Organizational Design and Technology Choice with Nonbinding Contracts

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Abstract. We present a new methodology for studying the problem of labor contracting within a firm's boundaries where contracts provide only a minimal commitment to wages and employment. Given the peculiar contractual incompleteness of labor contracts, the resulting wages and profits under an interesting class of complete information bargaining games distort the technological and organizational decisions facing the owner of the firm's capital. We show that in such settings where labor contracts are nonbinding, these decisions are distorted in an economically distinct way compared to the standard neoclassical firm. Among other things, we demonstrate that a firm with a nonbinding contractual basis will, relative to a neoclassical firm, (i) overemploy labor, (ii) underemploy capital, (iii) choose inefficient "frontloaded" technologies, (iv) de-emphasize scale and scope economies, and (v) inefficiently allocate labor across productive assets. We apply our analysis to product market competition, unionization, hierarchical management, and horizontal mergers.

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1. Introduction

Labor contracts are necessarily incomplete, with only a limited capability to bind either party to the relationship. Dissatisfied employees are generally free to quit at will, and firms are typically able to dismiss part or all of their labor force. If there are joint gains from trade, we expect firms with productive capital to hire labor, but the ability of any employee or the firm to re-enter into wage negotiations at any time prior to production places particular restrictions on the equilibrium wage profile. Focusing on the firm's optimal choices in such a bargaining framework, we propose to study various issues regarding organizational design, technology choice, and labor decisions using a novel bargaining methodology.

Following Coase's [1937] seminal work on the nature of the firm, economists have spent much time debating and defining the boundaries of the firm and the market place. Most recently, the papers of Grossman and Hart [1986] and Hart and Moore [1991] ("GHM") have refined the idea of the firm as essentially an allocation of property rights and residual rights of control over assets. Fundamental to the GHM "property rights" theory of the firm is that contracts are incomplete, and ex post bargaining generally results in return streams that are suboptimal for specific investments from an ex ante view point.[1] This line of research, however, has generally focused on the question of what determines the boundaries of a firm and how should assets optimally be allocated, taking as given the underlying asset structure, the number of productive agents, and their organizational relationships to one another. We seek an entirely different approach instead, asking questions regarding the optimal nature of assets, the number of productive agents, and the relationship of labor to capital in the optimal organizational design.

Our approach is similar in spirit to Jensen and Meckling [1976] who take the firm as a nexus of contracts, and then study the optimality of various underlying contractual arrangements and the outcome of the complex equilibrium process *within the firm*. We take as a working definition that a firm is a set of productive assets, which produces output that depends upon the allocation of laborers to those assets. To bring the analysis into clear focus, we assume that the firm's assets are owned by a single individual. Consequently, we put aside

[1] The property rights approach builds upon the work of Williamson [1975,1985], Klein, Crawford, and Alchian [1978], and Grout [1984], among others, who consider relationship-specific investments and ex post hold-up costs.

the question of optimal asset ownership, and rather devote our attention to other issues. To this extent, we do not address the issue of a firm's external boundary, but rather a formal theory of the firm's internal bargaining process and its economic ramifications.

Typically, employees can quit at will, regardless of contractual terms, and firms can layoff employees with only limited penalties. In this spirit, we take labor contracts as *nonbinding* in nature. The focus of our paper is on situations in which employees have the ability to bargain directly with the firm either because of their importance within the firm (managers, division heads, key engineers, etc.) or the small size of the firm. We model labor contracts as an agreement for a wage which the firm (here, the owner of the productive capital) pays the employee, providing that the employee actually carries out the contracted productive services. At any time before production takes place, an employee may approach the firm and enter into wage negotiations. Likewise, the firm may choose at any moment before production to call the employee in for wage negotiations. Hence, with labor contracts, the nature of the contractual incompleteness is the inability of either party to commit to a future wage and employment decision.[2] Given the contractual incompleteness, we consider wage contracts which are stable in the intra-firm negotiation environment.

In the framework of GHM, only asset-ownership matters because it alone commits the firm to particular residual income streams. We consider, however, that the firm's initial asset-employee assignments are costly to reverse (perhaps because of training costs, etc.), so an employee assigned to an asset can either work with that asset or quit, but cannot be reassigned elsewhere. Due to the irreversibility of labor assignments, such assignments provide a form of commitment and affect the residual income streams in the intra-firm bargaining environment.

In section 2, we proceed with a description of the contractual environment within our archetype firm which we refer to as a *wage-negotiable* firm, named such because rents will generally be appropriated by labor to some extent in wage negotiations, thereby distinguishing it from the standard neoclassical fixed-wage firm. We initially analyze a noncooperative bargaining process when there is only one asset. We find, strikingly, that our noncooperative bargaining solution is characterized by the Shapley value for a corresponding cooperative game, and has an elegant economic interpretation.

[2]Implicitly, we also assume that either the firm cannot commit (i.e., write a credible contract) to hire an agreed upon number of workers with a pre-specified form of organization and technology, or the employees are financially constrained so as to be unable to pay an entry fee to the firm. When neither assumption is true, the first best can be obtained by employees paying an entry fee equal to the present value of their excess bargaining rents and the firm, in turn, hiring at the first-best level using the first-best technology and labor allocation choices.

In section 3, we take the asset choice of the firm as given and examine the firm's optimal hiring decision compared to the neoclassical firm. With some very simple conditions on production functions, we demonstrate that excess employment always occurs. In essence, overemployment results to drive down the marginal employee's contribution to the firm's revenues (the marginal revenue product of labor), thereby reducing the firm's wage bill. We apply this intuition to an analysis of product market competition and the incidence of training costs.

In section 4, we consider the other side of the firm's maximization program by taking labor as fixed and considering the firm's optimal choice of asset. First, we examine the traditional case where the marginal product of labor can be affected through the choice of capital intensity. Here, we find that compared to its neoclassical counterpart, our firm will install a suboptimal level of capital. Second, we consider the more general situation where the asset productivity is not parameterized by a single variable such as capital level, but rather by a more complex notion of margin frontloading. We find that our firm will prefer inefficient frontloaded technology over the more efficient technology preferred by the neoclassical. This notion of frontloading has numerous applications, including a precise prediction about when a firm will prefer to negotiate with a single union rather than several.

In section 5, we consider further problems of organization. Suppose that the firm can choose two productive assets, A and B, each of which has an exclusive assignment of employees chosen by the firm. How will a firm tailor its asset choices to one another in light of nonbinding labor contracts? Specifically, is it in the interest of the firm to choose technology which exhibits economies of scope, or would some negative externality between assets serve a rent-extracting role? Additionally, will a firm ever find it advantageous to inefficiently allocate labor between the chosen assets? Answers to these questions will shed light on related questions such as Williamson's problem of selective intervention, as a merger of separate firms may change the underlying internal contracting process in a fundamental (and perhaps very costly) manner. We proceed by recasting our bargaining framework for multiple assets and labor groups, and examining the noncooperative solution. As in section 2, we show that this yields the Shapley value of the underlying cooperative game. We also find similar distortions as in the one-asset case, except now economies of scope and homotheticity of production play interesting roles.

Section 6 concludes by suggesting further applications and extensions using the present methodology.

2. The Contractual Environment within a Firm

2.1 The Simple One-Asset Bargaining Game

We examine a non-cooperative dynamic bargaining game with complete information. While we rigorously describe an extensive form game for the following bargaining environment (see Remark 1, below), the intuition for our results is more immediate if we first consider a looser description of the game. At date 1, the firm selects some N employees and offers each a wage contract for employment. These contracts are only minimally enforceable in the following sense: at any time before production takes place in period 2, the firm may fire an employee, in which case the employee is forced to return to the external labor market where he or she earns $\underline{w}$, or an employee may quit the firm and return to the external labor market. After period 1, but before production in period 2, the firm cannot hire employees from the external market. In this sense, the firm is constrained to buy labor from its own internal labor market which it endogenously creates at period 1. A plausible motivation for such setting is that it take one period to train an employee.

We allow both employees and the firm to engage in an unlimited number of pair-wise negotiations where any one employee can reopen negotiations over his or her individual wage contract with the firm.[3] Specifically, we look for a wage profile which neither an employee nor the firm will wish to alter. To determine the set of stable outcomes for our noncooperative setting, we need to establish the outcome of the pairwise bargaining subgames. Specifically, we assume that in any pairwise negotiation, *the firm and employee split the difference*, and thereby obtain the payoffs associated with the Nash bargaining solution.[4] This is not nec-

[3] We think of employees (e.g., mid-level managers) as the minimal bargaining units for labor. If, however, one instead wants to consider union negotiation, rather than individual employees, the analysis is the similar. We address the issue of union negotiations in section 4.

[4] We think the simplest motivation is to suppose that a repeated offer game similar to Rubinstein's [1982] is played between the bargaining pairs, but where there is some ϵ-probability that a rejection in an offer by either party will result in the termination of the game and the players receiving their outside payoffs. This game, for example, is considered in Binmore, Rubinstein, and Wolinsky [1986] and Sutton [1986]. In such a game, each party receives approximately half of the joint net surplus. Specifically, if the joint gain from trade is X, if each player's outside option is $\underline{v}_i$, if the probability that disagreement is followed by another period of offers is δ, and if player 1 makes the first offer, then the bargaining equilibrium is for player 1 to offer the split $\{\phi_1, \phi_2\}$, where $\phi_1 + \phi_2 = X$ and

$$\phi_1 = \frac{1}{1+\delta}X + \frac{\delta}{1+\delta}\underline{v}_1 - \frac{1}{1+\delta}\underline{v}_2.$$

essary for our general results as any weighted split of the surplus will provide qualitatively similar results; this notion is briefly explored in section 6. Since employees are symmetric in the bargaining game, we can characterize the equilibrium with a wage level, $\tilde{w}(n)$, where the argument n denotes the number of employees within the firm. After such a wage is found, the firm and its retained employees will enter the production phase, at which point the wage contracts become binding.

We first seek to understand this underlying game and the nature of equilibrium payoffs. We then use this bargaining game, which we find quite compelling on both theoretical and empirical levels, to analyze the firm's initial choice of organizational design, technological choice, and labor-hiring decisions.

For now we suppose that there are only N potential employees in the firm at date 1; we will endogenize the choice of N later. The productive nature of any asset can be described by a production function, $F(n) : \mathbb{N} \mapsto \mathbb{R}_+$, and $F \in \mathcal{F}$, where $\mathcal{F}$ if the set of available production technologies. Alternatively, we can define the first difference of the production function for the marginal employee n as $f_n \equiv F(n) - F(n-1)\ \forall n \in \mathbb{Z}$, $f_0 \equiv F(0)$, and therefore

$$F(n) = \sum_{i=0}^{n} f_i.$$

Initially, we inductively characterize the outcome of the bargaining game for the simple case of a single asset. Consider the outcome if only one employee accepted the contract at date 1, and either the employee or firm decides to open wage negotiations. The net surplus the employee receives from staying with the firm and taking a wage w is $w - \underline{w}$; the net surplus the firm receives from retaining the employee is $f_1 - w$. The total net surplus from exchange is $f_1 - \underline{w}$. In such a one-employee setting, the resulting equilibrium wage is

$$\tilde{w}(1) = \frac{1}{2}(f_1 + \underline{w}),$$

providing that $f_1 \geq \underline{w}$; otherwise, the firm dismisses the employee.

Now consider the outcome with two employees at date 1. Suppose an employee wishes to alter his or her wage with the firm, where the wage is currently at $\tilde{w}(2)$. The net surplus of the employee at a wage of w is $w - \underline{w}$. The firm's net surplus differs from the one-employee setting, however. If the employee leaves, the firm immediately losses f_2 on the margin, *but in*

As $\delta \to 1$, $(\phi_1 - \underline{v}_1) \to (\phi_2 - \underline{v}_2)$, satisfying our assumption.

addition a wage negotiation ensues with the remaining employee (who would otherwise have received $\tilde{w}(2)$). This secondary wage effect is significant. Rather than setting an equilibrium wage w such that $f_2 - w = w - \underline{w}$, the firm and employee negotiate for a wage $\tilde{w}(2)$ satisfying

$$f_2 - \tilde{w}(2) - [\tilde{w}(2) - \tilde{w}(1)] = \tilde{w}(2) - \underline{w},$$

providing again that $\tilde{w}(1) \geq \underline{w}$ and $\tilde{w}(2) \geq \underline{w}$. Substituting in the result for $\tilde{w}(1)$ we obtain the equilibrium wage:

$$\tilde{w}(2) = \frac{1}{3}[f_2 + \tilde{w}(1) + \underline{w}] = \frac{1}{3}f_2 + \frac{1}{6}f_1 + \frac{1}{2}\underline{w}).$$

Note that while the coefficients on the marginal products still sum to $\frac{1}{2}$, now $\tilde{w}(2)$ depends on *both* the marginal output f_2, as well as the inframarginal output f_1.

Proceeding inductively, for $n \geq 2$, the outcome can be characterized by the difference equation,

$$\tilde{w}(n) = \frac{1}{n+1}[f_n + (n-1)\tilde{w}(n-1) + \underline{w}], \tag{1}$$

providing that for all $i \leq n$, $\tilde{w}(i) \geq \underline{w}$. To simplify the analysis, we employ the following definition.

Definition 1 ***Feasibility.*** *Whenever the solution, $\{\tilde{w}(i)\}_{i=0}^{n}$, to a first-order difference equation, $w(i) = g(w(i-1), i)$ has the property that $\tilde{w}(i) \geq \underline{w}$ for all $i \leq n$, we say that the solution is* ***feasible*** *at n with respect to $\underline{w}$.*

A solution to the above difference equation that is feasible is also the unique equilibrium to the noncooperative bargaining game. When the solution is not feasible, we must be careful to include the possibility that for some intermediate number of employees, firing may be optimal from the firm's viewpoint. We will generally ignore this problem as it does not arise when the firm optimally chooses the level of employment and the underlying marginal revenue product of labor function is single peaked. With this definition, we are prepared to state are first equilibrium characterization result which we obtain by solving (1).

Theorem 1-A *Suppose that there are n employees in the firm and the solution to (1) is feasible. Then the non-cooperative equilibrium wage and profit functions are*

$$\tilde{w}(n, \underline{w}) = \frac{1}{n(n+1)} \sum_{i=1}^{n} i f_i + \frac{1}{2}\underline{w}, \tag{2}$$

$$\tilde{\pi}(n, \underline{w}) = \sum_{i=1}^{n} (1 - \frac{i}{n+1}) f_i - \frac{n}{2} \underline{w}. \tag{3}$$

Of particular interest is equation (2) which indicates that employee wages are given by a weighted average of marginal products, with decreasing weights the further infra-marginal the product. Whereas equations (2) and (3) give equilibrium wages and profits in terms of a weighted average of marginal production, we will find it useful to refer to an alternative equilibrium characterization using cumulative production functions. In particular, summing by parts equations (2) and (3) yields,[5]

$$\tilde{w}(n, \underline{w}) = \frac{1}{n} \left(F(n) - \frac{1}{n+1} \sum_{i=0}^{n} F(i) \right) + \frac{1}{2} \underline{w}, \tag{4}$$

$$\tilde{\pi}(n, \underline{w}) = \frac{1}{n+1} \sum_{i=0}^{n} F(i) - \frac{n}{2} \underline{w}. \tag{5}$$

By further rearranging (5) and using the definition of the neoclassical firm's profit function, $\pi(n, w) \equiv F(n) - wn$, we have our alternative solution.

Theorem 1-B *Suppose that there are n employees in the firm and the solution to (1) is feasible. Then the non-cooperative equilibrium profit and wage functions are*

$$\tilde{\pi}(n, \underline{w}) = \frac{1}{n+1} \sum_{i=0}^{n} \pi(i, \underline{w}), \tag{6}$$

$$\tilde{w}(n, \underline{w}) = \underline{w} + \frac{\pi(n, \underline{w}) - \tilde{\pi}(n, \underline{w})}{n}. \tag{7}$$

This is a very useful and powerful result: The profit to a firm with nonbinding contracts is the average of the neoclassical firm's payoffs from $i = 0, 1, \ldots, n$. Thus, unlike the neoclassical firm which cares only about the "marginal" $\pi(n, \underline{w})$, the wage-negotiable firm puts equal weight on the inframarginal profits as well. This will be particularly helpful in further characterizations of the wage-negotiable firm's decisions and will give rise to very distinct

[5] It is straightforward to show that the discrete version of the chain rule and integration by parts (what we call "summation by parts") are given as follows: Let u_i and v_i be real functions defined over the set of integers and let Δ be the difference operator defined as $\Delta f_i \equiv f_i - f_{i-1}$; then

$$u_i(\Delta v_i) + (\Delta u_i) v_{i-1} \equiv \Delta(u_i v_i),$$

$$\sum_{i=m}^{n} (\Delta u_i) v_{i-1} \equiv u_i v_i|_{i=m}^{n} - \sum_{i=m}^{n} u_i (\Delta v_i).$$

economic predictions.

Remark 1 As an alternative to describing the outcome to our bargaining game through this iterative process, we may obtain an identical outcome as the unique subgame perfect equilibrium of a simple, albeit incompletely specified, extensive-form game. In particular, consider a firm with n employees. Fix any ordering for the n employees to bargain with management over wages. If the employee and the firm do not reach an agreement, the employee drops out of the firm and all remaining employees (including those who have already negotiated with the firm) bargain again with the firm, in the previously specified order. Presume that the nature of bargaining is such that in each pair-wise meeting, the Nash outcome is obtained; we leave this part of the extensive form unspecified. It is easy to show that the unique subgame perfect outcome to this game – independent of the order chosen – is that specified in Theorem 1. This game, of course, simply captures the intuition behind our bargaining setup, in which we seek an outcome immune to any renegotiations which split the surplus relative to the outside option.

Remark 2 Although the results of this paper can be stated while rigorously taking into account integer concerns, we will find it useful at times to assume a continuous version of (6) above. The most natural extension is to think of units of labor as subdivisible, with each smaller unit capable of bargaining with the firm in pairwise meetings. Suppose that $F(n)$ is defined over $\mathbb{R}_+$, rather than only $\mathbb{Z}$. Then our noncooperative solution is defined for any finite subdivision, as also is the neoclassical firm's profit, $\pi(n, w)$. Formally, let n be subdivided into ν equal subdivisions of labor of size $\delta = \frac{n}{\nu}$. Then

$$\tilde{\pi}(n, w) = \frac{1}{\nu + 1} \sum_{i=0}^{\nu} \pi(i\delta, w) = \frac{1}{n + \delta} \sum_{i=0}^{n/\delta} \pi(i\delta, w)\delta.$$

Taking the limit as δ goes to 0, we obtain the following result.

Result 1 *Suppose that $F(n)$ is defined over $\mathbb{R}_+$, and labor is subdivisible, where each subdivision has the ability to negotiate with the firm in a pairwise meeting. Then $\tilde{\pi}(n, w)$ converges to*

$$\frac{1}{n} \int_0^n \pi(s, w) ds \tag{8}$$

as labor becomes infinitely subdivisible.

We now proceed to demonstrate a simple relationship between these payoffs and the Shapley value to the underlying *cooperative* game.

2.2 An Equivalence Result for Cooperative Games

As an alternative to our underlying noncooperative bargaining game, we demonstrate that the outcome is equivalent to the Shapley value for the associated cooperative game. In particular, consider the cooperative game, $(\mathbb{N}_N, v)$, where $\mathbb{N}_N$ is the section of natural numbers: $\mathbb{N}_N \equiv \{0, 1, 2, \ldots, N\}$; we let 0 index the firm and the positive integers index individual employees. v is the characteristic function which maps from subsets of agents to the value they can independently produce: $v: 2^{\mathbb{N}_N} \mapsto \mathbb{R}$, with $v(\emptyset) = 0$. Any coalition, $S \subseteq \mathbb{N}_N$, which does not include the owner of the firm does not have access to the firm's underlying production process and therefore $v(S) = |S|\underline{w}$. When S does include the owner of the firm, the value of the coalition is $v(S) = F(|S| - 1)$.[6] Theorem 2 states that the Shapley value[7] for this cooperative game is equivalent to the bargaining outcome of Theorem 1.

Theorem 2 *Suppose that there are n employees hired by the firm and the solution to (1) is feasible. Then the equilibrium wages and profit are given by the Shapley values of the underlying cooperative game $(\mathbb{N}_n, v)$.*

While we could compute Shapley values to the cooperative game directly and compare them to Theorem 1-B, it is perhaps more instructive to show equations (4) and (5) are first differences of a potential function for v, as defined in Hart and Mas Colell [1989]. Their paper demonstrates that there exists a unique function, called the potential, $P: (S, \mu) \mapsto \mathbb{R}_+$,

[6] We are implicitly assuming that when working in the firm, the outside reservation wage $\underline{w}$ is not obtainable; i.e., the firm cannot hire out its employees for the outside wage. Despite the fact that this characteristic function lacks super-additivity, agents obtaining their reservation values is ensured by equilibrium conditions. Additionally, it is sufficient to presume the firm can ex ante commit to not hiring out labor, because in our setting such commitment is beneficial to the firm insofar as this strengthens its bargaining position with its employees.

[7] The Shapley value is the unique cooperative game solution that satisfies axioms of symmetry, linearity and efficiency to carriers. The symmetry axiom holds that strategically identical players obtain identical outcomes; linearity is in outcomes obtained across games. The final axiom maintains that any carrier (a set containing all strategically relevant players in the game) jointly obtain the optimal outcome. With these axioms, the unique outcome for player i for the cooperative game v can be represented by,

$$\phi_i(v) = \sum_{S \subseteq \mathbb{N}_N - i} \frac{|S|!(N - |S|)!}{(N+1)!} \left[v(S \cup \{i\}) - v(S) \right]. \tag{9}$$

mapping from all games to the positive reals, such that for all games (S, μ),

$$\sum_{i \in S} (P(S, \mu) - P(S \backslash \{i\}, \mu)) = v(S),$$

and furthermore, the first difference of the potential P yields Shapley values.

Proof: It is sufficient to show that equation (4) is derived from the potential function, as then (5) immediately follows from the efficiency of Shapley values. Thus, we must show that if $P: (\mathbb{N}_N, v) \mapsto \mathbb{R}_+$ is the potential function, then $\forall S, i$ such that $0 \neq i \in S \subseteq \mathbb{N}_N$,

$$P(S, v) - P(S \backslash i, v) = \tilde{w}(|S| - 1). \tag{10}$$

Following Hart and Mas-Colell [1989], the unique potential function P can be written as,

$$P(S, v) = \frac{1}{|N|} \left[v(S) + \sum_{i \in S} P(S \backslash \{i\}, v) \right]. \tag{11}$$

If $0 \notin S$, it is clear from (11) that $P(S, v) = |S| \underline{w}$, and (10) is satisfied. When S consists of the firm and only 1 employee, equation (11) implies that $P(S, v) = \frac{1}{2}(F(1) + \underline{w}) = \tilde{w}(1)$, once again satisfying equation (10). Finally, we show by induction that (10) is satisfied $\forall S \ni 0$. Suppose (10) holds $\forall S \ni 0$ such that $|S| \leq s$. $\forall T \ni 0$ such that $|T| = s + 1$, (11) implies that $\forall i \neq 0$,

$$P(T, v) - P(T \backslash i, v) = \frac{1}{s+1} [F(s) + s\underline{w} + sP(T \backslash i, v)] - P(T \backslash i, v).$$

Since by induction, $\forall S \ni 0$ such that $|S| = s$, $P(S, v) = \pi(s-1) + (s-1)\underline{w}$ (because $P(S, v) - P(S \backslash 0, v) = \pi(s-1)$), it follows that,

$$[P(T, v) - P(T \backslash i, v)] = \frac{1}{s+1} \left\{ F(s) + s\underline{w} - [F(s-1) - (s-1)\tilde{w}(s-1) + (s-1)\underline{w}] \right\}.$$

Rearranging terms yields,

$$[P(T, v) - P(T \backslash i, v)] = \frac{1}{s+1} [f_s + (s-1)\tilde{w}(s-1) + \underline{w}] = \tilde{w}(s),$$

where the second equality follows from equation (1). ‖[8]

[8] Alternatively, following Myerson [1980] and Hart and Mas Colell [1989], it is sufficient to show that equation (4) is part of a payout structure over all subsets that induces *balanced contributions* and *efficiency*.

Note that this proof demonstrates that the difference equation (1) generated by our bargaining specification likewise follows from the condition that $\tilde{w}(s)$ is determined by $\tilde{w}(s-1)$ and the potential function for the cooperative game corresponding to F. In this sense, our solution concept is equivalent to presuming payoffs are generated according to the first difference of a potential, and hence must be given by Shapley values. Unlike the noncooperative games in Gul [1989] and Hart and Mas Colell [1992], however, Shapley values are presently obtained for *any ordering* of the employees, and not just in expectation over all orderings. Interestingly, this occurs because of our assumption that agents split the difference in every pair-wise meeting, rather than agents making take-it-or-leave-it offers to one another. We will see below in Theorem 3 that the firm hires a sufficient number of employees so that the outcome of this bargaining game is given by the outside wage.

3. Choice of Labor

We begin by taking the firm's choice of asset(s) as given. The neoclassical firm's profit function is given by $\pi(n, \underline{w}) \equiv F(n) - \underline{w}n$, which we assume is single-peaked, and so its optimal choice of labor is given by $N^*(\underline{w}) = \arg\max_{n\in\mathbb{Z}} \pi(n, \underline{w})$. The problem facing the firm bargaining with nonbinding labor contracts is different and has as its solution, $\tilde{N}^*(\underline{w}) = \arg\max_{n\in\mathbb{Z}} \tilde{\pi}(n, \underline{w})$. We will frequently compare $N^*(\underline{w})$ to $\tilde{N}^*(\underline{w})$, and on occasion we will omit $\underline{w}$ when no confusion would result. Finally, in order to simplify notation, we define the asymmetric binary relation, $x(n) \doteq y(n)$, to indicate that $x(n) - y(n) > 0 \geq x(n+1) - y(n+1)$. When this relation holds, we will say that *x equals y at n to within integer rounding over n.* With this definition, we proceed with our first characterization

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That is,

$$\phi_i(v, S) - \phi_i(v, S-j) = \phi_j(v, S) - \phi_j(v, S-i), \ \ \forall S \subseteq \mathbb{N}_N, \ \forall i \in S, \ \forall j \in S, \tag{12}$$

and

$$\sum_{i\in S} \phi_i(v, S) = v(S), \ \ \forall S \subseteq \mathbb{N}_N, \tag{13}$$

where $\phi_i(v, S)$ is the payoff to i when the game v is restricted to the coalition S. It is straightforward to check that (13) is satisfied when $0 \in S$ by the payoffs in Theorem 1-B. Letting $\phi_i(v, S) = \underline{w}$ whenever $0 \notin S$, (13) is obviously satisfied for all such sets as well. This also trivially implies that balanced contributions will be satisfied if $0 \notin S$. Finally, for $0 \in S$, (12) requires that,

$$\tilde{\pi}(|S|-1) - \tilde{\pi}(|S|-2) = \tilde{w}(|S|-1) - \underline{w},$$

which is immediately implied by (1). Intuitively, presuming the Nash solution to bargaining sessions (c.f. condition (12) and Remark 1), induces balanced contributions; the cost to the firm of losing one employee is equivalent to the employee's cost of leaving the firm.

result.

Theorem 3 *Suppose $N^* \geq 1$. For a given technology, the wage-negotiable firm chooses to hire either $\tilde{N}^*$ or $\tilde{N}^* + 1$ employees, where $\tilde{N}^*$ is uniquely defined by $\pi(\tilde{N}^*, \underline{w}) \doteq \tilde{\pi}(\tilde{N}^*, \underline{w})$. Additionally, $\tilde{N}^* \geq N^*$ (i.e., overemployment of labor) and $\tilde{w}(\tilde{N}^*) \doteq \underline{w}$ (i.e., no surplus rents to labor).*

Proof: As noted,

$$\tilde{\pi}(n) \equiv \frac{1}{n+1} \sum_{i=0}^{n} \pi(i),$$

and so $\tilde{\pi}(n)$ is the average of $\pi(n)$ over $i = 0, 1, \ldots, n$. But given that $\tilde{\pi}(0) = \pi(0)$ and π is single peaked in n, the maximum of $\tilde{\pi}$ occurs uniquely where the marginal function (π) intersects the average function ($\tilde{\pi}$) – i.e., $\tilde{N}^*$ or $\tilde{N}^* + 1$, where $\pi(\tilde{N}^*) \doteq \tilde{\pi}(\tilde{N}^*)$. Because at the optimum the marginal function must cut the average function from above, the point of intersection lies to the right of the maximum of the marginal function; i.e., $\tilde{N}^* \geq N^*$. Lastly, because $\pi(\tilde{N}^*) = F(\tilde{N}^*) - \underline{w}\tilde{N}^* \doteq \tilde{\pi}(\tilde{N}^*) = F(\tilde{N}^*) - \tilde{w}(\tilde{N}^*)\tilde{N}^*$, it must be that $\tilde{w}(\tilde{N}^*) \doteq \underline{w}$. $\|$

Note that this theorem did not require any assumption of feasibility because of the underlying single-peakedness of the neoclassical profit function.

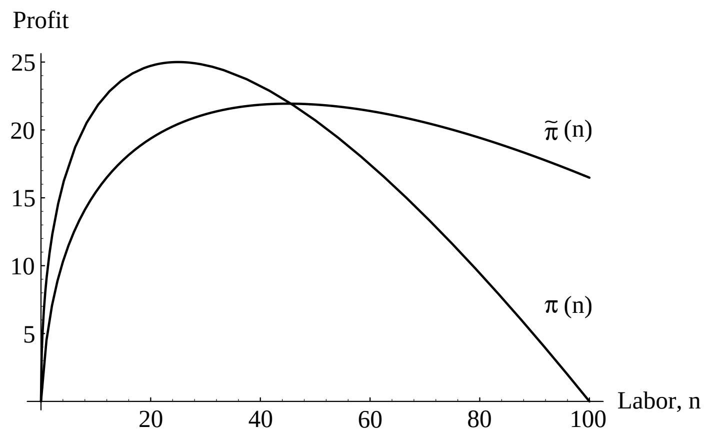
Example 1 Consider the following simple illustration where the asset yields a total output of 6 for one unit of labor and 7 for two or more units of labor:

$$\{f_0, f_1, f_2, f_3, f_4, \ldots\} = \{0, 6, 1, 0, 0, \ldots\}.$$

Suppose the outside reservation wage of employees is $\underline{w} = 2$. The neoclassical firm would choose to hire $N^* = 1$ and profits would be $\pi = 4$. If a firm with nonbinding contracts chose $N = 1$, the wage would be $\tilde{w}(1) = \frac{1}{2}(6+2) = 4$, and profits would be $\tilde{\pi}(1) = 6 - \tilde{w}(1) = 2$. If this latter firm instead chose $N = 2$, wages would be driven down to $\tilde{w}(2) = \frac{1}{3}(1+2) + \frac{1}{6}(6+2) = \frac{7}{3}$, and so profits would be $\tilde{\pi}(2) = 7 - 2\tilde{w}(2) = \frac{7}{3} > 2$. It is also straightforward to check that $\pi(2) \doteq \tilde{\pi}(2)$, and so $\tilde{N}^* = 2$ is indeed the optimal hiring decision. Overemployment results, and the internal wage differs from the reservation wage by only integer-rounding on the number of employees hired: $\frac{7}{3} = \tilde{w}(2) \doteq \underline{w} = 2$.

Example 2 Consider another example in which the equilibrium hiring decisions are larger, and therefore the problem is "smoother". Let $F(n) \equiv 10\sqrt{n}$ and $\underline{w} = 1$. A neoclassical firm would choose precisely $N^* = 25$ units of labor and obtain a profit of $\pi(25) = 50 - 25 = 25$. A nonbinding contracts firm will instead choose $\tilde{N}^* = 44$, resulting in a profit of $\tilde{\pi}(45) = 22.2$. Graphically, we illustrate both $\pi(n)$ and $\tilde{\pi}(n)$ in Figure 1.

Figure 1: The Optimal Choice of Labor



Application: The Cleansing Effect of Product-Market Competition. It is frequently argued that monopolists are plagued with incentive problems and *X-inefficiencies* which are visibly reduced by product market competition.[9] We provide an alternative story which suggests that such an empirical observation is equivalent to our model of a wage-negotiable firm where hiring decisions are second-best optimal (i.e., there is no X-inefficiency). Consider the simplest possible model where a firm's production is $q = N$ and its demand curve in the product market is given by $p(q) = \hat{\gamma} - q$; here, $\hat{\gamma}$ is a measure of the firm's (residual) market demand. Supposing that labor is divisible, it is straightforward to compute the revenue product of labor in such an environment: $\pi(n) \equiv n(\gamma - n)$, where $\gamma \equiv \hat{\gamma} - \underline{w}$. With some algebraic manipulation, it follows that $\tilde{\pi}(n) = \frac{1}{3}[\pi(n) + \frac{\gamma n}{2}]$. We wish to perform comparative statics on γ, where a decrease in γ can be interpreted as the result of Cournot competition where none existed before. Here, γ would be reduced by the quantity produced

[9] See for example Leibenstein [1966]. A similar story is frequently told about the cleansing effect of recessions: during a period of reduced demand, firms search for inefficiencies and waste within their organizations and dismiss employees with low marginal revenue product of labor. The model we construct below can equally well be applied to the recessionary setting.

by the firm's competitor. Although such an amount would be determined endogenously in a Nash equilibrium, we focus on the partial equilibrium comparison.[10] From the expression for $\tilde{\pi}$, it is clear that the wage-negotiable firm will overemploy labor by a marginal factor of γ, which directly measures the firm's residual market demand. Specifically,

$$\frac{dN^*}{d\gamma} = \frac{1}{2}, \qquad \frac{d\tilde{N}^*}{d\gamma} = \frac{dN^*}{d\gamma} + \frac{1}{4} = \frac{3}{4}.$$

In this case, the introduction of competition induces the wage-negotiable firm to reduce employment from the standard competition effect (i.e., $\frac{dN^*}{d\gamma}$), but also from an additional intra-firm rent extracting effect. As a result, a wage-negotiable firm will dismiss more employees than a neoclassical firm, and the wage-negotiable firm's dismissed employees will have a lower revenue product of labor than the neoclassical counterpart. This heightened layoff result occurs because competition reduces the marginal revenue product of labor in a manner which reduces the rent-extracting benefits of overemployment. Thus, inefficient employees whose primary purpose was to keep internal wages depressed, are no longer needed in as great numbers once competition is introduced.

Application: The Economic Incidence of Training Costs. We have assumed that the firm is locked into its current labor force at date 1. One such motivation for this lock-in is a training cost equal to the loss of one period of production; thus, the opportunity cost of firing an employee is the lost production of that employee for one period as someone else is trained in her place. Suppose additionally that there is some per employee training cost of C which must be paid. Consider the following timing prior to the initial stage of our bargaining game. First, a sufficiently large population of potential employees, $\overline{N}$, simultaneously announce payments which they will make to the firm in exchange for employment and training prior to date 1. After observing these payments, the firm simultaneously chooses some of the employees to hire and train, $\tilde{N}^* \leq \overline{N}$, and receives the payments which each hired employee has promised. Unchosen employees go to work in the neoclassical sector for the wage of $\underline{w}$. At this point, we begin our familiar bargaining game resulting in an internal wage of $\tilde{w}(\tilde{N}^*)$ and profit of $\tilde{\pi}(\tilde{N}^*)$.

[10] In the recessionary setting discussed in the previous footnote, γ is reduced in a recession due to lower product market demand and sticky wages.

Who pays C in this setting? If employees offered to pay the entire training cost themselves, the firm will overemploy as before and drive the internal wage down to $\underline{w}$, leaving the employees with a net wage $\underline{w} - C$, which is less then their outside option, $\underline{w}$. On the other hand, if the firm paid for the training without any payment by the employees, the firm would not hire as many employees, leaving an internal wage in excess of the outside option of the employees. In this case, employees would gladly pay some of the training costs in order to get hired and earn excess rents. To find the equilibrium payment, we must find a fixed point $(\tilde{N}^*, \theta)$, where θ is the firm's share of training costs, such that at this point the employees earn no excess rents in the firm given $\tilde{N}^*$ and the firm optimally chooses $\tilde{N}^*$ given its training costs fixed per employee θC. Formally,

$$\tilde{w}(\tilde{N}^*) - \underline{w} = (1 - \theta)C, \tag{14}$$

$$\tilde{N}^* \in \arg\max_n \tilde{\pi}(n) - \theta C n. \tag{15}$$

Ignoring integer problems, the first-order condition for (15) is $\pi(\tilde{N}^*) - \tilde{\pi}(\tilde{N}^*) = \theta C \tilde{N}^*$. By definition, $\pi - \tilde{\pi} \equiv N(\tilde{w} - \underline{w})$, and so $\tilde{w}(\tilde{N}^*) - \underline{w} = \theta C$. Combining this expression with (14) yields our result.

Result 2 *The firm and employee equally share in training costs, $\theta = \frac{1}{2}$. Moreover,*

$$\tilde{N}^* = \arg\max_n \tilde{\pi}(n, \underline{w} + C).$$

Not only are training costs equally absorbed by the firm and the employee, but the wage-negotiable firm's objective function is equivalent to the case where the outside wage has been raised to include training costs: $\underline{w}' = \underline{w} + C$. The second part of the result follows directly from comparing the problem of the firm with training costs (given that $\theta = \frac{1}{2}$) to that of a firm without such costs but with an outside wage of $\underline{w} + C$. Thus, the character of our results are unaffected by adding training costs in excess of lost production time.

These results also shed additional light on Becker's model of on-the-job training; Becker [1975]. Becker argues that workers pay for their general on-the-job training (i.e., training which increases marginal product at other firms as well as the current employer) because they alone will receive all of the benefits of such training through higher future wages, regardless of where they work. If training costs come out of wages received, these workers will have upward sloping wage profiles. On the other hand, specific training (training which only has value with the present employer's assets) does not increase the worker's outside wage, and so in a market with only exogenous labor turnover it is possible that the employer pays all of these costs, thereby inducing a flat wage profile. In the present paper, we are concerned solely with specific investment. Nonetheless, if long-term contracts for training cost repayment are not enforceable, we can obtain an upward sloping wage profile in a simple two-period version of the training cost model presented above: Worker's accept a lower wage today to cover part of the specific training costs because such training increases their future wage via increased bargaining power. Our result differs from Becker's because employees have some bargaining power over specific investments beyond the requirement that the internal wage generates at least the utility of their outside option.

4. Choice of Technology

We now focus our attention on the firm's choice of technology, $F \in \mathcal{F}$, taking labor as fixed.[11] The firm's problem when faced with choosing a single asset is

$$\max_{F \in \mathcal{F}} \frac{1}{1+N} \sum_{i=0}^{N} (F(i) - \underline{w}i).$$

4.1 Optimal Capital Intensity

We first consider a simple ordering over the set of available technologies, $\mathcal{F}$: technologies ordered by capital intensity. In particular, consider a set of technologies $F(N, K)$, which are indexed by capital intensity K that the firm can vary at a cost of r per unit with diminishing returns, $F_{KK} < 0$. Let $K^*(N)$ be the neoclassical firm's solution to the problem,

$$\max_{K} F(N, K) - \underline{w}N - rK,$$

for a given N. Let $\tilde{K}^*(N)$ be the bargaining firm's solution to the problem,

$$\max_{K} \frac{1}{N+1} \sum_{i=0}^{N} \{F(i, K) - \underline{w}i\} - rK,$$

also for a given N. The following result is immediate.

[11] We assume that the level of N is such that $N \leq \tilde{N}^*$ for the technology choices available. Because at any optimal choice of technology and labor such a relationship will hold, our assumption is unrestrictive and allows us to focus our attention on the technology decision of the firm.

Theorem 4 *$K^*(N) > \tilde{K}^*(N)$ if $F_{NK} > 0$, and $K^*(N) < \tilde{K}^*(N)$ if $F_{NK} < 0$.*

Proof: Fix N. The neoclassical firm chooses $K^*(N)$ such that $F_K(N, K^*) = r$. The bargaining firm chooses capital such that

$$\frac{1}{N+1}\sum_{i=0}^{N} F_K(i, \tilde{K}^*) = r.$$

If $F_{NK} > 0$, then $\frac{1}{N+1}\sum_{i=0}^{N} F_K(i, K) < F_K(N, K)$, and so $\tilde{K}^*(N) < K^*(N)$. Analogously, if $F_{NK} < 0$, then $\tilde{K}^*(N) > K^*(N)$. $\|$

Intuitively, by increasing capital, the wage negotiable firm obtains a weighted average of the marginal returns to capital of the various possible employment levels, ranging from 0 to N workers. When the marginal return to capital increases with labor, this weighted average is less than the marginal returns with N employees (which is what the neoclassical firm obtains). Consequently, the wage negotiable firm underemploys capital compared to the neoclassical firm. Likewise, when the marginal return to capital decreases with labor, the wage negotiable firm hires more capital than the neoclassical firm.

Although, $\tilde{K}^*(N) < K^*(N)$ does not imply $\tilde{K}^*(\tilde{N}^*) < K^*(N^*)$, with an additional weak assumption regarding single-peakedness, we can obtain the following result.

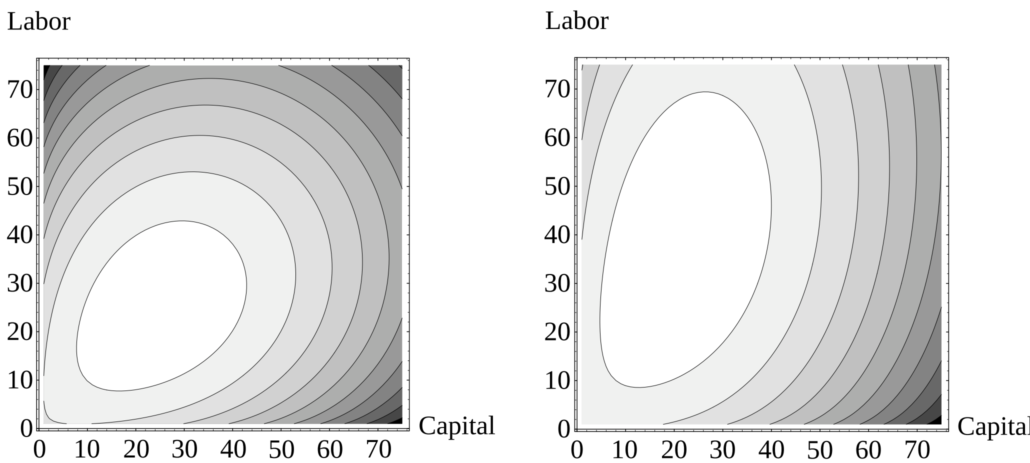
Corollary 1 *Suppose that $\tilde{\pi} = \frac{1}{N+1}\sum_{i=0}^{N}\{F(i, K) - \underline{w}i\} - rK$ is single peaked in (N, K). Then $K^* > \tilde{K}^*$ and $N^* \leq \tilde{N}^*$ if $F_{NK} > 0$; and $K^* < \tilde{K}^*$, $N^* \leq \tilde{N}^*$ if $F_{NK} < 0$.*

The corollary follows from noting that at (N^*, K^*), the gradient of $\tilde{\pi}$ points toward higher N and lower K when $F_{NK} > 0$. Since $\tilde{\pi}$ is globally quasi-concave, the maximum lies in this direction. The reverse is true when $F_{NK} < 0$. The following numerical example illustrates the distortion.

Example 3 Let $F(N, K) = 100\sqrt{NK} - N^2 - K^2$, $\underline{w} = 1$, and $r = 1$. The iso-profit contour plots of the neoclassical and contracting firms over choices of N and K are illustrated in Figure 2. As is clear, the contractual firm chooses a distorted capital-labor choice which underemploys capital and overemploys labor. This follows directly from Corollary 1.

A limitation of the ordering of assets in $\mathcal{F}$ with a single parameter K is that it fails to capture other economically interesting variations in technologies. We consider another interesting economic order presently.

Figure 2: Neoclassical Profits versus Wage-Negotiated Profits



4.2 Technology Choice and the Value of Frontloading

We now consider a more complex ordering on the set of available technologies, $\mathcal{F}$, which we refer to as frontloading.

Definition 2 *The* **frontload factor** *for technology F at N is given by the statistic*

$$\lambda(F,N) \equiv 1 - \frac{1}{\pi(N)(N+1)} \sum_{i=0}^{N} i\Delta\pi(i),$$

where $\pi(i) \equiv F(i) - \underline{w}i$ and $\Delta\pi(i) \equiv \pi(i) - \pi(i-1)$.

For most applications of interest, $\pi(N) > 0$ and the underlying solution to (1) is feasible. In these cases, $\lambda \in [0,1]$, where a higher λ represents greater frontloading.[12] We will sometimes find it of use to use the continuous version of the frontload factor for cases where labor is infinitely subdivisible. In such a case,

$$\lambda(F,N) \equiv 1 - \frac{1}{\pi(N)N} \int_0^N s d\pi(s).$$

Choose two technologies to compare: F^1 and F^2. Following this definition, if $\lambda(F^1,N) > \lambda(F^2,N)$, we say that technology F^1 is more frontloaded than F^2 at N. Consider two

[12] In other cases where $\pi(N) \leq 0$, a related notion of frontloading can be defined without dividing through by $\pi(N)$. Unfortunately, this notion is less elegant and more cumbersome. The underlying solution will be feasible in the relevant range whenever π is single peaked.

technologies that are equally efficient at N: $F^1(N) = F^2(N)$. Intuitively, one technology is frontloaded relative to the other when its higher marginal products are distributed more up "front" and less on the marginal employees; literally, the margins are frontloaded. With our definition of frontloading, we can show the following.

Theorem 5 ***(Preference for frontloading.)*** *Fix N and suppose that $F^1(N) = F^2(N)$. The wage-negotiable firm will strictly prefer F^1 to F^2 at N if and only if F^1 is frontloaded relative to F^2 at N.*

This result is easily seen by taking $\tilde{\pi}(N)$ and summing by parts, obtaining

$$\frac{1}{1+N}\sum_{i=0}^{N}\pi(i) = \pi(N) - \frac{1}{1+N}\sum_{i=0}^{N} i\Delta\pi(i) = \pi(N)\lambda(F,N). \tag{16}$$

As a consequence, profits from equally efficient technologies will only differ in the degree to which they affect the frontload factor, λ.[13]

Taking $\pi^j(n) \equiv F^j(n) - wn$, an immediate result from (16) is that for a given N, a wage-negotiable firm will choose an inefficient technology (i.e., $\pi^1(N) < \pi^2(N)$) if the technology has sufficient frontloading relative to the efficient technology (i.e., $\frac{\lambda(F^1,N)}{\lambda(F^2,N)} > \frac{\pi^2(N)}{\pi^1(N)}$).

Related to frontloading is the notion of concave transformations, which imply frontloading in the following sense.

Theorem 6 ***(Preference for concave technology)*** *Fix N and consider two equally efficient technologies at N: $F^1(N) = F^2(N)$. Let F^1 be a concave transformation of F^2 on the domain $n \in [0, N]$. Then F^1 is more frontloaded than F^2, and hence preferred by the firm.*

Proof: Define $F^1(0) = F^2(0) = a$, and $F^1(N) = F^2(N) = b$. Let $F^1 \equiv g(F^2)$ with $g'' < 0$. Define $\gamma(\alpha) \equiv \alpha - g(\alpha)$, which is strictly convex. Since strictly convex functions can have at most two roots, $\alpha - g(\alpha) < 0$ for all $n \in (0, N)$. Substituting $\alpha = F^2(i)$ and summing, $\frac{1}{N+1}\sum_{i=0}^{N} F^2(i) - \frac{1}{N+1}\sum_{i=0}^{N} F^1(i) < 0$. Because $F^1(N) = F^2(N)$, summation by parts implies that F^1 is more frontloaded then F^2. $\|$

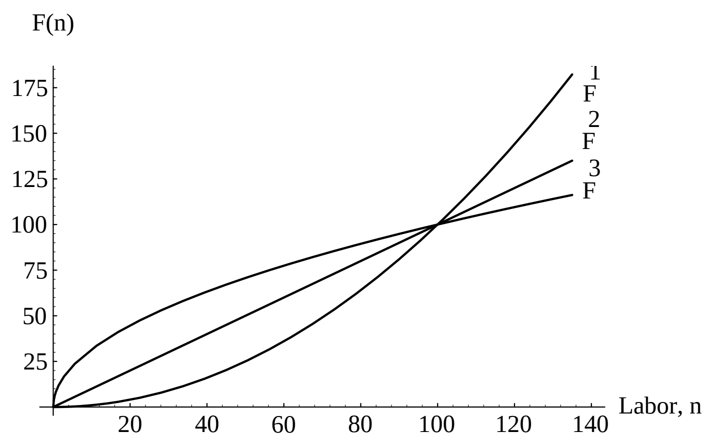
[13] If labor is sufficiently subdivisible, the continuous version of (16) is

$$\frac{1}{N}\int_0^N \pi(s)ds = \pi(N) - \frac{1}{N}\int_0^N s d\pi(s) = \pi(N)\lambda(F,N).$$

In the above sense, a firm will prefer technology that exhibits greater diminishing returns if the efficiency costs are not too great. Consider the following example.

Example 4 Let $F^1(n) = \frac{n^2}{100}$, $F^2(n) = n$, and $F^3(n) = 10\sqrt{n}$. This is graphically depicted in Figure 3. At $N = 100$, all of the technologies are efficient, but F^3 is frontloaded relative

FIGURE 3: FRONTLOADED TECHNOLOGIES



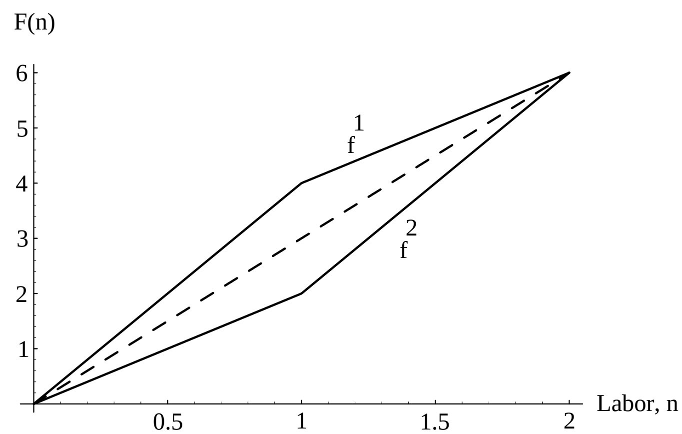
to F^2, and F^2 is frontloaded relative to F^1. This follows immediately because F^3 is concave relative to F^2, which is concave relative to F^1. At $\underline{w} = 0$, for example, the frontload factors for F^1, F^2, and F^3 are $\frac{1}{3}$, $\frac{1}{2}$, and $\frac{2}{3}$, respectively.

Application: Unionization as a Manifestation of Front-loading. Unionization is similar to a choice of technology. Consider a firm with several employees which has the option of dealing with each employee separately in the wage negotiations or dealing solely with a union. When will the firm prefer fragmented negotiations to centralized ones? The answer to this question is immediate upon Theorem 6. A union has the effect of linearizing the labor revenue product function, which means that if the original technology was convex, a union makes it more concave (and hence more frontloaded); and if the original technology was concave, unionization convexifies it. A union is desirable from the firm's point of view (and hence undesirable from the employee's point of view) whenever the underlying technology is convex, while the reverse holds true for concave technology.[14]

[14] A similar relationship between convexities and the desirability of unionization has been noted by Horn and Wolinsky [1988], although in a different bargaining environment with two employees (or groups of employees).

A numerical example with two employees illustrates this nicely, although it is quite generalizable. Consider two equally efficient technologies (for $N = 2$) characterized by their margins: $f^1 = \{0, 4, 2\}$ and $f^2 = \{0, 2, 4\}$. f^1 is concave; f^2 is convex. Suppose that the outside wage is normalized to 0, but that there are only two employees in the economy which can be hired. With the first technology, the wage-negotiation firm makes $\tilde{\pi}^1 = \frac{1}{3}(0)+\frac{1}{3}(4)+\frac{1}{3}(2+4) = \frac{10}{3}$ while with the second technology it makes only $\tilde{\pi}^2 = \frac{1}{3}(0) + \frac{1}{3}(2) + \frac{1}{3}(2 + 4) = \frac{8}{3}$. With a union, in both cases the firm is dealing with a single entity whose marginal product is identical to its total product: here, 6. The wage-negotiating firm facing a union makes $\tilde{\pi}^{union} = 3$, and so $\tilde{\pi}^1 > \tilde{\pi}^{union} > \tilde{\pi}^2$. The linearization is illustrated graphically in Figure 4.

Figure 4: Unionization and Convexities



5. Organizational Design

We now consider related problems of organizational design. So far we have focused our attention on a bargaining environment where the principal has a single technology, F, with which the N potential employees may produce output. This is clearly restrictive. The firm might, for example, have two different assets available, A and B, with N_A and N_B employees trained for each asset, respectively. In such an environment, how will a firm tailor its asset choices to one another given an environment of nonbinding labor contracts? We first solve for the solution to the underlying multiple-asset bargaining game, and then examine the nature of the firm's organizational design problem. Although we restrict our attention to the case of two productive assets, this is for simplicity as our results and theorems easily generalize to firms with any arbitrary number of assets.

5.1 Extension to Multiple-Asset Bargaining Games

We assume that employee reassignment is not possible – firms can only choose between retention of a trained employee or dismissal. To the extent that there are productive externalities (negative or positive) between the two assets, the ultimate payoffs to the firm and the employees in each group may vary. When no such externalities exist, the bargaining environment is no different from a multiple application of the simple bargaining game presented in section 2.

To capture these potential externalities, we denote the production function for the underlying assets A and B on which N_A and N_B employees are employed as $F(N_A, N_B) : \mathbb{N} \times \mathbb{N} \mapsto \mathbb{R}^+$. We denote the marginal output of an employee assigned to asset k, given there are a total of i employees using asset A and j employees using asset B, as $f^k(i,j)$. Thus, $f^A(i,j) \equiv F(i,j) - F(i-1,j)$ and $f^B(i,j) \equiv F(i,j) - F(i,j-1)$. Taking N_A and N_B as given, we seek to characterize the underlying equilibrium to the noncooperative game, which will be a triplet, $\{\tilde{w}^A(N_A,N_B), \tilde{w}^B(N_A,N_B), \tilde{\pi}(N_A,N_B)\}$.

Following previous inductive arguments, a set of first-order partial difference equations uniquely determine the noncooperative equilibrium of the bargaining game, providing that the solution is feasible (i.e., $\tilde{w}^k(i,j) \geq \underline{w} \;\; \forall (i,j) \in \mathbb{N}_{N_A} \times \mathbb{N}_{N_B}$).

$$\begin{aligned} f^A(i,j) - \tilde{w}^A(i,j) - (i-1)\left[\tilde{w}^A(i,j) - \tilde{w}^A(i-1,j)\right] - j\left[\tilde{w}^B(i,j) - \tilde{w}^B(i-1,j)\right] \\ = \tilde{w}^A(i,j) - \underline{w}, \end{aligned} \tag{17}$$

$$\begin{aligned} f^B(i,j) - \tilde{w}^B(i,j) - i\left[\tilde{w}^A(i,j) - \tilde{w}^A(i,j-1)\right] - (j-1)\left[\tilde{w}^B(i,j) - \tilde{w}^B(i,j-1)\right] \\ = \tilde{w}^B(i,j) - \underline{w}. \end{aligned} \tag{18}$$

Similar to the one equation case of section 2, we will demonstrate that the unique solution to this system of first-order partial difference equations also yields the Shapley value of the underlying cooperative game.

Theorem 7 *Suppose that N_A employees are assigned to asset A, N_B employees are assigned to asset B, and let $N \equiv N_A + N_B$. Furthermore, suppose that the solution to the system*

(17)-(18) is feasible. Then the equilibrium to the noncooperative game has wages and profit given by the Shapley values of the underlying cooperative game, where

$$\tilde{\pi}(N_A, N_B) = \frac{1}{N+1} \sum_{i=0}^{N_A} \sum_{j=0}^{N_B} \left(\frac{\binom{N_A}{i}\binom{N_B}{j}}{\binom{N}{i+j}} \right) \left[F(i,j) - (i+j)\underline{w} \right]. \qquad (19)$$

Proof: The proof proceeds as in Theorem 2 by showing that the noncooperative equilibrium payoffs are equivalent to the Shapley value, and then using this result to determine the actual noncooperative payoffs via (9).

Here it is most straightforward to use the balanced contributions proof as in footnote 8 rather than the potential function to demonstrate the equivalence with Shapley values. First note that the partial difference equations which determine the noncooperative equilibrium satisfy efficiency; that is, equation (13). Hence, we need only demonstrate balanced contributions to prove the equivalence with Shapley values. Because the solution to the equations exhibits symmetry for employees within a group, the contributions are balanced for $i, j \in \mathbb{Z}_A$ and $i, j \in \mathbb{Z}_B$, where $Z_X \equiv \{1, 2, \ldots, X\}$. Additionally, for either $i = 0,\ j \in \mathbb{Z}_A$ or $i = 0,\ j \in \mathbb{Z}_B$, balanced contributions are assured by (17) or (18), respectively. Lastly, we must show that contributions are balanced when $i \in \mathbb{Z}_A$ and $j \in \mathbb{Z}_B$. This is equivalent to showing

$$\tilde{w}^A(m,n) - \tilde{w}^A(m,n-1) = \tilde{w}^B(m,n) - \tilde{w}^B(m-1,n),\ \forall (m,n) \in \mathbb{Z}_{N_A} \times \mathbb{Z}_{N_B}.$$

Differencing (17) once and simplifying yields

$$2\Delta_2 \tilde{w}^A(m,n) = \Delta_1\Delta_2 F(m,n) - (m-1)\Delta_1\Delta_2\tilde{w}^A(m,n) - (n-1)\Delta_1\Delta_2\tilde{w}^B(m,n) - \Delta_1\tilde{w}^B(m,n),$$

where Δ_1 is the difference operator over the first argument, and Δ_2 operates over the second. Similarly, for (18) we have

$$2\Delta_1 \tilde{w}^B(m,n) = \Delta_1\Delta_2 F(m,n) - (m-1)\Delta_1\Delta_2\tilde{w}^A(m,n) - (n-1)\Delta_1\Delta_2\tilde{w}^B(m,n) - \Delta_2\tilde{w}^A(m,n).$$

Subtracting one of these equations from the other implies that,

$$\Delta_2 \tilde{w}^A(m,n) = \Delta_1 \tilde{w}^B(m,n),$$

which guarantees balanced contributions.

Having established the equivalence of Shapley values and the noncooperative equilibrium payoffs of the game, it is straightforward to verify from (9) that (19) is the Shapley value of the firm. ‖

As with the case of a single asset, we will find it useful to characterize the firm's profits given divisible labor. Analogous to Result 1 is the following.

Result 3 *Suppose that $F(n)$ is defined over $\mathbb{R}^2_+$, and labor is divisible, where each subdivision has the ability to negotiate with the firm in a pairwise meeting. Suppose further that there are N_A employees assigned to asset A, N_B employees assigned to asset B, and denote $N \equiv N_A + N_B$ and $\alpha \equiv \frac{N_A}{N_A+N_B}$. Then $\tilde{\pi}(N_A, N_B)$ converges to*

$$\frac{1}{N}\int_0^N \{F(\alpha s, (1-\alpha)s) - s\underline{w}\}\, ds \tag{20}$$

as labor becomes infinitely subdivisible.

The result looks very similar to the continuous version of the single-asset profit function in (8) except that the integral now proceeds along the diagonal of the rectangle with vertices $\{0, 0\}$ and $\{N_A, N_B\}$, rather than along the interval $[0, N]$. The game-theoretic inclined reader will recognize equation (20) as the Aumann-Shapley value (for transferable utility games) of the firm (see Aumann and Shapley [1974]). Intuitively, one can consider the heuristic of the firm getting its marginal contribution over random orderings of the firm and all employees. As labor becomes infinitely divisible, with probability approaching 1, the firm will be preceded by employees of types N_A and N_B in proportion with their population. While the proportion of different employees will thus be fixed in the limit, the total weight of employees preceding the firm will be uniformly distributed over $[0, N]$. Thus the firm's expected marginal contribution is found by integrating its marginal contribution along the diagonal of the rectangle of employees assigned to the two assets, as in equation (20) above.

Equation (20) yields a nice interpretation for how the nature of the problem of optimally allocating a given number of employees across different groups differs between a wage-negotiable and a neoclassical firm. For the latter, given N employees, the firm must choose α to maximize $F(\alpha N, (1-\alpha)N)$. That is, the firm compares *final production* over all employee assignments. Equation (20), however, indicates that the wage-negotiable firm compares *average production along the vector connecting the origin to an assignment* over all possible employee assignments. As is seen in Example 5 below, insofar as the optimal mix of employees (from the neoclassical firm's point of view) varies with firm size, the allocational decision of the wage-negotiable firm will diverge from that of the neoclassical firm.

It is important to note that going from a discrete number of employees to a continuum, we lose one potential economic effect not present in the single asset case. It is only in the continuum that the firm's profit for a given employee assignment is precisely given by the weighted production (net wages) along the diagonal. With a finite number of employees, off diagonal production levels will contribute to the firm's profit as well. While the weight given to the off-diagonal proportions fall off rapidly as one moves from the diagonal, in general, it is only in the limit that it's contribution can be ignored. With a characterization of the returns to the wage-negotiation firm, we can turn to our analysis of optimal organizational design.

5.2 The Organizational Design Problem

In the above profit formula for the wage-negotiable firm, (19), if $F_{N_A N_B} = 0$, the profits to a multiple-asset firm with non-binding labor contracts are identical to those of two single-asset firms. When the cross-partial is nonzero, however, additional interesting economic results emerge. We explore these results by first considering the more complicated finite labor case, and then turning to the less general (but simpler) continuous labor case.

5.2.1 The Finite Labor Case

The firm's decision now consists of three components: technology ($F \in \mathcal{F}$), scale of production (N), and labor allocation ($\alpha \in [0, 1]$).

First, consider the case where N and α are fixed, but the firm chooses technology. In the case of one asset, we found that a firm will favor frontloaded technologies, all else equal. For this reason, a wage-negotiable firm would prefer a technology which exhibited diseconomies of scale rather than economies of scale (given a fixed total production). With multiple assets, economies of scope play a similar role.

A simple example provides much of the intuition. Fix $N_A = 1$ and $N_B = 1$, and $\underline{w} = 0$. For example, there are two unions, each representing a minimal bargaining unit. Define the underlying scope economy from joint production as $\Delta_s F \equiv F(1, 1) - F(1, 0) - F(0, 1)$. That is, $\Delta_s F$ indicates the total additional revenue product of labor produced from the synergies of the two productive groups. A neoclassical wage-taking firm would choose technology, F, so as to maximize $\pi = F(1,1)$. Using our result for multi-asset bargaining (19), one can show that $\tilde{\pi} = \frac{1}{2}[F(1,1) - \frac{1}{3}\Delta_s F]$. It is immediate that the wage-negotiable firm will disfavor economies of scope in favor of increasing the margins associated with "front" employees which are independent of the other productive asset. Our frontloading notions have a straightforward, but more complicated, generalization to the multi-asset bargaining environment. Just as in the single-asset case where we found that the firm did not value economies of scale as much as a neoclassical firm (because the employees captured much of these returns), a multiple-asset firm similarly does not value economies of *scope* to the same extent as a neoclassical firm.

Application: The Cost of Horizontal Merger. Williamson's [1975] puzzle of selective intervention asks why one firm cannot merge two firms into a single entity and do at least as well as when separate (i.e., duplicate previous activities and selectively intervene whenever intervention would increase joint profits). We provide an answer. If two firms that as separate entities would otherwise impose positive externalities upon one another merged, the resulting single firm would be identical to our multi-asset firm with two productive groups of employees. Now, however, the employees can make claims on any positive externalities which previously existed and were unclaimable.[15] As such, two firms that impose externalities upon one another (and therefore are traditionally thought to be prime candidates for merger) may desire to remain separate so as to relax intra-firm bargaining pressures.

We now consider a wage-negotiable firm with a given technology, $F \in \mathcal{F}$, and number of employees, N, but which must decide how to allocate its work force via its choice of α. With our analysis extended to multiple assets, additional dimensions of choice (and hence distortion) exist. In particular, it is now possible that employee misallocation results. For example, fix $N = 2$, let $\underline{w} = 0$ and consider a technology, F, such that $F(0,0) = 0$, $F(1,0) = 0$, $F(0,1) = 3$, $F(1,1) = 4$, $F(0,2) = 3$, and $F(2,0) = 5$. With one employee, both the neoclassical and the wage-negotiable firm would allocate the employee to asset B. With two employees, the neoclassical firm would clearly allocate both workers to asset A yielding a profit of $\pi(2,0) = 5$. Its preferences over the three possible configurations of two workers

[15] For example, if the Chicago Bulls organization merged with the L.A. Lakers, presumably Michael Jordan could command a higher salary as his positive externalities on ticket sales at the L.A. Coliseum were previously unavailable to the firm, and hence non-negotiable. More common, there may be externalities between firms who supply retailers that compete in the same product market.

are simple to determine: $\pi(2,0) > \pi(1,1) > \pi(0,2)$. Nevertheless, it is straightforward to demonstrate that although the neoclassical firm would place both employees on asset A, our wage-negotiable firm would place both employees with asset B. Specifically, $\tilde{\pi}(2,0) = \frac{1}{3}(0)+\frac{1}{3}(0)+\frac{1}{3}(5) = \frac{5}{3}$, $\tilde{\pi}(1,1) = \frac{1}{3}[\frac{1}{2}(3)+\frac{1}{2}(0)+0+4] = \frac{11}{6}$, and $\tilde{\pi}(0,2) = \frac{1}{3}(0)+\frac{1}{3}(3)+\frac{1}{3}(3) = \frac{6}{3}$. As a consequence, although $\pi(2,0) > \pi(1,1) > \pi(0,2)$, for the wage negotiable firm, $\tilde{\pi}(2,0) < \tilde{\pi}(1,1) < \tilde{\pi}(0,2)$. In this example, the labor allocation preferences of the firms are reversed.

The example makes clear that the choice of employee-asset allocations generally may be distorted by a wage-negotiable firm. It is difficult to say anything more precise about the outcome in the finite case for general production functions. We instead turn to the continuous labor setting where it is simpler to generalize.

5.2.2 The Continuous Labor Case

We focus our attention on the wage-negotiable firm's profit function given by the integral in (20). Ignoring technological choice, the firm will solve

$$\max_{N,\alpha\in[0,1]} \ \tilde{\pi}(\alpha N, (1-\alpha)N) \equiv \frac{1}{N}\int_0^N \pi(\alpha s, (1-\alpha)s)ds.$$

We obtain the following theorem as a solution to this program.

Theorem 8 *Suppose that π is differentiable, $\frac{\partial}{\partial N_A}\pi(0, N_B) = \frac{\partial}{\partial N_B}\pi(N_A, 0) = \infty$, and that $\pi(\alpha N, (1-\alpha)N)$ is single-peaked in N for any $\alpha \in [0,1]$. Then the optimal $\{\alpha, N\}$ for the wage-negotiable firm satisfies*

$$\pi(\alpha N, (1-\alpha)N) = \tilde{\pi}(\alpha N, (1-\alpha)N), \tag{21}$$

and

$$\int_0^N s\frac{\partial \pi}{\partial N_A}(\alpha s, (1-\alpha)s)ds = \int_0^N s\frac{\partial \pi}{\partial N_B}(\alpha s, (1-\alpha)s)ds. \tag{22}$$

The result follows from classical optimization conditions. The first condition is the first-order condition for total hires, N. As in the single-asset case, labor is employed up to the point where the excess wage bill is driven to zero. Integrating the objective function by parts reveals that

$$\tilde{\pi}(\alpha N, (1-\alpha)N) \quad = \quad \pi(\alpha N, (1-\alpha)N) \tag{23}$$

$$- \quad \frac{1}{N}\int_0^N s\left\{\alpha\frac{\partial\pi}{\partial N_A}(\alpha s,(1-\alpha)s)+(1-\alpha)\frac{\partial\pi}{\partial N_B}(\alpha s,(1-\alpha)s)\right\}ds.$$

The second condition is the first-order condition in α and indicates that the allocation of employees to assets is chosen in order to equate the front-load factors of each asset. If one front-load factor is lower than the other, that asset's labor allocation is increased in order to reduce the rents accruing to labor. Combining equations (22)-(23), it is clear that N is chosen so as to drive the frontload factors for each asset to one.

Note that because the first condition is identical to the condition stated for the single-asset case, we obtain qualitatively similar results for the choice of N. In particular, overemployment will result for any given α. And if the marginal product of labor is increasing in the capital stock, capital investment will be inefficiently low. While we obtain qualitatively similar results for the questions of hiring and technology choice, α represents a new dimension of potential distortion.

The neoclassical firm will allocate labor to satisfy

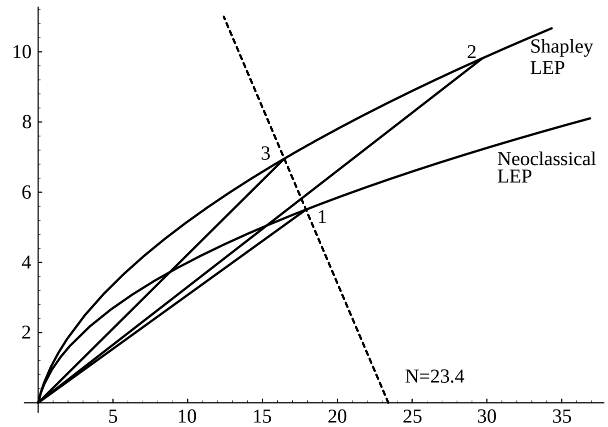
$$\frac{\partial\pi}{\partial N_A}(\alpha N,(1-\alpha)N)=\frac{\partial\pi}{\partial N_B}(\alpha N,(1-\alpha)N).$$

The wage-negotiable firm will choose α so as to satisfy the weighted expectation of this expression with labor varying from 0 to N. These choices for α will generally be different. Consider the following example.

Example 5 Suppose that the firm's production function is $F(N_A,N_B)=10\left(\frac{N_A}{N_A+1}\right)\sqrt{N_B}$. Suppose the outside wage is $\underline{w}=1$. In this case, the neoclassical firm will choose $N^*=23.4$ and $\alpha^*=.235$. The wage-negotiable firm, however, will choose $\tilde{N}^*=35.5$ and $\tilde{\alpha}^*=.248$, weighting asset A relatively more. Graphically, the neoclassical and wage-negotiable firms' choices are given as the vectors at points 1 and 2, respectively, in Figure 5. The labor-expansion-paths (LEP) for each firm's input choices are also illustrated. The wage-negotiable firm's LEP lies everywhere above the neoclassical firm's LEP, indicating that for any fixed N, the former will allocate relatively more employees to asset A. Point 3 in the figure represents the wage-negotiable firm's optimal choice of α, given N is constrained at $N=23.4$.

The example raises the importance of the labor expansion paths of each firm and suggests that homotheticity may play a role in the allocation distortions. Specifically, we consider a weaker variant of homotheticity which we refer to as *ray-homotheticity.*

Figure 5: Organizational Design



Definition 3 *A production function* $F(x, y)$ *is* **ray-homothetic** *if there exists an input price vector such that the input expansion path is linear through the origin.*

Note that a homothetic function is ray-homothetic *for every* price vector. Using this definition, we have an immediate result.

Corollary 2 *Suppose that the underlying production function is ray-homothetic for the unit input price vector. Then* $\alpha^* = \tilde{\alpha}^*$.

The absence of a distortion is clear. When a neoclassical firm's LEP is linear through the origin, the wage-negotiable firm's first-order condition is satisfied pointwise in labor at the same allocation. Thus, a homothetic firm does not introduce a labor-allocation distortion. Distortions in input decisions other than those associated with the single asset case can only occur in environments in which technology does not exhibit ray-homotheticity. Below, we consider a simple application in an environment with such technology.

Application: Span of Control in Hierarchies. Suppose that their are two levels of employees which bargain with the owners of a firm; let one be assigned to level A and the other to level B. We have the following corollary to Theorem 8.

Corollary 3 *Suppose that* $\tilde{\pi}$ *is single peaked in* (α, N) *and*

$$\frac{\partial \pi(\alpha^* n, (1-\alpha^*)n)}{\partial N_A} > \frac{\partial \pi(\alpha^* n, (1-\alpha^*)n)}{\partial N_B}, \quad \forall\, n < N^*,$$

where (α^*, N^*) *is the neoclassical optimum. Then* $\tilde{\alpha}^* > \alpha^*$*, where* $\alpha \equiv \frac{N_A}{N_A+N_B}$.

Thus, if A represents the senior-management level of the hierarchy, there will be top-heavy hierarchies (i.e., a relatively small span of control).[16] If on the other hand the relationship holds in the reverse direction, there will be a large span of control relative to the neoclassical firm. The basic insight is that in the optimal design of a hierarchy overemployment will be most useful on those levels where bargaining power is greatest: span of control is an effective instrument in mitigating bargaining power.

6. Concluding Remarks

We have presented a new methodology for studying the problem of labor contracting within a firm's boundaries where contracts provide only a minimal commitment to wages and employment. Given the peculiar contractual incompleteness of labor contracts, the resulting wages and profits under a large class of complete information bargaining games distort the technological and organizational decisions facing the owner of the firm's capital. In such settings where labor contracts are nonbinding, these decisions are distorted in an economically distinct way compared to the standard neoclassical firm. Among other things, a firm with a nonbinding contractual basis will, relative to a neoclassical firm, (i) overemploy labor, (ii) underemploy capital, (iii) choose inefficient "frontloaded" technologies, (iv) de-emphasize scale and scope economies, and (v) inefficiently allocate labor across productive assets.

We want to stress that the applicability of this new methodology is extensive, including such settings as product market competition, training costs, unionization, hierarchical management, and horizontal mergers. Other interesting areas of application include the use of debt to strengthen bargaining positions and the macroeconomic implications of demand shocks in hybrid economies with both neoclassical and wage-negotiable firms.

We also want to stress the generality of our results. Although we have assumed that parties always split net surplus in any pairwise meeting, this is not essential. Any nondegenerate split will give rise to an equilibrium profit function that places some weight on inframarginal profits. It is this concern for inframarginal profits, rather than just $\pi(n, \underline{w})$,

[16] Even if ray-homotheticity satisfied, a similar result of top-heavy organizations emerges if one believes that senior management are more costly to replace (in terms of lost production from quits and retraining) relative to lower-level employees. Alternatively, if low-level employees belong to a single union which negotiates with the firm, but upper management bargains individually, relative overemployment will result for upper management.

that generates the distortions we have found. As a consequence, any alternative bargaining game which exhibits this inframarginal feature will produce similar economic distortions. To be precise, suppose that in an alternative bargaining game the wage-negotiable firm places weights on inframarginal profits according to the distribution $\mu(s|n)$ on $[0, n]$. In such a case, profits are given by $\tilde{\pi}(n) = \int_0^n \pi(s) d\mu(s|n)$. [17] Integrating by parts, we obtain

$$\tilde{\pi}(n) = \pi(n) - \int_0^n \pi'(s)\mu(s|n)ds.$$

The distortions generated from this objective function are akin to those developed at length in this paper. The only important change is that now the appropriate notion of a front-load factor is constructed with a general distribution function rather than the simple uniform distribution weight of s. A preference for "quasi-frontloading" emerges along with the associated distortions, and so our results remain fundamentally unchanged. To this extent, we feel that the approach developed here is quite robust.

[17] In the simplest framework, for example, if the split of the net surplus is in the proportion $(\rho, 1-\rho)$ for the firm and employee, respectively, then $\mu(s|n) = \left(\frac{s}{n}\right)^{\frac{\rho}{1-\rho}}$. If $\rho = \frac{1}{2}$, then $\mu(s|n) = \frac{s}{n}$, which is the uniform distribution consistent with the results of this paper.

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