SETTLEMENT NEGOTIATIONS WITH TWO-SIDED ASYMMETRIC INFORMATION: MODEL DUALITY, INFORMATION DISTRIBUTION AND EFFICIENCY*

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Abstract

We analyze a settlement and litigation game in which both parties possess private information relevant to the value of a claim. The plaintiff knows the level of damages, while the defendant knows the probability he will be held liable for those damages. We consider two alternatives: (1) the plaintiff proposes a settlement, which the defendant accepts or rejects; and (2) the defendant proposes a settlement, which the plaintiff accepts or rejects. Despite the extensive symmetry of the model, these alternatives will generally result in different equilibrium expected frequencies of trial, and therefore different social efficiencies.

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I. Introduction

This paper has two main purposes. First, we will provide a reasonably general analysis of settlement and litigation wherein both parties have private information and act in sequence. The analysis involves a plaintiff, who is privately informed about the level of damages incurred, and a defendant, who is privately informed about the likelihood of being found liable for the damages. Two models are explored wherein the roles of settlement proposer and responder are alternately assigned to the plaintiff and the defendant. In one model the plaintiff makes a demand of the defendant, and the defendant chooses to accept or reject the demand. In the other model the defendant makes an offer to the plaintiff, and the plaintiff chooses to accept or reject the offer. In both models rejection means the case proceeds to trial. The models are highly symmetric and the equilibria exhibit a "label duality" (that is, either model and its equilibrium can be obtained from the other by simply switching symbols in a specified manner). We find that both models end up with the proposer's private information revealed, some responders' private information revealed (those who reject settlement and go to trial), and the rest of the responders (those who accept the proposed settlement) being indistinguishable.

The second purpose is to compare the relative efficiencies (in terms of the expected trial frequencies) of these two models and to link this to differences in the distribution of private information. The symmetry of the models allows us to attribute the differences in social efficiency, as measured by the expected frequency of trial, to the distribution of private information and to the specification of roles for the litigants. Finally, our analysis indicates which specification of roles is socially preferred. After discussing the related literature, we provide descriptions of the analysis, the results and the other sections of the paper.

The Settlement and Litigation Literature.

The reason for choosing to focus on sequential models as described above is that the vast majority of the extant literature[1] in this area employs one or the other of the above sequences, with one-sided asymmetric information, as the game form. For example, P'ng [1983], Bebchuk [1984], Salant [1984], Reinganum and Wilde [1986] and Daughety and Reinganum [1991] assume a single round of proposal/response with a specified proposer; some models specify that the defendant makes the proposal, while others specify that the plaintiff makes the proposal. Spier [1992] assumes finitely many rounds with the same proposer (the plaintiff) in each round.[2]

A general characteristic of the foregoing papers is that only one side was privately informed. Schweizer [1989] and Sobel [1989] allow for two-sided asymmetric information. Since this paper also assumes two-sided asymmetric information, and builds on this previous work, we briefly review Schweizer and Sobel and contrast our paper with theirs.

Schweizer's information structure involves each party receiving either "good news" or "bad news" about the likelihood that the plaintiff will prevail at trial; the extent of damages are known. Thus there are two potential types of each party (i.e., one strong and one weak). He assumes that the defendant is the proposer, while the plaintiff is the responder. Our information structure involves each

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[1] An alternative to modeling the negotiation process as a particular non-cooperative game is to employ a mechanism design approach; examples of this approach include Samuelson [1983], Spulber [1985] and Spier [1989]. We will not be considering mechanism design issues in this paper.

[2] In the labor area, Cramton and Tracy [1992] examine a continuous time model in which the firm is asymmetrically informed; the role of proposer alternates, beginning with the union.

party having private information about different aspects of the case (the defendant knows the likelihood or extent of liability π while the plaintiff knows the extent of damages δ), but these aspects combine in a symmetric fashion to determine the expected damages ($\pi\delta$). We feel that this information structure is a very natural one and, since we allow for a continuum of types of each party, each litigant's "strength" is a continuous variable. In addition to characterizing the equilibrium when the defendant is the proposer, we also characterize the equilibrium when the plaintiff is the proposer, demonstrate the label duality which exists between these two equilibria (given our information structure), and evaluate their relative social efficiencies in terms of the expected frequency of trial. One additional difference is that Schweizer employs the British scheme for allocating trial costs (loser pays all) while we employ the American scheme (each litigant pays his own).

Sobel examines the impact of discovery in a model of settlement and litigation which uses an interpretation of information which is similar to ours (the plaintiff knows damages, while the defendant knows the extent of negligence, and trial costs follow the American scheme), but again considers only two types of each party ("high" and "low"). The defendant is assumed to make an offer to the plaintiff, who accepts or rejects. If the plaintiff rejects the offer, then the analysis proceeds under one of the following assumptions: (1) the defendant must reveal his private information (mandatory disclosure) or (2) the defendant does not disclose his private information. Following the discovery phase, the plaintiff makes a demand, which the defendant accepts or rejects. Sobel shows that mandatory disclosure lowers the likelihood of trial, benefits plaintiffs and harms defendants.

Related Material from the Bargaining Literature

Most bargaining models assume that agents move sequentially; one agent is assumed to make a proposal to the other, who may either accept or reject the proposal. Upon rejection, some models specify that the identity of the proposer remains the same, while others assume that it alternates between agents. For example, Rubinstein [1982] describes a perfect information bargaining game in which the identity of the proposer alternates. Fudenberg and Tirole [1983] analyze a two-period bargaining game between a buyer and a seller with two-sided asymmetric information (each agent has two potential types); the seller is designated as proposer in both periods. Sobel and Takahashi [1983] analyze an infinite-horizon game with one-sided asymmetric information (the buyer's type belongs to a continuum); again, the seller is designated as proposer in all periods. Rubinstein [1985] examines a model of sequential bargaining with one-sided asymmetric information ("high" or "low") in which the identity of the proposer alternates (he describes the equilibrium outcome under both alternatives regarding the identity of the initial proposer). Cramton [1984] extends this model to two-sided asymmetric information, assuming that neither buyer nor seller knows the other's valuation (a continuous random variable), with the modification that the seller is designated as proposer in each period. Admati and Perry [1987] consider a game in which there is a single seller and a buyer who may be of two types, in which the amount of time between offers is a strategic variable. The role of proposer alternates, beginning with the seller. Cramton [1992] considers strategic delay in a model with two-sided asymmetric information and a continuum of types.

Matching and bargaining models in which the bargaining takes place under perfect information include Rubinstein and Wolinsky ([1985] and [1990]), Gale ([1986a],[1986b] and [1987]) and Binmore and Herrero ([1988a] and [1988b]). In these models, it is assumed that the identity of the proposer is determined by the flip of a fair coin. Wolinsky [1990] and Samuelson [1992] have developed models of matching and bargaining in which the bargaining takes place under asymmetric information. Wolinsky [1990] assumes that both parties make simultaneous announcements, and trade (should it occur) takes place at pre-specified prices. Samuelson's [1992] matching and bargaining model assumes a single type of seller and two potential types of buyers. In the bargaining phase, the seller is specified as proposer in each period, though Samuelson remarks in a footnote (fn. 6, p. 179) that "the identity of the agent making the offers could be randomly chosen in each period, as in Rubinstein and Wolinsky (1985), Gale (1986,1987), and Binmore and Herrero (1988a,b)."

A few studies have dealt with the determination of the roles of proposer and responder by modeling this as part of a non-cooperative game. Perry [1986] analyzes a bargaining game with two-sided incomplete information and alternating offers. Under the assumption that there is a fixed cost of making an offer[3], he shows that at most one offer will be made in equilibrium, and it will be made by the party with the lower cost of making an offer. Cramton [1992] employs the extent of delay as a strategic variable and thus the identity of the initial proposer is determined as part of the equilibrium: the less patient trader makes the first offer. If both traders were to make an offer at the same time, Cramton assumes that the actual proposer is determined by the flip of a fair coin (in the particular equilibrium Cramton describes, this is a zero-probability event). Daughety and Reinganum [1993] consider the settlement and litigation context allowing simultaneous as well as sequential play. In that paper a specific procedure for mapping simultaneous proposals into a single "outstanding proposal" is

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[3] Most other bargaining models assume that making an offer is costless but delay results in a shrinking pie due to discounting, though Rubinstein ([1982], [1985]) also considers the fixed cost case.

used. The basic result is that, for given informational endowments, both parties choose to propose simultaneously, but the equilibria are payoff-equivalent to those of the earlier sequential literature.

The Efficient Allocation of Roles

We wish to examine how the identity of the proposer ought to be determined; that is, what allocation of these roles is socially preferred? In the perfect information bargaining models cited above, bargaining is always concluded successfully in the first period (i.e., there is no efficiency loss due to delay or failure to agree). If the social objective function is either (a) indifferent about the distribution of wealth; or (b) calculated under a veil of ignorance, wherein each party is equally likely to be a buyer or seller (or, in the settlement and litigation context, a plaintiff or defendant), then society is indifferent about the allocation of roles (under perfect information). One consequence of this indifference is that society is willing to randomize, and a random allocation of roles involves no loss of efficiency.

It would seem natural to use randomization to determine the identity of the proposer in asymmetric information settings as well. Indeed, this has been suggested to us several times as a means of selecting a proposer in the absence of any obvious or "natural" order of moves (since the order of play also influences the allocation of bargaining power). However, when parties possess asymmetric information, there is always some efficiency loss due to either a failure to trade when gains existed or to strategic delay. It seems likely that a different selection of proposer will yield a different efficiency loss. Thus society may have a preferred proposer; if so, then society will be unwilling to randomize this choice.

To examine this question, we will use a (relatively) tractable model with two-sided asymmetric information, where each party's private information belongs to a continuum. For simplicity, we will use a single round of proposal/response.[4] In Cramton [1984], a privately informed seller makes a sequence of offers to a privately informed buyer, who accepts or rejects each offer. In each round, some (lower valuation) seller types make a revealing offer, while others pool together at an offer which is sure to be rejected in this round (anticipating a higher selling price in a later round). The revealing offer is accepted by some buyer types and rejected by others. In particular, Cramton's equilibrium involves (eventually) full revelation of the proposer's private information and incomplete revelation of the responder's private information. Our one-round model has precisely the same characteristics. Thus, for the issue at hand, a single-round model is sufficient.

In Section 2, we describe a settlement and litigation game in which both parties possess private information relevant to the value of a claim. The plaintiff knows the level of damages, while the defendant knows the probability he will be held liable for those damages. Thus each party's "type" belongs to a closed interval of the real line, and the trial outcome is the product of the parties' types. In Section 3 we consider two alternative models: (1) the plaintiff proposes a settlement, which the defendant accepts or rejects; and (2) the defendant proposes a settlement, which the plaintiff accepts or rejects. This section also provides a "label" duality between the two models as well as comparative

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[4] Alternatively, the Perry analysis in which there is a fixed cost of making an offer could be adapted to the legal bargaining context to provide an equilibrium justification for considering only a single round of proposal/response. Assuming the cost of making an offer is the same for the two parties would neutralize the potential effects of offer costs on the efficient determination of roles.

statics results. The value of this duality is to guarantee that any differences in the outcomes reflect either the characteristics of the distribution of information among the parties or the order of play. In Section 4, we show that despite the extensive symmetry of the model, these allocations of roles will generally result in different expected frequencies of trial in equilibrium. Finally, the Appendix contains proofs for the propositions and the derivation of the results for the model in which the defendant is the proposer.

II. Common Aspects of the Game

We consider a game with two players, the plaintiff (P) and the defendant (D). An event has occurred before this game which involves some level of damages to P. The level of damages, δ, is private information for P, and the probability of being held liable, π, is private information for D. From D's perspective, damages δ lie in the interval $[\delta_L, \delta_H]$, with $\delta_H > \delta_L > 0$. D's cumulative (prior) distribution on damages will be denoted F. Similarly, from P's perspective, D's liability π lies in the interval $[\pi_L, \pi_H]$, with $\pi_H > \pi_L > 0$. P's cumulative (prior) distribution on liability is denoted G. For the purposes of this discussion, both distributions are taken to be uniform distributions on their respective intervals. Both priors are common knowledge as are the court costs should the parties proceed to trial, namely k_P for P and k_D for D. For convenience, let $K \equiv k_P + k_D$.

Two issues concerning the nature of the bargaining process need to be addressed. First, as discussed in the Introduction, much of the literature on settlement and litigation has assumed that settlement negotiations follow a specific pattern, with one of the parties to the litigation making a settlement proposal, and the other party choosing to accept or reject the proposal, with rejection resulting in the case going to trial (thus we assume $\pi_L\delta_L \geq k_P$, so that even the least damaged P facing the least liable D would be prepared to go to court).[5] In the analysis to be developed below we consider both types of sequential settlement process: (1) P makes a demand of D, after which D chooses to accept or to reject; and (2) D makes an offer to P, after which P chooses to accept or to reject.

Second, settlement and litigation models typically do not employ Rubinstein-type bargaining processes with an infinite horizon, alternating offer format. This is because settlement processes are inherently costly (it is costly to make formal proposals and to evaluate them; small changes in a proposal means re-issuing the entire proposal, and possibly reconsidering the entire proposal as a whole) and because court availability is typically very lumpy, which requires that a court date be scheduled considerably ahead of time. Rescheduling the court date is therefore very costly. This focuses attention on the last two iterations of a finite horizon game with the penultimate period involving a proposal and the ultimate period a response limited to accept or reject. We will adhere to this format in our analysis.

In what follows we will provide the results for the P-model (wherein P makes a demand) and the D-model (wherein D makes an offer). To help the reader keep the results of the two analyses separate, we employ the convention that strategies, beliefs and payoffs in the P-model will be in upper case letters, while strategies, beliefs and payoffs in the D-model will be in lower case letters. Finally, we maintain the following assumptions.

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[5] For a model in which plaintiffs might drop a case rather than pursue it to trial, see Nalebuff [1987].

Assumption 1. $\delta_L(\pi_H - \pi_L) \geq K$.

Assumption 2. $\pi_H(\delta_H - \delta_L) \geq K$.

Assumptions 1 and 2 ensure the interiority of the equilibrium settlement demand and offer functions for the P-model and the D-model, respectively.

III. Analysis of the P- and D-Models

The P-Model

First, we consider the P-model in detail. Let S be the demand made by P of D. Denote D's beliefs about δ after observing S as B(S). This is D's expectation of the damages, incorporating any information that D might infer from P's demand S, i.e., $B(S) \equiv E[\delta \mid S]$. The interim expected payoff to a D of type π if P makes the demand S, denoted as $U_D(\pi, S; B(S))$, is:

$$U_D(\pi, S; B(S)) = \begin{cases} S & \text{if D accepts} \\ \pi B(S) + k_D & \text{if D rejects.} \end{cases}$$

Thus, if D accepts the demand he pays S, while if he rejects the demand he pays the expected damages[6] $\pi B(S)$ plus the costs of accessing and using the court k_D (which includes e.g., defense lawyer preparation costs, expert witness testimony, and so forth). D wants to minimize U_D and thus D accepts P's demand S if and only if $S \leq \pi B(S) + k_D$ (we will take indifference as cause for acceptance).

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[6] Note that this presumes that the court properly assesses the damage level. The effect of relaxing this assumption is addressed in Daughety and Reinganum [1991].

Alternatively, for all types $\pi < (S) \equiv (S - k_D)/B(S)$, D will reject the demand S and for all types $\pi \geq (S)$, D will accept P's demand; (S) is the "marginal type of defendant" for the demand S.

P anticipates this behavior by D in choosing his demand S. Since G is the prior for $\pi \in [\pi_L, \pi_H]$, it follows that Pr{S is rejected} = G((S)). Thus, the interim expected payoff to a P of type δ who demands S, denoted $U_P(\delta, S; (S))$, is:

$$U_P(\delta, S; (S)) = G((S))[\delta E(\pi \mid \pi < (S)) - k_P] + (1 - G((S)))S.$$

The first term on the right above is the probability of demand S being rejected times the expected return to P should he go to court. The second term is the probability that demand S is not rejected times the demand S (the expected return should P not go to court). Since $E[\pi \mid \pi < (S)] = \{\int_{\Omega(S)} \pi dG\}/G((S))$, where $\Omega(S) \equiv [\pi_L, (S))$, a minor amount of manipulation yields the following formula for P's payoff:

$$U_P(\delta, S; (S)) = \delta\int_{\Omega(S)} \pi dG + S - G((S))[S + k_P].$$

Given our assumptions on G, in a revealing[7] (or separating) equilibrium the optimal solution for $\max_S U_P(\delta, S; (S))$, denoted $S^*(\delta)$, satisfies $B(S^*(\delta)) = \delta$. Differentiating U_P with respect to S yields the first-order condition:

$$[\delta(S) - (S + k_P)]dG()'(S) + 1 - G((S)) = 0.$$

Using the facts that $B(S^*(\delta)) = \delta$ and that $G(\bullet)$ is the uniform distribution on $[\pi_L, \pi_H]$ yields the following differential equation characterizing (S):

$$K'(S) + (S) = \pi_H. \tag{1}$$

The appropriate boundary condition is that the weakest plaintiff type, δ_L, need not distort his demand. That is, $S^*(\delta_L)$ maximizes $U_P(\delta_L,S;(S-k_D)/\delta_L)$ subject to the constraint that $S \in [\delta_L\pi_L + k_D, \delta_L\pi_H + k_D]$.[8] This optimization yields $S^*(\delta_L) = \delta_L\pi_H - k_P$, which belongs to the required interval under Assumption 1.[9] Solving the differential equation with the boundary condition[10] yields $(S) = \pi_H -$

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[7] As in all signaling games, there may exist pure and semi-pooling equilibria as well. However, the equilibrium refinement D1 (see Cho and Kreps [1987]) suffices to eliminate all of these. Since these arguments are now quite standard, we omit them.

[8] The constraint ensures that $= (S - k_D)/\delta_L \in [\pi_L,\pi_H]$.

[9] In general, if P's type δ were common knowledge, then P's optimal demand would be given by $S^0(\delta) = \delta\pi_H - k_P$, where the superscript "0" indicates no distortion, since there is no need for P to signal type.

$(K/\delta_L)\exp\{-(S - S^*(\delta_L))/K\}$. Substituting this into $S^*(\delta) = \delta(S^*(\delta)) + k_D$ yields an implicit definition of $S^*(\delta)$:

$$S^*(\delta) = \delta[\pi_H - (K/\delta_L)\exp\{-(S^*(\delta) - S^*(\delta_L))/K\}] + k_D. \tag{2}$$

In the Appendix we show that there is a unique solution to equation (2) and that it is an increasing function of δ. Moreover, we prove the following result in the Appendix.

Proposition 1. The function $S^*(\bullet)$ is the unique revealing settlement demand function (surviving D1) for the P-model.

In other words, in the equilibrium the plaintiff's type is revealed. Furthermore, higher-damaged plaintiffs make higher demands and consequently are more likely to go to trial. Note that the type of the defendant is not fully revealed: defendant types are partitioned into two groups since the defendant either accepts or rejects the demand. Thus, some residual uncertainty remains, which is resolved at trial only for those cases that proceed to court.

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[10] The solution to equation (1) is of the form $(S) = \alpha\exp\{-S/K\} + \pi_H$, where α is a constant of integration. Since $(S^*(\delta_L)) = (\delta_L\pi_H - k_P - k_D)/\delta_L = \alpha\exp\{-S^*(\delta_L)/K\} + \pi_H$, it follows that $\alpha = -(K/\delta_L)\exp\{S^*(\delta_L)/K\}$.

The D-Model

The results for the D-model parallel those of the P-model. Let s be the offer made by D to P. Denote P's beliefs about π after observing s as b(s). The interim expected payoff to a P of type δ if D makes the offer s, written as $u_P(\delta, s; b(s))$, is:

$$u_P(\delta, s; b(s)) = \begin{cases} s & \text{if P accepts} \\ \delta b(s) - k_P & \text{if P rejects.} \end{cases}$$

P maximizes u_P and thus P accepts D's offer s if and only if $s \geq \delta b(s) - k_P$. Alternatively, for all types $\delta > (s) \equiv (s + k_P)/b(s)$, P will reject the offer s and for all types $\delta \leq (s)$, P will accept D's offer; (s) is the "marginal type of plaintiff."

D anticipates P's behavior in choosing his offer s. Since $\delta \in [\delta_L, \delta_H]$ is distributed according to F, Pr{s is rejected} = 1 - F((s)). Thus, the interim expected payoff to a D of type π who offers s, denoted $u_D(\pi, s; (s))$, is:

$$u_D(\pi, s; (s)) =[1 - F((s))][\pi E(\delta \mid \delta > (s)) + k_D] + F((s))s.$$

The first term on the right above is the probability of offer s being rejected times the expected return to D should he go to court. The second term is the probability that offer s is not rejected times the offer s (the expected return should D not go to court). Since $E[\delta \mid \delta > (s)] = \{\int_{\omega(s)} \delta dF\}/\{1 - F((s)\}$, where $\omega(s) \equiv ((s), \delta_H]$, a minor amount of manipulation yields the following formula for D's payoff:

$$u_D(\pi, s; (s)) = \pi\int_{\omega(s)} \delta dF + s - [1 - F((s))][s - k_D].$$

The derivation of the revealing equilibrium for this model is contained in the Appendix; there it is shown that the equilibrium offer function $s^*(\pi)$ is implicitly defined by:

$$s^*(\pi) = \pi[\delta_L + (K/\pi_H)\exp\{-(s^*(\pi_H) - s^*(\pi))/K\}] - k_P. \qquad (3)$$

As in the P-model, equation (3) has a unique solution, which is increasing in π (see Appendix). Moreover, we demonstrate a result analogous to Proposition 1, namely the following:

<u>Proposition 2</u>. The function $s^*(\bullet)$ is the unique revealing settlement offer function (surviving D1) for the D-model.

In this case it is the defendant's type that is fully revealed in equilibrium. Here, higher-liability defendants make higher offers and consequently are <u>less</u> likely to go to trial. Once again, however, uncertainty is not fully resolved. Since plaintiffs are responding by simply accepting or rejecting the defendant's offer, it is the plaintiff types that are now partitioned into two groups. Trial reveals the types of plaintiff that proceed to court and there is residual uncertainty about those who don't.

Linking the P- and D-Models

The symmetric and parallel construction of the two foregoing models allows us to link their results via a "label duality" and via comparative statics computations. First, we observe the following label duality: a pairing of symbols such that consistent replacement of one set of symbols in an equation by the other yields a formally correct equation.[11] The following table provides the matchings between symbols in the two models.

Table 1: Label Duality

$S \leftrightarrow s$ $\delta \leftrightarrow \pi$ $\leftrightarrow$ $K \leftrightarrow -K$ $G \leftrightarrow 1\text{-}F$

$k_D \leftrightarrow -k_P$ $k_P \leftrightarrow -k_D$ $\pi_L \leftrightarrow \delta_H$ $\pi_H \leftrightarrow \delta_L$ $< \leftrightarrow >$

For example, Assumption 1 requires that $\delta_L(\pi_H - \pi_L) \geq K$. Using the above matchings, this becomes $\pi_H(\delta_L - \delta_H) \leq -K$, which is readily transformed into Assumption 2. Thus, using the above substitutions, equation (1) becomes equation (A.1) and equation (2) becomes equation (3).

This relationship carries over to the following table of comparative statics, which provides the effect of infinitesimal changes in the basic parameters of the analysis (δ_L, δ_H, π_L, π_H, k_P and k_D) on the

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[11] For examples of other label dualities, see DeMorgan's Laws in set theory (Halmos [1960]), mechanical/electrical system analogues (Golomb and Shanks [1965]), dual queuing systems (Prabhu [1965]) and Cournot/Bertrand equilibria (Vives [1984]). Optimization dualities, such as in linear and nonlinear programming (e.g., Avriel [1976]) and in cost and production theory (Shephard [1970] and McFadden [1978]) involve label and property pairings that involve mathematical relationships among the paired labels. We raise this to clarify to the reader that the following only involves the pairing of labels, resulting in two results for the price of one, and not anything deeper, such as would be provided by an optimization duality.

equilibrium proposal functions $S^*(\delta)$ and $s^*(\pi)$, and on the marginal type functions (S) and (s). In the table an entry indicates the direction of influence of the parameter on the proposal function (i.e., the sign of the derivative); "?" indicates that the sign is indeterminate.

Table 2: Comparative Statics

		δ_L	δ_H	π_L	π_H	k_P	k_D
P-Model	$S^*(\delta)$	-	0	0	+	-	?
	(S) -	0	0	+	?	-	
D-Model	$s^*(\pi)$	+	0	0	-	?	+
	(s) +	0	0	-	+	?	

For example, the table indicates that (in the P-model) increasing π_H leads to an upward shift of the equilibrium demand function while increasing δ_L leads to a downward shift in the equilibrium demand function. To see why, observe that $S^*(\delta)$ and $s^*(\pi)$ can be written as follows:

$$S^*(\delta) = S^0(\delta) + [K - \delta(K/\delta_L)\exp\{-(S^*(\delta) - S^*(\delta_L))/K\}], \tag{4}$$

where $S^0(\delta) = \pi_H\delta - k_P$, and

$$s^*(\pi) = s^0(\pi) - [K - \pi(K/\pi_H)\exp\{-(s^*(\pi_H) - s^*(\pi))/K\}], \tag{5}$$

where $s^0(\pi) = \delta_L\pi + k_D$. Thus an equilibrium proposal is the sum of the "no-distortion" proposal (i.e., where the proposer's type is common knowledge, but the responder's is not) and an adjustment term reflecting the distortion necessary for the proposer to signal type. In the P-model, this distortion increases the demand, while in the D-model it reduces the offer.

Let us consider a few examples from the above table, all focused on the P-model. First consider an increase in π_H. This leads P to revise upward the no-distortion demand S^0. Moreover, the amount of distortion involved in signaling type has also increased. Thus, the overall effect is an upward shift of S* that is a composite of an increased expectation of the returns from court and the increased effort to signal type.

Now consider the effect of an increase in δ_L in the same model. In general, this cannot affect P's expectations about the expected returns from trial since P knows his type and the parameter change does not influence P's beliefs about D's type. Furthermore, less distortion is now required to prevent mimicry of the higher types by lower types. Thus the indicated sign reflects a pure reduction in distortion, as contrasted with the composite effect discussed above.

Why is there no impact on either function, and in either model, of changing δ_H or π_L? First, note that changes in π_L and δ_H do not affect the no-distortion demands. Second, changing π_L or δ_H does not change the extent of distortion required for δ to signal type. Analogous interpretations for the D-model are straightforward (and dual, in the sense that each direction is reversed).

IV. Social Preferences Over The Identity of The Proposer

Some models assume an objective function in which a lower expected frequency of trial translates into a lower cost to society. For instance, with risk-neutral parties under a complete veil of ignorance (i.e., without knowing either one's type or one's identity as P or D), the *ex ante* expected social loss is simply expected trial costs, which is itself equal to K times the ex ante expected frequency of trial. If different proposers lead to different ex ante expected frequencies of trial, then all parties would agree (under a veil of ignorance) that the order of play should minimize the expected trial frequency.[12] Although we are unable to obtain a closed-form solution for the equilibrium demand and offer functions $S^*(\delta)$ and $s^*(\pi)$ for the P-model and the D-model, respectively, we are able to compare the ex ante expected frequency of trial under each ordering.

First consider the P-model. The probability that a plaintiff of type δ ends up going to court is given by $G((S^*(\delta)))$. Thus the ex ante expected trial frequency in the P-model, which we denote by ETF^P, is given by:

$$ETF^P = \int_{\delta_L}^{\delta_H} G((S^*(\delta)))dF(\delta).$$

Substitution of $(S^*(\delta))$ and simplification yields:

$$ETF^P = 1 - [K/\delta_L(\pi_H-\pi_L)(\delta_H-\delta_L)]\int_{\delta_L}^{\delta_H} \exp\{-(S^*(\delta)-S^*(\delta_L))/K\}d\delta\ .$$

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[12] In a model involving endogenous care, a higher expected trial frequency might decrease social costs when care would otherwise be inadequate; see Polinsky and Rubinfeld [1988]. This follows since a credible threat of an increased probability of trial induces greater care-taking (which, of course, might lead to even fewer trials occurring overall).

Now consider the D-model. The probability that a defendant of type π goes to court is 1 - F((s*(π))). Thus the expected trial frequency in the D-model, denoted ETF^D, is given by:

$$ETF^D = \int_{\pi_L}^{\pi_H} [1 - F((s^*(\pi)))]dG(\pi)$$

Substitution of (s*(π)) and simplification yields:

$$ETF^D = 1 - [K/\pi_H(\pi_H-\pi_L)(\delta_H-\delta_L)]\int_{\pi_L}^{\pi_H} \exp\{-(s^*(\pi_H)-s^*(\pi))/K\}d\pi .$$

Finally, let $V \equiv (ETF^P - ETF^D)[(\pi_H-\pi_L)(\delta_H-\delta_L)]$. When $V > 0$ society prefers that D propose and when $V < 0$ society prefers that P propose. Since the integrands in ETF^P and ETF^D are independent of δ_H, it is straight-forward to show that, evaluated at $V = 0$, $\partial V/\partial\delta_H \mid_{V=0} = (S^*(\delta_H) - (\pi_H\delta_H + k_D))/\delta_H$. From equation (2), $S^*(\delta_H) < \pi_H\delta_H + k_D$, and thus the above derivative is negative. Similar considerations show that, evaluated at $V = 0$, $\partial V/\partial\pi_L \mid_{V=0} = ((\pi_L\delta_L - k_P) - s^*(\pi_L))/\pi_L$, and that this derivative is also negative (see equation (3)). This leads to the following proposition.

Proposition 3. Generically, there is a socially preferred allocation of roles. In particular, if (for a given parameter set) $V = 0$, then increases in δ_H and/or π_L favor the P-model over the D-model, while decreases in δ_H and/or π_L favor the D-model over the P-model.

Proposition 3, and the V-function, provide a notion of a socially neutral distribution of information among agents: for given k_P and k_D, a four-tuple of parameters (δ_L, δ_H, π_L, π_H) such that V = 0. Given such a socially neutral distribution, making P's (respectively, D's) information relatively more diffuse means that P (respectively, D) is socially preferred as proposer.[13] In general, the distribution of information is case-specific. It is not unreasonable, however, that there might cases that would tend to have similar distributions of information, thereby generating reasonably close values of V. If this is a readily characterizable collection of cases, then social efficiency might be improved by imposing some structure on settlement negotiations. For example, one might not only require a pre-trial conference, but also that the defendant always provide a "good-faith" proposal at such a conference.

In summary, the above analysis makes two contributions. First, it provides a reasonably general model of settlement negotiations with two-sided asymmetric information and a continuum of types. The symmetry of structure in the model provides a nice duality of results for the P- and D-models. Second, the models yield a comparison of their relative social efficiencies via the notion of a socially neutral distribution of information; socially non-neutral distributions imply a socially preferred proposer.

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[13] Proposition 3 therefore also implies that, in the context of asymmetric information bargaining games, using a randomization procedure (such as flipping a fair coin) to determine the identity of the proposer may induce a social inefficiency that may invalidate later assessments of comparative efficiencies in such models.

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APPENDIX

Lemma 1: For each $\delta \in [\delta_L, \delta_H]$, there exists a unique $S^*(\delta) \in [\delta_L\pi_H - k_P, \delta_H\pi_H + k_D]$ satisfying equation (2). Moreover, if $\delta' > \delta$, then $S^*(\delta') > S^*(\delta)$.

Proof: Let $M(\delta,S) = S - \delta[\pi_H -(K/\delta_L)\exp\{-(S - S^*(\delta_L))/K\}] - k_D$. Notice that $M(\delta_L,\delta_L\pi_H - k_P) = 0$ and $M(\delta,\delta_L\pi_H - k_P) < 0$ for all $\delta > \delta_L$. Moreover, $M(\delta,\delta_H\pi_H + k_D) > 0$ for all $\delta \in [\delta_L, \delta_H]$. Since $M(\delta,S)$ is a strictly convex function of S for given δ, there is a unique $S^*(\delta) \in [\delta_L\pi_H - k_P, \delta_H\pi_H + k_D]$ such that $M(\delta,S^*(\delta)) = 0$ (i.e., such that equation (2) is satisfied). Since $M(\delta',S) < M(\delta,S)$ for $\delta' > \delta$, it follows that $S^*(\delta') > S^*(\delta)$. We have thus verified that $S^*(\delta)$ is an increasing function. The proof is illustrated in Figure A-1.

Figure A-1

Proof of Proposition 1. Using the implied beliefs $B^*(S) = (S - k_D)/(S)$, differentiating U_P with respect to S yields equation (1), which is satisfied (by construction) by the function (S). Since $B^*(S^*(\delta)) = \delta$, the equation $(S^*) = (S^* - k_D)/\delta$ can be solved to obtain the (implicit) representation of the equilibrium demand function: $S^*(\delta) = \delta[\pi_H - (K/\delta_L)\exp\{-(S^*(\delta) - S^*(\delta_L))/K\}] + k_D$.

The second-order condition (upon noting that dG is simply a constant in the uniform case and P is maximizing) is equivalent to:

$$[\delta'(S) - 2]'(S) + [\delta(S) - (S + k_P)]''(S) < 0.$$

Evaluating at $S^*(\delta) = \delta(S^*(\delta)) + k_D$ and simplifying yields:

$$[\delta'(S^*(\delta)) - 2]'(S^*(\delta)) - K''(S^*(\delta)) < 0.$$

Since equation (1) must hold for all S, differentiating once again yields $-K''(S) = '(S)$. Substituting this result implies that the second-order necessary condition for $S^*(\delta)$ to provide a maximum reduces to $[\delta'(S^*(\delta)) - 1]'(S^*(\delta)) < 0$. Differentiating the relationship $S^*(\delta) = \delta(S^*(\delta)) + k_D$ and solving yields $dS^*(\delta)/d\delta = (S^*(\delta))/(1 - \delta'(S^*(\delta)))$, which is positive due to Lemma 1. Since the numerator is always positive, it follows that $1 - \delta'(S^*(\delta)) > 0$. Since $'(S^*(\delta)) > 0$, the required inequality is satisfied at the candidate equilibrium.

The Revealing Equilibrium for the D-Model.

The optimal solution for $\min_s u_D(\pi, s; (s))$, denoted $s^*(\pi)$, satisfies $b(s^*(\pi)) = \pi$. Differentiating u_D with respect to s yields the first-order condition:

$$[-\pi(s) + s - k_D]dF()(d/ds) + F((s)) = 0.$$

Using the facts that $b(s^*(\pi)) = \pi$ and that $F(\bullet)$ is the uniform distribution on $[\delta_L, \delta_H]$ yields the following differential equation characterizing the function (s):

$$K'(s) - (s) = - \delta_L. \tag{A.1}$$

The solution to equation (A.1) is of the form $(S) = \beta\exp\{s/K\} + \delta_L$, where β is a constant of integration. The appropriate boundary condition is that the weakest defendant type, π_H, need not distort his settlement offer. Thus $s^*(\pi_H)$ minimizes $u_D(\pi_H,s;(s + k_P)/\pi_H)$ subject to the constraint $s \in$

$[\delta_L\pi_H - k_P, \delta_H\pi_H - k_P]$.[14] Solving this optimization problem yields $s^*(\pi_H) = \delta_L\pi_H + k_D$, which belongs to the required interval under Assumption 2.[15]

Since $(s^*(\pi_H)) = (\delta_L\pi_H + k_D + k_P)/\pi_H = \beta\exp\{s^*(\pi_H)/K\} + \delta_L$, it follows that $\beta = (K/\pi_H)\exp\{-s^*(\pi_H)/K\}$. Thus $(s^*(\pi)) = \delta_L + (K/\pi_H)\exp\{-(s^*(\pi_H) - s)/K\}$, and $s^*(\pi)$ is given implicitly by $s^*(\pi) = \pi (s^*(\pi)) - k_P$, or $s^*(\pi) = \pi[\delta_L + (K/\pi_H)\exp\{-(s^*(\pi_H) - s^*(\pi))/K\}] - k_P$ (equation (3) in the text).

Lemma 2: For each $\pi \in [\pi_L, \pi_H]$, there exists a unique $s^*(\pi) \in [\delta_L\pi_L - k_P, \delta_L\pi_H + k_D]$ satisfying equation (3). Moreover, if $\pi' > \pi$, then $s^*(\pi') > s^*(\pi)$.

Proof: Let $m(\pi,s) = s - \pi[\delta_L + (K/\pi_H)\exp\{-(s^*(\pi_H) - s)/K\}] + k_P$. Notice that $m(\pi_H, \delta_L\pi_H + k_D) = 0$ and $m(\pi, \delta_L\pi_H + k_D) > 0$ for all $\pi < \pi_H$. Moreover, $m(\pi, \delta_L\pi_L - k_P) < 0$ for all $\delta \in [\delta_L, \delta_H]$. Since $m(\pi,s)$ is a strictly concave function of s for given π, there is a unique $s^*(\pi) \in [\delta_L\pi_L - k_P, \delta_L\pi_H + k_D]$ such that $m(\pi, s^*(\pi)) = 0$ (i.e., such that equation (3) is satisfied). Since $m(\pi',s) < m(\pi,s)$ for $\pi' > \pi$, it follows that $s^*(\pi') > s^*(\pi)$. We have thus verified that $s^*(\pi)$ is an increasing function. The proof is illustrated in Figure A-2.

Figure A-2

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[14] Again, the constraint ensures that $(s) = (s + k_P)/\pi_H \in [\delta_L, \delta_H]$.

[15] In general, if D's type π were common knowledge, then D's optimal offer would be given by $s^0(\pi) = \delta_L\pi + k_D$, where the superscript "0" indicates no distortion, since then there is no need for D to signal type.

<u>Proof of Proposition 2</u>. Using the implied beliefs $b^*(s) = (s + k_P)/(s)$, differentiating u_D with respect to s yields equation (A.1), which is satisfied (by construction) by the function (s). Since $b^*(s^*(\pi)) = \pi$, the equation $(s^*) = (s^* + k_P)/\pi$ can be solved to obtain the (implicit) representation of the equilibrium offer function: $s^*(\pi) = \pi[\delta_L + (K/\pi_H)\exp\{-(s^*(\pi_H) - s^*\pi))/K\}] - k_P$.

The second-order condition (upon noting that dF is simply a constant in the uniform case and D is minimizing) is equivalent to:

$$[-\pi'(s) + 2]'(s) + [-\pi(s) + s - k_D]''(s) > 0.$$

Evaluating at $s^*(\pi) = \pi(s^*(\pi)) - k_P$ and simplifying yields:

$$[-\pi'(s^*(\pi)) + 2]'(s^*(\pi)) - K''(s^*(\pi)) > 0.$$

Since equation (A.1) must hold for all s, differentiating once again yields $K''(s) = '(s)$. Substituting this result implies that the second-order necessary condition for $s^*(\pi)$ to provide a minimum reduces to $[-\pi'(s^*(\pi)) + 1]'(s^*(\pi)) > 0$. Differentiating the relationship $s^*(\pi) = \pi(s^*(\pi)) - k_P$ yields $ds^*(\pi)/d\pi = (s^*(\pi))/(1 - \pi'(s^*(\pi)))$, which is positive due to Lemma 2. Since the numerator is always positive, it follows that $1 - \pi'(s^*(\pi)) > 0$. Since $'(s^*(\pi)) > 0$, the required inequality is satisfied at the candidate equilibrium.

FIGURES

Available from authors upon request.