Dynamic Stability and Reform of Political Institutions

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Abstract

This paper studies dynamic, endogenous institutional change. We introduce the class of *dynamic political games (DPGs)*, dynamic games in which future political aggregation rules are decided under current ones, and the resulting institutional choices do not affect payoffs or technology directly. A companion paper (Lagunoff (2005b)) establishes existence of Markov Perfect equilibria of dynamic political games. The present paper examines issues of stability and reform when such equilibria exist. Which environments tend toward institutional stability? Which tend toward reform? We show that when political rules are *dynamically consistent* and private sector decisions are *inessential*, reform never occurs: all political rules are stable. Roughly, private sector decisions are inessential if any feasible "social" continuation payoff can achieved by public sector decisions alone. More generally, we identify sufficient conditions for stability and reform in terms of *recursive self selection* and *recursive self denial*, incentive compatibility concepts that treat the rules themselves as "players" who can strategically delegate future policy-making authority to different institutional types. These ideas are illustrated in an example of dynamic public goods provision.

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1 Introduction

Reforms of political institutions are common throughout history. They come in many varieties. In some cases, the reforms correspond to changes in the voting franchise. Periodic expansions of voting rights occurred in governments of ancient Athens (700- 338BC), the Roman Republic (509BC-25AD), and most of Western Europe in the 19th and early 20th centuries, to name just a few examples.[1]

In other cases, modifications were made to the voting procedure itself. Medieval Venice (1032-1300), for instance, gradually lowered the required voting threshold from unanimity to a simple majority in its Citizens' Council. Nineteenth century Prussia, where votes were initially weighted by one's wealth, eventually equalized the weights across all citizens. The U.S. changed its rules under the 17th Amendment to require direct election of senators.

In still other instances, the scope of a government's authority changed. For example, England and France privatized common land during the 16th and 17th century enclosure movement thus reducing scope for rules governing the commons.[2] The U.S. government, on the other hand, increased its scope under the 16th Amendment by legalizing federal income tax in 1913.

Numerous historical examples suggest that institutional change is often gradual and incremental. Consider the progress of reforms in the Roman Republic. In 509BC, the Senate and Assembly were founded; in 494 BC the Patricians conceded the right of the plebs (the "commoners") to participate in the election of magistrates; in 336 BC one of the consulships became available for election by plebians; and in 287 BC Hortensian Law was introduced which gave resolutions in the plebian council the force of law. Gradual reform also characterized expansion of voting rights in 19th century England. In 1830, the voting franchise restricted to 2% of the population. In 1832, the First Reform Act extended the franchise to 3.5% of population. The Second Reform Act of 1867 extended it to some 7.7%. By 1884 it had been extended to 15% of population. Universal suffrage only passed in 1928 (see Finer (1997).

The objective in this paper is to understand how and why institutional reform occurs. Most basically, which environments tend toward institutional stability? Which environments admit change? What are the relevant forces that drive this change?

To address these questions, we posit a broad framework that endogenizes the process of institutional change. We introduce a class of infinite horizon stochastic games in which the process of change is recursive: rules for choosing public decisions in period $t+1$ are, themselves, objects of choice in period t. The process is also instrumental: institutions do not affect payoffs or technology directly. We call games in this class *dynamic political games*

[1] See Fine (1983), Finer (1997), and Fleck and Hanssen (2003).

[2] See MacFarlane (1978) and Dahlman (1980).

(DPGs).

In a DPG, rules for choosing public decisions are themselves part of the decision process. At each date t, private and public sector decisions jointly determine the date $(t+1)$ distribution of "economic" and "political" states of the world. Economic states are substantive parameters that affect preferences and technology directly. Political states are "procedural" parameters that characterize the explicit *political rule* for determining public sector decisions. If, for example, the current political rule is a simple majority rule, then the feasible set of outcomes of majority rule is the set of Condorcet Winners — the choices which cannot be defeated by any alternative choice in a majority vote.

We study the Markovian equilibria of dynamic political games. A Markovian equilibrium in a DPG is a collection of state-contingent private and public sector decision rules such that (a) the private sector rule for each individual is optimal for him in each period and in each state, and (b) public sector decisions are consistent with the prevailing political rule in each period and in each state. The choice of restricting to Markovian equilibria follows the general rule of thumb: in an economy-wide context, the participants are less apt to coordinate on non payoff-relevant history than they would if they were in a small group.

A companion paper (Lagunoff, (2005b)) establishes existence of Markov Perfect equilibria for a class dynamic political games. The present paper examines properties of these equilibria. In particular, we ask: when do institutional reforms occur, or alternatively, when are rules stable over time?

By definition, a political rule *admits reform* whenever next period's political rule is chosen to be different than the present one. A rule is *stable* whenever no reform occurs. Our main result (Theorem 1) is that any political rule that satisfies two simple conditions, *dynamic consistency* and *private sector inessentiality*, is stable in every equilibrium. In other words, institutional reform *never* occurs if these two conditions are satisfied.

The first condition is a straightforward extension of the dynamic consistency idea in an individual decision problem. A political rule is *dynamically consistent* if it is rationalized by a time separable social welfare function that does not vary directly with the economic state in any period. Most dynamic voting games that admit the same median voter each period are dynamically consistent. However, voting models in which orderings of voters change through time may not be.

According to the second condition, the private sector is *inessential* in the current political rule if for any private sector decision, any social continuation that is feasible tomorrow can always be reached or exceeded today by a public sector decision. Hence, private decisions are inessential if their effects on social welfare can always be replaced by those of the public sector.

The intuition for the result may be seen in the special case of endogenous voting rights. An influential argument of Acemoglu and Robinson (2000, 2001) asserts that the elites of 19th century Europe chose to extend the voting franchise because policy concessions alone could not "buy off" the external threat of an uprising. In other words, franchise extension could only occur because the external threat (from the private sector) was *essential*. Of course, this condition provides only a necessary, not a sufficient condition for a reform such as a franchise expansion. Jack and Lagunoff (2003) display Acemoglu-Robinson logic as a special case in a recursive model in which gradual extension of the voting franchise is possible. The result therefore helps to make sense of their logic in a larger context; it identifies some necessary features of an environment in order for institutional change to occur.

More generally, by fixing a continuation value and an economic state in each period, one can interpret play of the dynamic political game in that period as a distinct normal form game. A rule is *recursively self-selected (RSS)* if, in this auxiliary game, the current rule never chooses to "delegate" decision authority to another rule for the subsequent period only. A rule is *recursively self-denied (RSD)* if it delegates authority to some other rule. Recursive self-selection and self-denial are institutional incentive constraints that treat each distinct rule as an institutional "player" in the induced normal form game with private individuals.

Recursive self-selection bears some relation to "self-selected rules" in the static social choice models of Koray (2000) and Barbera and Jackson (2000), and also to the infinite regress model of choice of rules by Lagunoff (1992). These all posit social orderings on the rules themselves based on the outcomes that these rules prescribe. Rules that "select themselves" do so on basis of selecting the same outcome as original rule. The present model has two differences. First, institutional choice occurs in real time — next period's rule is chosen by the present one. This makes possible an analysis of explicit dynamics of change. Second, the present model is more concrete; the trade-offs are explicitly derived from the interaction of economic fundamentals in the public and private sectors.

We show that recursively self-selected (RSS) rules are stable (Theorem 2). The converse, however, does not hold: there may be stable rules that are not RSS. In these instances, Rule A may choose not to delegate to Rule B, even though Rule A is recursively self-denied (RSD) by Rule B. Why? Because Rule B, with its decision authority, would subsequently delegate to Rule C which is unattractive from A's point of view. Given these intransitivities, Rule A remains stable. Consequently, we show in Theorem 2 that a rule admits institutional reform if either it is RSD by another RSS rule, or, alternatively, if it is RSD by every other rule.

The usefulness of RSS and RSD are illustrated in an example, based on a parametric example of Jack and Lagunoff (2003), of dynamic public goods provision that requires both private and public inputs. We show that there exists an equilibrium that converges globally to a steady state political rule. This rule is uniquely RSS. We show that every other political rule admits reform. In these cases, a time consistency problem arises when the current rule produces policies ill-adapted to private contributions. To alleviate this problem a reform occurs when the current rule commits future policy-making authority to another rule.

Ideally, we eventually hope to obtain a full characterization of those environments that admit reform and those that do not. Our results provide only sufficient conditions for each. Nevertheless, the notions of inessentiality and self-selection/denial seem more representative of general ideas than their application suggests.

There is a modest literature on dynamic, endogenous political institutions.[3] Informal discussions in North (1981) and Ostrom (1990) both hint at recursivity in the process of institutional change. In more formal work, Messner and Polborn (2002) examine an OLG model of endogenous changes to future voting rules under current ones. Lagunoff (2001) studies a dynamic recursive model of endogenously chosen civil liberties. Greif and Laitin (2004) model institutions as equilibrium outcomes in a repeated game. Acemoglu and Robinson (2000, 2001) examine endogenous voting rights in a dynamic game. In their model an enfranchised player representing "the elite" can choose in any period whether to make a once-and-for-all extension of voting rights to the other player representing "the masses". The extension of rights in their model is motivated by the elite's desire to head off social unrest. Models in which endogenous extensions of voting rights are gradual and incremental are proposed by Justman and Gradstein (1999), Roberts (1998, 1999), Barbera, Maschler, and Shalev (2001), Jack and Lagunoff (2003), and Gradstein (2003).

In certain respects, the model of Jack and Lagunoff (2003) is a prototype for the present framework. It shares the feature in common with the present framework that the choice of rule is both recursive and instrumental. The current paper extends their framework to produce new results and examine institutional choices other than voting rights. DPGs admit a broad array of institutional changes including changes in the voting rule (majority vs supermajority rules), changes in voting rights (e.g. larger vs smaller voting franchise), and changes in the scope of the public sector (e.g., expansions vs contractions of regulatory authority).

The paper is organized as follows. In Section 2, we present a less formal version of the model in order to highlight issues of institutional stability. Section 3 introduces the class of *political rules*. Political rules are the natural objects of choice in a recursive model of endogenous institutions. An "equilibrium" in the dynamic political game combines standard Markov Perfection in private decisions with a political fixed point requirement for public decisions. Section 4 gives the first main result — a set of sufficient conditions for institutional stability. Section 5 introduces RSS and RSD concepts. Section 6 gives an illustrative example. Section 7 summarizes the results, and Section 8 is an Appendix with the proofs of the main results.

[3] In focusing attention on dynamic models, I neglect a larger literature on static models of endogenous political institutions such as, for example, Lizzeri and Persico (2002) and Aghion, Alesina, and Trebbi (2002).

2 An Introductory Pure Public Sector Model

A less formal, "stripped down" version of the model is presented in this Section to highlight some basic issues in recursive institutional choice.

A society consists of a set $I = \{1, \ldots, n\}$ of infinitely lived individuals, at each date $t = 1, 2, \ldots$, this society must collectively choose policy p_t from a compact feasible set P. The stage payoffs of each individual depend on this policy and on a state variable ω_t drawn from a set Ω. Write $u_i(\omega_t, p_t)$ to denote the payoff of individual $i \in I$ at date t given state ω_t and policy p_t. The technology determines how current states and policies pin down distributions over future states. For example, ω_t could be distribution of date t incomes across individuals and p_t an income tax schedule. For now, omit (notationally) private sector behavior such as individuals' savings decisions.[4]

Each period, a political institution aggregates policy preferences to determine that period's policy p_t. We examine a scenario in which this institution itself is part of the decision problem. Let θ_t denote a parameter that determines a *political rule*. A political rule summarizes the political process by which public decisions are made. To simplify things for this example, suppose that there are two possible political rules which we categorize as "democracy" and "dictatorship." Formally, let $\theta_t \in \{\theta^A, \theta^B\}$ where, if $\theta_t = \theta^A$ then the policy is determined by a majority vote, and if $\theta_t = \theta^B$ then the policy p_t is imposed by a "dictator," whom we assume to be Individual $i = 1$. In particular, $\theta = \theta^A$ means that feasible political choices must be Condorcet Winners — outcomes that dominate any alternative in a pairwise majority vote. By contrast, $\theta = \theta^B$ means the only feasible political choices are those that maximize the payoff of individual $i = 1$.

The formal definitions of aggregation under θ^A and θ^B will be defined momentarily. The important aspect, however, is that the prevailing political rule at date t determines both the policy p_t *and* the subsequent political rule, θ_{t+1}.

The composite state at date t is denoted by $s_t = (\omega_t, \theta_t)$, consisting of the economic state and the political rule. A *strategy* is a pair (ψ, μ) of Markov decision functions that select a politically feasible choice in each state. Given s_t, ψ determines the policy $p_t = \psi(s_t)$ while μ determines next period's political rule $\theta_{t+1} = \mu(s_t)$. The institutional strategy μ is of particular interest since it describes a recursive process of institutional change.

The pair (ψ, μ) produces a *public sector decision* each period given by the pair (p_t, θ_{t+1}). The public sector decision includes the choice of institution for the following period. An individual i's *recursive payoff function* of the public sector decision in state s_t is

$$U_i(s_t;\ \psi, \mu)(p_t, \theta_{t+1}) \ \equiv\ (1-\delta)u_i(\omega_t, p_t) \ +\ \delta \int V_i(s_{t+1};\ \psi, \mu)dq(\omega_{t+1}|\ \omega_t, p_t) \qquad (1)$$

[4] Private sector behavior will be introduced in the next Section.

where δ is the discount factor, q denotes the stochastic transition function mapping current states and policies into probability distributions over future states, and V_i is i's continuation payoff given policy rule ψ.[5] Each individual therefore has payoff function $U_i(s_t;\psi)(\cdot)$ in (1) defined over (p_t, θ_{t+1}) pairs of public decisions. By construction, we must have,

$$U_i(s_t;\ \psi,\mu)(\psi(s_t),\mu(s_t)) \ = \ V_i(s_t;\ \psi,\mu).$$

A *profile* of payoff functions is denoted $U(s_t;\psi) \ = \ (U_i(s_t;\psi))_{i=1}^n$. Given any such profile, the political rules are summarized by a mapping C with:

$$C(\ U(s_t;\psi,\mu),\ s_t) = \begin{cases} \text{set of Condorcet Winners} & \text{if} \quad \theta_t = \theta^A \\ \\ \arg\max\limits_{p,\theta'} U_1(s_t;\psi,\mu)(p,\theta') & \text{if} \quad \theta_t = \theta^B \end{cases}$$

The operator C describes the institutional constraints.[6] Given these constraints, the strategy pair (ψ,μ) must satisfy the following "political fixed point" problem.

$$(\psi(s_t),\mu(s_t)) \in C(U(s_t;\ \psi,\mu), s_t),\ \forall s_t \qquad (2)$$

The companion paper of Lagunoff (2005b) focuses on finding solutions to the political fixed point problem. If, indeed, satisfactory solutions may be found, the model can determine if and when institutional change takes place. For instance, when is it true that dictators relinquish power, i.e., $\mu(\omega_t,\theta^B) = \theta^A$? When is it true that democracies turn over power to dictators, i.e., $\mu(\omega_t,\theta^A) = \theta^B$? In certain cases, the answer is available.

Proposition *Let (ψ,μ) be a political fixed point such that in state θ^A, there is some voter (e.g., the median voter) for whom the Condorcet Winning choice is this voter's most preferred decision for each ω. Then each political state $\theta \in \{\theta^A,\theta^B\}$ is politically stable in the sense that $\mu(\omega,\theta) = \theta$ for every ω.*

The proof is omitted since the result is a special case of Theorem 1 also in Section 4.[7] The intuition is: by maintaining the current political state, the current pivotal decision maker (either the dictator in θ^B or the pivotal voter in θ^A) holds on to power. By doing so, the decision problem reduces to a single agent dynamic programming problem. It is well known

[5]The transition q is assumed to satisfy the standard measurability assumptions.

[6]Recall that the set of Condorcet Winners is defined by all pairs (p_t,θ_{t+1}) that satisfy: for all $(\hat{p},\hat{\theta})$,

$$|\{i \in I:\ U_i(s_t;\ \psi,\mu)(\hat{p},\hat{\theta}) > U_i(s_t;\ \psi,\mu)(p_t,\theta_{t+1})\}| \ \leq \ \frac{n}{2}.$$

[7]A special case of this result also appears in Jack and Lagunoff (2003).

in such problems that the resulting sequence of decisions is optimal from the decision maker's point of view.

The Proposition makes use of two critical assumptions, one explicit, the other implicit. The explicit assumption is that in either political rule, the identity of the pivotal decision maker does not vary over time and states. To get a better sense of this explicit requirement, consider these examples.

Example 2a. Suppose n is odd and the stage game payoffs at date t are given by $u_i = \omega_t + y_i(1 - p_t)$, $\forall i$ where y_i is a fixed asset such as land holdings, p_t is a flat tax on income, and ω_t is a public good. The public good in period $t+1$ is produced (deterministically) from tax revenues in period t according to $\omega_{t+1} = (p_t \sum_j y_j)^\gamma, \quad \gamma < 1$. Individuals' land holdings are ordered: $y_1 < y_2 < \cdots < y_n$. With these payoffs, it is not hard to show that the Condorcet Winner is the preferred policy of the voter with median land value, y_m, and the identity of this individual never changes over time. Hence, it is easy to show that $\psi(\omega, \theta) = Ay_m^{-1/(1-\gamma)}$ if $\theta = \theta^A$ and $= Ay_1^{-1/(1-\gamma)}$ if $\theta = \theta^B$, and where $A = (\gamma\delta(\sum_j y_j)^\gamma)^{1/(1-\gamma)}$ is a constant. In either case, the political rule is stable since the pivotal decision maker never has an incentive to delegate decision authority to a different individual whose implicit preferences over policy is different than his own.

Example 2b. Now suppose that Example 1a is slightly modified so that assignments of land to individuals vary over time. To make the point as simply as possible, suppose that land rotates deterministically: in period t, individual i has land value y_{i+t}, modulo n.[8] With land rotation, the pivotal decision maker in the majority rule institution, θ^A, also rotates. In this case, the government is akin to a dynamically inconsistent decision maker, whose preferences over income streams changes over time. However, whenever Individual 1 has the median land value y_m under this rule, he is pivotal at that date, and so he will surely choose θ^B, thus remaining the dictator from that point onward. By choosing θ^B, Individual 1 can commit to installing himself as the permanent, dynamically consistent decision maker thereafter. It may also be the case that, say, Individual 2 with land y_2 close to y_1 may choose θ^B (receiving a "close-by" tax rate) rather than continue the rotation scheme of θ^A. In any case, θ^B is stable, while θ^A admits reform.

The implicit assumption is, of course, that there is only one decision maker in the game at each date. In particular, there is no private sector, no other individuals to make choices to offset the pivotal decision maker's choices. In the absence of a private sector, there is no time-consistency problem here. Sequentially rational decisions are optimal.

Significantly, the absence private decisions is the defining feature of a *totalitarian* government. This is the traditional definition, according to which inalienable rights of private

[8]If, for instance, $t = 154$ and $n = 40$, then $i = 5$ has income y_{39}.

individuals do not exist. Hence, even democracies can be totalitarian if *all* choices are filtered through the voting mechanism. The Proposition therefore asserts that both of these political rules are stable in a totalitarian state! Naturally, most governments are not thought to be totalitarian to this extreme, and so we introduce a private sector in the general model that follows.

3 The General Model

Here we introduce the general model. Our specification adds both private sector decisions and a fairly arbitrary space of political institutions to the existing framework of the previous section.

3.1 The Private Sector

We first introduce private sector decisions. Let e_{it} denote i's private decision at date t, chosen from a compact feasible set E. A profile of private decisions is $e_t = (e_{1t}, \ldots, e_{nt})$. These decisions may capture any number of activities, including labor effort, savings, or investment activities. They may also include "non-economic" activities such as religious worship or one's participation in a social revolt. The distinction between e_{it} and p_t is that while the latter is collectively determined, the former is chosen individually.

To express the dependence of payoffs and technology in private sector decisions, let $u_i(\omega_t, e_t, p_t)$ denote i's stage payoff and let $q(B|\ \omega_t,\ e_t,\ p_t)$ denote the probability that ω_{t+1} belongs to the (Borel measurable) subset $B \subseteq \Omega$, both given the economic state ω_t, the private decision profile e_t, and the policy p_t. Given that economic states evolve according to q, each individual's dynamic objective is to maximize average discounted payoff,

$$E\left[\sum_{t=0}^{\infty} \delta^t (1-\delta)\ u_i(\omega_t,\ e_t,\ p_t\ )\right] \tag{3}$$

Next, we specify a set Θ that represents the feasible political institutions. Obviously, there are many more institutions that one might consider than merely "dictatorships" and "democracies". We will assume only that Θ is a compact subset of a Euclidean space. As before, $s_t = (\omega_t, \theta_t)$ denotes the composite state at date t. Let $S = \Omega \times \Theta$ denote the composite state space. Finally, let $s_0 = (\omega_0, \theta_0)$ denote the initial state.

We now complete our specification of strategies and dynamic payoffs. To make the theory tractable, we restrict attention to Markov strategies. Such strategies only encode the payoff-relevant states of the game. Consequently, individuals are not required to coordinate on the history of past play.

Recall that $\psi : S \to P$ and $\mu : S \to \Theta$ describe the policy and institutional strategies, respectively. Together they constitute the *public sector strategies.* A *private sector strategy* for individual i is a function $\sigma_i : S \to E_i$ that prescribes private action $e_{it} = \sigma_i(s_t)$ in state s_t. Let $\sigma = (\sigma_1, \ldots, \sigma_n)$. The strategy profile is therefore summarized by the triple

$$\pi \;\equiv\; (\overbrace{\qquad\sigma,\qquad}^{\text{private sector profile}} \overbrace{\qquad\psi,\qquad}^{\text{policy strategy}} \overbrace{\qquad\mu\qquad}^{\text{institutional strategy}})$$

Individual i's *recursive payoff* in $s_t = (\omega_t, \theta_t)$ given profile π is defined by:

$$V_i(s_t;\ \pi) \;=\; (1-\delta)u_i(\omega_t, \sigma(s_t), \psi(s_t)\ ) \;+\; \delta \int V_i(\omega_{t+1}, \mu(s_t); \pi) dq(\omega_{t+1}|\ \omega_t, \sigma(s_t), \psi(s_t)\ ) \quad (4)$$

The function V depends on and varies with arbitrary Markov strategy profiles $\pi = (\sigma, \psi, \mu)$. Along an equilibrium path (defined below), the function V_i defines a Bellman's equation for citizen i.

Individual i's *recursive payoff function* in $s_t = (\omega_t, \theta_t)$ given profile π is a function $U_i(s_t, \pi) : P \times \Theta \to \mathbb{R}$ defined by

$$U_i(s_t, \pi)(p_t, \theta_{t+1}) \;\equiv\; (1-\delta)u_i(\omega_t, \sigma(s_t), p_t) \;+\; \delta \int V_i(\omega_{t+1}, \theta_{t+1}; \pi) dq(\omega_{t+1}|\ \omega_t, \sigma(s_t), p_t\ ) \quad (5)$$

3.2 Political Rules

From Equation (5), let $U(s_t, \pi) = (U_i(s_t, \pi)\ )_{i \in I}$ denote a profile of recursive payoff functions. An index θ describes a political institution that maps profiles $U(s_t, \pi)$ into public sector decisions. However, it is more useful to define a political institution that maps from arbitrary preference profiles — not just profiles $U(s_t, \pi)$ generated from the Markov strategies π in the dynamic model.

Following standard conventions, we will find it useful to drop time subscripts, and adopt instead the use of primes, e.g., θ' to denote subsequent period's variables, θ_{t+1}, and so on. An arbitrary payoff function is denoted by v_i, expressing the payoff $v_i(p, \theta')$ over current policy p and next period's political state, θ'. In the dynamic model of the previous section, v_i is a notational shorthand for $v_i(p, \theta') = U_i(s;\ \pi)(p, \theta')$. Let $\mathcal{V}$ denote the set of all profiles, $v = (v_1, \ldots, v_n)$.

A *class of political rules* corresponds to state-contingent social choice correspondence

$$C : \mathcal{V} \times S \rightarrow\rightarrow P \times \Theta$$

that associates to each state and each profile, v, of payoff functions, a set $C(v, s)$ of public decisions. If $(p, \theta') \in C(v, s)$, then (p, θ') is a feasible public sector decision under C. Each particular political rule in the class C is given by $C(\cdot, s)$.

The construction of C is broad enough to cover a large variety of institutional reforms, including those mentioned in the Introductory Section. Some examples below illustrate the degree of breadth (the Reader who prefers to skip the details of the examples can proceed, without losing the main ideas, to Section 3.3.)

Example 3a. Voting over the Voting Rule.

Supermajority rules are widely used, particularly to effect large or constitutional changes in policy. In formal work, Messner and Polborn (2002) examine endogenous supermajority rules in an OLG setting. The recursive approach in the present framework entails that current super-majority voting rule determines which supermajority rule is used in the future. In the present framework, operator C can capture this idea as follows. In each state $s = (\omega, \theta)$, the political state θ identifies the fraction, $\theta \geq 1/2$ of individuals required to pass a public decision. Let $(p, \theta') \in C(v, s)$ if for all $(\hat{p}, \hat{\theta}')$,

$$|\{i \in I : \ v_i(\hat{p}, \hat{\theta}') > v_i(p, \theta')\}| \ \leq \theta n$$

The example can be modified further so that the supermajority required for changing the policy is distinct from the supermajority for changing the current political rule (as in the U.S. constitution): let $\theta = (\theta^a, \theta^b)$, whereby θ^a is the supermajority required to determine policy, while θ^b is the supermajority required to determine the subsequent rule. Then $(p, \theta') \in C(v, s)$ if, for all $(\hat{p}, \hat{\theta}')$, EITHER

$$|\{i \in I : \ v_i(\hat{p}, \hat{\theta}') > v_i(p, \hat{\theta}')\}| \ \leq \theta^a n \ \text{ OR } \ |\{i \in I : \ v_i(\hat{p}, \hat{\theta}') > v_i(\hat{p}, \theta')\}| \ \leq \theta^b n$$

Example 3b. Voting over the Voting Franchise.

Consider the case of an endogenous voting franchise. The political state θ identifies the subset of individuals who currently possess the right to vote (the voting franchise). The chosen public decision is the one that is majority preferred within this restricted group. Each restricted voting franchise today uses a majority vote to determine what group of individuals have the right to vote tomorrow. Hence, the current voting franchise decides on a new voting franchise in the following period. This mechanism for change captures the phenomenon of franchise expansion which occurred throughout Europe in the 19th century and in which new groups of voters were "let in." Typically, these expansions took the form of relaxed wealth or property qualifications for voting. Formally, $\Theta \supseteq 2^I$, and let $(p, \theta') \in C(v, s)$ if for all $(\hat{p}, \hat{\theta}')$,

$$|\{i \in \theta : \; v_i(\hat{p}, \hat{\theta}') > v_i(p, \theta')\}| \; \leq \; \frac{1}{2}|\theta|$$

This case conforms precisely to the political model of Jack and Lagunoff (2003), but it also generalizes earlier models of endogenous enfranchisement by Justman and Gradstein (1999) and Acemoglu and Robinson (2000, 2001). In both of these models, $\Theta = \{S, I\}, S \subset I$. The interpretation is that an elite group S chooses either to maintain the status quo, or to make an irreversible, all-or-nothing extension of the vote to the entire population, I. In the Acemglu-Robinson model, the extension represents a commitment to lower taxes on the peasantry. The commitment is credible since the identity of the median voter is permanently altered. In the Justman-Gradstein OLG model a franchise extension is assumed to lower production costs. The key differences with both models here is that the process is potentially gradual (partial extensions are possible) and potentially reversible. In contrast to the Justman-Gradstein approach, the institutional decision here does not enter into either technology or preferences directly.[9]

An important subclass of these rules is the class of **Delegated Dictatorship** rules studied in Jack and Lagunoff (2003). Under these rules, θ varies only over the singletons $\{i\}$, $i = 1, \ldots, n$. In each such state, the current dictator chooses his most preferred policy and then delegates the decision to possibly a new dictator in the future. If the date t describes the length of a generation, then delegated dictator rule might be useful for describing a particular process of dynastic succession in which the king anoints his own successor.

Example 3c. Voting over the Scope of Government.

Finally, consider an endogenous choice of public sector scope. Each period, society draws a line between private and public sector. The political state therefore identifies the domain of public decisions. Privatization of public land, for example, reduce the public sector while broadening surveillance capabilities of police increases it. Formally, we set $\theta \subset P$ so that θ denotes the set of feasible policies. Let $(p, \theta') \in C(v, s)$ if $p \in \theta$ and for all $(\hat{p}, \hat{\theta}')$ satisfying $\hat{p} \in \theta$,

$$|\{i \in I : \; v_i(\hat{p}, \hat{\theta}') > v_i(p, \theta')\}| \; \leq \frac{1}{2}n$$

[9]This last attribute — whether or not institutional choice is an instrumental choice — also differentiates the present approach from the club choice models of Roberts (1998, 1999), and Barbera, Maschler, and Shalev (2001).

3.3 Dynamic Political Games

Define a *Dynamic Political Game (DPG)* to be the collection

$$G \equiv \langle \overbrace{(u_i)_{i\in I}, q, E, P, \Omega}^{\text{economic structure}}, \overbrace{\Theta, C}^{\text{political structure}}, \overbrace{s_0}^{\text{initial state}} \rangle$$

The class of dynamic political games (DPGs) constitutes a broad set of problems in which institutional changes occur endogenously and incrementally. The "economic structure," i.e., $(u_i)_{i\in I}, q, E, P$ and Ω is found in any standard stochastic game. The addition is the "political structure" given by Θ and C. The class of political rules is described by operator C. The operator C is defined on the profiles of recursive payoff functions in each state. In turn, these payoff functions (given by (5)) depend on future strategies.

For tractability we restrict attention in the rest of the paper to dynamic political games G that satisfy: Θ, P, and E are all compact, convex subsets of Euclidian spaces, Ω is a convex set, and u_i and q are continuous in (ω, e, p).

Definition 1 An *Equilibrium* of a dynamic political game, G, is a profile $\pi = (\sigma, \psi, \mu)$ of Markov strategies such that for all states $s = (\omega, \theta)$,

(a) *Private decision rationality*: For each citizen i, and each private strategy $\hat{\sigma}_i$,

$$V_i(s;\ \pi)\ \geq\ V_i(s;\ \sigma_{-i}, \hat{\sigma}_i, \psi, \mu\ ) \tag{6}$$

(b) *Political fixed points*: The public decision pair $(\psi(s), \mu(s)\ )$ satisfies the political fixed point problem

$$(\psi(s), \mu(s)\ )\ \in\ C\,(U(s, \pi),\ s\ ) \tag{7}$$

Part (a) is the standard Markov Perfection property of a stochastic game. Private sector actions are individually optimal in each state. Part (b) asserts the existence of political fixed points. Public sector decisions are required to be consistent with political rules in the class C. In keeping with the standard definition of a stochastic game, both types of decisions are simultaneous. Therefore, an equilibrium of a DPG requires Markov Perfection from individuals' private sector choices and recursive consistency of public sector choices with a political rule. The latter requirement must hold in each state s, and so the consistency condition also satisfies a "perfection" constraint.

4 Institutional Reform vs Institutional Stability

The term *institutional reform* refers to the idea that institutions are deliberately modified. Given an equilibrium $\pi = (\sigma, \psi, \mu)$, *a political state θ is said to admit institutional reform in π if there exists a (set of positive measure) ω such that $\mu(\omega, \theta) \neq \theta$.* Alternatively, *a political state θ is institutionally stable in π* if it does not admit reform, i.e, if $\mu(\omega, \theta) = \theta$ for (almost) every ω.

When do political rules admit reform? When are they stable? In this Section, we give some preliminary answers that depend on exogenous features of the dynamic political game. Our main result, stated just below, rests on two assumptions: *dynamic consistency of rules* and *inessentiality of the private sector* (underlined in Theorem 1). We first state the result without defining these two terms. The remainder of this Section then defines and discusses the importance of these assumptions, and elaborates upon the logic of the result. The proof is in the Appendix.

Theorem 1 *Consider a dynamic political game in which the class, C, of political rules is single valued. Suppose that*

(i) C is dynamically consistent; and

(ii) private decisions are inessential in some rule θ.

Then θ is institutionally stable in every equilibrium.

4.1 Dynamically Consistent Rules

It is well known what is meant for an individual decision maker to be dynamically consistent. Long taken for granted in decision models, dynamic consistency presumes that future decision makers' points of view coincide with that of the present decision maker if the latter were to called upon to make the decision in that future period. A large literature has emerged recently to evaluate dynamically *in*consistent decision-making from an individual's perspective. What does it mean for a *rule* to be dynamically consistent?

The most natural definition is one in which the political rule "inherits" the dynamic consistency of the individual participants. In turn, this entails that the political rule can be rationalized by a dynamically consistent social welfare criterion.

Formally, a class of rules C is *(partially) rationalized by* a social welfare function F :

$\mathbb{R}^n \times S \to \mathbb{R}$ if

$$C(v,s) = (\supseteq)\arg\max_{p,\theta'} F(v(p,\theta'),s)$$

By itself, rationalizing C by a social welfare function imposes very little restriction. Clearly, the Delegated Dictator Rule (Example 3b) is rationalized by v_θ where θ identifies the dictator. When political rules are voting rules, as in Examples 3a-3c, then two well known conditions, either single peaked preferences or order restriction, imply that C is rationalized by the preferences of a Median Voter.[10]

Defining dynamic consistency of a rule is now straightforward. Consider any additively separable public sector payoffs: payoffs function of the time separable form, $v(p_t,\theta_{t+1}) = (1-\delta)v^1(p_t) + \delta\int_{\omega_{t+1}} v^2(\omega_{t+1},\theta_{t+1})d\eta$ where η is a probability measure on the states ω_{t+1}. (Clearly, dynamic payoffs in the present model satisfy this requirement). A class of political rules, C, will be said to be *dynamically consistent* if it is partially rationalized by a continuous social welfare function F that satisfies in every state $s_t = (\omega_t,\theta_t)$,

$$F\left((1-\delta)v^1(p_t) + \delta\int_{\omega_{t+1}} v^2(\omega_{t+1},\theta_{t+1})d\eta,\ s_t\right) = (1-\delta)F\left(v^1(p_t),\ \theta_t\right) + \delta\int_{\omega_{t+1}} F\left(v^2(\omega_{t+1},\theta_{t+1}),\ \theta_t\right)d\eta$$

This definition embodies two critical properties. First, F is time and state separable and linearly homogeneous in the discount weights $(1-\delta,\delta)$.[11] Second, F does not vary directly with the economic state. Consequently, each political state θ is uniquely associated with a political rule. Both properties are needed to guarantee that the rate of intertemporal substitution between the current period payoff and next period coincides with the rate of substitution between any other pair of successive payoff-dates.

It is easy to see that the class of rules in Example 2a is dynamically consistent, while the class in Example 2b is not. The Examples 2a-3c are only dynamically consistent in particular circumstances (see Lagunoff (2005b), Theorem 2). Nevertheless, most dynamic models of government policy implicitly maintain assumptions of dynamic consistency.[12]

4.2 An Inessential Private Sector

In the Pure Public Sector Model of Section 2, it was shown that equilibria never admit institutional reform. It seems clear from that model that the private sector matters somehow. Here, we try to make clear what this means.

[10]The first conditions originated with Arrow (1951) and Black (1958). The second is due Rothstein (1990), although similar results can be found in applications of single crossing properties by Roberts (1977) and by Gans and Smart (1996).

[11]Broader definitions of dynamic consistency do not require the linear homogeneity in discount factor requirement. The results here are amenable to a more general definition, but at a cost in transparency.

[12]Two exceptions are Krusell, Kuruscu, and Smith (2002) and Lagunoff (2005a), both of which examine policies chosen by dynamically inconsistent governments.

First, we maintain the assumption C is rationalized by some F. However, dynamic consistency is not required for any of the subsequent definitions.

Next, the following notation is required. For any initial economic state ω, a *feasible* continuation payoff profile is a measurable function $x : \Omega \to [0, K)^n$ for which there exists an arbitrary pair of (measurable) Markov functions σ and ψ defined only on Ω such that for each $i = 1, \ldots, n$,

$$x_i(\omega) = E\left[\sum_{t=0}^{\infty}(1-\delta)\delta^t u_i(\omega_t, \sigma(\omega_t), \psi(\omega_t)) \;\middle|\; \omega = \omega_0\right],$$

where E is the expectation on $\{\omega_t\}$ induced by the transition law q. Feasible continuations are those that could be reached by any pair of Markov strategies, equilibrium or otherwise. Now given any feasible continuation x, define for each i,

$$H_i(\omega, e, p, x) \equiv (1-\delta)u_i(\omega, e, p) + \delta \int x_i(\omega')dq(\omega'|\omega, e, p).$$

Let $H = (H_i)_{i=1}^n$. Notice that $H(\cdot, e, p, x)$ is itself a feasible continuation profile.

Definition 2 Fix the political state θ. Formally, we will say that the *private sector decisions are inessential in* θ if the following holds for any economic state ω, and any feasible continuation profile, x: for every private sector profile e there exists a public sector decision p such that

$$F\left(H(\omega, e, p, x),\ s\right) \;\geq\; F\left(x(\omega),\ s\right) \tag{8}$$

We refer to $F\left(x(\omega),\ s\right)$ as a *feasible social continuation* in state $s = (\omega, \theta)$. In words, private sector decisions are inessential in θ if in any economic state and for any private sector decision, any feasible social continuation in any date $t+1$ can always be reached or exceeded in date t by a public sector decision. Informally, private decisions are inessential if their effects on social welfare can always be replaced by those of the public sector. The definition utilizes the recursive idea that a given social payoff available today, is always available tomorrow *if* the political aggregation rule (indexed by θ) is still available.

Example 4a. An extreme case was examined in Section 2. When there is no private sector (i.e., the totalitarian state), then private decisions are, by definition, inessential. A less extreme example is one where u, F, and θ satisfy:

$$\forall\omega,\ \forall e, \qquad \max_p F\left(u(\omega, e, p),\ s\right) \;=\; \max_{p,e} F\left(u(\omega, e, p),\ s\right) \tag{9}$$

In other words, the social welfare function, evaluated at the stage game payoffs, can achieve its optimum in each state by varying p alone. In this case, the "absence" of the private sector need only occur at the optimum. It is straightforward to verify that any dynamic political game satisfying (9) has an inessential private sector.

To specialize further, suppose stage payoffs can be expressed as $u_i(\omega, z)$ where $z = \max\{e_1, \ldots, e_n, p\}$. To be concrete, there are public and private internet service providers, and individuals care only about the one providing the highest quality. If $P \supset E$, then this satisfies (9), and so the private sector is inessential.

Example 4b. Private sector inessentiality may be less exceptional than the previous example suggests. Consider a case in which all decisions are filtered through the public sector as follows: let $P = E^n$ and let $BR_i(p_{-i}; v) = argmax_{p_i} v_i(p_i, p_{-i}, \theta')$ denoting the i's best response correspondence given payoff profile, v. Then define F by

$$F(v(p, \theta'), s) \;=\; -||p - BR(p; v)\;)||^2$$

In this example, the feasible policies are vectors of private decisions. The rule then prescribes public sector decisions that mimic exactly what each individual would do if the decision for his private action were his alone, taking as given the actions of others (the "Nash rule"). This is a case of a rule that decentralizes decision authority, effectively leaving the decisions to the individuals themselves. Because the rule formally governs all decisions, the private sector is inessential.[13]

4.3 Logic of the Result

The conclusion of the Theorem is that in any equilibrium, the type θ is institutionally stable, i.e., $\mu(\omega, \theta) = \theta$. According to the Theorem, reforms occurs only if either the rule is dynamally inconsistent or the private sector actions are *essential.* The Proposition in Section 2 is a special case.

The logic of the argument is straightforward. Interpret the parameter θ as a player — the social planner — whose payoff is $F(\cdot, \theta)$. We refer to this player as the "institutional type." In any state $s = (\omega, \theta)$, the institutional type θ chooses the public policy p and designates a subsequent decision maker, type θ', the following period. A standard result of dynamic programming is that if type θ faces a single agent decision problem, it need never designate the decision authority over future policies to another player. Its own choice of policies $\{p_t\}$ would be optimal in each realized state. To put it another way, Type θ is willing to relinquish its authority over future decisions *only if* its designated choice can induce a more favorable response from actions of others. Consequently, inessentiality of the private sector collapses the problem to a pure policy — hence single agent — decision problem. The "planner" of type θ can unilaterally reach any alternative social payoff using policies alone. Hence, this type need never relinquish decision making authority to another type $\hat{\theta} \neq \theta$. In such a case, the policy-path is optimal for type θ if that same type makes decisions each period. The institutional type θ is therefore stable.

[13]One can show, however, that this F is not dynamically inconsistent.

5 Recursively Self Selected Institutions

Even if the private sector is essential, change need not occur if the private sector response matters in a way that favors the current political institution. Moreover, it may be the case that change may or may not occur because the equilibrium selects "good equilibria" in certain states and "bad equilibria" in others.

This Section addresses some of these issues. In particular, we assume that all feasible rules are dynamically consisent and proceed to characterize stability and reform in terms of a simple incentive compatibility condition, treating institutional types as potential players in the game.

For any fixed continuation profile, x, and any state $s = (\omega, \theta)$, one can identify an $(n+1)$-player normal form game defined by the following payoffs,

$$(H_1(\omega, e, p, x), \ldots, H_n(\omega, e, p, x), F(H(\omega, e, p, x),\ \theta)\ )$$

Here, each Player $i = 1, \ldots, n$ is a participant in the original DPG and has payoff $H_i(\omega, e, p, x)$. With payoff $F(H(\omega, e, p, x), \theta)$, Player $n+1$ is the "social planner" of type θ who maximizes welfare function $F(\cdot,\ \theta)$. Following a convention in the stochastic games literature, we refer to this game as an *auxiliary game*. Let $N(s, x)$ denote the set of Nash equilibria of this auxiliary game. A standard result establishes that under stated assumptions, $N(s, x)$ is a closed and nonempty set. Since feasible choice sets are compact, $N(s, x)$ is as well.

Recall that the choice of next period's institutional type θ_{t+1} is viewed as a strategic delegation decision. In any auxiliary game, one can view the delegation decision as a choice between equilibria from the sets $N(\omega, \theta, x)$ and $N(\omega, \hat{\theta}, x)$ for any pair θ and $\hat{\theta}$. However, multiplicity of equilibrium potentially muddles the comparison. Suppose, for instance, the current type θ_t chose θ simply because a "good" equilibrium was subsequently played in $N(\omega, \theta, x)$ while a "bad" equilibrium was subsequently played in $N(\omega, \hat{\theta}, x)$. Then θ_t's choice would arise largely come from a coordination failure and little else. To avoid this "modeler's fiat" as a basis for the analysis, we restrict attention in each auxiliary game to Nash equilibria that satisfy

$$\max_{(e,p)\in N(\omega,\hat{\theta},x)} F\left(H(\omega, e, p, x),\ \theta\right) \tag{10}$$

Notice that if $\theta \neq \hat{\theta}$, then type θ is not the institutional player in the auxiliary game.

Consider a continuation profile x^θ that defines a fixed point of the map in (10), i.e, a profile x^θ that satisfies:

$$F(x^\theta(\omega),\ \theta) \quad = \quad \max_{(e,p)\in N(\omega,\theta,x^\theta)} F\left(H(\omega, e, p, x^\theta),\ \theta\right), \qquad \forall\omega$$

Here, $F(x^\theta(\omega),\ \theta)$ is the largest social welfare for type θ over all possible Nash equilibria of the auxiliary game with x^θ as the continuation profile and with the same type θ as institutional

Player. Existence of a fixed point problem in (10) is a nontrivial problem[14] Yet, finding a fixed point x^θ is tantamount to finding a "restricted equilibrium" of the DPG in which μ is required to be stable, i.e, $\mu(\omega, \theta) = \theta$ for all ω. Clearly, if there exists an (unrestricted) equilibrium in the DPG then there exists a "restricted equilibrium," and, consequently, a fixed point x^θ.

If (10) admits a fixed point, then there exists a function F^* defined on arbitrary triples $(\omega, \hat{\theta},\ \theta)$ by

$$F^*(\omega, \hat{\theta},\ \theta) \quad = \quad \max_{(e,p) \in N(\omega, \hat{\theta}, x^\theta)} F\left(H(\omega, e, p, x^\theta),\ \theta\right), \qquad \forall \omega \tag{11}$$

The payoff $F^*(\omega, \hat{\theta}, \theta)$ is the largest social welfare for type θ over all possible Nash equilibria of the auxiliary game in which $\hat{\theta}$ is the institutional player and x^θ is the continuation.

Definition 3 Type θ is *recursively self-selected (RSS)* if there exists a function F^* defined by (11) such that for all ω and all $\hat{\theta}$,

$$F^*(\omega, \theta,\ \theta) \ \geq\ F^*(\omega, \hat{\theta},\ \theta)$$

Type θ is *recursively self-denied (RSD)* by type $\hat{\theta} \neq \theta$ if for all ω,

$$F^*(\omega, \hat{\theta},\ \theta) \ >\ F^*(\omega, \theta,\ \theta)$$

Recursive self-selection describes an implicit incentive constraint on the institutional types. A recursively self-selected (RSS) θ is an institutional type that would never delegate decision authority in the public sector to another type, regardless of the realized state. A recursively self-denied (RSD) θ is a type that never delegates decision authority to itself in the subsequent period's game in any state.

Both RSS and RSD limit consideration only to one-shot delegation decisions, reverting back to auxiliary games in which type θ is the player thereafter. This restriction is sensible for checking for an institution's stability in light of the well-known one-shot deviation principle. Consequently, it is not difficult to show that if the private sector is inessential in θ, then θ is RSS. The converse does not generally hold, and so RSS is a weaker condition.

Surprisingly, recursively self-denied types do not necessarily admit reform. Consider, for example, a dynamic political game with three types: $\theta_1, \theta_2, \theta_3$. Suppose that θ_1 is recursively self-denied by type θ_2, which, in turn, is recursively self-denied by type θ_3. Suppose that $F^*(\omega, \theta_1, \theta_1) \ >\ F^*(\omega, \theta_3, \theta_1),\ \ \forall \omega$, and $\mu(\omega, \theta_2) = \theta_3$ for all ω. If δ is close enough to one, then it can easily be shown that type θ_1 is stable. Basically, type θ_1 does not delegate to θ_2 because θ_2 is expected to delegate to θ_3 the following period. The failure to induce reform is due the fact that recursive self-denial is not transitive. Type θ_1 *would* delegate to θ_2 if it were

[14] Results on this can be found in the companion paper, Lagunoff (2005b).

certain that θ_2 were stable. Generally, θ_2 cannot commit *not* to delegate further. The lack of commitment leads to stability of θ_1.

In a related paper, Jack and Lagunoff (2003) construct parametric examples with this time consistency problem: an elite chooses very conservatively to expand the voting franchise precisely because a larger franchise subsequently expands rights farther than is desirable for this elite. In light of these problems, the following result gives sufficient (though less than satisfactory) conditions for reform.

Theorem 2 *Fix a DPG in which C is a single valued and dynamically consistent class of political rules. Suppose the DPG admits at least one equilibrium. Then:*

(i) If a rule θ is recursively self-selected, then there exists an equilibrium in which θ is institutionally stable;

(ii) If a rule θ is recursively self-denied by another rule $\hat{\theta}$ which is itself recursively-self selected, then there exists an equilibrium in which θ admits institutional reform.

(iii) If a rule θ is recursively self-denied by every other rule, then there exists an equilibrium in which θ admits institutional reform.

The result provides two scenarios in which reforms can occur. First, a type admits reform if it is self-denied by a stable type. In that case, the aforementioned commitment problem does not arise. Second, a type admits reform if it is self-denied by all other types. In this case, the public sector is completely ineffective. So much so, that any form of delegation is preferred by type θ.

In both scenarios, future private sector decisions compensate for the loss of control by type θ in the policy arena. The best outcome from type θ's point of view is one where it delegates decision authority to another type. In order to induce the preferred alternative, the social planner must "buy-off" the private sector by delegating future decision authority to another "type" $\hat{\theta} \neq \theta$. The delegation represents a strategic commitment in which a type cedes control over public decisions to gain more favorable treatment from decisions over which it is has no direct control. Hence, "Player" θ designates a new "player", $\hat{\theta}$, in order to elicit the desired response from the private sector.

6 An Example

Consider a modification of Example 2a in Section 2 in which private sector behavior plays a role. In this example, based largely on an example in Jack and Lagunoff (JL) (2003, Section

5.1), $u_i = \omega_t + y_i(1-p_t) - e_{it}^2$ where e_{it} is effort at date t. The state evolves deterministically according to $\omega_{t+1} = (p_t \sum_j y_j)^\gamma \sum_j e_{jt}$ with $0 < \gamma < 1$. In Example 2a, next period's public good ω_{t+1} was produced from this period's tax revenue, $p_t \sum_j y_j$, generated from a fixed asset such as land. The policy instrument was a flat tax p_t on land value. In the present example, tax revenue combines with aggregate effort to produce a public good. A concrete example is public literacy. Everyone values general literacy, but it requires both public investment and individual effort.

As before, the land holdings of the n individuals are ordered according to $y_1 < \cdots < y_n$. For convenience, let $y_1 > 0$ and extend the class of rules as follows. Define C to behave *as if* the current political rule were a "dictator" with wealth anywhere in the interval $[y_1, y_n]$. Consequently, $\theta \in [y_1, y_n]$.

An equilibrium is found as follows. Let π be any Markov strategy, and $s = (\omega, \theta)$ the current state. The recursive payoff function of both policy and effort is $(1-\delta)[\omega + y_i(1-p) - e_i^2] + \delta V_i(s'; \pi)$ in which ω' is determined technologically and θ' is chosen by $\theta = y_i \in [y_1, y_n]$.

Claim *Institutional type $\theta = y_1$ is the unique, recursively self selected political rule. Any other type θ with $y_1 < \theta \le y_n$ is recursively self denied by some other type $\theta' < \theta$. Finally, if $\gamma > 1/2$ then an equilibrium $\pi = (\sigma, \psi, \mu)$ exists, and it is given by*

$$\begin{aligned} \psi(\theta) &= \left[\frac{2}{\delta^2 \gamma n (\sum_j y_j)^{2\gamma}}\right]^{1/(2\gamma-1)} \theta^{1/(2\gamma-1)} \equiv C\ \theta^{1/(2\gamma-1)}, \\ \sigma_j(\theta) &= \frac{\delta(\sum_j y_j)^\gamma}{2} C^\theta\ \theta^{\gamma/(2\gamma-1)}, \quad \textit{and} \end{aligned} \tag{12}$$

and $\mu(\theta) = \bar{\mu}\theta$ where $\bar{\mu} < 1$ is a constant given by the implicit solution to

$$\bar{\mu} = \frac{n}{2n-1}\left[\frac{1+\delta-\delta\bar{\mu}^{1/(2\gamma-1)}}{1-\delta\bar{\mu}^{1/(2\gamma-1)}}\right]\left[\frac{1-\delta\bar{\mu}^{2\gamma/(2\gamma-1)}}{1+\delta-\delta\bar{\mu}^{2\gamma/(2\gamma-1)}}\right] \tag{13}$$

The Claim asserts properties of RSS and RSD rules. Apart from these properties, the equilibrium is largely the same as the one derived in JL. Figure 1 illustrates the simple dymamics of the equilibrium institutional rule. In Figure 1, a unique stable political rule, $\theta = y_1$ (in the Figure, y_1 is small) is reached in finitely many periods. Intuitively, rich policy makers are recursively self-denied by poorer ones because the rich policy maker's chosen tax rate is too small to induce much private investment in literacy from the population.

To verify the Claim, observe first that the state enters linearly in the stage payoff. This means that a more convenient expression of the recursive payoff function can be derived by grouping the subsequent period's state with the current stage payoff. Define $V_i^*(s;\ \pi) \equiv V_i(s; \pi) - \omega$. Notice, that the state variable in the right hand side of this equation cancels

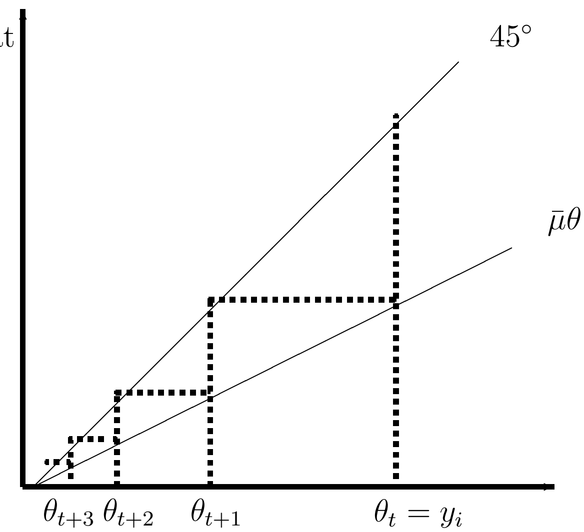

Figure 1: A Linear Rule in Land Holdings

out. Consequently, V^* does not vary with ω and can be expressed a function of θ alone. The recursive payoff function then becomes $(1-\delta)[y_i(1-p)-e_i^2+\delta(p\sum_j y_j)^\gamma \sum_j e_{jt}] + \delta V_i^*(\theta';\ \pi)$. Fixing $\theta = y_i$, the first order conditions in p and e_i imply the policy and private sector strategies of the form in (12). Notice that if $\gamma > 1/2$ then tax rates and private effort increase in the wealth, θ, of the (implicit) dictator. If $\gamma < 1/2$ then tax rates and effort decrease in θ.

With strategies in (12), the recursive payoff used by the current dictator is

$$V_i^*(\theta;\ \pi) \ = \ (1-\delta)\left[y_i(1-C\ \theta^{1/(2\gamma-1)}) + A\theta^{2\gamma/(2\gamma-1)}\right] \ + \ \delta V_i^*(\theta';\ \pi) \tag{14}$$

where $A \equiv \frac{1}{4}(2n-1)\delta^2 C^{2\gamma}(p\sum_j y_j)^{2\gamma}$, a positive constant.

Consider now the special case where θ is stable, i.e., $\mu(\theta) = \theta$. Then V_i^* reduces to

$$V_i^*(\theta\ \pi) = x_i^\theta = \left[y_i(1-C\ \theta^{1/(2\gamma-1)}) + A\theta^{2\gamma/(2\gamma-1)}\right]$$

It follows that when $\theta = y_i$ is the institutional type, then

$$F^*(\omega,\hat{\theta},\theta) = (1-\delta)\left[\theta(1-C\ \hat{\theta}^{1/(2\gamma-1)}) + A\hat{\theta}^{2\gamma/(2\gamma-1)}\right] + \delta\left[\theta(1-C\ \theta^{1/(2\gamma-1)}) + A\theta^{2\gamma/(2\gamma-1)}\right]$$

Now evaluate $dF^*(\omega,\hat{\theta},\theta)/d\hat{\theta} = 0$. A solution is given by linear function, $\hat{\theta} = \frac{n}{2n-1}\theta$. Hence, we have shown that institutional type θ is recursively self denied by any institutional type $\theta' \in [\frac{n}{2n-1}\theta, \theta)$. Since types are RSD only by lower types, it follows that the lowest type y_1 is recursively self selected, hence (by Theorem 2) stable. It also follows that all types $\theta \in (y_1, \frac{2n-1}{n}y_1]$ admit reform.

To find the equilibrium institutional rule, μ, we guess and verify a linear solution $\theta' = \bar{\mu}\theta$. Using the equation for V^* in (14), the recursive equilibrium payoff is of the form

$$V_i^*(\theta;\ \pi) = \sum_{t=0}^{\infty}(1-\delta)\delta^t \left[y_i(1 - C\ (\bar{\mu}^t\theta)^{1/(2\gamma-1)}) + A(\bar{\mu}^t\theta)^{2\gamma/(2\gamma-1)}\right] \tag{15}$$

Taking first order conditions and substituting for C and A, we verify that $\theta' = \bar{\mu}\theta$ is an equilibrium for $\bar{\mu}$ that satisfies (13). It remains to show that a solution to (13) exists. To verify this final step, observe that as $\bar{\mu}$ varies from 0 to 1, the left side (13) is continuously increasing from 0 to 1; the right side of (13) is continously decreasing from $n/(2n-1)$ and approaches 0 asymptotically. Hence, by the Intermediate Value Theorem, a solution $\bar{\mu} \in (0, 1)$ exists.

7 Summary

This paper introduces a dynamic recursive framework in which political institution are instrumental objects of choice each period. It is shown how the framework may be used evaluate questions of institutional stability and/or reform and equilibrium existence.

Political institutions are shown to be stable if they are dynamically consistent and the private sector is inessential or if rule is recursively self-selected. Alternatively, political institutions are shown to admit reform if the rule is recursively self-denied by a stable rule or by all rules. Clearly, the "intermediate" environments where the private sector is sometimes essential and rules are sometimes self selected are obvious focal points for future work.

In general, dynamic, recursive models of political aggregation are not new. One of the first is the pioneering work of Krusell, Quadrini, and Ríos-Rull (1997). More recent examples include Klein, Krusell, and Ríos-Rull (2002) and Hassler, et. al. (2003).[15] In this literature the institution itself is fixed. Usually, some form of majority voting is assumed, and so the "political fixed point" problem outlined above can be resolved in certain cases when the policy space is single dimensional.

A few papers examine dynamic models of voting that specifically allow for multi-dimensional choice spaces (though keeping the voting mechanism fixed). These include Bernheim and Nataraj (2002), Kalandrakis (2002), and Banks and Duggan (2003).

[15]See Persson and Tabellini (2001) for other references.

The present framework in necessarily multi-dimensional. The political fixed point problem is compounded by fact that different institutions each have possibly distinct requirements achieving recursive consistency. Nevertheless, under certain conditions, the problem is resolved when political rules are dynamically consistent.

Theorems 1 and 2 are a first step toward unifying a small but growing literature on dynamically endogenous institutions. Primarily, this literature concerns progressive expansion of voting rights. Under the "external conflict" explanation of Acemoglu and Robinson (2000), the voting franchise historically was extended by an elite to head off social unrest. In their model there are only two types: a restricted voting franchise of the elite and a full, universal manhood suffrage. Public decisions in the restricted franchise may be undercut by the threat of revolt. Consequently, the political state of the restricted franchise was self-denied by the stable state of the universal franchise.[16]

According to the "internal conflict" explanation of Lizzeri and Persico (2002), rights are extended to gain support when there is ideological or class conflict among the elite. Jack and Lagunoff (2003) construct an example of this in which taxes sustain investment in public literacy. Conflict between the median voter within the elite, and the population median individual's private investment in literacy leads to an expansion of voting rights. A variant of this example with "delegated dictatorship" is presented in Section 7.

There are, of course, many cases in which political rules are dynamically inconsistent. One useful set of examples is the set of wealth-weighted "voting" schemes examined by Jordan (2002). These rules give rise to dynamically inconsistent choice because they vary with current income distribution. Jordan characterizes the (static) core and stability of the "wealth-is-power" rule whereby policies are entirely determined by those with the most wealth.

There are also many more cases in which political rules are neither recursively self-selected or self-denied. So far, we produce no general results on these intermediate cases. Nevertheless, the intuition above suggests that reform or stability depends on a clearer understanding of the incentive constraints on the institutional "types." Endless possibilities exist for future research.

8 Appendix

Proof of Theorem 1 Fix a dynamic political game and a political state θ. Suppose that private decisions are inessential in state θ. Fix an equilibrium $\pi = (\sigma, \psi, \mu)$. We proceed to

[16] Though it is not a model of endogenous institutions per se, Powell (2003) constructs a dynamic game in which a temporarily weak government may lack credibility to induce another government to restrain its inefficient use of power such as launching a coup or attacking.

show that there will be no political reform in any economic state ω, i.e., $\mu(\omega,\theta)=\theta$ for all ω.

Since C is single valued and dynamically consistent, there is some social criterion F that rationalizes C and is invariant to the economic states ω. By definition,

$$\{(\psi(\omega,\theta),\mu(\omega,\theta))\}=C(U(\omega,\theta;\pi),s)=\arg\max_{p,\theta'}F(U(\omega,\theta;\pi)(p,\theta'),\theta)$$

In other words,

$$F(V(\omega,\theta;\pi),\theta)\ \equiv\ F(U(\omega,\theta;\pi)(\psi(\omega,\theta),\mu(\omega,\theta)),\ \theta)\ \geq\ F(U(\omega,\theta;\pi)(p,\theta'),\theta),\ \forall(p,\theta') \tag{16}$$

Inessentiality of private decisions means that for each ω', and each continuation x, for any private profile e there exists a public sector policy p such that (8) holds. In particular, choose some $\hat{\theta}\neq\theta$ and let $x(\cdot)=V(\cdot,\hat{\theta};\ \pi)$. Given some ω', let $e=\sigma(\omega',\theta)$. Using the definition of inessentiality, we assert that exists p the following string of equalities and inequalities holds.

$$\begin{aligned}
& F\left(V(\omega',\hat{\theta};\pi),\ \theta\right)\\
=\ & F(x(\omega'),\ \theta)\\
\leq\ & F(H(\omega',\sigma(\omega',\theta),p,x),\ \theta)\\
=\ & F\left((1-\delta)u(\omega',\sigma(\omega',\theta),p)+\delta\int x(\omega'')dq(\omega''|\ \omega',\sigma(\omega',\theta),p),\ \theta\right)\\
=\ & F\left((1-\delta)u(\omega',\sigma(\omega',\theta),p)+\delta\int V(\omega'',\hat{\theta};\pi)dq(\omega''|\ \omega',\sigma(\omega',\theta),p),\ \theta\right)\\
\leq\ & \max_{\tilde{p},\tilde{\theta}}\ F\left((1-\delta)u(\omega',\sigma(\omega',\theta),\tilde{p})+\delta\int V(\omega'',\tilde{\theta};\pi)dq(\omega''|\ \omega',\sigma(\omega',\theta),\tilde{p}),\ \theta\right)\\
=\ & F\left((1-\delta)u(\omega',\sigma(\omega',\theta),\psi(\omega',\theta))+\delta\int V(\omega'',\mu(\omega,\theta);\ \pi)dq(\omega''|\ \omega',\sigma(\omega',\theta),p),\ \theta\right)\\
=\ & F(V(\omega',\theta;\pi),\ \theta)
\end{aligned} \tag{17}$$

The first equality in (17) follows from the definition of feasible continuation x. The first *in*equality follows by Inessentiality of the private sector. The next two equalities follow from the definitions of payoff profile H and by x, respectively. The second (and last) inequality is obvious. The last two equalities follow from the definitions of equilibrium and the construction of recursive payoffs, respectively. We have therefore shown

$$F(V(\omega',\theta;\pi),\theta)\ \geq\ F\left(V(\omega',\hat{\theta};\pi),\theta\right) \tag{18}$$

However, since (18) holds for each economic state ω' and since $\hat{\theta}$ is arbitrary, it follows that for all $\hat{\theta} \neq \theta$,

$$\int F\left(V(\omega', \theta; \pi), \theta\right) \, dq(\omega' | \, \omega, \sigma(\omega, \theta), \psi(\omega, \theta)) \; \geq \; \int F\left(V(\omega', \hat{\theta}; \pi), \theta\right) \, dq(\omega' | \, \omega, \sigma(\omega, \theta), \psi(\omega, \theta)) \,. \tag{19}$$

Using dynamic consistency and Equation (19), we find that for all $\hat{\theta} \neq \theta$,

$$\begin{aligned}
& F\left(U(\omega, \theta; \pi)(\psi(\omega, \theta), \theta), \theta\right) \\
= \; & F\left((1-\delta)u(\omega, \sigma(\omega, \theta), \psi(\omega, \theta)), \theta\right) + \delta \int F\left(V(\omega', \theta; \pi), \theta\right) dq(\omega' | \, \omega, \sigma(\omega, \theta), \psi(\omega, \theta) \,) \\
\geq \; & (1-\delta) F\left(u(\omega, \sigma(\omega, \theta), \psi(\omega, \theta)), \theta\right) + \delta \int F\left(V(\omega', \hat{\theta}; \pi), \theta\right) dq(\omega' | \, \omega, \sigma(\omega, \theta), \psi(\omega, \theta) \,) \\
= \; & F\left(U(\omega, \theta; \pi)(\psi(\omega, \theta), \hat{\theta}), \theta\right)
\end{aligned} \tag{20}$$

Hence, we have shown that $\mu(\omega, \theta) = \theta$ satisfies (20). But because C is single valued, it is the only such solution. Consequently, μ *must* satisfy $\mu(\omega, \theta) = \theta$. We therefore conclude that inessentiality of private decisions implies no political reform. ∎

Proof of Theorem 2, Part (i).

The following definition will prove useful for subsequent results.

Definition 4 An equilibrium π is *institutionally optimal for* θ^* if for any other equilibrium $\hat{\pi}$ and for each $s = (\omega, \theta)$,

$$F(V(\omega, \mu(s); \; \pi), \; \theta^*) \; \geq \; F(V(\omega, \hat{\mu}(s); \; \hat{\pi}), \; \theta^*) \tag{21}$$

The institutionally optimal equilibrium is the one that generates the most preferred social welfare for type θ^* in each state among all equilibria. The idea behind institutional optimality extends the best-case comparison to the full dynamic model. Clearly, if an equilibrium exists, then it is clear from the definition that an institutionally optimal one exists as well.

We show that if θ is recursively self selected, then there exists an institutionally optimal equilibrium in which θ is stable. Let $\pi = (\sigma, \psi, \mu)$ denote any institutionally optimal equilibrium. Now let μ^* denote an institutional strategy that satisfies:

$$\begin{aligned}
\mu^*(\omega, \hat{\theta}) &= \mu(\omega, \hat{\theta}), \quad \forall \omega, \ \forall \hat{\theta} \neq \theta, \text{ and,} \\
\mu^*(\omega, \theta) &= \theta, \quad \forall \omega,
\end{aligned}$$

Clearly, μ^* differs from μ in that it is stable in political state θ.

Choose some $\hat{\theta} \neq \theta$. Fix an arbitrary ω' in the full measure set on which recursive self selection holds. We will proceed to verify that the following string of equalities and inequalities hold.

$$\begin{aligned}
& F(V(\omega, \theta; \pi), \ \theta) \\
= \ & (1-\delta) F\left(u(\omega, \sigma(\omega, \theta), \psi(\omega, \theta)), \ \theta\right) + \delta \int F\left(V(\omega', \mu(\omega, \theta); \ \pi), \ \theta\right) dq(\omega' |\, \omega, \sigma(\omega, \theta), \psi(\omega, \theta)) \\
\geq \ & (1-\delta) F\left(u(\omega, \sigma(\omega, \theta), \psi(\omega, \theta)), \ \theta\right) + \delta \int F\left(V(\omega', \theta; \ \pi), \ \theta\right) dq(\omega' |\, \omega, \sigma(\omega, \theta), \psi(\omega, \theta)) \\
= \ & (1-\delta) F\left(u(\omega, \sigma(\omega, \theta), \psi(\omega, \theta)), \ \theta\right) \\
& + (1-\delta)\delta \int F\left(u(\omega', \sigma(\omega', \theta), \psi(\omega', \theta)), \ \theta\right) dq(\omega' |\, \omega, \sigma(\omega, \theta), \psi(\omega, \theta)) \\
& + \delta^2 \int \int F\left(V(\omega'', \mu(\omega', \theta); \ \pi), \ \theta\right) dq(\omega'' |\, \omega', \sigma(\omega', \theta), \psi(\omega', \theta)) dq(\omega' |\, \omega, \sigma(\omega, \theta), \psi(\omega, \theta)) \\
\geq \ & (1-\delta) F\left(u(\omega, \sigma(\omega, \theta), \psi(\omega, \theta)), \ \theta\right) \\
& + (1-\delta)\delta \int F\left(u(\omega', \sigma(\omega', \theta), \psi(\omega', \theta)), \ \theta\right) dq(\omega' |\, \omega, \sigma(\omega, \theta), \psi(\omega, \theta)) \\
& + \delta^2 \int \int F\left(V(\omega'', \theta; \ \pi), \ \theta\right) dq(\omega'' |\, \omega', \sigma(\omega', \theta), \psi(\omega', \theta)) dq(\omega' |\, \omega, \sigma(\omega, \theta), \psi(\omega, \theta)) \\
\vdots \ & \ \vdots \\
\geq \ & F^*(\omega, \theta, \ \theta) \\
\geq \ & F^*(\omega, \hat{\theta}, \ \theta)
\end{aligned}$$

$$
\begin{aligned}
= &\ (1-\delta)F\left(u(\omega,\sigma(\omega,\hat{\theta}),\psi(\omega,\hat{\theta})),\ \theta\right)+\delta\int F^{*}(\omega',\theta,\ \theta)dq(\omega'|\,\omega,\sigma(\omega,\hat{\theta}),\psi(\omega,\hat{\theta}))\\
\geq &\ (1-\delta)F\left(u(\omega,\sigma(\omega,\hat{\theta}),\psi(\omega,\hat{\theta})),\ \theta\right)+\delta\int F^{*}(\omega',\mu(\omega,\hat{\theta}),\ \theta)dq(\omega'|\,\omega,\sigma(\omega,\hat{\theta}),\psi(\omega,\hat{\theta}))\\
\geq &\ (1-\delta)F\left(u(\omega,\sigma(\omega,\hat{\theta}),\psi(\omega,\hat{\theta})),\ \theta\right)\\
&+(1-\delta)\delta\int F\left(u(\omega',\mu(\omega,\hat{\theta})),\psi(\omega',\mu(\omega,\hat{\theta}))),\ \theta\right)dq(\omega'|\,\omega,\sigma(\omega,\mu(\omega,\hat{\theta})),\psi(\omega,\mu(\omega,\hat{\theta})))\\
&+\delta^{2}\int\int F^{*}(\omega'',\mu(\omega',\mu(\omega,\hat{\theta})),\ \theta)dq(\omega''|\,\omega',\sigma(\omega',\mu(\omega,\hat{\theta})),\psi(\omega',\mu(\omega,\hat{\theta})))dq(\omega'|\,\omega,\sigma(\omega,\hat{\theta}),\psi(\omega,\hat{\theta}))\\
\vdots &\ \ \vdots\\
\geq &\ F\left(V(\omega,\hat{\theta};\pi),\ \theta\right)
\end{aligned}
\tag{22}
$$

The string in equalities and inequalities in (22) are justified as follows. The first equality (second expression) in (22) follows by definition of $V(\omega,\theta;\pi)$ and dynamic consistency of F. The first *in*equality (in the third expression) follows by definition of equilibrium. The second equality (fourth expression) follows again by the definition of $V(\omega,\theta;\pi)$ and dynamic consistency of F (applied recursively). The second inequality (fifth expression) follows by again by definition of equilibrium applied recursively. The third inequality follows by repeating the above substitutions recursively and iterating forward. The fourth inequality (seventh expression) follows by recursive self selection (RSS) of θ. The third equality (eighth expression) expands the expression using dynamic consistency. The fifth and sixth inequalities (nineth and tenth expressions, resp.) follows from the (recursive) application of the hypothesis that θ is RSS. The last inequality (and last expression) follows by iterating forward.

Using the same argument as in the Proof of Theorem 1 after Inequality (18 and to the end of Expression (20), we find that

$$F\left(U(\omega,\theta;\pi)(\psi(\omega,\theta),\mu^{*}(\omega,\theta)\ ),\theta\right)\ \geq\ F\left(U(\omega,\theta;\pi)(\psi(\omega,\theta),\hat{\theta}),\theta\right)$$

or, equivalently, by dynamic consistency

$$
\begin{aligned}
&(1-\delta)F\left(u(\omega,\sigma(\omega,\theta),\psi(\omega,\theta)),\ \theta\right)+\delta\textstyle\int F\left(V(\omega',\mu^{*}(\omega,\theta);\ \pi),\ \theta\right)dq(\omega'|\,\omega,\sigma(\omega,\theta),\psi(\omega,\theta))\\
\geq &\ (1-\delta)F\left(u(\omega,\sigma(\omega,\theta),\psi(\omega,\theta)),\ \theta\right)+\delta\int F\left(V(\omega',\hat{\theta};\ \pi),\ \theta\right)dq(\omega'|\,\omega,\sigma(\omega,\theta),\psi(\omega,\theta))
\end{aligned}
\tag{23}
$$

But since

$$V(\omega, \mu^*(\omega,\theta);\ \pi) \equiv (1-\delta)u(\omega, \sigma(\omega,\theta), \psi(\omega,\theta)) + \delta \int V(\omega', \mu(\omega,\theta);\ \pi) dq(\omega'|\ \omega, \sigma(\omega,\theta), \psi(\omega,\theta))$$

we can, by dynamic consistency, iteratively substitute μ^* in place of μ. Iterating forward in this way, we show $\pi = (\sigma, \psi, \mu^*)$ is, in fact, an equilibrium in which θ is stable. ∎

Parts (ii) and (iii) of Theorem 2.

Suppose that θ is recursively self denied (RSD) by either (a) another recursively self selected (RSS) type, or (b) every other type. Our proof works for both cases. Once again, let π be an institutionally optimal equilibrium. Suppose, by contradiction, that θ is stable in π, i.e., $\mu(\omega,\theta) = \theta$ for almost all ω. Since θ is RSD by either (a) a RSS type $\hat{\theta}$, or by (b) every type $\hat{\theta} \neq \theta$, we choose $\hat{\theta}$ in one or the other category such that

$$F^*\left(\omega, \hat{\theta},\ \theta\right)\ >\ F^*\left(\omega, \theta,\ \theta\right),\ \ \forall \omega\ .$$

Since π is assumed stable, i.e., $\mu(\omega,\theta) = \theta$ on a set of ω with full measure, fix a state ω in that set. Then we verify that the following string of equalities and inequalities hold.

$$\begin{aligned}
& F(V(\omega,\theta;\ \pi),\ \theta\ ) \\
= \ & (1-\delta)F\left(u(\omega,\sigma(\omega,\theta),\psi(\omega,\theta)),\ \theta\right) + \delta \int F\left(V(\omega',\mu(\omega,\theta);\pi),\ \theta\right) dq(\omega'|\ \omega,\sigma(\omega,\theta),\psi(\omega,\theta)) \\
= \ & (1-\delta)F\left(u(\omega,\sigma(\omega,\theta),\psi(\omega,\theta)),\ \theta\right) + \delta \int F\left(V(\omega',\theta;\pi),\ \theta\right) dq(\omega'|\ \omega,\sigma(\omega,\theta),\psi(\omega,\theta)) \\
= \ & (1-\delta)F\left(u(\omega,\sigma(\omega,\theta),\psi(\omega,\theta)),\ \theta\right) \\
& +(1-\delta)\delta \int F\left(u(\omega',\sigma(\omega',\theta),\psi(\omega',\theta)),\ \theta\right) dq(\omega'|\ \omega,\sigma(\omega,\theta),\psi(\omega,\theta)) \\
& +\delta^2 \int\int F\left(V(\omega'',\theta;\ \pi),\ \theta\right) dq(\omega''|\ \omega',\sigma(\omega',\theta),\psi(\omega',\theta))dq(\omega'|\ \omega,\sigma(\omega,\theta),\psi(\omega,\theta)) \\
\vdots \ & \ \vdots
\end{aligned}$$

$$
\begin{aligned}
&= F^*(\omega,\theta,\ \theta\ ) \\
&< F^*(\omega,\hat\theta,\ \theta\ ) \\
&= (1-\delta)F\left(u(\omega,\sigma(\omega,\hat\theta),\psi(\omega,\hat\theta)),\ \theta\right)+\delta\int F^*(\omega',\theta,\ \theta)dq(\omega'|\,\omega,\sigma(\omega,\hat\theta),\psi(\omega,\hat\theta)) \\
&< (1-\delta)F\left(u(\omega,\sigma(\omega,\hat\theta),\psi(\omega,\hat\theta)),\ \theta\right)+\delta\int F^*(\omega',\mu(\omega,\hat\theta),\ \theta)dq(\omega'|\,\omega,\sigma(\omega,\hat\theta),\psi(\omega,\hat\theta)) \\
&< (1-\delta)F\left(u(\omega,\sigma(\omega,\hat\theta),\psi(\omega,\hat\theta)),\ \theta\right) \\
&\quad +(1-\delta)\delta\int F\left(u(\omega',\mu(\omega,\hat\theta)),\psi(\omega',\mu(\omega,\hat\theta))),\ \theta\right)dq(\omega'|\,\omega,\sigma(\omega,\mu(\omega,\hat\theta)),\psi(\omega,\mu(\omega,\hat\theta))) \\
&\quad +\delta^2\int\int F^*(\omega'',\mu(\omega',\mu(\omega,\hat\theta)),\ \theta)dq(\omega''|\,\omega',\sigma(\omega',\mu(\omega,\hat\theta)),\psi(\omega',\mu(\omega,\hat\theta)))dq(\omega'|\,\omega,\sigma(\omega,\hat\theta),\psi(\omega,\hat\theta)) \\
&\vdots\quad\vdots \\
&< F\left(V(\omega,\hat\theta;\pi),\ \theta\right)
\end{aligned}
\tag{24}
$$

The string in equalities and inequalities are justified as follows. The first equality (second expression) in (24) follows by definition and dynamic consistency of F. The second equality (in the third expression) follows from the hypothesis that the equilibrium π is stable in θ. The third equality (fourth expression) follows by recursive application of this hypothesis. The fourth equality (fifth expression) follows by iterative application of this hypothesis and the definition of F^*. The first *in*equality (sixth expression) follows by the hypothesis that θ is RSD by $\hat\theta$. The second inequality *in*equality (seventh expression) follows from the recursive application of the hypothesis that θ is RSD. Notice here that if $\hat\theta$ is itself RSS then the equilibrium is either stable in $\hat\theta$, i.e, $\mu(\omega',\hat\theta)=\hat\theta$ (such an equilibrium exists by the previous Theorem), OR $\mu(\omega',\hat\theta)\neq\hat\theta$ and $F^*\left(\omega',\mu(\omega,\hat\theta),\ \theta\right) > F^*(\omega',\theta,\ \theta)$ for all ω'. Otherwise, π could not have been institutionally optimal due to the availability of the $\hat\theta$-stable equilibrium. Of course, if θ is RSD against every other type $\hat\theta$, then clearly θ is RSS against $\mu(\omega',\hat\theta)$. The third equality (eighth expression) applies the prior reasoning (that θ is RSD by either $\hat\theta$ which is itself RSS or by every other type) recursively. Finally, the last *in*equality (and last expression) follows by iterating forward.

From (24) we therefore conclude

$$F\left(V(\omega,\hat\theta;\ \pi),\ \theta\right)\ >\ F(V(\omega,\theta;\ \pi),\ \theta\ )$$

And, since ω was chosen arbitrarily (from a full measure set), it follows that on a set of ω with positive measure, some set of

$$\int F\left(V(\omega',\hat{\theta};\pi),\ \theta\right) dq(\omega'|\,\omega,\sigma(\omega,\theta),\psi(\omega,\theta)) \ > \ \int F\left(V(\omega',\theta;\pi),\ \theta\right) dq(\omega'|\,\omega,\sigma(\omega,\theta),\psi(\omega,\theta))$$

But this clearly contradicts the supposition that $\mu(\omega,\theta)=\theta$ is an equilibrium institutional rule.

We conclude that $\mu(\omega,\theta)\neq\theta$ on a set ω with positive measure, and so θ admits institutional reform in $\pi=(\sigma,\psi,\mu)$. ∎

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