

SEEMINGLY UNRELATED REPEATED GAMES

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ABSTRACT: Suppose Player A is playing two apparently independent repeated games with two other people, B and C , with A randomly matched, each period, with either B or C . Each dyad maintains the maximum incentive-compatible level of cooperation within the dyad, even if cooperation has broken down in the other dyad. Thus, if A defects against B , say, then C is still willing to cooperate with A to the maximum incentive-compatible degree. Nevertheless, we show that the simple *presence* of each cooperative relationship can increase the maximum incentive compatible level of cooperation in the other dyad, due to a counterintuitive circular reasoning or “bootstrapping” effect. With more than two relationships, bootstrapping effects alternate with equally counterintuitive *reverse* bootstrapping effects.

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1. INTRODUCTION

Suppose Player A is playing two seemingly independent repeated games with two other people, B and C , with A randomly matched, each period, with either B or C . It is well understood that, if defection in one relationship causes cooperation to completely break down in the other relationship, then the presence of both relationships supports cooperation in each. For example, if C punishes A whenever A defects against B , then this clearly gives A a greater incentive to cooperate with B .¹

Suppose, however, that each dyad maintains the maximum incentive-compatible level of cooperation within the dyad, even if cooperation has broken down in the other dyad. Thus, if A defects against B , say, then C “minds his own business,” and so, is still willing to cooperate with A to the maximum incentive-compatible degree.²

Suppose also that there is complete information so, if A defects against B , say, then this reveals nothing to C about A 's type. That is, there are no reputation effects of the kind studied in Fudenberg and Kreps (1987).³ For example, C reacts to A in the same way if she sees B defect against A as if she sees A defect against B .

Thus, the obvious links between the two relationships have been severed. One would therefore think that the level of cooperation A can achieve with one partner is unaffected by the other relationship. However, this paper shows that the simple *presence* of each cooperative relationship can increase the achievable level of cooperation in the other dyad, even if these obvious links have been eliminated.

The underlying logic, here, is that A 's cooperation with C helps A 's cooperation with B because her cooperation with B helps her cooperation with C . Thus, if A defects

against B , then A loses this extra cooperation with C . This gives A an additional incentive to cooperate with B . Similarly, A is more willing to cooperate with C because her cooperation with C supports her cooperation with B . Thus, the presence of both relationships creates a counterintuitive circular reasoning or “bootstrapping” effect, which increases the potential level of cooperation in each relationship.

A second way to understand this bootstrapping effect is that, while each dyad tries to maintain the maximum incentive compatible level of cooperation, this maximum level of cooperation depends on whether cooperation has survived in the other relationship. Thus, although B and C do not *intend* to punish defections in the other relationship, some sort of *implicit* punishment may be unavoidable.

A third way to think about this is that, *if* cooperation in both relationships strengthens each, then A is more willing to cooperate in each relationship in order to strengthen the other, so cooperation in both relationships *does* strengthen each.

This bootstrapping effect, however, only exists if the *level* of cooperation has a sufficiently strong effect on the *payoff* from cooperation compared to the payoff from *cheating*. Also, the effect disappears if A can defect against B and C simultaneously. Finally, with multiple relationships, the bootstrapping effect alternates with an equally counterintuitive *reverse* bootstrapping effect as more relationships are added.⁴

While we focus on this bootstrapping effect primarily in the context of a long run Player A interacting with long run patient Players B and C , the same effect arises more generally. For example, B and C can each be replaced by highly impatient players, best responding to A 's expected play in each period.

This effect does not seem to have been noticed previously. Much work has been done on cooperation in societies of randomly matched individuals (Rosenthal, 1979, Kandori, 1992, Ellison, 1994), but these papers focus on cases where cheating in one relationship causes a *complete* breakdown of cooperation in other relationships. Next, Fudenberg and Kreps (1987) focus on reputational spillovers, showing that a “big” player may be reluctant to concede against one small player in a war of attrition, since other small players will conclude that the big player is a weak type, and play more aggressively. Again, bootstrapping does not depend upon such reputation effects.

Third, Bernheim and Whinston (1990) consider multimarket contact between two firms and show that, with asymmetry between markets, firms can use slack in one market’s incentive compatibility constraint to enforce greater collusion in the other market. Bootstrapping does not depend on any such asymmetries. In addition, Bernheim and Whinston focus on *simultaneous* defections in both markets. Bootstrapping breaks down if this is possible (see Section 4). Spagnolo (1999), finally, considers a multi-market contact model like Bernheim and Whinston’s, but his firms are averse to the intertemporal fluctuations in profits caused by cheating (higher cheating profits today, lower, noncooperative profits in the future). Multimarket contact then makes cheating less tempting since it amplifies these intertemporal fluctuations. Bootstrapping does not depend on any such aversion to intertemporal fluctuations.

If bootstrapping is reflected in actual behavior, this may suggest a new way in which relationships can be affected by their social context, beyond the usual punishment or reputational channels. Alternatively, one may consider the bootstrapping effect to

be so counterintuitive that it represents a paradox at the foundations of at least one plausible model of social capital as a network of repeated interactions.

The next section introduces bootstrapping through a simple numerical example. Section 3 considers a continuous action space version of the game, and also examines the effect of moving beyond two dyads. Remaining sections discuss extensions and limitations. Thus, Section 4 contrasts alternating with simultaneous interactions, and Section 5 briefly considers reputation. Section 6 concludes.

2. RANDOM MATCHING – A 3×3 EXAMPLE

This section uses a simple 3×3 game to illustrate the bootstrapping effect. It begins with a numerical example before turning to a more general case. The next section then considers a continuous action game.

2.1. A Simple Numerical Example

Consider the game in Figure 1. Here L , M and H represent low, medium and high levels of cooperation. Note that higher levels of cooperation generate higher temptations to cheat.⁵ Also, (L, L) is a dominant strategy Nash equilibrium.

	L	M	H
L	0, 0	60, -10	180, -20
M	-10, 60	10, 10	100, 0
H	-20, 180	0, 100	20, 20

Figure 1: A stage game with different possible levels of cooperation.

Suppose there are three players, A , B and C . In any given period, A is randomly matched with either B or C , with probability 0.5 each. However, unlike much of the random matching literature, all players know the outcome of all previous play by other players.⁶ Player A then plays the simultaneous move stage game in Figure 1 with whomever she is matched. Let r_A be Player A 's discount rate.

Suppose A treats each relationship in isolation, and cooperation between A and B , say, at level (M, M) , is enforced by the optimal trigger punishment strategy of (L, L) between A and B forever (Abreu, 1988). Player A then *plays* M if and only if

$$60 \leq 10 + 0.5 \frac{1}{r_A} 10, \quad \text{or } r_A \leq 0.10, \quad (1)$$

and similarly for B and C . The factor of 0.5 in the second term on the right is needed because, in any given future period, there is a 50% chance that A will be matched with the same person, B . Similarly, cooperation by A at level (H, H) requires

$$180 \leq 20 + 0.5 \frac{1}{r_A} 20, \quad \text{or } r_A \leq 0.0625, \quad (2)$$

and similarly for B and C .

Now, suppose B 's and C 's discount rates, r_B and r_C , are both less than 0.0625, but that $0.0625 < r_A \leq 0.10$. Thus, B and C are both sufficiently patient that they are willing to cooperate at level H if A does. However, if A considers each relationship in isolation, she is only patient enough to cooperate in each relationship at level M . Is it possible, in this context, for A to use the fact that she is actually interacting with *both* B and C to increase her cooperation with each, even if neither B nor C wants to *punish* A for defections against the other? It turns out that this is possible.

To see this, note first that, if Player A 's cooperation with one of B or C has broken down, then A will not be able to cooperate with the other at level H , since $r_A > 0.0625$. Thus, the maximum sustainable cooperation with the other in this case is (M, M) .⁷

Consider, then, the following trigger strategies: (i) If no one has defected in the past, everyone cooperates at the level H ; (ii) if a defection has occurred in one relationship, then each chooses the Abreu (1988) optimal punishment, L forever, in that relationship, and cooperates at the new maximum achievable level, M , in the other relationship; and (iii) if defections have occurred in both relationships, then everyone chooses the optimal punishment, L forever, in both relationships.

We want to see whether these strategies form an equilibrium, even though $r_A > 0.0625$, so A would not be able to cooperate at level H with either B or C separately. Note that these strategies have each player maintaining the maximum possible level of cooperation in surviving relationships, even if the other relationship has fallen apart. Is it nevertheless possible for these strategies to maintain A 's cooperation at level H ?

To see that it is, note first that B and C are very patient, so they are willing to follow all parts of the strategy. Next, A is willing to follow Part (ii) of the strategy profile since $r_A \leq 0.1$. Also, Part (iii) is a *static* Nash equilibrium, so A is willing to follow it as well. For Part (i), suppose that A has not yet defected against either player, and suppose A is currently facing B . Since no one has ever defected, A and B are supposed to play (H, H) . Player A will then *play* H if

$$180 + 0.5 \frac{1}{r_A} 10 \leq 20 + 0.5 \frac{1}{r_A} 20 + 0.5 \frac{1}{r_A} 20, \quad \text{or } r_A \leq 0.09375. \quad (3)$$

This says that the immediate payoff from cheating against B , 180, plus the future

payoff from reduced cooperation with C , 10 per interaction, is worth less to A than 20 from cooperating with B today plus the future payoff from continued cooperation with both B and C at level H . The same calculation applies to A 's cooperation with C .

Thus, if $r_A \leq 0.09375$ (so $r_A \leq 0.10$ as well), then A is willing to cooperate with B and C at level H . This is the case even though $r_A > 0.0625$, so A could not have achieved cooperation at level H with either player in isolation.

This effect arises because cheating in either relationship costs A not only a future cooperative payoff of 20 per interaction in that relationship, but also reduces the maximum achievable cooperative payoff in the other relationship from 20 to 10 per interaction. Thus, A 's relationship with C , say, strengthens A 's relationship with B because, if A defects against B , this hurts A 's relationship with C . This yields a counterintuitive circular reasoning or “bootstrapping” effect, where each relationship strengthens the other precisely because the second relationship strengthens the first.

A second way to think about this is that, even though B , say, does not *want* to punish A for defections against C , he is forced to at least implicitly impose *some* punishment, since A and B can no longer achieve (H, H) if cooperation with C has fallen apart. In standard cross-relationship punishments (Kandori, 1992), by contrast, C *completely* ceases cooperating with A if A defects against B , so (3) changes to

$$180 \leq 20 + 0.5 \frac{1}{r_A} 20 + 0.5 \frac{1}{r_A} 20, \quad \text{which requires only } r_A \leq 0.125. \quad (4)$$

In addition, (1) is no longer needed, so $r_A \leq 0.125$ is sufficient to support cooperation at level H . That is, if Players B and C explicitly *punish* A 's defection against the other, cooperation is easy to enforce.

Bootstrapping does not achieve as much cooperation, since it does not involve as much punishment. However, it does force some implicit punishment since, if A defects against one of B or C , this reduces her achievable level of cooperation with the other.⁸

Finally nothing here depends on Players B and C being extremely patient. For example, suppose B and C are very impatient, and that they are randomly matched with A in the game in Figure 2, with B and C occupying the column role. Since B and C are very impatient, they best respond to the expected play of A in this game.

	L	M	H
L	0, 10	60, 0	180, 0
M	-10, 0	10, 10	100, 0
H	-10, 0	0, 0	20, 10

Figure 2: A stage game with Player A choosing rows.

In this case, calculations identical to those above show that, if A ignores her relationship with C , say, then she needs $r_A \leq 0.0625$ to support H against B . And again, as long as $r_A \leq 0.09375$, A can support H against both B and C , since defecting to L against one will limit her future play to at best M against the other.

2.2. A General 3×3 Case

This subsection replaces the game in Figure 1 with a general symmetric game with actions L , M , and H and unique Nash equilibrium (L, L) . Assume

$$0 = u_K(L, L) < u_K(M, M) < u_K(H, H) \text{ for } K = A, B, C, \quad (5)$$

where $u_K(X, Y)$ is Player K 's payoff function from action profile (X, Y) . As before, L , M and H represent low, medium and high levels of cooperation, respectively. Let $u^{coop}(X) = u_A(X, X)$, $X = M, H$, and $\Delta u^{ch}(X) = u_A(R(X), X) - u_A(X, X)$, where $R(X)$ is A 's best reply to X . Here, $u^{coop}(X)$ is the payoff from cooperating at level X , and $\Delta u^{ch}(X)$ is the extra payoff from cheating, as opposed to cooperating, against X . By symmetry, nothing depends on these being expressed in terms of Player A 's payoffs, rather than B 's or C 's. Calculations similar to those above show that, if A considers each relationship in isolation, she can achieve cooperation at level M if and only if

$$r_A \leq \frac{u^{coop}(M)}{2\Delta u^{ch}(M)}, \quad (6)$$

and she can achieve cooperation at level H if and only if

$$r_A \leq \frac{u^{coop}(H)}{2\Delta u^{ch}(H)}. \quad (7)$$

Note that, if higher levels of cooperation are more difficult to achieve, then the right hand side of (7) will be smaller than the right hand side of (6). This was the case for the numerical example in Subsection 2.1.

Next, generalizing (3) above shows that, if A takes into account that she is cooperating with *both* B and C , she can cooperate at level H if and only if

$$r_A \leq \frac{2u^{coop}(H) - u^{coop}(M)}{2\Delta u^{ch}(H)}. \quad (8)$$

The bound in (8) exceeds the bound in (7) since $2u^{coop}(H) - u^{coop}(M) > u^{coop}(H)$. Thus, combining seemingly unrelated repeated games permits cooperation at level H

for a wider range of r_A 's than allowed in (7). The bootstrapping effect therefore generalizes quite widely.

We could, however, ask a somewhat different question. Suppose that the isolated-play incentive compatibility constraint binds as an equality for cooperation at level M . Thus, (6) holds as an equality. In this case will bootstrapping allow A to increase her level of cooperation from M to H ? This depends on whether the right hand side of (8) is bigger than the right hand side of (6), not (7). Thus if, in an isolated relationship, Player A is just indifferent to cooperating at level M , then bootstrapping allows her to cooperate at level H if and only if

$$\frac{u^{coop}(M)}{2\Delta u^{ch}(M)} \leq \frac{2u^{coop}(H) - u^{coop}(M)}{2\Delta u^{ch}(H)}.$$

Multiplying through by $2\Delta u^{ch}(H)/u^{coop}(M)$ and subtracting one, this becomes

$$\frac{\Delta u^{ch}(H) - \Delta u^{ch}(M)}{\Delta u^{ch}(M)} \leq 2 \frac{u^{coop}(H) - u^{coop}(M)}{u^{coop}(M)}.$$

Using $u^{coop}(M) = 2r_A\Delta u^{ch}(M)$ (since (6) holds as an equality), this becomes

$$\Delta u^{ch}(H) - \Delta u^{ch}(M) \leq \frac{1}{r_A} [u^{coop}(H) - u^{coop}(M)]. \quad (9)$$

This says that cooperative profits must rise sufficiently quickly, compared to cheating profits, as the level of cooperation rises.

Thus, if Player A 's incentive compatibility constraint exactly binds at cooperation level M , then bootstrapping only increases A 's attainable cooperation if (9) holds. A similar condition is central to the continuous action case considered next.

3. RANDOM MATCHING – CONTINUOUS ACTION SPACE

This section considers a game with a continuum of possible cooperation levels. Subsection 3.1 considers the two-dyad case, and derives a condition analogous to (9). Subsection 3.2 generalizes to n dyads, and illustrates a “reverse-bootstrapping” effect.

3.1. *The Two Dyad Case*

In the one-shot game, assume that players’ strategy sets are the nonnegative real numbers, $x \geq 0$. The single-period utility function for Player A , when playing x against y by Player K , $K = B, C$, is $u_A(x, y)$, and the payoff function for Player K is $u_K(y, x)$. Assume for specificity that the game is symmetric. Also, assume that $u_A(x, y)$ is increasing in y and continuously differentiable.⁹

Assume the unique Nash equilibrium of the one-shot game is $(x^*, y^*) = (0, 0)$, so the most severe punishment strategy in the repeated game (Abreu, 1988) is Nash reversion forever. Also, normalize by $u_A(0, 0) = u_B(0, 0) = u_C(0, 0) = 0$.

Assume that cooperation involves each player raising x by the same amount, so

$$u^{coop}(x) = u_J(x, x), \quad J = A, B, C, \quad (10)$$

is the payoff from cooperation at level x , where the payoff is independent of J by symmetry. Let $R(x)$ be an optimal one-shot reaction if the other player chooses x , so $R(x)$ is what a cheater chooses if the other player is cooperating at level x . Thus,

$$\Delta u^{ch}(x) = u_J(R(x), x) - u^{coop}(x), \quad J = A, B, C, \quad (11)$$

is the extra payoff a player gets from cheating, as opposed to cooperating, when the other player cooperates at level x . Again symmetry implies that this is the same for all

$J = A, B,$ and C . Assume that $u^{coop}(x)$ and $\Delta u^{ch}(x)$ are continuously differentiable and increasing in the relevant range.

In the repeated version of the game, Player A is randomly matched, each period, with either B or C , with equal probabilities. The discount rate for Player A is $r_A = r$, where we omit the subscript for brevity. Similarly, the discount rates for B and C are r_B and r_C . For simplicity assume that r_B and r_C are very small, so Players B and C are so patient that they are always willing to cooperate if A does. This allows us to focus on the incentives of Player A .

As before, assume that neither B nor C wants to explicitly punish A for defection against the other. That is, B and C are not “busybodies.” Thus, as long as two players have never defected against one another, they are willing to cooperate with each other to the maximum incentive compatible degree, even if cooperation has broken down in the other relationship. Nevertheless, the maximum level of cooperation in one relationship might depend on whether cooperation has survived in the other relationship. Thus, let x_B , say, be the level of cooperation between A and B if A is still cooperating with C at level x_C , and visa versa. Similarly, let x'_B be the level of cooperation between A and B if cooperation has broken down between A and C , and symmetrically for x'_C .

The players thus play the following trigger strategies: (i) if no one has defected in the past, everyone cooperates at the level x_K , with $K = B$ or C as appropriate, (ii) if a defection occurred in one relationship, then each chooses $x = 0$ in that relationship, and cooperates at level x'_K ($K = B, C$) in the other relationship, and (iii) if defection occurred in both relationships, everyone chooses $x = 0$ forever in their relationships.

We now find the maximum possible x_B , x_C , x'_B , and x'_C . First, suppose that cooperation has only survived between A and one player, K , $K = B$ or C . Then A 's cooperation with K at level x'_K requires

$$\Delta u^{ch}(x'_K) \leq \frac{1}{2r} u^{coop}(x'_K). \quad (12)$$

By symmetry, the maximum possible values for x'_K satisfy $x'_B = x'_C = x^{(1)}$, say, where $x^{(1)}$ satisfies

$$\Delta u^{ch}(x^{(1)}) = \frac{1}{2r} u^{coop}(x^{(1)}) \quad (13)$$

(the superscript (1) indicates that only *one* relationship is still cooperative). Also, if $x^{(1)}$ is really a maximum, then the slope of the $\Delta u^{ch}(x)$ curve should be at least weakly steeper than the slope of the $(1/2r)u^{coop}(x)$ curve at $x = x^{(1)}$ (compare the solid and dashed curves in Figure 3).

We now turn to A 's incentive-compatibility conditions for x_B and x_C . These are, for A 's cooperation with B :

$$\Delta u^{ch}(x_B) + \frac{1}{2r} u^{coop}(x^{(1)}) \leq \frac{1}{2r} u^{coop}(x_B) + \frac{1}{2r} u^{coop}(x_C), \quad (14)$$

and for A 's cooperation with C :

$$\Delta u^{ch}(x_C) + \frac{1}{2r} u^{coop}(x^{(1)}) \leq \frac{1}{2r} u^{coop}(x_B) + \frac{1}{2r} u^{coop}(x_C). \quad (15)$$

Again, at the maximum levels of cooperation, x_B and x_C are the same, so $x_B = x_C = x^{(2)}$, say. Thus, (14) and (15) yield

$$\Delta u^{ch}(x^{(2)}) = \frac{1}{r} u^{coop}(x^{(2)}) - \frac{1}{2r} u^{coop}(x^{(1)}). \quad (16)$$

Also, the slope of the $\Delta u^{ch}(x)$ curve should be at least weakly steeper than the slope of the $(1/r)u^{coop}(x) - (1/2r)u^{coop}(x^{(1)})$ curve at $x = x^{(2)}$.

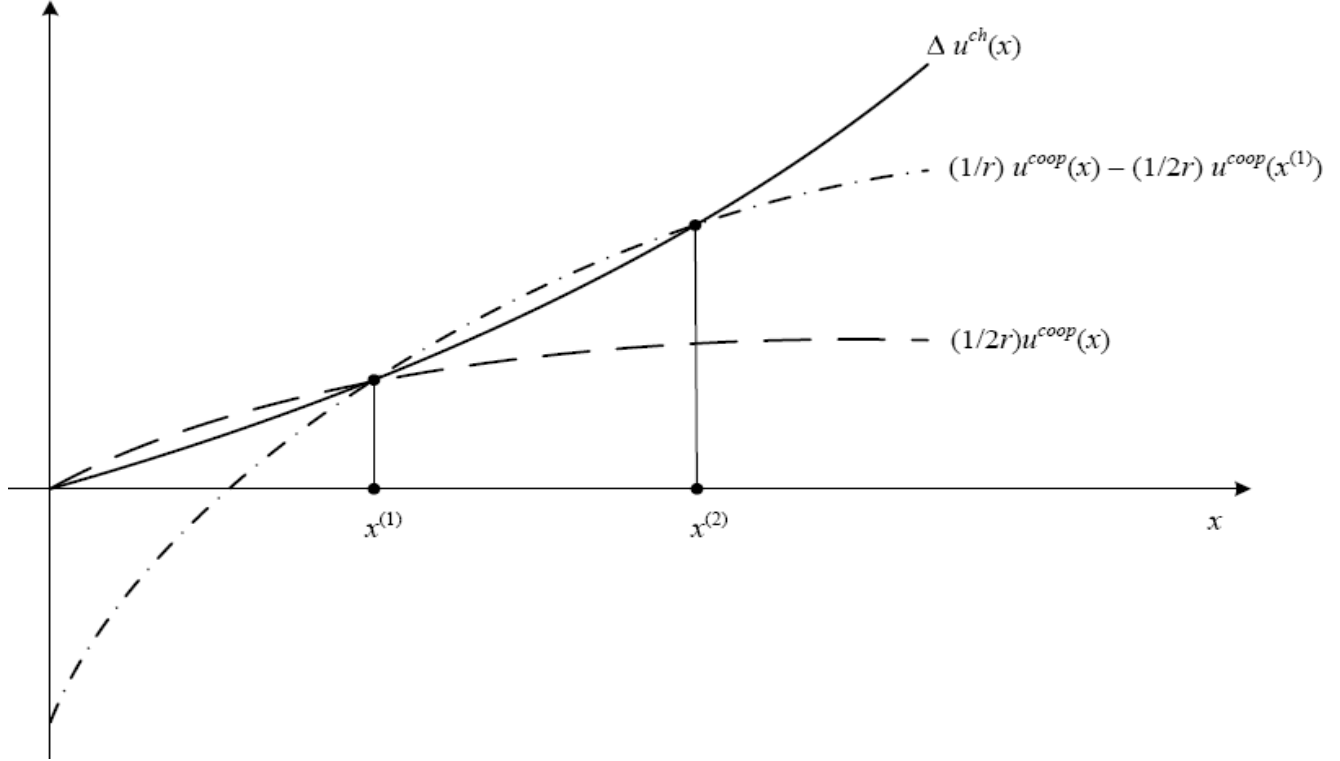


Figure 3: Payoffs to cheating and cooperation in (13) and (16).

To see that the presence of two seemingly unrelated relationships might allow increased cooperation in each, note first that the $\Delta u^{ch}(x)$ curve, the $(1/2r)u^{coop}(x)$ curve and the $(1/r)u^{coop}(x) - (1/2r)u^{coop}(x^{(1)})$ curve all cross at $x = x^{(1)}$ (the first two cross by (13), the second and third cross using a little algebra).

However, the $(1/r)u^{coop}(x) - (1/2r)u^{coop}(x^{(1)})$ curve is twice as steep as the $(1/2r)u^{coop}(x)$ curve at $x = x^{(1)}$. Thus, it is possible for the $\Delta u^{ch}(x)$ curve to be steeper than the $(1/2r)u^{coop}(x)$ curve at $x = x^{(1)}$, but *less* steep than the $(1/r)u^{coop}(x) - (1/2r)u^{coop}(x^{(1)})$ curve (compare the solid and dash-dot curves in Figure 3).

The $(1/r)u^{coop}(x) - (1/2r)u^{coop}(x^{(1)})$ curve is therefore above the $\Delta u^{ch}(x)$ curve in this case, for some values of x greater than $x^{(1)}$ (again see Figure 3). That is, A 's future benefit from cooperation in this range exceeds her current gain from cheating. Cooperation by A is therefore incentive compatible for these values of x , so $x^{(2)} > x^{(1)}$.

Thus, $x^{(2)}$ exceeds the single-dyad cooperation level, $x^{(1)}$, whenever the slope of $(1/r)u^{coop}(x) - (1/2r)u^{coop}(x^{(1)})$ in x exceeds the slope of $\Delta u^{ch}(x)$ at $x = x^{(1)}$, so

$$x^{(2)} > x^{(1)} \text{ if } \frac{1}{r} \frac{du^{coop}(x)}{dx} > \frac{d\Delta u^{ch}(x)}{dx} \text{ at } x = x^{(1)}. \quad (17)$$

Note that this is roughly the continuous-action analogue of (9). It is therefore easy to construct numerical examples, such as the following.

EXAMPLE: Suppose the game's payoff functions yield

$$\Delta u^{ch}(x) = \frac{10x^2}{1+x} \text{ and } u^{coop}(x) = 2x - \frac{1}{10}x^2. \quad (18)$$

Also, suppose that $r_A = r = 0.10$. Then the maximum achievable $x^{(1)}$ for each relationship in isolation is $x^{(1)} = 4$, and, for this $x^{(1)}$, cooperation in the *two* relationships is possible for $4 \leq x \leq (5 + \sqrt{57})/2 \approx 6.275$, so $x^{(2)} = (5 + \sqrt{57})/2$. Thus, the coexistence of two seemingly unrelated relationships allows increased cooperation in both.

The logic for this result is the same as that in Section 2. This therefore again creates a bootstrapping effect where A can increase her cooperation with B because increased cooperation with B supports increased cooperation with C , while increased cooperation with C also supports increased cooperation with B .

3.2. The n Dyad Case

Suppose now that A is randomly matched each period with one of Players B_1, B_2, \dots , or B_n , with probability $1/n$ each. Let $x^{(k)}$ be the maximum possible cooperation if only k out of n dyads are still cooperating. The analogue to equation (13) is then

$$u^{ch}(x^{(1)}) = \frac{1}{nr} u^{coop}(x^{(1)}), \quad (13')$$

and the analogue to (16) is

$$\Delta u^{ch}(x^{(k)}) = \frac{k}{nr} u^{coop}(x^{(k)}) - \frac{k-1}{nr} u^{coop}(x^{(k-1)}), \quad k = 1, 2, \dots, n. \quad (16')$$

Now, reasoning as above, if $x^{(1)}$ is truly the maximum achievable level of cooperation with only one dyad cooperating, then the slope of the left hand side of (13') must at least weakly exceed the slope of the right hand side. Also, as before,

$$x^{(2)} > x^{(1)} \text{ if } \frac{2}{nr} \frac{du^{coop}(x)}{dx} > \frac{d\Delta u^{ch}(x)}{dx} \text{ at } x = x^{(1)}. \quad (17')$$

However, even if (17') fails, there may well be a k with $2 < k \leq n$ such that

$$\frac{k}{nr} \frac{du^{coop}(x)}{dx} > \frac{d\Delta u^{ch}(x)}{dx} \text{ at } x = x^{(1)}, \quad (17'')$$

at which point $x^{(k)}$ will exceed $x^{(1)}$. Thus, with multiple dyads, the likelihood may increase that the bootstrapping effect will eventually manifest itself.

However, once the bootstrapping point is reached, cooperation levels begin to alternate in a counterintuitive way. For example, suppose $x^{(2)} > x^{(1)}$. Then it will often turn out that $x^{(3)}$ is much *less* than $x^{(1)}$. For (16') becomes

$$\Delta u^{ch}(x^{(3)}) = \frac{3}{nr} u^{coop}(x^{(3)}) - \frac{2}{nr} u^{coop}(x^{(2)}), \quad (16'')$$

where now the last term is big because $x^{(2)}$ is big. Since the last term is big, it becomes possible for the right hand side of (16'') to be less than the left hand side for *all* $x^{(3)} \geq 0$, so *no* cooperation is possible at all! Indeed, if we use the functional forms from the example at the end of Subsection 3.1, but with $n = 3$ and $r = 0.0666\dots$, then $x^{(1)} = 4$ and $x^{(2)} = (5 + \sqrt{57})/2$, as before, but $x^{(3)} = 0$.

The logic in this case is that, if $x^{(3)}$ is not bigger than $x^{(2)}$, then A does not lose any cooperation with B_2 and B_3 , say, if she defects against B_1 . But then there is no bootstrapping effect, so in fact, $x^{(3)}$ is no bigger than $x^{(1)}$. But then $x^{(3)} < x^{(2)}$, so defecting against B_1 actually *increases* her cooperation with B_2 and B_3 , so there is a kind of *reverse* bootstrapping effect, so $x^{(3)} < x^{(1)}$.

Of course, $x^{(4)}$ may then tend to be big again, since defecting against one of four partners reduces A 's achievable level of cooperation with the other three to $x^{(3)}$, which is very low. The $x^{(k)}$ presumably tend to alternate after that.

One could take this reverse bootstrapping effect in either of two ways. On the one hand, it could be taken as evidence of the fragility of the bootstrapping effect. Bootstrapping does not build on solid motivational forces like reputation or a desire to punish defections in other relationships. It may therefore not be surprising that it falls apart in an equally counterintuitive way. On the other hand, reverse bootstrapping might reflect a kind of “three (A, B, C) is company, four (A, B_1, B_2, B_3) is a crowd” effect. Some might find this plausible, though I do not currently see how.

4. ALTERNATING VERSUS SIMULTANEOUS INTERACTIONS

Consider again the two dyad case, so the reverse bootstrapping effect from the previous section is no longer an issue. To see how bootstrapping depends on the details of the random matching game, this section considers a model where Player A has deterministically alternating interactions with B and C , and also considers a related model where A interacts with B and C simultaneously. While the algebra is slightly more complicated, the basic bootstrapping results are the same in the alternating interactions case as in the random matching case above. On the other hand, if A can defect against B and C simultaneously then the bootstrapping effect disappears.

To increase comparability with the simultaneous defection case, we set up the alternations in a somewhat unusual way, though nothing essential depends on the details of this setup. Thus, assume the same one-shot game and notation as in Subsection 3.1. However, assume now that, in each period, A first plays with B , then C sees the play between A and B , then A plays with C , all in the same period. Then, in the next period, B sees the outcome of A 's previous play with C , and the process repeats itself.

Again we consider the maximum possible levels of cooperation in each relationship, conditional on whether cooperation still exists in the other relationship. As in the previous section, let x_K , $K = B, C$, be the level of cooperation between A and K , given that the other relationship is still cooperative, and let x'_K , $K = B, C$, be the level of cooperation between A and K , given that cooperation has broken down in the other relationship. In the present context, x'_K is incentive compatible for A if

$$\Delta u^{ch}(x'_K) \leq \frac{1}{r} u^{coop}(x'_K). \quad (19)$$

Similarly, x_B and x_C are jointly incentive compatible for A if

$$\Delta u^{ch}(x_B) + \frac{1+r}{r}u^{coop}(x'_C) \leq \frac{1}{r}u^{coop}(x_B) + \frac{1+r}{r}u^{coop}(x_C), \quad (20)$$

and

$$\Delta u^{ch}(x_C) + \frac{1}{r}u^{coop}(x'_B) \leq \frac{1}{r}u^{coop}(x_B) + \frac{1}{r}u^{coop}(x_C). \quad (21)$$

For the same reasons as in Subsection 3.1, the maximum possible x_B and x_C can exceed the maximum possible x'_B and x'_C if a condition like (17) holds. The straightforward details are available upon request.

Suppose however that, in any given period, Player C has to choose her level of cooperation with A *before* seeing how Player A played with B . This is equivalent to Player A interacting with B and C simultaneously. It is therefore possible for A to defect against both B and C at the same time. Thus, cooperation on A 's part requires an additional incentive compatibility condition:

$$\Delta u^{ch}(x_B) + \Delta u^{ch}(x_C) \leq \frac{1}{r}u^{coop}(x_B) + \frac{1}{r}u^{coop}(x_C). \quad (22)$$

In this case, the levels of cooperation, x_B and x_C cannot exceed the maximum possible $x'_K = x^{(1)}$, say, from (19). For suppose, e.g., that $x_B > x^{(1)}$. Then

$$\Delta u^{ch}(x_B) > \frac{1}{r}u^{coop}(x_B), \quad (23)$$

so

$$\Delta u^{ch}(x_C) < \frac{1}{r}u^{coop}(x_C), \quad (24)$$

by (22), so $x_C < x^{(1)}$ and $u^{coop}(x_C) < u^{coop}(x^{(1)})$. Thus (20) (with $x'_C = x^{(1)}$) yields

$$\Delta u^{ch}(x_B) < \frac{1}{r} u^{coop}(x_B), \quad (25)$$

which contradicts (23). Thus, if A can defect against B and C simultaneously, bootstrapping disappears and seemingly unrelated repeated games really are unrelated.¹⁰

This raises the question of how relevant the simultaneous move case is compared to the alternating move or random matching cases. If opportunities for profitable defections arise randomly, as in Sobel (1985) or Rotemberg and Saloner (1986), this may be well approximated by the random matching case, as in Sections 2 and 3.¹¹ Thus, the random matching case may have wide applicability, and the preconditions, at least, of the bootstrapping effect should be very common.

5. REPUTATION EFFECTS

This section briefly considers reputation effects, in order to more clearly disentangle bootstrapping from reputation effects. For this purpose, suppose Player A can be one of two types. Either A is *patient*, with discount rate r_P , or she is *impatient*, with discount rate $r_I > r_P$. The patient type is so much more likely than the impatient type that, unless Player A actually defects, B and C simply assume that A is patient.

However, if A ever defects against one of B or C , the other concludes that A is impatient for sure. Thus, letting $x_I^{(1)}$ be A 's maximum achievable level of cooperation if her partner believes she is impatient, equation (13) becomes

$$\Delta u^{ch}(x_I^{(1)}) = \frac{1}{2r_I} u^{coop}(x_I^{(1)}). \quad (13R)$$

Note that, since $r_I > r_P$, $x_I^{(1)}$ is less than the maximum level of cooperation, $x_P^{(1)}$, that A could achieve in a single relationship if the other player knew she was patient.

Next, let $x_R^{(2)}$ be Patient A 's maximum achievable level of cooperation with both B and C , given that reputation is a factor. If Player A is currently cooperating with both B and C , then defecting against one will lead the other to believe that she is impatient, so (16) becomes

$$\Delta u^{ch}(x_R^{(2)}) = \frac{1}{r_P} u^{coop}(x_R^{(2)}) - \frac{1}{2r_P} u^{coop}(x_I^{(1)}). \quad (16R)$$

This equation is illustrated in Figure 4.

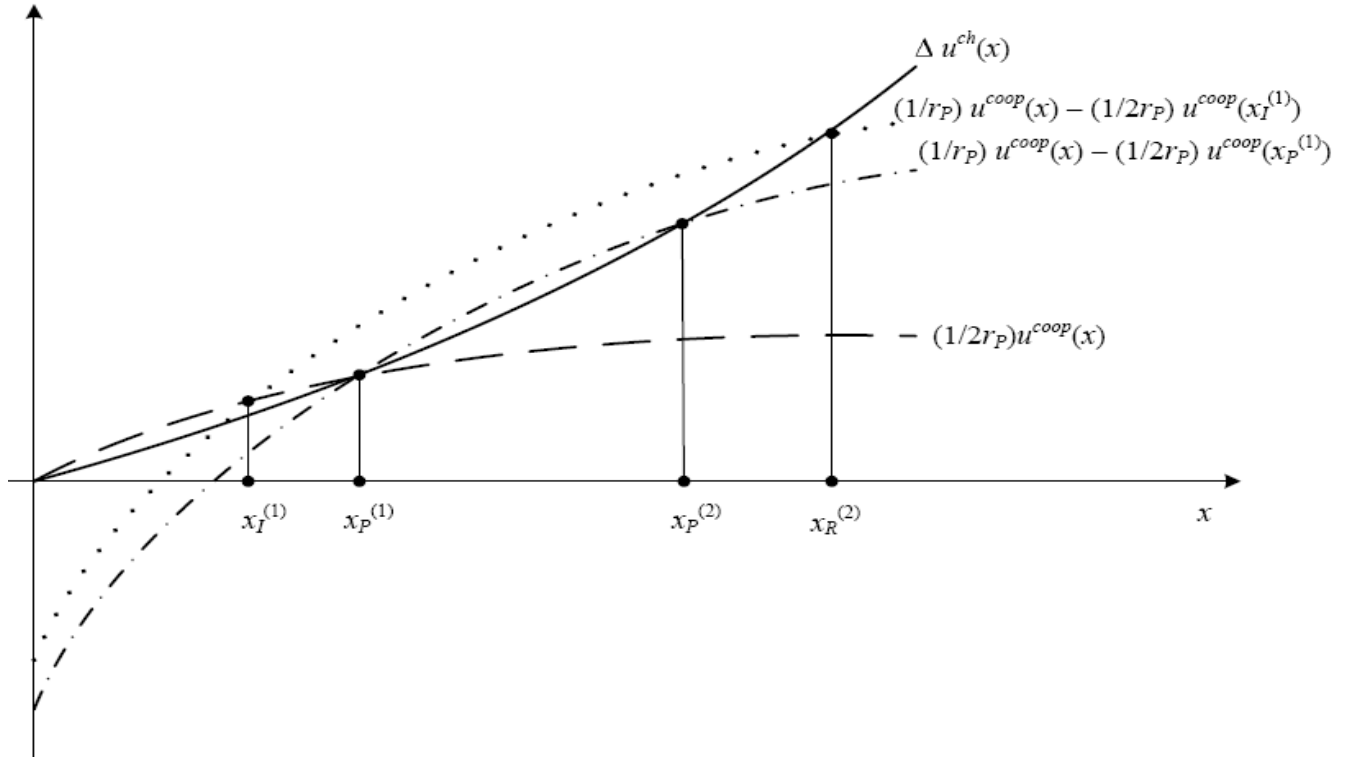


Figure 4: Payoffs to cheating and cooperation in (13_R) and (16_R).

Here the solid, dashed, and dash-dot curves are as in the no-reputation case in

Figure 3, where B and C remain convinced that A is patient for sure, even after a defection by A against one of them. The new curve is the dotted curve, $(1/r_P)u^{coop}(x) - (1/2r_P)u^{coop}(x_I^{(1)})$, which is the effect on A 's future payoffs of cooperating, as opposed to defecting, given A 's additional concern that defecting hurts her reputation.

Now, since $x_I^{(1)} < x_P^{(1)}$, it follows that $u^{coop}(x_I^{(1)}) < u^{coop}(x_P^{(1)})$, so the dotted curve in Figure 4 is above the dash-dot curve. Thus, as is clear from the diagram, the amount of cooperation a patient Player A can achieve with two relationships and reputational concerns, $x_R^{(2)}$, is greater than the amount, $x_P^{(2)}$, that she could achieve in the absence of reputation effects.

Thus, the bootstrapping effect raises the level of cooperation from $x_P^{(1)}$ to $x_P^{(2)}$, and the reputation effect raises it further from $x_P^{(2)}$ to $x_R^{(2)}$. Note here that, if condition (17) is not met, then there is no bootstrapping effect, but as long as $r_I > r_P$ there is a reputation effect. On the other hand, if (17) is met but $r_I = r_P$, the bootstrapping effect is present but the reputation effect is not.

6. CONCLUSION

The environment described here is ubiquitous. There are repeated relationships everywhere, and people often keep track of their acquaintances' relationships with other people. In addition, people often want to continue cooperating with acquaintances, even if those acquaintances have breakdowns in cooperation with other people. Finally, opportunities for profitable defections from cooperative relationships may often arrive randomly, as in Sobel (1985) and Rotemberg and Saloner (1986), so the random

matching element of the models in Sections 2, 3 and 5 may also be very widespread.

Thus, the preconditions for the bootstrapping effect are nearly universal. That said, the effect itself seems quite counterintuitive. If actual economic agents depend upon common-sense intuition to guide them through strategic interactions, it may not be easy for ordinary people to stumble across this effect. Thus, while the preconditions for this effect are almost certainly universal, it is not clear how often the effect itself actually arises in ordinary economic or social situations.

In addition, it may be difficult to distinguish bootstrapping from reputation effects (Fudenberg and Kreps, 1987). As with reputation, bootstrapping means that a defection in one relationship can reduce cooperation in a second relationship, even if both players in that second relationship want to cooperate as much as possible.¹² In fact, ordinary people may believe they are responding to reputation effects when their behavior actually reflects a combination of reputation and bootstrapping effects.

In any case, if bootstrapping influences actual behavior, this suggests that cooperation between two agents can be affected by the presence of others, beyond the usual punishment or reputational channels. On the other hand, one may decide that bootstrapping and reverse bootstrapping effects are so counterintuitive that they simply represent a paradox at the foundation of plausible models of social capital based on networks of repeated relationships.

NOTES

1. This mechanism is important in much of the random matching literature (Rosenthal, 1979, Ellison, 1994, and especially Kandori, 1992), which assumes that defection in one relationship leads to complete breakdown of cooperation in others.

2. This is in contrast to the random matching literature mentioned in footnote 1. It also contrasts with Bernheim and Whinston (1990), who assume that defection in one market leads to complete breakdown of collusion in others.

3. Recall that reputation effects require incomplete information. Specifically, there must be more than one “type” of Player A , with B and C uncertain about the true type of A . This would allow B and C to learn from A 's behavior whether A was a trustworthy or an untrustworthy type. Thus, in such a framework, if A defected against B , say, then C would attach more weight to A being an untrustworthy type.

However, in most of this paper we assume that there is only *one* type of Player A , so no such learning is possible. Thus, if A defects against B , then C may conclude that A simply made a mistake, which reveals nothing to C about A 's type. Since C 's belief about A 's type would not be affected, C would not expect A 's untrustworthy behavior to be repeated. Reputation effects *are* briefly considered in Section 5.

4. This paper does not consider bootstrapping in the context of renegotiation proof equilibria (Farrell and Maskin, 1989). However, it seems plausible that the effect will survive in that context. If A defects against B , say, then renegotiation proofness in the A - B relationship requires A to be punished while B is rewarded. Since A is being punished in the A - B relationship, this means that the A - B relationship cannot

deteriorate much more if A then defects in the $A-C$ relationship. Thus, if A is being punished in the $A-B$ relationship, this continues to weaken the $A-C$ relationship. This therefore still gives A an additional incentive to cooperate in the $A-B$ relationship.

5. This is therefore consistent with the Cournot case, as is the continuous action game considered in Section 3.

6. Rosenthal (1979), for example, assumes that each player only sees the previous choice of the player with whom she is currently matched. Kandori (1992) and Ellison (1994) consider models in which each player only sees the choices in games in which she herself participates, and Kandori (1992) also considers a model in which players get to see a “label” attached to the other player, which depends on the other player’s previous experiences and actions.

7. Note that it does not matter to the other player *why* A has ceased cooperating with the first player. For example, suppose A has ceased cooperating with B . Then C does not care whether A had previously defected against B or B had previously defected against A . Thus, A ’s *reputation* is not an issue here.

8. As Quan Wen has pointed out to me, it is possible for *both* B and C to *benefit* from carrying out a punishment if A defects against one of them. This happens if they use punishments with a renegotiation-proof flavor. For example, if A defects against B , then B and C could insist that A play H against *both* for a few periods, while B and C play L , before returning to (H, H) .

This suggests that B and C would not *mind* punishing A for defections against the other – they may benefit from being “busybodies.” However, the point of this paper

is not to find the optimal degree of cooperation between A , B , and C , but to point out how A 's relationship with one of B or C may be affected by the other relationship, even if they are *not* busybodies, so the obvious links between the relationships are broken.

9. Symmetry and smoothness are assumed for expositional reasons only. Nothing essential depends on these assumptions.

10. Note that standard Kandori (1992) cross-relationship punishment effects also tend to disappear in the simultaneous interactions case (see Bernheim and Whinston, 1990). It is therefore not surprising that the bootstrapping effect disappears in this case as well, since under bootstrapping, players do not even *want* to punish defections in others' relationships.

11. Bernheim and Whinston (1990) also consider a model where the temptation to cheat alternates randomly between the two markets. However, in their equilibrium, defection in one market leads to punishment in both, so the bootstrapping effect does not arise in their equilibrium.

12. Of course, in a reputation context, the level of cooperation between A and C , say, after A defects against B , will be lower than the level of cooperation between A and C if B defects against A . Under pure bootstrapping, by contrast, these two levels of cooperation coincide.

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