Collusion and Renegotiation in a Principal-Supervisor-Agent Relationship

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Abstract

This paper describes a principal-agent relationship with a supervisor who has information about the agent. The agent and the supervisor have the possibility to collude and misinform the principal. In accordance with the existing literature there exists an optimal contract which excludes collusion in equilibrium. The optimal contract exhibits, however, ex-post inefficient and creates scope for renegotiation. If a renegotiation-stage is incorporated in the game then for some parameter constellations the optimal contract is a contract which necessarily induces collusion. The paper thus shows that the principal's behavior toward ex-post inefficiencies may determine whether collusion occurs in equilibrium.

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1 Introduction

In recent years economists have extended the standard hierarchical principal-agent model by including a third player: the supervisor. The extension can be used to analyze situations in which the principal is able to acquire information about her agent from other economic agents. The problem which arises in these situations is the manipulation of information. Since the principal may use the supervisor's information to discipline the agent, the agent has an incentive to collude with the supervisor and manipulate the information which is sent to the principal. An important question is then whether in equilibrium the principal will offer contracts which exclude such forms of collusion. This paper shows that the answer to this question may depend on the principal's behavior toward contracts which turn out to be inefficient ex post. If she is expected to renegotiate such contracts, then she might strictly prefer collusion to take place in equilibrium.

The issue of collusion in a principal-supervisor-agent relationship was first studied in Tirole (1986). The paper analyzes a model, in which the supervisor may observe information about the agent's type. In this context Tirole derived the *collusion-proofness principle* by showing that if an optimal contract exists then there exists one which induces the supervisor and the agent not to collude in equilibrium. A central question in the collusion literature is then in which context does this collusion-proofness principle fail to hold? Some circumstances have been identified in which optimal contracts do induce collusion. In Tirole (1992) it is shown that the principal may strictly prefer to adopt contracts which induce collusion when there are different types of supervisors with different levels of scruple. By allowing collusion to take place by those types for which collusion is most costly to prevent the principal is able to screen between the different types. Depending on the parameter constellation screening may be optimal. Tirole further identifies other conditions, which may cause the collusion-proofness principle to break down. He argues that contracts need to be complete and a "separability" condition of the collusion-proofness constraints must hold. In Scheepens (1995) it is shown that collusion takes place in equilibrium when the principal can monitor collusion, but when monitoring is unverifiable. The collusion problem is then transformed into a standard inspection game. When monitoring effort is contractible, the collusion-proofness principle holds.

An important but fundamentally different reason why collusion may be beneficial is that collusion may have positive incentive effects which outweigh the negative collusion effect. Collusion may increase efficiency, or induce cooperation and coordination between agents. This way creating an extra surplus. If the principal is able to appropriate a large enough part of the surplus, then she may prefer collusion to take place. Itoh (1993) makes, therefore, the distinction between (strict) collusion models in which collusion is

only detrimental, and cooperation models in which colluding has positive side effects. We focus here on a strict collusion model: If the principal could, ex ante, choose between living in a world with collusion or without, she would choose the world without collusion.

The model we study is a procurement model with asymmetric information. The principal has a project of fixed size, which the agent can realize. The agent and the supervisor know the exact cost of the project, while the principal does not. The supervisor may report his information to the principal and the principal may condition the agent's payment on the report. This, however, creates collusive behavior between the agent and supervisor, since the agent wants those costs to be reported which result in the highest payment. To prevent collusion the principal may contract on a noisy signal which informs the principal about the likeliness that collusion occurred. The paper shows that if this signal is sufficiently informative then it is optimal for the principal to do so in order to reduce the attractiveness of collusion. The imperfectness of the signal, however, creates scope for renegotiation. Once the principal has ensured a truthful report, she prefers to change the contract. She does no longer want to condition the contract on the signal, since it may have given wrong indications. Preventing collusion and conditioning the contract on the external signal are incompatible and will lead to renegotiation. The principal has to choose between either allowing collusion to take place and to condition her contract on the external signal, or to prevent collusion and offer a contract which is not conditioned on the signal. We show that there exists a parameter constellation such that it is optimal for the principal to choose the former policy, i.e. to induce collusion to take place.

The rest of the paper is organized as follows. The next section introduces the model. Section 3 derives the optimal contract of the game. Section 4 points out the ex post inefficiency of the contract and analyzes the game with renegotiation. Section 5 discusses the main result.

2 The Model

The principal has a project which she values at R. A single agent can realize the project. Ex ante it is publicly known that the cost c of the project is c_l with probability ν and c_h with probability $1-\nu$, where $R > c_h > c_l$. Before contracting takes place the supervisor learns the agent's true cost. He is, therefore, a valuable source of information to the principal. After the principal contracts with the supervisor and the agent, the principal asks the supervisor to report the cost of the project. The report, specifying whether the cost is high or low, may be untruthful if the agent and the supervisor agree to collude.

In accordance with the literature on collusion we make the crucial assumption that

the report can only be untruthful if both the supervisor and the agent agree on sending a falsified report. This implies that the supervisor is unable to falsify the report without the voluntary help of the agent. And, conversely, the agent is unable to force the supervisor to send a falsified report. For a justification of this assumption see Kofman and Lawarree (1993). Collusion may be accompanied by a positive transfer (bribes) from the agent to the supervisor, but not vice versa.[1] The transfer is then part of a side-contract between the supervisor and the agent, specifying a payment conditional on the supervisor's report. Transfers are costly and to keep the problem tractable these costs are taken to be proportional to the size of the transfer. The parameter $k \in (0,1)$ expresses this cost. When the agent sends a transfer b, the supervisor receives only the amount kb.[2] We do not analyze the bargaining procedure by which the bribe b is determined. We assume that whenever there exists a surplus from colluding, collusion will indeed take place. Neither do we address the issue of enforceability of the side-contract. In accordance with standard collusion models we merely assume enforceability of side-contracts and only hint that the difficulties of enforcing the contract may be one of the reasons why bribing is costly.

After obtaining the report, the principal receives a signal $s \in \{b, n\}$, which is imperfectly correlated with the occurrence of collusion. When collusion has taken place the principal receives the signal $s = b$ with probability p and the signal $s = n$ with probability $1 - p$. When collusion did not take place the signal $s = b$ is received with probability q, while the signal $s = n$ is received with probability $1 - q$, where $0 < q < p < 1$. Since q is smaller than p a signal $s = b$ gives some indication that collusion has occurred. The signal s may, therefore, be informative for the principal. Parameters p and q are common knowledge.

We assume that the report r and the signal s are verifiable. The principal can, therefore, condition her contract on these observable variables. Consequently we denote a contract to the agent as a vector $w \equiv (w_{ln}, w_{lb}, w_{hn}, w_{hb})$ and the contract to the supervisor as a vector $t \equiv (t_{ln}, t_{lb}, t_{hn}, t_{hb})$, where the first subscripts denote the reported costs and the second the outcome of the signal s. Concerning admissible contracts we assume that the supervisor's liability is limited to zero: In none of the events the principal can force the supervisor to make a positive transfer. For the agent we assume a "no slavery condition": The agent cannot be contractually binded to execute the project. He can at any point in time decide to take the outside option not to undertake the project. Consequently, the contract w only specifies payments to the agent conditional on the

[1] The assumption that only the agent can send bribes is a simplifying assumption, which is not crucial for the analysis.

[2] Only when collusion is costly the principal's benefits of preventing collusion will outweigh her costs of preventing it.

realization of the project. Given that the agent can, at a later stage in the game, choose not to produce, the principal could also condition the supervisor payment on the agent's decision. For now we will disregard this possibility and later show that the principal does not benefit by conditioning the supervisor's payment on the agent's decision to produce.

In the following let U_P, U_A and U_S represent the payoff functions of the principal, agent and supervisor. We assume that all players are risk neutral and normalize all outside options to zero.

Timing in the game is as follows:

t=1: Nature chooses the cost of the project and reveals this to the agent and supervisor.

t=2: The principal offers a contract $w \in \mathbb{R}^4_+$ to the agent and a contract $t \in \mathbb{R}^4_+$ to the supervisor. These contracts are public information.

t=3: The supervisor decides whether to accept the contract.

t=4: The supervisor and agent decide whether to collude.

t=5: The supervisor reveals his report r.

t=6: The signal s is revealed.

t=7: The agent decides whether to execute the project.

t=8: Payoffs are realized.

Note that stage 3 of the game is redundant. The supervisor does not incur any costs. All contracts $t \in \mathbb{R}^4_+$ are therefore individual rational and will be accepted by the supervisor.

In stage 7 the agent has to decide whether to execute the project. He will do so if the wage he gets for realizing the project outweighs the costs. At stage 7 the wage and the cost are perfectly known to him. Consequently, his decision is straightforward. The project is realized when the relevant wage w is larger than or equal to the cost c. We introduce the following two indicator functions, which we will later use for expressing the agent's decision.

$$I_l(x) \equiv \begin{cases} 1 & \text{if} \quad x \geq c_l \\ 0 & \text{otherwise} \end{cases} \qquad I_h(x) \equiv \begin{cases} 1 & \text{if} \quad x \geq c_h \\ 0 & \text{otherwise.} \end{cases}$$

3 The Optimal Contract

Benchmarks

Before deriving the optimal contract for the game, we will briefly comment on simplified versions of the model in order to help to develop some intuition for the original game.

When the principal does not make use of a supervisor, she receives neither a report r nor the signal s. The optimal contract is given by a degenerated direct mechanism. The mechanism is degenerated in the sense that it prescribes identical schedules to both types of agents.[3] In the case that $R - c_h > \nu(R - c_l)$ it is optimal for the principal to offer a wage $w = c_h$ independent of the agent's announcement of his type. Under the parameter constellation $R - c_h < \nu(R - c_l)$ the optimal contract specifies a flat wage $w = c_l$.

The game is trivial when the players cannot forge the report. In this case the supervisor must truthfully reveal the cost of the project to the principal. The contract $t^* \equiv (0, 0, 0, 0)$, $w^*_{lb} \equiv w^*_{ln} \equiv c_l$ and $w^*_{hb} \equiv w^*_{hn} \equiv c_h$ gives the principal the payoff $U_P(w^*, t^*) = \nu(R - c_l) + (1 - \nu)(R - c_h)$. The contract (w^*, t^*) achieves the first best.

When the agent and supervisor are able to collude and forge the report, the contract (w^*, t^*) does no longer attain the first best. The low cost agent and the supervisor will collude in order to divide the surplus $\Delta c \equiv c_h - c_l$. This implies that under the contract (w^*, t^*) the principal receives a report $r = h$ whatever the cost of the project and has an expected payoff of $R - c_h$.

When collusion is possible, the principal has two options. She can design the contract (w, t) in such a way that there is no surplus from colluding. We will define such a contract as collusion-proof. The supervisor's report is truthful and the principal can make effective use of the report. A second option is to allow collusion to take place. In this case collusion will occur and the supervisor's report will not be truthful.[4] In the following we first show that there exists an optimal contract which is collusion-proof and compute the optimal contract. Note that in this section we explicitly assume that the principal can fully commit to her contracts and renegotiation does not take place.

Collusion-proofness

In order to ensure that collusion does not take place, the principal has to design the contract (w, t) in such a way that there does not exist a surplus between the agent and

[3] As Tirole (1992) notes the problem is identical to the classical pricing decision of a monopolist who faces two types of consumers with a different willingness to pay between which she cannot price-discriminate.

[4] A further option would be to allow collusion to take place with a certain probability. We here concentrate on pure actions only. Later we will shortly come back to the issue of probabilistic collusion.

the supervisor from colluding. In principle the principal has to prevent two forms of collusion. First, collusion may occur between the low cost agent and the supervisor and, second, the high cost agent and the supervisor may collude. Since optimal contracts will be weakly monotonic increasing with reported cost, the relevant threat of collusion comes from the low cost agent. A low cost agent will want to pass for a high cost agent in order to receive a higher payment. We will therefore concentrate on collusion by the low cost agent and ex post check whether the obtained optimal contract does not induce collusion between the high cost agent and supervisor.

Whether collusion occurs depends on the effect which the report has on the payoffs of the agent and supervisor. Let the project be of the low cost $c = c_l$. Then, if the supervisor reports the cost truthfully, this results in an expected payoff to the agent and the supervisor of

$$U_A^T(w) \equiv (1-q)(w_{ln} - c_l)I_l(w_{ln}) + q(w_{lb} - c_l)I_l(w_{lb})$$
$$U_S^T(t) \equiv (1-q)t_{ln} + qt_{lb}.$$

Collusion, on the other hand, implies that a forged report $r = h$ is sent. The expected payoffs gross of the bribe are consequently,

$$U_A^F(w) \equiv (1-p)(w_{hn} - c_l)I_l(w_{hn}) + p(w_{hb} - c_l)I_l(w_{hb})$$
$$U_S^F(t) \equiv (1-p)t_{hn} + pt_{hb}.$$

By assumption collusion cannot be accompanied by a negative transfer from the agent to the supervisor. A necessary condition for collusion is therefore $U_A^F(w) > U_A^T(w)$. In this case the agent is willing to send a non-negative bribe b of at most $b^{max} \equiv U_A^F(w) - U_A^T(w)$. In order for the supervisor to collude he has to receive a bribe of at least $b^{min} \equiv U_S^T(t) - U_S^F(t)$. It follows that collusion will not occur if

$$kb^{max} \leq b^{min}. \tag{1}$$

In this case the maximum transfer the agent is willing to give for collusion is not enough to induce the supervisor to cooperate. We can rewrite condition (1) as

$$\begin{aligned}(1-q)t_{ln} + qt_{lb} - (1-p)t_{hn} - pt_{hb} \geq k[&(1-p)(w_{hn} - c_l)I_l(w_{hn}) \\ &+p(w_{hb} - c_l)I_l(w_{hb}) - (1-q)(w_{ln} - c_l)I_l(w_{ln}) - q(w_{lb} - c_l)I_l(w_{lb})].\end{aligned} \tag{1'}$$

This leads to the following definition of collusion-proofness.

Definition 1 *The contract pair (w,t) is collusion-proof if and only if it satisfies the collusion-proofness constraint (1).*

A contract pair (w,t) which is collusion proof yields the principal a payoff

$$\begin{aligned} U_P^{CP} &= (1-\nu)[(1-q)((R-w_{hn})I_h(w_{hn})-t_{hn})+q((R-w_{hb})I_h(w_{hb})-t_{hb})] \\ &\quad +\nu[(1-q)((R-w_{ln})I_l(w_{ln})-t_{ln})+q((R-w_{lb})I_l(w_{lb})-t_{lb})]. \end{aligned} \tag{2}$$

A contract pair (w,t) which is not collusion proof yields the payoff

$$\begin{aligned} U_P^{NCP} &= (1-\nu)[(1-q)((R-w_{hn})I_h(w_{hn})-t_{hn})+q((R-w_{hb})I_h(w_{hb})-t_{hb})] \\ &\quad +\nu[(1-p)((R-w_{hn})I_l(w_{hn})-t_{hn})+p((R-w_{hb})I_l(w_{hb})-t_{hb})]. \end{aligned} \tag{3}$$

Proposition 1 *If an optimal contract exists then there exists an optimal contract pair* $(\hat{w},\hat{t})$ *with* $\hat{w} \geq c_l$.

Proof: We show that for every contract $(\hat{w},\hat{t})$ with $\hat{w}_i < c_l$ the principal can propose a contract $(w',\hat{t})$ with $w' \geq c_l$ which yields a weakly higher payoff. Let $(\hat{w},\hat{t})$ be such that $\hat{w}_i < c_l$ for some $i \in \{ln,lb,hn,hb\}$. Now consider the contract w' with $w'_i = c_l$ and $w'_j = \hat{w}_j$ for $j \neq i$. Although it holds that $I_l(\hat{w}_i) = 0$ and $I_l(w'_i) = 1$, it follows from the fact that $w'_i - c_l = 0$ that $U_A^T(\hat{w}) = U_A^T(w')$ and $U_A^F(\hat{w}) = U_A^F(w')$. This implies that $(w',\hat{t})$ is collusion proof if and only if $(\hat{w},\hat{t})$ is collusion proof. Since $R > c_l$ the principal is weakly better off proposing the contract pair $(w',\hat{t})$.

Q.E.D.

Proposition 2 *For any contract pair* (w,t) *with* $w \geq c_l$ *which is not collusion proof there exists a contract* (w,t') *which is collusion proof and yields the principal a weakly higher payoff.*

Proof: Let (w,t) be a contract pair with $w \geq c_l$ which induces the agent and the supervisor to collude. The principal's payoff for such a contract is given by (3).

Now consider the contract t' with $t' = (t'_{ln},t'_{lb},t'_{hn},t'_{hb}) = (t_{ln}+B,t_{lb}+B,t_{hn},t_{hb})$ and $B = kb^{max} - b^{min} > 0$. For the contract pair (w,t') it holds that $U_S^T(t') = (1-q)t'_{ln} + qt'_{lb} = U_S^T(t) + B$ and $U_S^F(t') = U_S^F(t)$. This implies that $b^{min'} = U_S^T(t') - U_S^F(t') = U_S^T(t) + B - U_S^F(t) = b^{min} + B = kb^{max}$. The contract pair (w,t') satisfies the collusion proofness constraint (1) and is therefore collusion proof. The principal's payoff associated with the contract pair (w,t') is given by (2). It follows that

$$\begin{aligned} &U_P(w,t') - U_P(w,t) = \\ &\quad = \nu[(1-q)(R-w_{ln}-t'_{ln})+q(R-w_{lb}-t'_{lb})] \\ &\qquad -\nu[(1-p)(R-w_{hn}-t_{hn})+p(R-w_{hb}-t_{hb})] \\ &\quad = \nu[(1-q)(-w_{ln}-t_{ln})+q(-w_{lb}-t_{lb})-(1-p)(-w_{hn}-t_{hn})-p(-w_{hb}-t_{hb})-B] \end{aligned}$$

$$
\begin{aligned}
&= \nu[-(1-q)(w_{ln}-c_l) - q(w_{lb}-c_l) + (1-p)(w_{hn}-c_l) + p(w_{hb}-c_l) \\
&\quad - (1-q)t_{ln} - qt_{lb} + (1-p)t_{hn} + pt_{hb} - B] \\
&= \nu[-U_A^T(w) + U_A^F(w) - U_S^T(t) + U_S^F(t) - (kb^{max} - b^{min})] \\
&= \nu(1-k)b^{max} > 0
\end{aligned}
$$

Q.E.D.

From proposition 1 and proposition 2 it follows that an optimal contract may be found in the set of contracts which are collusion proof. Thus an optimal contract pair is a contract pair $(\hat{w}, \hat{t})$ which maximizes

$$
\begin{aligned}
U_P(w,t) \;=\; & \nu((1-q)((R-w_{ln})I_l(w_{ln}) - t_{ln}) + q((R-w_{lb})I_l(w_{lb}) - t_{lb})) \\
& +(1-\nu)((1-q)((R-w_{hn})I_h(w_{hn}) - t_{hn}) + q((R-w_{hb})I_h(w_{hb}) - t_{hb})). \quad (4)
\end{aligned}
$$

subject to the collusion-proofness constraint (1). Two observations regarding the optimal contract pair follow immediately. First, the optimal contract satisfies $\hat{t}_{hn} = \hat{t}_{hb} = 0$, since the principal's payoff is decreasing in t_{hn} and t_{hb} and the collusion-proofness constraint is given more slack when t_{hn} or t_{hb} decreases. It implies that in the optimum the supervisor is paid an amount of zero for a report $r = h$. Second, the collusion-proofness constraint is binding at the optimum. If the collusion-proofness constraint does not bind then the principal is better off offering the supervisor a contract with a lower t_{ln} or t_{lb}, since her payoff is decreasing in t_{ln} and t_{lb}.[5] We can therefore treat the weak inequality in (1') as strict equality. Substituting (1') into the objective function (4) leads to

$$
\begin{aligned}
\max_w \quad U_P(w) = {} & (1-\nu)(1-q)(R-w_{hn})I_h(w_{hn}) - \nu k(1-p)(w_{hn}-c_l)I_l(w_{hn}) \\
& +(1-\nu)q(R-w_{hb})I_h(w_{hb}) - \nu kp(w_{hb}-c_l)I_l(w_{hb}) \\
& +\nu(1-q)(R-(1-k)w_{ln} - kc_l)I_l(w_{ln}) \\
& +\nu q(R-(1-k)w_{lb} - kc_l)I_l(w_{lb}).
\end{aligned} \quad (5)
$$

In the following the equilibrium will depend on the parameter constellation. To simplify notation we introduce the following function

$$S(a,b) \equiv a(1-\nu)(R-c_H) - b\nu\Delta c.$$

[5] Due to the monotonicity of the optimal contract we have $U_A^F(w) - U_A^T(w) \geq 0$. Consequently the constraint $t_{ln}, t_{lb} \geq 0$ is automatically satisfied when the collusion-proofness constraint (1') holds.

Proposition 3 *The optimal contract pair* $(\hat{w}, \hat{t})$ *depends on the parameter constellation in the following way*

i) If $S(1-q,(1-p)k) \leq 0$ *then* $(\hat{w}_{ln}, \hat{w}_{lb}, \hat{w}_{hn}, \hat{w}_{hb}) = (c_l, c_l, c_l, c_l)$ *and* $\hat{t} = (0,0,0,0)$. *The principal's payoff is* $U_P(\hat{w}, \hat{t}) = \nu(R - c_l)$.

ii) If $S(1-q,(1-p)k) \geq 0$ *and* $S(q,pk) \geq 0$ *then* $(\hat{w}_{ln}, \hat{w}_{lb}, \hat{w}_{hn}, \hat{w}_{hb}) = (c_l, c_l, c_h, c_h)$ *and* $(\hat{t}_{ln}, \hat{t}_{lb}, \hat{t}_{hn}, \hat{t}_{hb}) = (k\Delta c, k\Delta c, 0, 0)$.[6] *The principal's payoff is* $U_P(\hat{w}, \hat{t}) = \nu(R - c_l) + (1-\nu)(R - c_h) - k\nu\Delta c$.

iii) If $S(1-q,(1-p)k) \geq 0$ *and* $S(q,pk) \leq 0$ *then* $(\hat{w}_{ln}, \hat{w}_{lb}, \hat{w}_{hn}, \hat{w}_{hb}) = (c_l, c_l, c_h, c_l)$ *and* $(\hat{t}_{ln}, \hat{t}_{lb}, \hat{t}_{hn}, \hat{t}_{hb}) = (kp\Delta c, kp\Delta c, 0, 0)$. *The principal's payoff is* $U_P(\hat{w}, \hat{t}) = \nu(R - c_l) + (1-\nu)(1-q)(R - c_h) - \nu k(1-p)\Delta c$.

Proof: Note that the objective function in (5) is piece-wise linear in all w_i and has non-positive slopes. The function shows an upward jump at c_l for all w_i and a second upward jump at c_h for w_{hn} and w_{hb}. From these observations we conclude that the optimal contract is found for $w_{ln} = w_{lb} = c_l$ and $w_{hn}, w_{hb} \in \{c_l, c_h\}$. We have four cases to consider: Case 1: $w = w^1 \equiv (c_l, c_l, c_l, c_l)$ with $U_P(w^1) = \nu(R - c_l)$. Case 2: $w = w^2 \equiv (c_l, c_l, c_h, c_h)$ with $U_P(w^2) = \nu(R - c_l) + (1-\nu)(R - c_h) - \nu k\Delta c$. Case 3: $w = w^3 \equiv (c_l, c_l, c_h, c_l)$ resulting in $U_P(w^3) = \nu(R - c_l) + (1-\nu)(1-q)(R - c_h) - k(1-p)\nu\Delta c$. Case 4: $w = w^4 \equiv (c_l, c_l, c_l, c_h)$ with $U_P(w^4) = \nu(R - c_l) + (1-\nu)q(R - c_h) - p\nu k\Delta c$.

First note that the contract w^4 cannot be optimal. For w^4 to achieve the maximum payoff it is required that $U_P(w^4) > U_P(w^1)$ and $U_P(w^4) > U_P(w^2)$, which is equivalent to $q(1-\nu)(R - c_h) > p\nu k\Delta c$ and $(1-q)(1-\nu)(R - c_h) < (1-p)\nu k\Delta c$. But, since $0 < q < p$ it follows that $q(1-\nu)(R - c_h) > p\nu k\Delta c \Rightarrow (1-\nu)(R - c_h) > \nu k\Delta c \Rightarrow (1-q)(1-\nu)(R - c_h) > (1-p)\nu k\Delta c$. The two conditions are therefore incompatible.

Comparing the different payoffs to the principal we arrive at the following conditions i) $U_P(w^1) > U_P(w^2) \Leftrightarrow 0 > (1-\nu)(R - c_h) - \nu k\Delta c \Leftrightarrow S(1,k) < 0$, ii) $U_P(w^3) > U_P(w^2) \Leftrightarrow (1-\nu)(1-q)(R - c_h) - k(1-p)\nu\Delta c > (1-\nu)(R - c_h) - \nu k\Delta c \Leftrightarrow 0 > q(1-\nu)(R - c_h) - p\nu k\Delta c \Leftrightarrow S(q,pk) < 0$ and iii) $U_P(w^1) > U_P(w^3) \Leftrightarrow 0 > (1-\nu)(1-q)(R - c_h) - (1-p)\nu k\Delta c \Leftrightarrow S(1-q,(1-p)k) < 0$. It follows that if $U_P(w^2) > U_P(w^1)$ then $U_P(w^3) > U_P(w^1)$ and the proposition is immediate. Finally note that collusion by the high cost agent and the supervisor will indeed not occur under the contracts which the proposition specifies.

Q.E.D.

The statement of proposition 3 is illustrated in figure 1. The diagram depicts the

[6] Note that any contract t satisfying $(1-q)t_{ln} + qt_{lb} = k\Delta c$ is optimal. We take $t_{ln} = t_{lb}$, which has an intuitive interpretation: The principal is not interested in the signal s when the report is $r = l$. She then knows that the report is truthful and the signal s is non-informative.

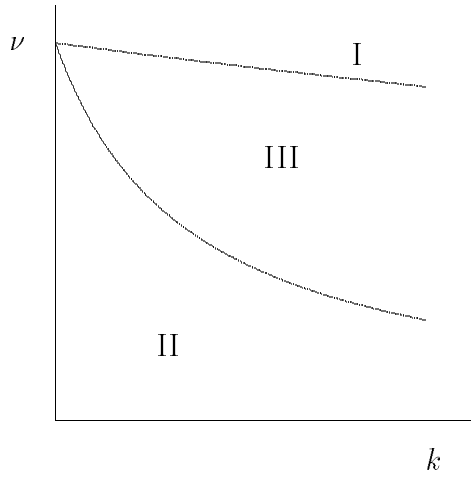

Figure 1: The optimal contracts

regions in which the different contracts are optimal. We will shortly discuss each of the regions.

The intuition behind the optimal contract is best explained by contrasting it to the first best contract (w^*, t^*). We showed that the contract (w^*, t^*) induces collusion and therefore does not attain the first best payoff. We further showed that there necessarily exists an optimal contract which is collusion proof and the principal's problem was to find such a contract. One may therefore view the principal's problem as dealing with collusion in some optimal way.

The principal can regulate the incentives concerning collusion in two ways. On the one hand she can reduce the stake in collusion by making the payment to the agent less sensitive to the supervisor's report. One the other hand the principal can outbid the agent's bribe by offering the supervisor a high wage in the case he reports that the cost are low. Note that this cannot tempt the supervisor to report low costs when the costs are actually high, because the falsification of the report is only possible with the consent of the agent. Proposition 3 shows that the principal's optimal use of these two methods depends on the parameter constellation and differ in the three regions in figure 1.

In region I the principal offers the agent a flat wage c_l and practices the method of stake reduction: The agent's payment does not depend on the supervisor's report and the agent has no stake in colluding with the supervisor. The costs of inducing the supervisor to report the truth are therefore zero. The drawback of the contract is, however, that the agent only executes the project when it is of low cost. Therefore, the contract can only be optimal if the expected cost of losing the project is relatively small as compared to the cost of preventing collusion. This means that ν and k must be relatively close to one. Note also that the principal conditions payments neither on the report r nor the

signal s.

In region II the principal uses outbidding as the method to prevent collusion. The stake of the agent is the same as under the first best contract (w^*, t^*). The threat of collusion is therefore big and the principal has to prevent collusion by inducing the supervisor to report the truth. This means that she has to outbid the agent's stake. The advantage of this method as compared to the contract used in region I is that the agent always executes the project. The drawback is that preventing collusion is relatively expensive when the bribing technology is efficient. Therefore, this method can only be optimal if the costs of preventing collusion are relatively low in comparison to the net value of the project. Note that here the principal uses only the report r to determine the payments and disregards the signal s.

In region III the principal conditions the agent's contract on the supervisor's report and the signal s and uses both methods at the same time. She offers the agent a high payment only if high costs are reported and the signal indicates that no bribing has taken place. A low cost agent will therefore not always receive the high payment when he colludes with the supervisor. Compared to the first best contract (w^*, t^*) his stake is therefore reduced. Since the stake is not reduced to zero, the principal also has to set incentives such that the supervisor reports the truth. The principal therefore practices also outbidding. Compared to the optimal contracts in regions I and II the contract is an intermediate one. The probability of non-production is smaller than for the type of contracts which are optimal in region I. The incentives costs for inducing the supervisor to report the truth are lower than for the type of contracts which are optimal in region II. Since the probability of non-production is increasing in q and the stake-reduction decreasing in p, the intermediate contract is especially attractive when q is small and p is large, i.e. when the signal s is highly informative.

Although the agent's decision to produce is in principle a contractible variable, we did not allow the principal to condition the supervisor's wage on this variable. It therefore seems that the contracts in proposition 3 are suboptimal and the principal can improve her situation by conditioning the supervisor's wage on the agent's decision. This is, however, not the case. The intuition for this is that under the contracts in proposition 3 non-production does not indicate that collusion has taken place. On the contrary when the agent does not produce then this is an indication that the supervisor's report was indeed truthful. If conditioning the supervisor's payment on the extra variable is helpful then it must be by promising him a high payment the case that production did not take place. This will reduce the attractiveness of collusion by making it more expensive for the agent to bribe the supervisor. The problem is, however, that non production takes place when the project is in fact of high cost. But if the project is of high cost then

inducing the supervisor to tell the truth is not a problem. The threat of collusion comes from the low cost agent and conditioning the supervisor's payment on the decision of the low cost agent does not help in dealing with collusion. Giving the principal the extra possibility to condition the supervisor's payment on the agent's decision to product does therefore not improve her situation.

Proposition 4 *The principal cannot do better by conditioning the supervisor's payment on the agent's decision whether to produce.*

Proof: Let $(w,t,\tau) \in \mathbb{R}^{12}$ be the optimal contract pair in the case that the supervisor's contract is also conditioned on the agent's decision to produce, where t is the relevant contract when the agent executes the project and τ the relevant contract when the agent does not execute the project. Then it is strictly better for the principal to condition the supervisor's payment on the agent's decision if and only if $U_P(w,t,\tau) > U_P(\hat{w},\hat{t})$ for all $\hat{w},\hat{t} \in \mathbb{R}^4_+$. We will show that this cannot be the case.

With some abuse of notation we may introduce the variables $U_S^T(t,\tau)$ and $U_S^F(t,\tau)$ as

$$\begin{aligned} U_S^T(t,\tau) &= (1-q)[t_{ln}I_l(w_{ln}) + \tau_{ln}(1-I_l(w_{ln}))] + q[t_{lb}I_l(w_{lb}) + \tau_{lb}(1-I_l(w_{lb}))] \\ U_S^F(t,\tau) &= (1-p)[t_{hn}I_l(w_{hn}) + \tau_{hn}(1-I_l(w_{hn}))] + p[t_{hb}I_l(w_{hb}) + \tau_{hb}(1-I_l(w_{hb}))] \end{aligned}$$

Then the contract (w,t,τ) is collusion proof if and only if

$$k(U_A^F(w) - U_A^T(w)) \leq U_S^T(t,\tau) - U_S^F(t,\tau). \quad (1'')$$

Furthermore, if (w,t,τ) induces no collusion then

$$\begin{aligned} U_P(w,t,\tau) &= (1-\nu)[(1-q)((R-w_{hn}-t_{hn})I_h(w_{hn}) - \tau_{hn}(1-I_h(w_{hn}))) \\ &\quad +q((R-w_{hb}-t_{hb})I_h(w_{hb}) - \tau_{hb}(1-I_h(w_{hb})))] \\ &\quad +\nu[(1-q)((R-w_{ln}-t_{ln})I_l(w_{ln}) - \tau_{ln}(1-I_l(w_{ln}))) \\ &\quad +q((R-w_{lb}-t_{lb})I_l(w_{lb}) - \tau_{lb})(1-I_l(w_{lb}))]. \end{aligned} \quad (2')$$

If (w,t,τ) induces collusion to take place between a low cost agent and the supervisor then

$$\begin{aligned} U_P(w,t,\tau) &= (1-\nu)[(1-q)((R-w_{hn}-t_{hn})I_h(w_{hn}) - \tau_{hn}(1-I_h(w_{hn}))) \\ &\quad +q((R-w_{hb}-t_{hb})I_h(w_{hb}) - \tau_{hb}(1-I_h(w_{hb})))] \\ &\quad +\nu[(1-p)((R-w_{hn}-t_{hn})I_l(w_{hn}) - \tau_{hn}(1-I_l(w_{hn}))) \\ &\quad +p((R-w_{hb}-t_{hb})I_l(w_{hb}) - \tau_{hb})(1-I_l(w_{hb}))]. \end{aligned} \quad (3')$$

Now consider the contract $\bar{t} \in \mathbb{R}^4$ which is not conditioned on the agent's decision with $\bar{t}_{ln} = t_{ln}I_l(w_{ln}) + \tau_{ln}(1-I_l(w_{ln}))$, $\bar{t}_{lb} = t_{lb}I_l(w_{lb}) + \tau_{lb}(1-I_l(w_{lb}))$, $\bar{t}_{hn} = t_{hn}I_h(w_{hn}) + \tau_{hn}(1-I_h(w_{hn}))$ and $\bar{t}_{hb} = t_{hb}I_h(w_{hb}) + \tau_{hb}(1-I_h(w_{hb}))$. Then $U_S^T(\bar{t}) = U_S^T(t,\tau)$ and

$U_S^F(\bar{t}) = U_S^F(t,\tau)$. This implies that the contract $(w,\bar{t})$ induces collusion between a low cost agent and the supervisor if and only if the contract (w,t,τ) induces collusion between a low cost agent and the supervisor. Note further that $U_P(w,\bar{t}) = U_P(w,t,\tau)$, which means that (w,t,τ) can be replicated by a contract $(w,\bar{t})$ which is not conditioned on the agent's decision.

Q.E.D.

4 Renegotiation

Consider a contract which is optimal in region I or III. Under these contracts the principal does not make full use of the supervisor's truthful report. For instance, when the principal receives a report $r = h$ and a signal $s = b$, she does not follow the supervisor's report, but sets a transfer c_l to the agent. When the actual cost of the project is c_h then this gives rise to an inefficiency. The agent is offered a transfer c_l for realizing the project, but will decline the offer, since his cost is larger than c_l. The agent does therefore not execute the project, even though the principal's willingness to pay R is greater than the cost of the project c_h. The optimal contracts for region I and III leave scope for inefficiencies. For the optimal contracts in region I these inefficiencies take place whenever the cost of the project is high, i.e. with probability ν. For optimal contracts in region III the inefficiency occurs with a probability $(1-\nu)q$.

The ex-post inefficiency prompts us to look at renegotiation and commitment. The important observation is that when the principal receives a report $r = h$ and a signal $s = b$ she in fact knows that the actual cost of the project is c_h. We showed that the optimal contract is collusion-proof. Upon receiving a report $r = h$ the principal will also conclude that the cost of the project is indeed c_h. As a consequence she realizes that the agent will refuse to execute the project if she sticks to a transfer c_l. After stage 6 she may have an incentive to renegotiate the contract and raise w_{hb} to c_h. Note that the rise is also weakly preferred by the agent and is therefore a Pareto improvement.

We incorporate renegotiation by introducing an intermediate stage $6^1/_2$, where we allow the principal to propose a new contract w and in which the agent may decide to accept the new contract or to stick to the old contract. In stage 7 the agent decides whether to execute the project given the contract which is relevant at that stage.

At the renegotiation stage the principal forms a belief about the cost of the project. Let $\sigma(w,t,r,s)$ represent the principal's belief that the cost of the project is c_l given the contract (w,t), the report r and the signal s. Then we can define renegotiation-proofness in the following way.

Definition 2 *A contract (w,t) is renegotiation-proof if it satisfies for all $r \in \{h,l\}$ and $s \in \{b,n\}$*

$$(a)\ (R-c_l)\sigma(w,t,r,s) > 0 \Rightarrow w_{rs} \geq c_l$$
$$(b)\ (R-c_l)\sigma(w,t,r,s) < R-c_h \Rightarrow w_{rs} \geq c_h.$$

Condition a) states that when the principal attaches positive probability to the event that the project is of low cost then she should offer at least a payment c_l for executing the project. The second condition states that when the principal believes she receives a higher expected payoff from offering a wage c_h instead of offering less than c_h, then she should at least offer a payment c_h. It is obvious that when these conditions are not satisfied by a contract (w,t) then there exists a contingency in which the contract is renegotiated. When the two conditions are satisfied no such contingency exists and the principal will not have an incentive to renegotiate.

Proposition 5 *Any payoff associated with a contract which is not renegotiation-proof can also be achieved by a contract which is renegotiation-proof.*

Proof: Consider a contract (w,t) which is not renegotiation-proof. Then it is common knowledge that this contract will be renegotiated into a contract (w',t) at a later stage. Rationality will ensure that all players act as if the relevant contract is the contract (w',t). Consequently the payoffs under the contract (w,t) and the contract (w',t) are identical.

Q.E.D.

It follows directly that we may assume without loss of generality that the optimal contract is renegotiation-proof.

In equilibrium the principal's beliefs should be consistent with the behavior of the agent and supervisor. Since the supervisor and agent do not collude given the contract $(\hat{w},\hat{t})$, consistency of beliefs requires that $\sigma(\hat{w},\hat{t},r,s) = 0$ for $r = h$ and $s \in \{n,b\}$. As a consequence the full-screening contract is not renegotiation-proof, since $\hat{w}_{hb} = c_l < c_h$. It follows that any contract (w,t) which is collusion-proof must specify $w_{hn} \geq c_h$ and $w_{hb} \geq c_h$ in order to be renegotiation-proof. This implies that independent of the parameter constellation the optimal collusion-proof contract which is also renegotiation-proof is the partial-screening contract $w_{lb} = w_{ln} = c_l$ and $w_{hb} = w_{hn} = c_h$.

Another implication of proposition 5 is that there may no longer exist an optimal contracts which is collusion proof as shown in proposition 2. In order to prove proposition

2 we showed that for every non-collusion-proof contract one can find a collusion-proof contract which achieves a weakly higher payoff. It is, however, not ensured that one can find for every non-collusion-proof contract which is renegotiation-proof a collusion-proof contract which is also renegotiation-proof. We therefore can no longer assume that there exists an optimal contract which is collusion-proof and must also consider contracts which are not collusion-proof.

Optimal non-collusion proof contract

If the agent and the supervisor collude then the report does not contain any information, because whatever the cost of the project the same report is sent. Consequently, the report is uninformative to the principal and she will offer the least costly contract which the supervisor accepts, i.e. $t = (0,0,0,0)$. Again we analyze the case in which the low cost agent and the supervisor collude. This implies that the principal always receives a report $r = h$. The principal's payoff is

$$\begin{aligned} U_P(w) &= \nu p(R - w_{hn})I_l(w_{hn}) + \nu(1-p)(R - w_{hb})I_l(w_{hb}) \\ &\quad +(1-\nu)q(R - w_{hn})I_h(w_{hn}) + (1-\nu)(1-q)(R - w_{hb})I_h(w_{hb}) \end{aligned} \tag{6}$$

Proposition 6 *In the game without renegotiation the contract (w^{ncp}, t^{ncp}) is optimal with respect to the set of contracts which are not collusion-proof, where $w_{ln}^{ncp} = w_{lb}^{ncp} = c_l$ and $t^{ncp} = (0,0,0,0)$. The optimal values for w_{hn}^{ncp} and w_{hb}^{ncp} depend on the parameter constellation in the following way,*

i) If $S(1-q, 1-p) \leq 0$ then $w_{hn}^{ncp} = w_{hb}^{ncp} = c_l$.

ii) If $S(q,p) \geq 0$ then $w_{hn}^{ncp} = w_{hb}^{ncp} = c_h$.

iii) If $S(1-q, 1-p) \geq 0$ and $S(q,p) \leq 0$ then $w_{hn}^{ncp} = c_h$ and $w_{hb}^{ncp} = c_l$.

Proof: The principal's payoff does not depend on w_{ln} and w_{lb}. When the low cost agent always colludes, the principal will never receive a report $r = l$ and the wages w_{ln} and w_{lb} can be set arbitrarily. For reasons which will become clear later we set $w_{ln} = w_{lb} = c_l$. Note, however, that the wages w_{ln}, w_{lb} do affect the decision regarding collusion and therefore influence payoffs indirectly. The function $U_P(w)$ is a discontinuous, piece-wise linear function. Again we have four cases to consider: a) $w_{hn} = w_{hb} = c_l$ leading to a payoff $U_P^a \equiv \nu(R - c_l)$. b) $w_{hn} = w_{hb} = c_h$ resulting in a payoff of $U_P^b \equiv R - c_h$. c) $w_{hn} = c_h$ and $w_{hb} = c_l$, which yields the principal a payoff $U_P^c \equiv \nu p(R - c_l) + \nu(1 - p)(R - c_h) + (1-\nu)(1-q)(R - c_h)$. d) $w_{hn} = c_l$ and $w_{hb} = c_h$ yielding the payoff $U_P^d \equiv \nu p(R - c_h) + \nu(1-p)(R - c_l) + (1-\nu)q(R - c_h)$. Comparing the principal's payoffs under the different wage contracts leads to the observation that the contract in case d) cannot be optimal. It would require that $U_P^d > U_P^a$ and $U_P^d > U_P^b$ which is equivalent to

requiring that $q(1-\nu)(R-c_h) > p\nu\Delta c$ and $(1-q)(1-\nu)(R-c_h) < (1-p)\nu\Delta c$. However, since $0 < q < p$ it follows that $q(1-\nu)(R-c_h) > p\nu\Delta c \Rightarrow (1-\nu)(R-c_h) > \nu\Delta c \Rightarrow (1-q)(1-\nu)(R-c_h) > (1-p)\nu\Delta c$. This implies that $S(q,p) > 0 \Rightarrow S(1-q,1-p) < 0$ and that both inequalities cannot hold simultaneously.

From comparing the payoffs in the cases a, b and c we may conclude that i) $U_P^a > U_P^c \Leftrightarrow (1-q)(1-\nu)(R-c_h) < (1-p)\nu\Delta c \Leftrightarrow S(1-q,1-p) < 0$ and ii) $U_P^b > U_P^c \Leftrightarrow q(1-\nu)(R-c_h) > p\nu\Delta c \Leftrightarrow S(q,p) > 0$.

From the fact that $S(q,p) > 0 \Rightarrow S(1-q,1-p) > 0$ it follows that if $S(q,p) > 0$ then $U_P^b > U_P^c > U_P^a$. By noting that under these contracts collusion by the low cost agent and supervisor will indeed occur, the proposition is then immediate.

Q.E.D.

Proposition 7 *The optimal contracts in proposition 6 are renegotiation-proof.*

Proof: It trivially holds that the contract specifying $w_{hn} = w_{hb} = c_h$ is renegotiation-proof. For the contract $w_{ln} = w_{lb} = w_{hb} = c_l$ and $w_{hn} = c_h$ it follows by Bayes' rule that

$$\sigma(w,t,h,b) = \frac{p\nu}{p\nu + q(1-\nu)}. \tag{7}$$

Note that the contract is optimal when $p\nu\Delta c \geq q(1-\nu)(R-c_h)$. This condition together with (7) leads to the conclusion that when the contract is optimal w.r.t. the set of non-collusion-proof contracts then it is also renegotiation-proof. By the same argument one may verify that when $w_{hn} = w_{hb} = c_l$ is part of the optimal non-collusion-proof contract then it is also renegotiation-proof.

Q.E.D.

Optimal contract in the game with renegotiation

Proposition 8 *The optimal contract in the game with renegotiation depends on the parameter constellation in the following way:*

i) If $S(1,k) \leq 0$ and $S(1-q,1-p) \leq 0$ then the optimal contract is neither conditioned on the report r nor on the signal s. The principal's payoff is $U_P = \nu(R-c_l)$.

ii) If $S(1,k) \geq 0$ and $S(q,k+p-1) \geq 0$ then the optimal contract is collusion-proof. The optimal contract is conditioned on the report r, but not on the signal s. The principal's payoff is $U_P = \nu(R-c_l) + (1-\nu)(R-c_h) - \nu k\Delta c$.

iii) If $S(1-q,1-p) \geq 0$ and $S(q,k+p-1) \leq 0$ then the optimal contract is not collusion-proof. The optimal contract is conditioned on both the report r and the signal s. The principal's payoff is $U_P = \nu p(R-c_l) + [\nu(1-p) + (1-\nu)(1-q)](R-c_h)$.

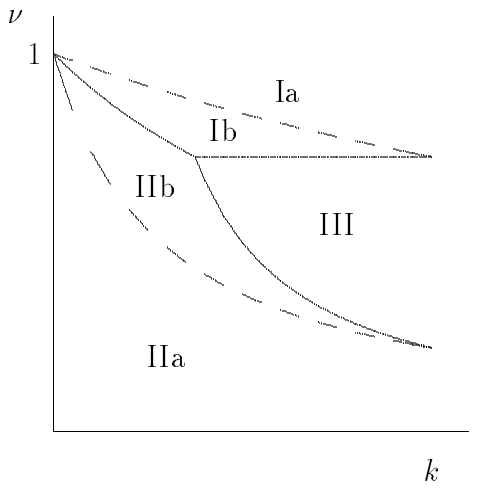

Figure 2: The optimal contracts with renegotiation

Proof: We compare the optimal contract which is collusion proof, U^{cp*}, to the optimal contract which is not collusion proof, U^{ncp*}. The optimal contract which is collusion proof is the contract $w^2 = (c_l, c_l, c_h, c_h)$. Therefore, $U^{cp*} = U_P^2 = \nu(R-c_l)+(1-\nu)(R-c_h) - k\nu\Delta c$.

i) For $S(1-q, 1-p) > 0$ the payoff U^{ncp*} is $U_P^a = \nu(R-c_l)$. It follows that $U^{cp*} > U^{ncp*}$ $\Leftrightarrow \nu(R-c_l)+(1-\nu)(R-c_h)-k\nu\Delta c > \nu(R-c_l) \Leftrightarrow (1-\nu)(R-c_h) > k\nu\Delta c \Leftrightarrow S(1,k) > 0$.

ii) For $S(q,p) \geq 0$ the payoff U^{ncp*} is $U_P^b = R - c_h$. This is always smaller than U_P^2, since $\nu(R-c_l)+(1-\nu)(R-c_h)-k\nu\Delta c > \nu(R-c_l)+(1-\nu)(R-c_h)-\nu\Delta c = (R-c_h)$.

iii) For $S(q,p) \leq 0$ and $S(1-q, 1-p) \geq 0$ the payoff U^{ncp*} is $U_P^c = \nu p(R-c_l) + \nu(1-p)(R-c_h) + (1-\nu)(1-q)(R-c_h)$. It follows that $U_P^2 > U_P^c \Leftrightarrow \nu(R-c_l) + (1-\nu)(R-c_h) - k\nu\Delta c > \nu p(R-c_l) + \nu(1-p)(R-c_h) + (1-\nu)(1-q)(R-c_h) \Leftrightarrow -k\nu\Delta c > -\nu(1-p)(R-c_l)+\nu(1-p)(R-c_h)-q(R-c_h) \Leftrightarrow q(R-c_h) < (p+k-1)\nu\Delta c \Leftrightarrow S(q, p+k-1) < 0$.

Q.E.D.

Figure 2 depicts the regions for which the different contracts are optimal. In region Ia it is optimal for the principal to practice stake-reduction and to offer a flat wage independent of the report r and the signal s. The payoff associated with this contract is equal to the maximum payoff of the principal in the game without renegotiation. In fact under this parameter constellation the principal is not interested in employing a supervisor or receiving the signal s. Also in the region Ib the principal does not condition the wages on the report r or the signal s. The contract, however, differs from the optimal contract in the game without renegotiation. In the game without

renegotiation the optimal contract conditions wages on the report r and the signal s. Such a contract creates ex post inefficiencies and is therefore not renegotiation proof. As a consequence, it will be renegotiated in the game with renegotiation and is no longer optimal. Instead it is now optimal for the principal to offer a flat contract to the agent. Note that the principal does strictly worse when she is known to renegotiate ex post inefficiencies.

In region II the principal offers a collusion-proof contract in equilibrium. The region can be divided into two subregions. The area IIa depicts the region in which the optimal contract is identical to the optimal contract which we obtained in the game without renegotiation. Consequently, also the payoffs are the same in both games. In the region IIb the optimal contract in the game with renegotiation differs from the optimal contract in the game without renegotiation. When the principal can commit not to renegotiate she offers a contract which also conditions the agent's contract on the signal s. The contract is, however, not renegotiation-proof and no longer optimal. Instead it is optimal for the principal to offer a different collusion-proof contract. It follows that the maximum payoff of the principal is lower in the game with renegotiation than in the game without renegotiation.

The area of most interest is region III. For this region the optimal contract also differs from the optimal contract in the game without renegotiation. The interesting point is that the optimal contract has changed from a collusion-proof contract into a contract which is not collusion-proof. We obtain that it is optimal for the principal to set contracts such that collusion between the agent and the supervisor is induced.

The intuition behind the result becomes clear by looking more closely at the principal's dilemma. The principal would like to obtain information about the cost of the project, because with this information she is able to finance the project at a lower cost. However, obtaining the information lures her into renegotiating the contract. Such behavior is anticipated and will distort incentives ex ante. The principal has therefore two options. Her first option is to set contracts such that she will obtain the information during the game and can use it to realize the project at a lower cost. The disadvantage is, however, that many of these contracts make the principal renegotiate. Her second option is to offer a contract which will not lead to the revelation of the supervisor's information. In this case the principal is unable to tailor the payment of the agent to the exact costs. The advantage is however that without obtaining the information she is never tempted to renegotiate at a latter stage in the game. In region II it is optimal for the principal to use the first option and offer a collusion-proof contract pair. In the region III the parameter constellation is such that it is optimal for the principal to choose the second option. It is better for her not to obtain the information so that she is not lured into

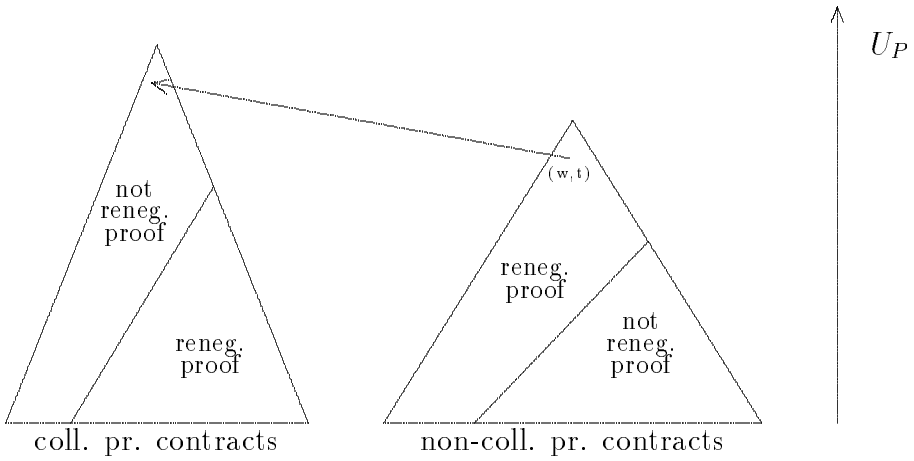

Figure 3: Collusion proof vs. non-collusion proof contracts

renegotiating the contract too easily.

The reasoning is schematically explained in figure 3 The figure shows two triangles which represent sets of contracts. The left triangle depicts the set of contracts which are collusion proof. The right triangle shows the set of contract which are not collusion proof. The sets themselves are ordered in such a way that contracts which are on a higher horizontal plane yield a larger payoff to the principal. The figure shows that the principal can, in principle, attain a higher payoff by offering a collusion proof than by offering a non-collusion proof contract. This is illustrated by the fact that the right triangle reaches higher than the left triangle. Each set is further divided into a subset which contains the renegotiation proof contracts and its complement. As these subsets are drawn it is clear that if renegotiation is an issue then the maximum payoff is attained by a contract which is not collusion proof. In this case the principal has to choose a contract which is renegotiation proof. Being restricted to contracts which are renegotiation proof the contract which achieves the highest payoff is a contract which induces collusion.

The figure further shows a mapping as meant in the proof of proposition 2. There it was shown that for every contract which is not collusion-proof one could find a contract which is collusion proof and yields a higher payoff. The arrow in the figure represents such a mapping and shows what goes wrong when renegotiation is an issue. The mapping namely does not guarantee that the collusion proof contract is also renegotiation proof. As shown here the original contract is renegotiation proof, while its collusion proof counterpart is not.

5 Conclusion and Discussion

This paper showed that when the principal renegotiates ex post inefficient contracts then the optimal contract may necessarily be such that it induces collusion. When the principal can commit not to renegotiate ex post inefficient contracts, then there always exists an optimal contract which is collusion-proof. The optimal contract may, however, produce ex post inefficiencies.

In order to discuss the result we refer to two often used principles. First there is the principle of renegotiation. This principle says that if an optimal contract exists then there exists an optimal contract which is renegotiation proof. Second, there exists the principle of collusion proofness. This principle says that if an optimal contract exists then there exists an optimal contract which is collusion-proof. We have shown that in the game without renegotiation the principle of collusion proofness holds. We have further shown that when renegotiation becomes an issue, then the principle of renegotiation takes precedence over the principle of collusion-proofness and the latter principle may fail to hold.

We may give an alternative explanation for our result. It was shown that when a collusion-proof contract is offered the principal recognizes the inefficiency and wants to renegotiate. In contrast the principal is not certain enough about the inefficiency to renegotiate when she offers a contract which does induce collusion. Since the principal's attitude toward renegotiation is common knowledge, it affects the behavior of players ex ante. This worsens the principal's situation by such a degree that she is better off allowing collusion to take place instead of preventing it. It is the fact that a contract is collusion-proof which informs the principal about the actual state of the world and makes her fully aware of the inefficiency. The information embodied in the collusion-proof contract is harmful to the principal. Interpreting the result in this way it becomes clear that the result is closely linked to a general theme in game-theory that information may worsen a player's situation, when it is common knowledge that this player has information. Extra information changes the behavior of a player and this change is anticipated by other players in the game. In the present paper obtaining information coincides with offering a collusion-proof contract. Since the principal is aware that the extra information worsens her situation, she will not offer a collusion-proof contract. Instead she allows collusion to take place.

This paper focused on pure strategies only. Strausz (1995) looks at the case in which the agent and the supervisor are allowed to collude with a certain probability. It is shown that in this case other equilibria exist, but these equilibria also entail contracts which are necessarily not collusion-proof. More importantly however the contracts are only weakly

renegotiation-proof: There exist some best-response of the coalition "Agent-Supervisor" which induces renegotiation to take place.

The paper makes its point by analyzing a rather simple screening model. It must nevertheless be clear that the obtained result is general. Whenever renegotiation is an issue, it may be that optimal contracts necessarily induce collusion in equilibrium. Scope for renegotiation was introduced by assuming a non slavery condition on part of the agent. This is a straightforward but rather blunt way of introducing renegotiation, but kept the analysis of the problem relatively simple. The no-slavery condition is, however, a natural way of modeling the situation in which the contract between the principal and the agent can only be written at the end of the game after reports, bribes and signals are received. For instance, one may envisage a setting in which the supervisor is a long term employee of the principal, whose task it is to find an appropriate agent for the principal's project. On his report the principal decides which agent to contract for the job. The agent and the supervisor will anticipate the future's contract of the agent and make their decision about collusion appropriately. Such a situation would be identical to our model with renegotiation in which collusion occurs in equilibrium.

The model can also easily be transformed into an optimal taxation problem in the vein of Border and Sobel (1987) when auditing is performed externally. Consider a firm who can either pay a tax $t \in \{t_1, t_2\}$, where $\Delta t \equiv t_2 - t_1 > 0$. The government does not observe t, but has to decide to claim either t_1 or t_2. When the government demands the higher tax t_2, while the firm can only pay the tax t_1, the firm will go bankrupt which indirectly hurts the government by an amount $D > \Delta t$. If the firm is able to pay the tax t_2, then this is preferred by the government. Assume there is a tax collector who knows the type of the firm and plays a similar role as in our procurement model. If there exists an imperfect signal which is correlated with collusion then identical results are obtained: Given certain parameter constellations the fact whether the government offers a collusion-proof contract depends on her attitude towards ex-post inefficiencies. The inefficiency in this setting is whether the government makes the firm go bankrupt when she knows ex post that her tax levy is too high.

References

Border, K.C. and Sobel, J. (1987), "Samurai Accountant: A Theory of Auditing and Plunder", *Review of Economic Studies* 54, p. 525-540.

Dewatripont, M. (1986), "Renegotiation and Information Revelation over Time in Optimal Labor Contracts" Chapter 1, *On the Theory of Commitment, with Appli-*

cations to the Labor Market, Ph.D. dissertation, Harvard University.

Itoh, H. (1993), "Coalitions, Incentives, and Risk Sharing", *Journal of Economic Theory* 60, p. 410-427.

Kofman, F. and Lawarrée, J. (1993), "Collusion in Hierarchical Agency", *Econometrica* 61, p. 629-656.

Laffont, J.J. (1990), "Analysis of hidden gaming in a three-level hierarchy", *Journal of Law, Economics and Organization* 6, p. 301-324.

Laffont, J.J. and Tirole, J. (1991), "The politics of Government Decision-Making: A Theory of Regulatory Capture", *Quarterly Journal of Economics* 106, p. 1087-1127.

Maskin, E. and Tirole, J. (1992), "The Principal Relationship with an informed Principal, II: Common Values", *Econometrica* 60, p. 1-42.

Scheepens, J. (1995), "Collusion and Hierarchy in Banking" Chapter 4, *Financial Intermediation and Corporate Finance*, Ph.D. dissertation, Tilburg University.

Strausz, R. (1995), "Collusion and Renegotiation in a Principal-Supervisor-Agent Relationship", Discussion Paper No. 9548, CentER, Tilburg University.

Tirole, J. (1986), "Hierarchies and Bureaucracies: On the Role of Collusion in Organizations", *Journal of Law, Economics and Organization* 2, p. 181-214.

Tirole, J. (1992), "Collusion and the Theory of Organizations", in: J.J. Laffont (ed.), *Advances in economic theory*, 6th World Congress of the Econometric Society, Vol. II, Cambridge, p. 151-206.