1 Introduction

Costless pre-play communication is one avenue through which players might achieve coordination of their expectations in a game. Does this mean that pre-play communication guarantees that a Nash equilibrium will be played in a game? Does pre-play communication favor efficient Nash equilibria? Can communicating players guarantee themselves their favorite Nash equilibria? Intuitively, pre-play communication should have at least some influence on the focalness of an equilibrium. However, such communication may not be decisive if there are other factors affecting the focalness of an equilibrium.

Aumann [1990] has argued that pre-play communication may not lead to Nash equilibrium, even if the underlying game has a unique strict and efficient equilibrium, if that equilibrium is risky.[1] It seems that the proper way to address the questions raised above and to formalize Aumann's conjecture is to formally add communication moves to the underlying game and to analyze the resulting communication game. Upon doing so it is immediately clear it is of no help to consider all the Nash equilibria in the communication game. Communication enlarges the set of equilibria; in particular, every equilibrium outcome in the underlying game can be supported through an equilibrium in the communication game; furthermore, commonly used refinements are of little help. An approach that has proved successful in establishing a role for costless pre-play communication in games is the evolutionary one, as exemplified by Wärneryd [1991], Kim and Sobel [1993] and Sobel [1993]. Roughly, if player's preferences over equilibria in the underlying game are identical, inefficient equilibria are not evolutionarily stable, and if the underlying game has a unique efficient payoff, there exists a stable set of strategies supporting only that payoff.

The evolutionary approach to pre-play communication does not distinguish among equilibria according to their risk. In games with common interests the common interest outcome cannot be destabilized through evolutionary forces. Intuitively this is so because the evolutionary process moves the population via mutations which affect only a small fraction of the entire population. Since the mutant population is small, the strategic problem faced by the mutants reduces essentially to an optimization problem. The mutants face no significant strategic uncertainty. This point is expressed most clearly in Sobel [1993] where one player

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[1] Farrell [1988] argues that pre-play communication need not lead to a Nash equilibrium even in a game with a unique Nash equilibrium.

at a time gets to adjust her strategy while the strategies used by the rest of the population remain unchanged.

This paper looks at simple games with one round of pre-play communication followed by a simultaneous-move, complete-information game. It is shown that the *curb (closed under rational behavior)* concept (Basu and Weibull [1991]) distinguishes among efficient equilibria in these games according to their risk. *Curb retracts* are minimal sets of strategies closed under inclusion of best replies. An equilibrium is a curb equilibrium if it is a member of a curb retract.

The paper contains two major results. The first deals with two-player games in which only one of the players can send a message. Assume that there is a unique strategy combination which maximizes the communicating player's payoff and that this strategy combination is a strict equilibrium, i.e. it is its own unique best reply, in the underlying game. Then, for given sizes of the message space and of the underlying game, the corresponding payoffs will be the only curb equilibrium payoffs in the communication game, provided the communicating player's risk at that equilibrium is sufficiently low. The risk measure employed in the statement of this result is inspired by the Harsanyi-Selten [1988] notion of *risk dominance.* It is more general in that it applies to all finite two-player games; it is more conservative in that it takes into account the worst possible outcome from playing the equilibrium strategy and the best outcome from playing any alternative strategy. One can construct a less conservative risk measure, more closely in line with risk dominance. It turns out this measure comes into play if one restricts the size of the message space. With only two messages, if the less conservative risk measure is less than one, then in the above setting, the communicating player will get her maximal payoff in any curb equilibrium of the communication game. In a symmetric 2×2 game the less conservative risk measure coincides with Harsanyi and Selten's notion of risk dominance. Hence if such a game is preceded by one round of pre-play communication in which one player talks and the other listens, if the cardinality of the message space equals two, then the efficient equilibrium of the underlying game is the unique curb equilibrium in the communication game if and only if it is risk dominant in the sense of Harsanyi and Selten.

Perhaps one should note the emergence of risk dominance in other contexts. Risk dominance plays a role when it is possible to transit from one strict equilibrium to another. In Ellison [1993], Kandori, Mailath and Rob [1993] and Young [1993] this transition is made possible through perpetual mutations. Here the transition is made possible through communication. Under both scenarios the world is in flux, either because there are exogenous shocks or because there is indeterminacy because of indifference. Finally, risk dominance plays a role in 2×2 games with small payoff uncertainty and incomplete information. This has been demonstrated in a recent paper by Carlsson and van Damme [1993].

The second major result concerns multi-sided communication in N-player games in which all players can talk. It is shown that even in games with common interests there is a fundamental difference between one-sided and multi-sided communication. With multi-sided communication, efficient outcomes are less vulnerable to risk considerations than with only one-sided communication. To obtain this result I add an arbitrarily small amount of inertia to the game. Examples in the literature where games are analyzed after adding such inertia include Blume, Kim an Sobel [1993] and Hurkens [1993]. The comparison with Hurkens' work shows that the form this inertia takes matters. I assume that communicating players display a slight preference for strategies where messages communicate only their intent to play according to their preferred equilibrium. The action taken under such a strategy does not depend on the other player's message; it is always the action associated with the preferred equilibrium. It does not matter whether only some or all such strategies are preferred; therefore it is possible but not necessary to distinguish messages. With this inertia the curb concept distinguishes two-sided from one-sided communication. Modifying payoffs in this way does not affect the results with one-sided communication. However with two-sided communication, and a unique strict common interest equilibrium in the underlying game, only the payoffs of that equilibrium are curb equilibrium payoffs in the communication game.

2 Examples: Dodo and Stag Hunt

This section introduces two different underlying games and compares the role of one-sided pre-play communication in these games. Both of these are symmetric 2×2 games with two strict Pareto-ranked equilibria. In one of these games the Pareto-efficient equilibrium is also the risk dominant one, in the other there is a tension between risk dominance and Pareto dominance. For future reference it is useful to recall Harsanyi and Selten's definition of risk dominance for symmetric games. Consider the game $\boldsymbol{G_0}$ below and assume that $a > c$ and $b < d$ such that (U, L) and (D, R) are two strict Nash equilibria. According to Harsanyi and Selten (U, L) risk dominates (D, R) if $a - c > d - b$ and vice versa. It is also true that (U, L) is risk dominant exactly when the probability of the column player playing

L which makes the row player indifferent is less than 1/2.

	L	*R*
U	a,a	b,c
D	c,b	d,d

$\boldsymbol{G_0}$

Consider the game $\boldsymbol{G_1}$ which Binmore calls *Dodo.* If players could rely on a commonly understood language, we would expect communication to be effective, in the sense of achieving the efficient outcome, in *Dodo.* If believed, an announcement by the row player to play *U* will induce the column player to play *L* in *Dodo.* There is no reason for the column player to not believe such an announcement; if believed it leads to the preferred outcome for both players *and* if the column player intended to play *D* she would not announce *U* if she expected such an announcement to be believed.

	L	*R*
U	3,3	0,0
D	0,0	1,1

$\boldsymbol{G_1}$

	L	*R*
U	9,9	0,8
D	8,0	7,7

$\boldsymbol{G_2}$

The situation in *Dodo* seems so clear cut that it appears unnecessary to rely on communication to achieve efficiency. The efficient outcome seems focal even without prior communication. Van Huyck, Gillette and Battalio [1992] present experimental evidence to show that it is difficult for a mediator to coordinate players on the (D, R) outcome in games like *Dodo*. Therefore let me construct a scenario in which the (U, L) outcome is less obvious and suggest how pre-play communication might help in attaining it. Consider two (equally) large populations of *row* and *column* players. Let players from one population be repeatedly randomly matched with players from the other population to play *Dodo.* Suppose that initially nine tenths of each population are programmed to play *D* and *R* in their respective roles as row and column players. The remaining players are rational utility maximizers. Suppose that over time the preprogrammed players die out. I believe that it is not implausible to think that this procedure can lead to a situation where eventually all players know that only rational players are left and all players play either *D* or *R* depending on their role. From a purely a priori viewpoint (U, L) may be focal, but if for some reason (say history, in the above construction), players are convinced that everyone else will play according to (D, R), then they will do so themselves. The role of communication in this case could be to undo the focal effect of history. If a player is given the opportunity to talk before playing against her randomly chosen partner, she should have no trouble convincing him that it is in both players interest to play according to the efficient equilibrium.

In game $\boldsymbol{G_2}$, which is a version of Rousseau's *Stag Hunt* like in game $\boldsymbol{G_1}$ players have common interests; there is a unique efficient payoff vector. Nevertheless Aumann [1990] has used the *Stag Hunt* to argue that pre-play communication need not lead to Nash equilibrium. Consider the same scenario as above. So without pre-play communication players would be coordinated on the inefficient (D, R) equilibrium. Would it help if a player were able to make a pre-play announcement? Not necessarily. Presumably the row player wants to convince the column player to play L and thus might announce to play U herself. The problem is that she wants to induce the column player to play L no matter what she intends to play herself. If she is not completely convinced that her announcement is successful she may well play D. If that possibility is given sufficient weight by the column player, he will play R himself. According this argument pre-play communication is not successful in the *Stag Hunt* game because a message does not reveal anything about the intentions of the announcer.

This paper is an attempt to formalize the intuition that pre-play communication is more likely to succeed in games like *Dodo* than in games like the *Stag Hunt.* Like Hurkens [1993], I will use Basu and Weibull's [1991] *curb (closed under rational behavior) sets* as the solution concept. A *curb set* is set of strategies, such that each player's component is closed and convex, it is closed under inclusion of best replies, and it is minimal in the class of such sets. What distinguishes this work from Hurkens' is that throughout messages will be costless; at some point I will introduce small payoff variations in the communication game but they will not be linked to costly messages.

To see how *curb sets* distinguish *Dodo* from *Stag Hunt,* let each of these games be preceded by one round of pre-play communication in which only the row player can talk and is restricted to two messages, m_1 and m_2. The reduced normal forms of the communication games corresponding to *Dodo* and *Stag Hunt,* denoted by $_{\mathbf{1}}$ and $_{\mathbf{2}}$ are shown below.

	LL	*LR*	*RL*	*RR*
(m_1, U)	3,3	3,3	0,0	0,0
(m_1, D)	0,0	0,0	1,1	1,1
(m_2, U)	3,3	0,0	3,3	0,0
(m_2, D)	0,0	1,1	0,0	1,1

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	LL	*LR*	*RL*	*RR*
(m_1, U)	9,9	9,9	0,8	0,8
(m_1, D)	8,0	8,0	7,7	7,7
(m_2, U)	9,9	0,8	9,9	0,8
(m_2, D)	8,0	7,7	8,0	7,7

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In $_{\mathbf{1}}$ the unique minimal set Q of strategies closed under inclusion of best replies consists of all mixtures of (m_1, U) and (m_2, U) for the column player and of all mixtures of *LL*, *LR* and *RL* for the row player. To verify that any curb set must include Q start at any strategy combination in the game and iterate best replies. Conversely to check that strategies outside of Q need not be included observe first that for beliefs concentrated on $\{(m_1, U), (m_2, U)\}$, *RR* is never a best reply; its expected payoff is zero while there is a strategy whose payoff against such beliefs is always strictly positive. Similarly, if beliefs of the row player are concentrated on $\{LL, LR, RL\}$, (m_1, D) cannot be a best reply; for any beliefs such that (m_1, D) yields a positive (zero) payoff, (m_2, U) $((m_1, U))$ is a better reply. An analogous argument shows that (m_2, D) is never a best reply against such beliefs. Finally, all equilibria in the game restricted to Q yield payoffs $(3, 3)$: this is the unique *curb equilibrium* payoff vector in the game.

In $_{\mathbf{2}}$, the same set Q of strategies is *not* closed under inclusion of best replies. If the row player's beliefs assign no weight to *LL*, and assign equal weight to *LR* and *RL*, the strategy (m_1, D) is a best reply. The problem arises because in this case the row player knows that one of her messages will induce the column player to play left; but she does not know which one. It is safer to play down than to gamble on communication being successful.

3 The Solution Concept

For a finite strategic form game G let S_i be player i's set of pure strategies, with typical element s_i and let Σ_i be her set of mixed strategies, with typical element σ_i. S and Σ are the sets of pure and mixed strategy profiles. σ is a strategy profile and σ_{-i} a partial profile which excludes player i's mixed strategy. Denote the convex hull of any set Z by $co(Z)$. $BR_i(\cdot)$ is player i's (mixed) best reply correspondence and $BR(\sigma) = \times_{i=1}^{N} BR_i(\sigma_{-i})$. $Q \subset \Sigma$ is a retract if $Q = \times_{i=1}^{N} Q_i$ where $Q_i \subset \Sigma_i$ is nonempty, closed and convex. A retract Q is a *curb retract* if for all $\sigma \in Q$, $BR(\sigma) \subset Q$, and Q is a minimal retract with that property. A Nash equilibrium contained in a *curb retract* is called a *curb equilibrium.*

In every finite game there is at least one *curb retract* and a *curb equilibrium. Curb retracts* are spanned by pure strategies; i.e. with any mixed strategy $\sigma_i \in Q_i$, Q_i contains all pure strategies in the carrier of σ_i. Therefore any *curb retract* can be identified with a set of pure strategies. The intersection of two retracts closed under inclusion of best replies is itself closed under inclusion of best replies.[2]

4 One-Sided Communication

In this section I will generalize the observation about *Dodo* versus *Stag Hunt* made in Section 2. I will first prove a result about one-sided communication in a two-player game with an arbitrary finite underlying game G, and an arbitrary finite message space M. Then I specialize this result to the case of only two messages, and finally I specialize it further to the case where the underlying game is a symmetric 2×2-game. In each of these cases the result is that communication is effective if the *risk* of the preferred equilibrium is low relative to a *standard of comparison* that depends on the size of the underlying game and the size of the message space. In the general setting the standard of comparison is inversely related to both the size of the message space and the size of the underlying game. Both of these factors increase the number of possible strategies which can be interpreted as increasing strategic uncertainty. With only two message the standard of comparison becomes independent of the size of the underlying game (and trivially the message space.) If the underlying game is in addition a symmetric 2×2-game, then the appropriate condition reduces to Harsanyi and Selten's [1988] *risk dominance* criterion.

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[2] For more on this see Basu and Weibull [1991], Balkenborg [1992] and Hurkens [1993].

Denote the strategies of the row (column) player in the underlying game G by $i \in I$ ($j \in J$.) Let the row player be player one and the column player be player two with payoffs $u_k(i, j)$, $k = 1, 2$. Assume that the underlying game has multiple strict Nash equilibria; this is the interesting case. Assume also that there is a unique strategy combination $(\hat{\imath}, \hat{\jmath})$ which maximizes the row player's payoff, i.e.

$$(\hat{\imath}, \hat{\jmath}) = \arg\max_{(i,j)} u_1(i, j),$$

and that this strategy combination is a strict Nash equilibrium. Examples of such games are *Dodo*, *Stag Hunt* and the *Battle of the Sexes.* Let M be the message space available to player one in the communication game $\Gamma_1(G, M)$. In this game player one first sends a message from the set M, then both players play the game G. Player one's strategies in the reduced normal form of the communication game are of the form (m, i) with $m \in M$ and $i \in I$. Player two's strategies are functions f which map messages $m \in M$ into actions $j \in J$. Let $F(G, M)$ be the set of pure strategies of player two in the communication game induced by G and M. Define

$$\hat{F}(G, M) := \{f \in F(G, M) | (\exists m \in M : f(m) = \hat{\jmath})\}.$$

Denote the cardinality of a set X by $\#(X)$. Measure the risk of the equilibrium $(\hat{\imath}, \hat{\jmath})$ in the game G by

$$\rho((\hat{\imath}, \hat{\jmath}), G) := \max_{i \neq \hat{\imath}} \frac{\max_j u_1(i, j) - \min_j u_1(\hat{\imath}, j)}{u_1(\hat{\imath}, \hat{\jmath}) - \max_j u_1(i, j)}.$$

This risk measure requires some explanation. It measures only the risk of one of the players at the equilibrium $(\hat{\imath}, \hat{\jmath})$. It is suitable only for measuring the risk of an equilibrium that maximizes player one's payoff; only then the denominator is guaranteed to be positive.[3] The risk of the equilibrium $(\hat{\imath}, \hat{\jmath})$ to player one decreases if her payoff at that equilibrium increases or if the worst outcome from playing her equilibrium strategy $\hat{\imath}$ improves. Relative to any alternative strategy i, the risk increases if the maximum payoff from that strategy increases. Finally, only the maximal risk relative to any alternative strategy matters. The latter two properties suggest that this risk measure is fairly conservative; it takes into account the worst outcome for the equilibrium strategy in question and the best outcome over all alternatives. As shown below, a less stringent criterion can be used in special

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[3] This risk measure cannot be used to compare the risk of multiple Pareto-ranked equilibria. For a related measure which allows such comparisons, see below.

cases. It is worth pointing out that this measure is invariant with respect to positive affine transformations of the payoff function.

The first result states that, for a given size of the message space, if this risk measure is sufficiently small, then communication will be effective. I will construct a curb retract Q which for the communicating player is spanned by all strategies of the form $(m, \hat{\imath})$, and for the silent player is spanned by all strategies which respond with $\hat{\jmath}$ to *some* message. The condition relating the risk of $(\hat{\imath}, \hat{\jmath})$ to the size of the message space ensures that strategies of the form (m, i), $i \neq \hat{\imath}$, are never best replies against any beliefs over Q. The max and min operators in the risk measure come from constructing a worst case scenario for beliefs concentrated on Q, in which player one believes that the message m induces player one's favorite reply given i, and an alternative message l induces player one's worst reply given $\hat{\imath}$ with high probability. Even in such a worst case scenario one can always find a message l which induces the reply $\hat{\jmath}$ with probability of at least $1/\#(\hat{F}(G, M))$. $\#(\hat{F}(G, M))$ is simply the number of strategies of the silent player in Q and is an increasing function of the number of messages; no belief concentrated on Q can assign weight less than the reciprocal of that number to all strategies in $\hat{F}(G, M)$. Thus, for any beliefs concentrated on Q, the communicating player can ensure that the weight of $u_1(\hat{\imath}, \hat{\jmath})$ in her expected payoffs is at least $1/\#(\hat{F}(G, M))$ for one of her strategies in Q, say $(l, \hat{\imath})$. Since $(\hat{\imath}, \hat{\jmath})$ is the unique profile which maximizes her payoff, if the possible payoffs from other strategy profiles (i, j), $i \neq \hat{\imath}$, in the underlying game are not "too high," this suffices to make $(l, \hat{\imath})$ a strictly better reply than (m, i). The risk measure makes the meaning of "too high" precise.

Proposition 1 *For any G and M, if*

$$\frac{1}{\#(\hat{F}(G, M)) - 1} > \rho((\hat{\imath}, \hat{\jmath}), G),$$

then (a) $Q = Q_1 \times Q_2$

$$:= \{co\{(m, \hat{\imath})\}_{m \in M}\} \times \{co\{f \in F(G, M) | (\exists m \in M : f(m) = \hat{\jmath})\}\}$$

is the unique curb retract in $\Gamma_1(G, M)$*, and (b) the payoffs in all curb equilibria are* $u_k(\hat{\imath}, \hat{\jmath})$, $k = 1, 2$.

Proof **:** By assumption, $\hat{\jmath}$ is the unique best reply to $\hat{\imath}$ in the underlying game.

Thus, against beliefs concentrated on Q_1 any strategy f' with $f'(m) \neq \hat{j} \;\; \forall m \in M$ has a strictly lower expected payoff than $\hat{f}$ where $\hat{f}(m) = \hat{j} \;\; \forall m \in M$.

Suppose that $(m, i), i \neq \hat{\imath}$, is a best reply against beliefs λ concentrated on Q_2. Let $\lambda(f)$ be the probability assigned to f by λ. Let $\hat{F}(m,j) := \{f \in \hat{F}(G, M) | f(m) = j\}$. Then

$$\sum_{j \in J} \sum_{f \in \hat{F}(m,j)} u_1(i,j)\lambda(f) \geq \sum_{j \in J} \sum_{f \in \hat{F}(l,j)} u_1(\hat{\imath},j)\lambda(f) \;\; \forall l \neq m.$$

Note that

$$\max_{f \in \hat{F}(G,M)} \lambda(f) \geq \frac{1}{\#(\hat{F}(G,M))} > 0.$$

Let $\tilde{f} \in \arg\max_{f \in \hat{F}(G,M)} \lambda(f)$ and without loss of generality let $\tilde{f} \in \hat{F}(l, \hat{j})$. Then

$$u_1(i, \tilde{f}(m))\lambda(\tilde{f}) + (1 - \lambda(\tilde{f})) \max_j u_1(i,j) \geq$$

$$u_1(\hat{\imath}, \hat{j})\lambda(\tilde{f}) + (1 - \lambda(\tilde{f})) \min_j u_1(\hat{\imath}, j)$$

from which

$$\frac{\lambda(\tilde{f})}{1 - \lambda(\tilde{f})}(u_1(\hat{\imath}, \hat{j}) - \max_j u_1(i,j)) \leq \max_j u_1(i,j) - \min_j u_1(\hat{\imath}, j),$$

which implies

$$\frac{1}{\#(\hat{F}(G,M)) - 1} \leq \rho((\hat{\imath}, \hat{j}), G).$$

Thus, for a strategy outside of Q_1 to be a best reply to beliefs concentrated on Q_2, the condition in the proposition must be violated. If it holds Q is closed under inclusion of best replies.

To see that Q is minimal among retracts closed under inclusion of best replies consider the following: The strategy $f(m) = \hat{j} \;\; \forall m \in M$ is a best reply to any strategy in Q_1. All strategies in Q_1 are best replies to $f(m) = \hat{j} \;\; \forall m \in M$. Any strategy in Q_2 is a best reply to some strategy in Q_1 by construction.

To establish uniqueness it suffices to show that for any set $\tilde{Q} \neq Q$ which is closed under best replies, $Q \cap \tilde{Q} \neq \emptyset$: If $(\tilde{m}, i) \in \tilde{Q}_1$, $i \neq \hat{\imath}$, and j is a best reply to i, then there exists an $f \in \tilde{Q}_2$ such that $f(\tilde{m}) = j$, and $f(m) = \hat{j} \;\; \forall m \neq \tilde{m}$. Thus for all $m' \neq \tilde{m}$, $\{(m', \hat{\imath}), f\} \in Q \cap \tilde{Q}$. This shows that Q is the unique curb retract in the communication game.

It remains to show that all equilibria in the game restricted to Q have payoffs $u_k(\hat{\imath},\hat{\jmath})$. Since $(\hat{\imath},\hat{\jmath})$ is a strict Nash equilibrium in the underlying game, $u_2(\hat{\imath},\hat{\jmath})$ is player two's maximal payoff in the communication game restricted to Q. Player two can guarantee himself that payoff in Q. Whenever player two gets $u_2(\hat{\imath},\hat{\jmath})$, player one gets $u_1(\hat{\imath},\hat{\jmath})$ in the game restricted to Q. □

With a message space consisting of only two messages a somewhat stronger result holds. To state this result I will introduce an alternative risk measure $\tilde{\rho}$. One can measure the risk of the equilibrium $(\hat{\imath},\hat{\jmath})$ in the game G by

$$\tilde{\rho}((\hat{\imath},\hat{\jmath}),G) := \max_{i\neq\hat{\imath}} \frac{u_1(i,\hat{\jmath}) - \min_{j\neq\hat{\jmath}} u_1(\hat{\imath},j)}{u_1(\hat{\imath},\hat{\jmath}) - \max_{j\neq\hat{\jmath}} u_1(i,j)}.$$

It is easily verified that $\rho((\hat{\imath},\hat{\jmath}),G) \geq \tilde{\rho}((\hat{\imath},\hat{\jmath}),G)$. In this sense the risk measure $\tilde{\rho}$ is less conservative than ρ. Like ρ this measure is invariant under positive affine transformations. The differences between the two measures are threefold. $u_1(i,\hat{\jmath})$ replaces $\max_j u_1(i,j)$ in the numerator, which means that less weight is given to the best possible outcome under an alternative strategy i. The other two differences concern the ranges of j over which the min in the numerator and the max in the denominator are taken. Under $\tilde{\rho}$ in both cases $\hat{\jmath}$ is excluded which means that a (weakly) higher payoff is considered under the status quo and a weakly lower payoff under the alternative. In the symmetric 2 × 2 game discussed in Section 2 there is only one alternative and therefore there is no need to maximize over alternatives. Moreover in this case the limitations on the range of the max and min operators imply that these operators can be dropped. By doing so one arrives at what one might call the Harsanyi-Selten risk measure

$$\rho_{HS}((\hat{\imath},\hat{\jmath}),G) := \frac{u_1(i,\hat{\jmath}) - u_1(\hat{\imath},j)}{u_1(\hat{\imath},\hat{\jmath}) - u_1(i,j)}$$

for 2 × 2 games. Note that in such a game the equilibrium $(\hat{\imath},\hat{\jmath})$ is risk dominant in the Harsanyi-Selten sense if

$$u_1(\hat{\imath},\hat{\jmath}) - u_1(i,\hat{\jmath}) > u_1(i,j) - u_1(\hat{\imath},j),$$

Which is equivalent to

$$\rho_{HS} < 1.$$

The following result resembles Proposition 1, with three differences. They are the cardinality of the message space (two messages instead of finitely many), the risk measure ($\tilde{\rho}$ instead of ρ) and the standard of comparison (1 instead of $\frac{1}{\#(\hat{F}(G,M))-1}$).

Proposition 2 *For any G and* $\#(M) = 2$ *if*

$$1 > \tilde{\rho}((\hat{\imath},\hat{\jmath}),G),$$

then (a) $Q = Q_1 \times Q_2 :=$

$$\{co\{(m,\hat{\imath})\}_{m\in M}\} \times \{co\{f \in F(G,M)|(\exists m \in M : f(m) = \hat{\jmath})\}\}$$

is the unique curb retract in $\Gamma_1(G,M)$*, and (b) the payoffs in all curb equilibria are* $u_k(\hat{\imath},\hat{\jmath})$*,* $k = 1,2$*.*

Proof **:** The proof of Proposition 2 is identical to the proof of Proposition 1 with one exception that concerns showing that Q is closed under inclusion of best replies.

Let $M := \{m_1,m_2\}$. Suppose that (m_1,i), $i \neq \hat{\imath}$, is a best reply against beliefs λ concentrated on Q_2. With only two messages it is convenient to represent a strategy of player two as a vector (j^1,j^2) with the first (second) element being the response to message m_1, (m_2). Let $\lambda(j^1,j^2)$ be the probability assigned to strategy (j^1,j^2) by λ. Then, since (m_1,i), $i \neq \hat{\imath}$, must be at least as good a reply as $(m_1,\hat{\imath})$ we have

$$\sum_{j\neq\hat{j}} u_1(i,\hat{\jmath})\lambda(\hat{\jmath},j) + \sum_{j\neq\hat{j}} u_1(i,j)\lambda(j,\hat{\jmath}) \geq$$

$$\sum_{j\neq\hat{j}} u_1(\hat{\imath},\hat{\jmath})\lambda(\hat{\jmath},j) + \sum_{j\neq\hat{j}} u_1(\hat{\imath},j)\lambda(j,\hat{\jmath}).$$

Note that we do not need all the λs because unlike in the proof of the previous proposition we make no use of the fact that there is a lower bound on the maximum.

Therefore

$$[u_1(\hat{\imath},\hat{\jmath}) - u_1(i,\hat{\jmath})]\sum_{j\neq\hat{j}} \lambda(\hat{\jmath},j) \leq$$

$$[\max_{j\neq\hat{j}} u_1(i,j) - \min_{j\neq\hat{j}} u_1(\hat{\imath},j)]\sum_{j\neq\hat{j}} \lambda(j,\hat{\jmath}).$$

Together with the condition in the proposition this implies

$$\sum_{j\neq\hat{j}} \lambda(j,\hat{\jmath}) > \sum_{j\neq\hat{j}} \lambda(\hat{\jmath},j),$$

since at least one of these sums must be positive for (m_1, i), $i \neq \hat{\imath}$, to be a best reply.

Since (m_1, i), $i \neq \hat{\imath}$, must be at least as good a reply as $(m_2, \hat{\imath})$ we also have the following condition

$$\sum_{j\neq\hat{j}} u_1(i,\hat{j})\lambda(\hat{j},j) + \sum_{j\neq\hat{j}} u_1(i,j)\lambda(j,\hat{j}) \geq$$
$$\sum_{j\neq\hat{j}} u_1(\hat{\imath},j)\lambda(\hat{j},j) + \sum_{j\neq\hat{j}} u_1(\hat{\imath},\hat{j})\lambda(j,\hat{j}).$$

Therefore

$$[u_1(\hat{\imath},\hat{j}) - \max_{i\neq\hat{j}} u_1(i,j)] \sum_{j\neq\hat{j}} \lambda(j,\hat{j}) \leq$$
$$[u_1(i,\hat{j}) - \min_{j\neq\hat{j}} u_1(\hat{\imath},j)] \sum_{j\neq\hat{j}} \lambda(\hat{j},j).$$

Together with the condition in the proposition this implies

$$\sum_{j\neq\hat{j}} \lambda(\hat{j},j) > \sum_{j\neq\hat{j}} \lambda(j,\hat{j}).$$

Therefore we have reached a contradiction. Hence, (m_1, i), $i \neq \hat{\imath}$, cannot be a best reply to any beliefs λ concentrated on Q_2. An analogous argument works for (m_2, i), $i \neq \hat{\imath}$. □

We have seen that $\tilde{\rho}$ is a generalization of the Harsanyi-Selten risk measure ρ_{HS} while ρ is not. For that reason let me examine why ρ and not $\tilde{\rho}$ is the appropriate risk measure in games with more than two messages. Consider the following version of *Dodo*

	L	*R*
U	x,x	0,0
D	0,0	1,1

where $2 > x > 1$. Consider the communication game where only the row player can talk and the cardinality of the message space equals three. The reduced normal form of this game is given by

	LLL	LLR	LRL	LRR	RLL	RLR	RRL	RRR
(m_1, U)	x,x	x,x	x,x	x,x	0,0	0,0	0,0	0,0
(m_1, D)	0,0	0,0	0,0	0,0	1,1	1,1	1,1	1,1
(m_2, U)	x,x	x,x	0,0	0,0	x,x	x,x	0,0	0,0
(m_2, D)	0,0	0,0	1,1	1,1	0,0	0,0	1,1	1,1
(m_3, U)	x,x	0,0	x,x	0,0	x,x	0,0	x,x	0,0
(m_3, D)	0,0	1,1	0,0	1,1	0,0	1,1	0,0	1,1

It is evident that in this game any curb retract which supports the efficient outcome in the underlying game must contain all strategies which support the efficient outcome in the communication game. This means that for example the strategies *RLR* and *RRL* for player two must be included in the curb set. However, if player one believes that player two uses only these two strategies and uses them with equal probability, then the strategy (m_1, D) is a best reply, from which it follows easily that in fact all strategies will be in any curb retract. Only if we made x larger than 2 would this argument not work. It is this distinction which is captured by the risk measure ρ. Note that $\tilde{\rho}(x) = 0\ \forall x > 1$ and $\rho(x) = \frac{1}{x-1}\ \forall x > 1$; the risk associated with the equilibrium *UL* gets large as x approaches one.

This example does not show why in addition to changing the risk measure we need to change our standard of comparison as we increase the number of messages since $\rho(x) < 1$ is equivalent to the condition that $x > 2$. The reader may check however that if we add another message, x needs to increase for it to be the unique curb equilibrium payoff in the communication game.

The various risk measures introduced up to this point are all structurally similar to ρ. They work well for our purpose, however they rely on $(\hat{\imath}, \hat{\jmath})$ being the unique strategy profile which maximizes player one's payoff in the underlying game. Therefore it is worth pointing out that instead of $\tilde{\rho}$ we could have considered an alternative measure in Proposition 2 which works just as well there and can be used to compare the risks of multiple Pareto-ranked Nash equilibria. This measure is

$$\rho_{GHS} := \max_{i \neq \hat{\imath}} \frac{\max_{j \neq \hat{j}} u_1(i,j) - \min_{j \neq \hat{j}} u_1(\hat{\imath},j)}{u_1(\hat{\imath},\hat{j}) - u_1(i,\hat{j})}.$$

I will refer to this as the Generalized Harsanyi-Selten measure of risk. Section 6 contains a brief discussion on the relation of this measure to some examples from the literature on stochastic evolutionary game dynamics.

5 Multi-Sided Communication

In this section I will look at multi-sided communication and compare it with one-sided communication. For the main result I will adopt an (arbitrarily small) modification of payoffs in the communication game. This modification favors those strategies which use the “efficient action" after all messages of the other players. The message used with such a strategy can be thought of as affirming an intent to play according to the preferred equilibrium. I will show that while the ability to affirm one’s intent in this way is of no consequence with one-sided communication, it has a significant effect if all players can talk. If all players can talk, communication can reduce strategic uncertainty enough to ensure that a unique efficient equilibrium in the underlying game will in fact be played.

Since I will prove the main result for an N player game, I will adopt a somewhat more general notation in this section than in the previous one. Let each player’s space of pure strategies in the underlying game be denoted by S_n, $n \in N$, and let s_n be a typical element of player $n's$ strategy space. Analogous to the previous section assume that there is a strategy combination $\hat{s} = \{\hat{s}_n\}_{n=1}^{N}$ in the underlying game such that

$$\hat{s} = \arg\max_{s} u_n(s) \;\; \forall n \in N;$$

i.e., for every player $n \in N$, $\hat{s}$ is the unique strategy combination which gives n’s maximum payoff in the game.

I will assume that each player has preferences in the communication game which add a small reward for the use of strategies in which the message *affirms the intent* to use the unique efficient action in the underlying game. I will refer to such preferences in the underlying game as *AI* preferences. To be precise, assume that each player $n \in N$ receives a bonus of $\epsilon > 0$ when she uses a strategy of the form (m_n, f_n) where $f_n(m_{-n}) = \hat{s}_n \; \forall m_{-n}$. If we denote payoffs in the communication game by $U_n(m, f)$, then

$$U_n(m,f) = \begin{cases} u_n(f(m)) + \epsilon, & \text{if } f_n(\tilde{m}_{-n}) = \hat{s}_n \ \ \forall \tilde{m}_{-n}; \\ u_n(f(m)), & \text{otherwise.} \end{cases}$$

The arguments which establish Propositions 1 and 2 in the previous section go through virtually unchanged with AI preferences. Therefore AI preferences have no effect with one-sided communication.[4] If on the other hand all players can communicate, AI preferences introduce enough inertia that only the efficient outcome is stable, without any reference to risk considerations. This is shown in the next proposition.

Denote by $\Gamma_N(G, M)$ the communication game in which the play of the underlying game G is preceded by one round of simultaneous communication in which all N players announce a message from their respective message spaces M_n.

Proposition 3 *For any underlying game G and message spaces M_n, $n \in N$, if all players have AI preferences in $\Gamma_N(G, M)$, then $Q = \times_{n=1}^{N} Q_n$ where $Q_n := co\{(m_n, f_n)|m_n \in M_n,\ f_n(m_{-n}) = \hat{s}_n \ \ \forall m\}$ is the unique curb retract in $\Gamma_N(G, M)$, for all $\epsilon > 0$.*

Proof **:**Against any belief concentrated on Q_{-n} player n can achieve a payoff of $u_n(\hat{s}) + \epsilon$ by using one of the strategies in Q_n. Any other strategy will at most yield a payoff of $u_n(\hat{s})$. This show that Q is closed under inclusion of best replies.

Every strategy combination in Q gives player n her maximal payoff in the communication game. Therefore no strict subset of Q is closed under inclusion of best replies, which shows that Q is minimal.

It remains to show uniqueness. I will show that if $\tilde{Q}$ is curb, then $Q \cap \tilde{Q} \neq \emptyset$. Let $(m, f) \in \tilde{Q} \backslash Q$. Then there exists a strategy $(m', f') \in \tilde{Q}$ with $f'_n(\tilde{m}_{-n}) = \hat{s}_n \ \ \forall n,\ \forall \tilde{m}_{-n} \neq m_{-n}$. Against (m', f') player n can guarantee that the other players will play $\hat{s}_{-n}$ in the underlying game by not sending message m_n. Hence, there exists $(m'', f'') \in \tilde{Q}$ with $f''(m'') = \hat{s}$ which implies that there exists $(m'', f''') \in \tilde{Q}$ with $f'''_n(\tilde{m}_{-n}) = \hat{s}_n \ \forall n,\ \ \forall \tilde{m}_{-n}$. □

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[4] If instead of only the communicating players, all players got a bonus from planning to use only the “good action," then a result analogous to the one below would hold also with one-sided communication. The point here is that it is explicitly assumed that it is communication which makes a difference. It is assumed that communication signals some (arbitrarily small) commitment to taking the “good action." This is much like assuming a small preference for truth; truth is not an issue for a silent player.

With multi-sided communication, there is no result of the form: “If the risk of an efficient equilibrium in the stage game is sufficiently low, it will be the only curb equilibrium in the communication game." To see this consider the following example of a communication game derived from a version of *Dodo*.

	m'_1LL	m'_1LR	m'_1RL	m'_1RR	m'_2LL	m'_2LR	m'_2RL	m'_2RR
(m_1, UU)	x,x	x,x	0,0	0,0	x,x	x,x	0,0	0,0
(m_1, UD)	x,x	x,x	0,0	0,0	0,0	0,0	1,1	1,1
(m_1, DU)	0,0	0,0	1,1	1,1	x,x	x,x	0,0	0,0
(m_1, DD)	0,0	0,0	1,1	1,1	0,0	0,0	1,1	1,1
(m_2, UU)	x,x	0,0	x,x	0,0	x,x	0,0	x,x	0,0
(m_2, UD)	x,x	0,0	x,x	0,0	0,0	1,1	0,0	1,1
(m_2, DU)	0,0	1,1	0,0	1,1	x,x	0,0	x,x	0,0
(m_2, DD)	0,0	1,1	0,0	1,1	0,0	1,1	0,0	1,1

In this game two players simultaneously exchange messages before playing *Dodo;* with $x > 1$. The cardinality of each player’s message space equals two. It is easily checked that any curb set which supports the efficient outcome must include all strategies which are consistent with efficiency. However the game induced by the retract which is formed by all such strategies contains an equilibrium in which the row player uses m_1UD and m_2DU with equal probability and the column player uses m'_1LR and m'_2RL with equal probability. Thus, for any x there is a curb equilibrium with payoffs $\frac{x+1}{2}$ for each player.

6 Related Literature

This section discusses three strands of literature relevant to this paper. It first looks at some experimental evidence on pre-play communication. Then follows an overview of the evolutionary approach to pre-play communication. The section concludes with comments on the literature on stochastic evolutionary game dynamics and its relation to generalized risk measures.

Cooper, DeJong, Forsythe and Ross (CDFR) [1992] report on experiments they conducted on a version of the *Stag Hunt* game ($a = 1000,\ b = 0,\ c = 800, d = 800$ in game $\boldsymbol{G_0}$). They repeatedly let players play one-shot communication games. Each communication game consists of a communication stage followed by the play of the underlying game. They consider three different communication environments: no communication, one-way communication and two-way communication. They find that without communication the risk dominant equilibrium will be played. With one-way communication the frequency of the Pareto-efficient equilibrium increases but there are also a significant number of coordination failures. Two-way communication resolves the coordination problem; almost exclusively the Pareto-efficient equilibrium is played. In light of the theory proposed here it is interesting to note the difference between one and two-way communication. We saw in the previous section that *AI* preferences made no difference with one-sided communication but guaranteed efficiency with multi-sided communication. The interpretation is that if communication serves to affirm the intent to play according to the efficient equilibrium this is enough to reduce strategic uncertainty if all players do it; on the other hand, if only one player communicates she cannot be as confident of the response of the listener. In the words of CDFR: “This doubt about the action of a receiver is overcome by the two-way communication design since both players receive information about the likely play of their opponents." [1992, p.757]

While refinements in the spirit of strategic stability have little power in games with pre-play communication the evolutionary approach yields sensible predictions in common interest games. Several authors have used versions of Maynard Smith and Price’s [1973] notion of an *evolutionarily stable strategy (ESS)* in these games. Roughly, in a symmetric game a strategy is evolutionarily stable if it is a symmetric Nash equilibrium and, if it is played by all members of a large population, it cannot be invaded by a small population of mutants who use a different strategy. An *ESS* must be a best reply to itself, and it must be a better reply to the post-entry population than any potential entrant.

Wärneryd [1991] studies *Dodo* preceded by one round of pre-play communication in which each player sends a message from a common finite message space which contains at least two messages. The *EES* concept has no power in these games; mixed strategy equilibria can be invaded by their constituent pure strategies and pure strategy equilibria can be invaded by strategies which differ only at unreached information sets. For that reason Wärneryd adopts *neutrally stable strategies (NSS)*, a variant of *ESS's*, as his solution concept. *NSS's* need only be weakly better in the post-entry population. Wärneryd shows that any *NSS* in *Dodo* preceded by one round of simultaneous pre-play communication leads to the efficient equilibrium in the underlying game. Wärneryd's analysis does not extend to more general games because there the use of unused messages may get penalized deterring players from introducing them.

Kim and Sobel [1993] obtain similar results as Wärneryd for more general games than *Dodo* by using a set-valued solution concept, Swinkels' [1992] *Equilibrium Evolutionarily Stable (EES) Set.* Set valuedness makes it possible that an equilibrium in which the use of unused messages is penalized *drifts* to one where no stigma is attached to unused messages. At that point it may be possible for more efficient equilibria to *invade.* Kim and Sobel obtain two main results, an existence theorem for games with common interests, and an efficiency theorem for games with equilibrium common interests. They consider finite two-player games preceded by one round of simultaneous communication. Each player sends a message from a large finite message space. A common interest game has a unique efficient payoff vector. In such a game there will always be an *EES* set containing all strategies which support the efficient outcome. In a game with equilibrium common interest both players have identical preferences over the equilibria in a game. In such a game no inefficient equilibrium can be part of an *EES* set. The argument which establishes the efficiency result has two parts. First, any equilibrium can drift to a version of itself which does not use some of the message and in which the use of such messages is not penalized. Second, once such unused messages are available they can be used as a secret code. Members of an invading population can use these messages to identify themselves to their fellow invaders. Against the original population they may continue to play according to the status quo, against other invaders they may play an efficient equilibrium in the underlying game. Drift moves the population to a point where a successful invasion becomes possible.

Kim and Sobel give an example to show that pre-play communication need not lead to efficiency, even in a game with a unique efficient Nash equilibrium in the underlying game.

The evolutionary argument differs from the argument presented in this paper, in that only a small fraction of the population moves at any given point in time. This reduces the possibility for coordination failures significantly. In the case of common interest games, once the evolutionary process reaches the efficient point, any potential invader must also play an efficient strategy in order to maximize its payoff against the post-entry population. The problem of a mutant in this situation is reduced to a simple optimization problem with no strategic component.

Sobel [1993] has a particularly transparent version of the evolutionary argument. In his model only one player at a time adjusts her strategy. A strategy profile *replaces* another if it differs only in the component corresponding to one of the players, and this player is weakly better off in the replacement. A set of strategy profiles is called a *nonequilibrium evolutionarily stable (NES)* set if it includes all such replacements and is minimal with respect to this property. In games with one round of pre-play communication there exists a unique *NES* set. If the underlying game has common interest the *NES* set consists only of profiles which yield the efficient payoff.

The efficient outcome in common interest games is the unique stable outcome if the strategic problem faced by the population is broken down into a sequence of single player decision problems. With common interests, if only one player can adjust at a time, in a sense, the social problem is solved in small increments. Each agent faces a simple decision problem because he does not have to worry about possible simultaneous adjustments by other members of the population.

With simultaneous adjustments as in the present paper, either one has to acknowledge the role of risk and strategic uncertainty, or one must introduce some other source of inertia if one wants to ensure that only efficient outcomes are stable.

In Section 5 we examined one such form of inertia. Hurkens [1993] gets very strong result for the same solution concept, curb retracts, and a different form of inertia. Hurkens considers finite games with one round of simultaneous pre-play communication in which a subset J of the player set N can send a message and players in $N \backslash J$ cannot talk. He assumes that messages, except for one, come with a nominal cost which is positive, may be arbitrarily small and differs across messages. He finds that if the communicating players have common interests, then they obtain their preferred outcome in any curb equilibrium of the communication game. In two-player games the same result holds for persistent equilibria, which were defined by Kalai and Samet [1984]. Since messages are costly in Hurkens' paper his work does also provide a link with the work on “burning money" by Ben-Porath and Dekel [1992]. In both papers, the players who are given the opportunity to burn money can achieve their preferred outcome, and no money is actually burned in equilibrium. Ben-Porath and Dekel use a different solution concept, iterative deletion of weakly dominated strategies, and require that message costs be nonnegligible.

Risk becomes an issue in this paper because of the relative ease with which one can travel from one strategy combination to another. This is a consequence of the low entry requirements embedded in the solution concept (curb retracts) and the absence of other sources of friction such as message costs, as in Hurkens' work.[5] One reason it is so easy to travel from one profile to another is that communication turns strict equilibria into weak ones. Thus communication creates an escape route from strict equilibria. An alternative escape route is analyzed in the literature on stochastic evolutionary game dynamics, as for example in Kandori, Mailath and Rob [1993], Young [1993] and Ellison [1993]. There too risk plays a role. Kandori, Mailath and Rob for example prove that in two-player two-strategy coordination games the limit of the stationary distributions of their dynamics as the noise vanishes puts all probability weight on the risk dominant equilibrium. Young [1993] examines a similar kind of dynamics and comes to similar conclusions. He also points that while in two-strategy games a characterization of stochastically stable equilibria in terms of risk dominance is possible no such characterization may be available for games with three or more strategies. He gives the following example.

	L	*C*	*R*
U	6,6	0,5	0,0
M	5,0	7,7	5,5
D	0,0	5,5	8,8

Young points out that while the equilibrium (D, R) pairwise risk dominates the other two pure strategy equilibria in this game, there are plausible dynamics under which (M, C) is the unique stochastically stable equilibrium. Against this background it is perhaps interesting to calculate the Generalized Harsanyi-Selten measures for this game. They are $\rho_{GHS}(U, L) = 7$, $\rho_{GHS}(M, C) = 3/2$, and $\rho_{GHS}(D, R) = 7/3$. Therefore the equilibrium picked by Young's dynamics is also

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[5] Hurkens' results holds also for the weaker solution concept "closed under inclusion of better replies." Such a solution concept poses even less stringent entry requirements than curb retracts. In Hurkens work this permissiveness is balanced by the cost of messages. For a discussion and characterization of this solution concept see Ritzberger and Weibull [1993].

the one with the lowest global risk measure. While Ellison's [1993] discussion makes clear that this identification will not hold for all dynamics of this kind, the global risk measure defined here seems to identify some of the dynamic forces acting on multiple Pareto-ranked strict Nash equilibria.

7 Conclusion

The chances for pre-play communication to allow players to coordinate on efficient equilibria are tied to the risk associated with these equilibria. This observation can be formally expressed through the use of *curb sets* and *curb equilibria* as the solution concepts. This contrasts with and complements results from the evolutionary literature on pre-play communication in which risk plays no role. One interpretation could be that the dynamic forces captured in these approaches differ. *Curb sets* use a permissive “entry" condition, and because they are retracts seem to allow for simultaneous adjustments and miscoordination. One benefit of this approach is that it permits one to make a distinction between one- and two-sided communication which parallels some experimental results.

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