makes a take-it-or-leave-it offer to A that A can observe only by paying his ex-ante cost.

CASE 4: The extensive form for this case can be simply obtained swapping the names of A and B in the description of Case 3.

Our next case is that of an extensive form obtained from a randomization between the extensive forms described in Cases 1 and 2.

CASE 5: The two parties observe the outcome of a public randomization device — a coin toss for example — which has outcomes A with probability $\psi \in (0,1)$ and B with probability $1 - \psi$. If the outcome of the public randomizing device is A, the negotiation proceeds according to the extensive form described in Case 1 above. Conversely, if the outcome is B the negotiation proceeds according to the extensive form described in Case 2.

Notice that for Proposition 8 below to hold in Case 5 above, it is of critical importance that the parties cannot sink the ex-ante costs (c_A, c_B) *before* the outcome of the public randomization is known. This is in keeping with our discussion above in which we emphasized that the ex-ante costs must be payable *immediately before* any offers, counter-offers and decisions to accept or reject are taken by the agents.[21] Notice however, that we could imagine an *additional* tier of ex-ante costs, payable before the outcome of the randomization is known, which would be interpreted as the cost of setting up the randomization we have just described. In this case, Proposition 8, which tells us that a contract will not be drawn-up in equilibrium would still apply.

[21]Of course, if either $\psi < c_A$ or $1 - \psi < c_B$ a contract will not be drawn-up even if the ex-ante costs are payable before the outcome of the randomization is known. Our assumption that they are paid after the randomization guarantees that Proposition 8 below holds for *any* value of $\psi \in (0,1)$.

We now turn to two dynamic extensive forms for the negotiation of the distribution parameter. One involves a finite number, N, of 'alternating offers', and the other a potentially infinite number of them.[22] The two extensive forms described in Cases 6 and 7 below are designed to embody the possibility that each party has the ability to make a counter-offer in the case in which an offer of compensating transfer has been made by the other agent and rejected. For the sake of clarity, in the description of the next two cases, we divide each time period in three consecutive stages: stage I in which costs are paid, stage II in which offers are made, and stage III in which offers can be accepted or rejected. We denote by c_i^n, with $i \in \{A, B\}$, the ex-ante costs that the agents have to pay in order to make and accept/reject offers in period $t = n$.

We are now ready to describe the extensive form for the compensating transfers negotiation game with N rounds of alternating offers.[23]

CASE 6: We only deal with the case in which N is even, and agent A makes the first offer. Proposition 8 is valid for the other three cases as well, in which B makes the first offer and/or N is odd. The details of the other three cases can be obtained in the obvious way from the particular case we describe.

The game starts in period 0. In stage I of period 0 both parties decide, simultaneously and independently, whether to sink the ex-ante costs (c_A^0, c_B^0). If either agent (or both) decides not to sink this costs then the game moves to period $t = 1$. If, on the other hand, both parties pay the costs then we enter stage II of period 0. At this point party A makes an offer $\lambda^0 \in [0, 1]$ to B, specifying a value for the distribution parameter. The game then moves to stage III, when B has the possibility to accept or reject A's offer. If agent B accepts the offer, the surplus-generating contract is drawn-up and the game ends with the agents receiving pay-offs of $\lambda^0 - c_A$ and $1 - \lambda^0 - c_B$ respectively. If, instead, B rejects the offer the negotiation moves to period $t = 1$.

[22] Both the formulation and the analysis of Case 6 and 7 below are closely related to a vast literature on alternating offers models of bargaining sparked off by Rubinstein (1982) and subsequently enriched by the contribution of Shaked and Sutton (1984) among *many* others.

[23] It is easy to generalise Case 6 to allow for discounting of future payoffs. We do not provide the details for reasons of space.

The description of the game in period $t = 1$ is the same as in period 0, except that the agents' roles are exchanged. It is now B who can make an offer λ^1 to A (if the ex-ante costs have been paid by both), and then A who has the chance to accept or reject.

The following periods up to and including period $t = N-1$ are the same as the first two, with the agents making offers in turn. If the game ends with an offer being accepted at time $t = 0, \ldots, N-1$, the agents pay-offs are $\lambda^t - \gamma_A$ and $1-\lambda^t-\gamma_B$, where γ_A and γ_B are the total ex-ante costs paid by the two agents respectively. If the game ends in period $N-1$ with no offer being accepted, the agents' pay-offs are $-\gamma_A$ and $-\gamma_B$ respectively.

We can now proceed to the description of the last case we consider, in which the alternating offers negotiation of the distribution parameter may last indefinitely. Case 7 which follows is the extension to a potentially infinite number of rounds of the extensive form we have described in Case 6 above.[24]

CASE 7: We describe the game for the case in which party A makes the first offer. Proposition 8 also applies to the symmetric case in which the first offer is made by B.

The game starts in period $t = 0$. In stage I of period 0, both agents decide, simultaneously and independently, whether to sink the ex-ante costs (c_A^0, c_B^0). If either party (or both) decides not to pay this cost, the game moves directly to period $t = 1$. If both A and B pay these ex-ante costs, the game moves to stage II of period 0. Now A can make an offer to B of a value $\lambda^0 \in [0,1]$ for the distribution parameter. In stage III of period 0, B can decide to accept or reject A's offer. If B accepts A's offer, the surplus-generating contract is drawn-up and the game ends with the agents receiving a pay-off of $\lambda^0 - c_A^0$ and $1-\lambda^0-c_A^0$ respectively. However, if B rejects A's offer, the game moves on to period 1.

[24]Note that in Case 7 allowing for time discounting of pay-offs raises a problem about how the Assumption that drawing-up a contract must be socially efficient after all ex-ante costs are taken into account (cf. Assumption 1 above). If the game lasts indefinitely, the discounted value of the final surplus from the risk-sharing agreement actually goes to zero in the limit. It is then not clear how any positive ex-ante costs paid before the game ends could be 'covered' by the final surplus so as to ensure that a contract would be socially efficient ever after all ex-ante costs have been paid. On the other hand, it is also worth emphasizing that the logic of the proof of Proposition 8 for Case 7 does generalize to the case in which discounting is allowed.

The description of period 1 is essentially the same as period 0, save for the fact that the roles of the two agents are exchanged. It is now B who (provided the ex-ante costs (c_A^1, c_B^1) are paid by both agents) can make an offer λ^1 to A, who then has the chance to accept or reject.

All odd periods $3,5,7,\ldots$ are essentially the same as period 1, and all even periods $2,4,\ldots$ are essentially the same as period 0. However, recall that if at any time t an offer of λ^t is accepted, the surplus-generating contract is drawn-up and the game ends with pay-offs $\lambda^t - \gamma_A$ and $1 - \lambda^t - \gamma_B$ where γ_A and γ_B are the total ex-ante costs paid during the entire game by A and B respectively.

To complete the description of Case 7, we stipulate that if the agents never reach an agreement on a value for the distribution parameter, and therefore the game does not terminate in finite time, then the agents receive pay-offs of $-\gamma_A$ and $-\gamma_B$ respectively.

We are now ready to state our next proposition. It states that, whenever the negotiation of the distribution parameter is carried out according to one of the extensive forms we have described, the two parties face a hold-up problem which will prevent the agents from drawing-up the surplus-generating contract, even though doing so would be socially efficient after all ex-ante costs are taken into account.

PROPOSITION 8: *Consider any of the extensive forms described in Cases 1 through 7 above. Assume that all 'tiers' of ex-ante costs are strictly positive. Then the unique equilibrium outcome of the model involves both agents not paying any of the ex-ante costs, and hence yields the no contract outcome.*

We conclude this section with two observations. First, notice that the intuition behind the proof of Proposition 8 for all the cases considered is very much the same. After the ex-ante costs have been paid, they are *sunk*. Any offer which comes after these costs have been paid will therefore not take these costs into account, and will leave one of the two agents with a 'deficit' which will not be covered in future stages of the game. Thus, the ex-ante costs will not be paid, and the no contract outcome will obtain.

Secondly, the hold-up problem identified in Proposition 8 is in a sense more acute than the one described in Section 3 above. In fact, whenever the negotiation of the sharing of the surplus is disciplined by one of the extensive forms described in Cases 1 through 7 above, the contract will never be drawn-up whatever the ex-ante costs, provided they are positive.

6. CONTRACTS OVER CONTRACTS, ... OVER CONTRACTS

6.1. Simple Ex-Ante Compensating Transfers

In Section 3 we have argued that ex-ante contractual costs may give rise to a version of the hold-up problem which in turn generates an inefficient (no contract) outcome. As we mentioned in the Introduction, in all the previous literature of which we are aware (Grossman and Hart 1986, Hart and Moore 1988, Hart and Moore 1990, among others) the *reason* a hold-up problem might arise in the first place is that the parties are *constrained* in their ability to write contracts: given that certain variables are not negotiable ex-ante (or that only limited ex-ante negotiation is feasible because of the constraints imposed by the possibility of renegotiation ex-post (Hart and Moore 1988)) the parties' 'relationship specific' investment(s) will be inefficiently low.

In a model with 'relationship specific' investments and incomplete contracting, the hold-up problem typically has a 'contractual solution'. If either the assumption that the parties are constrained to write incomplete contracts is removed (for example by increasing the information which the enforcing agency can verify as in Nöldeke and Schmidt (1995)), or if a contracting stage is added to the model in which the parties can write a 'grand' ex-ante contract in which either the amounts of relationship specific investment are specified or alternatively reported by the parties to the enforcing agency as in Maskin and Tirole (1995), or finally if the parties can commit to a given renegotiation procedure as in Aghion, Dewatripont, and Rey (1994), then the hold-up problem is resolved, and an efficient outcome is guaranteed.

The next natural question to ask is then whether a contractual solution to the hold-up problem is generally available in the present set up. In other words: is it possible to add another stage to our model (say $t = -1$), prior to the stage in which the ex-ante costs are incurred, in which the agents can negotiate a 'grand contract', which will resolve the hold-up problem and hence restore efficiency?

The answer to the above question is trivially 'yes', if at $t = -1$ a *truly grand* contract can be negotiated *costlessly*, which specifies everything, including the payment of the ex-ante costs, *and* the division of the actual surplus at time $t = 1$. The answer, however, changes dramatically if the 'grand contract' is itself costly.

We specify two crucial details of the grand contract stage. First of all we assume, as seems plausible in the present context, that in order to be able to negotiate a contract at $t = -1$ a fresh set of ex-ante costs must be incurred by the parties before $t = -1$, say at $t = -2$. Secondly, we restrict the agents to negotiate a *compensating transfer* at $t = -1$. In other words, we take a specific view on the agreements which the agents can enter at $t = -1$. Indeed, we restrict them to be transfers *contingent* on the payment of ex-ante costs at $t = 0$. This seems to be in the spirit of our reduced form model of Section 3, in that, in principle, it allows the agents to effectively transfer surplus between them, but it keeps the distribution of surplus in the last stage of the contracting process, $t = 1$, exogenously fixed, as before.[25]

It is worth emphasizing at this point that we find that the presence of any *strictly positive* 'second tier' ex-ante costs is sufficient to keep the addition of a grand contract stage from resolving the hold-up problem of Section 3. We view this as a strength of the results we present in this section. Indeed, in many situations it would be sensible to assume that the second tier ex-ante costs are in fact at least as large as the 'first tier' ex-ante costs, on the grounds that a 'contract over a contract', in an intuitive sense, is a more complex object than the contract itself.

Formally, we modify the model of Section 3 as follows. There are now four time periods, $t \in \{-2, -1, 0, 1\}$. The sequence of decisions and events for the two agents (depicted schematically in Figure 3) is as follows. In period $t = -2$, the two agents decide simul-

[25]While the assumption that a fresh set of ex-ante costs arises at $t = -2$ is crucial for our result (Proposition 9), we conjecture that the restriction to the negotiation of *compensating transfers* is not. The parties could, for instance, negotiate which extensive form to use in the following stage of the game. This could be 'payoff equivalent' to a compensating transfer of the type we analyze here.

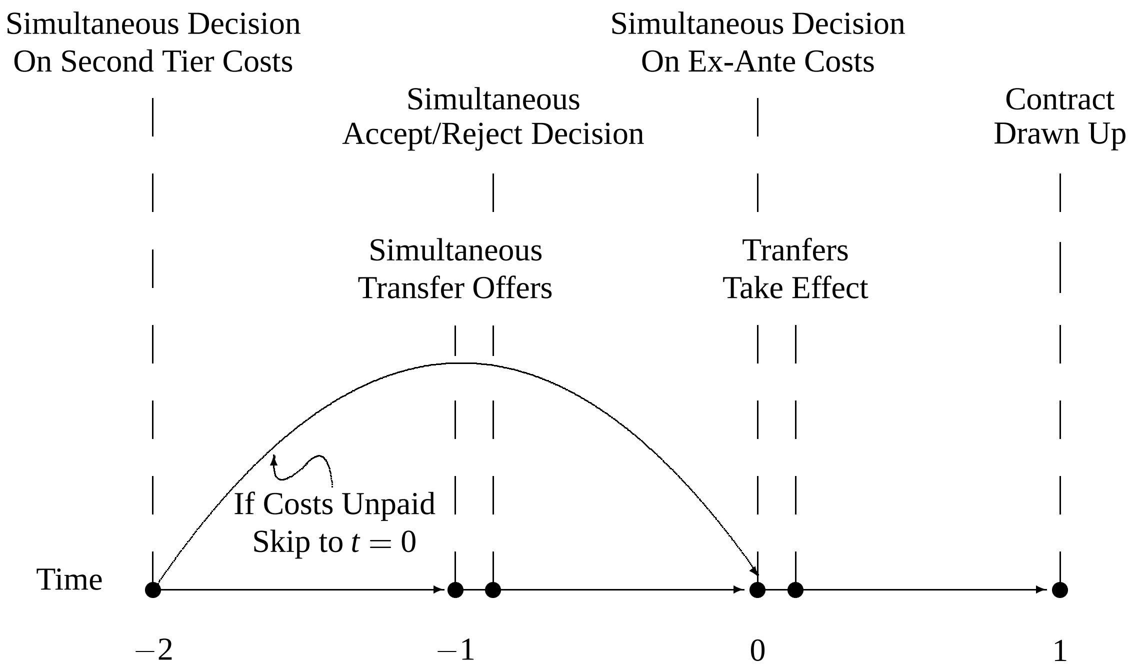

Figure 3: Timing in the two tier contracting model.

taneously whether to pay the second tier ex-ante costs (c_A^2, c_B^2). If either or both agents decide not to pay these ex-ante costs, the period $t = -1$ compensating transfers to be described shortly are automatically set equal to 0, and the agents effectively move directly to time $t = 0$. If, on the other hand both agents pay the second tier ex-ante costs, then period $t = -1$ compensating transfers can be negotiated.

For simplicity, we assume that (provided that both agents pay the second tier costs) at $t = -1$, both agents make simultaneous offers of contingent compensating transfers to each other.[26] Formally, each agent $i \in \{A, B\}$ chooses a real number $\sigma_i \geq 0$, which is interpreted as a commitment to transfer the amount of wealth σ_i to the other agent, $j \neq i$, if and only if agent j pays the first tier ex-ante cost c_j^0 in period $t = 0$. Immediately after choosing σ_i, still in period $t = -1$, A and B simultaneously choose whether to accept or reject the

[26]In Anderlini and Felli (1996a) we also explore a variety of different extensive forms for the negotiation of compensating transfers. In essence, our results of this subsection generalize to the analogs of the extensive forms described in Cases 1 to 7 of Section 5 above, modified so that each offer of a value for the distribution parameter is changed to be an offer of compensating transfer.

other agent's compensating transfer offer. Those offers which are accepted at this stage are binding in period $t = 0$.

The decisions and events in periods $t = 0$ and $t = 1$ are analogous to those described in Subsection 3.1. At $t = 0$, both agents choose simultaneously and independently whether to pay the first tier ex-ante costs (c_A^0, c_B^0). Each agent $i \in \{A, B\}$ then incurs an ex-ante cost of c_i^0 at this time, and subsequently receives a compensating transfer of σ_j from agent $j \neq i$. Only if both agents have paid the first tier ex-ante costs the $t = 1$ surplus-generating contract becomes possible.

Provided both agents have paid their first tier ex-ante costs their pay-offs are $\lambda - \gamma_A$ and $1 - \lambda - \gamma_B$ respectively, where γ_i denotes the total ex-ante costs paid by agent $i \in \{A, B\}$ during the entire game, minus any compensating transfer received from agent $j \neq i$, and plus any compensating transfers paid by i to j. If the surplus-generating contract is not drawn-up, the the two agents pay-offs are simply $-\gamma_A$ and $-\gamma_B$ respectively.

The assumption that the total (for both tiers) of ex-ante costs must be low enough so that it is socially efficient for the parties to enter a contingent contract is easy to state for this version of our model.

ASSUMPTION 2: *Let* $c_i = c_i^2 + c_i^0$ *for* $i \in \{A, B\}$. *Then* $c_A + c_B < 1$.

It is apparent from the description of our reduced form model with simple compensating transfers above (cf. Figure 3) that our model in this subsection, viewed from $t = 0$, is in fact identical to the simple reduced form model of Subsection 3.1, whenever both agents have chosen not to pay the second tier ex-ante costs. We can therefore ask whether the parameters of our model with compensating transfers are such that either Proposition 1 or Proposition 2 guarantee that, in the absence of compensating transfers, the no contract outcome is the unique equilibrium of the model. This motivates our next definition.

DEFINITION 2: *Assume that either* $c_A^0 < \lambda$ *or* $c_B^0 < 1 - \lambda$ *so that, provided that neither agent has paid the second tier ex-ante cost then the only equilibrium outcome of the model is the no contract outcome (see Propositions 1 and 2 above). Then we say that the reduced* *form model with simple compensating transfers 'yields the no contract outcome in the final stage'.*

We are now ready to state our next proposition. It tells us that, if the parameters of the reduced form model of Subsection 3.1 yield the no contract outcome, then adding a new stage to the model, with a second tier of positive ex-ante costs and compensating transfers will not solve the hold-up problem generated by the first tier ex-ante costs — the unique equilibrium outcome of the reduced form model with compensating transfers is still the no contract outcome.

PROPOSITION 9: *Consider the reduced form model with simple compensating transfers. Suppose that c_A^0, c_B^0 and λ yield the no contract outcome in the final stage (cf. Definition 2), and assume that the second tier ex-ante costs are strictly positive for both agents ($c_i^2 > 0$ for $i \in \{A, B\}$). Then the unique equilibrium outcome of the model involves both agents paying neither the second nor the first tier ex-ante costs, and hence yields the no contract outcome.*

Given Proposition 9 it is clear how the analogs of Propositions 1 and 2 hold for the reduced form model with simple compensating transfers. We state them without proof as they are immediate corollaries of Propositions 1, 2 and 9.

COROLLARY 1: *Consider the reduced form model with simple compensating transfers and assume that $c_i^2 > 0$ for $i \in \{A, B\}$. Then there exist a range of values Λ for the distribution parameter λ such that, whenever $\lambda \in \Lambda$, then the unique equilibrium outcome of the model involves both agents paying neither the second nor the first tier ex-ante costs, and hence yields the no contract outcome.*

COROLLARY 2: *Consider the reduced form model with simple compensating transfers. Let any strictly positive second tier ex-ante costs and any value for the distribution parameter λ be given. Assume that $c_A^2 + c_B^2 < 1$. Then there exists a range $\mathcal{C}$ of pairs of first tier ex-ante costs c_A^0 and c_B^0 such that $c_A = c_A^2 + c_A^0$ and $c_B = c_B^2 + c_B^0$ satisfy Assumption 2, and such that, whenever $(c_A^0, c_B^0) \in \mathcal{C}$ the unique equilibrium outcome of the model involves* *both agents paying neither the second nor the first tier ex-ante costs, and hence yields the no contract outcome.*

The results of this subsection are straightforward to summarize. Adding a new stage to the model of Section 3, in which the agents can agree to compensating transfers, will not solve the hold-up problem identified there, provided that the second tier contract has positive ex-ante costs for both agents. We view this as meaning that a 'contractual solution' to the hold-up problem of Section 3 is not available in this model.

The intuition which drives our results in this subsection is easy to outline. The hold-up problem identified in Section 3 will prevent the agents from entering into a contract unless some compensating transfers can be agreed. However, after the second tier ex-ante costs, necessary for the feasibility of an agreement on compensating transfers, have been paid, they are *sunk*. This means that, in equilibrium, it is impossible that the compensating transfers will take into account the second tier ex-ante costs. Therefore, one of the two agents will find it profitable not to pay the second tier ex-ante cost for which he would not possibly be compensated. This, in turn, means that compensating transfers will not be observed in equilibrium, and therefore yields the no contract outcome.

6.2. A Hierarchy of Contracts Over Contracts

Given the intuition behind our results of Subsection 6.1, it is natural to ask whether they generalize to a model which includes a whole hierarchy of 'contracts over contracts'. In this subsection, we examine this question in two separate, but essentially 'nested', models, and find that our results of Subsection 6.1 generalize in both cases. We begin with the simpler of the two models.

Consider adding N time periods prior to period 0 to the model of Section 3, rather than 2 as we did in Subsection 6.1. There are now $N+2$ time periods $t \in \{-N, -N+1, \ldots, 0, 1\}$.

For ease of exposition, we divide again each period from $-N$ to -1 into three consecutive *stages*, called I, II and III respectively. In stage I of period $t = -N$ both agents decide whether to pay the $(N+1)$-th tier ex-ante costs. Formally, each agent i decides whether to pay the cost c_i^N, where c_i^N is a given, strictly positive constant. If one or both agents do not pay c_i^N, the agents effectively move directly to period $-N+1$, skipping stages *II* and *III* of period $-N$. Only if both agents have paid the $N+1$-th tier ex-ante costs, in stage *II* of period $-N$, they can make compensating transfer offers (σ_A^N, σ_B^N). In stage *III* of period $-N$, exactly as in Subsection 6.1 each agent can then accept or reject the other's offer. Those offers which are accepted in stage *III* of period $-N$ become binding.

The agents then move to period $-N+1$, and must decide whether to pay the N-th tier ex-ante costs. In stage *I* of period $-N+1$, each agent decides whether to pay the cost c_i^{N-1}, where c_i^{N-1} is a given, positive constant. Immediately after, still in stage *I* of period $-N+1$, each agent i receives a compensating transfer of σ_j^N, if such transfer was agreed on at time $t=-N$. Only if both agents pay c_i^{N-1}, in stages *II* and *III* of period $-N+1$ a fresh set of compensating transfers can be offered and accepted or rejected respectively. Otherwise, the agents move directly to period $-N+2$.

The same structure of moves is then repeated up to period $t=-1$. In periods $t=0$ and $t=1$, the model is identical to the one in Subsection 6.1 (cf. Figure 3). Below, we refer to the model we have just described as the 'reduced form model with N tiers of simple compensating transfers'.

The second model we consider in this subsection is a generalization of the reduced form model with N tiers of simple compensating transfers which we have just described. The structure of time periods, stages, endowments and preferences is unchanged. We assume that, provided the ex-ante costs have been paid by both agents in stage *II* of period $t=-n$, then each agent i can make compensating transfer offers to $j \neq i$ for *all subsequent* tiers $(m=1,\ldots,n)$ of ex-ante costs; all offers are made at the same time and the two agents make offers simultaneously in stage *II* of period $t=-n$. In stage *III* of period $-n$, the agents simultaneously decide which offers to accept and which ones to reject. Those offers which are accepted then become binding. We denote the offer of compensating transfer which $i \in \{A,B\}$ makes at time t relative to the m-th tier of ex-ante costs by $\sigma_i^{t,m}$. Whenever i actually pays a tier of ex-ante costs, he receives, immediately after the payment, the *sum* of the compensating transfer offers previously agreed for that tier of ex-ante costs. Below, we refer to the model we have just described as the 'reduced form model with N tiers of multiple compensating transfers'.

Let Let $c_i = \sum_{n=0}^{N} c_i^n$ for $i \in \{A, B\}$. The equivalent of Assumption 2, stipulating that it is socially efficient for the parties to enter into a contingent contract even when all tiers of ex-ante costs are payable, is also easy to state for the two models we have just described.

ASSUMPTION 3: *The total ex-ante costs c_A and c_B satisfy $c_A + c_B < 1$.*

The purpose of exploring the two models with N tiers of compensating transfers described above is to show that the possibility of such transfers will not resolve the hold-up problem of Section 3, if the parameters of the model are such that a hold-up problem in fact exists. Therefore, as in Subsection 6.1, we assume that c_A^0, c_B^0 and λ are such that the model 'yields the no contract outcome in the final stage' (cf. Definition 2).

The following two propositions tell us that adding N tiers of compensating transfers to the model of Section 3 does not solve the hold-up problem identified there.

PROPOSITION 10: *Consider the reduced form model with N tiers of simple or multiple compensating transfers. Suppose that c_A^0, c_B^0 and λ are such that this model yields the no contract outcome in the final stage, and assume that all tiers of ex-ante costs are strictly positive for both agents ($c_i^n > 0$ for $i \in \{A, B\}$ and $n = 0, \ldots, N$). Then the unique equilibrium outcome of the model involves both agents not paying any of the ex-ante costs, and hence yields the no contract outcome.*

Exactly as in Subsection 6.1 Propositions 1 and 2 together with Proposition 10, give us two immediate corollaries. We state them without proof.

COROLLARY 3: *Consider the reduced form model with N tiers of simple or multiple compensating transfers. Let $c_A = \sum_{n=0}^{N} c_A^n$, $c_B = \sum_{n=0}^{N} c_B^n$ satisfying Assumption 3 be given. Assume further that $c_i^n > 0$ for $i \in \{A, B\}$ and $n = 0, \ldots, N$. Then there exist a range Λ of values for the distribution parameter λ such that, whenever $\lambda \in \Lambda$ the unique equilibrium outcome of the model involves both agents not paying any tier of ex-ante costs, and hence yields the no contract outcome.*

COROLLARY 4: *Consider the reduced form model with N tiers of simple or multiple compensating transfers. Let any $\lambda \in (0,1)$, and any array of strictly positive ex-ante costs for tiers $2,\ldots,N$ satisfying $\sum_{n=1}^{N}(c_A^n + c_B^n) < 1$ be given. Then there exists a range C of pairs of first tier ex-ante costs such that whenever $(c_A^0, c_B^0) \in C$, c_A and c_B satisfy Assumption 3 and the unique equilibrium outcome of the model involves both agents not paying any tier of ex-ante costs, and hence yields the no contract outcome.*

The results of this subsection are a generalization of the findings of Subsection 6.1. Adding N tiers of possible compensating transfers still does not resolve the hold-up problem identified in Section 3, if each tier of compensating transfers carries a positive ex-ante cost for each agent. The reason is, again, that once a tier of ex-ante costs are paid these costs are *sunk*. It is therefore impossible that in equilibrium the compensating transfers will take into account the previous set of ex-ante costs. This, in turn, means that the ex-ante costs will not be paid and therefore yields the no contract outcome.

7. A PRINCIPAL-AGENT MODEL WITH INCOMPLETE CONTRACTS

7.1. Preamble

In this section we analyze a simple principal-agent problem in which the two contracting parties' shares of the surplus depend on actual economic parameters. Once ex-ante contractual costs are added to this model, we obtain an example of an environment in which the basic hold-up problem that we have identified may or may not obtain, as a function of these economic parameters.

The interplay of the agent's limited liability and of the incentive compatibility constraint which the equilibrium contract needs to satisfy, endogenously determines the distribution of surplus between the principal and the agent. The key economic parameter in the analysis is the cost of the agent's effort.

Before we proceed it is worth emphasizing one particular way to relate the models of this and of the next Section to our analysis so far. In Section 3 we kept the distribution of surplus exogenously fixed. In Section 5 we allowed the agents to negotiate the distribution of surplus via a variety of extensive forms. In either case the *size* of the surplus was given, and therefore *independent* of its distribution across the two agents. In the models of this and of the next Section, the extensive form through which the contract is negotiated is the standard one for principal-agent models, namely a take-it-or-leave-it offer by the principal to the agent. The reason why the distribution of surplus now varies with the relevant economic parameters is intuitively simple. Via the incentive compatibility and the limited lability constraints, we are introducing a new determinant in the size of the surplus which the contract generates: its distribution between the two agents. Because of the incentive compatibility and limited liability constraints, the surplus from the contract will shrink to zero unless the principal releases some of it to the agent. Therefore, even though the principal has the ability to make a take-it-or-leave-it offer to the agent, in equilibrium he might find it in his interest to give positive surplus to the agent.

7.2. A Simple Principal-Agent Model

Consider a principal who hires an agent to run a very simple stochastic technology. The agent when hired may decide to exert a productive effort e that may take one of the two values 0 and $\phi > 0$. We assume that the high level of effort $e = \phi$ entails a cost to the agent of size ϕ, while the low level of effort $e = 0$ entails no cost.[27]

If the agent chooses effort $e = 0$ then output, y, is equal to 1 with probability p_0, and $y = 0$ with probability $1 - p_0$. On the other hand, if the agent exerts effort $e = \phi$, then $y = 1$ with probability p_1, and $y = 0$ with probability $1 - p_1$. We assume $p_1 > p_0$.

The value of e is private information of the agent, while the amount of output y is verifiable in court. Therefore an employment contract for the agent specifies a pair of wages (w_0, w_1) depending on whether output is high $y = 1$ or low $y = 0$.

Finally, we assume that the agent has a reservation wage $\bar{w}$ normalized to be 0, and that any contract offered to the agent needs to satisfy a *limited liability* constraint specifying that the agent cannot be paid a negative amount $w_h \geq 0$, $h = 0, 1$.

Both the principal and the agent are risk neutral, and the negotiation of the employment contract takes the following form. The principal makes an offer of a contract to the agent

[27] In what follows we let party B act as the principal and party A as the agent.

that the agent may accept or reject. In the event of acceptance the contract is enforced. In the event of a rejection the agent gets its reservation wage while the principal obtains a payoff of 0.

7.3. *The Optimal Contract*

In the contractual environment described above, *B*'s problem consists of choosing an employment contract (w_0, w_1) for *A* that maximizes his expected profit. This contract will depend on whether *B* finds it profitable to offer a contract that induces *A* to choose a high level of effort or a low one.

Assume for the time being that *B* wants to induce *A* to choose the effort level $e = 1$. In this case *B*'s decision problem takes the following form:

$$\max_{w_1, w_0} \quad p_1(1 - w_1) - (1 - p_1)w_0 \tag{4}$$

$$\text{s.t.} \quad p_1 w_1 + (1 - p_1)w_0 - \phi \geq 0 \tag{5}$$

$$p_1 w_1 + (1 - p_1)w_0 - \phi \geq p_0 w_1 + (1 - p_0)w_0 \tag{6}$$

$$w_0 \geq 0, \quad w_1 \geq 0 \tag{7}$$

In other words, the optimal contract is the one that maximizes *B*'s expected payoff and yields *A* at least his reservation wage, (5), induces *A*'s to choose $e = \phi$, (6), and satisfies the limited liability constraint (7).

The solution to problem (4), (w_0^*, w_1^*), is simple to characterize. It is in fact clear that $w_0^* = 0$ while the wage level associated with $y = 1$ can be obtained from (6): $w_1^* = \phi/(p_1 - p_0)$.

Notice that when *B* wants to induce a high level of effort, not all three constraints (5), (6) and (7) are binding. In particular in equilibrium, for any positive value of ϕ, constraint (5) is *not* binding. In other words, given the limited liability constraint (7), in order to meet the incentive compatibility constraint (6) the principal has to reward the agent with a level of expected utility that strictly exceeds the agent's reservation level. In particular this implies that the agent strictly prefers to accept the contract if no ex-ante cost needs to be paid.

The equilibrium payoffs associated with contract (w_1^*, w_0^*) are:

$$\Pi_A^* = \frac{p_0 \phi}{p_1 - p_0} \tag{8}$$

for the agent, and

$$\Pi_B^* = p_1 \left[1 - \frac{\phi}{p_1 - p_0}\right] \tag{9}$$

for for the principal, while the total surplus can easily be computed as

$$\Pi^* = \Pi_A^* + \Pi_B^* = p_1 - \phi$$

Consider now the case in which B decides to induce a low level of effort. The principal's problem is now the following

$$\begin{aligned} \max_{w_1, w_0} \quad & p_0(1 - w_1) - (1 - p_0)w_0 \\ \text{s.t.} \quad & p_0 w_1 + (1 - p_0)w_0 \geq 0 \\ & w_0 \geq 0, \quad w_1 \geq 0 \end{aligned} \tag{10}$$

It is immediate to see that the solution, (w_0^{**}, w_1^{**}), to problem (10) satisfies $w_1^{**} = w_0^{**} = 0$, which of course does not satisfy constraint (6). The payoffs to the parties in this case are $\Pi_A^{**} = 0$ and $\Pi_B^{**} = p_0$, while the total surplus is $\Pi^{**} = p_0$.

The equilibrium contract is now obtained from the comparison of Π_B^* and Π_B^{**} — B's payoff when the contract offered induces a high level of effort, (w_0^*, w_1^*), and B's payoff when the contract offered induces a low level of effort (w_0^{**}, w_1^{**}), respectively. In general, both outcomes are possible. The following observations are in order.

First, it is *socially* optimal to induce the agent to exert high effort whenever $\Pi^* \geq \Pi^{**}$. This occurs if and only if

$$\phi \leq p_1 - p_0 \tag{11}$$

Second, B will strictly prefer to induce high effort whenever $\Pi_B^* \geq \Pi_B^{**}$. This occurs if and only if the following condition is satisfied.[28]

$$\phi \leq \frac{(p_1 - p_0)^2}{p_1} \tag{12}$$

Notice next that the contract which induces high effort differs from the one which induces low effort in one crucial respect for our purposes. The contract (w_0^*, w_1^*) is contingent on the value of output y, while (w_0^{**}, w_1^{**}) is not.

As an application of our general model of ex-ante contractual costs, we now proceed to assume that the contract (w_0^*, w_1^*) is only feasible if both parties, simultaneously and independently, decide to sink the ex-ante costs c_B and c_A respectively. If either party decides not to pay this cost, only contracts which do not depend on output are feasible. Clearly, in this case the contract between B and A will simply be (w_0^{**}, w_1^{**}).

In what follows we restrict ourselves to values of the parameter ϕ such that it is both socially efficient and in the interest of the principal to induce the agent to exert a high level of effort. From (11) and (12), we know that this occurs whenever

$$0 \leq \phi \leq \frac{(p_1 - p_0)^2}{p_1} \tag{13}$$

Notice that for any ϕ satisfying condition (13) the total surplus will exceed p_0 by at least $p_0(p_1 - p_0)/p_1$:

$$\Pi^* - p_0 \geq p_0 \frac{p_1 - p_0}{p_1}$$

[28]Notice that conditions (11) and (12) imply that there exist parameter values for which B offers A a socially inefficient contract. Indeed, for any value of ϕ such that $(p_1 - p_0)^2/p_1 < \phi < p_1 - p_0$ it is socially efficient to induce the agent to exert a high level of effort but the principal, constrained by the need to satisfy incentive compatibility in the presence of limited liability, in equilibrium receives a payoff which is too low to make him willing to choose the efficient contract. For the analysis of an analogous situation see Innes (1990).

Therefore, (13) together with

$$c_A + c_B < p_0 \frac{p_1 - p_0}{p_1} \tag{14}$$

is enough to guarantee that it is socially efficient for the parties to sink the ex-ante costs and write a contingent contract (this is the analogue of Assumption 1 in Section 3 above), which in what follows we assume to be the case.

It follows from (8) that for values of ϕ sufficiently close to 0, the payoff to A is not high enough to induce him to sink the ex-ante cost c_A. In other words, there exist values of ϕ which satisfy (13) and such that

$$\Pi_A^* = \frac{p_0 \phi}{p_1 - p_0} < c_A.$$

Furthermore, from (9) it follows that for values of ϕ which satisfy (13) but are sufficiently close to $(p_1 - p_0)^2/p_1$ the payoff to B will not be high enough to induce him to sink the ex-ante cost associated with a contingent contract. In other words, there exists values of ϕ which satisfy (13) and such that

$$\Pi_B^* - \Pi_B^{**} = p_1 \left[1 - \frac{c}{p_1 - p_0}\right] - p_0 < c_B$$

We can now summarize our findings as follows. There exists two distinct values $\underline{\phi}$ and $\overline{\phi}$ of the parameter ϕ such that for every principal-agent problem in which $\phi \in [0, \underline{\phi}) \cup (\overline{\phi}, 1]$, the contracting parties choose to write a socially inefficient constant wage contract (w_0^{**}, w_1^{**}) rather than a contingent contract (w_0^*, w_1^*) on the sole account that it is not strategically optimal for one of the two parties to sink the ex-ante contractual cost.

8. AN INTERTEMPORAL MODEL WITH SHORT-TERM CONTRACTS

8.1. Preamble

In this section we present a further example of an economic environment to which the basic logic of our findings for the reduced form model of Section 3 applies. We consider a multi-period principal-agent model in which the choice of ex-ante contractual costs determines the maximum length of the contract which the principal and the agent can write. We find that, in general, the equilibrium outcome of the model involves the parties writing a contract which is shorter than would be socially optimal, after ex-ante contractual costs are taken into account.

8.2. An Intertemporal Principal-Agent Model

We start with a description of the model without any ex-ante contractual costs. Time is discrete, and runs for infinitely many periods, $t = 1, 2 \ldots$. The principal, B, hires an agent, A, to run a simple multi-period technology as follows. In period 1, A chooses a level of effort e, which can take one of the discrete number of values $\{\phi_0, \phi_1, \ldots, \phi_T, \ldots, \phi_\infty\}$ with $\infty > \phi_\infty > \phi_T > \phi_{T-1}$ for all $T = 1, 2, \ldots$, and $\phi_0 = 0$. Exerting effort level ϕ_T has a cost for the agent, which we simply take to be equal to ϕ_T. The agent's effort is non-contractible by assumption, so that any contract between the principal and the agent can only be contingent on output. Output is described as a sequence of i.i.d. binary random variables as follows. In each period t, the output y_t can be either high ($y_t = 1$) or low ($y_t = 0$). If A chooses effort level ϕ_0, then in all periods $y_t = 1$ with probability p_0 and $y_t = 1$ with probability $1 - p_0$. If the agent chooses $e = \phi_T$, then in each period $t = 1, \ldots, T$ $y_t = 1$ with probability p_1 and $y_t = 0$ with probability $1 - p_1$, while for all periods $t \geq T + 1$, $y_t = 1$ with probability p_0 and $y_t = 0$ with probability $1 - p_0$. Finally, if A chooses $e = \phi_\infty$, then in all periods $y_t = 1$ with probability p_1 and $y_t = 0$ with probability $1 - p_1$.

We assume that $p_1 > p_0 > 0$, and we denote by h_t the history of realized output values up to and including period t, $(y_1, \ldots, y_t)$. The set of all possible histories of length t denoted by H_t, while H_T denotes $\cup_{t=1}^{T} H_t$, and $H = \cup_{t=1}^{\infty} H_T$. The probability of history h_t, when A chooses effort level ϕ_T, is denoted by $p_1(h_t, T)$.

A contract between B and A can now be seen as a sequence $W = \{w(h_t)\}_{h_t \in H}$, specifying a wage to be paid to the agent for each possible sequence of outputs h_t at time t. For simplicity, we assume that both parties discount the future at the same rate, and denote by δ their common discount factor. Moreover, for simplicity again, we make the extreme assumption that both A and B are risk-neutral, although our results generalize fairly easily to the case in which the agent is risk-averse and the principal is either risk-neutral or 'less risk-averse' than the agent.

Given a wage package W and a level of effort ϕ_T, we can write the expected utility of A as

$$\sum_{h_t \in H} \delta^{t-1} p(h_t, T) w(h_t) - \phi_T$$

Therefore, A will choose to set $e = \phi_T$ only if W is such that for all $T' \neq T$

$$\sum_{h_t \in H} \delta^{t-1} p(h_t, T) w(h_t) - \phi_T \geq \sum_{h_t \in H} \delta^{t-1} p(h_t, T') w(h_t) - \phi_{T'} \tag{15}$$

The expected pay-off to B when the agent chooses $e = \phi_T$ and the wage package is W can be written as

$$\sum_{h_t \in H} \delta^{t-1} p(h_t, T) \left[y_t(h_t) - w(h_t) \right] \tag{16}$$

where $y_t(h_t)$ is output at time t, as specified in the history h_t.

As in Section 7, we assume that the agent has limited liability in the sense that the principal cannot pay him a negative wage in any eventuality. The principal's problem, under the assumption that he wants to induce effort ϕ_T from the part of the agent is therefore the following: maximize (16) by choice of W and e subject to the constraints (15) and $w(h_t) \geq 0$ for all $h_t \in H$. Throughout the rest of this section, we denote the solution to this maximization problem by W^*, with typical element $w^*(h_t)$.

In Subsections 8.3 and 8.4 below, we will be interested in the case in which the principal and the agent are *constrained* to write a contract which does not exceed a given length T. Therefore we need to specify what the principal's problem is in this case. For simplicity, we assume that once the contract expires B has all the bargaining power. This is because the agent's effort is decided once and for all at $t = 1$, and after date T the agent's effort is sunk. It follows that if the parties write a contract which lasts up to and including period T, for all $t > T$ the principal will pay the agent a wage of 0 in all possible eventualities.

In other words, the principal's problem when the maximum contract length is T can be written as

$$\begin{aligned}
\max_{W,\, T\in\{1,\ldots,T\}} \quad & \sum_{h_t\in H_T} \delta^{t-1}\, p(h_t,T)\,[y_t(h_t)-w(h_t)] + \frac{\delta^T}{1-\delta}\, p_0 \\
\text{s.t.} \quad & \sum_{h_t\in H_T} \delta^{t-1}\, p(h_t,T)w(h_t) - \phi_T \geq \\
& \qquad \sum_{h_t\in H_T} \delta^{t-1}\, p(h_t,T')w(h_t) - \phi_{T'} \quad \forall\, T' \neq T \\
& w(h_t) \geq 0 \qquad \forall\, h_t \in H_T
\end{aligned} \tag{17}$$

Throughout the rest of this section, we denote the solution to problem (17) by $\{T^*, W^{*T}\}$, with $w^{*T}(h_t)$ a typical element of W^{*T}.

We make two assumptions on the parameters of problem (17). First of all we will assume that A's cost of effort increases at a decreasing rate. This is designed to ensure that the variation in social surplus from increasing the agent's effort from ϕ_T to ϕ_{T+1} is positive for all $T = 1, 2, \ldots$.[29] Formally, we assume that

$$\exists k > 0 \qquad \text{such that} \qquad \delta^{T-1}k > \phi_T - \phi_{T-1}\; \forall\, T = 1, 2, \ldots \tag{18}$$

Our second assumption guarantees that the principal finds it profitable to induce effort ϕ_1, rather than ϕ_0 when the maximum length of the contract is $T = 1$. This coupled with (18) is enough to guarantee that the principal will always want to induce the agent to choose effort ϕ^T when the maximum possible length of the contract is T; in other words $T^* = T$ for all $T = 1, 2, \ldots$. Since our model in this section when $T = 1$ is the same as the model we analysed in Section 7, the condition which guarantees that this is the case is the same as (12) when we set $\phi = k$. Therefore we assume that

$$k \leq \frac{(p_1 - p_0)^2}{p_1} \tag{19}$$

[29] Recall that the cost ϕ_T is incurred at $t = 1$, while the benefits of an increase from ϕ_{T-1} to ϕ_T accrue at $t = T$.

We give a characterization of the solution to problem (17) and of the principal's problem when there is no limit imposed on the length of the contract in Lemmas A.4, A.5 and A.6 in the Appendix. Here, we limit ourselves to a summary of the features of these solutions which we will use in the next two subsections.

Let Π_B^*, $\Pi_B^{*\mathcal{T}}$, Π_A^* and $\Pi_A^{*\mathcal{T}}$ be the principal's and the agent's expected pay-offs when $W = W^*$ and $W = W^{*\mathcal{T}}$ respectively. What matters for the analysis that follows are the following three facts. First of all, we have that the optimal contract satisfies $T^* = \mathcal{T}$ ($T^* = \infty$ when there is no limit to the length of the contract). Secondly, the principal's pay-off is strictly increasing in $\mathcal{T}$. Formally

$$\Pi_B^* > \Pi_B^{*\mathcal{T}+1} > \Pi_B^{*\mathcal{T}} \quad \forall\, \mathcal{T} = 1, 2, \ldots \tag{20}$$

Finally, the agent's pay-off is also strictly increasing in the maximum possible length of the contract $\mathcal{T}$. Formally

$$\Pi_A^* > \Pi_A^{*\mathcal{T}+1} > \Pi_A^{*\mathcal{T}} \quad \forall\, \mathcal{T} = 1, 2, \ldots \tag{21}$$

The intuition behind this characterization of the optimal contract between the principal and the agent is simple to outline. Because (18) and (19) hold, B will always want to induce the highest possible level of effort from the part of the agent given $\mathcal{T}$. Since it is always open to the principal to induce any other effort level ϕ_T with $T < \mathcal{T}$ by offering A the wage package W^{*T} which would guarantee him a pay-off of Π^{*T}, then it must be that $\Pi_B^{*\mathcal{T}} > \Pi_B^{*T}$ for all $T < \mathcal{T}$. Since B always wants to 'buy the maximum effort' from A, he has to give him appropriate incentives to choose $e = \phi_{\mathcal{T}}$. Because of the limited liability constraint we have imposed on the agent's payments, this guarantees that A's expected pay-off increases with $\mathcal{T}$.

8.3. *Ex-ante Costs and the Length of the Contract*

We are now ready to introduce ex-ante contractual costs in the multi-period principal-agent model we have described in Subsection 8.2 above. We do so in the simplest possible way which allows us to capture the idea that longer contracts will cost the parties more than shorter ones. This can be justified in a number of ways. One obvious candidate which springs to mind is that longer contracts must specify more contingencies and hence are more difficult and complex to draw-up.

We imagine that in period 0 the two parties simultaneously and independently choose numbers t_A and t_B. If $i \in \{A,B\}$ chooses t_i, he incurs a cost of $c_i t_i$ (with $c_i > 0$ for $i \in \{A,B\}$) at this time. The agents' choices of t_A and t_B determine the maximum length of the contract which they are allowed to write. We take this to be given by the following expression.[30]

$$T = \min\{t_A, t_B\} \tag{22}$$

The description of our inter-temporal principal-agent model with ex-ante contractual costs is now complete. We are interested in the sub-game perfect equilibria of the two stage game in which the two agents first choose t_A and t_B as described, and then proceed to write the optimal contract, given T, as described in Subsection 8.2. For simplicity we assume no discounting between period 0 and period 1.[31] Therefore, the pay-off to agent $i \in \{A,B\}$ when the pair (t_A, t_B) is chosen can be written as

$$\Pi_i^{*T} - c_i t_i \tag{23}$$

where T is given by (22).

8.4. *Short-Term Equilibrium Contracts*

We are now ready to show that, in general, the model which we described in Subsections 8.2 and 8.3 above yields equilibrium contracts which are short-term in the sense that they are shorter than would be socially efficient, after the ex-ante contractual costs are taken into account.

[30] Equation (22) clearly embodies the assumption that the agents' ex-ante costs are perfect complements. We discussed the cases of partial or perfect substitutes in Section 3 above, in the context of our reduced form model. Our results for the present inter-temporal model generalize fairly easily to the case of ex-ante costs which are partial substitutes.

[31] A simple re-scaling of pay-offs would be sufficient to take such discounting ito account.

Given (22), it is clearly never socially efficient to choose $t_A \neq t_B$. Given this observation, the socially efficient length of the principal-agent contract is easy to characterise. Let $\Delta\Pi_i^T = \Pi_i^{*T} - \Pi_i^{*T-1}$ for $i \in \{A, B\}$ and notice that by (20) and (21) we have that $\Delta\Pi_i^T > 0$ for all T, and $\lim_{T\to\infty} \Delta\Pi_i^T = 0$. The socially optimal level of T guarantees that the sum of the pay-offs in (23) is highest. Let this value of T be denoted by T^*. It is then easy to see that T^* must satisfy

$$\Delta\Pi_A^{T^*} + \Delta\Pi_B^{T^*} - c_A - c_B \geq 0 \quad \text{and} \quad \Delta\Pi_A^{T^*+1} + \Delta\Pi_B^{T^*+1} - c_A - c_B \leq 0 \tag{24}$$

In any equilibrium of the model, again because of (22), we must also have that $t_A = t_B$. Now let

$$T_i = \max\left\{T \;\middle|\; \Delta\Pi_i^T - c_i \geq 0 \text{ and } \Delta\Pi_i^{T+1} - c_i \leq 0\right\}$$

It is then clear that the maximum length of any equilibrium contract will never exceed

$$T^E = \min\{T_A, T_B\} \tag{25}$$

Comparing (24) and (25) it is immediate to conclude that $T^E \leq T^*$. Moreover, by inspection of the two conditions (24) and (25), it is also easy to see that 'in general' we will have $T^E < T^*$. Where the qualification 'in general' refers to the fact that we could have $T^E = T^*$ either if $T_A = T_B$, or because of 'integer problems'. Aside from integer problems it is clear that, in general, the parameters of our multi-period principal-agent model will be such that $T_A \neq T_B$. We omit the details for reasons of space.

Our findings can therefore be summarized as follows. 'In general', our multi-period principal-agent problem with ex-ante contractual costs yields equilibrium contracts which are shorter than would be socially optimal, after the ex-ante costs are taken into account. The intuition behind this result is very similar to the one we described for the continuous costs reduced form model of Section 4 above. The inefficiency stems from the fact that the 'marginal conditions' which determine each party's choice of ex-ante cost do not, in general, coincide with the 'marginal conditions' for social efficiency.

9. CONCLUDING REMARKS

If the parties to a contract need to sink some ex-ante contractual costs before they can reach the contract-negotiating phase of their interaction, the ex-ante costs may generate a version of the hold-up problem. If the distribution of ex-ante costs and the distribution of the surplus generated by the contract are sufficiently 'mis-matched', one of the two parties to the contract will not find it to his advantage to pay the ex-ante contractual cost, even though the surplus generated by the contract would be sufficient to cover the total ex-ante costs associated with it. Therefore, in equilibrium the contract will not be written. We have verified this claim in a variety of simple models, including a number of extensive forms for the negotiation of the distribution of surplus among the agents.

Unlike many other versions of this problem, under appropriate conditions, the hold-up problem generated by ex-ante contractual costs is unlikely to have a 'contractual solution'. This is because a 'contract over a contract' is likely to generate a fresh set of ex-ante contractual costs and hence a new hold-up problem. We have found this to be true in a variety of settings which include a whole hierarchy of 'contracts over contracts'.

Lastly, we have explored two examples of economic situations in which the distribution of surplus from the contract between a principal and an agent is 'pinned down' by the interplay of incentive compatibility and limited liability constraints. Since the economic parameters of these two models fix the distribution of surplus, these provide two examples of economic situations which generate the hold-up problem we identified before. Our second example concerns a dynamic principal-agent model. The equilibrium contracts which we obtain in this case are short term in the sense that they are shorter than would be socially efficient, even after contractual costs are taken into account.

APPENDIX

PROOF OF PROPOSITIONS 4 AND 5: We start by arguing that the reduced form model with partial substitutes has an equilibrium in which a contract is drawn-up if and only if

$$\lambda \geq \underline{c}_A \quad \text{and} \quad 1-\lambda \geq \underline{c}_B \tag{A.1}$$

Moreover, whenever (A.1) does not hold, the only equilibrium is for both agents to pay zero ex-ante costs.

Clearly, if either of the two inequalities in (A.1) is violated, one of the two agents will have as a dominant strategy to pay zero ex-ante cost. Since the best response to the opposing agent paying zero ex-ante cost is to pay zero ex-ante cost, it is clear that if (A.1) is violated then the unique equilibrium involves both agents paying zero ex-ante costs.

Suppose now that (A.1) is satisfied. Then, since $\underline{c}_A + \underline{c}_B < \overline{c} \leq 1$, it is always possible to find a pair (c_A^*, c_B^*) such that

$$\lambda \geq c_A^* \geq \underline{c}_A \quad \text{and} \quad 1-\lambda \geq c_B^* \geq \underline{c}_B \quad \text{and} \quad c_A^* + c_B^* = \overline{c} \tag{A.2}$$

It is now immediate to check that such pair of ex-ante costs is indeed an equilibrium and therefore our initial claim is proved.

To conclude the proof of Proposition 4 it is now enough to notice that, given a pair of minimum ex-ante costs $(\underline{c}_A, \underline{c}_B)$ it is always possible to find a range of values of λ such that (A.1) is violated.

Similarly, to conclude the proof of Proposition 5 it is now enough to notice that, for any given value of $\lambda \in (0,1)$ we can find a pair of minimum ex-ante costs such that (A.1) is violated. ■

PROOF OF PROPOSITION 6: Since either $\lambda < c_A$ or $1-\lambda < c_B$, it is clear that there is no equilibrium in which both agents pay the ex-ante cost at $t = 0$.

We only show that it is not possible that in equilibrium A alone pays the ex-ante cost at $t = 0$. Any equilibrium in which B alone pays the ex-ante cost at $t = 0$ can be ruled out in a symmetric way and we omit the details.

Suppose then that there is an equilibrium in which only A pays the ex-ante cost at $t = 0$. There are two cases to consider. Either B pays his cost to see A's offer or he does not.

Suppose next that there is an equilibrium in which A only pays the ex-ante costs at $t = 0$ and subsequently B either accepts or rejects A's offer without seeing it. Note that in this case B cannot condition his decision to accept or reject on the value of ℓ since he does not pay to see it. If B accepts in equilibrium, clearly A will set $\ell = -\varepsilon$. But this would give an equilibrium pay-off of $-\varepsilon$ to B, and therefore yields a contradiction since B can always guarantee himself a pay-off of zero by not paying any costs and rejecting any offer. If B rejects

A's offer blind in equilibrium, then A's equilibrium pay-off is $-c_A$ since no contract is drawn-up and A pays his ex-ante cost at $t = 0$. This is again a contradiction since A can guarantee himself a pay-off of zero by not paying the ex-ante cost at $t = 0$ (and rejecting any offers made by B if he pays his ex-ante cost).

Lastly, consider the possibility of an equilibrium in which A alone pays the ex-ante costs at $t = 0$ and subsequently B pays his ex-ante cost to see the value of ℓ, ad then accepts or rejects A's offer. Notice that now B can condition his decision to accept or reject A's offer on the actual value of ℓ. Using subgame perfection, it is immediate to see that, in equilibrium, it must be the case that B accepts all offers which guarantee that $1 - \ell > 0$ (his ex-ante cost is *sunk* when the accept/reject decision is made). Therefore, in equilibrium, A will offer precisely $\ell = 1$. It follows that in any equilibrium in which A alone pays the ex-ante cost at $t = 0$ and subsequently B pays to see A's offer, B's pay-off is at most $-c_B$. But this is a contradiction since B, as before, can guarantee himself a pay-off of zero by not paying any costs and rejecting any offer. ■

PROOF OF PROPOSITION 8 IN CASE 1: Assume that the proposition is false and hence that there is an equilibrium in which both agents pay their ex-ante costs.

Consider now the subgame in which both parties have already sunk the ex-ante costs (c_A, c_B). Clearly B will accept any offer from A which guarantees that

$$1 - \lambda > 0 \tag{A.3}$$

It now follows that the highest offer which A will possibly make in equilibrium guarantees that

$$1 - \lambda = 0 \tag{A.4}$$

But equation (A.4) implies that B's pay-off in any equilibrium in which a contract is drawn-up is at most $-c_B$. This is a contradiction since B can guarantee himself a pay-off of 0 by simply not paying his ex-ante cost. This is enough to prove the proposition. ■

PROOF OF PROPOSITION 8 IN CASE 2: The proof for this case can be obtained simply by swapping the names of agents A and B and substituting λ for $1 - \lambda$ in equations (A.3) and (A.4) in the proof of Proposition 8 for Case 1 above. ■

PROOF OF PROPOSITION 8 IN CASE 3: The argument is a simple modification of the proof of Proposition 8 for Case 1. Observe that B cannot condition his paying c_B on the actual offer λ (since this is un-observable before c_B is paid). In the subgame which starts after both agents have paid the ex-ante costs, B will accept any offer which satisfies (A.3). Therefore, the rest of the proof of Proposition 8 for Case 1 applies. ■

PROOF OF PROPOSITION 8 IN CASE 4: The proof for this case can be obtained simply by swapping the names of agents A and B in the proof of Proposition 8 for Case 3. ■

PROOF OF PROPOSITION 8 IN CASE 5: Using sub-game perfection, the proof follows immediately from the proofs of Proposition 8 in Cases 1 and 2 above. We omit the details. ■

PROOF OF PROPOSITION 8 IN CASE 6: We prove the result by backward induction. Note that the extensive forms in Cases 1 and 2 are equivalent to Case 6 when $N = 1$ and A or B make the first offer respectively. Therefore, from Proposition 8 in Cases 1 and 2 we know that the claim is true for $N = 1$. It is therefore enough to show that if the claim is true for N then it is also true for $N+1$.

We prove the backward induction step for the case in which it is A who makes an offer to B in the $N+1$-th period, $t = N$. The details for the case in which B makes an offer to A at $t = N$ are symmetric and therefore omitted.

Assume, by way of contradiction, that the claim is true for N, but false for $N+1$. Consider any equilibrium of the game with $N+1$ rounds of negotiation in which either A or B does not sink his ex-ante cost c_A^N or c_B^N. Since the claim is true for N, clearly this equilibrium yields the no contract outcome. Therefore if the claim is true for N and false for $N+1$, then there must be an equilibrium for the model with $N+1$ rounds of negotiation in which in the last period both agents sink the ex-ante costs c_A^N and c_B^N. Consider now the subgame which starts in stage II of period $t = N$ in which both agents have already sunk these costs. Clearly, at this point B will accept any offer from A which guarantees that

$$1 - \lambda^N > 0$$

It therefore follows that the highest offer which A will possibly make in this equilibrium guarantees that

$$1 - \lambda^N = 0 \tag{A.5}$$

Notice next that (A.5) implies that in any equilibrium in which the statement is true for N but false for $N+1$, B's pay-off in the entire game is at most $-c_B^N$. But this is clearly a contradiction since B can guarantee himself a pay-off of 0 by not paying any of his ex-ante costs. ■

PROOF OF PROPOSITION 8 IN CASE 7: Fix any subgame perfect equilibrium of the game, and let $\pi(A,n)$ and $\pi(B,n)$ be the continuation pay-offs which A and B get respectively in the subgame which starts in stage I of period $n = 0,1,\ldots$. Suppose that the equilibrium we have fixed involves the parties drawing-up the surplus-generating contract. Then it must be that at some time $n = 0,1,\ldots$ both parties sink the ex-ante costs (c_A^n, c_B^n), and subsequently an offer λ^n is made and accepted. We provide the details of the argument for the case in which n is even, so that it is agent A who makes an offer to B in period n. The details for the case of n odd are symmetric and therefore omitted.

Consider the subgame which starts in stage II of period n, after the ex-ante costs (c_A^n, c_B^n) have been paid. By an argument completely analogous to the one used in the proof of Proposition 8 in Case 6, we now know that A's offer must ensure that

$$1-\lambda^n = \pi(B, n+1) \tag{A.6}$$

using (A.6) it is then immediate that

$$\pi(B,n) = \pi(B, n+1) - c_B^n \tag{A.7}$$

Notice now that agent B, can clearly guarantee himself a pay-off of $\pi(B, n+1)$ by not paying the ex-ante cost c_B^n in stage I of period n and hence moving the game directly to period $n+1$. Therefore (A.7) yields a contradiction, and this clearly enough to establish that Proposition 8 holds for Case 7. ■

LEMMA A.1: *Consider the reduced form model with simple compensating transfers described in Subsection 6.1. If there exists an equilibrium of the model in which both* $\sigma_A^1 > 0$ *and* $\sigma_B^1 > 0$*, then there exists another, payoff equivalent, equilibrium of the model in which the transfers take the values* $\tilde{\sigma}_A^1 = \sigma_A^1 - \sigma_B^1$ *and* $\sigma_B^1 = 0$ *if* $\sigma_A \geq \sigma_B^1$*, and* $\tilde{\sigma}_A^1 = 0$ *and* $\tilde{\sigma}_B^1 = \sigma_B^1 - \sigma_A^1$ *if* $\sigma_B^1 \geq \sigma_A^1$*.*

PROOF: We examine only the case in which $\sigma_A^1 \geq \sigma_B^1$. The other case is a simple re-labelling of this one. To construct the new equilibrium, let the strategies of both agents be identical to the strategies in the original equilibrium, except for the way actions are conditioned on the other agents' compensating transfer offer. In the new equilibrium, each agent $i \in \{A, B\}$ responds to any offer $\tilde{\sigma}_j^1$ (with $j \neq i$) exactly as he would respond to the offer $\tilde{\sigma}_j^1 + \sigma_i^1$ in the original equilibrium. ■

PROOF OF PROPOSITION 9: We only deal with the case in which $1-\lambda < c_B^0$. The case in which $\lambda < c_A^0$ is a simple re-labelling of this one and we omit the details.

Since we are assuming that the parameters of the model yield the no contract outcome in the final stage, any equilibrium which yields a contingent contract as an outcome must have both agents paying both tiers of ex-ante costs.

Assume by way of contradiction that such an equilibrium exists and denote by a superscript '$*$' the equilibrium values of all variables in this equilibrium. Notice first of all that if $\sigma_B^* \geq \sigma_A^*$ we have an immediate contradiction since in this case $\gamma_B^* > c_B$ and therefore B's equilibrium pay-off must be negative. Since B can guarantee a pay-off of zero by not paying any of the ex-ante costs this is a contradiction.

By Lemma A.1, we can then assume without loss of generality that $\sigma_A^* > 0$ and $\sigma_B^* = 0$.

Next, consider the subgame that starts after both agents have paid the second tier ex-ante costs and the offers $\sigma_A^* > 0$ and $\sigma_B^* = 0$ have been made. We now claim that it must be that case that

$$1 - \lambda + \sigma_A^* - c_B^0 = 0 \tag{A.8}$$

To see this notice that the continuation strategy for B in this subgame must be to accept any offer σ_A which guarantees that

$$1-\lambda+\sigma_A-c_B^0>0 \tag{A.9}$$

since if he rejects any such offer his continuation pay-off would be at most zero (if he does not pay c_B^0). Therefore, any offer σ_A which satisfies (A.9) cannot be pay-off maximizing for A.

It follows directly from (A.8) that B's pay-off in this equilibrium is $-c_B^2$. But this is a contradiction since, as we noted above, B can guarantee a pay-off of zero by not paying any ex-ante costs. This is enough to prove the proposition. ■

LEMMA A.2: *Consider the reduced form model with N tiers of simple compensating transfers described in Subsection 6.2. Given any equilibrium of the model, let $\ell_n = \arg\min_{i\in\{A,B\}}\sigma_i^n$, for $n=1,\ldots,N$. Then there exists another, payoff equivalent, equilibrium of the model in which the compensating transfers are $\tilde{\sigma}_i^n=\sigma_i^n-\sigma_{\ell_n}^n$.*

PROOF: The new equilibrium can be constructed in a way which is completely analogous to the one in the proof of Lemma A.1. We omit the details. ■

PROOF OF PROPOSITION 10 (Simple Transfers): We only deal with the case in which $1-\lambda<c_B^0$. The case in which $\lambda<c_A$ is a simple re-labelling of this one and we omit the details.

Notice that when $N=1$, the reduced form model with N tiers of simple compensating transfers of Subsection 6.2 is identical to the reduced form model with simple compensating transfers of Subsection 6.1. It follows that Proposition 9 implies that Proposition 10 is true when $N=1$. To prove the proposition it then remains to show that, if it is true for a given value of N, then it is also true for $N+1$.

Suppose then that the proposition is true for a given value of N, and assume, by way of contradiction, that it is false for $N+1$. Since the proposition is true for N, if there is an equilibrium of the reduced form model with $N+1$ tiers of simple compensating transfers in which some ex-ante costs are paid, it must be that in such equilibrium both agents pay the time $t=-N-1$ ex-ante costs. Let the values of all choice variables in such equilibrium be denoted by superscript '$*$'.

In a way analogous to the proof of Proposition 9, using Lemma A.2 we can now assume without loss of generality that $\sigma_A^{N+1*}>0$ and $\sigma_B^{N+1*}=0$. Again as in the proof of Proposition 9 this implies that B's pay-off in this equilibrium is $-c_B^{N+1}$. But this is clearly a contradiction since B can guarantee himself a payoff of zero by not paying any tier of ex-ante costs. The proof of Proposition 10 for the case of N tiers of simple compensating transfers is therefore complete. ■

LEMMA A.3: *Consider the reduced form model with N tiers of multiple compensating transfers described in Subsection 6.2. Given any equilibrium of the model, let* $\ell_{n,m} = \arg\min_{i\in\{A,B\}} \sigma_i^{n,2m}$*, for* $n = 1,\ldots,N$ *and* $m = 0,\ldots,n-1$*. Then there exists another, payoff equivalent, equilibrium of the model in which the compensating transfers are* $\tilde{\sigma}_i^{n,m} = \sigma_i^{n,m} - \sigma_{\ell_{n,m}}^{n,m}$.

PROOF: The new equilibrium can be constructed in a way which is completely analogous to the one in the proof of Lemma A.1. We omit the details. ■

PROOF OF PROPOSITION 10 (Multiple Transfers): We only deal with the case in which $1-\lambda < c_B^0$. The case in which $\lambda < c_A$ is a simple re-labelling of this one and we omit the details.

Notice that when $N = 1$, the reduced form model with N tiers of multiple compensating transfers of Subsection 6.2 is identical to the reduced form model with simple compensating transfers of Subsection 6.1. It follows that Proposition 9 implies that Proposition 10 is true when $N = 1$. To prove the proposition it then remains to show that, if it is true for a given value of N, then it is also true for $N+1$.

Suppose then that the proposition is true for a given value of N, and assume, by way of contradiction, that it is false for $N+1$. Since the proposition is true for N, if there is an equilibrium of the reduced form model with $N+1$ tiers of multiple compensating transfers in which some ex-ante costs are paid, it must be that in such equilibrium both agents pay the time $t = -N-1$ ex-ante costs. Let the values of all choice variables in such equilibrium be denoted by superscript '$*$'.

In a way analogous to the proof of Proposition 9, using Lemma A.3 we can now assume without loss of generality that $\sigma_A^{N+1,m*} > 0$ for some $m = 0,\ldots,N$ and $\sigma_B^{N+1,m*} = 0$ for all $m = 0,\ldots,N$. Again as in the proof of Proposition 9 this implies that B's pay-off in this equilibrium is $-c_B^{N+1}$. But this is clearly a contradiction since i can guarantee himself a payoff of zero by not paying any tier of ex-ante costs. The proof of Proposition 10 for the case of N tiers of multiple compensating transfers is therefore complete. ■

LEMMA A.4: *Consider the multi-period principal-agent problem (17) described in Subsection 8.2, for a given maximum length of the contract* T*. For each* $n = 1,2,\ldots,T$*, let*

$$H_n^* = \left\{ h_n^* \in H_n \;\middle|\; h_n^* \in \arg\max_{h_n\in H_n} \frac{p(h_n,n)}{p(h_n,n)-p(h_n,n-1)} \right\}$$

Then the solution to problem (17) satifies the following. The agent's level of effort is the maximum possible, that is $e = \phi_T$*, or equivalently* $T^* = T$*. Moreover, for all* $n = 1,\ldots,T$ *whenever* $h_n \notin H_n^*$ *then* $w^{*T}(h_n) = 0$*. Lastly, for all* $n = 1,\ldots,T$

$$\sum_{h_n^*\in H_n^*} w^{*T}(h_n^*) = \frac{p(h_n^*,n)}{\delta^{n-1}[p(h_n^*,n)-p(h_n^*,n-1)]}\xi_{T,n} \tag{A.10}$$

where

$$\xi_{T,n} = (T-n+1)\phi_T - \sum_{\ell=n-1}^{T-1} \phi_\ell \tag{A.11}$$

PROOF: We start by arguing that it must be that $T^* = T$. This is almost immediate if we consider problem (17) for two consecutive values of the maximum length of the contract, say T and $T+1$. When the maximum length of the contract is $T+1$ it is open to B to offer A the wage package W^{*T} for all periods up to and including T, and to make the wage in period $T+1$ only contingent on y_{T+1}, and equal to the optimal contract when $T=1$. Because of (19), this both satisfies the constraint which guarantees that $e=\phi_{T+1}$ and guarantees that B's pay-off is strictly greater than Π_B^{*T}. Since the latter is the maximum which B can obtain if he induces effort $e=\phi_T$, this is clearly enough to show that if the claim is true for $T=1$ then it is true for all T. Since (19) guarantees directly that the claim is true for $T=1$, this is enough to show that $T^*=T$ for all T.

Notice that since $T^*=T$, the incentive constraints in problem (17) can be re-written as

$$\sum_{h_t \in H_T} \delta^{t-1} p(h_t,T)w(h_t) - \phi_T \geq \sum_{h_t \in H_T} \delta^{t-1} p(h_t,n)w(h_t) - \phi_n \quad \forall n < T \tag{A.12}$$

Consider (A.12) when $n=T-1$. Since $p(h_t,T)=p(h_t,T-1)$ for all $h_t \in H_{T-1}$, this constraint can be simply re-written as

$$\sum_{h_T \in H_T} \delta^{T-1} p(h_T,T)w(h_T) - \phi_T \geq \sum_{h_T \in H_T} \delta^{T-1} p(h_T,T-1)w(h_T) - \phi_{T-1} \tag{A.13}$$

where the summations now extend only to histories of length T. Clearly, in the solution to problem(17), constraint (A.13) must bind, since otherwise B could reduce A's wage after some histories of length T and still satisfy all the appropriate constraints. Using now (A.13) with equality, we can re-write (A.12) when $n=T-2$ as

$$\begin{aligned}\sum_{h_{T-1} \in H_{T-1}} \delta^{T-2} p(h_{T-1},T-1)w(h_{T-1}) - 2\phi_T \geq \\ \sum_{h_{T-1} \in H_{T-1}} \delta^{T-2} p(h_{T-1},T-2)w(h_{T-1}) - \phi_{T-1} - \phi_{T-2}\end{aligned} \tag{A.14}$$

Since (A.14) must also clearly bind in the solution to problem (17), we can now proceed backwards to re-write all the constraints in (A.12) together as

$$\sum_{h_n \in H_n} \delta^{n-1} p(h_n,n) w(h_n) - (T-n+1)\phi_T \geq \sum_{h_n \in H_n} \delta^{n-1} p(h_n,n-1) w(h_n) - \sum_{\ell=n-1}^{T-1} \phi_\ell \quad \forall\, n < T \tag{A.15}$$

Therefore, the principal's problem is equivalent to maximizing the objective function in (17), subject to constraints (A.15) and non-negative wages. Since both the objective function and the constraints in this problem are linear in wages, this is enough to show that whenever $h_n \notin H_n^*$ then $w^{*T}(h_n) = 0$, and that (A.10) and (A.11) hold. This is enough to conclude the proof of the lemma. ■

LEMMA A.5: *Equation (20) holds.*

PROOF: The claim that Π_B^{*T} is increasing in T follows immediately from the fact that $T^* = T$ for all T. Since the principal's problem without any limit to the length of the contract can be seen as the problem (17) with (infinitely many) additional constraints (setting $w(h_t) = 0$ for all h_t with $t > T$), it is clear that $\Pi_B^* \geq \Pi_B^{*T}$ for all T. To see that the inequality is strict, it is enough to recall that $\Pi_B^{*T} < \Pi_B^{*T+1}$. This concludes the proof of the claim. ■

LEMMA A.6: *Equation (21) holds.*

PROOF: Recall that ϕ_T is increasing in T by assumption. This is enough to show that ξ_{Tn} is increasing in T for any given n. Given (A.10) this implies that the expected wage of A in any given period is increasing with T, which is of course enough to show that Π_A^{*T} is increasing in T. The rest of the claim follows from the fact that $\Pi_A^* = \lim_{T\to\infty} \Pi_A^{*T}$. We omit the details. ■

REFERENCES

ABREU, D., AND A. RUBINSTEIN (1988): "The Structure of Nash Equilibrium in Repeated Games with Finite Automata," *Econometrica*, 56, 1259–81.

AGHION, P., M. DEWATRIPONT, AND P. REY (1994): "Renegotiation Design with Unverifiable Information," *Econometrica*, 62, 257–82.

AGHION, P., AND J. TIROLE (1997): "Formal and Real Authority in Organizations," *Journal of Political Economy*, forthcoming.

ANDERLINI, L., AND L. FELLI (1996a): “Costly Contingent Contracts,” Theoretical Economics Discussion Paper TE/96/313, STICERD, London School of Economics.

——— (1996b): “Incomplete Contracts and Complexity Costs,” CPNSS discussion paper 20/96, London School of Economics.

CHUNG, T. Y. (1991): “Incomplete Contracts, Specific Investments, and Risk Sharing,” *Review of Economic Studies*, 58, 1031–42.

COASE, R. H. (1960): “The Problem of Social Cost,” *The Journal of Law and Economics*, 3, 1–44.

DYE, R. A. (1985): “Costly Contract Contingencies,” *International Economic Review*, 26, 233–50.

GROSSMAN, S. J., AND O. D. HART (1986): “The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration,” *Journal of Political Economy*, 94, 691–719.

HART, O. D., AND J. MOORE (1988): “Incomplete Contracts and Renegotiation,” *Econometrica*, 56, 755–85.

——— (1990): “Property Rights and the Nature of the Firm,” *Journal of Political Economy*, 98, 1119–58.

HOLMSTRÖM, B. (1982): “Moral Hazard in Teams,” *Bell Journal of Economics*, 13, 324–340.

INNES, R. D. (1990): “Limited Liability and Incentive Contracting with Ex-ante Action Choices,” *Journal of Economic Theory*, 52, 45–67.

MASKIN, E., AND J. TIROLE (1995): “Dynamic Programming, Unforseen Contingencies, and Incomplete Contracts,” forthcoming.

NÖLDEKE, G., AND K. M. SCHMIDT (1995): “Option Contracts and Renegotiation: A Solution to the Hold-Up Problem,” *RAND Journal of Economics*, 26, 163–179.

PICCIONE, M. (1992): "Finite Automata Equilibria with Discounting," *Journal of Economic Theory*, 56, 180–193.

PICCIONE, M., AND A. RUBINSTEIN (1993): "Finite Automata Play a Repeated Extensive Game," *Journal of Economic Theory*, 61, 160–168.

ROGERSON, W. P. (1992): "Contractual Solutions to the Hold-Up Problem," *Review of Economic Studies*, 59, 777–793.

ROSE, F. D. (ed.) (1987): *Blackstone's Statutes on Contract, Tort and Restitution*. London: Blackstone Press Limited. 5-th Edition.

RUBINSTEIN, A. (1982): "Perfect Equilibrium in a Bargaining Model," *Econometrica*, 50, 97–110.

——— (1986): "Finite Automata Play the Repeated Prisoner's Dilemma," *Journal of Economic Theory*, 39, 83–96.

SEGAL, I. (1995): "Complexity and Renegotiation: A Foundation for Incomplete Contracts," Harvard University, mimeo.

SHAKED, A., AND J. SUTTON (1984): "Involuntary unemployment as a Perfect Equilibrium in a Bargaining Game," *Econometrica*, 52, 1351–1364.

WILLIAMSON, O. (1985): *The Economic Institutions of Capitalism*. New York: Free Press.