An Evolutionary Approach to Tacit Communication in Van Huyck, Battalio, and Beil's Game Experiments

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Proposed Running Head: Tacit Communication in Game Experiments

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Abstract

This paper provides an evolutionary interpretation of Van Huyck, Battalio, and Beil's experimental results on coordination games with auction. A set-valued solution concept is defined for a finite population model under the best response evolutionary dynamics and applied to their games. It is shown that our solution concept captures the role of auction as a tacit communication device and predicts the equilibria chosen by players in the real experiments.

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1. Introduction

Recently a lot of work has been produced on the coordination problem in games. We say that players face the coordination problem in game situations if there are multiple intuitively plausible Nash equilibria. Since many Nash equilibria in these games survive standard refinement criteria of traditional game theory, we have to look for other approaches to analyze the coordination problem.

The growing body of experimental literature on coordination provides us with important information about how agents solve the coordination problem; see for example Cachon and Camerer (1992), Cooper, DeJong, Forsythe, and Ross (1992, 1993); and Van Huyck, Battalio, and Beil (1990, 1991, 1993; VHBB below). The results of these studies suggest that players' initial beliefs in such games are normally widely dispersed; that in the face of this dispersion players' previous experience with analogous games is an important determinant of their decisions; and that the learning from experience can exert a strong and lasting influence on coordination outcomes.

Evolutionary game theory (see for example Maynard Smith (1982)) is designed to describe the long-run tendencies of the strategy frequencies in populations in which an individual's expected reproduction rate is jointly determined by its own strategy and the strategies of the other individuals in the population. Almost by definition, history plays an important role in evolutionary game theory, and coordination has always been its primary focus. This is in sharp contrast to traditional game theory which has avoided coordination problems. As a result, evolutionary game theory has significant comparative advantages in studying coordination.

Having these observations we can naturally expect that evolutionary game theory and game experiments can go hand in hand in the analysis of the coordination problem in games. For example Crawford (1991, 1992) has recently shown that some of VHBB's (1990, 1991) experimental results on coordination games can be explained by the solution concept used in evolutionary game theory (evolutionarily stable strategy (ESS below)) and a learning dynamic suggested by evolutionary dynamics.

In VHBB's (1990, 1991) earlier experiments, groups of 2 to 27 subjects played series of one-stage simultaneous-move symmetric coordination games, which can be called the group minimum effort game and the group median effort game. Most of the games had multiple strict Pareto-ranked pure-strategy Nash equilibria. As is well known in game theory, a strict Nash equilibrium survives almost all the refinement criteria suggested in traditional game theory. But in these experiments they find that players systematically discriminate among strict Nash equilibria: In all group minimum games players' strategy choices always converge to the most inefficient Nash equilibrium if the group size is large; in many group median games, which VHBB called the game Γ and will be explained in section 2 below, somewhat more efficient equilibria are chosen depending on initial histories, but the most efficient Nash equilibrium is never chosen. Crawford (1991, 1992) finds that the structure of the experiment is the same as the "playing-the-field" model used in evolutionary game theory, and provides both evolutionary game theoretic interpretation and game learning theoretic explanation about their experimental data.

VHBB's (1993) new experimental results on coordination games with auction are still waiting for theoretical explanations. This paper provides an evolutionary game theoretic explanation about these experimental results.

In the new experiments, the right to play 9-person median effort games are auctioned off in groups of 18 before each play. Players' bids in these auctions can be viewed as a form of costly pre-play communication. They determine who gets to play the median game and also signal players' strategy choices in the median game implicitly. Compared with previous experiments one would expect auctioning the right to play to yield somewhat more efficient outcomes in the underlying game, because the auction ensures that the most optimistic players are selected to play a game in which optimism enhances efficiency. The results in the real experiments are more surprising; players quickly bid the price of the right to play the underlying game up to a level that could only be justified by the payoffs of its most efficient equilibrium, and then rapidly converged to that equilibrium.[1)]

The effects of players' bids on their expectations suggest a simple mechanism by which competition promotes efficiency. Auction plays the role of a tacit communication device and influences players' expectations greatly. The tacit communication here is similar to the forward induction argument in traditional game theory. Recent evolutionary game theoretic work also shows that several modified versions of ESS and some evolutionary learning dynamics capture stronger forward induction arguments than Kohlberg and Mertens' (1986) strategic stability; for example, Gilboa and Matsui (1991), Kim and Sobel (1992), Matsui (1992), Nöldeke and Samuelson (1993), Samuelson (1991), Sobel (1993), and Swinkels (1992). Following this literature we define a set-valued solution concept for a finite population model based on the best response evolutionary dynamics, and show that only those strategies, which choose the Pareto efficient equilibrium action in the second stage game, are stable in VHBB's median effort games with auction.

The rest of the paper is organized as follows. Section 2 summarizes VHBB's experimental designs and results. Section 3 introduces the solution concept for our analysis. The solution concept is a discrete-time version of Gilboa and Matsui's cyclically stable set in a finite population model and we use the same name. Section 4 shows how our solution concept selects the Nash equilibria chosen by subjects in the experiment among many Nash equilibria of the game. Finally section 5 is the concluding remarks.

2. Van Huyck, Battalio, and Beil's Experimental Design and Results

This section summarizes VHBB's (1993) experimental design and results.

2.1 Game Rule: a two-stage median effort game with auction

The game G is a N-player game which consists of two stages. N is an even number, N = 2n, where n is an odd number. In the first stage of the game, N players play an auction game to buy the right to participate in the second stage game Γ. Players simultaneously and independently make bids b_1, b_2,..., b_N, where $b_i \in B$ and B is a finite set of all possible bids. Among all the players n highest bidders are selected and play the second stage coordination game. These n players must pay the price p, which VHBB call the asset price, and the asset price is determined in the following way: if the n-th highest bid is higher than the (n+1)-th highest bid, p equals the smallest possible bid higher than the (n+1)-th highest bid; if the n-th and the (n+1)-th highest bids are the same, then p is the n-th highest bid.[2] When there is a tie among players' bids, winners are chosen randomly.

The second stage game Γ is a coordination game in which n players simultaneously choose their efforts e_1, e_2,..., e_n, where $e_i \in E$ for all i = 1,2,...,n. The strategy space E equals {1, 2,..., $\bar{e}$}, where $\bar{e}$ is the largest feasible integer. Let M be the median of these actions. The payoff function π(.) for each of the n players in the coordination game Γ is as follows:

$$\pi(e_i,M) = aM - b(M-e_i)^2 + c,$$

where a > b > 0. Note that a player's payoff is decreasing in the distance between his effort choice e_i and the median M, and is increasing in the median M.

Then the payoff function for player i, i = 1,2,...,N, in the game G is as follows:

$$u_i(b_1,b_2,...,b_N;\ e_1,e_2,...,e_n) = 0, \text{ if he does not play the median game,}$$
$$= -p + \pi(e_i,M), \text{ if he plays the median game.}$$

The payoff function and strategy space are common knowledge among players, but players are not allowed to engage in explicit pre-play communication.

2.2 Experimental Design

In VHBB's experiment the multiple unit English Clock auction is used as the first stage game. Initially when the asset price is 65 cents all subjects hold up their bid cards. Every five second the price ticks up an increment, and when the price goes above the price a subject is willing to pay that subject lowers his or her bid card. This process continues until only nine subjects remain and these nine subjects proceed to play the median effort game Γ.

The English auction is a multi-stage open-bid auction game, but it is strategically equivalent to the simultaneous-move sealed-bid auction game in simple game situations.[3)] In the theoretical analysis of section 4 we will use the rule described in subsection 2.1. We also assume that a player can choose a bid price from the set B = {b| b = k/100, where k is an integer and 65 ≤ k ≤ 150}. Without loss of generality we impose the upper bound of 1.50 on the bid price space for simplicity.

The payoff function of the second stage game is defined under the following parameter values: a = \$.10, b = \$.05, c = \$.60, and $\bar{e}$ = 7. Eighteen players make bids in the first stage game, and among them nine players participate in the second stage median effort game. That is, N = 18 and n = 9.

The payoff matrix for the median game is as shown in Table 1.

		Group median effort						
		7	6	5	4	3	2	1
	7	1.30	1.15	0.90	0.55	0.10	-0.45	-1.10
	6	1.25	1.20	1.05	0.80	0.45	0.00	-0.55
Subject's	5	1.10	1.15	1.10	0.95	0.70	0.35	-0.10
effort	4	0.85	1.00	1.05	1.00	0.85	0.60	0.25
	3	0.50	0.75	0.90	0.95	0.90	0.75	0.50
	2	0.05	0.40	0.65	0.80	0.85	0.80	0.65

1	-0.50	-0.05	0.30	0.55	0.70	0.75	0.70

Table 1

2.3 Experimental Results

In the baseline experiments, when subjects play the median game without auction, the initial outcome is never the payoff-dominant equilibrium and the initial median is extremely stable in the repeated play of the game.

VHBB explained that this result is due to the perceived "riskiness" of selecting the highest effort 7 in the first period. Initially subjects are uncertain about what value the median will take, and choosing action 7 may result in a large disequilibrium loss. Consequently players respond to the strategic uncertainty cautiously by choosing middle values. But once an inefficient median is observed it becomes an important historical precedent and the initial median determines the median in later periods in many cases. This result is a replication of VHBB's (1991) earlier experimental results and shows the importance of history as a coordination device.

In the two-stage game, where players participate in auction in the first stage, players show totally different behaviors than in the baseline experiments. Whether players have the experience of playing the median game or not, initial prices of the participation right are between $.95 and $1.24 and initial medians in the second stage game are below 7 except one case. But as time goes by, the asset price rises and the group median also increases until it reaches the highest effort 7. That is, competition among subjects increases the asset price and the higher asset price plays an important role as a coordination device in the second stage game.

3. The solution Concept

The solution concept used in this paper is a discrete-time version of Gilboa and Matsui's (1991) cyclically stable set (CSS below). Sobel (1993) also introduces a new evolutionary solution concept for a finite population model under the discrete time dynamics. To introduce our solution concept we redefine Gilboa and Matsui's CSS in Sobel's framework.[4)]

We consider a game played in a single finite population of N players. These players use only pure strategies S, where S is a finite set, and play the N-person game repeatedly.[5)] Since N can be a fairly large number in our model, we will assume that players do not use repeated game strategies. The full dynamic specification of the evolutionary process will not be provided, but it will be assumed that at most one member of the population can change his strategy in a period. When a player changes his strategy, he chooses an optimal response against the existing population strategy profile. The stability condition described below is one way to capture the idea of evolution that bad strategies die out, new strategies have the potential to enter the population, and the successful strategies may be adopted by more members of the population.

Since only pure strategies are allowed, a population strategy profile is a list $(s_1,s_2,...,s_N)$ of pure strategies for each of the N players in the population. According to our dynamic, a population strategy profile $\theta' = (s_1,s_2,...,s_i,...,s_N)$ can replace another population strategy profile $\theta = (s_1,s_2,...,s_i',...,s_N)$, if θ and θ' differs in player i's strategy choice for some i, i = 1,2,...,N, and the new strategy s_i' best responds against the current population state. We also assume that a population strategy profile θ can replace itself. Let $U(s_i|s_{-i})$ be the payoff of the strategy s_i when the population state is $\theta = (s_1,s_2,...,s_i,...,s_N)$ and $s_{-i} = \theta\backslash\{s_i\}$. Formally, $\theta' = (s_1,s_2,...,s_i',...,s_N)$ can replace $\theta = (s_1,s_2,...,s_i,...,s_N)$, if

$$s_i' \in \underset{t_i}{\operatorname{argmax}} \; U(t_i|s_{-i}).$$

Definition: A set Θ of population strategy profiles is a **cyclically stable set** (CSS below) if it is a minimal nonempty subset of S^N with respect to the following property:

(B) if $\theta \in \Theta$ and θ' can replace θ, then $\theta' \in \Theta$.

This definition suggests how the population might evolve over time. If a given population strategy profile is an element of a CSS, then so is a population strategy profile obtained by changing one individual's strategy in a way that makes him best respond against the current population state. We also define accessibility as follows, which will be used to describe the evolution of the population state later.

Definition: A population strategy profile θ_n is **accessible** from θ_0 if there exists a finite sequence $\{\theta_i\}_{i=1}^{n-1}$ such that θ_{i+1} can replace θ_i for all i = 1,2,...,n-1.

It is easy to see that a nonempty set Θ is a CSS if and only if it is a subset of S^N which is closed under mutual accessibility.

The new solution concept is similar to several other solution concepts used in evolutionary game theory. It differs from ESS since it is set-valued. It also differs from other set-valued solution concepts like evolutionarily stable set (Thomas (1985)) or equilibrium evolutionarily stable set (Swinkels (1992)) in that it does not impose the Nash equilibrium condition. It is similar to Gilboa and Matsui's cyclically stable set (hence the same name) and Sobel's nonequilibrium evolutionarily stable set (NES set below). But it differs from Gilboa and Matsui's CSS in that we consider discrete-time dynamics in a finite population model, and it also differs from Sobel's NES set in that we use the best response dynamics.[6)]

A CSS always exists in a finite population game as in Gilboa and Matsui. Since we only allow pure strategies, the state space S^N is a finite set and we do not need Zorn's lemma to prove the existence of CSS.

Proposition: In a population game there exists a CSS.

Proof: Let us define $R(\theta)$ to be the set of all strategy profiles which are accessible from $\theta \in S^N$. It is easy to see that $R(\theta)$ is nonempty and closed for all $\theta \in S^N$. Since accessibility is a transitive relation, $\theta' \in R(\theta)$ implies $R(\theta') \subset R(\theta)$.

To find a CSS in the game, pick an element $\theta_0 \in S^N$ and find $R(\theta_0)$. Then pick an element $\theta_1 \in R(\theta_0)$ and find $R(\theta_1)$. If $R(\theta_1)$ is a strict subset of $R(\theta_0)$, pick an element $\theta_2 \in R(\theta_1)$ and find $R(\theta_2)$; if $R(\theta_1)$ is the same as $R(\theta_0)$, pick another element $\theta_1' \in R(\theta_1)$ and find $R(\theta_1')$. Continue this process until we find a set Θ such that $R(\theta) = \Theta$ for all $\theta \in \Theta$. The iteration converges in finitely many steps since there are only finitely many $R(\theta)$'s and finitely many elements in all $R(\theta)$. The set Θ is nonempty since every θ is accessible from itself. The set Θ is a CSS in the game since it is closed under mutual accessibility. Q.E.D.

In some games there are mutiple CSS's. A strict Nash equilibrium in a game is a CSS as a singleton, and a game in which there are multiple strict Nash equilibria has many CSS's. In general there may be non-Nash equilibrium elements in a CSS. For example, in the game Paper-Scissors-Rock there is a unique CSS, no element of which is a Nash equilibrium.

Example:

	P	S	R

P	0, 0	-1, 1	1, -1
S	1, -1	0, 0	-1, 1
R	-1, 1	1, -1	0, 0

Table 2

If N=3, the unique CSS is the set (PSS, SSR, SRR, PRR, PPR, PPS) in the Paper-Scissors-Rock game shown in Table 2. Here we ignore the possible different orderings of strategies, since players are in symmetric positions. Whatever the initial population strategy profile may be, the population state reaches one element of the CSS and cycles forever from then on under the best response dynamic. In this game Sobel's NES set is the set of all possible strategy combinations for three players. Since a NES set uses a weaker entry condition than a CSS, it contains more elements than a CSS. We can also see that the MES set defined in footenote 6 is the same as the NES set in this game. Later in the next section we can see the difference among these solution concepts in the analysis of VHBB's minimum game and median game.

4. Analysis

VHBB's median effort game with auction G is a two-stage game, and a player's strategy consists of the bid in the first-stage game and effort choices in the second-stage game. The strategy in the second-stage game must be a complete specification of effort choices for all possible prices. Formally, player i's strategy s_i is of the form $(b_i, e_i(p))$, where b_i is player i's bid in the auction and $e_i(p)$ gives his effort choices for each price $p \le b_i$. Here we assume that a player's effort choice in the second stage game depends only on the asset price, not on the subset of players who play the median game. It is a natural specification of strategies considering that the game is symmetric.[7)]

In VHBB's game G there are many different Nash equilibria (and subgame perfect Nash equilibria). One thing common to all these equilibria is that a

Nash equilibrium in the game G must induce a Nash equilibrium in the median game if the subgame is reached with positive probability during the play of the game. Since there are both pure strategy and mixed strategy Nash equilibria in the median game, the game G also has many different sorts of Nash equilibria. Among many Nash equilibria in the game G, the set of Nash equilibria which support pure strategy Nash equilibrium outcomes in the median game are especially important in the evolutionary analysis. For a pure strategy Nash equilibrium in the game G, which supports a particular pure strategy Nash equilibrium in the median game, there are two different classes of bidding strategies in the auction game: in one class of Nash equilibria at least 9 players choose bid prices greater than or equal to the equilibrium payoff in the median game; in another class of Nash equilibria 10 players choose the highest possible bid price lower than the equilibrium payoff in the median game and eight players choose bid prices higher than or equal to the equilibrium payoff. In the first class of Nash equilibria the asset price is equal to or lower than the equilibrium payoff in the median game depending on the values of low bids; in the second class of Nash equilibria the asset price is a little bit lower than the equilibrium payoff in the median game. In many of the first class of Nash equilibria there are nine players who cannot participate in the median game and receive zero payoff in the game G (the remaining nine players may receive a positive payoff depending on the asset price), but in the second class of Nash equilibria every player participates in the median game with positive probability and those players who participate in the median game receive positive payoffs. We summarize this observation in Lemma 1. Here we reorder players such that the highest bidder is player 1, the second highest bidder is player 2, and so on. When there is a tie among players' bids we can order them randomly. Since players' positions are symmetric we do not lose anything by using this numbering of players.

Lemma 1: The following two classes of strategy profiles are Nash equilibria in the game G, which support the pure strategy Nash equilibrium (e*,e*,...,e*) in the median game for all e* = 1,2,...,7:

(i) In the first stage game, for player i = 1,2,...,9, $b_i \geq \pi(e^*,e^*)$, for player i = 10,11,...,18, $b_i \leq \pi(e^*,e^*)$, and if $b_{10} = \pi(e^*,e^*)$, $b_9 = \pi(e^*,e^*)$; in the second stage game, for all i with $b_i \geq b_{10}$,

$$e_i(p) = e^*, \text{ if } p \geq b_{10},$$

$$= e \leq e^*, \text{ if } p < b_{10},$$

and for all i with $b_i < b_{10}$, $e_i(p)$ is any effort choice for all p.

(ii) In the first stage game, for player i = 1,2,...,8, $b_i \geq \pi(e^*,e^*)$, and for player i = 9,10,...,18, $b_i = \pi(e^*,e^*)-0.01$; in the second stage game, for all i = 1,2,...,18,

$$e_i(p) = e^*, \text{ if } p = \pi(e^*,e^*)-0.01,$$

$$= e \leq e^*, \text{ if } p \neq \pi(e^*,e^*)-0.01.$$

Besides the Nash equilibria characterized in Lemma 1, there are more pure strategy Nash equilibria which support the same outcome, since some players may choose higher effort levels for nonequilibrium prices in an uncoordinated way.

As was explained in section 3, in general games our solution concept CSS need not contain a Nash equilibrium, but it will be shown that in VHBB's game G a CSS must consist of Nash equilibria. It will be also shown that no equilibrium in which players choose an inefficient Nash equilibrium in the second stage game is an element of a CSS. If the initial state of the population is an equilibrium with a low bid and an inefficient equilibrium in the median effort game, new strategies, which follow the equilibrium actions for the current equilibrium price but choose higher efforts for higher asset prices, can enter the population following the entry condition (B). When a large enough number of new entrants are in the population, the asset price goes up and players choose higher efforts in the group median game. Repeated application of this invading procedure shows that no equilibrium with low effort choices in the second stage game is cyclically stable.

On the other hand there are some Nash equilibria in which everybody chooses the highest effort in the second stage game and makes a small but positive profit in the overall game. Since everybody makes a positive profit, nobody wants to change his action choices at least on the equilibrium path, and the set of Nash equilibria which support this outcome is a CSS. The experimental data show that this equilibrium outcome is chosen eventually in almost all experiments.

The mechanism described above shows how players make use of the price in auction as a coordination device and why competition among players increase both the asset price and the effort choice in the second-stage game. We will prove these results one by one below.

To do that let us analyze VHBB's median game Γ first. In this game our set-valued solution concept reduces to a single-valued solution concept and an ESS induces a CSS. We can observe that no polymorphic population can be an element of a CSS in the median game. In a polymorphic population those players who do not choose the current median are not best responding to the current population state. They should change their strategies to the current median under the best response dynamics. This replacement process will continue until the population state reaches a pure strategy Nash equilibrium. After the population state reaches a pure strategy Nash equilibrium nobody wants to change his strategy under the best response dynamics. Hence it is a CSS as a singleton. This is summarized in Lemma 2.

Lemma 2: A population strategy profile is a CSS in the median game Γ if and only if it is a pure strategy Nash equilibrium.

We can also show that all the pure strategy Nash equilibria are CSS's as a singleton in the minimum game. This is in sharp contrast to an ESS (or a MES set) in a finite population model which predicts the most inefficient pure strategy Nash equilibrium. In the case of ESS (or MES set) a new strategy can enter the population if it does better than the population average. A strategy which chooses a low effort can destabilize a more efficient Nash equilibrium by hurting other players more seriously than itself. Crawford calls this effect the beggar-thy-neighbor policy. But the beggar-thy-neighbor policy does not work for a CSS. An invader cannot enter the population unless it best responds against the current population state. Sobel's NES set allows more entrants, but it cannot destabilize a pure strategy Nash equilibrium in the minimum game and all the pure strategy Nash equilibria are NES sets as a singleton.

With these preliminary results let us analyze VHBB's two-stage game G. First we will characterize a particular CSS in Theorem 1, which is an existence theorem of CSS in the game G. In this CSS players must choose the efficient Nash equilibrium in the median game. This CSS plays an important role in the subsequent analysis, since it can destabilize many other population states.

Theorem 1: The set of population strategy profiles described below, Θ, is a CSS in VHBB's median game with auction. In the first stage game, for player i = 1,2,...,8, $b_i \geq 1.30$, and for player i = 9,10,...,18, $b_i = 1.29$; in the second stage game, for all i = 1,2,...,18, $e_i(1.29) = 7$ and $e_i(p)$ is any effort choice for all $p \neq 1.29$.

Proof: First, we have to prove that no strategy profile outside the set Θ is accessible from any element of Θ. Observe that the ten players who bid the low price do not have an incentive to change their bids. And they cannot change their effort choices for the price p = 1.29. If they change their bids or effort choice e(1.29), their expected payoffs become nonpositive. They may change their effort choices for other asset prices, but the new strategy profiles belong to the set Θ in this case.

Likewise the remaining eight bidders cannot make a bid lower than $1.30. They cannot change their effort choice e(1.29) either. In either case they will lose. On the other hand, if they change their bid prices higher than $1.29 and change their effort choices for corresponding asset prices, the new strategy profiles belong to the set Θ. This shows that no strategy outside the set Θ is accessible from any element of the set Θ.

Next, we have to prove that the set Θ satisfies the set minimality condition. Since any change in any player's action choices off the equilibrium path cannot change the best responses of other players, any element $\theta \in \Theta$ is accessible from another element $\theta' \in \Theta$. This completes the proof of Theorem 1.

Q.E.D.

Here we ignore the labeling of players. If we consider labeling seriously we can find several different CSS's by permutating the role of players. In this CSS the first eight players who bid at least $1.30 receive the equilibrium payoff $0.01, and the remaining ten players participate in the median game with probability 0.1 and receive the expected payoff $0.001. Since everybody is receiving positive payoff nobody has an incentive to change their strategy on the equilibrium path. Changes in players' action choices off the equilibrium path do not matter because they do not change the outcome of the game.

Next we will prove a lemma for the game G, which is a generalization of Lemma 2 for the simple median game. Lemma 3 shows that in the second stage game which is reached with positive probability players must choose the same pure strategy in a CSS.

Lemma 3: If a population strategy profile θ does not induce a pure strategy Nash equilibrium in the second stage game, then there exists a Nash equilibrium θ' of class (ii) in Lemma 2, which is accessible from θ.

Proof: First, pick up the ninth highest bidder in the population and change his strategy in the following way to best respond to the current population state:

$b_9 = \pi(e^*,e^*)-0.01$ for some e*, and

$e_9(p) = e^*$, if $p = \pi(e^*,e^*)-0.01$,

$= 1$, if $p \neq \pi(e^*,e^*)-0.01$.

After this change if a new player becomes the ninth highest bidder, let the new ninth highest bidder change his strategy in the same way. Iterate this process until the ninth bidder has the same rank after his strategy change.

Next, let the tenth highest bidder take the same strategy as the ninth highest bidder. It is easy to see that the tenth highest bidder is best responding to the current population state and that the asset price $p = \pi(e^*,e^*)-0.01$.

Finally, let all the other players change their strategies sequentially to strategies as described in class (ii) of Lemma 1. Call the new population strategy profile θ'. It is easy to see that all these changes satisfy the condition (B) and that θ' is accessible from θ.

Q.E.D.

There are some Nash equilibria in the game G, which induce pure strategy Nash equilibria in the second stage game but do not belong to class (i) or (ii) in Lemma 1. It is straightforward to show that some Nash equilibria of class (i) or (ii) are accessible from these Nash equilibria, since for this movement some players need to change their action choices off the equilibrium path and the best response dynamics allow players to change their actions at unreached information sets fairly freely. Moreover, the following lemma shows that a Nash equilibrium of class (ii) is accessible from a Nash equilibrium of class (i), which induces the same Nash equilibrium outcome in the second stage game.

Lemma 4: If a population strategy profile θ is a Nash equilibrium of class (i), then there exists another population strategy profile θ' of class (ii), which is accessible from θ and induces the same Nash equilibrium outcome in the second stage game.

Proof: First, let all the players change their effort choices in the second stage game as follows;

$e_i(p) = e^*$, if $p = \pi(e^*,e^*)$ or $\pi(e^*,e^*)-0.01$,

$= e \leq e^*$, otherwise.

Next, let the nine lowest bidders choose the bid price $b_i = \pi(e^*,e^*)-0.01$ for all $i = 10, 11, ..., 18$. These changes are possible under the best response dynamics, since the changes do not result in a new asset price and players receive zero payoffs throughout the changes. Then let the ninth highest bidder also choose the same bidding strategy, which is his best response under the new population situation. And we find a new Nash equilibrium of class (ii).

Q.E.D.

Now we can observe that a CSS does not impose serious restrictions on a player's behavior when a subgame is not reached with positive probability. Hence players' action choices at unreached subgames drift freely and in particular players may select good equilibria at unreached subgames. Exploiting this property we can show that the set of Nash equilibrium of class (ii) in Lemma 1, which induces an efficient Nash equilibrium in the second stage game, are not elements of a CSS.

Lemma 5: If a population strategy profile θ is a Nash equilibrium of class (ii) and does not induce the efficient Nash equilibrium in the median game, then there exists an element $\theta' \in \Theta$ which is accessible from θ.

Proof: First, let the eight highest bidders choose the strategy described in Theorem 1. Then the ninth bidder has many different best responses and we can let him choose the strategy as in Theorem 1. From this point on, the strategy of the CSS is a best response for all the remaining players.

Q.E.D.

In the proof of Lemma 5 we allow players to make higher bids and increase their effort choices for higher prices. The increase in the asset price is the only direction that the population can move toward if the initial population state is a Nash equilibrium of class (ii). There is a local ratchet effect in the asset price at least for these strategy profiles.

Lemma 6: If a population strategy profile θ is a Nash equilibrium of class (ii) in Lemma 1, then no population strategy profile θ' which results in an asset price below $\pi(e^*,e^*)-0.01$ can replace θ.

The evolutionary dynamics described above show that the CSS characterized in Theorem 1 is the unique CSS in the game G. Combining Theorem 1, Lemmas 3, 4 and 5, and related observations, we can conclude that whatever the initial population state may be the population state can move to the CSS in Theorem 1 and never leaves it thereafter. If the time horizon is sufficiently long we may say that the population state reaches the CSS with probability one and remains there forever. In this sense we prove both the uniqueness of CSS and the global convergence to the CSS. This is summarized in Theorem 2.

Theorem 2: The CSS Θ described in Theorem 1 is the unique CSS in the game G up to the permutation of players.

The CSS in Theorem 1 is also the unique NES set in the game G. We can also prove all the lemmas for the NES set. But we believe that subjects in the experiment behave in a smarter way than is assumed in the usual evolutionary analysis. We decide to use CSS in the paper, since it assumes a higher degree of rationality than a NES set does. On the other hand, if we use the MES set as the solution concept, we have a big MES set and it is difficult to provide a sharp explanation about the experimental results. As was explained before, the MES set allows more entrants and some elements of the CSS is vulnerable to the invasion of strategies which use the beggar-thy-neighbor policy.

5. Conclusion

We have introduced a new solution concept for a finite population model and shown that it predicts the Nash equilibria chosen by players in VHBB's experiments of coordination games with auction. In the analysis we have used several assumptions including pure strategies for individual players, a finite strategy space in the auction game, and a strong restriction on entrants to the population.

The assumption that players must choose pure strategies may be a strong one, but we conjecture that allowing mixed strategies will not change the main results of the paper.

If the action space of the bidding game is a continuum, our CSS disappears. Every Nash equilibrium in the new game must be of class (i). Since the half of the population receive zero payoff in a Nash equilibrium, they can change their strategy choices in many different ways. Hence the population state can drift freely and the CSS will become a huge set. This is a general theoretical problem of the auction game under complete information when the strategy space is a continuum (we can find a similar problem in the Bertrand oligopoly model too). Related to this problem we should mention that the asset price in the experiments tends to be higher than the CSS predicts. In many cases it fluctuates between $1.29 and $1.30.[8)]

If we allow many players to change their strategies simultaneously, a CSS can be destabilized in many games (for example the Paper-Scissors-Rock game in section 3) and it is more difficult to provide a sharp prediction about what will happen in the population. Considering that several players change their strategies simultaneously in a period in the experiment, one may say that the assumption that only one player can change his strategy at a time may be a restrictive one in our analysis. But for VHBB's median game with auction the CSS has a strong stability property. It remains stable even if many players can change their strategies simultaneously, as long as new strategies best respond against the current population state.

Our analysis does not provide any explanation about players' strategy choices in the initial period. It does not provide an explicit dynamic analysis of players' learning over time either. A complete analysis based on some kind of explicit learning dynamics should be done in the future. A generalization of Crawford (1992) or some modification of Nöldeke and Samuelson (1993) may be one way to the explicit dynamic analysis. But we believe that the analysis in the paper captures an important aspect of the role of competition in coordination games using a static solution concept of evolutionary game theory.

Cachon and Camerer (1992) also run experiments similar to VHBB and replicate many results in VHBB. Besides they find that not only the outside option but also some kind of sunk-cost fallacy argument helps players to coordinate on more efficient equilibria. One important difference between VHBB and Cachon and Camerer is that in Cachon and Camerer the first stage game is a binary choice problem and that there are several Nash equilibria in the second stage game which give higher payoffs than the cost of participation. Because of this problem, even after players decide to pay the cost players face a new coordination problem, and we cannot expect the general efficiency result in their environment. However at least for the median effort game with an outside option the solution concept CSS can provide a theoretical explanation consistent with their findings.[9)]

Cooper, DeJong, Forsythe and Ross (1992, 1993, CDFR below) run several game experiments, in which the forward induction argument can play an important role, and find only limited support for the forward induction hypothesis. In particular, in the battle-of-the-sexes game with an outside option there are many players who choose the outside option instead of trying to communicate their intention in the battle-of-the-sexes game. This is in contrast, to some extent, to the prediction of evolutionary work which implies a stronger forward induction property than Kohlberg and Mertens. We can make an analogy between CDFR's experiments and the pairwise random matching model used in evolutionary game theory. But we think that there is a difference between many experimental environments of CDFR and evolutionary game models. In CDFR's experiments players usually adjust their strategies based on their individual histories. On the other hand, many evolutionary game models assume that at least a small proportion of players can observe the current population state or a part of the population history and change their strategies with this information.[10)] In a few experiments of CDFR, where the aggregate history of play is known to players, players show significantly different behaviors than in other experiments. The new behaviors are more consistent with the forward induction hypothesis.

If we allow players to participate in auction before they play the minimum game, the mechanism used in the analysis of the median game still works and we can expect an efficient equilibrium in the minimum game with auction. In other words, if our solution concept is a more appropriate one than the MES set in environments like VHBB's experiments with auction, we can expect a drastically different result from that in the minimum game without auction. To run an experiment like this will be an interesting test about the robustness of our solution concept.

In contrast to tacit communication analyzed in the paper explicit communication can also play an important role as a coordination device. Kim and Sobel (1992), Matsui (1991), Sobel (1993), and Wärneryd (1991) provide evolutionary analyses of the role of pre-play communication. A new experiment can be designed to see the effectiveness of explicit communication in VHBB's environment by allowing players to send nonbinding messages before playing the median or minimum game.

Footnotes

1) Cachon and Camerer (1992) also run experiments on the median game, the minimum game, and other related games. In many median and minimum game experiments they replicate VHBB's results. When they force players to pay some fixed entry fee, they find that subjects' effort choices tend to increase in the median game, but not in the minimum game. They also find that an outside option has an effect of increasing players' effort choices in both games. We will provide some evolutionary explanations about their results later in the paper.

2) The pricing rule described here is the one most consistent with VHBB's real experiments. Different pricing rules may change the set of Nash equilibria in the game a little bit, but do not affect the main results of the paper.

3) Strictly speaking, the rule of VHBB's multiple unit English Clock auction is somewhat different from the auction described here. In their experiments price increments are 5 cents intially until only 11 subjects remain, at which time, price increments are 1 cent until only 9 subjects remain. In our theoretical analysis we will not consider subtle problems due to this specific auction rule.

4) The new solution concept can also be considered as a discrete time version of entry resistant set introduced by Blume, Kim, and Sobel (1993).

5) The population game model explained in this section is very general and it includes both the pairwise-random-matching model and the playing-the-field model used in evolutionary game theory. As Crawford (1991) has explained VHBB's games are examples of the playing-the-field model.

6) Different solution concepts use different entry conditions. Sobel's NES set uses the entry condition that a new strategy s_i' can enter the population $\theta = (s_1, s_2, ..., s_i, ..., s_N)$ if $U(s_i'|s_{-i}) \geq U(s_i|s_{-i})$. On the other hand, if we restrict an individual's strategy space to the pure strategy space, a set-valued generalization of ESS in a finite population model can be defined using the following entry condition: $\theta' = (s_1, s_2, ..., s_i', ..., s_N)$ can replace $\theta = (s_1, s_2, ..., s_i, ..., s_N)$, if

$$U(s_i'|s_{-i}) \geq 1/(N-1) \sum_{j \neq i} U(s_j|s_{-j}'),$$

where $s_{-j}' = \theta' \backslash \{s_j\}$. We call a solution defined under this entry condition a modified evolutionarily stable set (MES set below). We may also define a new solution concept under the imitator dynamics in which players can only imitate the best strategy in the current population. We will discuss the relevance of these solution concepts to VHBB's experiments later.

7) Strictly speaking, a strategy of the median game in the real experiments should be a function of the history of bids in the multiple unit English Clock auction. This more general history dependence of a strategy does not add anything substantial to our analysis.

8) VHBB also run other experiments in which the multiple unit Dutch Clock auction is used. These experimental results are not reported in the published version of their work, but the asset price in these experiments tends to be lower than $1.30 and sometimes an inefficient Nash equilibrium is selected in the median game. How to explain this difference in the asset price is an interesting theoretical question which we do not answer in this paper.

9) In some of Cachon and Camerer's experiments subjects were forced to pay some fixed entry fee before playing the minimum or median game. Theoretically speaking, the entry fee results in a positive linear transformation of

players' payoffs, which should not change the strategic property of the game. But considering that players face a serious coordination problem in these games we can say that the entry fee plays the role of focal point in Schelling's (1960) sense. This kind of focal point effect cannot be captured by a static solution concept in current evolutionary game theory.

10) In VHBB's experiments subjects did not know the complete aggregate history of the game either. But they knew important summary statistics of the past game play including asset prices in many experiments.

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