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DECENTRALIZATION AND THE COORDINATION PROBLEM[*]

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Abstract

This paper addresses the relation between the degree of decentralization in a population and the probability of coordinating on an efficient outcome. An evolutionary type learning mechanism with a group structure that allows players to "vote with their feet" is introduced. In contrast to most of the recent literature (e.g. Kandori, Mailath and Rob, 1993) in which the risk dominant equilibrium is shown to prevail in the long run, in this paper it is demonstrated that given a general probability distribution over initial states the evolutionary learning process converges almost always to the efficient equilibrium if interaction is decentralized enough. Furthermore, it is shown how the model can be applied to the problem of product standardization.

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I. INTRODUCTION

The idea that efficiency can be enhanced by decentralization is an old one in economics. Generally there are two reasons why this could happen. First, decentralization means smaller groups and in smaller groups individuals are usually better able to coordinate because communication is easier and agreements based on reciprocity are feasible because behavior is more likely to be observable (see e.g. Olson, 1965). Furthermore, there may exist concern for other group members that makes altruistic actions possible. Second, even if the above factors do not apply (for example when interaction is anonymous) efficiency may still be improved by decentralization because decentralization may result in a competition among groups, for example through members "voting with their feet".[1] It is this second possibility that I will address in this paper.

The coordination problem considered here is the simplest possible: a symmetric 2x2 game with two strict Nash equilibria. Recently, the problem of equilibrium selection for such games has received much attention by a growing literature on evolutionary, stochastic learning processes.[2] A common feature of these models is that players are repeatedly and anonymously matched with other players from a large population to play a normal form game.[3] Without knowing the payoffs of their opponents players observe past strategy profiles and behave myopically in the sense that they seek to maximize their current payoffs.

Kandori, Mailath and Rob (1993) (KMR) and Young (1993) show that for 2x2 games the learning process converges to one of the Nash equilibria of the game. Which of the Nash equilibria the process converges to depends on the initial condition. Through the introduction of mutations or mistakes (that is, players use a suboptimal strategy with some probability) a unique equilibrium can be selected in the sense that this equilibrium will be observed "almost all the time" in the long-run. In 2x2 games with two strict Nash equilibria this approach selects the risk dominant equilibrium.

With respect to games in which the risk dominant and the efficient equilibria do not coincide this implies a quite pessimistic view regarding the ability of the population to reach an efficient equilibrium. In the model without mutations the absorbing state of the best reply process depends on the initial condition. In the model with mutations the unique stationary distribution

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[1] See Hirschman's *Exit, Voice and Loyalty* (1970), Tiebout's (1956) model of local public goods and Caplin and Nalebuff's (1992) model on the choice among institutions.

[2] For example Foster and Young (1990), Canning (1990), Kandori, Mailath and Rob (1993) and Young (1993) to name just a few.

[3] See however Canning (1992) and Nöldeke and Samuelson (1993) for applications to extensive form games.

puts probability one on the inefficient, but risk dominant equilibrium even if the population was initially at the efficient equilibrium.

I argue in this paper that decentralization may enable the population to achieve an efficient outcome. To demonstrate this I present a model in which the population is partitioned into many *groups*. As in the above models by KMR and Young (1993) players are randomly drawn from the population to play a coordination game. However, in this model they are matched with members of their own group only. Players can not only choose which action to take in the coordination game but also which group they want to join. It is this assumption that drives the competition among groups.

The main result of the paper is that coordination on the efficient equilibrium becomes more likely the more decentralized the interaction of players in the population is. The term "more likely" here means that in a more decentralized population the process converges to the efficient equilibrium for a larger set of initial conditions (where an initial condition specifies for every group the number of players taking each action).

The degree of decentralization can be measured in two different ways. On way to increase decentralization is by reducing the *absolute* group size while keeping the size of the total population constant. An alternative way is to decrease *relative* group size by increasing the number of groups (and thus total population) while keeping absolute group size constant. Both notions of decentralization will be considered below.

With respect to relative group size a fairly strong result can be proved. The probability that the process converges to the efficient equilibrium can be made arbitrarily close to one when the size of the population is increased while keeping group size constant. For absolute group size the result are somewhat more limited. The probability that the sufficient condition for convergence to the efficient equilibrium is satisfied increases when group size is reduced.

The paper is structured as follows. The next section contains the presentation of the model including the statement of the sufficient condition for convergence to the efficient equilibrium. An alternative best reply process is introduced in section III. In section IV I give the main results on the relationship between decentralization and efficiency. In section V the relationship to the literature and some possible applications will be discussed followed by a short conclusion in section VI.

II. THE MODEL

Consider the symmetric coordination game CG which entails the choice between two actions, A_1 and A_2.

	A_1	A_2
A_1	a	b
A_2	c	d

Coordination Game CG

Assume that $a + b < c + d$, $a > b$, $a > d$ and $d > c$ so that (A_1,A_1) and (A_2,A_2) are both strict Nash equilibria with the first being efficient and the second being risk dominant (Harsanyi and Selten, 1988). Further assume that there is a large but finite population of N players. This population is partitioned into s *groups* $G = \{G_1, G_2,...,G_j,...,G_s\}$. Once within a group players interact only with other players in their group. The *state* of the system is a list of how many players take each particular actions in each of the groups and can be described by a vector $\mathbf{n} = \{n_{ij}\}_{i=1,2,\ j=1...s}$, where n_{ij} is the number of players taking action A_i in group G_j. Let **N** denote the (finite) set of all such states.

Each period players are randomly matched with one player from their own group to play CG. A *strategy* S_{ij} in the stage game consists of choosing an action A_i *and* of selecting a group G_j.[4] The stage game is repeated infinitely often.

The assumption of the population being finite causes various complications with respect to integer problems and requires a more involved notation. I think, however, that the assumption should be taken seriously: real groups are finite and especially in small groups it matters whether a player realizes that his own strategy is part of the profile he observes. Thus, since players cannot be matched against themselves, I will assume that a player first excludes himself from the population profile when calculating the matching probabilities α. Let α_{-kl} denote the relative frequency distribution over strategies faced by a player who currently plays strategy k in group l, $\alpha_{-kl} = \{\alpha_{ij}\}_{i=1,2,\ j=1...s}$, where $\alpha_{ij} = n_{ij}/n_j$ if $j \neq l$, $\alpha_{il} = n_{il}/(n_l - 1)$ if $i \neq k$, and $\alpha_{kl} = (n_{kl} - 1)/(n_l - 1)$.[5]

The one-period expected payoff from taking action A_1 and A_2, respectively, in group j for a player who used strategy S_{kl} in the previous period are given by

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[4] To avoid confusion note that A_1 and A_2 are called *actions* to differentiate them from the *strategies* which entail the choice of a group and of an action.

[5] If $n_j = 0$ or $n_j = 1$, a group is empty or contains just one player. It seems reasonable to assume that in this case the payoff is equal to the one in case of miscoordination, i.e. b if A_1 is chosen and c if A_2 is chosen.

$$\Pi_{1j}(\alpha_{-kl}) = \alpha_{1j}\ a + \alpha_{2j}\ b$$
$$\Pi_{2j}(\alpha_{-kl}) = \alpha_{1j}\ c + \alpha_{2j}\ d$$

The set of best replies is $BR(\alpha_{-kl}) \equiv \{S_{ij} \in S \mid S_{ij} \in argmax_{ij}\ \Pi_{ij}(\alpha_{-kl})\}$.

Assumption 1: In the initial state $\mathbf{n}^{(0)}$ the population of size N is partitioned into s groups of size n.

The general idea of evolutionary game theory - not surprising given its roots in biology - is that strategies that do well should increase in frequency whereas strategies that don't do well should decrease and eventually become extinct. I will follow Samuelson (1991) and Kandori and Rob (1993) in formalizing this general idea by assuming that players are myopic optimizers who - infrequently - have the opportunity to adjust their strategies to the current environment. Infrequent adjustment seems to be a reasonable assumption in situations with adjustment costs that vary stochastically over time. For example, many people buy a new car only when their old car breaks down. Taking the break-down time of cars to be stochastic the (opportunity) cost of switching to a different model is lower at some infrequent points in time.

Infrequent adjustment is not a necessary assumption for this model but it gives some plausibility to the assumption of myopic behavior. If adjustment is infrequent players are almost correct in assuming that the strategy profile tomorrow looks like the profile today. If in addition the future is discounted heavily, then seemingly myopic behavior could be based on rational decisions.

Assumption 2: Every period each player receives the opportunity to adjust his strategy with probability $\theta > 0$. If the opportunity arises, a player observes the current strategy profile $\mathbf{n}$ and chooses a best reply against it. If there are several best replies, a player may choose any of them with positive probability unless he is already playing a best reply in which case he remains at his current strategy.

The probability θ may depend on the state and on the strategy the player is currently using, $\theta(\mathbf{n},S_{ij})$. For example, it may be plausible to specify that players with very bad strategies can switch more frequently. The results in this paper are unaffected by this modification as long as all $\theta(\mathbf{n},S_{ij})$ are strictly positive.[6] Assumption 2 gives rise to a stationary Markov chain on the state space $\mathbf{N}$. I will say that the process has converged to an *absorbing state* in period τ if $\mathbf{n}^{(t)} = \mathbf{n}^{(t+1)}$ for all periods $t \geq \tau$.

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[6] See also Kandori and Rob (1993) for this observation.

From Assumption 2 it is obvious that a state is absorbing if and only if all players use best replies. While in general such best reply processes may converge to a mixed Nash equilibrium,[7] the following proposition shows that for the class of games considered here the process always converges to either of the pure Nash equilibria of the game.

Proposition 1: The best reply process reaches an absorbing state in finite time with probability one. In an absorbing state either all players take action A_1 or all players take A_2.

Proof: I will show first that a state in which both actions are used in some group cannot be an absorbing state, which proves that a mixed Nash equilibrium profile cannot constitute an absorbing state.

Suppose to the contrary that both actions are used in some group, say group j = 1, in an absorbing state. Since in an absorbing state all players use best replies, it must hold that

$$\Pi_{11}(\alpha_{-11}) \geq \Pi_{21}(\alpha_{-11}) \tag{1}$$

$$\Pi_{21}(\alpha_{-21}) \geq \Pi_{11}(\alpha_{-21}) \tag{2}$$

Let μ^* denote the mixing probability for A_1 in the symmetric mixed Nash equilibrium of CG, that is $\mu^* = (d - b) / (a + d - b - c)$. Note that if a player faces a strategy profile in which the proportion of A_1-players is higher than μ^*, then A_1 yields a higher payoff than A_2 for this player.

(1) implies that the proportion of A_1-players faced by a current A_1-player, $(n_{1j} - 1) / (n_j - 1)$, must be at least μ^*, which implies that $n_{1j} / (n_j - 1) > \mu^*$, where $n_{1j} / (n_j - 1)$ is the proportion of A_1-players faced by a current A_2-player. Hence, $\Pi_{11}(\alpha_{-21}) > \Pi_{21}(\alpha_{-21})$, contradicting (2).

Next consider a state in which in some groups everyone takes A_1 and in the remaining groups everyone takes A_2. Clearly, the A_2-players could switch to a group in which A_1 is being played and increase their payoff. Hence, in an absorbing state either everyone takes A_1 or everyone takes A_2.

It remains to prove that the process always converges starting from an arbitrary initial condition. I will demonstrate this by showing that all non-absorbing states are transient, which implies that an absorbing state must be reached eventually. To show that all non-absorbing states are transient I will construct a sequence of possible transitions that lead to an absorbing state.

Consider any non-absorbing state $\mathbf{n}$ and some group j in which $n_{1j} > 0$ and $S_{2j} \notin BR(\alpha_{-2j})$. Such a group must exist unless the A_1- and A_2-players are separated into different groups, a case in which an absorbing state can easily be reached when all A_2-players get the opportunity to move. Now suppose all A_2-players in group j (and no one else) get the opportunity to switch strategies The result is a group consisting entirely of A_1-players who receive a payoff of a. In a possible next

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[7] For conditions under which this is possible see Oechssler (1994).

step all players who do not receive a payoff of a get the opportunity to switch strategies, which yields an absorbing state with everyone playing A_1.■

Even though in general the outcome of the best reply process is determined by both the initial condition and the realizations of the stochastic process, there is some range of initial conditions for which the process converges deterministically to a particular absorbing state.

Proposition 2: If in the initial state there exists a group in which the proportion of A_1-players

$$\mu_j \equiv \frac{n_{1j}}{n} \geq \frac{\delta n+1}{n}, \textit{ where } \delta \equiv \frac{d-b}{a-b},$$

then the process will subsequently converge to an absorbing state in which the efficient equilibrium (A_1,A_1) is played by all non-empty groups.

Proof: Let group j be the group with the highest proportion of A_1-players at time t (denoted by μ^t_{max}) and assume that $\mu^0_{max} > (\delta n + 1) / n$. I will show that μ^t_{max} cannot decrease with t and with positive probability strictly increases every period. Thus, $\mu^t_{max} = 1$ is reached eventually. Subsequently an absorbing state must be reached as soon as all players had the opportunity to adjust their strategies once.

The highest payoff that can possibly be achieved playing A_2 anywhere is d (in a group that consists entirely of A_2-players). If there exists a group j in which $\mu_j > (\delta n + 1) / n$, then A_2 cannot be a best reply anywhere since all players can receive a higher payoff by playing A_1. Consider first players who are not currently playing S_{1j}. When these players choose S_{1j} they can receive a payoff of $\Pi_{1j}(\alpha_{-2j})$ if they are currently in group j or $\Pi_{1j}(\alpha_{-kl})$ if they are currently in some other group $j \neq l$. Both payoffs are higher than the highest payoff they could receive from playing A_2 anywhere:

$$\Pi_{1j}(\alpha_{-2j}) > \Pi_{1j}(\alpha_{-kl}) > d, \ \forall\, l \neq j$$

The first inequality follows from the fact that $\frac{n_{1j}}{n-1} > \frac{n_{1j}}{n}$. The second inequality follows because if $\mu_j \geq (\delta n + 1) / n > \delta$, then

$$\Pi_{1j}(\alpha_{-kl}) > \delta a + (1-\delta)b = \frac{d-b}{a-b}a + \left(1-\frac{d-b}{a-b}\right)b = d.$$

It remains to show that a player who currently plays S_{1j} cannot gain from taking action A_2 anywhere. Since $n_{1j} \geq \delta n + 1$ in group j,

$$\Pi_{1j}(\alpha_{-1j}) = \frac{n_{1j}-1}{n-1}a + \left(1-\frac{n_{1j}-1}{n-1}\right)b \geq \frac{\delta n}{n-1}a + \left(1-\frac{\delta n}{n-1}\right)b = \frac{dn-b}{n-1} > d.$$

This proves that taking A_2 cannot be a best reply for any player which implies that μ^t_{max} cannot decrease and will increase will with positive probability.■

Proposition 2 gives a sufficient condition for convergence to the efficient equilibrium depending only on the initial starting condition $\mathbf{n}^{(0)}$. It should be noted that the condition of Proposition 2 is not necessary as there are initial states not satisfying this condition from which convergence to the efficient equilibrium is possible. In some cases the path (and the absorbing state) of the process depends on the order in which players have the opportunity to adjust their strategies.

It is interesting that an equivalent sufficient condition for convergence to the inefficient equilibrium does not exist. Even if already every player in all groups but one use action A_2, there is still the possibility that all A_2-players in the remaining group switch to another group leaving only A_1-players in that group. Subsequently, the process would converge to the efficient equilibrium.[8]

A less desirable feature of Proposition 2 is, however, that the sufficient condition depends on the group size n. Since some of the results in section IV are based on the comparison of populations with different group sizes, such a restriction is unfortunate. In the next section I will, therefore, suggest an alternative best reply process, which does not have this feature.

III. AN ALTERNATIVE BEST REPLY PROCESS

Consider an extreme version of the best reply process above with $\theta = 1$, that is, a process in which players can adjust their strategies every period. If in some state $\mathbf{n}$ there is a unique best reply for all players, this process moves deterministically to the next state $\mathbf{n}'$. But even if there are multiple best replies all possible successor states of $\mathbf{n}$ have the property that the number of players taking a certain strategy will increase only if there are more players who want to switch to this particular strategy than there are players who want to switch away from it. And if a strategy is a best reply for the players who are currently using it, it cannot be reduced in frequency as no one wants to switch away. It is this general feature (but not necessarily the possibility of switching every period) that forms the basis for the suggested alternative best reply process B (the old process is henceforth called best reply process A).

Let $R(S_{ij})$ denote the set of strategies whose current players would want to switch to S_{ij}, that is $R(S_{ij}) = \{S_{kl} \in S \mid S_{ij} \in BR(\alpha_{-kl}), S_{kl} \neq S_{ij}\}$.

Assumption 3: The transition probability from state $\mathbf{n}$ to some other state $\mathbf{n}'$ is positive if and only if

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[8] Even if there was only one A_1-player left in this group, the process converges to the efficient equilibrium if any other player gets the opportunity to switch his strategy before this lone A_1-player.

1. $S_{ij} \in BR(\alpha_{-ij}) \Rightarrow n'_{ij} \geq n_{ij}$ and
2. if $S_{ij} \notin BR(\alpha_{-ij})$, then

$$\sum_{S_{kl} \in R(S_{ij})} n_{kl} \geq n_{ij} \quad \Rightarrow \quad n'_{ij} \geq n_{ij} \quad \text{and} \quad n_{ij} \geq \sum_{S_{kl} \in R(S_{ij})} n_{kl} \quad \Rightarrow \quad n'_{ij} \leq n_{ij}$$

Condition 1 states that the number of players of a strategy that is best reply for the players who currently play this strategy cannot go down. Condition 2 implies that a strategy weakly increases in frequency if there are at least as many players who want to switch to this strategy as there are players who want to switch away from it and vice versa. The latter condition is the main difference between best reply processes A and B. Note that it imposes a rule on aggregate behavior that - although it seems to be a plausible assumption - cannot be derived from purely individual myopic behavior with independent θ's.

Proposition 3: The best reply process B converges with probability one to an absorbing state in which either all players take action A_1 or all players take action A_2.

Proof: The proof is divided in three parts. First I will show that no state can be absorbing in which both actions are used in the same group. Second, I demonstrate the impossibility of an absorbing state in which in some group everyone uses A_1 and in some other group everyone uses A_2. And finally, I will prove that the process always converges to an absorbing state.

(1) Suppose to the contrary that in an absorbing state **n** both actions are used in some group j. Consider three cases.

(a) $\mu_j > \mu^*$. This implies that $S_{2j} \notin BR(\alpha_{-kl})$, for all $kl \neq 1j$. If S_{2j} is a best reply for the current S_{1j}-players, the process can move to a state with $n'_{2j} > n_{2j}$ since $\sum_{S_{kl} \in R(S_{2j})} n_{kl} = n_{1j} > n_{2j}$. Hence **n** cannot be absorbing. On the other hand if S_{2j} is not a best reply for current S_{1j}-players, $R(S_{2j})$ is empty and the process can move to a state in which $n'_{2j} < n_{2j}$.

(b) $\mu_j < \mu^*$. In this case $S_{1j} \notin BR(\alpha_{-kl})$, for all $kl \neq 2j$. Again, if S_{1j} is not a best reply for the current S_{2j}-players or if $\mu_j \geq 1/2$, $\sum_{S_{kl} \in R(S_{1j})} n_{kl} \leq n_{1j}$ and the process can move to a state in which $n'_{1j} < n_{1j}$. If $\mu_j \leq 1/2$ and $S_{1j} \in BR(\alpha_{-2j})$, there are more players who want to switch to S_{1j} than there are players who want to switch away from it and, consequently, a state can be reached in which $n'_{1j} > n_{1j}$.

(c) If no group exists in which (a) or (b) applies, all groups in which both actions are used must have a proportion of A_1-players of exactly μ^*. Note that none of the strategies in these groups is a best reply to itself as every player can gain by switching to the other strategy in his group. Hence, the state is not absorbing.

(2) Suppose there is an absorbing state in which in some group everyone plays A_1 and in some other group everyone plays A_2. Clearly, for all S_{2j}, $\sum_{S_{kl} \in R(S_{2j})} n_{kl} = 0 < n_{2j}$. Therefore, the process can move to a state in which $n'_{2j} = 0$, yielding a contradiction.

(3) To prove that the process always converges to an absorbing state I will again construct a sequence of possible transitions to an absorbing state starting from any non-absorbing state. Any non-absorbing state **n** must have a group j with a strategy $S_{ij} \notin BR(\alpha_{-ij})$, $\sum_{S_{kl} \in R(S_{ij})} n_{kl} \leq n_{ij}$ and $n_{kj} > 0$, $k \neq i$, unless the A_1-players and the A_2-players are separated in different groups, a case just dealt with in (2). A new state **n'** can be reached in which $n_{ij} = 0$. If k = 1, all players who do not receive a payoff of a may then switch in the next period to S_{1j} and an absorbing state is reached. If k = 2, then either $S_{2j} \in BR(\alpha_{-gh})$ and an absorbing state in which everyone plays A_2 can be reached or $S_{2j} \notin BR(\alpha_{-gh})$. In the latter case there must exist some strategy $S_{1h} \in BR(\alpha_{-2j})$, which implies that $S_{1h} \in BR(\alpha_{-2h})$. Thus, if all S_{2h}-players get to move, $n_{2h} = 0$, and an absorbing state can be reached if all other players switch to S_{1h}.■

Proposition 4: If in the initial state there exists a group in which the proportion of A_1-players

$$\mu_j > \delta$$

then the process will subsequently converge to an absorbing state in which the efficient equilibrium (A_1,A_1) is played by all non-empty group.

Proof: The proof is very similar to the one of Proposition 2. Let μ^t_{max} be defined as above and assume that $\mu^0_{max} > \delta$. I will show that more players would like to switch to S_{1j} than want to switch away from it (if any) and that no one wants to switch to S_{2j} but all players currently playing S_{2j} want to switch away from it. Given the definition of the best reply process B this implies that n_{1j} can only increase and n_{2j} can only decrease. Hence, μ^t_{max} cannot decrease and will increase with strictly positive probability.

To show that the number of S_{1j}-players cannot decrease note that S_{1j} is a best reply for all players but (possibly) the ones currently playing S_{1j}. It suffices to show that $\Pi_{1j}(\alpha_{-2j}) > \Pi_{1j}(\alpha_{-kl}) > d$ since d is the highest payoff that can possibly be achieved by playing A_2. The first inequality follows from the fact that

$$\frac{n_{1j}}{n-1}a + \left(1 - \frac{n_{1j}}{n-1}\right)b \quad > \quad \frac{n_{1j}}{n}a + \left(1 - \frac{n_{1j}}{n}\right)b.$$

The second inequality follows because $\mu^0_{\max} > \delta$ and therefore,

$$\Pi_{1j}(\alpha_{-kl}) > \delta a + (1-\delta) b = \frac{d-b}{a-b} a + \left(1 - \frac{d-b}{a-b}\right) b = d\,.$$

Thus, $\sum_{S_{kl} \in R(S_{1j})} n_{kl} = N - n_{1j} > n_{1j}$, which implies that $n'_{1j} \geq n_{1j}$.

It remains to prove that $S_{2j} \notin BR(\alpha_{-kl})$ for any kl. Since $\mu^0_{\max} > \delta > \mu^*$, $\Pi_{1j}(\alpha_{-kl}) > \Pi_{2j}(\alpha_{-kl})$ for all $kl \neq 1j$. Thus the only candidate remaining for having S_{2j} as a best reply is an A_1-player in group j.

Recall that $\mu^0_{\max}$ is the proportion of A_1-players faced by an outsider to the group and note that $\mu^0_{\max} > \delta$ implies that $\mu^0_{\max} \geq (|n\delta| + 1) / n$, where $|x|$ denotes the largest integer less than or equal to x. After excluding himself a player currently playing S_{1j} faces a proportion of A_1-players that is higher than $|n\delta| / (n-1)$. Thus,

$$\Pi_{2j}(\alpha_{-1j}) < \frac{|n\delta|}{n-1} c + \left(1 - \frac{|n\delta|}{n-1}\right) d\,.$$

If the same player went instead to join any other group k, he would face a proportion of at most $|n\delta| / n$ A_1-players (because in any other group there is at least one A_1-player less than in j). Therefore, $\Pi_{2k}(\alpha_{-1j}) > \Pi_{2j}(\alpha_{-1j})$ which implies that $S_{2j} \notin BR(\alpha_{-1j})$.■

IV. DECENTRALIZATION AND EFFICIENCY

The purpose of this section is to clarify the relation between the degree of decentralization and the likelihood of achieving efficiency. Since both, the degree of decentralization and the likelihood of convergence to the efficient equilibrium, allow a variety of interpretations, I will first discuss these terms.

The degree of decentralization can be measured either in terms of absolute or relative group size. In particular, one could compare the property of the best reply process for a given N but varying absolute group sizes n. Alternatively, one could keep the absolute group size n constant but increase N (and thereby the number of groups s). Decentralization would then be higher in the sense that group size relative to the total population decreases. Both interpretations will be considered below.

Measuring the likelihood of convergence to the efficient equilibrium depends crucially on the assumed distribution over initial states $\mathbf{n}^{(0)}$, i.e. the distribution of A_1-players in the initial state. When the degree of decentralization is measured in terms of relative group size, i.e. when N is changed for a given n, I will suppose that there exists some distribution function $F(\mu, n) = \text{prob}\{n_{1j}/n \leq \mu\}$ over the proportion of A_1-players in each group j, with $F(\mu, n) < 1$ for all $\mu < 1$. Note that $F(\mu, n)$ may depend on the group size but not on the identity of the group. It is assumed that distributions across groups are independent of each other.

Probably the most natural assumption to make about this distribution function is that in the initial state each player uses each action with certain probability, i.e. each individual player is "born" with an initial action - with probability p action A_1 and with probability $q = 1- p$ action A_2. If groups are formed randomly, then for each group μ has a binomial distribution with

$$F(\mu, n) = B(n, \mu n, p) = \sum_{y=0}^{\mu n} \binom{n}{y} p^y q^{n-y}.$$

Even though a binomial distribution seems to be the most natural assumption, other distributions are conceivable. For example, one could assume that when groups are formed any number of A_1-players between 0 and n is equally likely to occur, which gives rise to a uniform (flat) distribution for μ with $F(\mu,n) = \frac{|n\delta + 1|}{n + 1}$.

The next proposition shows that if the degree of decentralization is measured in terms of *relative* group size, a strong statement can be made about the probability of convergence to the efficient equilibrium. Roughly speaking, if group size becomes very small relative to N, then the process converges almost always to the efficient equilibrium regardless of the distribution over initial states. The proposition holds for both best reply processes, A and B.

Proposition 5: Fix a group size n and a distribution function $F(\mu,n)$. Then, for any $\varepsilon > 0$, there exists an N large enough such that the probability of convergence to (A_1,A_1) is higher than 1-ε.

Proof: By Proposition 2 a sufficient condition for convergence of best reply process A to (A_1,A_1) is the existence of at least one group in which the proportion of A_1-players exceeds $(\delta n +1) / n$. The equivalent sufficient condition for best reply process B given in Proposition 4 is the existence of a group with a proportion of A_1-players of at least δ. Since $(\delta n + 1) / n > \delta$, process B converges to the efficient equilibrium whenever process A does. Let $\Lambda(N)$ denote the probability that a group with $\mu_j > (\delta n + 1) / n$ exists. Since the condition is sufficient, $\Lambda(N) > 1- \varepsilon$ implies that the probability of convergence to (A_1,A_1) exceeds $1- \varepsilon$. The probability that all groups have less than or equal to $\delta n + 1$ A_1-players is $F(| n\delta +1| / n, n)^{N/n}$. Hence, $\Lambda(N) = 1 - F(| n\delta +1| / n, n)^{N/n} > 1 - \varepsilon$ for N large enough since $\delta < 1$ and $F(\mu, n) < 1$ for all $\mu < 1$ by assumption.■

I will now turn to the second way in which decentralization can be measured, that is in terms of *absolute* group size. Since in this case group size n is changed for a given total population N, I will assume that there exists a distribution function over the proportion of A_1-players in the total population, $G(\mu,N)$ with $G(\mu,N) < 1$ for all $\mu < 1$. In three aspects the results for this notion of decentralization are more limited in scope than the results with respect to relative group size. First, results will only be given for changing group size by an integer factor. Second, results concern only the probability with which the sufficient condition for convergence to the efficient equilibrium is satisfied. Note that this does not necessarily imply a statement about the probability of the efficient equilibrium occurring. And finally, Proposition 6 holds only for best reply process B.

Proposition 6: Let x be the number of A_1-players in the total population at the initial state. Given any x the probability that the sufficient condition for convergence of best reply process B to the efficient equilibrium given in Proposition 4 is satisfied strictly decreases when the group size n is doubled unless this probability is zero or one, in which case it decreases weakly.

An analogous argument applies if group size is multiplied by any integer instead of doubled.

Proof: Note first that for $x < |\delta n| + 1$ the sufficient condition cannot be satisfied even if all A_1-players are concentrated in one of the small groups. Thus the probability of the sufficient condition being satisfied is zero for both small and large groups. Likewise, if $x > |2\delta n|\, s/2$, there are so many A_1-players that the sufficient condition is satisfied with probability one even with large groups. In the following it is shown that the probability of the sufficient condition being satisfied decreases strictly when group size is doubled for all x with $|\delta n| + 1 \leq x \leq |2\delta n|\, s/2$.

Consider the ways in which the x A_1-players can be ordered in the total population. The N players can be arranged in N! ways. Since we do not have to distinguish the A_1- and A_2-players amongst themselves, there are in fact only $\binom{N}{x}$ distinguishable permutations. All of these permutations are equally likely.

Without loss of generality let players 1 through n belong to group one, players n + 1 through 2n to group etc. When group size is doubled let the new, larger groups contain players 1 through 2n, 2n + 1 through 4n etc.

Since all permutations are equally likely, a statement is more likely if it is true for a larger number of permutations. Therefore, all one has to show is that for any permutation for which the sufficient condition is satisfied with large groups it is also satisfied with small groups but that the reverse is not generally true. The sufficient condition is satisfied for large groups if at least one group contains at least $|2\delta n| + 1$ A_1-players. If $|2\delta n| + 1$ is an even number, then $|2\delta n| = 2|\delta n| + 1$. Thus even if both subgroups contain both exactly half of the A_1-players from the large group, the sufficient condition is satisfied because both small groups have at least $\frac{|2\delta n|+1}{2} = |\delta n| + 1$ A_1-players. If $|2\delta n| + 1$ is odd, $|2\delta n| = 2|\delta n|$. Thus even if A_1-players are split into groups of ½ |

$2\delta n \mid + 1$ and $\frac{1}{2} \mid 2\delta n \mid$, the sufficient condition is satisfied because at least one small group must have at least $\frac{1}{2} \mid 2\delta n \mid + 1 = \mid \delta n \mid + 1$.

In order to show that the reverse of the above statement is not true I will construct for every x such that $\mid \delta n \mid +1 \leq x \leq \mid 2\delta n \mid s/2$ a permutation $\mathbf{n}^{(0)}$ for which the sufficient condition is satisfied for small groups but not for large ones. If $x < \mid 2\delta n \mid s/2$ or if $\mid 2\delta n \mid$ is even, consider a permutation defined by

$$n_{1j} \leq \mid \delta n \mid +1, \ \forall \ j$$
$$n_{1j} < \mid \delta n \mid , \ \forall \ j \text{ odd}$$
$$n_{12} = \mid \delta n \mid +1$$

For small groups the sufficient condition is clearly satisfied in group two. However, for large groups the maximal number of A_1-players in any group is $2 \mid \delta n \mid$ which does not exceed the critical value.

If $x = \mid 2\delta n \mid s/2$ and $\mid 2\delta n \mid$ is odd, take a permutation with $n_{1j} = \mid \delta n \mid +1$ for j even and $n_{1j} = \mid \delta n \mid$ for j odd. Thus, the number of A_1-players in any of the large groups is $2 \mid \delta n \mid +1 = \mid 2\delta n \mid$ which again does not satisfy the sufficient condition.■

Corollary: If the number of A_1-players in the total population is distributed according to a binomial distribution, then the probability that the sufficient condition is satisfied decreases when group size is doubled.

Proof: If x is distributed binomially with B(N,x,p), it is independent of the group size n. The result then follows immediately from Proposition 5 since for all $x < \mid \delta n \mid +1$ and for all $x > \mid 2\delta n \mid s/2$ the probability of the sufficient condition is unchanged and for all $\mid \delta n \mid +1 \leq x \leq \mid 2\delta n \mid s/2$ the probability decreases when group size is doubled.■

V. POSSIBLE APPLICATIONS AND RELATED WORK

One of the most important fields of applications for models of the type analyzed in this paper is the economics of standardization (see Oechssler, 1993b for more details). The literature on standardization based on network externalities addresses the issue from two different viewpoints. One approach considers the supply side, in particular, the way standards are chosen and promoted by producers (see Katz and Shapiro, 1985 and 1986). The other approach starts from the demand side and considers the coordination problem among consumers who - before making their purchases - have to anticipate which standard is going to prevail in the future (see Farrell and Saloner, 1985 and 1986). Especially for the latter approach I see a potential to apply evolutionary game theory in a productive way.

Farrell and Saloner show that the standardization problem can often be reduced to a simple coordination game with multiple, strict equilibria. In such a game players face the difficulty of forming expectations about which equilibrium is going to be adopted. In contrast to the traditional game theory of equilibrium refinements, which cannot select from a set of strict Nash equilibria,[9] evolutionary game theory provides a useful model with which to study the consumers' problem in this case.

Consumers faced with the problem to decide which of two competing technologies will be predominant in the future (and thus will produce higher network externalities) might capitulate given the complexities of such forecasts. Even expert industry analysts generally have a hard time predicting which of two competing systems will prove more successful (who could have predicted whether VHS or Beta would become the standard for VCR's?). It is probably a fair assumption that consumers decide quite myopically in such cases, which makes the application of evolutionary game theory possible.

A simple example of how a standardization problem can be modeled as a coordination game might be helpful. Consider a situation in which consumers have the choice between two incompatible products, e.g. computers with different operating systems. Suppose that consumers interact frequently with each other and positive network externalities result when a consumer meets another with the same operating system (e.g. because they can swap data or software). If they meet a user of the other operating system, no network externalities materialize. Assume further that product 1 is more expensive, $P_1 > P_2$, but that it also yields higher network externalities (U_i) which more than compensates for the higher price, $U_1 - P_1 > U_2 - P_2$. The intrinsic value (autarky value) of both products is the same and is omitted from the payoff matrix. If we denote $a = U_1 - P_1$, $b = - P_1$, $c = - P_2$, and $d = U_2 - P_2$, we get exactly the assumed payoff structure of the coordination game CG above.

An alternative application for evolutionary models with a group structure is the labor-management relationship in firms as suggested by Boyer and Orléan (1992) and Vega-Redondo (1993). In each firm employees play a coordination game with the strategies "cooperate" and "don't cooperate". On the group level firms compete in a market for their products. Although the modeling approach in these papers differs quite significantly from the one presented here, both papers find that competition among firms helps to achieve the efficient equilibrium.

Several other papers have made use of a group structure to show that efficient equilibria can be the outcome of myopic learning processes. Independently Mailath, Samuelson and Shaked (1993) and Oechssler (1993a) developed the approach followed in the current paper, in which the

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[9] Harsanyi and Selten's (1988) theory is an exception but it does not really belong to the traditional refinements literature.

competition among groups is driven by the freedom of players to choose a group to which they want to belong.[10] Canals and Vega-Redondo (1993) present a similar model in which the best reply process is replaced by an imitation process, which operates not only on the individual level but also on the group level (at certain times a group is dissolved and then replaced by a new group that imitates the (mixed) strategy profile of the currently most successful group).[11]

Another related approach is based on cheap-talk (Matsui 1991, see also Kim and Sobel 1992, and Sobel 1993). In these models a player can send a costless and non-binding message regarding the strategy he proposes to play. Cheap-talk works as a "secret handshake" that allows players to escape from inefficient equilibria in common interest games and symmetric coordination games. In these models cheap-talk fulfills a role very similar to the group structure in the other models by allowing players to interact more with players who use the same cooperative strategy as they do.

VI. CONCLUSION

This paper investigates the relation between the degree of decentralization in a population and the probability with which the population can coordinate on an efficient equilibrium. The general setup was the following. Players from a large but finite population are repeatedly and randomly matched to play a 2x2 normal form game with two pure equilibria, one of which being efficient, the other being risk dominant. Interaction is decentralized in the sense that the population is split into many groups and players interact only with other players from their own group. A strategy in this game consists of both the choice of an action in the 2x2 game as well as the choice of a group.

In contrast to most of the recent papers on evolutionary type learning mechanisms (e.g. Kandori, Mailath and Rob, 1993 and Young, 1993) in which the risk dominant equilibrium is shown to prevail in the long run, in this paper it is demonstrated that the learning process almost always converges to the efficient equilibrium if interaction is decentralized enough.

Possible applications of the model include the economics of standardization. In the case of two competing product standards the probability that the more efficient standard is eventually adopted is higher if consumers are more decentralized.

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[10] See, however, Ellison (1993) who applies KMR's model to a setting in which players interact only with their neighbors. Since in his model players cannot choose their neighbors, there is no competition among groups and the risk dominant equilibrium prevails in the long run.

[11] The general idea of group selection has been used in biology for quite some time (Hamilton, 1964). Compare also Axelrod (1984) and Robson (1993).

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