The Killing Game: Reputation and Knowledge in Politics of Succession*

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Preliminary, comments are very welcome

Abstract

The winner of a battle for a throne can either execute or spare the loser; if the loser is spared, he contends the throne in the next period. Executing the losing contender gives the winner an additional quiet period, but then his life is at risk if he loses to some future contender. The trade-off is analyzed within an infinite-time complete information game. Conditions which govern equilibrium behavior of agents are identified. Our theory predicts that we would witness more killings along the succession lines in countries where a 'circle of potential contenders' is limited and that executions of the predecessor are autocorrelated. In particular, with a dynastic rule in place, incentives to kill the predecessor are much higher than in a non-hereditary dictatorships, e.g. in 19th century Latin America. Our analysis of historical material demonstrates that long succession lines indeed exhibit patterns as predicted by our model.

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"[Domitian] used to say that the lot of princes was most unhappy, since when they discovered a conspiracy, no one believed them unless they had been killed."

Suetonius "Life of Domitian"

1 Introduction

On December 23, 1989, Nicolae Ceauşescu, a Romanian communist leader for 22 years, fled his residence in the presidential palace in Bucharest and was captured by army troops that revolted after mass protests against his rule erupted the day before. On December 25, after a two-hour military trial, he and his wife, a former first deputy prime minister and the President of Romanian Academy of Science, faced a firing squad.[1] What did those who captured and executed Ceauşescu have in mind? Why would not they wait for a regular process of justice, which might have very likely ended the same way? This kind of perfectly real problems involved in leadership dynamics (Bueno de Mesquita et al., 2003, Gallego and Pitchik, 2004, Acemoglu and Robinson, 2003) outside the democratic world (Olson, 1993, Tullock, 1987, Acemoglu and Robinson, 2005) enables us to attempt to assess fundamental theoretical issues: what is reputation and what is knowledge in historical perspective, and what are their workings in generation of history-dependence (North, 1981).[2]

What is the downside of executing the enemy when it is possible and then enjoying a period of quietness? The downside is that in that case, the current decision-maker might

[1] Though not a necessary consequence of a coup d'etat, a violent death of the fallen leader in a short period after the coup is definitely not an isolated phenomenon. Examples of countries that have witnessed at least two killings of the fallen leaders during the last 50 years include Afghanistan (Mohammad Daoud, 1978, Mohammad Taraki, 1979, Hafizullah Amin, 1979), Bangladesh (Mujibur Rahman, 1975, Khalid Musharaf, 1975, Ziaur Rahman, 1981), Iraq (Faisal II, 1958, Nuri as-Said, 1958, Abdul Karim Kassem, 1963), Nigeria (Abubakar Tafawa Balewa, 1966, Johnson Aguiyi-Ironsi, 1966, Murtala Ramat Muhammad, 1974), Comoros (Ali Soilih, 1978, Ahmed Abdallah Abderemane, 1989), and Liberia (William Tolbert, 1980, Samuel Doe, 1990). For numerours historical examples see Section .4.

[2] Recent theoretical models of dictatorships include Bueno de Mesquita *et al* (2003), Wintrobe (1998), Grossman and Noh (1990), Acemoglu, Robinson and Verdier (2004), Galetovic and Sanhueza (2000), Overland, Simmons, and Spagat (2000), and Restrepo and Spagat (2001). Empirical investigation of military coup dynamics include Longredan and Poole (1990) and earlier works by Luttwak (1979), Ferguson (1978), and O'Kane (1978). Domínguez (2002) contains an excellent overview of descriptive political science literature on modern dictatorships (see also Linz and Chehabi, eds., 1998).

be executed himself once removed from power. Sparing the life of a person who lost a fight against the incumbent makes his rule more troubling in the short-run (he will for certain have a powerful enemy alive), but then he will enjoy a higher probability of being spared himself once he loses a fight in the future. Formally, any decision by a rational agent incorporates future enemies' opinion of him as a result of his actions. If dictator A executed his predecessor, then dictator B, who eventually takes over A, is likely to kill A, being concerned about bloody reputation of A. This reputation would indeed matter for B, the decision-maker at the moment, since if A is spared then, upon taking power back from B, A would likely execute B. (Or, more precisely, this is what B would most naturally expect from A basing on A's reputation.) One immediate result is that once somebody takes over a bloody dictator, he might be 'bound' to become a bloody dictator himself.

Economists are often concerned with problems that could be conceptualized best in classic economic terms. E.g., games of reputation are analyzed as games of a long-term monopoly against players who play only once but observe the entire previous history (Kreps and Wilson, 1982, Milgrom and Roberts, 1982, Fudenberg and Kreps, 1982, Fudenberg and Levine, 1989, Sorin, 1999, and Ely, Fudenberg, and Levine, 2004). The game setup we consider has a political science origin and has no straightforward IO parallel; still, the machinery developed within economic theory is most suitable for our analysis. And while economic theorists' assumptions are often stylized to the point where they hardly approximate real decision-making problems, in reality people do face the binary problem we investigate. As General Gelu Voican Voiculescu, appointed by the Romanian revolutionary government to supervise the trial and burial of the Ceauşescus, testified in 1995: "The decision to try the couple was dictated by desire to survive – either them, or us."

What definitely makes our theory a part of the 'reputation' literature is that the cost of executing a certain action is associated with the equilibrium response of a future player. There is a long tradition in economic science to study reputation in games with incomplete information, starting from seminal contributions of Kreps and Wilson (1982) and Milgrom and Roberts (1982). We depart from this tradition and argue that many behavioral aspects of reputation could be successfully studied in a complete information environment. Fudenberg and Kreps (1987) compare a standard reputation-building model (in which prior reputations are fixed) to a model in which the opponents do not observe how the long-term player played against other opponents.[3] Another departure from the economic tradition in our model is that we have an infinite number of (potentially) long-term players, though at each moment only two of them interact.

What social traits should be captured by a general theory of reputation? First, reputation is something that changes over time. Second, a proper conceptualization of reputation requires some sort of monotonicity: the more incidents of a certain action are committed by the reputation-builder, the stronger is his reputation.[4] However, reputation makes little sense unless we allow at least one of the two actions (the one that enhances reputation and the one that ruins it) to be verifiable someway. In the situation we focus on, it is easy to pretend to be bloody (e.g. by revealing a dead body), and hard to pretend to be enlightened (unless one is truly enlightened). Once we have introduced this asymmetry, it becomes reasonable to focus exclusively on equilibria in which contenders do not 'believe' in actions that can be faked. In our model, this rules out equilibria where the dictator can reduce the probability of being executed by committing more killings. This brings the intuitively appealing monotonicity with respect to decisions players make.

Though there is indeed a certain similarity between classic 'reputation' literature and this model, we stress that here, reputation concern is more important for the players. In contrast with the model of a long-term player facing a sequence of short-term ones, in our model every agent cares about his reputation. Moreover, he needs to take into account the effect of his actions as an input into future players' decision processes. Indeed, the decision to kill or to spare the current loser requires understanding of not only the loser's strategy (if he is spared), but also of future contenders' strategies. In the long-term monopoly models, the short-term players need to think about future short-term players' strategies, since this might affect the current monopoly's incentives to maintain reputation, but they do not care about their own reputation.

Of course, in the real world, the same reputation might be good in countering some types of threats and detrimental in other situations. For example, a reputation of 'toughness' or

[3] In economic theory, strategic dynamic interaction is most often modeled as a repeated game with a fixed number of participating agents. In our setup, in every decision node the deciding agent could always leave forever the interaction with *this particular* opponent. Lagunoff and Matsui (1995, 2004) introduce a concept of an 'asynchronously repeated game'. Our model is an example of such game.

[4] Of course the consequences of reputation might be non-monotonic as, e.g., in Diamond (1991) or, most recently, in Ely and Valimaki (2004).

'cruelty' has at least two effects for a ruler: First, potential contenders might be less willing to become actual ones. Second, if potential contenders might be either strong or weak, the reputation for toughness makes the selection of actual contenders more strong. The balancing effect of a tough reputation on the incumbent's tenure is, therefore, unclear. In our model, this complication does not arise, since the 'supply side' of contenders is exogenous.

Recently, Acemoglu and Robinson (2001) and Acemoglu (2003) (see also Bueno de Mesquita et al., 2003 and Lagunoff, 2004a) have developed a workable framework for dynamic analysis of political transitions. However, the reliance on Markov-type dynamic models limit their ability to focus on path-dependence, a key concept in North (1989) institutional approach to history.[5] Our focus on reputational concerns allows us to go beyond the existing models of path-dependence.

For our analysis, we restrict the set of equilibria by letting each agent's strategy to depend on the number of killings committed by participants who are active at the moment rather than on the entire history of the game. (See, e.g., Maskin and Tirole, 1998, who argue that it is plausible to restrict the set of perfect equilibria in such a game by allowing agents' strategies to depend on 'pay-off relevant' histories only.) Our focus is on *perfect* equilibria: in any decision node, each agent's strategy is optimal given strategies of other agents. Furthermore, we require that all equilibria are 'fake-murder proof': effectively, this requirement imposes monotonicity with respect to killings of losers. In any such equilibrium, each additional murder implies a higher probability of being punished (so there is no forgiveness or indulgence), until the probability of punishment reaches one. Once it does, fear of additional chance of being punished is no longer in effect, and the agent opts to execute every time.

It appears that these relatively mild restrictions on equilibria allow to get meaningful comparative statics even though the number of equilibria is still large. We define 'maximum patience' of equilibria as the maximum number of killings a dictator can commit while still facing a non-zero probability of being spared if overthrown. This value is well defined (it is always a finite number) and has intuitive properties. The maximum patience is increasing in the probability that a new contender appears in the next period despite execution of the previous one, and is decreasing in the incumbent's survival rate. It also increases in the cost of losing life and decreases in the utility of being in power.

[5] Another approach that allows to go beyond Markov-type dynamic models is suggested in Section 5.7 of Acemoglu and Robinson (2005) and Acemoglu (2005).

Historically, understanding of idea of reputation for executions manifested itself in various laws and constitutional clauses. The idea of restraint in killing defeated political opponents might be the most basic of all constitutional constraints. The problem is addressed in the Hittite Telipinu Proclamation (pp. 194–198, The Context of Scripture I, edited by W.Hallo), which may be the world's oldest existing document of the West. After discussing the excessive number of revolts and civil wars, King Telipinu stipulates that high nobles should not be killed in secret, but should only be killed after a trial before the Council of the realm. Also, when someone is convicted of a treason, his innocent family members should not be killed, and his property should not be confiscated. The reason for the first rule is apparently to avoid provoking pre-emptive revolts, and the second rule could prevent forcing rebel's family members to support his rebellion (which would spur a sequence of reputation-based killings). Abolishment of confiscations might also be aimed at eliminating an incentive for palace bureaucrats to falsely accuse a noble of a treason.

We illustrate the existence of different equilibrium paths and comparative statics results by drawing upon historical examples, including detailed descriptions of the Osmanli dynasty of the Ottoman Empire in 1281–1922 and military dictatorships in Venezuela in 1830–1964. Methodological concerns about analyzing historical narrative and historical data are discussed in detail in a companion paper Egorov, Nye, and Sonin (2005). The dictators' desire to survive and its impact on the quality of government is analyzed in Egorov and Sonin (2004). Acemoglu and Robinson (2005) provide, alongside with numerous historical illustrations, a most up-to-date analytical account of relationship between democracy and dictatorship.

The rest of this paper is organized as follows. In Section 2, we introduce the formal game. Section 3 contains analysis of the game. In Section 4 we illustrate our theory, e.g., the possibility of two pure-strategy equilibria, with historical examples. Section 5 concludes.

2 Formal Setup

2.1 Players, Payoffs, and Timing

We assume time to be discrete, $t = 1, 2, \ldots \infty$. Each player comes and fights with the incumbent; the fight is modeled as a lottery. If a player is the winner, he decides what to do with the loser, kill or spare. The spared loser becomes the contender in the next period. If the loser is executed, there might be no contender in the next period. However, in two periods, a new contender arrives with certainty. We formalize this as follows: there is an infinite sequence of identical players $i = 1, 2, \ldots \infty$, each of which joins the active part of the game sequentially. In each period t, there is one player (the identity of this player is $D_t \in \mathbb{N}$) who is the incumbent dictator in this period. In each period, there may be a player $C_t \in \mathbb{N}$, *the contender* (we write $C_t = 0$ if there is no contender in period t).

In period t, $D_t = 1$ and $C_t = 2$. For each period t, let N_t denote the identity of player with the least number who had not joined the active part of the game yet. For example, $N_1 = 3$. For each period t, let W_t and L_t denote the winner and the loser, respectively.

Denote the instantaneous utility player i receives in period t by $U_t(i)$. We assume that if $i \neq D_t$ and $i \neq C_t$, then $U_t(i) = 0$. In other words, only actively participating players can get a non-trivial utility in the current period. At each period, agent i (actually, only agent W_t) maximizes his life-time utility $U(i) = \sum_{\tau=1}^{\infty} \beta^{\tau} U_{\tau}(i)$, where $\beta < 1$ is the discount factor. In each period t, the sequence of actions and events is as follows.

1. If $C_t \neq 0$, then the contender attempts to become the dictator. If $C_t = 0$, then $W_t = D_t$, $L_t = C_t = 0$, $C_{t+1} = A_t$, and $A_{t+1} = A_t + 1$, and in this case steps $2 - 4$ are skipped.

2. The fight breaks out, and the contender wins with probability $0 < p < 1$. In other words, $\mathbf{P}(L_t = D_t) = \mathbf{P}(W_t = C_t) = p$, and $\mathbf{P}(W_t = D_t) = \mathbf{P}(L_t = C_t) = 1 - p$.

3. W_t decides on his action A_t, whether to execute $(A_t = E)$ or spare $(A_t = S)$ the loser L_t.

4. If $A_t = E$, then $U_t(L_t) = -D$, and with probability $\mu < 1$ there is still a successor in the next period ($C_{t+1} = A_t$ and $A_{t+1} = A_t + 1$), and with probability $1 - \mu$ there is no successor ($C_{t+1} = 0$). If $A_t = S$, then $U_t(L_t) = 0$, and $C_{t+1} = L_t$.

5. The winner gets $U_t(W_t) = Y$, and becomes the next dictator, i.e. $D_{t+1} = W_t$.

2.2 Strategies

First, we introduce the history h_t of period t, which is the 6-tuple $h_t = (D_t, C_t, W_t, L_t, N_t, A_t)$. Also, we denote the projection of h_t on its first 5 components by $\hat{h}_t$, so $\hat{h}_t =$

$(D_t, C_t, W_t, L_t, N_t)$. By the time W_t has to make his decision on A_t, he knows $H_t = \left(\bigtimes_{k=1}^{t-1} h_k\right) \times \hat{h}_t$. We call history H_t *feasible* if it can be a result of some path of the game. The set of feasible histories is denoted by Ξ. For history $\xi \in \Xi$, let $|\xi|$ denote the number of periods in the history (so $|H_t| = t$), and let D_t^ξ, C_t^ξ etc. denote the dictator, contender etc. of period t in history ξ. A strategy of player i is an element of $\bigtimes_{\xi \in \Xi} A_\xi$, where $A_\xi \in \Delta(\{E, S\})$ if the last winner in this history $W_{|\xi|}^\xi = i$ and $L_{|\xi|}^\xi \neq 0$, and $A_\xi \in \{N\}$ otherwise.

The case we are most interested in is where in equilibrium, players pursue strategies from a narrower set. Namely, denote the number of actions $A = E$ chosen by agent i in history ξ by $E_\xi(i)$. Mathematically, $E_\xi(i) = \sum_{t=1}^{|\xi|-1} I\left(W_t^\xi = i, A_t^\xi = E\right)$, where $I(\cdot)$ is an indicator function.

In this paper, we focus on *perfect* equilibria: in any decision node, each agent's strategy is optimal given strategies of other agents. We further restrict the set of equilibria by letting agents strategies depend on the number of killings committed by participants who are active at the moment. (Similarly, Maskin and Tirole, 1998, restrict the set of perfect equilibria in a dynamic duopoly game by allowing agents' strategies to depend on 'pay-off relevant' histories only.) Specifically, we call a strategy $A \in \bigtimes_{\xi \in \Xi} A_\xi$ *stationary*, if for any two histories $\xi, \eta \in \Xi$ such that $E_\xi\left(W_{|\xi|}^\xi\right) = E_\eta\left(W_{|\eta|}^\eta\right)$ and $E_\xi\left(L_{|\xi|}^\xi\right) = E_\eta\left(L_{|\eta|}^\eta\right)$ the equality $A_\xi = A_\eta$ holds.

Denote the expected future utility of winner $W_{|\xi|}^\xi$ of the last period of history ξ by $U_\xi^S\left(W_{|\xi|}^\xi\right)$ if he spares, and by $U_\xi^E\left(W_{|\xi|}^\xi\right)$ if he executes. Denote the expected future utility of the loser by $U_\xi\left(L_{|\xi|}^\xi\right)$ (note that he does not determine his own fate).

In the case of stationary equilibria, we can simplify the notation. For $n \in \mathbb{N} \cup \{0\}$, denote $n^+ \equiv n + 1$. For $m, n \in \mathbb{N} \cup \{0\}$ such that $mn = 0$, let $U_{mn}^E = U_\xi^E\left(W_{|\xi|}^\xi\right)$, if ξ satisfies $W_{|\xi|}^\xi = m$ and $L_{|\xi|}^\xi = n$. This definition is correct, in the sense that U_{mn}^E does not depend on the history, and there exists at least one history that satisfies these properties. Similarly, denote $U_{mn}^S = U_\xi^S\left(W_{|\xi|}^\xi\right)$, and $V_{nm} = U_\xi\left(L_{|\xi|}^\xi\right)$ (the latter is the utility of a loser who has just lost the fight, so index n is the first one to reflect that it is the loser who gets this utility). Denote $U_{mn} = \max\left(U_{mn}^E, U_{mn}^S\right)$. Let $W_{mn} = (1-p)\,U_{mn} + pV_{mn}$, which is simply the expected utility of an incumbent dictator before engaging in a fight. Finally, let α_{mn} be such that $A_\xi = \alpha_{mn}E + (1-\alpha_{mn})\,S$ if ξ satisfies the properties stated above (evidently, this is just the probability of being executed, as perceived by the loser). Since numbers α_{mn}

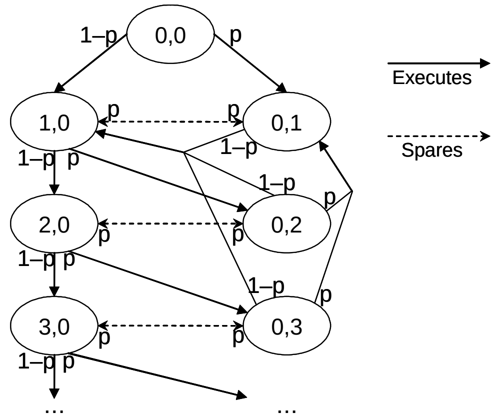

Figure 1: Switching of States of the Game.

define stationary strategies uniquely, we will refer to a stationary strategy by $\boldsymbol{\alpha}$.

Since with stationary strategies everything depends on the number of executions performed by the current dictator and the current opponent, we will say that the game is in the state (m,n) if the current dictator killed m times and his opponent killed n times. In that case, the switching of the game between different states is summarized on Figure 1 (note that neither sparing nor winning changes the state, and corresponding arrows are not shown for the sake of simplicity).

Definition 1 *We say that a stationary equilibrium satisfies the* single-crossing condition *(Milgrom and Shannon, 1994) if for any histories $\xi, \eta \in \Xi$ such that $E_\xi \left(W^\xi_{|\xi|}\right) = E_\eta \left(W^\eta_{|\eta|}\right)$, $E_\xi \left(L^\xi_{|\xi|}\right) < E_\eta \left(L^\eta_{|\eta|}\right)$, and $U^S_\xi \left(W^\xi_{|\xi|}\right) < U^E_\xi \left(W^\xi_{|\xi|}\right)$, the inequality $U^S_\eta \left(W^\eta_{|\eta|}\right) < U^E_\eta \left(W^\eta_{|\eta|}\right)$ is satisfied. Equivalently, for any $m \geq 0$, $U^E_{m0} > U^S_{m0}$ implies $U^E_{m^+0} > U^S_{m^+0}$.*

Definition 2 *A stationary equilibrium is called* monotonic *if for any $m \geq 0$, $\alpha_{m0} \leq \alpha_{m^+0}$.*

Definition 3 *A stationary equilibrium is said to have* non-increasing (with additional murder) utility, *if for any $m \geq 0$, $U_{m0} \geq U_{m^+0}$.*

Suppose now that at any time a player makes a decision whether to spare or execute, he can commit a incidental murder M, instead of playing E or S. This action yields the same payoffs in that period as action S (the loser is not executed, and therefore the winner gets no period of safe rule). However, action M counts against the number of murders he

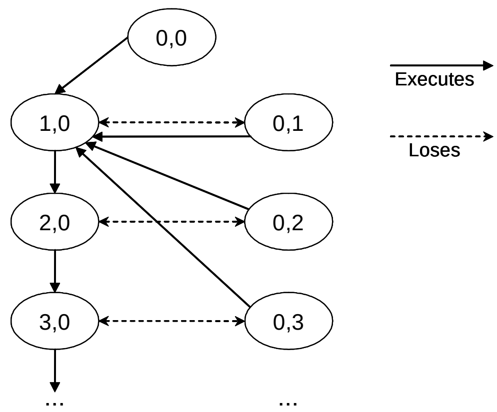

Figure 2: Switching of States as Perceived by the Winner.

committed, i.e. $E_\xi(i)$ is defined as $\sum_{t=1}^{|\xi|-1} I\left(W_t^\xi = i, A_t^\xi \in \{E, M\}\right)$ and gives him no extra utility for any $m \geq 0$, $U_{m0} \geq U_{m^+0}$.

Definition 4 *A stationary equilibrium is said to be* fake murder-proof, *if for any* $m \geq 0$, $U_{m0}^S > U_{m0}^M$, *i.e. if a murder yields no direct utility, it is unprofitable to commit it.*

The rest of the paper is devoted to study of symmetric equilibria in single-crossing stationary strategies. In particular, we shall prove the following general result: *Single-crossing, monotonic, non-increasing utility, and fake murder-proof conditions define the same set of symmetric stationary equilibria* (Theorem 1)*.*

3 Analysis

3.1 Necessary Equilibrium Conditions

Suppose that the winner of the fight finds himself in state (m, n). On Figure 2, we depict, what the state of the world after the next battle will be depending on the action chosen and the result of the next battle.

The analogs of the Bellman equation for our problem look as follows:

$$
\begin{aligned}
U_{mn} &= \max\left(U^E_{mn}, U^S_{mn}\right) && (1)\\
W_{mn} &= (1-p)\,U_{mn} + pV_{mn} && (2)\\
U^E_{mn} &= Y + \beta\left((1-\mu)\left(Y + \beta W_{m^+0}\right) + \mu W_{m^+0}\right) && (3)\\
U^S_{mn} &= Y + \beta W_{mn} && (4)\\
V_{mn} &= (1-\alpha_{mn})\,\beta\left((1-p)\,V_{mn} + pU_{mn}\right) - \alpha_{mn} D && (5)
\end{aligned}
$$

We start with deriving necessary conditions to characterize equilibria that consist of stationary strategies. The purpose of the analysis is twofold. First, it is a prerequisite to full description of equilibria. Second, this will allow us prove the equivalence of equilibria with single-crossing, monotonic, non-increasing utility, and fake murder-proof strategies.

Lemma 1 *Suppose X is one of variables $U_{mn}, U^S_{mn}, U^E_{mn}, V_{mn}, W_{mn}$. Then $-D \leq X \leq \frac{Y}{1-\beta}$.*

Proof. X is the expectation of a discounted sum of numbers, each of which is equal to either $-D, 0$, or Y; the negative term $-D$ may occur only once in this sum. The sum of any series that satisfies this property lies between $-D$ and $\frac{Y}{1-\beta}$. ■

Now consider the function

$$S(\alpha) \equiv \frac{Y\left(1-(1-\alpha)\,\beta\,(1-p)\right) - \beta\alpha p D}{\left(1-\beta\,(1-p)\right) - (1-\alpha)\,\beta\left(1-\beta\,(1-p) - (1-\beta)\,p\right)}.$$

As we will prove, this function is the lower bound for the incumbent's life-time utility.

Lemma 2 *$S(\alpha)$ is a strictly decreasing function.*

Proof. Compute the derivative:

$$\frac{dS(\alpha)}{d\alpha} = -\frac{\beta p\left(\beta p Y + D\,(1-\beta)\,(1-\beta+2\beta p)\right)}{\left(\left(1-\beta\,(1-p)\right) - (1-\alpha)\,\beta\left(1-\beta\,(1-p) - (1-\beta)\,p\right)\right)^2} < 0.$$

■

We will need the following information on the values of the function $S(\alpha)$ later on.

$$
\begin{aligned}
S(0) &\equiv \frac{Y\left(1-\beta\,(1-p)\right)}{(1-\beta)\,(1-\beta+2p\beta)},\\
S(1) &\equiv \frac{Y-\beta p D}{1-\beta\,(1-p)}.
\end{aligned}
$$

The next lemma states its relation to the utility of an agent who opts to spare.

Lemma 3 *$U_{mn} \geq S(\alpha_{mn})$. Moreover, if $U^S_{mn} \geq U^E_{mn}$, then*

$$U_{mn} = S(\alpha_{mn}). \tag{6}$$

Proof. First of all, from (5) it follows that

$$V_{mn} = \frac{(1-\alpha_{mn})\,\beta p U_{mn} - \alpha_{mn} D}{1-(1-\alpha_{mn})\,\beta\,(1-p)}.$$

Also, we use (2) to find that

$$W_{mn} = \frac{1-p-\beta\,(1-\alpha_{mn})\,(1-2p)}{1-(1-\alpha_{mn})\,\beta\,(1-p)} U_{mn} - \frac{\alpha_{mn} p D}{1-(1-\alpha_{mn})\,\beta\,(1-p)}. \tag{7}$$

Therefore, from (4) we get

$$U^S_{mn} = Y + \beta \frac{1-p-\beta\,(1-\alpha_{mn})\,(1-2p)}{1-(1-\alpha_{mn})\,\beta\,(1-p)} U_{mn} - \frac{\beta \alpha_{mn} p D}{1-(1-\alpha_{mn})\,\beta\,(1-p)}.$$

Taking into account inequalities $U_{mn} \geq U^S_{mn}$ and

$$\beta \frac{1-p-\beta\,(1-\alpha_{mn})\,(1-2p)}{1-(1-\alpha_{mn})\,\beta\,(1-p)} < 1,$$

we obtain

$$U_{mn} \geq \frac{Y - \beta \frac{\alpha_{mn} p D}{1-(1-\alpha_{mn})\beta(1-p)}}{1 - \beta \frac{1-p-\beta(1-\alpha_{mn})(1-2p)}{1-(1-\alpha_{mn})\beta(1-p)}} = S(\alpha_{mn}) \tag{8}$$

(the last equality is proved by multiplication of both the numerator and the denominator by $1-(1-\alpha_{mn})\,\beta\,(1-p)$). This proves the first part of the statement.

If $U^S_{mn} \geq U^E_{mn}$, then $U_{mn} = U^S_{mn}$. In that case, the inequality (8) turns into equality, and yields $U_{mn} = S(\alpha_{mn})$. ■

The next two lemmas establish that the 'talion law' holds in any equilibrium: for any number of previous killings n, a person who has executed contenders n times is executed by one who has no killings in his record with probability one if and only if he executes a newcomer.

Lemma 4 *For any $n \geq 0$, $\alpha_{n0} = 1$ implies $U^E_{n0} > U^S_{n0}$, and $\alpha_{0n} = 1$ implies $U^E_{0n} > U^S_{0n}$.*

Proof. Assume the contrary. Consider the case where $\alpha_{n0} = 1$, but $U^E_{n0} \leq U^S_{n0}$ (the remaining case may be treated in a similar way). Therefore, $U_{n0} = S(1)$. If, however, he chooses to kill once, then he gets $U^E_{n0} = Y + \beta\left((1-\mu)\,(Y + \beta W_{n^+0}) + \mu W_{n^+0}\right) = (1 + \beta\,(1-\mu))\,Y + \beta\,(\beta\,(1-\mu) + \mu)\,W_{n^+0}$. Observe that

$$W_{n^+0} = (1-p)\,U_{n^+0} + pV_{n^+0} \geq (1-p)\,S(1) - pD.$$

Here, we used inequalities $U_{n^+0} \geq S(\alpha_{n^+0}) \geq S(1)$, which holds because $S(\alpha)$ is a decreasing function, and $V_{n^+0} \geq -D$ by lemma (1). Therefore,

$$\begin{aligned} & U_{n0}^{E} - U_{n0} \\ \geq \; & (1+\beta(1-\mu))\,Y + \beta(\beta(1-\mu)+\mu)\,((1-p)\,S(1) - pD) - S(1) \\ = \; & \frac{\beta p (1-\mu)(Y + D(1-\beta))}{1-\beta(1-p)} > 0. \end{aligned}$$

However, in an equilibrium, $U_{n0} \geq U_{n0}^{S}$ must hold. This contradiction completes the proof. ■

Lemma 5 *In a stationary equilibrium, for any $n \geq 0$, four conditions $U_{n0}^{E} > U_{n0}^{S}$, $U_{0n}^{E} > U_{0n}^{S}$, $\alpha_{0n} = 1$, and $\alpha_{n0} = 1$ are equivalent.*

Proof. Evidently, $U_{n0}^{E} > U_{n0}^{S}$ implies $\alpha_{0n} = 1$, and $U_{0n}^{E} > U_{0n}^{S}$ implies $\alpha_{n0} = 1$. The remaining implications are proved in lemma 4. ■

3.2 The Monotonicity Theorem

Theorem 1 *For any symmetric stationary equilibrium, the following four conditions are equivalent:*

(i) equilibrium satisfies the single-crossing condition: if the winner strictly preferred to kill the loser rather than to spare when the number of killings he committed before was m, he still strictly prefers to kill when the number of killings he committed is $m+1$;

(ii) equilibrium strategies are (weakly) monotonic with respect to the number of murders committed by the winner: the larger is the number of killings already committed, the higher is the probability that he executes the newcomer;

(iii) equilibrium satisfies non-increasing utility conditions;

(iv) equilibrium is fake murder-proof: if the winner is allowed to make fake killings, which do not remove contenders, but count toward his reputation, he finds this option no better than sparing.

Proof. (i) $\Longrightarrow$ (ii) *Any stationary fake murder-proof equilibrium is a stationary monotonic equilibrium.*

Assume the contrary, i.e. $\alpha_{m^+0} < \alpha_{m0}$. This implies $\alpha_{m^+0} < 1$, and thus $\alpha_{0m^+} < 1$, therefore $U_{m^+0}^{S} \geq U_{m^+0}^{E}$. In that case, $U_{m^+0} = U_{m^+0}^{S} = S(\alpha_{m^+0})$. Obviously, that's also equal to U^M_{m0}, for $U^M_{m0} = Y + \beta W_{m^+0}$. Fake murder-proofness implies $U^S_{m0} \geq S(\alpha_{m^+0}) > S(\alpha_{m0})$ (the latter holds because $\alpha_{m^+0} < \alpha_{m0}$). Therefore, $U^E_{m0} > U^S_{m0}$, for otherwise $U^S_{m0} = S(\alpha_{m0})$. This, in its turn, implies $U_{m0} > S(\alpha_{m0})$, which contradicts Lemma 3.

(iii) $\Longrightarrow$ (ii) *Any stationary non-increasing utility equilibrium is a stationary monotonic equilibrium.*

Assume the contrary, i.e. $\alpha_{m^+0} < \alpha_{m0}$. As above, we obtain $U_{m^+0} = U^S_{m^+0} = S(\alpha_{m^+0})$. Non-increasing utility condition implies $U_{m0} \geq U_{m^+0} \geq S(\alpha_{m^+0}) > S(\alpha_{m0})$. As demonstrated in the proof of the previous claim, this leads to a contradiction.

(ii) $\Longrightarrow$ (i)*Any stationary monotonic equilibrium satisfies the single-crossing condition.*

Suppose that $U^E_{m0} > U^S_{m0}$. Then, by lemma 5, $\alpha_{m0} = 1$. Monotonicity condition implies $\alpha_{m^+0} = 1$, and we use lemma 5 once again to get $U^E_{m^+0} > U^S_{m^+0}$,

We showed that any of the three refinements (ii)-(iv) leads to equilibria satisfying single-crossing condition. Now we establish some lemmas about the properties of such equilibria, which are important per se and will allow us to demonstrate that these equilibria are fake murder-proof, non-increasing utility and monotonic. This would complete the proof of equivalence of these refinements. Proofs of these lemmas are relegated to Appendix.

Lemma 6 *If $\alpha_{n0} = 1$, then $\alpha_{n^+0} = 1$. In particular, $\alpha_{m0} = 1$ for all $m > n$.*

Lemma 7 *There exists $\varepsilon > 0$ such that if $\alpha_{n^+0} < 1$, then $S(\alpha_{n0}) > S(\alpha_{n^+0}) + \varepsilon$ (which implies $\alpha_{n0} < \alpha_{n^+0}$).*

Lemma 8 *There exists $k \geq 0$ such that $\alpha_{0n} = 1$ and $\alpha_{n0} = 1$ for $n \geq k$, and $\alpha_{0n} < 1$ and $\alpha_{n0} < 1$ for $n < k$.*

The two previous lemmas imply that the 'reputation' sequence $\{\alpha_{n0}\}$ is strictly increasing until it reaches 1, and once it does, it stabilizes. Intuitively, it means that in any equilibrium under consideration, each additional murder implies a higher probability of being punished (so there is no forgiveness or indulgence), until the probability of punishment reaches its maximum. Once it does, fear of additional chance of being punished is no longer in effect, and the agent opts to execute every time (and so $\alpha_{0n} = 1$ once $\alpha_{n0} = 1$).

Another corollary is that in the sequence $\{\alpha_{n0}\}$, only the first term α_{00} may equal 0. Other terms are strictly positive. This means that in the equilibria under consideration, no murder can be completely forgiven, and anyone who has executed at least once is subject to a non-zero probability of punishment.

Lemma 9 *Assume that* $0 < \alpha_{m0} < \alpha_{m^+0} < 1$*. Then* $\alpha_{0m} = \alpha_{0m^+}$*.*

Denote

$$A \equiv \frac{(1+\beta(1-\mu))Y - \beta(\beta(1-\mu)+\mu)pD}{1-\beta(\beta(1-\mu)+\mu)(1-p)}.$$

Lemma 10 *If* m *is such that* $\alpha_{m0} = 1$*, then* $U_{m0} = U^E_{m0} = A$*. Moreover,* $A > S(1)$*.*

Lemma 11 *If* m *is such that* $\alpha_{m^+0} = 1$*, then* $U^E_{mn} = A$*.*

Now we are ready to finish the proof of Theorem 1.

(i) $\Longrightarrow$ **(iii)** *Any stationary single-crossing equilibrium satisfies the non-increasing utility property.*

For any $m \geq 0$, either $\alpha_{m^+0} = 1$ or $\alpha_{m^+0} < 1$. In the first case, $U^E_{m0} = A$, and also $\alpha_{m^{++}0} = 1$, which implies $U^E_{m^+0} = A$ as well. Since $\alpha_{m^+0} = 1$, $U_{m^+0} = U^E_{m^+0} = A = U^E_{m0} \leq U_{m0}$. In this case, non-increasing utility property is satisfied. In the latter case, $U_{m^+0} = S(\alpha_{m^+0})$, while $U_{m0} = S(\alpha_{m0})$, since $\alpha_{m0} < \alpha_{m^+0} < 1$. Therefore, in this case, $U_{m^+0} = S(\alpha_{m^+0}) < S(\alpha_{m0}) = U_{m0}$.

(i) $\Longrightarrow$ **(iv)** *Any stationary single-crossing equilibrium is fake murder-proof.*

For any $m \geq 0$, either $\alpha_{m^+0} = 1$ or $\alpha_{m^+0} < 1$. In the first case, $\alpha_{0m^+} = \alpha_{m^{++}0} = 1$, and therefore $U^M_{m0} = Y + \beta((1-p)U_{m^+0} + pV_{m^+0}) = Y + \beta((1-p)A - pD) \leq Y + \beta((1-p)U_{m0} + pV_{m0}) = U^S_{m0}$ (because $V_{m0} \geq -D$). In the latter case, $U^M_{m0} = Y + \beta((1-p)U_{m^+0} + pV_{m^+0}) = S(\alpha_{m^+0}) < S(\alpha_{m0}) = Y + \beta((1-p)U_{m0} + pV_{m0}) = U^S_{m0}$. In both cases, fake-murder proof condition holds.

As for monotonicity condition in stationary single crossing equilibria, it trivially follows from lemmas 6 and 7. This completes the proof of equivalence of all four equilibria refinements. ∎

3.3 Best Responses

To analyze best responses, we introduce mapping $T : \mathbb{R} \to \mathbb{R}$ given by

$$T(x) = (1-\mu)Y + (\beta(1-\mu)+\mu)x.$$

Evidently, this mapping is contracting to the point $\frac{Y}{1-\beta}$. For $x < \frac{Y}{1-\beta}$ we have $T(x) > x$. Denote $T^n = \underbrace{T \circ \ldots \circ T}_{n \text{ times}}$. Similarly we denote T^0 to be the trivial mapping, and T^{-n} to be such that $T^{-n} \circ T^n \equiv T^0$. Evidently, mapping T satisfies

$$\left(\frac{Y}{1-\beta} - T^n(x)\right) = (\beta(1-\mu) + \mu)^n \left(\frac{Y}{1-\beta} - x\right) \tag{9}$$

for any x.

Before proceeding, we prove a theorem which distinguishes between two main cases: where there is only one equilibrium, where players choose to execute at every decision node, and where there are multiple equilibria (the most interesting case).

Theorem 2 *Set of strategies $\alpha_{mn} = 1$ for all m and n always constitutes an equilibrium. Moreover, if $S(0) < A$, it is the only equilibrium. If $S(0) \geq A$, there are at least two different equilibria.*

Proof. By Lemma 5, it is always rational to execute in the state (m, n), because $\alpha_{nm} = 1$. This proves the first part of the proposition.

If $S(0) < A$, then there exists another equilibrium, where $\alpha_{00} = 0$, and $\alpha_{mn} = 1$ for $m + n > 0$. The rational for execution is literally the same. However, if $S(0) \geq A$, then the player who chooses to spare gets $S(0)$ by definition, while he who opts to execute gets $U_{m0}^{E} = A \leq S(0)$. Therefore, at $(0, 0)$ it is best response to spare, and thus this constitutes an equilibrium. ■

Note that if $S(0) < A$, there is also a third equilibrium, given by $\alpha_{00} = S^{-1}(A)$, and $\alpha_{mn} = 1$ for $m + n > 0$. In this case, a person in state $(0, 0)$ is indifferent between executing and sparing.

Now, in order to characterize equilibria, it is useful to summarize best responses on strategies played by other people. Of course, we may restrict ourselves to strategies that satisfies the necessary conditions obtained in the previous subsection. Furthermore, as usual in dynamic games, it is sufficient here to consider one-shot deviations only, i.e. every player considers his future actions as given by the profile of strategies under consideration.

Lemma 12 *Let $\boldsymbol{\alpha}$ be a profile of strategies satisfying lemmas of the previous subsection. If $\alpha_{mn} < 1$, then in the state (m, n), best response to strategies played by other people is*

$$BR_{mn}(\boldsymbol{\alpha}) = \begin{cases} \{E\}, & \textit{if } S(\alpha_{mn}) < X; \\ \{S\}, & \textit{if } S(\alpha_{mn}) > X; \\ \Delta(\{E, S\}), & \textit{if } S(\alpha_{mn}) = X, \end{cases} \tag{10}$$

where $X = T(S(\alpha_{m+0}))$ *if* $\alpha_{m+0} < 1$*, and* $X = A$ *if* $\alpha_{m+0} = 1$*. If* $\alpha_{mn} = 1$*, then* $BR_{mn}(\boldsymbol{\alpha}) = \{E\}$ *(i.e. the above formula holds).*

Proof. If $\alpha_{mn} < 1$, then $U^S_{mn} = S(\alpha_{mn})$. If $\alpha_{m+0} = 1$, then by Lemma 11 $U^E_{mn} = A$. If $\alpha_{m+0} < 1$, then from (4) we get $S(\alpha_{m+0}) = U^S_{m+0} = Y + \beta W_{m+0}$; eliminating W_{m+0} from this and from (3), we get $U^E_{mn} = T(S(\alpha_{m+0}))$. Therefore (10) simply means that a player chooses the action that yields higher utility. If $\alpha_{mn} = 1$, then by Lemma 4 $U^E_{mn} > U^S_{nn}$, i.e. best response is to execute. Note that it satisfies (10), for X is either equal to A or to $T(S(\alpha_{m+0}))$, and both $A > S(1)$ and $T(S(\alpha_{m+0})) > S(\alpha_{m+0}) \geq S(1)$, since $S(\alpha_{m+0}) < \frac{Y}{1-\beta}$. ■

The last statement lays out some necessary equilibrium conditions.

Definition 5 *Suppose that profile* $\boldsymbol{\alpha}$ *constitutes an equilibrium. Denote* $\pi(\boldsymbol{\alpha}) = \max\{n \mid \alpha_{n0} < 1\}$*. (If there is no such* n*, i.e.* $\alpha_{00} = 1$*, we write* $\pi(\boldsymbol{\alpha}) = -\infty$*). We call this number the* patience *of equilibrium.*

In this interpretation, patience is the maximum number of murders one can commit to still have a non-zero chance of being spared in the future.

Lemma 13 *In any equilibrium given by* $\boldsymbol{\alpha}$*,* $\pi(\boldsymbol{\alpha})$ *satisfies*

$$T^{\pi(\boldsymbol{\alpha})}(A) \leq S(0). \tag{11}$$

Proof. For any $m \leq \pi(\boldsymbol{\alpha})$, $\alpha_{m0} < 1$, and thus $\alpha_{0m} < 1$. Therefore, $S \in BR_{m0}(\boldsymbol{\alpha})$, and by Lemma 12, $S(\alpha_{m0}) \geq T(S(\alpha_{m+0}))$ for $m < \pi(\boldsymbol{\alpha})$, and $S(\alpha_{m0}) \geq A$ for $m = \pi(\boldsymbol{\alpha})$. Combining these inequalities, we obtain $S(0) \geq T^{\pi(\boldsymbol{\alpha})}(A)$. This completes the proof. ■

Evidently, this implies that the equilibrium patience function, $\pi(\boldsymbol{\alpha})$, is bounded from above. The following theorem gives the exact boundary.

Theorem 3 *Denote*

$$\bar{\pi} = \log_{\frac{1}{\beta(1-\mu)+\mu}} \left(\frac{\frac{Y}{1-\beta} - A}{\frac{Y}{1-\beta} - S(0)} \right). \tag{12}$$

Let $M = \lfloor \bar{\pi} \rfloor$*, where brackets mean rounding down to the nearest integer. Then for any equilibrium* $\boldsymbol{\alpha}$ *we have* $\pi(\boldsymbol{\alpha}) \leq M$*.*

Proof. Substituting $x = A$ and $n = \pi(\boldsymbol{\alpha})$ in the equation (9), we find (using (11)) that $\left(\frac{Y}{1-\beta} - S(0)\right) \leq (\beta(1-\mu)+\mu)^{\pi(\boldsymbol{\alpha})} \left(\frac{Y}{1-\beta} - A\right)$. Since $0 < \beta(1-\mu)+\mu < 1$, we have $\pi(\boldsymbol{\alpha}) \leq \bar{\pi}$. Now $\pi(\boldsymbol{\alpha}) \leq M$ follows from the fact that $\pi(\boldsymbol{\alpha})$ is an integer. ■

3.4 Equilibria

Our next step is to characterize all stationary equilibria. In particular, we demonstrate that if $M \geq 0$, then there exists an equilibrium $\boldsymbol{\alpha}$ such that $\pi(\boldsymbol{\alpha}) = M$, i.e. this boundary is always achieved. Thus, we can call M *maximum patience* for a given set of parameters. Note that if $S(0) \geq A$, which is the case if and only if there are multiple equilibria (Theorem 2), then $M \geq 0$. We characterize equilibria with a given level of patience $m \leq M$. We consider it useful to discuss $m < 1$ (a simple case) and $m \geq 1$ (an interesting case) separately.

Theorem 4 *(i) Profile $\boldsymbol{\alpha}$ such that $\alpha_{mn} = 1$ for every m, n always constitutes an equilibrium such that $\pi(\boldsymbol{\alpha}) = -\infty$.*

(ii) If $M \geq 0$, then there exist one or two equilibria with patience $\pi(\boldsymbol{\alpha}) = 0$. In these equilibria, $\alpha_{mn} = 1$ for all (m, n) except for $(0, 0)$; α_{00} is either 0 or $S^{-1}(A)$. If $S^{-1}(A) > 0$, these equilibria are different, otherwise they coincide.

Proof. *(i)* This was actually proved as part of Theorem 2; it is anyway easy to check that if one's opponents play such strategy, best response is to execute (Lemma 12).

(ii) The fact that all α_{mn}'s should equal to 1, while α_{00} should not, follows from definition of patience. If $\alpha_{00} > 0$, then $BR_{00}(\boldsymbol{\alpha})$ includes both E and S, and then by Lemma 12, $S(0) = A$. Consequently, α_{00} is either 0 or $S^{-1}(A)$ (the latter is not equal to 1, for that would imply $S(1) = A$, which violates Lemma 10). It is easy to check, using Lemma 12, that for any of the two α_{00}'s, we obtain an equilibrium. Of course, if both numbers are equal (which is the case if and only if $\bar{\pi} = M = 1$[6]), equilibria coincide. ■

Now consider the case $m \geq 1$. In this case, in particular, $0 < \alpha_{10} < 1$, and $\alpha_{01} < 1$. On the other hand, $\alpha_{n0} = \alpha_{0n} = 1$ for $n > m$. We consider cases $\alpha_{01} \neq 0$ and $\alpha_{01} = 0$ separately.

Theorem 5 *Assume $M \geq 1$ and $1 \leq m \leq M$.*

(i) If $S^{-1}(T^m(A)) > 0$, then there are two and only two equilibria that satisfy $\pi(\boldsymbol{\alpha}) = m$ and $\alpha_{01} \neq 0$. In both of them, for $1 \leq n \leq m$, $\alpha_{n0} = S^{-1}(T^{m-n}(A))$, and $\alpha_{0n} = S^{-1}(T^m(A))$. In one equilibrium $\alpha_{00} = S^{-1}(T^m(A))$, while in the other one $\alpha_{00} = 0$. If, however, $S^{-1}(T^m(A)) = 0$ (which is the case if and only if $m = \bar{\pi} = M$[7]), there are no such equilibria.

[6] This is discussed in the proof of Theorem 5 in a more general case.

[7] This is a degenerate case, since this requires, in particular, $\bar{\pi}$ (found from (12) to be integer.

(ii) There always exists at least one equilibrium such that $\pi(\boldsymbol{\alpha}) = m$ and $\alpha_{01} = 0$. In any equilibrium satisfying these conditions, for $0 \leq n \leq m$, $\alpha_{0n} = 0$ (in particular, $\alpha_{00} = 0$), and $\alpha_{10} = S^{-1} \circ T^{-1} \circ S(0)$. As for α_{n0} for $1 < n \leq m$ if $m \geq 2$, these can be any numbers satisfying $\alpha_{m0} \leq S^{-1}(A)$ and $\alpha_{n0} \leq S^{-1} \circ T \circ S(\alpha_{n+0})$ if $n < m$. Any $\boldsymbol{\alpha}$ satisfying these conditions constitutes an equilibrium. If $m = \bar{\pi} = M$ or $m = 1$, then this series yields a unique equilibrium. Otherwise, there are continuum equilibria in this series.

Proof. *(i)* All probabilities α_{0n}, $1 \leq n \leq m$, are equal (Lemma 9). If $\alpha_{01} \neq 0$, they are non-zero (but not equal to one either, by Lemma 5 and the assumption that $\pi(\boldsymbol{\alpha}) = m$), and therefore for $1 \leq n \leq m$, $BR_{n0}(\boldsymbol{\alpha})$ includes both E and S. Hence, it is easy to prove by induction, using Lemma 12, that for $1 \leq n \leq m$, $\alpha_{n0} = S^{-1}(T^{m-n}(A))$. Since $\pi(\boldsymbol{\alpha}) \geq 1$, $\alpha_{10} < 1$, and hence, by the same Lemma, $\alpha_{0n} = S^{-1}(T^m(A))$ for $1 \leq n \leq m$. If $\alpha_{00} \neq 0$, then by Lemma 9 it equals $S^{-1}(T^m(A))$. It is straightforward to check that for both $\alpha_{00} = 0$ and $\alpha_{00} = S^{-1}(T^m(A))$, equations (10) are satisfied. Evidently, these equilibria differ in α_{00}. If $S^{-1}(T^m(A)) = 0$, then α_{01} should equal 0, which violates $\alpha_{01} \neq 0$. Finally, $S^{-1}(T^m(A)) = 0$ is equivalent to $T^m(A) = S(0)$, which is only possible if $m = \bar{\pi}$ (and thus equals $M = \lfloor \bar{\pi} \rfloor$); similarly, $m = \bar{\pi}$ implies $\bar{\pi}$ is integer and $T^m(A) = S(0)$.

(ii) By Lemma 9, $\alpha_{0n} = 0$ for $1 \leq n \leq m$. Similarly, $\alpha_{00} = 0$ (because if it is not the case, by the same lemma, $\alpha_{00} = \alpha_{01} = 0$). Since $0 < \alpha_{10} < 1$, $BR_{01}(\boldsymbol{\alpha})$ includes both E and S, which implies $S(0) = T(S(\alpha_{10}))$, and therefore, $\alpha_{10} = S^{-1} \circ T^{-1} \circ S(0)$. Furthemore, since $S \in BR_{n0}(\boldsymbol{\alpha})$ for $1 \leq n \leq m$, we conclude (recalling that $S(\alpha)$ is a decreasing function) that $\alpha_{n0} \leq S^{-1}(A)$ for $n = m$, and $\alpha_{n0} \leq S^{-1} \circ T \circ S(\alpha_{n+0})$ for $n < m$. It is straigtforward to check that any such profile constitutes an equilibrium. Evidently, there is at least one set of numbers α_{n0}, $1 \leq n \leq m$ satisfying these conditions: $\alpha_{n0} = S^{-1} \circ T^{-n} \circ S(0)$ (it is easy to check, as before, that $\alpha_{m0} \leq S^{-1}(A)$). These numbers are determined uniquely either if $m = 1$ (so there is no ambiguity), or if $m = \bar{\pi} = M$ (so all inequalities for α_{n0}, $2 \leq n \leq m$, become equalities). If neither is the case, then there are continuum equilibria with $\pi(\boldsymbol{\alpha}) = m \geq 1$, and $\alpha_{01} = 0$. ■

3.5 Comparative Statics

Substituting A and $S(0)$ into (12), we can rewrite $\bar{\pi}$ as

$$\bar{\pi} = \frac{\ln \frac{(1-(1-\mu)(1-\beta))(1-\beta(1-2p))(1+(1-\beta)R)}{1-\beta(1-p)(1-(1-\mu)(1-\beta))}}{\ln \frac{1}{\beta(1-\mu)+\mu}},$$

where $R = \frac{D}{Y}$. Evidently, M is affected by changes in $\bar{\pi}$ only if $\bar{\pi}$ is a non-negative number. The denominator is always positive, so when we analyze comparative statics, we may consider the numerator to be non-negative as well.

Now we can analyze comparative statics of $\bar{\pi}$ and M. Empirical support for these results is provided in Section 4.

Theorem 6 *The maximum equilibrium patience M is increasing in $\frac{D}{Y}$ and μ (the probability that a new contender appears in the next period despite execution of the previous one), and decreases in p (incumbent's survivorship rate). In particular, M increases with D (the cost of losing life) and decreases with Y (incremental utility of being in power).*

Proof. Since natural logarithm is an increasing function, the proposition is evident as far as R (and, therefore, D and Y) is concerned. Variable p does not appear in the denominator. Differentiating the fraction of which the logarithm in the numerator is taken, we obtain

$$\frac{\beta (1-\beta)(1+(1+\beta)(1-\mu))(1-(1-\mu)(1-\beta))(1+R(1-\beta))}{(1-\beta(1-p)(1-(1-\mu)(1-\beta)))^2} > 0.$$

As for μ, it is necessary to consider both the numerator and the denominator. Differentiation of the same expression in the numerator yields

$$\frac{(1-\beta)(1-\beta(1-2p))(1+(1-\beta)R)}{(1-\beta(1-p)(1-(1-\mu)(1-\beta)))^2} > 0.$$

At the same time, $\ln \frac{1}{\beta(1-\mu)+\mu}$ is obviously decreasing with respect to μ. Since both the numerator and the denominator are positive, we obtain the necessary comparative statics with respect to μ. ■

In other words, patience increases with the size of punishment, which is very intuitive, since a harder punishment makes a person more fearful of it, and increases incentives to spare. A higher p implies less stability of the dictator's position and less expected time until losing the fight, which also decreases incentives to execute. Finally, a higher μ means that one is less likely to experience a period of safe rule in the case of execution. This also makes execution less profitable.

3.6 Comparing Equilibrium Paths

In this subsection, we characterize equilibrium paths of various equilibria in the game. In most interesting cases agents play mixed strategies, so the actual game path is random. Still, it is possible to derive a number of general comparative statics results.

Theorem 7 *(i) Consider two equilibria E and E' given by $\{\alpha_{mn}\}$ and $\{\alpha'_{mn}\}$. If for any m, n we have $\alpha_{mn} \geq \alpha'_{mn}$[8], then for any m, n the following inequality holds: $U_{mn} \leq U'_{mn}$. In other words, a less violent equilibrium yields higher utility for all the dictators along the equilibrium path.*

(ii) At any equilibrium, if $\alpha_{m0} < 1$, then $U_{m0} \geq U_{m^+0}$ (and if $\alpha_{m^+0} < 1$, then $U_{m0} > U_{m^+0}$). In other words, an additional murder will decrease utility as measured from the period where a new enemy will emerge (but increases instantaneous next period's expected utility. If $\alpha_{m0} = 1$, then these utilities are equal.

Proof. *(i)* If $\alpha_{mn} < 1$, then $\alpha'_{mn} < 1$, and hence $U_{mn} = S(\alpha_{mn})$, $U'_{mn} = S(\alpha'_{mn})$. The necessary inequality follows from monotonicity of function $S(\alpha)$. If $\alpha_{mn} = 1$, then there are two possible cases. If $n = 0$, then by Lemma 10 $U_{mn} = A$. At the same time, U'_{mn} is either equal to A or to $S(\alpha'_{mn})$ (if the latter is the case, then $\alpha'_{mn} \leq S^{-1}(A)$), so in both cases $U'_{mn} \geq A = U_{mn}$. Finally, if $m = 0$, $n > 0$, it is sufficient to demonstrate that $W'_{10} \geq W_{10}$, for that would imply (from (3)) $U'_{mn} \geq U'^E_{mn} \geq U^E_{mn} = U_{mn}$. Both W_{10} and W'_{10} may be found from (7). Its right-hand side increasing with respect to U_{mn} (that's trivial), and decreasing with respect to α_{mn} (differentiation yields

$$-\frac{p\left(D\left(1-\beta\right)+p\beta\left(U_{mn}+D\right)\right)}{\left(1-\left(1-\alpha_{mn}\right)\beta\left(1-p\right)\right)^2} < 0,$$

since $U_{mn} \geq -D$). However, $\alpha'_{10} \leq \alpha_{mn}$, and we have already proved that $U'_{10} \geq U_{10}$. Therefore, $W'_{10} \geq W_{10}$, and hence $U'_{mn} \geq U_{mn}$ in the remaining case, too.

(ii) If $\alpha_{m0} < 1$, then $U_{m0} = S(\alpha_{m0})$. At the same time, if $\alpha_{m^+0} < 1$, then $\alpha_{m^+0} > \alpha_{m0}$ (Lemma 7), and hence $U_{m^+0} = S(\alpha_{m^+0}) < S(\alpha_{m0}) = U_{m0}$. If, however, $\alpha_{m^+0} = 1$, then $U^E_{m0} = A$ (Lemma 11) and $U_{m^+0} = A$ (Lemma 10), and we get $U_{m0} \geq U^E_{m0} = U_{m^+0}$. Finally, if $\alpha_{m0} = 1$, then $\alpha_{m^+0} = 1$ (monotonicity), and hence both U_{m0} and U_{m^+0} are equal to A. ■

Despite the multiplicity of equilibria, equilibrium paths of the game may be naturally split into two major groups. Evidently, all equilibria where $\alpha_{00} = 0$ lead to a trivial equilibrium path, which is depicted on Figure 3. The winner always spares, and even if he loses, the game returns back to the state $(0, 0)$.

[8] This is the case, for example, if $\alpha_{00} \geq \alpha'_{00} > 0$. This follows from full description of equilibria (Theorems 4 and 5).

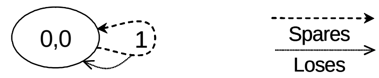

Figure 3: Equilibrium Path If $\alpha_{00} = 0$.

In other words, the state $(0, 0)$ remains forever.[9] The average duration of each rule is given by Theorem 8.

Theorem 8 *If* $\alpha_{00} = 0$*, then on the equilibrium path players* 1 *and* 2 *always spare each other, and replace each other on the dictator's position. The mean duration of each subsequent rule equals* $\frac{1}{p}$*.*

Proof. The first part of the statement is evident, since the game is stuck in the state $(0, 0)$. The mean duration is calculated as

$$1 + \sum_{n=1}^{\infty} n (1-p)^{n-1} p = p \sum_{n=0}^{\infty} n (1-p)^{n-1} = -p \frac{d}{dp} \sum_{n=0}^{\infty} (1-p)^n = -p \frac{d}{dp} \left(\frac{1}{p} \right) = \frac{1}{p}.$$

■

Conversely, if $\alpha_{00} \neq 0$, there may be a variety of paths, parametrized by patience of equilibrium $m \leq M$. In general, the switching of states along such paths is depicted on Figure 4. In the picture, the probabilities of different actions (executing or sparing) in each state are shown. However, the winner of the current period may lose with probability p in the next battle, and thus with probability p the game does not follow the arrow chosen by the winner, but rather turns to a dotted line.

It is also informative to depict probabilities of committing a murder and being murdered as considered by a person who committed n murders in the past, when facing an innocent

[9] One can also consider the situation where $\alpha_{00} = 0$, but the game (for any reason, say, a deviation of any player) comes to the state other than $(0, 0)$. In that case, if $\alpha_{01} \neq 0$, then the game evolves as if the corresponding equilibrium with $\alpha_{00} = S^{-1}(T^m(A))$ was played (m is the patience of the equilibrium), and the latter will be considered later in this subsection. If, however, $\alpha_{01} = 0$, then the following happens. If the state of the game is $(n, 0)$, then the incumbent executes until he eventually loses to the contender, which brings the game to the state $(0, k)$, $k \geq n$. In such states, the contender eliminates the violent dictator, bringing the game to the state $(0, 1)$. If the game is at the state $(n, 0)$ or $(0, n)$, where $1 \leq n \leq m$, then it evolves as follows. The dictator who killed at least once before never kills again. However, he is killed by the contender who never killed before with probability $\alpha_{n0} > 0$. If this happens, the game again moves to the state $(0, 1)$, and the contender who just killed never kills again, but is subject to a murder.

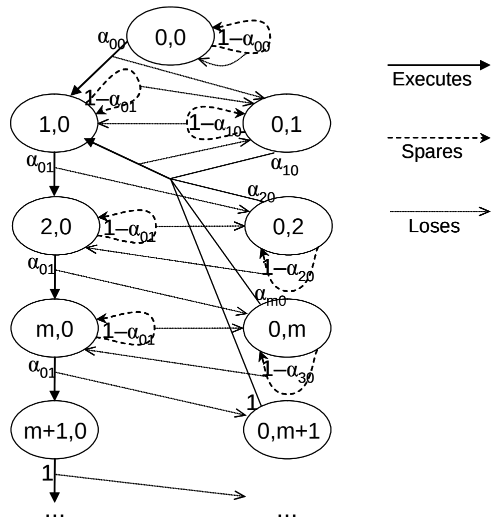

Figure 4: Equilibrium Path If $\alpha_{00} \neq 0$.

opponent. We choose a certain set of parameters ($Y = 10$, $D = 5$, $p = 0.5$, $\beta = 0.95$, $\mu = 0.2$), in that case, maximum patience $M = 6$. On Figure 5, we show probabilities α_{0n} and α_{n0} as functions of n (the number of murders committed in the past) for $m = 2$ and $m = 5$.

As one can see from Figure 5, in equilibria with non-trivial equilibrium paths, the probability that a bloody dictator will execute is a function that has two values, one of which is 1. In the beginning, bloody dictators execute and spare with constant probabilities (depending on parameters of the model and the patience of equilibrium, of course), being aware that every additional murder increases their chance to be executed (since probabilities of being murdered by innocent contenders) are monotonically increasing. When this fear is no longer in effect, both probabilities of murdering and of being murdered simultaneously reach 1 and stabilize there.

It is easy to see from Figure 5 that bloody dictator's probabilities of being executed are convex functions (until they reach 1). This can be justified formally, and the intuition behind this observation is very plain. What we need is to understand, why a greater increase in probability of revenge is needed to prevent fifth murder (i.e. to make one indifferent whether to commit it or not) as compared to preventing second murder. But it is of course easy to understand that a certain increment of the probability of being murdered is more dangerous for a person who committed one murder than to one who committed four murders before,

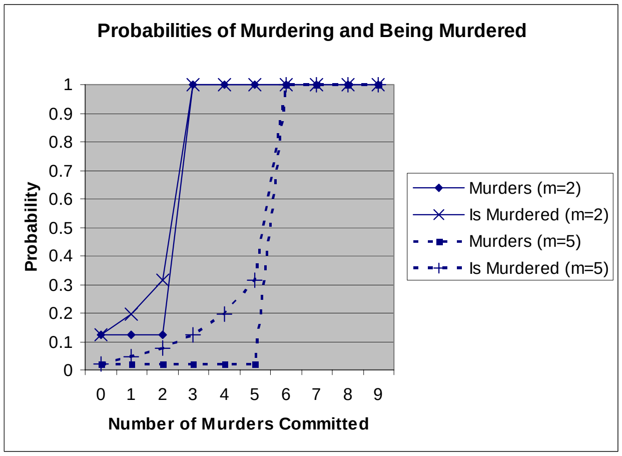

Figure 5: Probabilities of Executions for Different Patience Parameters.

because this 'Sword of Damocle' would hang upon him for a longer time, thus decreasing his expected utility by the same amount, but for a longer time. The reason for that is of course that a person who already committed four murders is more likely to be murdered soon anyway.

To finish our discussion of equilibrium paths and compararison between them, we give a reference to a paper where a similar (yet simpler) game is used to analyze agency problems along different paths. Egorov and Sonin (2004) analyze a static game, where a dictator hires an agent of a certain competence, trading off benefits of having a smart vizier and costs of a possible betrayal. (A more competent agent is more able to distinguishing the enemy type, and, therefore, less loyal in equilibrium.) Egorov, Nye, and Sonin (2005) demonstrate how this static game may be built into a particular type of the killing game considered here. In particular, it appears that the more killings happen along the equilibrium paths (which essentially correspond to the pure-strategy equilibria in this paper), the lower is the quality of governance.

4 Historical Illustrations[10]

What does our theory say about cross-country comparison of dictatorships?[11] To illustrate the existence of two paths with markedly different characteristics, which correspond to the only pure-strategy equilibria in our model, we first discuss general patterns in Europe and Latin America, and then focus on two particular examples, Venezuela, 1830–1970 and the Ottoman Empire, 1230–1922. One difference between the two examples is of course that the Ottoman Empire was a hereditary monarchy and Latin American countries were not. However, an Ottoman Empire was not a place where succession was automatic. Often, there were brothers succeeding the deposed dictator; the fact that most of sultans had many children made competition for succession serious. The difference which can be predicted by our comparative statics is that in a monarchy, the set of potential contenders is limited and pre-defined (e.g., all brothers, sons, and nephews); accordingly, it really makes sense to try to eliminate all potential contenders.

European monarchies of the era witnessed significantly fewer executions and killings of kings, though it was not impossible. (And, of course, the fate of numerous contenders was very often miserable.) E.g., Richard III of England was immortalized by Thomas More and William Shakespeare for slaying the baby-king Edward V and his brother Richard in an attempt to secure the crown for himself.[12] The young princes, sons of the previous king Edward IV of England, were declared illegitimate by the Act of Parliament known as Titulus Regius; however, the act of parliament was legally reversible, which make them a potential

[10] In this preliminary version, this Section is based partially on Egorov, Nye, and Sonin (2005).

[11] Footnote 1 provides a list of rulers killed after being ousted from power in countries that witnessed at least two such incidents during the last fifty years. Other rulers killed during this period include Melchior Ndadaye in Burundi (1993), Carlos Castillo Armas in Guatemala (1957), Thomas Sankara in Burkina Faso (1987), Salvatore Allende in Chile (1973), Long Boreth in Cambodia (1975), Sylvanus Olimpio in Togo (1963), François Tombalbaye in Chad (1975). Violent deaths of leaders not associated with a serious attempt to change the regime, e.g. of Anwar as-Sadat in Egypt (1981), Indira Gandhi in India (1984), René Moawad in Lebanon (1989), Yitzhak Rabin in Israel (1995), or Loran Kabila in Congo (2001) are not such examples. Both samples are truncated since they do not take into account unsuccessful contenders that were killed during a coup or executed thereafter.

[12] Historians do not universally take More's account as authentic. Still, there is no doubt that the 'Princes of the Tower' were killed, and the primarily motive was elimination of potential heirs to the throne, either for the benefit of Richard III, Henry VII, or even of Duke of Buckingham, also an Edward III descendant.

threat for Richard III.[13] Execution of Mary Stuart of Scots by Elizabeth I of England was apparently aimed at reducing the possibility of a pro-Stuart coup. In 1685, James Crofts, Duke of Monmouth, a bastard child of Charles II of England, was executed within nine days after capture (cf. typically long trials of other British royals in 17th century). The apparent reason for that was he proclaimed himself James II, the king of England, thus endangering the power of the existing king, also James II, his uncle.[14] Even executions of kings committed by revolutionaries (e.g. Charles I of England, Louis XVI of France, and Nikolai II of Russia, whose mere existence – even after abdication – made them contenders to power), were in part motivated by consideration highlighted by our theory.[15]

In a fundamental study of patterns of political succession in England, Bartlet (2002) notes that "between the 11th and early 14th, defeated political opponents of high birth were ... scarcerly ever maimed or killed in cold blood." However, in the Celtic part of British Isles, "the kings and princes of Wales, Ireland, and Gaelic Scotland continued to employ blinding, maiming, and killing in their conflicts with rivals from both with-in and with-out their families" (Bartlet, 2002). Bartlet and earlier medieval studies such as Pollock and Maitland (1898) contrast the virtual absence of royal-member executions in Norman and Angevin England with the bloodiness of the later Middle Ages and Tudor period. Figure 6 at the end of the paper demonstrates that, by 1486, the last year of the War of Roses, the only surviving male from both houses of York and Lancaster was the king, Henry VII. Both chronological (in 1455–1485) and geographical (in 1075–1225) concentration of killings support the idea of history-dependence highlighted by our theoretical model.

To further illustrate how different degree of security affected the winner's attitude to the mere presence of potential contenders, compare accessions of two young and vivacious women, Elizabeth I of Russia in 1741 and Catherine II (the Great) in 1762.[16] Each of them was brought to power by a military coup organized by young officers of elite guard divisions.

[13] Bartlet (2002) provides an example of situation when, against all odds, King Stephen did not kill 6-years old William Marshal (a future regent of England), given to Stephen as a hostage by his father John.

[14] As in the case of Princes of the Tower, being a bastard (a legal term at that time) does not automatically exclude the person from the set of legitimate contenders.

[15] A famous example of the 'elimination strategy' is the Slaughter of Innocent. Upon hearing the prophecy that he would be dethroned by the just-born 'King of the Jews', King Herod ordered to kill all male children under two years of age in Bethlehem.

[16] Women occupied the Russian throne for more than a half of the 18th century, and half of them were brought to power by a military coup.

Elizabeth I removed the one-year old tsar, Ivan VI, and his parents, who were the regents designated by the predecessor, Empress Anna, Elizabeth's cousin. Upon Elizabeth's accession, Ivan VI was not killed but isolated and guarded in different fortresses, and his parents were exiled. Catherine II removed from power her husband, Emperor Peter III, who was designated as a successor by Elizabeth. Though Peter III was not executed immediately after the removal from power, he was assassinated within six months by people Catherine sent to 'watch him'. Archives contain a hand-written note by one of the assassins, where he proudly reports the death of Catherine's husband. Furthermore, Ivan VI, who had been spared by Elizabeth and kept in prison for 22 years, was killed soon after Catherine's accession by the guards, fearful of a plot to rescue him.

The crucial difference in attitude of Elizabeth and Catherine to those whom they removed from power and who could have been expected to become if not the center, but at least a focal element of opposition, might be in that Elizabeth was a daughter of Peter the Great, the Russian tsar in 1696–1725, and thus was a heir at least as legitimate as those whom she replaced. In contrast, Catherine was a daughter of an obscure count in Prussia (Germany), and was 'imported' to marry Peter III, who was a great-grandchild of Peter the Great and a nephew of the reigning Empress Elizabeth. Thus, Catherine, the 'illegitimate' ruler, had to take much more care of contenders' fates than Elizabeth.

4.1 Dynastic Succession

The House of Osman ruled the Ottoman Empire from 1281 to 1922; officially, the sultan was the sole source of governmental authority in the empire. During more than six centuries covered in the list of the Ottoman sultans, there was almost no hostile comebacks. The main reason for this was that, after a coup, the loser, be it the incumbent or the contender, was usually executed.[17] Still, by many standards, the Ottoman Empire resembled European

[17] The first 'comeback' in the Ottoman Empire was not a result of a hostile fight between the predecessor, Murad II, and his son, Mehmed II. Indeed, During his first reign, seeing the upcoming Battle of Varna, Mehmed sent for his father, Murad II, asking him to claim the throne again to fight the enemy, only to be refused. Enraged at his father, who was then retired to rest in southwestern Anatolia, Mehmed in his famous letter wrote to his father: "If you are the sultan, come and lead your armies. If I am the sultan I hereby order you to come and lead my armies." It was upon this letter that Murad II led the Ottoman army in the Battle of Varna in 1444. The second 'comeback' (1622) also appears to be very specific, as Mustafa I, reportedly mentally retarded (Alderson, 1982), was merely a 'façade' king.

monarchies. Alderson (1982) notes that average duration of an Osmanli sultan's reign (17 years), compares favorably with Roman emperors (7 years), Byzantine emperors (12), Abbasid caliphs (12) and is close to European monarchies such as Russia (18), France (21) and Britain (23).[18]

Executions of predecessors, failed contenders, or just potential contenders such as younger brothers, was indeed wide-spread. Of the 36 sultans, 16 abdicated or were disposed. Among these 16, 11 were killed during or soon after a hostile disposition from power. Of 5 sultans that were not killed upon the accession of a new sultan, four were the last sultans of the Empire in the period of 1876–1922; one of them, Murad V, ruled for only 93 days in 1876 and was widely believed to suffer mental illness. Therefore, it is safe to conclude that killing the predecessor was a typical strategy in the Ottoman Empire.

Another course of actions, typical for the Ottoman Empire and all but unseen in Venezuela, was killing of potential heirs to the throne. Beyazid I (1389–1402), Mehmed I (1413–1421), Mehmed II (1444–1445, 1451–1481) Murad II (1421–1444, 1445–1451), Selim I (1512–1520), Suleiman I, (1520–1566), Mustafa IV (1807–1808), Mahmud II (1808–1839) put some of their brothers or sons (and often other relatives) to death. Of course Mehmed III (1595–1603) stays notorious even in this long list for having his sixteen brothers killed upon his accession. His son, Ahmed I (1603–1617) broke with the pattern, refusing to execute his mentally retarded brother Mustafa I (1617–1618, 1622–1623).

The tradition to kill potential contenders persisted even in 19th century. In 1808, a janissary revolt brought to power Mustafa, a son of Abd-ul-Hamid I. Mustafa ordered execution of his brother Selim, the disposed sultan, as well as his another brother, Mahmud. Selim was killed, but Mahmud, the only remaining male member in line for succession, escaped, revolted against Mustafa, and had him executed upon succession to the throne.

It is plausible again to juxtapose the Ottoman Empire experience with that of the Russian Empire. As in the Ottoman Empire, the Russian Empire has had a well-defined sequence of absolute rulers for a prolonged period, since early 14th century, and the difference in average durations is minimal (17 and 18, respectively). In Russia, we witness both (i) less deaths of predecessors and (ii) more examples of people being removed from power, but spared. There might be only single execution of a heir by the current ruler (prince Alexei, who was accused of conspiring in the plot against his father Peter the Great; the ultimate circumstances of

[18] Alderson (1982), McCarthy (1997), Palmer (1992), Morby (2001).

Alexei's death are unknown, e.g., Massie, 1980)[19] and a single episode of a son successfully, though passively, participating in a plot against his father (Alexander I against his father Pavel I in 1801). There were examples that people were removed from power or succession line and not executed (Vasily Shuisky, Sophia, a half-sister of Peter the Great, Peter the Great himself, Ivan VI, Konstantin).[20]

The Russian history provides another example of the sparing equilibrium, a power struggle of two boyar clans each of which has a boy contender to the throne (chronology: Massie, 1980). In 1669, Maria Miloslavskaya, the first wife of tsar Alexis died, survived by 4 children, including her ailing sons Fedor and Ivan. Two years later, in 1671, Alexis married Natalya Naryshkina, and the whole clan of Miloslavskys, including Maria Miloslavskaya's father, but not Fedor and Ivan, went into exile. In 1672, son Peter is born. In 1676, Alexis died, Fedor was proclaimed tsar, and Miloslavskys (including grandparents of Fedor) were returned, while Naryshkins were exiled (but not killed).

In 1682, tsar Fedor died, and his half-brother Peter, aged ten, was proclaimed tsar. Fedor's brother Ivan, 16, who was burdened with several chronic illnesses, conceded to Peter's accession and was kept in the Kremlin palace unharmed, while the Naryshkins (including the former first minister Artamon Matveev, tsarina Nataliya's guardian) were returned from exile. In a few days, most impotant Naryshkins, including Artamon Matveev, were killed in a military uprising, Sophia Miloslavskaya became regent, officially on behalf of Ivan and Peter. Though Peter, a single royal male in the Naryshkins clan, was in hands of Miloslavskys, his life was not threatened. In 1689, Peter acceded to power in bloodless coup; Sophia is kept under home arrest for rest of her life (until 1704). The essence of the story is that, despite very strong incentives and excellent opportunities on each side to kill the heir representing the rival clan, they deliberately abstained from that.

[19] Ivan the Terrible's son Ivan was killed by his father in a quarrel; however, there is a lot of evidence that it was accidental.

[20] In contrast with the Ottoman Empire, killing of non-rivaling siblings was near-taboo in Russia since the very early years: in 1217, Prince Gleb Vladimirovich of Ryazan was thrown out by citizens of his state after ordering to kill his brothers, princes of neighbouring states, at a dinner table. The first Russian saints of the Orthodox Church were young and innocent princes Boris and Gleb, killed by their brother Svyatopolk. Of course, Russian princes have had less legitimate consanguineous brothers than Ottoman rulers.

4.2 Nonhereditary Succession

The history of Venezuela since early 19th century – we start with the year of 1830, the first year of full state independence – serves us as a vivid example that dozens of dictators that come and go need not be necessarily harsh on their predecessors.[21] One reason for relative exsanguinity at the very top was, as we argue, the equilibrium behavior of winners. Correctly anticipating a high probability to be removed from power, they opted for a mild treatment of their predecessors. Another reason was the absence of a royal family and/or aristocratic tradition, which made it impossible to significantly reduce the set of contenders (μ is high, in terms of the model). It could be argued that Venezuela in 1830–1964 provides a typical example of a Latin American country and polity; we identified comeback military rulers in almost every Latin American country.

For example, of 54 presidents and provisional rulers of Mexico in the 19th century, 17 have held this positions more than one time, and 7 came back to power at least two times (Cahoon, 2004). General de Santa Anna, "the Napoleon of Mexico", came back at least 5 times (and 11 by some accounts).[22] In Chile, General Ramon Freire came back 5 times. In Cuba, the last comeback dictator was Fulgencio Batista, who came to power twice (in 1933 and 1952) by means of a military coup (Domínguez, 1998). In Venezuela, among 56 changes in leadership (this figure includes all constitutional leaders – elected, military, and provisional), there were 14 comebacks by 10 leaders who had previously been constitutional leaders of the country. Needless to say, a comeback is the most visible sign that the person had not been executed after removal from power the previous time. On the other hand, some of the rulers indeed died in office or shortly after removal from power.

Among the generals that ruled Venezuela during 20 years after 1830, there are Jose Paez (1830–1835 and 1839–1843, president, 1861–1863, supreme dictator), Carlos Soublette (1837–1839, provisional president, 1843–1847, president, 1858, provisional president), Jose Tadeo Monagas (1847–1851, 1851–1855, 1868) and Pedro Gual (1859, provisional president, 1861, president). In 1837, 1848, 1858, 1859, 1861 (twice), 1863, and 1868, transition of power

[21] Our brief overview of Venezuelan leadership in historical perspective is based primarily on Munro (1950), Levine (1978), and Rudolph and Rudolph (1996). For modern military coups in Latin America, see Linz and Stephen, 1978; empirical patterns are analyzed in Luttwak (1979), Ferguson (1978), O'Kane (1978), and Farcau (1994).

[22] This would not be surprising if General de Santa Anna were a democratic politician, coming back and forth via elections. However, most of power changes were military coups.

was hostile. Still, even in the exceptionally bloody 1858 turmoil that started the Federal War, the outgoing dictator Monagas, who had forced Paez in exile, was allowed to find a refuge first in the French embassy, and then to retire to France (only to come back in ten years).[23] Julian Castro, the president that removed Monagas in 1858 and was removed by Gual in 1859, was convicted for treason, but absolved. Páez staged unsuccessful coups in 1848 and 1849 against Monagas and was exiled (1850–58). In 1861, Páez returned to become the supreme dictator.

The end of the Federal War brought General Antonio Guzman Blanco in to the center of Venezuelan political arena. He first became president in 1870 (before that he was acting president temporarily replacing General Falcon, his military principal during the Federal War, and the president in 1864–1868), ousting José Ruperto Monagas, a son of José Tadeo Monagas (the Monagas who was President in 1847–1851 and 1855–1858), who brought to power by the coup of 1868. In 1877, Guzman left the office and went to France. In 1880, after the death of President Linares (in 1878) and ouster of his short-lived successor Jose Gregorio Varela, Guzman returned to Venezuela and took power by a coup again. Another comeback, Joaquin Crespo, was president first in 1884–86 (replacing Guzman Blanco who left for Paris only to be back in 1886) and from 1892 to 1898, when he was killed during a revolt against his desired successor. Joaquin Crespo appears to be the only ruler of Venezuela in two centuries to be killed during or shortly after a coup; there has been no evidence that he was executed rather than killed in a fight.

The first half of the 20th century was the era of Juan Vincente Gomez, who took power in 1908 and was formally a president or a provisional president in 1908–1914, 1915–1929 and 1931–1935. However, this might not be an illustrative example of a comeback president, since historians agree that, whatever has been his official position, he was the undisputed ruler of Venezuela since 1908 until his death in 1935.

The second half of the 20th century has changed the patterns of dictatorship, though the phenomenon of 'comeback rulers' persisted even with democracy gaining a more solid ground since the presidency of Rómulo Betancourt, who himself was a comeback military leader. After taking power in 1945, he was ousted by a military coup in 1948, was returned by another coup in 1958 and voluntarily left presidency in 1964. Since 1964, two Venezuelan

[23] It would be only fair to note that his younger brother, Jose Gregorio Monagas, who was president in 1851–1855, was put to jail after the coup of 1858 and died the same year.

politicians were presidents twice (they were barred from running for a second consecutive term by constitution), but each time they were elected in a democratic election.[24]

5 Conclusion

Most advanced analysis of political and economic history draws insights from studies of large-scale institutional change. The challenge we took upon in this paper is to reconcile big historical processes with micro decisions made by significant decision-makers at critical points in history. In this paper, we study reputation and knowledge in a complete information game with an infinite number of players. The rational winner in a power struggle determines the fate of the loser. His choice of equilibrium strategy decisions is motivated by two basic considerations: first, he is willing to increase the probability of survival by reducing the set of potential contenders. Second, he fears that a bad reputation would serve him poorly should he in turn become the loser. One conclusion that we are able to illustrate employing a historical narrative is the existence of markedly different equilibria paths. Between 1830 and nowadays, Venezuela witnessed a larger number of successful hostile comebacks of leaders that were disposed earlier. In a drastic contrast, in the Ottoman Empire, a hereditary monarchy, a typical move by a new ruler was to try to kill all the potential contenders to the throne.

[24] Still, Carlos Andres Pérez, who was president in 1974–1978, and was elected again in 1989, survived two military coups during his second term, and was finally suspended from the office under charges of corruption.

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6 Appendix

Proof of Lemma 6. $\alpha_{n0} = 1$ implies $\alpha_{0n} = 1$. From the proof of lemma 5 it follows that $U^E_{n0} > U^S_{n0}$ (because $U^S_{n0} \geq U^E_{n0}$ leads to a contradiction). By the single-crossing condition, we find that $U^E_{n^+0} > U^S_{n^+0}$. This implies $\alpha_{0n^+} = 1$, and hence $\alpha_{n^+0} = 1$. This completes the proof. ■

Proof of Lemma 7. Since $\alpha_{n^+0} < 1$, $\alpha_{n0} < 1$ as well. Therefore, $U_{n0} = S(\alpha_{n0})$ and $U_{n^+0} = S(\alpha_{n^+0})$. Consider, however, U^E_{n0}. We have

$$\begin{aligned} U^E_{n0} &= Y + \beta\left((1-\mu)\left(Y + \beta W_{n^+0}\right) + \mu W_{n^+0}\right) \\ &= (1-\mu)\,Y + ((1-\mu)\,\beta + \mu)\left(Y + \beta W_{n^+0}\right) \\ &= (1-\mu)\,Y + ((1-\mu)\,\beta + \mu)\,U^S_{n^+0}. \end{aligned}$$

Assume that $S(\alpha_{n0}) \leq S(\alpha_{n^+0}) + \varepsilon$, we obtain

$$\begin{aligned} U^E_{n0} - U_{n0} &= (1-\mu)\,Y + ((1-\mu)\,\beta + \mu)\,S(\alpha_{n^+0}) - S(\alpha_{n0}) \\ &\geq (1-\mu)\,Y + ((1-\mu)\,\beta + \mu)\,S(\alpha_{n^+0}) - S(\alpha_{n^+0}) - \varepsilon \\ &\geq (1-\mu)\left(Y - S(\alpha_{n^+0})(1-\beta)\right) - \varepsilon \\ &\geq (1-\mu)\left(Y - S(0)(1-\beta)\right) - \varepsilon \\ &\geq \frac{(1-\mu)\,\beta p Y}{(1-\beta+2p\beta)} - \varepsilon. \end{aligned}$$

Therefore, if we take $\varepsilon < \frac{(1-\mu)\beta p Y}{(1-\beta+2p\beta)}$, assertion that $S(\alpha_{n0}) \leq S(\alpha_{n^+0}) + \varepsilon$ would lead to a contradiction. Then $S(\alpha_{n0}) > S(\alpha_{n^+0}) + \varepsilon$, and, since S is strictly decreasing, $\alpha_{n0} < \alpha_{n^+0}$. This observation completes the proof. ■

Proof of Lemma 8. By previous lemmas, it is sufficient to demonstrate that $\alpha_{n0} = 1$ for some n. If it were not true, however, then we would find that $S(\alpha_{00}) - S(\alpha_{N0})$ may be an arbitrarily large number. This contradicts the fact that both numbers lie between $S(1)$ and $S(0)$. ■

Proof of Lemma 9. An agent in the states $(0, m)$ and $(0, m^+)$ is indifferent whether to execute or spare if $U^E_{0m} = U^S_{0m}$ and $U^E_{0m^+} = U^S_{0m^+}$. The utility from execution on either of these states are equal to

$$U^E_{0m} = Y + \beta\left((1-\mu)\left(Y + \beta W_{10}\right) + \mu W_{10}\right) = U^E_{0m^+}.$$

Therefore, $S(\alpha_{0m}) = U^S_{0m} = U^S_{0m^+} = S(\alpha_{0m^+})$. Since $S(\alpha)$ is a strictly decreasing function, $\alpha_{0m} = \alpha_{0m^+}$. ■

Proof of Lemma 10. Since $\alpha_{m0} = 1$, the agent at least does not strictly prefer to spare, and thus $U_{m0} = U^E_{m0}$. Since $\alpha_{m^+0} = 1$,

$$\begin{aligned} W_{m^+0} &= \frac{1-p-\beta(1-\alpha_{m^+0})(1-2p)}{1-(1-\alpha_{m^+0})\beta(1-p)} U_{m^+0} - \frac{\alpha_{m^+0} pD}{1-(1-\alpha_{m^+0})\beta(1-p)} \\ &= (1-p) U_{m^+0} - pD, \end{aligned}$$

and hence

$$\begin{aligned} U_{m0} &= U^E_{m0} = (1+\beta(1-\mu)) Y + \beta(\beta(1-\mu)+\mu) W_{m^+0} \qquad (13) \\ &= (1+\beta(1-\mu)) Y + \beta(\beta(1-\mu)+\mu)((1-p) U_{m^+0} - pD). \end{aligned}$$

Let us demonstrate that $U_{m0} = U_{m^+0}$. If we substitute U_{m^+0} in (13) for $(1+\beta(1-\mu)) Y + \beta(\beta(1-\mu)+\mu)\big((1-p) U_{(m+2)0} - pD\big)$ etc., then after n iterations (denote $\gamma = \beta(\beta(1-\mu)+\mu)(1-p))$ we will get

$$U_{m0} = ((1+\beta(1-\mu)) Y - \beta(\beta(1-\mu)+\mu) pD) \frac{1-\gamma^n}{1-\gamma} + \gamma^n U_{(m+n)0}.$$

Similarly,

$$U_{m^+0} = ((1+\beta(1-\mu)) Y - \beta(\beta(1-\mu)+\mu) pD) \frac{1-\gamma^n}{1-\gamma} + \gamma^n U_{(m+n+1)0}.$$

Therefore,

$$|U_{m0} - U_{m^+0}| = \gamma^n \left|U_{(m+n)0} - U_{(m+n+1)0}\right| \le \gamma^n \left(\frac{Y}{1-\beta} + D\right).$$

Since $0 < \gamma < 1$, and n can be chosen arbitrarily large, we conclude that $U_{m0} - U_{m^+0} = 0$. Substituting in (13) U_{m^+0} for U_{m0}, we will find $U_{m0} = A$.

To prove that $A > S(1)$, simply subtract these values.

$$\begin{aligned} & A - S(1) \\ =& \frac{(1+\beta(1-\mu)) Y - \beta(\beta(1-\mu)+\mu) pD}{1-\beta(\beta(1-\mu)+\mu)(1-p)} - \frac{Y-\beta pD}{1-\beta(1-p)} \\ =& \frac{\beta p(1-\mu)(Y+D(1-\beta))(1-\mu) p\beta}{(1-\beta(\beta(1-\mu)+\mu)(1-p))(1-\beta(1-p))} > 0. \end{aligned}$$

This completes the proof. ■

Proof of Lemma 11. Evidently,

$$\begin{aligned} U^E_{m0} &= (1+\beta(1-\mu)) Y + \beta(\beta(1-\mu)+\mu) W_{m^+0} \\ &= (1+\beta(1-\mu)) Y + \beta(\beta(1-\mu)+\mu)((1-p) A - pD) = A. \end{aligned}$$

The last equality follows from definition of A. ■

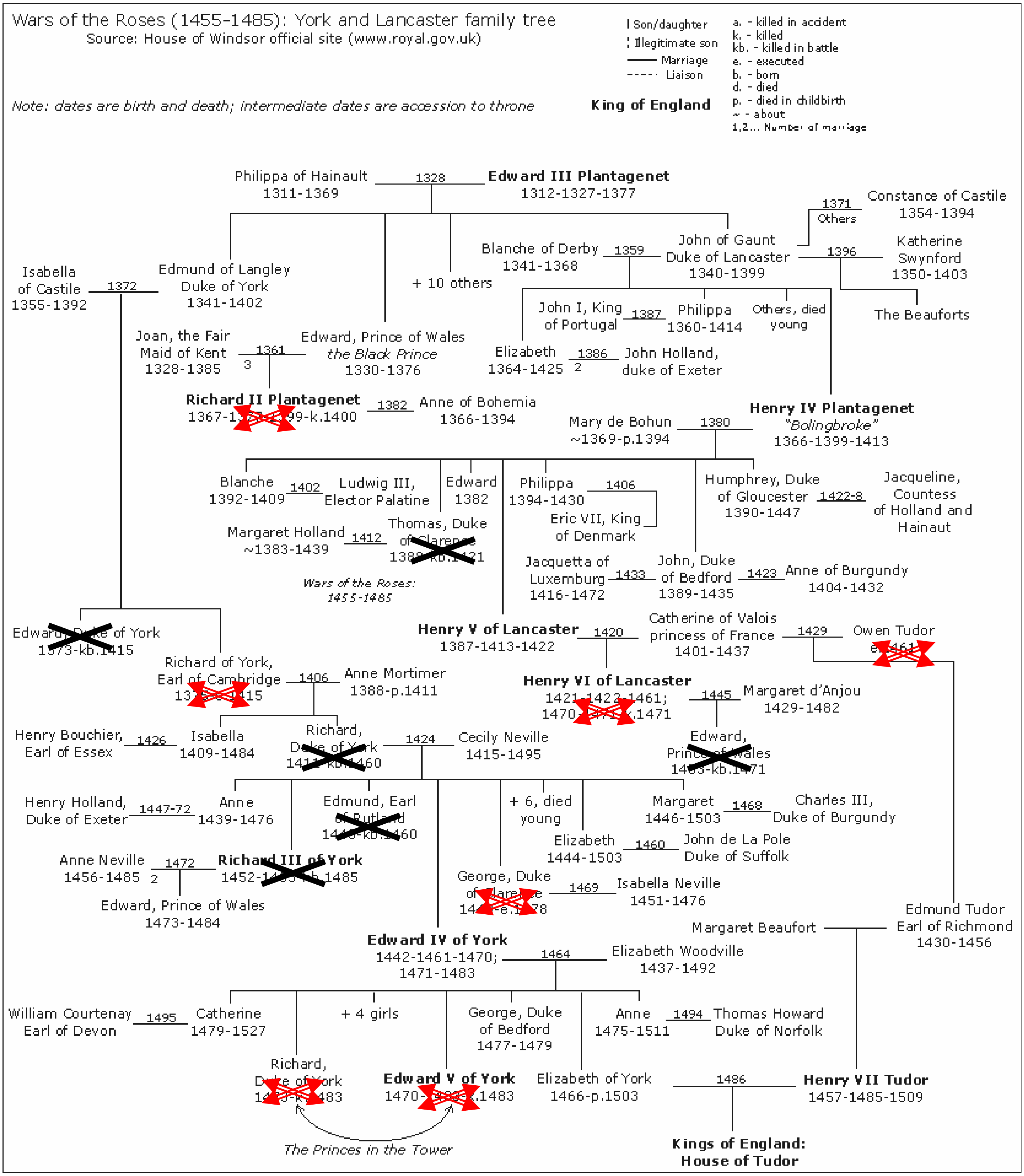

Figure 6: `Killing' equilibrium in the War of Roses (1455 – 1485)

killed in battle , **executed**