SHOULD CENTRAL BANKS BURST BUBBLES?

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ABSTRACT

Policy towards speculative bubbles is examined in a model of a finite horizon "greater fool" bubble, with rational agents, asymmetric information and short-sales constraints. This model permits the use of standard tools of comparative dynamics and welfare economics to analyze bubble policies.

Government policy is modeled as deflating overpriced assets by revealing information about whether or not the asset is overpriced. It is shown that such a policy tends to improve welfare if it protects less-informed buyers from "bad" sellers, who know the asset is overpriced. However, if policy deflates prices only in "strong bubbles," where *all* private agents know the asset is overpriced, this tends to reduce welfare. This is because, in those states where the central bank turns out *not* to deflate prices, bad sellers become more confident of selling the asset. That is, bubble bursting protects bad sellers from each other, which, in turn, can exacerbate the lemons problem in states where the asset is valuable (*JEL* D82, E52, G14).

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Asset prices have fluctuated wildly in recent years, and many have attributed these fluctuations to asset price bubbles (Higgins and Osler, 1997, Shiller, 2000, Ofek and Richardson, 2003).[1] There has also been a heated debate about whether central banks should try to deflate these bubbles (Bernanke and Gertler, 1999, 2001, Cecchetti and coauthors, 2000, 2003, Bordo and Jeanne, 2002, Hunter, et al. 2003, Bean, 2004). During the Internet boom, for example, the *Economist* (1998) opined that "the Fed made a mistake in not raising interest rates last year to let some air out of the bubble."

Unfortunately, there have been few theoretical models in which to examine the welfare implications of policies towards bubbles, so economic theory has so far been relatively quiet on bubble policy. Standard models of rational bubbles use an infinite-horizon framework, where agents hold overpriced assets because they believe these assets will be overpriced forever in expected value.[2] These models, however, violate market participants' intuition that bubbles eventually burst.[3] In addition, bubbles generally *improve* welfare in these models.[4] Finally, as pointed out by Kent and Lowe (1997, p. 18), "the size of the bubble is indeterminate" in these models, so "there is no way of tying down the size" of any response of bubbles to policy actions.

For these reasons, most studies of bubble policy simply assume some sort of exogenous gap between the market price of an asset and its fundamental value (Kent and Lowe, 1997, Bernanke and Gertler, 1999, 2001, Cecchetti and coauthors, 2000, 2003, and Dupor, 2002). However, since the process driving the bubble is never explicitly modeled in these papers, it is difficult to relate the welfare effects of bubble policy to the market failures that generate the bubble.

This paper therefore analyzes bubble policy using an explicit, fully endogenous model of a bubble. Specifically, we assume a "greater fool" model of asset price bubbles, where investors hold overpriced assets in hopes of selling them to someone else – a "greater fool" – before asset prices collapse.[5]

It is difficult, however, to capture this greater fool dynamic in standard economic models, where all agents are perfectly rational. Fortunately, a major breakthrough in modeling greater-fool bubbles with rational agents was achieved a decade ago by Allen, Morris and Postlewaite (1993). These authors consider a finite horizon model, so any bubbles must burst eventually, consistent with the intuition of market participants. They then use asymmetric information and short sale constraints to model a "strong bubble," where everyone knows that an asset is overpriced. Agents hold an asset they know is overpriced because, with asymmetric information, no one knows whether anyone else also knows the asset is overpriced. Thus, everyone hopes to sell the asset to someone else, yielding a greater fool bubble.

Allen et al. (1993) presents a very promising approach to modeling asset price bubbles. Unfortunately, the Allen et al. example is too complicated to work with easily. Recently, Conlon (2004) simplified the Allen et al. approach, making it more straightforward to analyze issues related to asset price bubbles. In particular, it is now possible to get the first real look at appropriate policies towards greater fool bubbles.

We therefore analyze asset deflation policies in a simplified Allen et al. (1993) greater fool model. Welfare analysis is especially convenient in greater fool models with *rational* agents since standard tools of welfare economics then apply. In particular, welfare analysis can be based on utility functions which agents themselves maximize.[6]

Our bubble is structured as follows. First, there are several possible states of the world, and agents have incomplete information about which one of these states is the true state. In some states, half the agents are "good sellers," whose asset might be valuable, and half are "buyers." In other states, half are "bad sellers," who know their asset is worthless, and half are buyers. In still other states, all agents are bad sellers.

Buyers are willing to buy because they do not know whether sellers are good or bad. In addition, there are nontrivial gains from trade if the seller is good – due to hedging, say – which compensate for the danger of buying from a bad seller. It is therefore rational for these buyers to risk becoming the "greater fools" that bad sellers hope to sell to. That is, bad sellers create a lemons problem, since they cause buyers to trust good sellers less (Akerlof, 1970), but gains from trade are large enough to at least partially overcome this lemons problem. A strong bubble in the sense of Allen et al. (1993) is then a state of the world in which bad sellers hope they are facing these greater-fool buyers, but they are actually just facing other bad sellers.

One might conjecture that bursting these bubbles would improve welfare by reducing volatility faced by risk averse investors. However, this may not be the case. Bubbles exist here only if there are gains from trade in risky assets due, say, to hedging motives. Thus, reducing asset volatility may not improve investor welfare.[7]

Nevertheless, a policy of deflating overpriced assets does *affect* welfare. First, it affects the distribution of wealth between agents. In states of the world where overpriced assets are deflated, bad sellers are prevented from selling these overpriced assets. This benefits buyers at the expense of bad sellers. It also affects asset prices in *other* states, where buyers trust sellers more, and so, bid prices up. This benefits sellers at the expense of buyers. We call such effects "transfer effects."

Next, if assets are *produced* then policy can influence the allocation of production. For example, bubble bursting can reduce wasted resources in bubble states themselves. Also, since these policies affect the level of trust between buyers and sellers, they can affect the lemons problem created by the presence of bad sellers, and so, influence the volume of production and gains from trade in other states. Produced assets may include real estate (e.g., office space), where investors can generate additional units through construction. Equity may also be an example of a produced asset. Thus, the high equity prices of the late 1990's may have encouraged entrepreneurs to expand their firms more rapidly in anticipation of an IPO.[8]

To separate the transfer effect from the production-allocation effect, we consider two cases: a fixed endowments case and a production case. The fixed endowments case allows us to study the transfer effect in isolation. We can then study the effects of policy on production and the lemons problem in isolation from the transfer effect.

In the present model, the only policy tool capable of influencing asset prices is the release of information. This is because the discount rate is fixed (at zero), and the elasticity of demand for assets is infinite, so agents bid prices up to the certainty equivalent of expected future prices, regardless of supply. Open market operations, for example, would have no effect here, beyond the information revealed about central bank beliefs. We therefore simply represent the central bank's policy as the release of this information, and ignore other aspects of central bank policy.

We must then specify what the central bank knows. This paper assumes that the central bank only believes an asset is overpriced if it really is overpriced. Thus, the central bank is never wrong in believing an asset is overpriced. However, we assume that the central bank only knows an asset is overpriced if some *private* agents *also* know this. That is, the central bank is never the only one to know the asset is overpriced.

Within this context, we consider two extreme information structures for the central bank. In the first, the central bank is relatively smart in the sense that it can know an asset is overpriced even some private agents do *not* know this. If the central bank then deflates these overpriced assets, we call this a policy of "general deflation of overpriced assets." Note that this is not yet the bursting of an Allen et al. strong bubble, since some agents – buyers, say – may not know the asset is overpriced.

In this case, since the central bank may know more than some buyers, a policy of general deflation of overpriced assets can protect these buyers from bad sellers who know the asset is worthless. This makes it easier for *good* sellers, who believe the asset may be valuable, to sell the asset, and so, reduces the lemons problem.

This extreme case is contrasted to the opposite extreme, i.e., bubble bursting proper. In this case the central bank only knows an asset is overpriced if there is a strong bubble, so all private agents *also* know the asset is overpriced. That is, the central bank is no better informed about fundamentals than any private agent.

Thus, bubble bursting announcements reveal nothing to private agents about fundamentals. However, since the central bank only knows an asset is overpriced if everyone else does, the central bank's announcement tells bad sellers that all other agents are also bad sellers. A policy of bursting bubbles therefore protects these bad sellers from each other. Thus, in states where the central bank turns out *not* to announce a bubble, bad sellers become more confident of selling the asset, exacerbating the lemons problem faced by good sellers. This negative effect can outweigh the positive effect of preventing bad sellers from wasting resources in bubble states.

Thus, while bubbles may be a symptom of asymmetric information, which is a bad thing, eliminating this particular symptom may make the underlying problem worse.

These extreme cases should illuminate the major issues which would also arise in less extreme intermediate cases. Of course, other cases, such as where the central bank sometimes *wrongly* believes that an asset is overpriced, are also of interest. This is therefore an important topic for future research (see the conclusion).

In addition, this paper focuses only on the *microeconomic* aspects of bubble policy. Future work should study endogenous bubbles in a *macroeconomic* context, to shed light on their role in countercyclical policy (Bernanke and Gertler, 1999, 2001, Bordo and Jeanne, 2002, Cecchetti and coauthors, 2000, 2003).

Finally, this paper examines policy in the simplest possible models. For example, we limit our analysis to a three-period world, with a bubble only in period one. This makes the timing of policy very rigid. In particular, "bubble bursting" really means bubble *prevention* – i.e., prevention of the first-period bubble. Thus, we cannot study the effects of *delayed* policy actions. While our results should generalize, it is important to determine what other issues also arise in more complicated models.

The next section introduces the basic asset market model. Section II studies general deflation of overpriced assets while Section III considers bubble bursting proper. Section IV briefly discusses agent irrationality and Section V concludes.

I. PRELIMINARIES

This section presents the basic asset market model. The framework is similar to Milgrom and Stockey (1982), Allen et al. (1993) and Conlon (2004).

There are two risk neutral individuals in the market, Ellen and Frank, and a finite set of states of the world, Ω. A typical state of the world is $\omega \in \Omega$. We also use symbols such as b, e^B, f_2^G, etc., to denote states of the world. Ellen and Frank have a common prior probability distribution, $\pi(\omega)$, over Ω. In addition, Ellen and Frank have state-dependent marginal utilities, $MU_E(\omega)$ and $MU_F(\omega)$. We can think of $MU_E(\omega)$ and $MU_F(\omega)$ as differing because Ellen and Frank have different underlying future wealths in different states of the world, due, say, to risky future labor income. However, the marginal utility of wealth in each state is at least locally constant, independent of the outcome of trade in this market. That is, $MU_E(\omega)$ and $MU_F(\omega)$ depend only on ω, and not on the wealth obtained from this market. Thus, the utility function must be at least piecewise linear (see Allen et al., 1993).

Let $M_E(\omega) = MU_E(\omega)\pi(\omega)$ and $M_F(\omega) = MU_F(\omega)\pi(\omega)$ be shadow state prices indicating the *ex ante* value that Ellen and Frank attach to a unit of consumption in state ω. We condense sums like $M_E(\omega_1) + ... + M_E(\omega_k)$ as $M_E(\omega_1,\ ...,\ \omega_k)$ for short, and similarly for $M_F(\omega_1,\ ...,\ \omega_k)$.

The market lasts for three periods, denoted $t = 1,\ 2,\ 3$, but there is no discounting. There is a riskless asset (money), and a risky asset. A unit of the risky asset ultimately pays a single dividend of $d(\omega)$ in state ω.

This paper allows the risky asset to be a produced asset. For example, office buildings can be constructed and entrepreneurs can expand their firms in anticipation of an IPO. Thus, in certain states, $\omega \in \Omega_E$, Ellen can produce an amount of the asset, a, at cost $c(a)$. In other states she cannot produce. Similarly, Frank can only produce in states $\omega \in \Omega_F$, also at cost $c(a)$. All production occurs before period $t = 1$, but after the central bank makes any announcements. For the fixed endowment case, $c(a)$ is zero up to the endowment point, and infinity thereafter. The initial amounts of the risky asset, after production, are denoted by $a_0^E(\omega)$ and $a_0^F(\omega)$, for Ellen and Frank, respectively. Of course, $a_0^E(\omega) = 0$ for $\omega \notin \Omega_E$, and similarly for Frank. Ellen and Frank also begin with state-dependent endowments of money, $m_0^E(\omega)$ and $m_0^F(\omega)$.

Denote Ellen's and Frank's net sales of the risky asset in period t by $x_t^E(\omega)$ and $x_t^F(\omega)$. Thus, if $a_t^E(\omega)$ and $a_t^F(\omega)$ are their holdings of the risky asset at the end of period t, then $a_t^E(\omega) = a_{t-1}^E(\omega) - x_t^E(\omega)$ for Ellen, and similarly for Frank. In the same way, if $m_t^E(\omega)$ and $m_t^F(\omega)$ are Ellen's and Frank's money holdings at the end of period t, and $p_t(\omega)$ is the price of the risky asset in period t and state ω, then $m_t^E(\omega) = m_{t-1}^E(\omega) + p_t(\omega)x_t^E(\omega)$ for Ellen, and similarly for Frank. Assume that there are no short sales of the risky asset, so $a_t^E(\omega) \geq 0$ and $a_t^F(\omega) \geq 0$ for all ω and t.[9]

Assume that the price of the consumption good, in terms of money, is fixed at one. Since marginal utilities are locally constant, the overall expected payoff to Ellen, say, based on the value of her portfolio in period 3, is then

$$E\big[MU_E(\omega)[m_3^E(\omega) + a_3^E(\omega)d(\omega) - c(a_0^E(\omega))]\big], \tag{1}$$

where E is the expectation with respect to the prior π, and similarly for Frank.

The models below have rich information structures. As is common in such models, we represent agents' information using information partitions.[10] A *partition* of the set Ω is a set of subsets, S_i, of Ω, such that the subsets are all disjoint ($S_i \cap S_j = \emptyset$ for

$i \neq j$), but they cover Ω ($\cup_i S_i = \Omega$). The partition $\{S_i\}$ is an agent's *information partition* if the agent knows which subset, S_i, the true state is in, but she cannot distinguish between different elements of S_i. For example, if ω_1 is the actual state of the world, and $\omega_1 \in S_i$, then the agent knows that the state is in S_i, but she does not know whether the true state is ω_1 or some other state, ω_2, say, in S_i. The subsets, S_i, of an information partition are called "cells" or "information sets." These information partitions can represent rich information structures, as shown in the following example.

Example: Suppose $\Omega = \{\omega_1,\ \omega_2,\ \omega_3,\ \omega_4\}$, and assume that Ellen has an information partition $\{E_1,\ E_2\}$ with $E_1 = \{\omega_1,\ \omega_3\}$ and $E_2 = \{\omega_2,\ \omega_4\}$. This indicates that, if the true state of the world is ω_2, for example, then Ellen knows that either ω_2 or ω_4 is the true state, but she does not know which one.

Assume that Frank's information partition is $\{F_1,\ F_2\}$, where $F_1 = \{\omega_1,\ \omega_2,\ \omega_3\}$ and $F_2 = \{\omega_4\}$. Assume also that these information partitions are "common knowledge," in the sense that Ellen knows Frank's information partition, Frank knows Ellen's information partition, Ellen knows that Frank knows Ellen's information partition, and so on. Thus, each of their information *structures* is common knowledge, but they are not told the actual piece of information the other received. This sort of common knowledge assumption is standard in the literature.

Suppose the true state is ω_1. Then Ellen knows the state is one of ω_1 or ω_3. Thus, Ellen knows the state is in Frank's cell $F_1 = \{\omega_1,\ \omega_2,\ \omega_3\}$, so she knows that Frank thinks the state might be ω_2. Also, since Frank thinks the state might be ω_2, he incorrectly thinks that Ellen might think the state might be ω_4. Thus, Ellen knows the state is not ω_4, but she also knows that Frank thinks she *might* think the state might be ω_4. This type of "higher order" thinking is essential for greater fool bubble models (see Allen et al., 1993, Morris et al., 1995, Brunnermeier, 2001, or Conlon, 2004).

Ellen's and Frank's information partitions evolve over time as they get new information, with their underlying information partitions in period t given by $\mathbf{E}_t = \{E_{it}\}$ and $\mathbf{F}_t = \{F_{it}\}$. These partitions incorporate any previous information that Ellen or Frank have (e.g., from Ω_E, Ω_F, $m_0^E(\omega)$ and $m_0^E(\omega)$). They also become (at least weakly) more informative over time, so Ellen and Frank do not forget. Ellen and Frank can also learn from current and previous market prices. The partitions also incorporating this additional price information will be denoted by $\mathbf{E}_t^P = \{E_{it}^P\}$ and $\mathbf{F}_t^P = \{F_{it}^P\}$. Finally, assume that all information is revealed by period 3.

A competitive equilibrium in this market consists of a state-dependent pricing function, $p_t(\omega)$, and a pair of state-dependent net sales functions, $x_t^E(\omega)$ and $x_t^F(\omega)$, for Ellen and Frank, such that:

(i) $p_t(\omega)$ depends only on information possessed by Ellen or Frank at time t, i.e., on information in their underlying information partitions $\mathbf{E}_t$ and $\mathbf{F}_t$,

(ii) each agent's net trades depend only on information he/she actually possesses at the time of trade, so Ellen's (Frank's) net trades in period t depend on information in her (his) price-refined partition, $\mathbf{E}_t^P$ ($\mathbf{F}_t^P$),

(iii) the market clears, so $x_t^E(\omega) + x_t^F(\omega) = 0$, and

(iv) each agent's net trades are optimal, given his/her information, the set of state-dependent prices, the short-sales constraints, and his/her (correct) beliefs about the other's strategy rule.

Follow Allen et al. (1993) by saying that a *strong bubble* exists at a state, ω, if all agents know that the risky asset is overpriced for sure. Thus, if a strong bubble exists at state ω and time t, then Ellen, say, knows that the asset is overpriced, so $\omega \in E_{it}^P$ implies that, for all $\omega' \in E_{it}^P$, $p_t(\omega') > d(\omega')$. That is, if the state is ω, Ellen might not know that the state is ω, but she does know which cell, E_{it}^P, ω is in, and, for *every* state, ω' in E_{it}^P, the asset is overpriced. A similar condition must hold for Frank.

We next derive formulas for $p_t(\omega)$, $t = 1,\ 2,\ 3$. For $t = 3$, price equals the dividend, since both agents have complete information. This means that $p_3(\omega) = d(\omega)$.

To obtain $p_2(\omega)$, suppose a buyer, Ellen say, is considering buying one more unit of the risky asset at information set E^P_{i2} in period 2. Since E^P_{i2} incorporates price information, $p_2(\omega)$ will be constant on E^P_{i2}. Denote this constant by p_2. Then if Ellen buys this unit, her expected utility will change by

$$\Delta EU_E = \sum_{\omega' \in E^P_{i2}} MU_E(\omega')\pi(\omega')d(\omega') - \sum_{\omega' \in E^P_{i2}} MU_E(\omega')\pi(\omega')p_2. \tag{2}$$

This is the expected marginal utility, from dividends, of holding one more unit of the asset in period 3, minus the expected marginal utility cost of holding p_2 units less money, both at information set E^P_{i2}.

Ellen buys if $\Delta EU_E \geq 0$, but she has infinite demand if $\Delta EU_E > 0$. She therefore buys a positive finite amount only if $\Delta EU_E = 0$. This yields

$$p_2(\omega) = \frac{\sum_{\omega' \in E^P_{i2}} MU_E(\omega')\pi(\omega')d(\omega')}{\sum_{\omega' \in E^P_{i2}} MU_E(\omega')\pi(\omega')} = \frac{\sum_{\omega' \in E^P_{i2}} M_E(\omega')d(\omega')}{\sum_{\omega' \in E^P_{i2}} M_E(\omega')} \tag{3}$$

for all $\omega \in E^P_{i2}$. This is the equilibrium period 2 price if Ellen is buying, or more generally, if Ellen is not short-sale constrained. Similarly, if Ellen is not short-sale constrained in period 1 and information set E^P_{i1}, then

$$p_1(\omega) = \frac{\sum_{\omega' \in E^P_{i1}} M_E(\omega')p_2(\omega')}{\sum_{\omega' \in E^P_{i1}} M_E(\omega')} \text{ for all } \omega \in E^P_{i1}. \tag{4}$$

Similar formulas hold if Frank is not short-sale constrained.

Note that the elasticity of demand is infinite at the equilibrium price, so expected consumer surplus must be zero. Expected welfare therefore simply equals the appropriately weighted expected *producer* surplus.

II. GENERAL DEFLATION OF OVERPRICED ASSETS

This section presents a simple example of a bubble. It also examines a policy of "general deflation of overpriced assets," where, if any investors know an asset is overpriced, then, with probability λ, the central bank *also* knows this, and announces its information. This is not yet "bubble bursting," since some investors may *not* know the asset is overpriced. Thus, general deflation of overpriced assets can protect these uninformed investors from the informed investors, and so, tends to increase welfare. We first present the basic model and equilibrium, and then analyze policy.

A. *Basic Setup and Equilibrium*

We first give some intuition for the bubble model. There are two traders, Ellen and Frank. In some states of the world Ellen is a "bad seller" who wants to sell Frank an asset she knows is worthless. In other states Ellen is a "good seller," who believes that the asset may be valuable, but is willing to sell it to Frank because he is willing to pay more for it than she is. Frank is willing to buy the asset from Ellen, even though she might be a bad seller, because there are potential gains from trade if she is good, and Frank cannot distinguish between states where Ellen is a good versus a bad seller.

Symmetrically, there are certain states in which Frank is a good or bad seller, and Ellen is willing to buy in some of these states, since she cannot distinguish between states where Frank is good versus bad. Finally, in certain of the bad states, both know the asset is worthless, but each is willing to hold it in the (mistaken) belief that he/she will be able to sell it later. A strong bubble therefore exists in those states.

In certain of the bad states, the central bank also knows the asset is worthless. This subsection assumes that the central bank does not reveal its information, while the next subsection assumes the central bank announces these states if they occur.

Assume that there are twelve possible states of nature,

$$\Omega = \{b,\ b_{CB},\ e^B,\ e^B_{CB},\ f^B,\ f^B_{CB},\ e^G_1,\ e^G_2,\ e^G_3,\ f^G_1,\ f^G_2,\ f^G_3\}. \tag{5}$$

The letter b indicates a potential bubble state, while the letters e versus f indicate whether Ellen alone or Frank alone can produce the asset in that state. Superscripts

B versus G indicate whether the seller is bad (so he/she knows the asset is worthless) or good (so he/she thinks it might be valuable), and the subscript CB indicates that the central bank knows the asset is worthless in that state. More specifically, assume Ellen (Frank) can produce the asset in states $\omega \in \Omega_E$ ($\omega \in \Omega_F$), where

$$\Omega_E = \{b,\ b_{CB},\ e^B,\ e^B_{CB},\ e^G_1,\ e^G_2,\ e^G_3\},$$
$$\Omega_F = \{b,\ b_{CB},\ f^B,\ f^B_{CB},\ f^G_1,\ f^G_2,\ f^G_3\}, \tag{6}$$

and the central bank knows whether or not the state of the world is in

$$\Omega_{CB} = \{b_{CB},\ e^B_{CB},\ f^B_{CB}\}. \tag{7}$$

Assume the asset only pays a nonzero dividend in states e^G_3 and f^G_3, and this dividend is $d(e^G_3) = d(f^G_3) = d$. Note that the dividend is zero in Ω_{CB}. Thus, if the central bank learns that $\omega \in \Omega_{CB}$ and announces this, this information becomes common knowledge, and the price falls to zero. However, this subsection assumes that the central bank does *not* make any announcements.

For simplicity, assume symmetry in probabilities and marginal utilities. Thus, for probabilities assume $\pi(e^B) = \pi(f^B)$, $\pi(e^B_{CB}) = \pi(f^B_{CB})$, and $\pi(e^G_i) = \pi(f^G_i)$ for $i = 1,\ 2,\ 3$. Similarly, for marginal utilities assume symmetries such as $MU_E(b) = MU_F(b)$, $MU_E(e^B) = MU_F(f^B)$, $MU_E(f^G_i) = MU_F(e^G_i)$, and so on. These symmetries imply the following symmetries for the shadow state prices M_E and M_F:

$$M_E(b) = M_F(b),\ M_E(b_{CB}) = M_F(b_{CB}),\ M_E(e^B) = M_F(f^B),$$
$$M_E(f^B) = M_F(e^B),\ M_E(e^B_{CB}) = M_F(f^B_{CB}),\ M_E(f^B_{CB}) = M_F(e^B_{CB}), \tag{8}$$
$$M_E(e^G_i) = M_F(f^G_i),\ \text{and}\ M_E(f^G_i) = M_F(e^G_i),\ i = 1,\ 2,\ 3.$$

We next indicate what Ellen and Frank know, using their information partitions, $\mathbf{E}_t = \{E_{it}\}$ and $\mathbf{F}_t = \{F_{it}\}$. Let Ellen's underlying period 1 information partition be

$$E^B_{Seller} = \{b,\ b_{CB},\ e^B,\ e^B_{CB}\},\quad E^G_{Seller} = \{e^G_1,\ e^G_2,\ e^G_3\},$$

(9)

$$E_{Buyer} = \{f^B,\ f^B_{CB},\ f^G_1,\ f^G_2,\ f^G_3\},$$

while Frank's underlying information partition is

$$F^B_{Seller} = \{b,\ b_{CB},\ f^B,\ f^B_{CB}\},\ F^G_{Seller} = \{f^G_1,\ f^G_2,\ f^G_3\}$$

(10)

$$F_{Buyer} = \{e^B,\ e^B_{CB},\ e^G_1,\ e^G_2,\ e^G_3\}.$$

These partitions are illustrated in Figure 1.

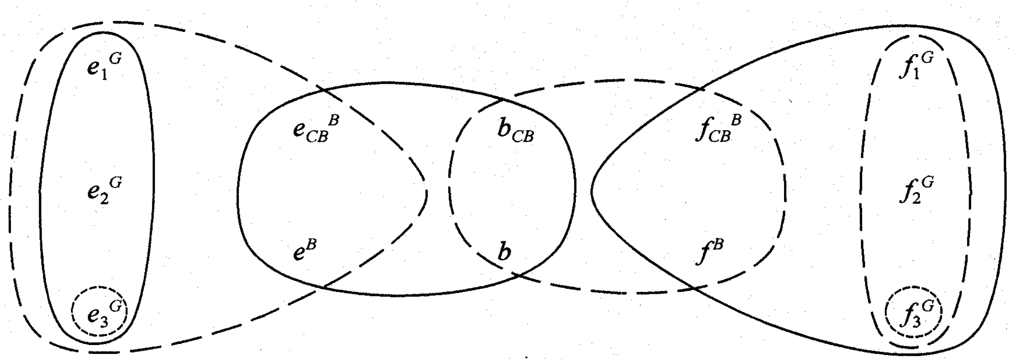

Figure 1: Information Partitions: Ellen's – Solid Lines, Frank's – Dashed Lines. Dividend Paying States – Dotted Lines.

Note that we left the time subscript $t = 1$ off of the information sets for simplicity. Note also that Ellen can produce the risky asset in the cells E^B_{Seller} and E^G_{Seller}, and Frank can produce it in the cells F^B_{Seller} and F^G_{Seller}. Thus, both agents know whether or not they can produce the risky asset.

The cell E^B_{Seller} will contain states where Ellen is a *bad seller*, who hopes to sell an asset she knows is worthless, and E^G_{Seller} will contain states where Ellen is a *good seller*, who hopes to sell an asset she thinks may be valuable, i.e., may pay a positive dividend. Also, note that Frank – in cell F_{Buyer} – cannot distinguish between the good states in E^G_{Seller}, and two bad states, e^B and e^B_{CB}, from E^B_{Seller}. This is why Frank may be willing to buy from Ellen in states e^B and e^B_{CB}, where she is bad. Similarly, Ellen may be willing to buy from Frank in states f^B and f^B_{CB}, where he is bad.

In period 2, both players learn the true state, ω, if it is b, b_{CB}, e^G_1, or f^G_1. Ellen's underlying information partition in period 2 is therefore:

(11)
$$E^0_{12} = \{b\},\ E^0_{22} = \{b_{CB}\},\ E^0_{32} = \{e^G_1\},\ E^0_{42} = \{f^G_1\}$$
$$E^B_{Seller2} = \{e^B,\ e^B_{CB}\},\ E^G_{Seller2} = \{e^G_2,\ e^G_3\},\ E_{Buyer2} = \{f^B,\ f^B_{CB},\ f^G_2,\ f^G_3\},$$

and Frank's underlying partition in period 2 is

(12)
$$F^0_{12} = \{b\},\ F^0_{22} = \{b_{CB}\},\ F^0_{32} = \{f^G_1\},\ F^0_{42} = \{e^G_1\}$$
$$F^B_{Seller2} = \{f^B,\ f^B_{CB}\},\ F^G_{Seller2} = \{f^G_2,\ f^G_3\},\ F_{Buyer2} = \{e^B,\ e^B_{CB},\ e^G_2,\ e^G_3\}.$$

Note that we include the time subscript $t = 2$. Also, in the cells E^0_{12}, E^0_{22}, E^0_{32}, E^0_{42}, F^0_{12}, F^0_{22}, F^0_{32}, and F^0_{42}, it is common knowledge that the asset is worthless, so the price will be zero. Thus, the only interesting cells are $E^B_{Seller2}$, $E^G_{Seller2}$, and E_{Buyer2} for Ellen, and $F^B_{Seller2}$, $F^G_{Seller2}$, and F_{Buyer2} for Frank (see Figure 2). In period 3 Ellen and Frank learn the true state no matter what it is.

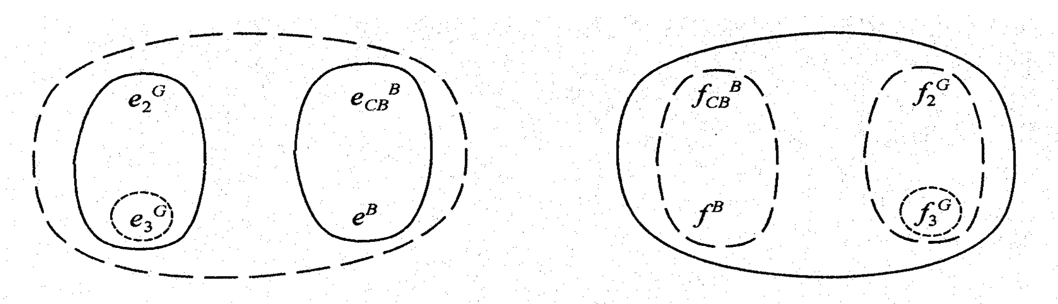

Figure 2: The Interesting Period 2 Information Sets: Ellen's – Solid Lines, Frank's – Dashed Lines. Dividend Paying States – Dotted Lines.

Recall that the asset only pays a nonzero dividend in states e_3^G and f_3^G. Thus, when Ellen observes the event $E^B_{Seller} = \{b,\ b_{CB},\ e^B,\ e^B_{CB}\}$, she knows that the asset is actually worthless, and when Frank observes the event $F^B_{Seller} = \{b,\ b_{CB},\ f^B,\ f^B_{CB}\}$, *he* knows that the asset is worthless. This implies that, in states b and b_{CB}, both Ellen and Frank know that the asset is worthless, though neither knows that the other knows. Thus, if the price of the asset is nevertheless positive in these states, this will represent a strong bubble in the sense of Allen et al. (1993).

We now construct an equilibrium with a strong bubble in states b and b_{CB}. Table 1 presents the general pattern of prices in this equilibrium. Proposition 1 determines equilibrium prices p_1 and p_2, and indicates the conditions needed to sustain this equilibrium. Throughout we focus primarily on the states b, b_{CB}, e^B, e^B_{CB} and the e_i^G, where Ellen hopes to sell to Frank. By symmetry (see (8)), the same results will automatically apply to states where Frank hopes to sell to Ellen.

TABLE 1: EQUILIBRIUM PRICES

ω	b	b_{CB}	e^B	e^B_{CB}	f^B	f^B_{CB}	e_1^G	e_2^G	e_3^G	f_1^G	f_2^G	f_3^G
$p_1(\omega)$	p_1	p_1	p_1	p_1	p_1	p_1	p_1	p_1	p_1	p_1	p_1	p_1
$p_2(\omega)$	0	0	p_2	p_2	p_2	p_2	0	p_2	p_2	0	p_2	p_2
$p_3(\omega)$	0	0	0	0	0	0	0	0	d	0	0	d

PROPOSITION 1: Suppose that M_E and M_F satisfy the three conditions

(13)
$$\frac{M_E(e_2^G,\ e_3^G)}{M_E(e_1^G,\ e_2^G,\ e_3^G)} = \frac{M_E(e^B,\ e^B_{CB})}{M_E(b,\ b_{CB},\ e^B,\ e^B_{CB})},$$

(14)
$$\frac{M_E(e_2^G,\ e_3^G)}{M_E(e_1^G,\ e_2^G,\ e_3^G)} \geq \frac{M_F(e^B,\ e^B_{CB},\ e_2^G,\ e_3^G)}{M_F(e^B,\ e^B_{CB},\ e_1^G,\ e_2^G,\ e_3^G)},$$

and

(15)
$$\frac{M_F(e_3^G)}{M_F(e^B,\ e^B_{CB},\ e_2^G,\ e_3^G)} \geq \frac{M_E(e_3^G)}{M_E(e_2^G,\ e_3^G)}.$$

Then the prices in Table 1 form an equilibrium, where p_1 and p_2 are given as

$$p_1 = \frac{M_E(e_2^G,\ e_3^G)}{M_E(e_1^G,\ e_2^G,\ e_3^G)} p_2 \tag{16}$$

and

$$p_2 = \frac{M_F(e_3^G)}{M_F(e^B,\ e_{CB}^B,\ e_2^G,\ e_3^G)} d. \tag{17}$$

Ellen and Frank produce a^* in Ω_E and Ω_F, respectively, where $a^* = a^*(p_1)$ satisfies $c'(a^*) = p_1$. In period 2, Ellen sells a^* to Frank in states e^B, e_{CB}^B, e_2^G, and e_3^G and Frank sells a^* to Ellen in states f^B, f_{CB}^B, f_2^G, and f_3^G.

PROOF: See Appendix A.

Note that conditions symmetrical to (13), (14) and (15) follow automatically from the symmetries in (8). These conditions have the following interpretations:

Condition (13) says that, in period 1, Ellen bids up the price, p_1, to the same levels at information sets $E_{Seller}^B = \{b,\ b_{CB},\ e^B,\ e_{CB}^B\}$, and $E_{Seller}^G = \{e_1^G,\ e_2^G,\ e_3^G\}$. This condition is necessary so that bad Ellen pools with good Ellen, i.e., Ellen's behavior in period 1 does not reveal to Frank whether she is good or bad. This requires a coincidence between different shadow state prices for Ellen, so this equilibrium is not robust to variations in model parameters (see Subsection 3A below).

Condition (14) says that future *seller* Ellen bids *first* period price p_1 up more at information set $E_{Seller}^G = \{e_1^G, e_2^G, e_3^G\}$ (and so, also at $E_{Seller}^B = \{b,\ b_{CB},\ e^B,\ e_{CB}^B\}$) than does future buyer Frank at his overlapping information set $F_{Buyer} = \{e^B, e_{CB}^B, e_1^G, e_2^G, e_3^G\}$. This requires Frank to put more weight on state e_1^G, where p_2 will fall to zero, than does good seller Ellen. Frank is then short-sale constrained in period 1 at F_{Buyer}, so his preferences do not affect the market price. Thus bad seller Ellen, at $E_{Seller}^B = \{b,\ b_{CB},\ e^B,\ e_{CB}^B\}$, cannot tell whether Frank is a buyer at F_{Buyer}, or a bad seller at $F_{Seller}^B = \{b,\ b_{CB},\ f^B,\ f_{CB}^B\}$.

Finally, condition (15) says that, in period 2, Frank, at information set $F_{Buyer2} = \{e^B,\ e^B_{CB},\ e^G_2,\ e^G_3\}$, is willing to buy from good Ellen, at her information set $E^G_{Seller2} = \{e^G_2,\ e^G_3\}$, even though Frank thinks that Ellen might be a bad seller at information set $E^B_{Seller2} = \{e^B,\ e^B_{CB}\}$. This requires there to be strictly positive gains from trade between Frank and good Ellen, which requires Frank to put more weight on the dividend-paying state, e^G_3, than does good Ellen, so $M_F(e^G_3)/M_F(e^G_2) > M_E(e^G_3)/M_E(e^G_2)$. For example, Frank might expect very low future labor income in the dividend paying state e^G_3, and so, may consider the asset to be a good hedge against his future labor income, while Ellen may consider the asset to be a bad hedge against *her* future labor income.

Thus, when these conditions are met, future sellers bid p_1 up to (16) in the first period, and buyers bid p_2 up to (17) in the second period, and purchase at that price.[11]

As discussed at the end of the next subsection, there is also a second nonbubble equilibrium, with price always zero in states b, b_{CB}, e^B, e^B_{CB}, f^B, and f^B_{CB}.

B. *Policy Analysis*

We now examine the welfare effects of a policy of "general deflation of overpriced assets." Recall that b_{CB}, e^B_{CB}, and f^B_{CB} are states where the central bank knows the asset is worthless, and assume now that the central bank announces whether ω is one of these states. The announcement is made before production, so central bank announcements – or their absence – can influence agents' production decisions. We analyze how this policy affects an agent's welfare in her different interim situations – good seller, bad seller, or buyer. We also analyze her *ex ante* expected utility, i.e., from the viewpoint of an agent who has not yet received any information, so she does not know whether she is a good seller, a bad seller, or a buyer.

Let $0 < \lambda < 1$ be a parameter indicating the probability that the central bank knows the asset is overpriced, if at least one private agent does. Specifically, let

(18) $\pi(b_{CB}) = \lambda\ \pi(b,\ b_{CB}),\ \ \pi(e^B_{CB}) = \lambda\ \pi(e^B,\ e^B_{CB}),\ \ \pi(f^B_{CB}) = \lambda\ \pi(f^B,\ f^B_{CB}).$

Also, to simplify the analysis, assume that the states b and b_{CB}, etc, are identical in terms of marginal utilities, so $MU_k(b) = MU_k(b_{CB})$, $MU_k(e^B) = MU_k(e^B_{CB})$, and $MU_k(f^B) = MU_k(f^B_{CB})$, for $k = E,\ F$. This, combined with (18), implies that

$$M_k(b_{CB}) = \lambda\ M_k(b,\ b_{CB}),\ \ M_k(e^B_{CB}) = \lambda\ M_k(e^B,\ e^B_{CB}),\ \text{and}$$
(19)
$$M_k(f^B_{CB}) = \lambda\ M_k(f^B,\ f^B_{CB}),\ \text{for}\ k = E,\ F.$$

Suppose the central bank follows a policy of deflating overpriced assets, so it announces whether or not the state is one of b_{CB}, e^B_{CB}, or f^B_{CB}. Suppose also that the public understands this policy. Then, if the central bank announces b_{CB}, e^B_{CB} or f^B_{CB}, it becomes common knowledge that the asset is worthless, and the price collapses.

Suppose, on the other hand, that the central bank turns out not to announce b_{CB}, e^B_{CB} or f^B_{CB}. This is equivalent to announcing that the true state is *not* b_{CB}, e^B_{CB} or f^B_{CB}. This reduces the probability of a bubble, where both know that the asset is worthless, by a factor of $1 - \lambda$, from $\pi(b,\ b_{CB})$ to $\pi(b)$. It also reduces, by the same factor, $1 - \lambda$, the probability that Ellen alone knows the asset is worthless, from $\pi(e^B,\ e^B_{CB})$ to $\pi(e^B)$, and similarly for Frank.

Thus, given that there is at least *one* bad seller who knows the asset is worthless, the central bank's information is not correlated with the *number* of bad sellers who know this. The next section considers the opposite extreme – bubble bursting – where the central bank's information is highly correlated with the number of bad sellers.

Since the central bank's information is uncorrelated with whether one versus two sellers are bad in this section, the central bank's announcement policy does not influence a bad seller's probability assessment of whether *other* agents are bad sellers. The central bank's policy therefore does not affect bad sellers' confidence levels, so bad sellers continue to pool with good sellers, and an equilibrium like the one in Proposition 1 continues to exist, as shown in Proposition 2.

PROPOSITION 2: If shadow state prices satisfy (19) for some λ between zero and one, then, under a policy of general deflation of overpriced assets, an equilibrium like that in Proposition 1 continues to exist, but with $p_1(\omega) = 0$ at $\omega = b_{CB}$, e^B_{CB} and f^B_{CB}, with $p_2(\omega) = 0$ at $\omega = e^B_{CB}$ and f^B_{CB}, and with (17) replaced by

$$p_2 = \frac{M_F(e^G_3)}{M_F(e^B,\ e^G_2,\ e^G_3)}d. \tag{17'}$$

PROOF: First, equations (13) and (19) above imply that

$$\frac{M_E(e^G_2,\ e^G_3)}{M_E(e^G_1,\ e^G_2,\ e^G_3)} = \frac{M_E(e^B)}{M_E(b,\ e^B)}, \tag{13'}$$

since (19) implies that the right hand side of (13′) equals the right hand side of (13). Bad sellers therefore continue to pool with good sellers. Also, the analogue of (14) continues to hold if e^B_{CB} is removed, since the right hand side becomes smaller. Similarly, the analogue of (15) continues to hold, since the left hand side becomes bigger. **QED**

Thus, a bubble equilibrium continues to exist in the presence of a policy of general deflation of overpriced assets. This policy rule has four major effects:

(a) In those states where an overpriced asset is deflated, producers do not waste resources producing the asset. This improves welfare.

(b) Bad sellers, who know the asset is worthless, cannot sell the asset if the central bank reveals it to be worthless. This hurts bad sellers, but helps buyers.

(c) In states where the central bank does *not* make a price deflating announcement, buyers become more confident that the asset is valuable, so they bid up p_2, so p_1 also rises. This hurts buyers but helps sellers.

(d) Effect (c) encourages production in states where the central bank does nothing.

Effects (b) and (c) are pure "transfer effects," while (a) and (d) influence production. For buyers, these effects must perfectly cancel, since their demand is infinitely elastic, so their expected consumer surplus remains constant at zero. For sellers, these effects may not cancel. The lower probability of selling worthless assets (effect (b)) hurts sellers, but the higher price (effect (c)) helps them. Thus, seller welfare could rise or fall.[12] However, suppose transfer effects (b) and (c) exactly cancel. Then we will show that the improved allocation of production (less output in bad-seller states, from effect (a), more output in remaining states, from effect (d)) helps welfare.

Thus, consider Ellen's expected utility. Since her elasticity of demand is infinite in all states where she buys, her expected consumer surplus is zero in those states, whether the policy is in effect or not.[13] Thus, her expected benefit from this market is her expected profit from states where she produces. Since, by (16), she is indifferent between selling her output in period 1 and holding it for period 2, we can imagine, when calculating her expected utility, that she sells her output in period 1. Also, her profit from states where she produces is $p_1 a^*(p_1) - c(a^*(p_1)) = \Pi(p_1)$.

We must compare her expected welfare with no deflation policy (NDP) to that with deflation policy (DP). Denote the value of p_1 in these two cases as p_1^{NDP} and p_1^{DP}, respectively. Then using (16), (17) and (17′),

$$\frac{p_1^{DP}}{p_1^{NDP}} = \frac{M_F(e^B,\ e^B_{CB},\ e^G_2,\ e^G_3)}{M_F(e^B,\ e^G_2,\ e^G_3)}, \tag{20}$$

so $p_1^{DP} > p_1^{NDP}$. This yields effect (c) above.

In the no-deflation-policy case, Ellen produces in states b, b_{CB}, e^B, e^B_{CB}, e^G_1, e^G_2, and e^G_3, so her expected welfare from this market is

$$EU_E^{NDP} = \Pi(p_1^{NDP}) M_E(b,\ b_{CB},\ e^B,\ e^B_{CB},\ e^G_1,\ e^G_2,\ e^G_3). \tag{21}$$

On the other hand, in the deflation-policy case, she produces only in states b, e^B, e^G_1,

e_2^G, and e_3^G, so her expected welfare becomes

$$EU_E^{DP} = \Pi(p_1^{DP}) M_E(b,\ e^B,\ e_1^G,\ e_2^G,\ e_3^G). \tag{22}$$

To isolate transfer effects (b) and (c), consider first the case of a fixed endowment e, so $\Pi(p_1) = ep_1$. Proposition 3 determines the welfare effect of policy in this case.

PROPOSITION 3: In the fixed endowment case, general deflation of overpriced assets increases agents' expected welfare if and only if

$$\frac{M_E(e_{CB}^B)}{M_E(e^B,\ e_2^G,\ e_3^G)} < \frac{M_F(e_{CB}^B)}{M_F(e^B,\ e_2^G,\ e_3^G)}. \tag{23}$$

PROOF: See Appendix B.

Proposition 3 says that, for the fixed endowment case, the welfare of Ellen, say, will increase with a policy of general deflation of overpriced assets if Ellen attaches less weight to $\{e_{CB}^B\}$ relative to $\{e^B,\ e_2^G,\ e_3^G\}$ than does Frank. Of course, in the opposite case the central bank's information revelation hurts expected welfare.[14]

This may be understood as follows. First, in the fixed endowment case, effects (a) and (d) above go away, so just effects (b) and (c) remain. The policy then hurts Ellen but helps Frank in state e_{CB}^B, since it prevents Ellen from selling Frank a worthless asset in that state. On the other hand, the policy helps Ellen but hurts Frank by raising the price in states e^B, e_2^G and e_3^G. For Frank, these effects must cancel, to keep his overall consumer surplus constant at zero. Thus, Ellen benefits on average if, as in (23), she puts less weight, relative to Frank, on state e_{CB}^B, where she suffers, than on states e^B, e_2^G and e_3^G, where she benefits. That is, if (23) holds, policy transfers wealth from Ellen to Frank when Frank's marginal utility of consumption is relatively high, and from Frank to Ellen when Ellen's marginal utility of consumption is relatively high.

Finally, conditions (13) through (15) in Proposition 1 do not force inequality (23) to go either way, since they say nothing about how much weight Ellen puts on her good states, e_2^G and e_3^G, relative to her bad state, e_{CB}^B.

Next take as a baseline the case where shadow state prices are chosen to eliminate transfer effects, so policy has no effect on expected welfare under fixed endowments. This is analogous to the standard practice of ignoring pure lump sum transfers in consumer/producer surplus analysis. In this case, (21) equals (22) for $\Pi(p_1) = ep_1$, so

$$\text{(24)} \qquad \textit{Baseline Case}: \ \frac{p_1^{DP}}{p_1^{NDP}} = \frac{M_E(b,\ b_{CB},\ e^B,\ e^B_{CB},\ e^G_1,\ e^G_2,\ e^G_3)}{M_E(b,\ e^B,\ e^G_1,\ e^G_2,\ e^G_3)}.$$

Suppose (24) holds. Then, in the case where production is possible, a policy of general deflation of overpriced assets improves welfare, as shown in Proposition 4.

PROPOSITION 4: Suppose shadow state prices are such that a policy of general deflation of overpriced assets does not affect overall expected welfare in the fixed endowment case. Then if production is possible, so the supply curve $a^*(p_1)$ is upward sloping, a policy of general deflation of overpriced assets will increase welfare.

PROOF: First, by Hotelling's Lemma, $\Pi'(p_1) = a^*(p_1)$ so, since $a^*(p_1)$ is increasing in p_1, the function $\Pi(p_1)$ is strictly convex. Also, $\Pi(0) = 0$. Thus, since $p_1^{DP} > p_1^{NDP}$, it follows that $\Pi(p_1^{DP})/\Pi(p_1^{NDP}) > p_1^{DP}/p_1^{NDP}$. From this and (24) it follows that (22) is bigger than (21). **QED**

Intuitively, if (24) holds, so there is no transfer effect, then the only remaining welfare consequence of the price-deflation policy is better production decisions. This improvement has two aspects. First, since the central bank sometimes reveals when the asset is worthless, bad sellers waste less resources producing worthless assets (effect (a)). Second, in those states where the central bank makes *no* announcement, the lemons problem is reduced, so producers can produce more confidently (effect (d)).

Up to now, we have focused on the case where the policy rule shifts the economy from one bubble equilibrium to another. However, a nonbubble equilibrium also continues to exist in the presence of the policy, with price equal to zero in states b, b_{CB}, e^B, e^B_{CB}, f^B, and f^B_{CB}. While we cannot determine which equilibrium will prevail, the above framework allows us to analyze the effect of the policy in each case, regardless of whether the policy shifts the market into or out of a bubble equilibrium. This is because a nonbubble equilibrium is identical to the equilibrium in Proposition 2, but with $\lambda = 1$ (so the central bank always deflates an asset someone knows is overpriced). For example, if the policy shifts the market from a bubble to a nonbubble equilibrium, the effect is as in Propositions 2 through 4, but with $\lambda = 1$, so the effect is stronger. If the policy shifts the market from a *nonbubble* to a *bubble* equilibrium, the effect is *reversed* – like *reducing* λ from $\lambda_0 = 1$ to some $\lambda_1 < 1$, and so on.

Of course, the fact that policy may shift the economy between bubble and nonbubble equilibria is important, since it may help to explain why policy sometimes has such unpredictable effects on asset markets.

In any case, unless a policy of general deflation of overpriced assets causes the economy to shift from a nonbubble to a bubble equilibrium, it improves production allocation decisions. This policy will therefore be beneficial overall unless the transfer effect is negative. By contrast, a *bubble bursting* policy is likely to *worsen* production allocation decisions, as shown next.

III. BURSTING BUBBLES AND THE LEMONS PROBLEM

A. *Comment on Robustness*

The above equilibria were not robust since they required the coincidence (13). However, this is not an inescapable problem in this kind of bubble model. Instead, it is simply the price we pay for the convenience of a finite state space.

The coincidence in (13) causes the bad seller to pool with the good seller, and slight variations in parameters can break this coincidence. However, if we allow for a continuum of different types of good and bad sellers, then it becomes possible for each type of bad seller to pool with *some* type of good seller, even if the parameters of the model vary. Details are available upon request.

This is relevant for the bubble bursting policies considered below, since these policies would break the coincidence in (13). However, we do not want the effect of the policy to be driven solely by the lack of robustness of our finite state space model. Thus, we must increase the robustness of the model. At the same time, we want to keep the modified model as simple as possible. Therefore, instead of introducing a continuum of types, we consider a finite state extension that is only slightly more complicated than the previous model. Specifically, we still assume one type of bad seller, but allow for *two* types of good seller, a low-confidence type and a high-confidence type.

It then turns out that a bubble bursting policy rule increases the confidence of the bad seller. We therefore choose parameters such that the bad seller pools with the low-confidence good seller in the absence of the policy, but with the high-confidence good seller if the policy rule is in place.

B. *Basic Setup and Equilibrium*

The previous section examined a policy where the central bank deflates asset prices when some agents know the asset is worthless, even if others do not. However, central banks may know less about fundamentals than *all* private agents in the economy. For example, the central bank may only learn that an asset is overpriced in states where all other agents already know this, so the asset is in an Allen et al. (1993) strong bubble.

The policy then becomes one of *bursting bubbles.* This section shows that such a policy protects bad sellers from each other, so they can more confidently exploit buyers. For consider a bad seller who knows the asset is worthless. She also knows that, if the asset is in a bubble, then the central bank might announce this. Thus, if the central bank makes no announcement, she becomes more confident that she is *not* in a bubble. That is, she becomes more confident that some other agent does not know the asset is worthless, so she can sell him the asset. She therefore more closely mimics those among the good sellers who are confident that the asset is valuable. This exacerbates the lemons problem faced by the more confident of the good sellers, which distorts production decisions. In short, while *general* deflation of overpriced assets tends to improve production decisions, a *bubble bursting* policy may *hurt* production decisions.

This analysis requires a modification of the above bubble model. As explained in the previous subsection, we need to posit two different types of good seller, with two different confidence levels. Bad sellers can then pool with low-confidence good sellers if there is no bubble-bursting policy, and pool with high-confidence good sellers if there is a bubble bursting policy. If bad sellers could not pool with good sellers, asset prices would collapse, and there would be no bubble.

Let the two confidence levels for the good types of seller be L, for low confidence, and H, for high confidence, and let the states of the world be

$$\begin{aligned}\Omega = \{b,\ b_{CB},\ e^B,\ f^B,\ e^G_{1L},\ e^G_{2L},\ e^G_{3L},\ f^G_{1L},\ f^G_{2L},\ f^G_{3L},& \\ e^G_{1H},\ e^G_{2H},\ e^G_{3H},\ f^G_{1H},\ f^G_{2H},\ f^G_{3H}\}.&\end{aligned} \tag{25}$$

The asset pays dividend $d(\omega) = d$ in states $\omega = e^G_{3L},\ f^G_{3L},\ e^G_{3H}$, and f^G_{3H}, and zero otherwise. Ellen can produce quantity a of the asset, at cost $c(a)$, in the states b, b_{CB}, e^B, and the e^G_{iI} states, $i = 1,\ 2,\ 3,\ I = L,\ H$, and symmetrically for Frank. Any announcements again occur before production, so central bank announcements, or lack thereof, can influence agents' production decisions.

Suppose that, in the first period, and prior to the central bank announcement, Ellen has four information sets:

$$E^B_{Seller} = \{b,\ b_{CB},\ e^B\},\ E^{LG}_{Seller} = \{e^G_{1L},\ e^G_{2L},\ e^G_{3L}\},\ E^{HG}_{Seller} = \{e^G_{1H},\ e^G_{2H},\ e^G_{3H}\}$$
$$E_{Buyer} = \{f^B,\ f^G_{1L},\ f^G_{2L},\ f^G_{3L},\ f^G_{1H},\ f^G_{2H},\ f^G_{3H}\},$$

and symmetrically for Frank. Here E^B_{Seller} is Ellen's "bad seller" information set, E^{LG}_{Seller} her "low-confidence good seller" information set, E^{HG}_{Seller} her "high-confidence good seller" information set, and E_{Buyer} her "buyer" information set. In period 2, the states b, b_{CB}, e^G_{1L}, e^G_{1H}, f^G_{1L}, and f^G_{1H} are revealed to all players. In period 3, all information is revealed and any dividends are paid.

Below, "high confidence" will mean that

$$\frac{M_E(e^G_{2H},\ e^G_{3H})}{M_E(e^G_{1H},\ e^G_{2H},\ e^G_{3H})} > \frac{M_E(e^G_{2L},\ e^G_{3L})}{M_E(e^G_{1L},\ e^G_{2L},\ e^G_{3L})}. \tag{26}$$

This means that, in the *first* period, high-confidence good sellers attach greater weight to states with positive *second* period price than do low-confidence good sellers.

As in Section II, assume symmetry between Ellen and Frank, so $M_E(b) = M_F(b)$, $M_E(b_{CB}) = M_F(b_{CB})$, $M_E(e^B) = M_F(f^B)$, $M_E(f^B) = M_F(e^B)$, and, for $i = 1, 2, 3$, and $I = L, H$, $M_E(e^G_{iI}) = M_F(f^G_{iI})$, and $M_E(f^G_{iI}) = M_F(e^G_{iI})$. We can therefore focus on the states where Ellen can produce and sell:

$$\Omega_E = \{b,\ b_{CB},\ e^B,\ e^G_{1L},\ e^G_{2L},\ e^G_{3L},\ e^G_{1H},\ e^G_{2H},\ e^G_{3H}\}. \tag{27}$$

We want the model to have nice equilibria whether or not the central bank announces b_{CB}. Specifically, we want a bubble equilibrium to exist where bad sellers pool with some type of good seller, whether or not a bubble-bursting rule is in effect. The equilibrium structure that works is presented in Tables 2 and 3. Table 2 presents the structure of equilibrium prices without a bubble bursting policy (NP), and Table 3 presents the structure with a bubble bursting policy (BP).

TABLE 2: EQUILIBRIUM PRICES – NO BUBBLE BURSTING POLICY

State	b	b_{CB}	e^B	e^G_{1L}	e^G_{2L}	e^G_{3L}	e^G_{1H}	e^G_{2H}	e^G_{3H}	f^B	f^G_{1L}	f^G_{2L}	f^G_{3L}	f^G_{1H}	f^G_{2H}	f^G_{3H}
$t = 1$	p^{NP}_{1L}	p^{NP}_{1L}	p^{NP}_{1L}	p^{NP}_{1L}	p^{NP}_{1L}	p^{NP}_{1L}	p^{NP}_{1H}	p^{NP}_{1H}	p^{NP}_{1H}	p^{NP}_{1L}	p^{NP}_{1L}	p^{NP}_{1L}	p^{NP}_{1L}	p^{NP}_{1H}	p^{NP}_{1H}	p^{NP}_{1H}
$t = 2$	0	0	p^{NP}_{2L}	0	p^{NP}_{2L}	p^{NP}_{2L}	0	p^{NP}_{2H}	p^{NP}_{2H}	p^{NP}_{2L}	0	p^{NP}_{2L}	p^{NP}_{2L}	0	p^{NP}_{2H}	p^{NP}_{2H}
$t = 3$	0	0	0	0	0	d	0	0	d	0	0	0	d	0	0	d

TABLE 3: EQUILIBRIUM PRICES – BUBBLE BURSTING POLICY

State	b	b_{CB}	e^B	e^G_{1L}	e^G_{2L}	e^G_{3L}	e^G_{1H}	e^G_{2H}	e^G_{3H}	f^B	f^G_{1L}	f^G_{2L}	f^G_{3L}	f^G_{1H}	f^G_{2H}	f^G_{3H}
$t=1$	p^{BP}_{1H}	0	p^{BP}_{1H}	p^{BP}_{1L}	p^{BP}_{1L}	p^{BP}_{1L}	p^{BP}_{1H}	p^{BP}_{1H}	p^{BP}_{1H}	p^{BP}_{1H}	p^{BP}_{1L}	p^{BP}_{1L}	p^{BP}_{1L}	p^{BP}_{1H}	p^{BP}_{1H}	p^{BP}_{1H}
$t=2$	0	0	p^{BP}_{2H}	0	p^{BP}_{2L}	p^{BP}_{2L}	0	p^{BP}_{2H}	p^{BP}_{2H}	p^{BP}_{2H}	0	p^{BP}_{2L}	p^{BP}_{2L}	0	p^{BP}_{2H}	p^{BP}_{2H}
$t=3$	0	0	0	0	0	d	0	0	d	0	0	0	d	0	0	d

Note that the main difference between Tables 2 and 3 is in the b, b_{CB}, e^B, and f^B columns. Specifically, bubble bursting reduces the price to zero in state b_{CB}, and causes bad sellers, in states b, e^B, and f^B, to switch from pooling with low-confidence types (so $p_t = p^{NP}_{tL}$), to pooling with high-confidence types (so $p_t = p^{BP}_{tH}$). Note also that, if $p^{NP}_{1L} \neq p^{NP}_{1H}$ and $p^{BP}_{1L} \neq p^{BP}_{1H}$, then the buyer, by observing $p_1(\omega)$, can figure out whether the seller, if good, has high or low confidence, so prices reveal information about seller confidence. The prices themselves, and the conditions for these prices to be an equilibrium, are given in Proposition 5.

PROPOSITION 5: Suppose the following conditions, analogous to conditions (13) through (15) in Proposition 1, are met:

$$(13_L) \qquad \frac{M_E(e^G_{2L},\ e^G_{3L})}{M_E(e^G_{1L},\ e^G_{2L},\ e^G_{3L})} = \frac{M_E(e^B)}{M_E(b,\ b_{CB},\ e^B)},$$

$$(13_H) \qquad \frac{M_E(e^G_{2H},\ e^G_{3H})}{M_E(e^G_{1H},\ e^G_{2H},\ e^G_{3H})} = \frac{M_E(e^B)}{M_E(b,\ e^B)},$$

$$(14') \qquad \frac{M_E(e^G_{2I},\ e^G_{3I})}{M_E(e^G_{1I},\ e^G_{2I},\ e^G_{3I})} \geq \frac{M_F(e^B,\ e^G_{2I},\ e^G_{3I})}{M_F(e^B,\ e^G_{1I},\ e^G_{2I},\ e^G_{3I})},$$

with $I = L,\ H$, and

$$(15') \qquad \frac{M_F(e^G_{3I})}{M_F(e^B,\ e^G_{2I},\ e^G_{3I})} \geq \frac{M_E(e^G_{3I})}{M_E(e^G_{2I},\ e^G_{3I})},$$

with $I = L,\ H$. Then, if the central bank does *not* follow a bubble-bursting policy, an equilibrium exists with prices as in Table 2, where

$$(16') \qquad p_{1I}^{NP} = \frac{M_E(e_{2I}^G,\ e_{3I}^G)}{M_E(e_{1I}^G,\ e_{2I}^G,\ e_{3I}^G)} p_{2I}^{NP}, \quad I = L,\ H,$$

$$(17_{NP}) \qquad p_{2L}^{NP} = \frac{M_F(e_{3L}^G)d}{M_F(e^B,\ e_{2L}^G,\ e_{3L}^G)} \quad \text{and} \quad p_{2H}^{NP} = \frac{M_F(e_{3H}^G)d}{M_F(e_{2H}^G,\ e_{3H}^G)},$$

and where we must also assume that (16′) and (17_{NP}) yield $p_{1L}^{NP} \neq p_{1H}^{NP}$, so prices reveal information about good seller confidence.

If the central bank *does* follow a bubble bursting policy, then prices are as in Table 3, with p_{1I}^{BP}, $I = L,\ H$, given by (16′), but with p_{tI}^{NP} replaced by p_{tI}^{BP}, with

$$(17_{BP}) \qquad p_{2L}^{BP} = \frac{M_F(e_{3L}^G)d}{M_F(e_{2L}^G,\ e_{3L}^G)} \quad \text{and} \quad p_{2H}^{BP} = \frac{M_F(e_{3H}^G)d}{M_F(e^B,\ e_{2H}^G,\ e_{3H}^G)},$$

and where again we assume that (16′) and (17_{BP}) yield $p_{1L}^{BP} \neq p_{1H}^{BP}$.

Finally, Ellen produces $a^*(p_1(\omega))$ (where $a^*(p_1)$ satisfies $c'(a^*(p_1)) = p_1$), in states $\omega = b,\ b_{CB},\ e^B$, and the e_{iI}^G states, $i = 1,\ 2,\ 3,\ I = L,\ H$, and symmetrically for Frank, where $p_1(\omega)$ is given in Table 2 in the no bubble bursting case, and Table 3 in the bubble bursting case. In period 2, Ellen sells her output to Frank in states ω where she produced and $p_2(\omega)$ is positive, and symmetrically for Frank selling to Ellen.

PROOF: Similar to the proof of Proposition 1.

Proposition 5 says that, given conditions (13_L), (13_H), (14′), and (15′), bad sellers pool with low-confidence good sellers in the absence of a bubble bursting policy, but, in the presence of a bubble-bursting policy, pool with high-confidence good sellers if no announcement is actually made. Thus, bubble bursting policies tend to lead bad sellers to pool with the more confident of the good sellers.

Condition (13_L) is analogous to condition (13) of Proposition 1, except here the bad seller is pooling with the low-confidence good seller when the bad seller thinks the state might be b_{CB}. Condition (13_H) is analogous to (13) except that here the bad seller pools with the *high*-confidence good seller when the bad seller is sure that the state is *not* b_{CB}. Comparing the right hand sides of (13_L) and (13_H) shows that the weight attached to selling in the second period implied by (13_H) is higher than the weight implied by (13_L), which is consistent with (26). Thus, (13_H) and (13_L) really do represent confidence levels of high and low-confidence types, respectively.

Condition (14′) is analogous to (14) in Proposition 1. It says that good sellers of each confidence level, $I = L,\ H$ (and so, bad sellers who pool with them), bid up first period price beyond what the buyers are willing to pay, given that buyers do not know whether sellers are good or bad but can figure out the confidence level of sellers if good (by observing the price). This condition is needed so that the bad sellers, in period 1, cannot figure out whether they are facing buyers or other bad sellers.

Condition (15′) is analogous to (15) in Proposition 1. It says that the buyer, knowing the confidence level of the seller if good, is willing to pay more than the good seller in period 2, even if the buyer believes that the seller might be bad. This assures that the buyer really does buy from the good seller, even when the bad seller is pooling with the good seller. Of course, it follows from this that the buyer is even more willing to buy from the good seller when he is sure that the seller is *not* bad.

C. *Welfare Analysis: Bursting of Actual Bubbles*

To examine the welfare effects of policy, remember that, since the elasticity of demand is infinite, consumer surplus is zero, so welfare from the market equals expected producer surplus. Also, as before, sellers are indifferent between selling in periods 1 or 2, so for the purpose of calculating expected welfare, we can imagine that they sell in period 1. Thus, in the absence of a bubble-bursting policy, Ellen's *ex ante* expected welfare, averaging over all her information sets, is

$$(28) \qquad \Pi(p_{1L}^{NP})\ M_E(b,\ b_{BC},\ e^B,\ e_{1L}^G,\ e_{2L}^G,\ e_{3L}^G)\ +\ \Pi(p_{1H}^{NP})\ M_E(e_{1H}^G,\ e_{2H}^G,\ e_{3H}^G).$$

Similarly, if the central bank follows a bubble-bursting rule, Ellen's welfare will be

$$\Pi(p_{1L}^{BP})\ M_E(e_{1L}^G,\ e_{2L}^G,\ e_{3L}^G)\ +\ \Pi(p_{1H}^{BP})\ M_E(b,\ e^B,\ e_{1H}^G,\ e_{2H}^G,\ e_{3H}^G). \tag{29}$$

These are *ex ante* expected utilities, from Ellen's point of view, before she knows her own type, etc. We can break this up into contributions to expected utility through her bad seller type, her low-confidence good seller type, and her high-confidence good seller type. Proposition 6 treats the welfare contribution through her bad seller type, while Propositions 7 and 8 treat the contributions through her two good seller types.

PROPOSITION 6: In the fixed endowment case, the bubble bursting policy will have no effect on the expected welfare of bad sellers if and only if policy does not affect the actual second period sales price received by bad sellers, so $p_{2L}^{NP} = p_{2H}^{BP}$. In this case, the bubble bursting policy *helps* bad sellers when production is possible.

PROOF: See Appendix C.

Thus, if the transfer effect alone does not hurt bad sellers, then a bubble-bursting policy helps them. This is because the central bank bursts bubbles in states where bad sellers would not be able to sell anyway. The policy therefore does not affect the *ex ante* probability that bad sellers actually sell their assets. It only gives them some information, before production, about whether they will be able to sell the asset. Thus, if the policy does not affect the actual sale price, its only effect is to allow bad sellers to make better informed production decisions.

We now turn to the effect on high and low-confidence types of *good* seller. Since the bubble bursting policy causes bad types of seller to pool with high, rather than low-confidence types of good seller, bubble bursting helps low-confidence good sellers, but hurts high-confidence good sellers, as shown in the following Proposition:

PROPOSITION 7: A bubble-bursting policy helps low-confidence types of good seller, but hurts high-confidence types of good seller.

PROOF: Obvious, since $p_{1L}^{BP} > p_{1L}^{NP}$ and $p_{1H}^{BP} < p_{1H}^{NP}$ (compare (17_{BP}) to (17_{NP}) and use (16′)). **QED**

Proposition 7 raises the question of which of the two effects – higher welfare for low-confidence good sellers, or lower welfare for high-confidence good sellers – dominates. Of course, if agents put much more weight on their high-confidence types $(M_E(e_{1H}^G,\ e_{2H}^G,\ e_{3H}^G) >> M_E(e_{1L}^G,\ e_{2L}^G,\ e_{3L}^G))$, then a bubble-bursting policy will hurt good sellers on average, and visa versa. The transfer effect is therefore straightforward.

To focus on production distortions, we choose weights so that the transfer effects on high and low-confidence good sellers perfectly cancel in the fixed endowment case. When we do this, we find that the negative production effects for high-confidence good sellers tend to dominate when production is possible.

PROPOSITION 8: Suppose the shadow prices, $M_E(\omega)$ and $M_F(\omega)$, are such that, in the fixed endowment case, overall expected welfare of good sellers is unaffected by a bubble bursting policy. Suppose also that the greater confidence of the high-confidence good sellers is sufficient so that

(30) $$p_{1H}^{BP} > p_{1L}^{NP} \text{ and } p_{1H}^{NP} > p_{1L}^{BP}.$$

Then, in the production case, the negative effect of the bubble bursting policy on high-confidence types dominates the positive effect of the policy on low-confidence types, so the overall expected effect of the bubble bursting policy on good sellers is negative.

PROOF: See Appendix D.

The first half of (30) compares p_1 for high-confidence good sellers to p_1 for low-confidence good sellers, given that, in both cases, the bad seller is pooling with the good seller. Similarly, the second half of (30) compares p_1 for high-confidence versus low-confidence good sellers, given that the bad seller is *not* pooling with the good seller. Thus, both halves state that, holding all else equal in the appropriate sense, high-confidence good sellers bid p_1 up higher than low-confidence good sellers, which makes sense.

Combining Propositions 6 and 8, a bubble bursting policy tends to improve production decisions for bad sellers, but distort production decisions for good sellers, by shifting the lemons problem from low to high-confidence good sellers. Intuitively, a bubble is a situation where bad sellers are hurting other bad sellers, and this interferes with their ability to exploit buyers. Thus, a bubble bursting policy, by protecting bad sellers from each *other*, allows them to more confidently exploit buyers. This exacerbates the lemons problem faced by the more confident of the good sellers. While the overall effect is ambiguous, one can question the value of a policy whose main benefit is to help bad sellers to more efficiently exploit uninformed buyers.

In summary, bubbles tend to exist in environments of asymmetric information. This asymmetric information hurts welfare by creating a lemons problem. However, the most extreme *symptom* of this asymmetric information – the bubble – does not, itself, necessarily hurt welfare. Thus, curing the symptom may make the underlying problem worse.

IV. WELFARE WITH IRRATIONAL INVESTORS

The above assumed that all agents were rational. However, the analysis can easily be extended to certain types of irrationality. In particular, while trade in the above model was induced by a hedging motive, it may be more plausible to assume that trade is driven by overconfidence, as in Scheinkman and Xiong (2003). In this case, the shadow prices $M_E(\omega)$ and $M_F(\omega)$ may differ, not because marginal utilities differ, but because probabilities differ, as in Allen et al. (1993) and Conlon (2004).

However, if agents are irrational, it is not clear how one should measure welfare. One could measure welfare according to the policy maker's supposedly true model, yielding what we might call (following De Long et al., 1989, p. 690), a "paternalistic" approach to welfare analysis. Alternatively, one could measure the welfare of each agent according to that agent's own possibly mistaken model.

The present paper is more in line with this second approach. Indeed, if shadow prices differ because probability beliefs differ, then Pareto improvements in the present framework become Pareto improvements with each agent's welfare evaluated using that agent's own probabilities, rather than any true probabilities. If policy is evaluated from this point of view, then all the above results go through exactly as before.

Of course, if one prefers to measure welfare according to the policy maker's probabilities, then there might be a stronger argument for protecting buyers from bad sellers. On the other hand, if investors are overconfident, then lemons problems may be a *good* thing, since they may make buyers less confident, and so, may reduce mispricing caused by overconfidence. In any case, one cannot understand the effects of asset deflation policies without tracing the effects of these policies on the lemons problem.

V. CONCLUSION AND POSSIBLE EXTENSIONS

This paper analyzes bubble-bursting policy in the context of an Allen et al. (1993) greater fool bubble model. This allows us to study greater fool bubbles in a framework with rational agents, which, in turn, allows us to separate the role of policy as responding to distorted incentives from the role of policy as protecting agents from their own irrationality. Of course, the case of agent irrationality is also important.

The main lesson of this study is that, while asymmetric information is bad (since it creates a lemons problem), and asymmetric information tends to create bubbles, bubbles themselves are not necessarily bad. Thus, a policy which reduces the adverse effects of asymmetric information on uninformed buyers, such as general deflation of overpriced assets, may be a good thing, but a policy which protects bad sellers from each other, such as the bursting of actual *bubbles*, may be harmful.

Since this paper provides only a first look at policy in asymmetric information bubble models, it is obviously incomplete in important ways. One obvious extension would be to consider what happens if the central bank mistakenly tries to deflate the price of an asset which is not, in fact, overpriced. Presumably the effect of the policy would depend on how much private agents trust the central bank's judgment.

Second, it would be useful to incorporate a richer model of the monetary policy instrument. This would potentially include mechanisms other than central bank announcements, such as an effect through open market operations. Of course, given the importance of asymmetric information in the present framework, information will be an important part of any transmission mechanism.

A third extension would be to examine a model with more periods. This would allow us to analyze delayed policy, and the effects of expected future announcements on current resource allocation. For example, in the above framework, any bubbles already exist in the first period, and the central bank either does or does not deflate overpriced assets right away. In contrast, one could imagine a model where, as prices gradually rise, more and more investors come to realize that the asset is overpriced, as in Abreu and Brunnermeier (2003). In this case, early action by the central bank would deflate prices before all agents knew they were overpriced, as in general deflation of overpriced assets, while delayed action would mean the bursting of an actual strong bubble. It would be interesting to see how the above conclusions change when carried over to this more complicated environment.

Finally, future work should vary the information structure, and also modify the assumption that the elasticity of demand for the asset is infinite. In addition, the potential consequences of investor risk aversion should be examined more seriously.

APPENDIX A: PROOF OF PROPOSITION 1

Start with period 2. Equation (17) uses Frank's version of (3) to give Frank's willingness to pay for the asset at information set $F_{Buyer2} = \{e^B,\ e^B_{CB},\ e^G_2,\ e^G_3\}$, given the asset only pays a dividend (of d) in state e^G_3. Inequality (15) says that Ellen, at her information set $E^G_{Seller2} = \{e^G_2,\ e^G_3\}$, values the asset less than Frank at F_{Buyer2}. Ellen also values the asset less at her other information set $E^B_{Seller2} = \{e^B,\ e^B_{CB}\}$, since at that information set, Ellen knows that the asset is worthless. Thus, Ellen is willing to sell at the price p_2 from (17) in the states e^B, e^B_{CB}, e^G_2, and e^G_3, and Frank bids the price up to exactly that value in those states. By symmetry, the same price applies at states f^B, f^B_{CB} f^G_2, and f^G_3. At the other states, b, b_{CB}, e^G_1, and f^G_1, it is common knowledge in period 2 that the asset is worthless, so $p_2(\omega)$ must be zero in those states.

In period 1, the price p_1 from (16) gives Ellen's willingness to pay at her information set $E^G_{Seller} = \{e^G_1,\ e^G_2,\ e^G_3\}$, by (4). Equation (13) says that Ellen has the same willingness to pay at $E^B_{Seller} = \{b,\ b_{CB},\ e^B,\ e^B_{CB}\}$. Meanwhile, (14) says that Frank is willing to pay less than this at his information set $F_{Buyer} = \{e^B,\ e^B_{CB},\ e^G_1,\ e^G_2,\ e^G_3\}$ (where he is short sale constrained at zero), while at the information set $F^B_{Seller} = \{b,\ b_{CB},\ f^B,\ f^B_{CB}\}$, Frank is willing to pay exactly p_1 by symmetry. Thus, at the states b, b_{CB}, e^B, e^B_{CB}, e^G_1, e^G_2, and e^G_3, Ellen bids the price up to p_1 in (16), and Frank is willing to pay less than or equal to this at these states. A similar argument applies to f^B, f^B_{CB}, f^G_1, f^G_2, and f^G_3.

Thus, in both periods 1 and 2, the price is bid up to the highest willingness to pay, and the other side is either indifferent to trade or short-sale constrained at the equilibrium trade. Also, each period's price is constant on that period's information sets, so the price reveals no new information. The volume of output and trade also reveal no new information. This therefore yields an equilibrium. **QED**

APPENDIX B: PROOF OF PROPOSITION 3

PROOF: Using $\Pi(p_1) = ep_1$ shows that (22) is bigger than (21) when

$$\frac{M_E(b,\ b_{CB},\ e^B,\ e^B_{CB},\ e^G_1,\ e^G_2,\ e^G_3)}{M_E(b,\ e^B,\ e^G_1,\ e^G_2,\ e^G_3)} < \frac{p_1^{DP}}{p_1^{NDP}} = \frac{M_F(e^B,\ e^B_{CB},\ e^G_2,\ e^G_3)}{M_F(e^B,\ e^G_2,\ e^G_3)},$$

where the last step uses (20). Subtracting one from the left and right hand sides gives

$$(B1)\qquad \frac{M_E(b_{CB},\ e^B_{CB})}{M_E(b,\ e^B,\ e^G_1,\ e^G_2,\ e^G_3)} < \frac{M_F(e^B_{CB})}{M_F(e^B,\ e^G_2,\ e^G_3)}.$$

Finally, the left hand side of $(B1)$ equals the left hand side of (23), since the numerator and denominator both differ by the same factor, $M_E(e^B_{CB})/M_E(b_{CB},\ e^B_{CB}) = M_E(e^B)/M_E(b,\ e^B)$ (by (19)) $= M_E(e^G_2,\ e^G_3)/M_E(e^G_1,\ e^G_2,\ e^G_3)$ (by (13′)). **QED**

APPENDIX C: PROOF OF PROPOSITION 6

The bubble-bursting policy causes the contribution to welfare through the bad types of seller to change from

$$(C1)\qquad EU_B^{NP} = \Pi(p_{1L}^{NP})M_E(b,\ b_{BC},\ e^B) \ \text{ to } \ EU_B^{BP} = \Pi(p_{1H}^{BP})M_E(b,\ e^B).$$

In the fixed endowment case, $\Pi(p_1) = ep_1$, so (16′) and (13_L) show that $EU_B^{NP} = e\, p_{2L}^{NP}\ M_E(e^B)$. Similarly, by (16′) and (13_H), $EU_B^{BP} = e\, p_{2H}^{BP}\ M_E(e^B)$. Thus, $EU_B^{NP} = EU_B^{BP}$ (so the policy rule does not affect bad sellers' average welfare) if and only if $p_{2L}^{NP} = p_{2H}^{BP}$, that is, if and only if bubble policy does not affect bad sellers' p_2.

Next, if bubble bursting has no effect on bad seller welfare in the fixed endowment case, then $p_{1L}^{NP} M_E(b,\ b_{BC},\ e^B) = p_{1H}^{BP} M_E(b,\ e^B)$, so

$$(C2)\qquad p_{1H}^{BP}/p_{1L}^{NP} = M_E(b,\ b_{BC},\ e^B)/M_E(b,\ e^B).$$

This shows that $p_{1H}^{BP} > p_{1L}^{NP}$. Thus, since $\Pi(p_1)$ is strictly convex in the production case, and $\Pi(0) = 0$, it follows that $\Pi(p_{1H}^{BP})/\Pi(p_{1L}^{NP}) > p_{1H}^{BP}/p_{1L}^{NP}$. This, combined with $(C2)$, shows that EU_B^{BP} is larger than EU_B^{NP} in this case. **QED**

APPENDIX D: PROOF OF PROPOSITION 8

Let $A_L = M_E(e^G_{1L},\ e^G_{2L},\ e^G_{3L})$ and $A_H = M_E(e^G_{1H},\ e^G_{2H},\ e^G_{3H})$. Then if the overall expected welfare of good sellers is unaffected by a bubble bursting policy in the fixed endowment case, this means that $A_L p^{NP}_{1L} + A_H p^{NP}_{1H} = A_L p^{BP}_{1L} + A_H p^{BP}_{1H}$, so

$$A_L[p^{BP}_{1L} - p^{NP}_{1L}] = A_H[p^{NP}_{1H} - p^{BP}_{1H}]. \tag{D1}$$

Let

$$\delta_L = \frac{\Pi(p^{BP}_{1L}) - \Pi(p^{NP}_{1L})}{p^{BP}_{1L} - p^{NP}_{1L}} \quad \text{and} \quad \delta_H = \frac{\Pi(p^{NP}_{1H}) - \Pi(p^{BP}_{1H})}{p^{NP}_{1H} - p^{BP}_{1H}}. \tag{D2}$$

Since production is possible, $\Pi(p_1)$ is strictly convex. This, combined with $p^{BP}_{1L} > p^{NP}_{1L}$, $p^{NP}_{1H} > p^{BP}_{1H}$, and (30) implies that $\delta_H > \delta_L$. Thus,

$$\begin{aligned} A_L[\Pi(p^{BP}_{1L}) - \Pi(p^{NP}_{1L})] &= A_L\delta_L[p^{BP}_{1L} - p^{NP}_{1L}] = A_H\delta_L[p^{NP}_{1H} - p^{BP}_{1H}] \\ &< A_H\delta_H[p^{NP}_{1H} - p^{BP}_{1H}] = A_H[\Pi(p^{NP}_{1H}) - \Pi(p^{BP}_{1H})]. \end{aligned} \tag{D3}$$

Here the first step follows from the definition of δ_L and the second step follows from $(D1)$. The third step follows since $p^{NP}_{1H} - p^{BP}_{1H} > 0$ (by Proposition 7) and $\delta_L < \delta_H$, and the fourth step uses the definition of δ_H. Inequality $(D3)$ then shows that

$$A_L\Pi(p^{BP}_{1L}) + A_H\Pi(p^{BP}_{1H}) < A_L\Pi(p^{NP}_{1L}) + A_H\Pi(p^{NP}_{1H}),$$

so the bubble bursting policy reduces the overall expected welfare of the good sellers in the production case. **QED**

NOTES

1. Of course, there are strong disagreements on whether bubbles even exist. See Kindleberger (2000) and Garber (2000) among many others.

2. See Samuelson (1958), Blanchard and Watson (1982), Tirole (1985), Santos and Woodford (1997), or LeRoy (2004). Other popular approaches assume either agent irrationality (Abreu and Brunnermeier, 2003, Scheinkman and Xiong, 2003), or moral hazard problems (Allen and Gorton, 1993, Allen and Gale, 2000). See Chapter 2 of Brunnermeier (2001) for a review of the literature on bubbles.

3. Warren Buffett (2001) describes investors in bubble markets as resembling "Cinderella at the ball. They know that overstaying the festivities ... will eventually bring on pumpkins and mice" but they "all plan to leave just seconds before midnight." Unfortunately, "the clocks have no hands." Similarly, Kindleberger (2000, p. 15) suggests that "the word ... *bubble* foreshadows the bursting" (emphasis his).

4. For example, in Samuelson (1958), a bubble in fiat currency makes it possible for people to save for old age. In Tirole (1985), a bubble in an intrinsically useless asset allows people to save without wasting resources overproducing capital. It is unlikely that recent boom-bust cycles in asset prices served either of these functions.

5. Kindleberger (2000), traces an explicit statement of this theory as far back as 1890, when the *Chicago Tribune* editorialized about "men who bought property at prices they knew perfectly well were fictitious, but who ... knew that some still greater fool could be depended on to take the property off their hands and leave them with a profit" (p. 111; see also Chancellor, 2000, p. 95).

Many recent bubble models have a greater fool flavor (Harrison and Kreps, 1978, Allen et al., 1993, Abreu and Brunnermeier, 2003, Scheinkman and Xiong, 2003, Conlon, 2004). Also, greater fool models are consistent with evidence that asset price booms put pressure on brokers' loans (Rappoport and White, 1993, 1994) and put options (Bates, 1991), since these suggest that some agents anticipate a crash. In addition, stocks which are expensive to short have lower expected returns (Jones and Lamont, 2002), which is also consistent with a greater fool dynamic. The negative effect of short *interest* (the *volume* of short selling) on expected returns is weak (Asquith et al., 2004), but Jones and Lamont argue that short interest confounds supply and demand factors. Also, Henry (2005) argues that *informed* short interest has a stronger negative effect on expected future returns. Finally, Temin and Voth (2004), Brunnermeier and Nagel (2004) and Dhar and Goetzmann (2005) argue that many traders followed greater fool strategies during the South Sea bubble and the Internet boom.

6. While rational bubble models are therefore a natural place to begin, models based on irrationality are also important (Abreu and Brunnermeier, 2003, Scheinkman and Xiong, 2003). As explained in Section IV, the present analysis may cast some light on this case as well.

7. However, we do not consider the effects of bubble policies on non-investors, such as workers whose employment is affected by bubble-induced fluctuations. That is, we do not address the macroeconomic issues discussed by Bernanke and Gertler (1999, 2001), Bordo and Jeanne (2002), Cecchetti and coauthors (2000, 2003), and others. Hopefully this paper will provide a step in that direction.

8. This paper considers the costly production case (upward sloping supply) since otherwise the volume of trade is determined by short sale constraints alone, independent of lemons problems, etc. This is because, for simplicity, we assume preferences that yield infinite elasticities of demand for assets. Similar results should follow, however, if asset quantities are fixed but elasticities of demand are finite. The case with upward sloping supplies *and* downward sloping demands should then also yield similar results.

Regarding the production case, there has been some disagreement as to whether managers increase investment in response to overpriced assets. Blanchard et al. (1993,

p. 115) find that overpricing plays "at most a limited role in affecting investment decisions," while Chirinko and Schaller (2001), Panageas (2003), and Gilchrist, et al. (2004) find stronger evidence that asset overpricing encourages managers to invest.

9. Many models of asset markets assume short-sale constraints (Harrison and Kreps, 1978, Tirole, 1982, Allen et al., 1993). As Shiller (2000), p. 244, explains, "[w]hen a ridiculous fad develops for some stocks ... most investors ... do no more than avoid those stocks: They do not take the kind of massive short positions ... that would fully offset the overly exuberant prices that the fad investors would create." See also Ofek and Richardson (2003) and Jones and Lamont (2002) who relate short selling costs to asset overpricing. More generally, if short sales are especially difficult in certain markets, e.g., real estate, then bubbles may be more likely in those markets.

Put options may play a role similar to short sales. Asquith et al. (2004), p. 30, however, argue that "[h]edge fund managers and other practitioners involved in short selling maintain that they can not effectively use the options market. In interviews, they repeatedly claimed that the options market provides less liquidity and is more expensive than the short sales market when trying to establish a large position."

10. See Milgrom and Stokey (1982). For expository treatments, see Huang and Litzenberger (1988), Binmore (1992), or Samuelson (2004).

11. In terms of economic significance, (13) is not needed in richer models (see Subsection 3A). Condition (14) assures that trade occurs in period 2, not period 1, and (15) simply generates a motive for trade in period 2.

12. These transfer effects are absent if information is symmetric, or if "no trade theorems" apply. They have therefore received little attention in the past.

13. This also follows directly from (17), (17′), and symmetry (equation (8)).

14. However, this effect is different from that discussed in Hirshleifer (1972), p. 568, where information revelation simply disrupts insurance markets.

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