Two Types of Collusion in a Model of Hierarchical Agency

MEHMET BAC AND SERKAN KUCUKSENEL*

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Abstract

We introduce supervision costs and a new, ex-ante, occasion for collusion whereby the supervisor stops monitoring for a transfer payment from the agent, in addition to ex-post collusion possibilities conditional on the monitoring outcome. Extending TIROLE's [1986] model of hierarchy, we study the links between ex-ante collusion, ex-post collusion and the supervisor's monitoring incentives. Ex-ante collusion and the supervisor's incentive constraint can be ignored if monitoring costs are small and the probability of successful detection is large. To prevent ex-ante collusion the principal increases the difference between the wages paid when the report is empty and when it contains productivity evidence.
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1. *Introduction*

An important benefit from supervision activities is generation of information about organization members' outputs and job-relevant characteristics such as productivity. Supervision will be effective and produce truthful reports only if incentives are designed to prevent agent-supervisor collusions. Building on the insights of TIROLE [1986], LAFFONT [1990], KOFMAN and LAWARREE [1993], and LAFFONT and MARTIMORT [1997], a huge literature is devoted to the analysis of the costs and benefits of supervision in hierarchical organizations. In this literature, it is a standard practice to assume only one type of collusion, which arises from *ex-post* concerns, upon realization of the monitoring outcome where the supervisor is about to submit a report about a relevant parameter or action of the monitored agent. Though in some models the timing of ex-post collusion precedes monitoring activities, the collusive agreements which the principal aims to prevent all bear on the specific potential final outcomes of the monitoring game. In all these models, supervision or monitoring is assumed costless.

Another important occasion for collusion would arise before the supervisor exerts any monitoring effort: the agent can offer the supervisor a side transfer for not monitoring him. A supervisor-agent agreement to this effect would "close the supervisor's eyes" and thus eliminate the possibility of generating evidence about the agent's relevant characteristics. If this *ex-ante* type of collusion occurs, the supervisor's report is bound to be "empty" and ex-post collusion prevention considerations become irrelevant. Examples of ex-ante collusion are often reported in the news media, for instance, between the police and the mafia, where the police receives a transfer and does not monitor the redlight district in return.[1] Ex-ante collusion is certainly not an unknown practice to students of organizations.

The first objective of this paper is to introduce ex-ante collusion in a three-layer hierarchy model and investigate its impact on the optimal incentive scheme. The second objective is to explore the links between the two types of collusion possibilities. Obviously, under ex-ante collusion, an ex-post collusion to conceal the supervisor's information cannot occur because the supervisor's hands are empty. Does prevention of all types of ex-post collusion automatically imply prevention of

[1] See, for example, KLITGAARD [1988] presenting an account of such collusion in the context of Hong Kong police during early 70s.

ex-ante collusion? Third, what makes ex-ante collusion more likely, or its prevention, more costly?

The answers to these questions should depend on the information structure, the feasible contract space and timing of events in the agency problem under consideration. The setup we choose for this task is TIROLE's [1986] well-known model of three-layer hierarchy where the agent's effort and productivity parameter additively determine his output and are private information. The supervisor's task is to observe the productivity parameter and report it to the principal. In the next section we extend this model to include a monitoring cost, an incentive compatibility constraint for the supervisor and an ex-ante collusion proofness constraint.

Introduction of a positive monitoring cost brings in the need to motivate the supervisor and adds a corresponding incentive constraint to the principal's problem. In Section 3 we show that the supervisor's incentive constraint is not binding if the probability of successful monitoring is large, high productivity realization for the agent is a likely event and monitoring costs are sufficiently small. The intuition is rooted in the supervisor's optimal wage contract when monitoring costs and ex-ante collusion are ignored, as in TIROLE's original formulation, where the supervisor receives the highest wages when she successfully monitors and reports the agent's productivity. Then a more effective monitoring technology with larger probability of success means stronger monitoring incentives and relaxes the supervisor's incentive constraint. By the same token, because the supervisor is paid the largest wage when she documents high productivity for the agent, increasing the likelihood of this outcome enhances monitoring incentives. If the cost of monitoring is zero, the supervisor's incentive constraint would be automatically satisfied because her wages are smallest when she fails to produce evidence. This conclusion continues to hold for positive but small monitoring costs.

When the supervisor's incentive constraint is not binding and thus, is satisfied at no additional cost, the supervisor reaps a surplus over her expected payoff from ex-ante collusion. The larger this "noncooperation" surplus of the supervisor, the larger is the side transfer that the agent has to offer the supervisor to stop monitoring. Thus, the factors that relax the supervisor's incentive constraint also relax the ex-ante collusion proofness constraint. These results highlight a new, additional important beneficial impact of a more effective monitoring technology: in addition to reducing monitoring costs, an improved monitoring technology reduces the surplus from, therefore can eliminate the possibility of, ex-ante collusion.

When the supervisor's incentive constraint is binding, however, the principal may have to leave a positive rent to the supervisor and her participation constraint can become nonbinding. We show that, while the ex-ante collusion constraint may not be binding, depending on the factors we identify, the ex-post collusion constraints that are binding in TIROLE's solution continue to be binding in the present extended version. We close the paper with a discussion and some concluding remarks. The proofs are gathered in the Appendix.

2. The Model

In TIROLE's (1986) principal-supervisor-agent model the agent is the productive unit with productivity parameter θ, generating the profit or output x by exerting an effort $e \geq 0$ according to the technology $x = \theta + e$.[2] Given a wage W and an effort e, the agent's utility is $U(W - g(e))$, where $g(e)$ is a strictly convex, increasing effort disutility function with $g(0) = g'(0) = 0$ and U is a strictly concave wealth utility function. His reservation wage and utility are respectively W_0 and $\overline{U} \equiv U(W_0)$. The efficient level of effort, denoted $\hat{e}$, satisfies $g'(\hat{e}) = 1$.

The principal faces a hidden action problem because only the agent can observe his effort. There is also a hidden information problem. The agent's productivity parameter can take two values, $\underline{\theta}$ and $\overline{\theta}$, where $0 < \underline{\theta} < \overline{\theta}$. It is common knowledge that $\theta = \overline{\theta}$ with probability π. Only the agent learns the value of θ, after signing the contract with the principal.

The supervisor's task is to monitor the agent and submit a report r to the principal about θ. We postulate a simple monitoring technology where monitoring has a fixed effort cost z, allowing the supervisor to observe (obtain hard evidence about) θ with probability $0 < \rho < 1$. The supervisor is not able to observe anything if she does not monitor, but also if she monitors without success. Her report then is bound to be empty, that is, $r = \emptyset$. If monitoring is successful, the supervisor's report can be $r \in \{\theta, \emptyset\}$: she has the choice of truthfully reporting the observation or suppressing it. However, she cannot fabricate false evidence: if monitoring fails, the only possible report is $r = \emptyset$. We assume that the principal cannot observe the supervisor and that the agent cannot submit the principal hard evidence on the supervisor's monitoring choice. Given a wage S, the supervisor's utility if she

[2] We refer the reader to TIROLE's paper for discussion and justification of the setup and its assumptions.

monitors the agent is $V(S-z)$ where V is increasing and strictly concave, with $V'(0)=\infty$. The supervisor's reservation wage is S_0 with corresponding reservation utility $\overline{V} \equiv V(S_0)$.

The principal is risk neutral. He designs and offers contracts to the supervisor and the agent. The contracts can specify the wages S and W as functions of the contractible variables, the profit x and the supervisor's report r.

The sequence of events is as follows: Following acceptance of the contracts offered by the principal, the parties move to the collusion stage. There are two types of collusion possibilities for the supervisor and the agent. The first, *ex-ante*, type of collusion is an agreement whereby the agent makes a transfer to the supervisor, who in return commits not to monitor the agent. The second, *ex-post* type of collusion bears on the potential outcomes of the monitoring game, called "states" (see below). Under ex-post collusion the two parties misrepresent the state when possible, to increase their joint payoffs. After the collusion stage, the agent privately learns his productivity θ and determines his effort e while, absent ex-ante collusion, the supervisor simultaneously decides on whether or not to monitor the agent. Thus the supervisor may unilaterally (without inducement by the agent) decide not to monitor the agent. Next, the outcome of the monitoring game (the state) is realized. The last event is the supervisor's submission of a report to the principal and execution of contract terms.

In the absence of ex-ante collusion, if the supervisor monitors the agent, one of the following four states will arise:

State 1: agent and supervisor observe $\theta = \underline{\theta}$.

State 2: agent observes $\theta = \underline{\theta}$, supervisor does not observe the θ.

State 3: agent observes $\theta = \overline{\theta}$, supervisor does not observe the θ.

State 4: agent and supervisor observe $\theta = \overline{\theta}$.

The probabilities of these states, if the supervisor monitors the agent, are respectively $p_1 = (1-\pi)\rho$, $p_2 = (1-\pi)(1-\rho)$, $p_3 = \pi(1-\rho)$, $p_4 = \pi\rho$. If the parties collude ex-ante or if the supervisor unilaterally decides not to monitor the agent, only states 2 and 3 can arise, with probabilities $\tilde{p}_2 = 1-\pi$ and $\tilde{p}_3 = \pi$, respectively.

3. Analysis of the Principal's Problem

We first present the constraints to the principal's problem of designing contracts that maximize net expected profits. The agent's and the supervisor's participation constraints are

$$(APC): \qquad \sum_i p_i U(W_i - g(e_i)) \geq \overline{U},$$

$$(SPC): \qquad \sum_i p_i V(S_i - z) \geq \overline{V}.$$

There are two incentive-compatibility constraints, one for the agent to choose the contract intended for state 3 and reveal correctly his high productivity when monitoring fails, the other for the supervisor to incur the cost z and monitor effectively:

$$(AIC): \qquad W_3 - g(e_3) \geq W_2 - g(e_2 - \Delta\theta)),$$

$$(SIC): \qquad \sum_i p_i V(S_i - z) \geq (1-\pi)V(S_2) + \pi V(S_3).$$

When monitoring fails and the principal lacks evidence on the agent's productivity θ, the high-productivity agent can produce the same output $x_2 = e_2 + \underline{\theta}$ as the low-productivity agent by exerting $\Delta\theta = \overline{\theta} - \underline{\theta}$ less effort. (AIC) prevents this imitation strategy. (SIC) is a new constraint, ensuring that monitoring the agent is in the supervisor's own interest.

Three ex-post collusion constraints are added to rule out the possibility of agent-supervisor cooperation against the principal at the post-monitoring phase. (CIC1) rules out the possibility that the supervisor-agent coalition conceals the agent's low productivity in state 1. Similarly, (CIC2) guarantees truthful revelation of the agent's high productivity in state 4. In state 3, (CIC3) prevents the supervisor from bribing the agent to pick the incentive scheme for the low-productivity agent in state 2 where the wages are W_2 and S_2 respectively.[3]

$$(CIC1): \qquad S_1 + W_1 - g(e_1) \geq S_2 + W_2 - g(e_2),$$

$$(CIC2): \qquad S_4 + W_4 - g(e_4) \geq S_3 + W_3 - g(e_3),$$

$$(CIC3): \qquad S_3 + W_3 - g(e_3) \geq S_2 + W_2 - g(e_2 - \Delta\theta).$$

To these, we add the ex-ante collusion proofness constraint which rules out the possibility of cooperation to eliminate monitoring. Thus, the change in the agent's

[3]Note that the agent-supervisor coalition will not misrepresent state i as state j if and only if their total wealth in state i is larger than in state j.

and the supervisor's certainty equivalents, if they jointly switch from noncooperation to collusion, must sum up to a nonnegative number. We write this constraint in terms of expected utilities, which generates qualitatively the same results and avoids introduction of additional notation:[4]

$$(XCIC): \qquad \sum_i p_i U(W_i - g(e_i)) - [(1-\pi)U(W_2 - g(e_2)) + \pi U(W_3 - g(e_3))]$$

$$+ \sum_i p_i V(S_i - z) - [(1-\pi)V(S_2) + \pi V(S_3)] \geq 0.$$

The left hand side of this constraint represents the sum expected utilities under noncooperation where the supervisor monitors and reports truthfully, net of the utilities resulting from an agreement to stop the monitoring activity. Reformulating (XCIC) in terms of the parties' net expected utilities from noncooperation will be useful. Let EU^N and EV^N denote the agent's and the supervisor's expected utility from ex-ante collusion. Then (XCIC) can be written as

$$[\sum_i p_i U(W_i - g(e_i)) - EU^N] + [\sum_i p_i V(S_i - z) - EV^N] \geq 0.$$

Note that the supervisor's part of this constraint, $\sum_i p_i V(S_i - z) - EV^N$, is simply the constraint (SIC). Satisfaction of (SIC) is a positive step for (XCIC). However, if the agent's net expected benefit from noncooperation, $\sum_i p_i U(W_i - g(e_i)) - EU^N$, is negative and larger in absolute value than the supervisor's corresponding benefit from noncooperation, the agent can still bribe the supervisor and induce her to stop monitoring in return. Therefore the principal must first ensure that (SIC) holds and next, that the agent's net expected benefit from noncooperation is not negative enough to violate (XCIC).

The principal's problem is to choose $\{W_i, e_i, S_i\}$ that maximize $\sum_i p_i(\theta_i + e_i - W_i - S_i)$ subject to the contraints above. To better expose the impact of the new

[4]The amount that the agent is willing to offer for collusion (the change in his certainty equivalent, from noncooperation to collusion) and the difference between his expected utility from collusion and noncooperation always move together. Similarly the change in the supervisor's certainty equivalent and her corresponding change in expected utility move together. The question as to whether the principal should increase or decrease the agent's and the supervisor's wages in state i when the ex-ante collusion proofness constraint is violated has the same answer under both formulations of the ex-ante collusion-proofness constraint. Note that the two formulations are exactly equivalent only if the supervisor's and the agent's (Bernouilli) wealth utility functions are identical. We are grateful to a referee for this point.

constraints (SIC) and (XCIC), we first present as a benchmark the properties of the solution by ignoring (SIC) and (XCIC). We denote this solution $\{W_i^*, e_i^*, S_i^*\}$ and call it the *(*)-mechanism.* Ex-post collusion is possible and should be prevented. The case $z = 0$ corresponds to the problem studied by TIROLE [1986].

Proposition 1: The solution to the principal's problem without the supervisor incentive constraint (SIC) and the ex-ante collusion proofness constraint (XCIC) has the following properties:

a) $S_4^* > S_1^* > S_2^* = S_3^*$ and $dS_i^*/dz = 1$, $i = 1, 2, 3, 4$;
b) $W_3^* - g(e_3^*) > W_4^* - g(e_4^*) > W_1^* - g(e_1^*) > W_2^* - g(e_2^*)$;
c) $S_4^* + W_4^* = S_3^* + W_3^*$ and by (a), $W_3^* > W_4^*$;
d) $e_1^* = e_3^* = e_4^* = \hat{e} > e_2^*$.

The properties listed in parts (a)-(d) of Proposition 1, except for the impact of the monitoring cost on the supervisor's wages, are as given in TIROLE [1986]. A positive monitoring cost z shifts up the supervisor's wages S_i^* in all four states to keep her participation constraint satisfied. Let us briefly go over the intuition behind the properties of the (*)-mechanism. Note that the principal induces the first-best effort level $\hat{e}$ in states 1, 3 and 4. The exception is in state 2 where the agent's productivity is low and the supervisor fails to monitor successfully, thus submits an empty report. In this case the agent is induced to exert a suboptimal effort and obtains the lowest utility among all states. As in the standard adverse selection mechanism design problem, this is due to (AIC), the constraint preventing the high-productivity agent from imitating the low-productivity agent when monitoring fails. (AIC) is binding, $W_3^* - g(e_3^*) = W_2^* - g(e_2^* - \Delta\theta)$, and the high-productivity agent obtains a rent over his outside option thanks to her productivity advantage. Part (c) follows from (CIC2) and the fact that the same efficient effort level is induced in states 3 and 4.

The supervisor submitting an empty report receives the smallest wage ($S_2^* = S_3^*$). This result is rooted in the ex-post collusion proofness constraints. Keeping S_3 small and S_4 large reduces the high-productivity agent's ability to bribe the successful supervisor for submitting an empty report (thereby misrepresenting state 4 as state 3). But reducing S_3 too much would prompt another collusion where the supervisor whose monitoring fails bribes the high-productivity agent to choose the incentive scheme of the low-productivity agent. Setting $S_3^* = S_2^*$ eliminates this possibility. Also, keeping $S_1 > S_2$ induces the successful supervisor to report the agent's low productivity and at the same time serves to satisfy the participation constraint (SPC). All constraints are binding, except (CIC1), which is automatically satisfied.

The next step is to introduce the constraint (SIC) and study its impact on the constraints and the incentive scheme. The (*)-mechanism described in Proposition 1 satisfies (SPC) with equality. It also satisfies (SIC) if

$$\sum_i p_i V(S_i^* - z) = \overline{V} \geq V(S_2^*) = V(S_3^*)$$

or, using the expressions for p_i and rearranging terms, if

$$\rho\{(1-\pi)V(S_1^* - z) + \pi V(S_4^* - z)\} + (1-\rho)V(S_2^* - z) \geq V(S_2^*). \qquad (1)$$

Note that (1) holds as $z \to 0$, irrespective of π and ρ. Because $dS_i^*/dz = 1$ as stated in part (a) of Proposition 1, $S_i^* - z$ and the left-hand side of (1) remain constant while the right-hand side increases when z becomes larger. By continuity there exists a critical monitoring cost, denoted $z^C(\pi, \rho)$, such that (1) holds with equality. The (*)-mechanism satisfies (SIC)for $z \leq z^C(\pi, \rho)$: it is in the supervisor's best interest to incur the (low) monitoring cost and potentially get the high wages S_1^* or S_4^* rather than the low wage in states 2 and 3 ($S_2^* = S_3^*$) which she gets for sure if she does not monitor. The critical monitoring cost $z^C(\pi, \rho)$ is strictly positive and increasing in both π and ρ. A larger value for π means a larger expected benefit from monitoring, thus a larger value for $z^C(\pi, \rho)$, mainly because state 4, the state in which the supervisor gets the largest wage under the (*)-mechanism, becomes more likely. Finally, an increase in the probability of successful monitoring, ρ, makes states 1 and 4 more likely. Then, $z^C(\pi, \rho)$ is large and the (*)-mechanism is more likely to automatically satisfy the supervisor's incentive constraint (SIC).

Let the *(SI)-mechanism* be the collection of wages and effort levels that solve the principal's problem including (SIC), but excluding (XCIC). Recall, if $\overline{V} \geq V(S_2^*) = V(S_3^*)$, the (*)-mechanism satisfies (SIC) and the two mechanisms, (*) and (SI), yield exactly the same incentive scheme presented in Proposition 1. The alternative case is presented in Proposition 2.

Proposition 2: Suppose $\overline{V} < V(S_2^*) = V(S_3^*)$. The (SI)-mechanism has exactly the same properties as the (*)-mechanism listed in Proposition 1, except that

$dS_i^{SI}/dz \neq 1$. The supervisor's incentive constraint (SIC), as well as (AIC), (APC), (CIC2) and (CIC3) are binding. The supervisor's participation constraint (SPC) may no longer be binding.

When the (*)-mechanism violates (SIC), the principal modifies the incentive scheme to induce monitoring by the supervisor. To prevent the supervisor opt for the "sure" outcome where she gets the wage $S_2^{SI} = S_3^{SI}$ from not monitoring the agent, the principal increases the difference between $S_2 = S_3$ and the wages paid in states 1 and 4, when monitoring is successful. While this modification is done to satisfy (SIC) at minimum expected incentive costs, the principal also modifies the agent's wages to keep (CIC2) and (CIC3) binding. However, while (SIC) is binding the supervisor's participation constraint (SPC) may now be nonbinding, that is, it is possible to have $\sum_i p_i V(S_i^{SI} - z) > \overline{V}$.[5]

Let us, as the last step, include the possibility of ex-ante collusion. We have shown that when monitoring costs are large and exceed $z^C(\pi, \rho)$, under the (SI)-mechanism (SIC) is binding, which leaves the supervisor with zero net expected payoff from noncooperation. Then the (SI)-mechanism is ex-ante collusion-proof if $\sum_i p_i U(W_i^{SI} - g(e_i^{SI})) \geq EU_N^{SI}$, or, using the expressions for p_i and EU_N, if

$$(1-\pi)[U(W_1^{SI}-g(e_1^{SI}))-U(W_2^{SI}-g(e_2^{SI}))]-\pi[U(W_3^{SI}-g(e_3^{SI}))-U(W_4^{SI}-g(e_4^{SI}))] \geq 0.$$

Let π^C denote the value of π satisfying the above condition with equality. Thus, if the monitoring cost is larger than $z^C(\pi, \rho)$ while $\pi \leq \pi^C$, the principal can ignore the ex-ante collusion proofness constraint, which is automatically satisfied by the supervisor's incentive constraint under the (SI)-mechanism. The intuition is simple. Now that the supervisor's net surplus from noncooperation is zero, ex-ante collusion is prevented by ensuring a nonnegative net surplus from noncooperation for the agent. The agent gets the largest payoff in state 3, which under ex-ante collusion occurs with probability π, and smallest payoff in state 2, which under ex-ante collusion occurs with probability $1-\pi$. A small π means the agent is less likely

[5] Though the (*)-mechanism satisfies the participation constraint (SPC) with equality, under the (SI)-mechanism this constraint may be nonbinding because the principal may not be able to bind a larger number of constraints with the same number of instruments. He has to increase the supervisor's expected utility from monitoring (which, initially under the (*)-mechanism was equal to $\overline{V}$) to exceed the expected utility from not monitoring the agent, while binding exactly the same set of other constraints.

to get the large payoff $W_3 - g(e_3)$ and more likely to get the small payoff $W_2 - g(e_2)$, thus implies a smaller expected payoff for the agent under ex-ante collusion.

We consider now the case where (XCIC) is violated by the above mechanisms. The incentive scheme has to be modified. Denote the solution to the principal's problem under full set of constraints, including (XCIC), *the (X)-mechanism.*

Proposition 3: Suppose the mechanisms considered above violate (XCIC). Then, under the (X)-mechanism the principal decreases the agent's wages in states 2 and 3 and increases in states 1 and 4. The supervisor's participation constraint may not be binding, as under the (SI)-mechanism. Exactly the same ranking of effort levels and supervisor and agent wages obtain under the (X)-mechanism. (XCIC) and the same set of constraints given in Proposition 2 are binding.

The main impact of the ex-ante collusion-proofness constraint is modification of the agent's wages: W_2 and W_3 should be made small relative to W_1 and W_4 while keeping all other constraints satisfied. The modification in the agent's wages needed to eliminate the possibility of ex-ante collusion does not relax the two ex-post collusion-proofness constraints (CIC2) and (CIC3), which, as in propositions 1 and 2, continue to be binding. (CIC1) is not binding, as in the cases considered in propositions 1 and 2.

Proposition 3 thus provides an answer to the question we posed in the Introduction, on the link between the two types of collusion. In this canonical adverse selection setup, the modifications in the optimal wages brought about by the need to prevent ex-ante collusion, if necessary as stated in Proposition 3, do not relax the ex-post collusion proofness constraints. Both ex-ante and ex-post collusion proofness constraints are binding.

Would the principal benefit from using a "bounty hunter" incentive scheme for the supervisor, where supervision is motivated by allowing agent-supervisor ex-post collusion? Under this mechanism, the supervisor's reward from monitoring would be the potential bribe the agent would pay ex-post, for misrepresenting the state. In this setup, the answer is "no". For instance, if (CIC2) is ignored and ex-post collusion is allowed in state 4, the supervisor will never report a high productivity and the two parties will act as if they are in state 3. But when (CIC2) is not ignored and thus is binding the total wage bill is the same in states 3 and 4: we have $W_3 + S_3 = W_4 + S_4$ because the same, efficient, level of effort is induced by the agent in these states. Since total wage bill is the same and the output is the same, there is no potential benefit the principal can hope to get from allowing ex-post collusion in state 4.

If (CIC3) is ignored and ex-post collusion is allowed in state 3, the parties will act as if they are in state 2. The principal's total wage bill in state 2 is lower, but so is the output. Moreover, allowing for this type of collusion where the high-productivity agent picks the low-productivity agent's contract when monitoring fails, beats the whole purpose of the adverse selection problem: there would be no reason to employ the supervisor in the first place, if the agent is allowed to fully enjoy the rents from his private information and misrepresent his type.

These observations regarding the link between ex-ante and ex-post collusion proofness constraints should not be expected to hold universally. While the use of a bounty hunter scheme cannot benefit the principal in this basic setup, it may well be beneficial in others. In the present model ex-post collusion constraints are always binding. In a pure moral hazard problem the ex-post collusion constraint may automatically be satisfied and be nonbinding whenever the ex-ante collusion constraint is binding.

4. Concluding Remarks

A crucial objective in designing incentives in hierarchies should be prevention of agent-supervisor ex-ante collusion. Under ex-ante collusion the supervisor does not monitor and ex-post collusion prevention considerations also become irrelevant. We have chosen the hidden information - hidden action setup by TIROLE [1986] to analyze the impact of introducing the possibility of ex-ante collusion. The analysis has shown that for sufficiently small supervision costs the optimal incentive scheme that assures agent-supervisor noncooperation also provides sufficient incentives for the supervisor to monitor the agent. Then the principal can ignore the supervisor's incentive constraint. Moreover, if the agent's net surplus from noncooperation is positive or not too negative, the ex-ante collusion proofness constraint is also automatically satisfied. As supervision costs increase, ex-ante collusion becomes an attractive option and should be prevented. We also show that, at least in this canonical setup, introduction of the possibility of ex-ante collusion does not relax any of the ex-post collusion constraints that were binding without the ex-ante collusion constraint in the problem statement.

The analysis highlights two adverse effects of larger monitoring costs. Besides the obvious impact on the supervisor's wages, which must be increased to keep inducing effective monitoring, a larger monitoring cost increases the principal's cost of preventing collusion. When the monitoring cost is large, the supervisor's expected surplus from monitoring is small, thus, she is willing to accept a smaller "bribe" from the agent to stop monitoring in return.

A question for future research is whether and how the presence of a second source of information, besides the supervisor, modifies the relation between ex-ante and ex-post collusion possibilities. This extension would generate a larger number of states, most importantly it would bring about the possibility of punishing the supervisor for not reporting when the second source reports the productivity parameter. Another interesting class of models to study the issues we raised in this paper involves hidden action and noncontractible output, or a noncontractible output attribute such as quality. In these models, as in the corruption- or shirking-prevention hierarchical model in BAC [1996], the agent's actions or inputs have to be monitored. Monitoring of actions would be important for two reasons, first, for the information it generates to the principal and second, for its direct influence on the monitored agent's behavior. In the present setup where output is contractible the agent's behavior (effort) is influenced by output-dependent wage payments. Supervision activity only plays the indirect role of preventing the agent from misrepresenting his productivity level.

Appendix

In this Appendix we prove propositions 1, 2 and 3. Below is the Lagrangean and the corresponding first-order conditions for the problem including full set of constraints but (CIC1). We will later show that (CIC1) is satisfied.

$$L^c = \sum_i p_i(\theta_i + e_i - W_i - S_i) + \nu(\sum_i p_i V(S_i - z) - \overline{V})$$

$$+\psi(\sum_i p_i V(S_i - z) - (1-\pi)V(S_2) - (\pi)V(S_3))$$

$$+\mu(\sum_i p_i U(W_i - g(e_i)) - \overline{U}) + \gamma(W_3 - g(e_3) - W_2 + g(e_2 - \Delta\theta))$$

$$+\Pi(S_3 + W_3 - g(e_3) - S_2 - W_2 + g(e_2 - \Delta\theta))$$

$$+\epsilon(S_4 + W_4 - g(e_4) - S_3 - W_3 + g(e_3))$$

$+\lambda(\sum_i p_i U(W_i - g(e_i)) + \sum_i p_i V(S_i - z) - (1-\pi)U(W_2 - g(e_2)) - (\pi)U(W_3 - g(e_3)) - (1-\pi)V(S_2) - (\pi)V(S_3))$

Taking the derivatives with respect to S_i, W_i, e_i gives

$$(\nu + \psi + \lambda)V'(S_1 - z) = 1 \tag{2}$$

$$(\nu + \psi + \lambda)V'(S_2 - z) - \frac{1-\pi}{p_2}(\psi + \lambda)V'(S_2) = 1 + \frac{\Pi}{p_2} \tag{3}$$

$$(\nu + \psi + \lambda)V'(S_3 - z) - \frac{1-\pi}{p_2}(\psi + \lambda)V'(S_3) = 1 + \frac{\epsilon - \Pi}{p_3} \tag{4}$$

$$(\nu + \psi + \lambda)V'(S_4 - z) = 1 - \frac{\epsilon}{p_4} \tag{5}$$

$$(\mu + \lambda)U'(W_1 - g(e_1)) = 1 \tag{6}$$

$$(\mu + \lambda(1 - \frac{1-\pi}{p_2}))U'(W_2 - g(e_2)) = 1 + \frac{\gamma + \Pi}{p_2} \tag{7}$$

$$(\mu + \lambda(1 - \frac{\pi}{p_3}))U'(W_3 - g(e_3)) = 1 - \frac{\gamma + \Pi - \epsilon}{p_3} \tag{8}$$

$$(\mu + \lambda)U'(W_4 - g(e_4)) = 1 - \frac{\epsilon}{p_4} \tag{9}$$

$$(\mu + \lambda)U'(W_1 - g(e_1))g'(e_1) = 1 \tag{10}$$

$$(\mu + \lambda(1 - \frac{1-\pi}{p_2}))U'(W_2 - g(e_2))g'(e_2) - \frac{\gamma + \Pi}{p_2}g'(e_2 - \Delta\theta) = 1 \tag{11}$$

$$(\mu + \lambda(1 - \frac{\pi}{p_3}))U'(W_3 - g(e_3))g'(e_3) + \frac{\gamma + \Pi - \epsilon}{p_3}g'(e_3) = 1 \tag{12}$$

$$(\mu + \lambda)U'(W_4 - g(e_4))g'(e_4) + \frac{\epsilon}{p_4}g'(e_4) = 1. \tag{13}$$

Proof of Proposition 1: For Proposition 1, the Lagrangean expression and the corresponding first order conditions are obtained by setting $\lambda = \psi = 0$ above.

Characterization of the optimal incentive scheme follows TIROLE [1986], proof of Lemma 1: Using equations (6), (7), (8) and (9) respectively in equations (10), (11), (12), and (13) we get the ranking of efforts as $e_1^* = e_3^* = e_4^* = \hat{e} > e_2^*$.

Suppose, to show a contradiction, (AIC) is not binding: $W_3 - g(e_3) > W_2 - g(e_2 - \Delta\theta)$, hence $\gamma = 0$. Note that $(1-\pi)/p_2 = \pi/p_3 = 1/(1-\rho)$, so, comparing the wage first-order conditions (3), (4) with respectively (7), (8) reveals that $S_2 < S_3$. But this implies that (CIC3) is not binding, i.e., $\Pi = 0$. Using $\Pi = 0$ in (7) and (8) yields $W_3 - g(e_3) \leq W_2 - g(e_2)$ which violates (AIC) because $W_2 - g(e_2) < W_2 - g(e_2 - \Delta\theta)$.

Suppose (CIC2) is not binding, so $\epsilon = 0$. Used in (8) and (9), this implies $W_4 - g(e_4) < W_3 - g(e_3)$, while from (4) and (5) we get $S_3 > S_4$ which violates (CIC2). Therefore $\epsilon > 0$.

Given $\gamma > 0$ and $\epsilon > 0$, suppose $\Pi = 0$. Then, (3) and (4) can be respectively written as $V'(S_2 - z)\nu = 1$ and $V'(S_3 - z)\nu = 1 + \epsilon/p_3$, implying $S_3 < S_2$. Given that (AIC) is binding, (CIC3) will be violated. Therefore $\Pi > 0$.

Having established $\gamma > 0$, $\epsilon > 0$ and $\Pi > 0$, from the agent's wage first-order conditions (6), (7), and (9) we obtain $W_4^* - g(e_4^*) > W_1^* - g(e_1^*) > W_2^* - g(e_2^*)$. From (CIC2), $W_3^* + S_3^* = W_4^* + S_4^*$, which implies $W_3^* > W_4^*$ given $S_4^* > S_3^*$. Therefore, $W_3^* - g(e_3^*) > W_4^* - g(e_4^*) > W_1^* - g(e_1^*) > W_2^* - g(e_2^*)$. Similarly the supervisor's wage first-order conditions (2), (3), (4) and (5) can be combined with the fact that (AIC) and (CIC3) are binding to yield the wage order $S_4^* > S_1^* > S_2^* = S_3^*$. The reader can now verify that (CIC1) is satisfied given the above ranking of wages and efforts. *Q.E.D.*

Proof of Proposition 2: Suppose, as assumed in the proposition, $\lambda = 0$ ((XCIC) is ignored) and $\overline{V} < V(S_2^*) = V(S_3^*)$ (the (*)-mechanism violates (SIC)). Then, under the new, (SI)-mechanism, (SIC) must be binding and thus $\psi > 0$.

The result $e_4^{SI} = e_3^{SI} = e_1^{SI} = \hat{e} > e_2^{SI}$ can be shown by using (6)-(9) in (10)-(13). Below we show that under the (SI)-mechanism (CIC2), (CIC3) and (AIC) are binding.

A useful observation which we use below and in the proof of proposition 3 to compare the wages S_2 and S_3 is that, for $\psi > 0$, the left-hand side of (3) must be positive as $S_2 \to z$ from above. The left hand sides of (3) and (4) are of the form $Y \equiv AV'(S - z) - BV'(S)$. Clearly, $S \to z$ implies $Y > 0$ because $V'(0) = \infty$. Note that for $S \in (z, \infty)$, Y is continuous and monotonic in S. Because $V'(S - z)$ approaches $V'(S)$ as $S \to \infty$, we have $Y \to (A - B)V'(\infty)$, which must be finite (with sign depending on $A - B$) by strict concavity of $V(.)$. Therefore Y must be monotonically decreasing in S.

Suppose, first, that (CIC2) is not binding, i.e., $\epsilon = 0$. Then the right-hand sides of (3) and (4) are respectively given by $1 + \Pi/p_2$ and $1 - \Pi/p_3$. Consider each of the following possibilities: First, suppose $\Pi > 0$, which implies $S_3 + W_3 - g(e_3) = S_2 + W_2 - g(e_2 - \Delta\theta)$. From (3) and (4) where $\lambda = \epsilon = 0$, we get $S_3 > S_2$. But then, $W_3 - g(e_3) < W_2 - g(e_2 - \Delta\theta)$, violating (AIC). Second, suppose $\Pi = 0$. Then from (3) and (4) we get $S_3 = S_2$ and thus, $W_3 - g(e_3) > W_2 - g(e_2 - \Delta\theta)$. This means $\gamma = 0$. Using $\gamma = \epsilon = \Pi = 0$ in the first-order conditions (7) for W_2 and (8) for W_3 yields $W_2 - g(e_2) = W_3 - g(e_3)$, which violates (AIC). Since both cases are shown to be impossible, we conclude that $\epsilon > 0$. Along with the fact that $e_3^{SI} = e_4^{SI}$, (CIC2) binding implies $S_4^{SI} + W_4^{SI} = S_3^{SI} + W_3^{SI}$.

Suppose now $\gamma = 0$, so (AIC) is not binding and $W_3 - g(e_3) > W_2 - g(e_2 - \Delta\theta) > W_2 - g(e_2)$. Therefore $U'(W_2 - g(e_2)) > U'(W_3 - g(e_3))$. Then, (7) and (8) imply $\Pi/p_2 > (\epsilon - \Pi)/p_3$. Using this in (3) and (4) we have $\Pi/p_2 > (\epsilon - \Pi)/p_3 \Longrightarrow S_3 > S_2$. Then (CIC3) must be nonbinding and $\Pi = 0$. If $\Pi = 0$, however, (7) and (8) imply $U'(W_2 - g(e_2)) < U'(W_3 - g(e_3))$, a contradiction. Thus, $\gamma > 0$.

To show that (CIC3) is binding, suppose, on the contrary, $\Pi = 0$ and $S_3 + W_3 - g(e_3) > S_2 + W_2 - g(e_2 - \Delta\theta)$. Since $\gamma > 0$ we have $W_3 - g(e_3) = W_2 - g(e_2 - \Delta\theta)$, which implies $S_3 > S_2$. But then from (3) and (4) we obtain $\epsilon/p_3 < 0$, impossible because $\epsilon > 0$. Thus, $\Pi > 0$.

The agent's participation constraint (APC) must be binding for otherwise, if $\mu = 0$, conditions (6)-(9) will all be violated. As for the supervisor's participation constraint (SPC), it may not be binding because the associated Lagrange multiplier ν and the Lagrange multiplier ψ for (SIC) only appear in (2)-(5), additively (it is their sum that counts). These conditions and the results above will all hold given $\psi > 0$ and $\nu = 0$. *Q.E.D.*

Proof of Proposition 3: Suppose that the (SI)-mechanism violates (XCIC). Then the principal must modify the incentive scheme and use the (X)-mechanism under which (XCIC) must be binding, that is, $\lambda > 0$ (for if $\lambda = 0$, the two mechanisms would be identical, which would imply that the (SI)-mechanism does not violate (XCIC), contradicting the assumption that it does). The same arguments in the proofs of propositions 1 and 2 can be used to show that $e_1^X = e_3^X = e_4^X = \hat{e} > e_2^X$. Using the expressions for p_2 and p_3, the first-order conditions for the agent's wage, (7) and (8) can be written as

$$(\mu - \lambda\frac{\rho}{1-\rho})U'(W_2 - g(e_2)) = 1 + \frac{\gamma + \Pi}{(1-\pi)(1-\rho)} \tag{14}$$

$$(\mu - \lambda\frac{\rho}{1-\rho})U'(W_3 - g(e_3)) = 1 - \frac{\gamma + \Pi - \epsilon}{\pi(1-\rho)} \tag{15}$$

Clearly, $\mu > 0$, for otherwise the marginal utility in state 2 (see (14)) must be nonpositive, which is impossible. Therefore (APC) is binding. Comparing (14) and (15) with conditions (7) and (8) under the (SI)-mechanism with $\lambda = 0$ reveals that the agent's wages in states 2 and 3 must be decreased when $\lambda > 0$. Since $\mu > 0$ and (APC) is binding, we must also have $W_1^X > W_1^{SI}$ and $W_4^X > W_4^{SI}$.

That $\gamma > 0$ follows from the same arguments in the proof of Proposition 1: If $\gamma = 0$ we have $W_3 - g(e_3) > W_2 - g(e_2)$, thus $U'(W_3 - g(e_3)) < U'(W_2 - g(e_2))$ and from (7) and (8), $\Pi/p_2 > (\epsilon - \Pi)/p_3$. Combining this with (3) and (4) yields $S_2 < S_3$, which implies $\Pi = 0$. However, $\Pi = \gamma = 0$, used in (14) and (15) imply $W_3 - g(e_3) \leq W_2 - g(e_2)$, violating (AIC). Thus $\gamma > 0$.

Next, to show that (CIC2) and (CIC3) are both binding, we consider the three cases in which this is not true.

First, suppose (CIC3) is binding but (CIC2) is not. Then, $\Pi > 0$ and $\epsilon = 0$. Given that (AIC) is binding and $\gamma > 0$, from (CIC3) we get $S_2 = S_3$. But using $\Pi > 0$ and $\epsilon = 0$ in (3) and (4) reveals that $S_2 < S_3$, which implies $\Pi = 0$, a contradiction.

Second, suppose (CIC2) is binding but (CIC3) is not. Then, $\Pi = 0$ and $\epsilon > 0$. Under these conditions the right-hand side of (3) is equal to one while the right hand side of (4) is equal to $1 + \epsilon/p_3$, which implies $S_3 < S_2$, and given that (AIC) is binding, violates (CIC3).

Finally, suppose neither (CIC2) nor (CIC3) is binding. Given that (AIC) is binding, from (CIC3) we get $S_3 > S_2$. But when $\Pi = \epsilon = 0$, from (3) and (4) we get $S_2 = S_3$, contradicting the assumption that (CIC3) is not binding. We conclude that $\Pi > 0$ and $\epsilon > 0$. Given these results, (CIC1) is automatically satisfied.

Q.E.D.

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Professor Mehmet Bac
Sabanci University
Faculty of Arts and Social Sciences
Orhanli, Tuzla, Istanbul, 81474
Turkey

Dr. Serkan Kucuksenel
California Institute of Technology
Division of Humanities and Social Sciences, 228-77
Pasadena, CA 91125, U.S.A.