Collusion as an Informed Principal Problem*

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Abstract

In this paper we address the question of collusion in mechanisms under asymmetric information. We develop a methodology to analyze collusion as an informed principal problem. First, if collusion occurs after the agents accept or reject the principal's offer; the dominant-strategy implementation of the optimal contract without collusion is collusion proof. Second, we look at a different timing, assuming that the agents' decision to accept or reject the principal's offer is taken after collusion, so agents can collude on their participation decisions. We also assume that the collusion offer includes a punishment strategy, to be used whenever the other agent rejects the side contract. We establish the conditions that have to be satisfied for a contract to be collusion proof and we show that the optimal contract without collusion is no longer collusion proof. The optimal collusion proof contract is asymmetric, both in transfers and in quantities.

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1 Introduction

In a one-principal-multi-agent environment, the Revelation Principle states that, in the absence of collusion and in a complete contract framework, the optimal allocation can be reached by a centralized organization in which the principal contracts simultaneously with each agent. Any implementable allocation can be replicated by a mechanism in which agents are asked to announce their private information and have incentives to tell the truth. The possibility of collusion, however, may modify the set of achievable outcomes for the principal, because agents may have incentives to collectively deviate from truth-telling. Knowing this, the principal has to find the optimal response to the possibility of collusion.

Collusion seems quite possible in many examples. In an auction the mechanism designer exacerbates competition among agents in order to obtain a high price for the good. Bidders may have incentives to coordinate their bids in order to bid as low as possible. In a firm in which agents work together, it seems quite easy for the agents to coordinate their actions and take advantage of the firm's offer. Nevertheless, collusion may not be an easy task. Asymmetries of information among agents introduce frictions at the time of signing an agreement. Under symmetric information, any bargaining process will maximize the agents' joint utility. When information is asymmetric, each agent may want to conceal his private information in order to increase his own utility, and this could go against the maximization of joint utility. The relevant question is, then, what collusion under asymmetric information can achieve.

In this paper we address the question of collusion in mechanisms by assuming that one of the colluding parties offers a side contract to the other one. Thus, collusion has to be analyzed as an informed principal problem. We consider an organization composed by three parties: a principal, and two agents. Given a contract offered by the principal, agents can agree on a collusion contract. We assume that one of the agents has all the bargaining power in collusion and makes an offer to the other agent (to call this contract we will use the words collusion contract and side contract interchangeably). The principal is interested in a final good, that is produced using two inputs and each of the agents produces one input. Agents are privately informed about their marginal cost of production and their private information is independent.

Laffont and Martimort (1997) and (2000) develop a methodology to analyze the case of optimal contracting when agents can sign side contracts under asymmetric information. They assume that an uninformed third party, interested in the maximization of joint utility, organizes side contracting between agents. The advantage of this methodology is that the revelation principle applies at the collusion stage. On the down side, the third-party approach minimizes the frictions within the coalition and overstates the internal efficiency of the collusive agreement. They show that when agents' private information is statistically independent the optimal contract without collusion is implementable even when collusion is possible. This is no longer true when the agents' private information is correlated.

Dequiedt (2002) uses the methodology developed by Laffont and Martimort, but assuming that agents accept the principal's offer after having agreed on a collusion contract. Looking at the case of an independent private value auction, he shows that if quantity reallocations are not feasible at the collusion stage, then the principal can benefit from offering asymmetric mechanisms, because the presence of asymmetries increases the frictions among the colluding parties. However, when quantity reallocations are feasible, the principal can no longer exploit asymmetries.

Another piece of literature related to our paper concerns the mechanism design by an informed party. This is exactly what we observe at the collusion stage, where a privately informed agent offers a collusion contract to another agent, also privately informed. Myerson (1983) shows that there is always an equilibrium of the informed principal game and he states the *inscrutability principle*, which implies that there is no loss of generality in restricting attention to pooling mechanisms, that is, equilibria in which the principal, whatever his type offers the same mechanism to the agent. Maskin and Tirole (1990) and (1992) characterize the set of equilibria of the informed principal game and show that it depends on whether the principal's private information is an argument of the agent's utility function or not.

In our model, as in Laffont and Martimort (1997) and (2000), the Revelation Principle has to be replaced by a Collusion-Proofness Principle, which allows us to restrict attention to direct revelation mechanisms that do not leave any scope for collusion.

We first show in Section 3 that if collusion occurs after the agents accept or reject the principal's offer, then the optimal contract without collusion (the second-best contract) is collusion-proof. This shows that in this particular case, the use of a third party gives a good approximation of what the principal can obtain when collusion is actually organized by one of the colluding parties. Moreover, even if we give more power to the coalition by assuming that one agent can commit to a punishment strategy if the other agent refuses to collude, the result is not modified: the optimal contract without collusion is still collusion-proof.

In Section 4, we consider a different timing. As in Dequiedt (2002), we assume that the agents collude before accepting the principal's offer. Very often there is some delay between the principal's offer and the agents' agreement, which may be used to collude against the principal. This makes this analysis particularly relevant. The new feature of this model is that agents share their private information before accepting the principal's offer, meaning that they know each other's type when the participation decision is taken. Implicitly, this is equivalent to assuming that agents can collude on their participation decisions. Furthermore, we assume that the collusion offer includes a punishment strategy, to be used whenever the other agent rejects the collusion contract. We establish the conditions that have to be satisfied for a contract to be collusion-proof and we show that the second-best is not collusion-proof and hence it is not implementable. Whenever the principal wants to implement the second-best contract, the agent with the bargaining power finds a way to undo the principal's offer and increase his own utility. The principal has to modify her offer with respect to the second-best in order to account for the collusion constraints. Special incentives have to be provided to the agent who has the bargaining power at the collusion stage. To do so, the principal offers an asymmetric contract in which the expected utility of the agent who offers the collusion contract is always higher than that of the other agent.

Suppose instead, that we follow Laffont and Martimort's third-party approach, while retaining the assumption that agents collude before accepting the principal's offer. Furthermore, assume that the third party can impose punishments on the agents if they reject the side contract. Then, we would find that the second-best contract is collusion-proof and hence implementable. The reason for such a result is that the principal always prefers to deal with a more efficient coalition to avoid the problem of double marginalization of rents.[1] A third

[1] This is analogous to the industrial organization literature, which argues that it is socially more efficient to have a vertically integrated monopolist rather than two vertically disintegrated monopolists.

party who is interested in joint utility maximizes the internal efficiency of the coalition, given the constraints imposed by asymmetric information. Thus, assuming a third party that maximizes a weighted sum of the agents' utilities would increase the frictions inside the coalition, making the analysis more flexible. Moreover, the relative weight given to each agent could be interpreted as a measure of his relative bargaining power.[2]

The paper is organized as follows. In Section 2 we describe the main features of the model and we derive a weak collusion-proofness principle that states that we can restrict attention to direct revelation mechanisms for which no collusion is an equilibrium.

In Section 3 we assume that agents collude after having accepted the principal's offer. We provide some conditions for a mechanism to be weakly collusion-proof and we show that the optimal contract without collusion is weakly collusion-proof. In Section 4 we assume that collusion occurs before the agents accept or reject the contract and that the agent who offers the side contract commits to a punishment strategy whenever the other agent rejects his offer. We give conditions for a contract to be collusion-proof in this context and we show that the optimal contract without collusion is not implementable. We characterize the optimal collusion-proof mechanism. We discuss some assumptions of the model in Section 5. Finally, we conclude in Section 6. All the proofs that are not in the text are in the Appendix.

2 The model

We consider an organization composed of three agents. A principal (P) produces a final good and contracts with two input suppliers (A^1 and A^2). Agent A^k's contribution to a quantity q of the final good is q^k. We assume a Leontief production technology: $q = \min\{q^1, q^2\}$, so $q = q^1 = q^2$ in any optimal allocation.

The principal obtains a monetary revenue from selling the output equal to $S(q)$ and

[2] If we considered a third party who gives all the weight to the utility of a single agent, we would actually find the same results as in our informed principal model in which one agent has all the bargaining power. This is not a priori obvious, given the potential inefficiencies introduced by the informed principal problem. Che and Kim (2004) show that if collusion is assumed to be the outcome of the maximization of a weighted sum of the utilities of the members of the coalition, the second best can virtually always be implemented when agents accept the principal's offer before colluding.

pays a monetary transfer of t^k to A^k. So the principal's total profit is

$$W = S(q) - \left(t^1 + t^2\right).$$

We assume that the function $S(\cdot)$ is strictly increasing and strictly concave and satisfies the Inada conditions: $S'(0) = +\infty$ and $S'(+\infty) = 0$.

Agent k receives a monetary payment t^k and incurs a linear production cost $\theta^k q^k$ when producing q^k units. So A^k's total utility is

$$u^k = t^k - \theta^k q^k.$$

Agent k's marginal cost, θ^k, is his private information. We assume that marginal costs are independent and identically distributed, drawn from a common knowledge distribution with support $\Theta = \{\theta_1, \theta_2\}$, $\theta_1 < \theta_2$, $\Delta\theta = \theta_2 - \theta_1$, and we call ν_i the probability of observing θ_i: $\nu_i = \Pr\left(\theta^k = \theta_i\right)$, $\nu_1 + \nu_2 = 1$.[3] We assume that at all the optimal solutions the principal's profit is positive, so it is never optimal to shut down production, even if both agents turn out to be high-cost.

We depart from the previous literature in assuming that at the collusion stage, it is an informed party, A^1, who makes the offer, so the way to analyze this problem is to look at the informed principal literature and we will refer to agent 1 in the collusion game as the sub-principal. For a given offer by the principal, a collusion contract is a manipulation of reports function, ϕ, side transfers, y, from A^1 to A^2 and potentially a punishment strategy, x if A^2 rejects collusion. The manipulation of reports function and the punishment strategy define a probability distribution over possible reports in the mechanism offered by the principal. This implies that we allow for stochastic collusion contracts.[4]

We need to introduce some definitions. For a direct revelation mechanism, define the null collusion contract as the contract in which there is no manipulation of reports and side transfers are null: $\phi(m^1, m^2) = (m^1, m^2)$ and $y(m^1, m^2) = 0$ for any announcements made at the collusion stage. We denote by $\emptyset$ the null contract.

[3] The two-types assumption is made only for simplicity of exposition. All the results, except those about characterization, extend to more general type spaces. For an extension to a continuous type space, see Quesada (2004).

[4] Since both agents are risk neutral, we can restrict ourselves to deterministic transfers in side contracting. Indeed, we could replace any random transfer by its mean without changing anything.

Definition 1 *A grand contract GC is* weakly collusion-proof *if there exists one equilibrium of the collusion game in which payoffs are the same as with the null collusion contract.*

Definition 2 *A grand contract GC is* strongly collusion-proof *if it is weakly collusion-proof and in any equilibrium of the collusion game, the sub-principal's payoff is the same as under the null collusion contract.*

Myerson (1983) has shown that there is always at least one equilibrium of the informed principal game. So, for any offer GC there is at least one equilibrium of the collusion game. In order to analyze the outcome of the collusion game, we invoke Myerson's *inscrutability principle* (Myerson (1983)), which states that, when looking at the informed sub-principal game, there is no loss of generality in restricting attention to pooling equilibria. This implies that on the equilibrium path the collusive offer by agent 1 does not modify the beliefs of agent 2.

We are interested in the best contract for P, knowing that collusion is possible between the two agents. To do that, we prove a weak collusion-proofness principle.

We introduce some notation in order to explain this principle. Consider any grand contract $GC = \{M^1, M^2, t^1(\cdot,\cdot), t^2(\cdot,\cdot), q(\cdot,\cdot)\}$, where M^i is the set of messages the principal chooses to be available for agent i. Denote by $\mathcal{GC}$ the set of grand contracts that can be offered by the principal, so GC is a typical element of $\mathcal{GC}$. Following an offer GC, the agents agree on a side contract. Applying the *revelation principle* for Bayesian games we know that any equilibrium outcome of the collusion game that follows GC is a side contract $SC_{GC} = \{\Theta, \Theta, \phi(\cdot,\cdot), x(\cdot), y(\cdot,\cdot)\}$.[5] Denote by $\mathcal{SC}_{GC}$ the set of side contracts that can follow the offer of a general contract $GC \in \mathcal{GC}$, so SC_{GC} is a typical element of $\mathcal{SC}_{GC}$. For any $GC \in \mathcal{GC}$ there is at least one equilibrium collusion contract $SC^*_{GC} \in \mathcal{SC}_{GC}$. Therefore, if the principal offers mechanism GC, which is followed by SC^*_{GC}, the equilibrium payoffs of all the parties are determined by the combination of GC and SC^*_{GC}. The vector of equilibrium payoffs when the principal offers GC is $v^*_{GC} = (W^e(GC, SC^*_{GC}), u^{1e}(GC, SC^*_{GC}, \theta^1), u^{2e}(GC, SC^*_{GC}, \theta^2))$, where e stands for expectation. Call $\mathcal{V}_{GC}$ the set of equilibrium payoffs if the principal offers

[5] The revelation principle for Bayesian games implies that we can restrict the set of messages in the side contract to be equal to the set of types.

mechanism GC and $\mathcal{V} = \bigcup_{GC \in \mathcal{GC}} \mathcal{V}_{GC}$ the set of all possible equilibrium payoffs.[6] Finally, call $\mathcal{WCP} \subset \mathcal{GC}$ the set of Weakly Collusion-Proof Direct Revelation Mechanisms.

Proposition 1 Weak collusion-proofness principle. $\mathcal{V} = \bigcup_{GC \in \mathcal{WCP}} \mathcal{V}_{GC}$. *That is, the set of all possible equilibrium payoffs coincides with the set of equilibrium payoffs following a weakly collusion-proof direct revelation mechanism.*

In words, this principle states that when looking for the optimal mechanism for the principal, we can restrict attention to direct revelation mechanisms that do not leave any scope for collusion. The idea behind this result is that if the principal can induce a particular payoff vector with any offer followed by collusion, she can induce exactly the same payoff vector through a direct revelation mechanism for which the null collusion contract is an equilibrium of the collusion subgame.

However, the collusion-proofness principle is only weak, because there may be multiple equilibria of the collusion game. There are two sources of multiplicity of equilibria. First, Myerson (1983) and Maskin and Tirole (1992) have proved that, in general, the equilibrium of the informed (sub-)principal game is not unique. Second, the conditions for a mechanism to be weakly collusion-proof depend on the outcome of the game in the (out-of-equilibrium) event in which agent 2 rejects the side contract, because this is what determines the status quo at the collusion stage. This outcome, in turn, depends on the out-of-equilibrium beliefs of agent 1 about agent 2's type following a rejection. A usual assumption in the literature on collusion is that beliefs are passive, in the sense that beliefs following a rejection of the side contract are equal to the prior beliefs.[7]

In contrast with previous collusion-proofness results, we cannot ensure that, given a weakly collusion-proof mechanism, the null contract would be the unique equilibrium *even if we restrict ourselves to passive beliefs*. Indeed, due to the multiplicity of equilibria of the informed sub-principal game our collusion-proofness principle is weak even for passive beliefs and therefore, at the collusion stage other equilibria different from the null contract could be

[6]If there are multiple equilibria of the collusion subgame following GC, the set $\mathcal{V}_{GC}$ contains the equilibrium payoffs for all those equilibria.

[7]See, for instance, Laffont and Martimort (1997), (1998), (2000) and Faure-Grimaud, Laffont, and Martimort (2003).

selected. That a strong version of the principle cannot be proved is straightforward, because whenever the principal's offer GC is weakly collusion-proof but fails to be strongly collusion-proof, the principal may actually obtain the payoff corresponding to GC if the non-collusive equilibrium is selected.

2.1 Some efficiency concepts

Collusion-proofness implies some kind of efficiency of the null contract at the collusion stage. Exactly which kind of efficiency is something that we will determine afterwards. For the time being, let us concentrate on some efficiency definitions, borrowed from Maskin and Tirole (1992) and adapted to this particular context.

Definition 3 *An allocation* $\left(\overline{\phi}_{k\ell}, \overline{y}_{k\ell}\right)_{\substack{k=1,2\\ \ell=1,2}}$ *is* weakly interim efficient *(WIE) if (a) it is interim incentive compatible for* A^1 *and (b) there is no allocation satisfying (a) that,* regardless of A^1's type, *is incentive compatible for* A^2 *and gives* A^2 *at least as much utility. That is, a WIE allocation is, for some vector of positive weights,* $\{w_k\}_{k=1,2}$ *a solution to problem I defined as:*[8]

$$\max_{(\phi_{k\ell}, y_{k\ell})_{\substack{k=1,2\\ \ell=1,2}}} \sum_k w_k \sum_\ell \nu_\ell \left(t^1\left(\phi_{k\ell}\right) - y_{k\ell} - \theta_k q\left(\phi_{k\ell}\right)\right)$$

subject to

$$\sum_\ell \nu_\ell \left(t^1\left(\phi_{i\ell}\right) - y_{i\ell} - \theta_i q\left(\phi_{i\ell}\right)\right) \geq \sum_\ell \nu_\ell \left(t^1\left(\phi_{i'\ell}\right) - y_{i'\ell} - \theta_i q\left(\phi_{i'\ell}\right)\right) \quad \forall\left(i, i'\right) \in \{1,2\}^2,$$

$$t^2\left(\phi_{i\ell}\right) + y_{i\ell} - \theta_\ell q\left(\phi_{i\ell}\right) \geq t^2\left(\phi_{i\ell'}\right) + y_{i\ell'} - \theta_\ell q\left(\phi_{i\ell'}\right) \quad \forall\left(i, \ell, \ell'\right) \in \{1,2\}^3,$$

$$t^2\left(\phi_{i\ell}\right) + y_{i\ell} - \theta_\ell q\left(\phi_{i\ell}\right) \geq t^2\left(\overline{\phi}_{i\ell}\right) + \overline{y}_{i\ell} - \theta_\ell q\left(\overline{\phi}_{i\ell}\right) \quad \forall\left(i, \ell\right) \in \{1,2\}^2.$$

Definition 4 *An allocation* $\left(\hat{\phi}_{k\ell}, \hat{y}_{k\ell}\right)_{\substack{k=1,2\\ \ell=1,2}}$ *is a* Rothschild-Stiglitz-Wilson *(RSW) allocation relative to the status quo allocation* $(\phi^0_{k\ell}, y^0_{k\ell})_{\substack{k=1,2\\ \ell=1,2}}$ *if and only if, for all k:* $(\phi_{k\ell}, y_{k\ell})_{\substack{k=1,2\\ \ell=1,2}} =$

[8]Because we allow a stochastic manipulation of reports, the function $t^i\left(\phi_{k\ell}\right)$ (resp. $q\left(\phi_{k\ell}\right)$) has to be interpreted as an expectation with respect to the distribution of reports in the grand contract induced by $\phi_{k\ell}$. Similarly, $S\left(q\left(\phi_{k\ell}\right)\right)$ is the expectation of the function S.

$\left(\hat{\phi}_{k\ell}, \hat{y}_{k\ell}\right)_{\substack{k=1,2 \\ \ell=1,2}}$; *and* $\forall k$, $\left(\hat{\phi}_{k\ell}, \hat{y}_{k\ell}\right)_{\ell=1,2}$ *is obtained as a solution to problem* II_k *defined as*

$$\max_{\left(\phi_{k\ell}, y_{k\ell}\right)_{\substack{k=1,2 \\ \ell=1,2}}} \sum_{\ell} \nu_\ell \left(t^1\left(\phi_{k\ell}\right) - y_{k\ell} - \theta_k q\left(\phi_{k\ell}\right)\right)$$

subject to

$$\sum_{\ell} \nu_\ell \left(t^1\left(\phi_{i\ell}\right) - y_{i\ell} - \theta_i q\left(\phi_{i\ell}\right)\right) \geq \sum_{\ell} \nu_\ell \left(t^1\left(\phi_{i'\ell}\right) - y_{i'\ell} - \theta_i q\left(\phi_{i'\ell}\right)\right) \quad \forall (i, i') \in \{1,2\}^2$$

$$t^2\left(\phi_{i\ell}\right) + y_{i\ell} - \theta_\ell q\left(\phi_{i\ell}\right) \geq t^2\left(\phi_{i\ell'}\right) + y_{i\ell'} - \theta_\ell q\left(\phi_{i\ell'}\right) \quad \forall (i, \ell, \ell') \in \{1,2\}^3$$

$$t^2\left(\phi_{i\ell}\right) + y_{i\ell} - \theta_\ell q\left(\phi_{i\ell}\right) \geq t^2\left(\phi^0_{i\ell}\right) + y^0_{i\ell} - \theta_\ell q\left(\phi^0_{i\ell}\right) \quad \forall (i, \ell) \in \{1,2\}^2.$$

As Maskin and Tirole (1992) have shown, any RSW allocation is WIE and, therefore, incentive compatible for agent 1. Moreover, any WIE allocation is RSW relative to itself. This result turns out to be important for the characterization of weakly collusion-proof mechanisms.

Definition 5 *An allocation* $\left(\overline{\phi}_{k\ell}, \overline{y}_{k\ell}\right)_{\substack{k=1,2 \\ \ell=1,2}}$ *is* interim efficient relative to beliefs $\hat{\nu}$ *(IE($\hat{\nu}$)) if and only if for some vector of positive weights,* $\{w_k\}_{k=1,2}$ *it is a solution to problem III defined as*

$$\max_{\left(\phi_{k\ell}, y_{k\ell}\right)_{\substack{k=1,2 \\ \ell=1,2}}} \sum_{k} w_k \sum_{\ell} \nu_\ell \left(t^1\left(\phi_{k\ell}\right) - y_{k\ell} - \theta_k q\left(\phi_{k\ell}\right)\right)$$

subject to

$$\sum_{\ell} \nu_\ell \left(t^1\left(\phi_{i\ell}\right) - y_{i\ell} - \theta_i q\left(\phi_{i\ell}\right)\right) \geq \sum_{\ell} \nu_\ell \left(t^1\left(\phi_{i'\ell}\right) - y_{i'\ell} - \theta_i q\left(\phi_{i'\ell}\right)\right) \quad \forall (i, i') \in \{1,2\}^2$$

$$\sum_{i} \hat{\nu}_i \left(t^2\left(\phi_{i\ell}\right) + y_{i\ell} - \theta_\ell q\left(\phi_{i\ell}\right)\right) \geq \sum_{i} \hat{\nu}_i \left(t^2\left(\phi_{i\ell'}\right) + y_{i\ell'} - \theta_\ell q\left(\phi_{i\ell'}\right)\right) \quad \forall (\ell, \ell') \in \{1,2\}^2$$

$$\sum_{i} \hat{\nu}_i \left(t^2\left(\phi_{i\ell}\right) + y_{i\ell} - \theta_\ell q\left(\phi_{i\ell}\right)\right) \geq \sum_{i} \hat{\nu}_i \left(t^2\left(\overline{\phi}_{i\ell}\right) + \overline{y}_{i\ell} - \theta_\ell q\left(\overline{\phi}_{i\ell}\right)\right) \quad \forall \ell \in \{1,2\}.$$

The collusion-proofness principle implies that in order to characterize the principal's set of feasible offers we need to find necessary and sufficient conditions for an allocation to be weakly collusion-proof. In the next two sections we characterize the set of weakly collusion proof mechanisms under two alternative timings and we obtain the contract that maximizes the principal's payoff.

3 Ex post collusion

In this section, we follow the approach in Laffont and Martimort (1997), in the sense that we assume that collusion, if any, occurs after accepting the principal's offer. The timing is as follows:

1. P offers a grand contract, GC in the set of direct revelation mechanisms, to A^1 and A^2, $GC = \{\Theta, \Theta, t^1(\cdot,\cdot), t^2(\cdot,\cdot), q(\cdot,\cdot)\}$.
2. A^1 and A^2 simultaneously accept or reject P's offer. If one of them rejects, the game is over and everyone obtains 0. If they both accept, they go to stage 3.
3. A^1 offers a collusion contract, SC, to A^2, $SC = \{\Theta, \Theta, \phi(\cdot,\cdot), y(\cdot,\cdot)\}$.
4. A^2 accepts or rejects A^1's offer. If A^2 rejects the offer, A^1 and A^2 go to stage 6. If A^2 accepts A^1's offer, they go to stage 5.
5. A^1 and A^2 simultaneously announce their types to each other.
6. A^1 and A^2 simultaneously announce their types to the principal (according to the function ϕ if they had agreed on a collusion contract).

The informed sub-principal problem that we have to analyze at the collusion stage corresponds to the common value framework as defined by Maskin and Tirole (1992), because A^2's status quo level at the collusion stage is given by the utility he would obtain under GC, whenever he rejects the side contract, which, in turn, is a function of A^1's private information. Although there is a problem of multiplicity of equilibria, Maskin and Tirole (1992) have shown that there is a lower bound to the payoff of the sub-principal that can be easily characterized.

An important result of the informed sub-principal model is that, regardless of his type, the sub-principal can always guarantee himself the payoff corresponding to the RSW allocation. Indeed, he can propose the allocation that solves problem II_k. Notice, then, that whatever the beliefs of agent 2 about agent 1's type, A^2 accepts the contract and is truthful, and therefore, A^1's payoff is the RSW payoff. This implies that in order to look for the set of equilibria of the collusion game, only allocations that give at least the RSW payoff to both types of A^1 can be candidates. We will call $\hat{u}_k^1$ the RSW payoff of type k agent 1.

In order to look for the set of weakly collusion-proof mechanisms, we can restrict attention to (interim) incentive compatible direct revelation mechanisms, because, on the equilibrium path, collusion does not occur.

Proposition 2 *Assume that the RSW allocation is IE relative to some strictly positive beliefs and that the null contract satisfies agent 2's incentive constraints whatever agent 1's type. Then, a) if the null side contract is RSW relative to the status quo allocation, the mechanism GC is weakly collusion-proof; b) the converse is also true provided that agent 1 is truthful even if agent 2 rejects the side contract.*

Proof. a) If the RSW allocation is IE relative to strictly positive beliefs, the RSW allocation is an equilibrium of the game (Theorem 1* in Maskin and Tirole (1992)). But the RSW allocation is the null contract itself. Therefore, the null contract is an equilibrium of the game and GC is weakly collusion-proof.

b) Since A^1 can guarantee the RSW payoff, we know that in any equilibrium of the collusion game, any type k A^1 gets $\tilde{u}_k^1 \geq \hat{u}_k^1$. Now, because A^1 is truthful on GC when A^2 rejects the side contract, the status quo payoff of A^2 is the null contract (by assumption, A^2 is truthful whatever his beliefs about A^1's type). Suppose that the null contract is not RSW relative to itself. Then, it is not weakly interim efficient. Now, since the null contract satisfies the ex post incentive compatibility constraints of A^2, it satisfies all the constraints in program I (it satisfies A^1's incentive compatibility constraints because GC is interim incentive compatible), but it is not RSW, so $\forall k$, $\hat{u}_k^1 \geq u_k^{10}$, where u_k^{10} is the utility of type k A^1 under the null contract. Moreover, $\exists k$ such that $\hat{u}_k^1 > u_k^{10}$ because the null contract is not WIE. Thus, the null contract cannot be an equilibrium of the collusion game, because it gives strictly less utility than the RSW allocation to at least one type. Therefore, GC is not weakly collusion-proof. ■

Proposition 2 gives, then, necessary and sufficient conditions for the grand contract to be weakly collusion-proof, provided that the RSW allocation is interim efficient for some strictly positive beliefs. Loosely speaking, a mechanism is weakly collusion-proof if and only if the null side contract corresponding to this mechanism is an RSW allocation in the collusion game.

There are two obvious cases in which agent 1 is truthful out of the equilibrium path. One is the already mentioned case of passive beliefs. Because the original offer is incentive compatible for the prior beliefs, whenever the out-of-equilibrium beliefs are equal to the priors, agent 1's incentive compatibility constraints are satisfied also out of the equilibrium path. The second case, is the case in which agent 1's incentive compatibility constraints are satisfied whatever the type of agent 2. This case is particularly interesting, because the fact that agent 1 is truthful out of the equilibrium path is independent of the specific out-of-equilibrium beliefs.

3.1 The second-best contract

If collusion between agents is impossible (for instance, non-enforceable), the best the principal can do is to offer the second-best contract, $GC^{sb} = \left(t^{1sb}, t^{2sb}, q^{sb}\right)$ The second-best contract is characterized by the following conditions:

$$\begin{aligned}
\sum_{\ell} \nu_{\ell} t_{2\ell}^{1sb} = \sum_{k} \nu_{k} t_{k2}^{2sb} &= \theta_2 \sum_{k} \nu_k q_{k2}^{sb}, \\
\sum_{\ell} \nu_{\ell} t_{1\ell}^{1sb} = \sum_{k} \nu_{k} t_{k1}^{2sb} &= \theta_1 \sum_{k} \nu_k q_{k1}^{sb} + \Delta\theta \sum_{k} \nu_k q_{k2}^{sb}, \\
S'\left(q_{11}^{sb}\right) &= 2\theta_1, \\
S'\left(q_{12}^{sb}\right) = S'\left(q_{21}^{sb}\right) &= \theta_1 + \theta_2 + \frac{\nu_1}{\nu_2}\Delta\theta, \\
S'\left(q_{22}^{sb}\right) &= 2\theta_2 + 2\frac{\nu_1}{\nu_2}\Delta\theta.
\end{aligned}$$

Since agents are both risk neutral, the principal has some degrees of freedom in choosing the transfers, because only the expected transfer given to each agent is determined by the optimal contract. The quantities, however, are uniquely determined in the second-best contract and downward distorted with respect to the full information case.

One interesting property of the second-best contract is that it can be implemented in dominant strategies.[9] That is, we can find transfers such that the incentive and participation constraints of agent i are satisfied whatever agent j's type:

$$\begin{aligned}
t_{1i}^{1ds} &= t_{i1}^{2ds} = \theta_1 q_{1i}^{sb} + \Delta\theta q_{2i}^{sb}, \\
t_{2i}^{1ds} &= t_{i2}^{2ds} = \theta_2 q_{2i}^{sb}, \qquad (1)
\end{aligned}$$

[9] See Laffont and Tirole (1993) chap. 7 and Mookherjee and Reichelstein (1992).

We are now ready to state the main result of this section.

Proposition 3 *If $S'''(\cdot) \geq 0$, the dominant-strategy implementation of the second-best contract is weakly collusion-proof.*

According to Proposition 3, the principal can still implement the second-best contract in dominant strategies even if agents can collude. The reason is that when the second best contract is offered, then the null contract maximizes the agents' joint utility: The informational rents that the principal is giving in the second-best offer increase agent 2's outside option in a way that agent 1 cannot compensate it without losing himself. This is also the result in Laffont and Martimort (1997) when they allow for non-anonymous contracts. The difference with their paper is not in the result, but in the methodology. We provide here some insights on how one should look at the problem of collusion under asymmetric information, when it is organized by one of the colluding parties. This is a first step towards the analysis of collusion as a bargaining process under asymmetric information.

The dominant-strategy implementation of the second-best contract is weakly collusion-proof, so there is one equilibrium in which collusion does not occur. However, given that the collusion game has potentially multiple equilibria, an interesting question is whether the second-best contract is strongly collusion-proof. We can easily eliminate one source of multiplicity of equilibria.

Remark 1 *The dominant-strategy implementation of the second-best contract is weakly collusion-proof whatever the out-of-equilibrium beliefs following a rejection by agent 2.*

Since the contract is implemented in dominant strategies, the incentive constraints of agent 1 when there is no collusion are satisfied *for any beliefs about agent 2's type.* Therefore, suppose agent 2 rejects the side contract. Then, no matter what agent 1 infers from this action, he has always incentives to tell the truth. Thus, from type j agent 2's point of view the probability that agent 1 announces θ_i is equal to ν_i, so the incentive constraint writes

$$\sum_i \nu_i \left(t_{ij}^2 - \theta_j q_{ij}\right) \geq \sum_i \nu_i \left(t_{i\ell}^2 - \theta_j q_{i\ell}\right),$$

which is satisfied for any j, because the contract is incentive compatible in dominant strategies also for agent 2.

To eliminate the second source of multiplicity we have to show that the null side contract is the unique equilibrium of the collusion game.

Proposition 4 *For given out-of-equilibrium beliefs following a rejection, if the null contract is an RSW allocation relative to the status quo and is interim efficient relative to the prior beliefs, then the GC is strongly collusion-proof.*

Proof. Because the RSW allocation is interim efficient relative to the prior beliefs (which are strictly positive), then the set of equilibrium allocations is the set of allocations that satisfy the interim incentive compatibility constraints of the two agents and the interim participation constraints of agent 2 and Pareto dominate (from agent 1's view point) the RSW allocation. Even if the RSW allocation is not unique, the RSW payoffs for A^1 are. Now, because the null contract is an RSW allocation and is interim efficient relative to the prior beliefs, in any equilibrium of the collusion game, A^1's payoff is the RSW payoff. Therefore, A^1 cannot do strictly better by offering a non-null collusion contract. ■

When the RSW allocation is interim efficient relative to the prior beliefs, the set of equilibria of the informed sub-principal game shrinks to the RSW allocation. Therefore, once agent 1's beliefs about agent 2's type following a rejection of the side contract are fixed (therefore, agent 2's status quo payoff is also fixed), if the null contract is RSW and interim efficient relative to the prior beliefs, it has to be the unique equilibrium.

Proposition 5 *If the second-best contract is offered and the null contract is RSW relative to itself, then, the null contract is interim efficient relative to the prior beliefs.*

Whenever the null contract is RSW relative to itself, the principal can make the null side contract the unique equilibrium at the collusion stage with fixed out-of-equilibrium beliefs following a rejection by agent 2. By doing so, the principal eliminates other equilibria at the collusion stage and thus, can ensure the second-best payoff. Moreover, the Revelation Principle implies that the second-best payoff represents an upper bound for what the principal can get in any mechanism. Hence, collusion in this context can never be beneficial for the principal. Therefore, if the collusion game has multiple equilibria, including the null contract, the principal's payoff when the null contract is selected is at least as high as in any other equilibrium. Since the second-best contract is strongly collusion-proof, the principal can guarantee that the unique equilibrium of the collusion game, when the dominant-strategy implementation of the second-best contract is offered, is the null contract. In this way, she is sure to obtain the second-best payoff, even when collusion is possible.

The result of this section implies that the coalition is not powerful enough to be able to change the outcome when the second-best contract is offered. We could imagine that collusion would be more powerful if agent 1 could commit to punish agent 2 whenever the latter rejects the side contract. For instance, he could commit to announce a high cost if the side contract is not accepted (assuming that he can indeed commit to such an announcement). This reduces agent 2's outside option because agent 2's rent decreases when agent 1 is inefficient and gives more power to agent 1 to implement collusion. Indeed, the dominant-strategy implementation of the second-best is not RSW relative to the new status quo of agent 2. A coalition of (θ_1, θ_2) would like to mimic (θ_2, θ_2). With this manipulation of reports agent 1 can impose a small penalty to agent 2 whenever the true state is (θ_1, θ_1). If the punishment is credible, type 1 agent 2 accepts the penalty, because his utility is lower when agent 1 punishes by announcing a high cost. However, there is another implementation of the second-best contract that is RSW. In order to avoid a manipulation of reports as mentioned before, the collusion constraint of a (θ_1, θ_2) coalition who pretends to be (θ_2, θ_2) has to be binding. Following the proof of Proposition 3, the following asymmetric transfers would do the job:[10]

$$\begin{aligned}
t_{11}^{1sb} &= \theta_1 q_{11}^{sb} + \nu_1 \Delta\theta q_{12}^{sb} + \nu_2 \Delta\theta q_{22}^{sb}, & t_{11}^{2sb} &= \theta_1 q_{11}^{sb} + \Delta\theta q_{21}^{sb}, \\
t_{12}^{1sb} &= \theta_1 q_{12}^{sb} + \nu_1 \Delta\theta q_{12}^{sb} + \nu_2 \Delta\theta q_{22}^{sb}, & t_{12}^{2sb} &= \theta_2 q_{12}^{sb}, \\
t_{21}^{1sb} &= \theta_2 q_{21}^{sb} + \nu_1 \Delta\theta \left(q_{21}^{sb} - q_{22}^{sb}\right), & t_{21}^{2sb} &= \theta_1 q_{21}^{sb} + \Delta\theta q_{22}^{sb}, \\
t_{22}^{1sb} &= \theta_2 q_{22}^{sb} - \frac{\nu_1^2}{\nu_2} \Delta\theta \left(q_{21}^{sb} - q_{22}^{sb}\right), & t_{22}^{2sb} &= \theta_2 q_{22}^{sb}.
\end{aligned}$$

Thus, the same conclusions are drawn with a model in which collusion is more powerful in the sense that agent 1 can punish agent 2 following a rejection.[11] Risk neutrality on the agents gives the principal enough degrees of freedom to find (asymmetric) transfers such that the second-best contract is weakly collusion-proof. The whole trick consists in rewarding (punishing) an inefficient agent 1 when he meets an efficient (inefficient) agent 2 and give a

[10] Take $\mu_{21} = \mu_{11} = \gamma_1 = 0$.

[11] Notice that here we do not need any condition on $S'''(\cdot)$.

constant rent to an efficient agent 1 (independent of both agents' types). The second-best contract is still implemented in dominant strategies for agent 2. Because all the bargaining power at the collusion stage belongs to agent 1, there is no point in changing the implementation for agent 2. It is enough to offer a contract such that agent 1 cannot improve upon by offering a collusion contract. Nevertheless, notice that agent 1 gets a negative utility when the state of nature is (θ_2, θ_2) in order to discourage him from proposing such a manipulation of reports.

4 Ex ante collusion

We have shown in the previous section that whether or not agent 1 can punish agent 2, collusion has no effect on the principal's payoff. This result depends, however, on a strong assumption about communication. It is assumed that agents accept the principal's offer before collusion takes place. Implicitly, this means that, before collusion, there is communication between the principal and the agents, but this communication is limited. Indeed, if the principal can communicate with the agents before the agents communicate with each other, what prevents her from forcing the agents to send messages about their types at the same time? This would make any collusion impossible. A more plausible assumption is that collusion between the agents happens before accepting or rejecting the principal's offer.[12] Following Dequiedt (2002), we assume in this section that agents collude before accepting or rejecting the principal's offer. The new feature is that agents can collectively decide whether to accept or reject the offer, so the principal is forced to design a contract that satisfies ex post participation constraints. Therefore, if agents get negative utility in some state, they would collectively decide to refuse the principal's offer. We know from the results of the previous section that if $S'''(\cdot) \geq 0$ and there is no punishment the principal can still implement the second-best contract with ex post participation constraints. She just needs to offer the dominant-strategy implementation. Things might be different if we allow the sub-principal

[12] This is typically the case in auctions, in which participation takes place with some delay after the design, giving time to the potential participants to communicate with each other before deciding whether to participate or not.

to punish the other agent whenever the latter rejects the side contract. [13] The new timing is as follows:

1. P offers a mechanism, GC, to A^1 and A^2, $GC = \{\Theta, \Theta, t^1(\cdot,\cdot), t^2(\cdot,\cdot), q(\cdot,\cdot)\}$.

2. A^1 offers a collusion contract, SC, to A^2, $SC\,\{\Theta, \Theta, \phi(\cdot,\cdot), x(\cdot), y(\cdot,\cdot)\}$

3. A^2 accepts or rejects A^1's offer. If A^2 rejects the offer, A^1 and A^2 go to stage 5. If A^2 accepts A^1's offer, they go to stage 4.

4. A^1 and A^2 simultaneously announce their types to each other.

5. A^1 and A^2 simultaneously accept or reject P's offer. If one of them rejects, the game is over and everyone obtains 0. If they both accept, they go to stage 6.

6. A^1 and A^2 simultaneously announce their types to the principal (according to the function ϕ if they agreed on a collusion contract or according to the function x for A^1 if they did not).

First, notice that the second-best contract without collusion is not affected by this change in the timing, because when there is no collusion, agents have to accept or reject the principal's offer under asymmetric information, so participation constraints are still interim.

[13] Punishing agent 2 may be an ex post suboptimal strategy for agent 1. However, he can find some means to commit to play such an ex post suboptimal strategy. Consider, for instance, the following mechanism designed by agent 1. Following the principal's offer, agent 1 offers a contract to agent 2 and to an uninformed witness. This contract consists of a side contract between agent 1 and agent 2 and specifies the punishment strategy if agent 2 rejects the offer and a small payment ε to the witness if he accepts the offer. Agent 1's offer includes a bank deposit equal to π, which goes back to agent 1 if agent 2 accepts the offer. The contract specifies that whenever agent 2 rejects the offer, if agent 1 follows the punishment strategy, he can get the deposit back. If agent 1 deviates from the punishment strategy, the witness can enforce the contract and get π for himself. If agent 1 deviates from the punishment strategy and the witness does not enforce the contract, the money goes to charity. The penalty π can always be made large enough in order to force agent 1 to implement the punishment strategy whatever his type. The witness always accepts the offer because he is sure to get ε, and, therefore the punishment strategy becomes enforceable even though it is not *a priori* ex post optimal. With this mechanism there is no scope for renegotiation between agent 1 and the witness.

Second, the status quo utility at the collusion stage, given by the payoff of A^2 if he rejects the collusion offer, is determined by the punishment strategy of A^1.

Observe that in the first stage we are imposing that the principal conditions the allocation only on the types revealed by the agents. We claim that the principal cannot benefit from using more complicated contracts.

Suppose that the principal's contract specifies a message space $\Theta \times M^i$ for each agent i. So each agent is asked to report his own type and (potentially) some other message. Given this offer, the collusion contract offered by agent 1 consists of the manipulation of reports function, $\phi : \Theta \times \Theta \to \Theta \times M^1 \times \Theta \times M^2$, the punishment strategy, $x : \Theta \to \Theta \times M^1$ and the side payment function, $y : \Theta \times \Theta \to \mathbb{R}$. This implies that the additional message M^1 is not of any use, because it will be set by the collusion contract, through the function ϕ if agent 2 accepts it or through the function x if he rejects it. On the other hand, the additional message M^2 does not help either if agent 2 accepts the collusion offer (on the equilibrium path). However, it could be of some use in the out of equilibrium event in which agent 2 rejects the collusion offer, since in this case agent 2 is not bound by any contract.

There are two pieces of information that the principal would want to extract from agent 2: his decision to accept or reject the collusion offer and the type of punishment strategy that agent 1 has committed to.

The decision of agent 2 to accept or reject the collusion contract could be useful if the principal could reward agent 2 or punish agent 1 (or both) in the case of rejection (rejection is always an out-of-equilibrium event, so the principal cannot lose by rewarding agent 2 at this point). In this way, the principal could force agent 1 to offer a "more generous" collusion contract. First note that punishing agent 1 in case of rejection does not change the decision of agent 2 of accepting or rejecting and therefore, cannot help the principal. The second option is to reward agent 2 if he announces rejection. But in this case, agent 2 should announce rejection even when he has actually accepted the contract. Thus, the reward has no effect in changing incentives and the principal ends up always paying the reward. The last possibility is to ask both agents this information, reward agent 2 if they announce rejection and punish strongly both agents if they disagree. Then, agent 1 should offer a collusion contract in which both agents commit to announce rejection if agent 2 has accepted and agent 1 commits to announce acceptance if agent 2 has rejected. In this case, the incentives to accept the collusion offer are modified but in a way that makes acceptance more attractive, which hurts the principal.

Now, suppose that the principal asks agent 2 about the type of punishment strategy that agent 1 has offered. The idea is that agent 2 can reveal the announcement that agent 1 is supposed to make in case of rejection. The principal could then compare this announcement with the true announcement of agent 1 and reward agent 2 if the two coincide. However, the principal wants to pay this reward only if agent 2 has rejected the offer. But then we are back to the previous problem, because the principal cannot extract this information truthfully.

The fact that the principal is forced to play first by making an offer implies that agent 1 can optimally adjust his own collusion offer in order to make any information other than the agents' types irrelevant.

Thus, the only information that is useful for the principal is the information about types. Finally, if the punishment strategy is independent of A^1's type, the common value component of the collusion game disappears and the problem can be analyzed under the private value framework. We will show that it is indeed optimal for agent 1 to commit to set his punishment strategy independent of his type.[14]

Proposition 6 *It is optimal for agent 1 to commit himself to a punishment strategy independent of his type.*

By committing to a type independent punishment strategy, agent 1 transforms the collusion game in an informed sub-principal problem with private values. A property of the private value case is that the equilibrium is unique.

[14] The proof of the Collusion-Proofness Principle (Proposition 1) shows that we can restrict attention to incentive compatible grand mechanisms that are followed by the null side contract, with a punishment strategy $x(\theta^1) = \theta^1$. Clearly this punishment strategy is not type independent, but truthful. However, as a methodology to find the optimal collusion-proof grand contract we will use the type independent punishment strategy, and show at the end that there is an equivalent truthful punishment strategy that would give the same equilibrium payoffs.

4.1 Type independent punishment strategy

Suppose that agent 1 sets a punishment strategy independent of his private information:

$$x(\theta_1) = x(\theta_2) = x \in \{\theta_1, \theta_2\}.$$

Then, the utility of a type j agent 2 if he rejects the collusion contract is given by

$$u_0^2(\theta_j / x) = \max\left\{0, \max_{m^2 \in \{\theta_1, \theta_2\}} \left[t^2\left(x, m^2\right) - \theta_j q\left(x, m^2\right)\right]\right\},$$

that is, agent 2 chooses the best strategy in the grand contract, given the punishment strategy of agent 1. Agent 2 has always the possibility of rejecting the grand contract and, then, can always guarantee himself a 0 payoff. Because the punishment strategy is independent of agent 1's type, the problem has to be analyzed in the private value context. Using the results in Maskin and Tirole (1990) and Quesada (2003) we can characterize the solution of the collusion game.

Proposition 7 *a) If the punishment strategy is $x = \theta_2$ then a grand contract is collusion-proof if and only if it satisfies the following individual constraints:*

$$\begin{aligned} t_{11}^2 - \theta_1 q_{11} &= t_{12}^2 - \theta_1 q_{12}, \\ t_{21}^2 - \theta_1 q_{21} &= t_{22}^2 - \theta_1 q_{22} \geq \max\left\{0, u_0^2(\theta_1 / \theta_2)\right\}, \\ t_{12}^2 - \theta_2 q_{12} &= \max\left\{0, u_0^2(\theta_2 / \theta_2)\right\}, \\ t_{22}^2 - \theta_2 q_{22} &= \max\left\{0, u_0^2(\theta_2 / \theta_2)\right\}, \end{aligned} \tag{2}$$

$$t_{ij}^1 - \theta_i q_{ij} \geq 0 \quad \forall i, \forall j, \tag{3}$$

and the coalition constraints: $\forall i, \forall j$

$$\begin{aligned} t_{11}^1 + t_{11}^2 - 2\theta_1 q_{11} &\geq t_{ij}^1 + t_{ij}^2 - 2\theta_1 q_{ij}, \\ t_{21}^1 + t_{21}^2 - (\theta_1 + \theta_2) q_{21} &\geq t_{ij}^1 + t_{ij}^2 - (\theta_1 + \theta_2) q_{ij}, \\ t_{12}^1 + t_{12}^2 - (\theta_1 + \theta_2) q_{12} - \frac{\nu_1}{\nu_2}\Delta\theta q_{12} &\geq t_{ij}^1 + t_{ij}^2 - (\theta_1 + \theta_2) q_{ij} - \frac{\nu_1}{\nu_2}\Delta\theta q_{ij}, \\ t_{22}^1 + t_{22}^2 - 2\theta_2 q_{22} - \frac{\nu_1}{\nu_2}\Delta\theta q_{22} &\geq t_{ij}^1 + t_{ij}^2 - 2\theta_2 q_{ij} - \frac{\nu_1}{\nu_2}\Delta\theta q_{ij}. \end{aligned} \tag{4}$$

b) Given any collusion-proof grand contract, agent 1 weakly prefers to set his punishment strategy at $x = \theta_2$.

We have found necessary and sufficient conditions for a grand contract to be collusion-proof, provided that agent 1's punishment strategy is independent of his own type. From an ex ante point of view (before knowing agent 2's type), the optimal punishment strategy is to announce θ_2 whenever agent 2 rejects the side contract. The intuition is that agent 1 wants to decrease agent 2's outside option at the collusion stage, and this is obtained through a harsh punishment in case of rejection ($x = \theta_2$). There is another possible punishment that we did not consider here. Agent 1 could punish agent 2 by rejecting the principal's offer. However, we can easily see that agent 1 cannot do better with such a punishment. Threatening agent 2 with a rejection of the grand mechanism certainly decreases the outside option of a low-cost agent 2, who gets a strictly positive utility if the contract is signed in any collusion-proof contract. Nevertheless, the participation constraints of a low-cost agent 2 are non-binding in the optimal collusion contract and, thus, decreasing the low-cost agent 2's outside option does not benefit agent 1. Therefore, the only benefit could come from a decrease of the outside option of a high-cost agent 2. But the principal wants to minimize agent 2's utility and, thus, any optimal collusion-proof contract will have the inefficient agent 2's utility equal to 0 and agent 1 cannot decrease it further.

Collusion is efficient (i.e., maximizes the sum of the agents' utilities) whenever agent 2 is efficient. As usual, there are no gains from introducing inefficiencies in side contracting when agent 2 is efficient. However, a distortion in side contracting appears whenever agent 2 is inefficient. This distortion helps reducing the informational rent that agent 1 has to give up in order to obtain truthful revelation from a low-cost agent 2. When agent 2's production cost is high, the coalition does not maximize joint utility, but a "virtual" joint utility, in which the virtual (aggregate) cost is higher than the total production cost because it includes the (expected) cost of the informational rent from A^1 to A^2. The difference between the virtual cost and the production cost is a measure of the frictions in side contracting created by asymmetric information. The private value feature of the model and the assumption of risk neutrality imply that the distortion is the same that would arise in a model in which agent 1's private information were known by agent 2. At this point, it is important to bring attention to the problem of multiple equilibria at the side contracting stage. It is explicitly shown in Quesada (2003) that when both parties are risk neutral, the informed sub-principal problem has an infinity of equilibria. Nevertheless, the sub-principal's payoff, whatever his type, is the same in all equilibria. This property allows us to neglect the problem of equilibrium selection without loss of generality. Indeed, suppose that, given a grand contract GC, there is one equilibrium in which agent 1 strictly gains by offering a non-null collusion contract. Then, the null contract cannot be an equilibrium collusion contract, because agent 1 would also gain by offering any other side contract that would be an equilibrium of the collusion game following GC. Similarly, if the null contract is one equilibrium of the collusion game given GC, no other equilibrium of the collusion game would give A^1 a higher payoff than the null contract and, therefore, GC is strongly collusion-proof.

Another interesting point is that the incentive constraints for agent 1 are always satisfied, even though they have been neglected in the analysis. This is also due to the private value framework.

With all these ingredients we are ready to show that collusion is now powerful enough to improve with respect to the second-best contract.

Proposition 8 *The second-best contract is not collusion-proof.*

Proof. First, notice that for the second-best contract to be accepted at stage 5 it has to satisfy the ex post participation constraints for both agents. Then, we have

$$\begin{aligned} t_{21}^{1sb} &= t_{12}^{2sb} = \theta_2 q_{12}^{sb}, \\ t_{22}^{1sb} &= t_{22}^{2sb} = \theta_2 q_{22}^{sb}, \end{aligned}$$

because $q_{21}^{sb} = q_{12}^{sb}$.
Conditions (2) imply also that

$$\begin{aligned} t_{11}^{2sb} &= \theta_1 q_{11}^{sb} + \Delta\theta q_{12}^{sb}, \\ t_{21}^{2sb} &= \theta_1 q_{12}^{sb} + \Delta\theta q_{22}^{sb}. \end{aligned}$$

$q_{21}^{sb} = q_{12}^{sb}$ together with the collusion constraints imply that

$$t_{12}^{1sb} + t_{12}^{2sb} = t_{21}^{1sb} + t_{21}^{2sb},$$

or

$$t_{12}^{1sb} = t_{21}^{1sb} + t_{21}^{2sb} - t_{12}^{2sb} = \theta_1 q_{12}^{sb} + \Delta\theta q_{22}^{sb}.$$

Finally, the second-best contract satisfies with equality the interim incentive constraint of an efficient agent 1:

$$\nu_1 \left(t_{11}^{1sb} - \theta_1 q_{11}^{sb} \right) + \nu_2 \left(t_{12}^{1sb} - \theta_1 q_{12}^{sb} \right) = \Delta\theta \left(\nu_1 q_{12}^{sb} + \nu_2 q_{22}^{sb} \right),$$

so

$$t_{11}^{1sb} = \theta_1 q_{11}^{sb} + \Delta\theta q_{12}^{sb}.$$

So, actually, the second-best contract has to be implemented in dominant strategies. Now, agent 1 has incentives to offer a collusion contract in which in state (θ_1, θ_2) the coalition announces (θ_2, θ_2), and punish agent 2 by announcing θ_2 if he rejects. Indeed, he does not need to compensate a high-cost agent 2 for this change of announcement because the latter will get 0 anyway. On the contrary, he can impose a penalty to agent 2 in state (θ_1, θ_1), $y_{11} = -\Delta\theta \left(q_{12}^{sb} - q_{22}^{sb} \right) < 0$. ■

When agents collude before accepting the principal's offer, agent 1 can always find a collusion contract different from the null contract that makes him better off if the principal offers the second-best contract. Therefore, the second-best contract is not collusion-proof The difference with the case in which agents accept or reject the grand contract before colluding is that the principal loses some degrees of freedom that have to be used to satisfy the ex post participation constraints instead of the collusion constraints.

It is the interaction of both the ex post participation constraints and the punishment strategy what makes the second-best contract non-implementable when collusion is possible. Indeed, we have proven in the previous section that only one of those conditions is not enough to prevent the principal from implementing the second-best contract. On the one hand, if agent 1 does not have access to a punishment technology, the principal can satisfy ex post participation constraints by increasing the status quo utility level of agent 2 in order to make any collusion offer very expensive for agent 1. On the other hand, if a punishment technology is available but participation occurs at an interim stage, the principal can still implement the second-best contract by offering a contract that gives a negative utility to agent 1 whenever both agents are inefficient, destroying the stakes for collusion. However, when agent 1 has access to a punishment technology and participation is ex post, the principal cannot manage anymore to implement the second-best contract. Agent 1 can always take advantage of the second-best contract with a side contract in which a coalition of (θ_1, θ_2) announces (θ_2, θ_2).

By doing so, he relaxes the incentive constraint of a low-cost agent 2, because $q_{22}^{sb} < q_{12}^{sb}$, and, therefore, can reduce the transfer (ask for a fee) to a low-cost agent 2. Finally, agent 2 is willing to accept to pay such a fee because agent 1 threatens to punish him by announcing a high cost in the grand mechanism.

The result of Proposition 8 does not rely on the two-types assumption. It will go through with any (symmetric) discrete or continuous type space. The intuition is the same. With ex ante collusion, the only way to implement the second-best contract is in dominant strategies for agent 2. This, together with the punishment, implies that there is always a non-zero measure subset of types that will collectively gain by deviating from truthful revelation whenever the principal wants to implement the second best. Therefore, the second-best is not collusion-proof.[15]

The second-best contract, then, cannot be an optimal contract for the principal because it cannot be implemented when collusion is possible. In Proposition 7 we have obtained the characteristics of the set of collusion-proof (implementable) contracts. We now look at the best offer of the principal.

Proposition 9 *The best collusion-proof contract is characterized by the following conditions on transfers:*

$$t_{11}^{1c} = \theta_1 q_{11}^c + \Delta\theta\left(q_{21}^c - q_{12}^c + q_{22}^c\right) + \tfrac{\nu_1}{\nu_2}\Delta\theta\left(q_{12}^c - q_{22}^c\right), \qquad t_{11}^{2c} = \theta_1 q_{11}^c + \Delta\theta q_{12}^c,$$
$$t_{12}^{1c} = \theta_1 q_{12}^c + \Delta\theta q_{22}^c + \tfrac{\nu_1}{\nu_2}\Delta\theta\left(q_{12}^c - q_{22}^c\right), \qquad t_{12}^{2c} = \theta_2 q_{12}^c,$$
$$t_{21}^{1c} = \theta_2 q_{21}^c + \tfrac{\nu_1}{\nu_2}\Delta\theta\left(q_{12}^c - q_{22}^c\right), \qquad t_{21}^{2c} = \theta_1 q_{21}^c + \Delta\theta q_{22}^c,$$
$$t_{22}^{1c} = \theta_2 q_{22}^c, \qquad t_{22}^{2c} = \theta_2 q_{22}^c.$$

Optimal quantities are given by:

$$\begin{aligned} S'\left(q_{11}^c\right) &= 2\theta_1, \\ S'\left(q_{21}^c\right) &= \theta_1 + \theta_2 + \frac{\nu_1}{\nu_2}\Delta\theta, \\ S'\left(q_{12}^c\right) &= \begin{cases} \theta_1 + \theta_2 + \frac{1-\nu_2^2}{\nu_2^2}\Delta\theta, & \textit{if} \quad \nu_2 \geq \frac{\sqrt{5}-1}{2} \\ \theta_1 + \theta_2 + \frac{1}{\nu_2}\Delta\theta, & \textit{if} \quad \nu_2 < \frac{\sqrt{5}-1}{2} \end{cases} \\ S'\left(q_{22}^c\right) &= \begin{cases} 2\theta_2 + \frac{1-\nu_2^2}{\nu_2^2}\frac{\nu_2-\nu_1}{\nu_2}\Delta\theta & \textit{if} \quad \nu_2 \geq \frac{\sqrt{5}-1}{2}, \\ 2\theta_2 + \frac{\nu_1}{\nu_2}\Delta\theta & \textit{if} \quad \nu_2 < \frac{\sqrt{5}-1}{2}. \end{cases} \end{aligned} \tag{5}$$

[15] See Quesada (2004) for a formal proof in the continuous case.

Remark 2 *If the principal offers the contract in Proposition 9, Agent 1 can set a truthful punishment strategy, $x(\theta^1) = \theta^1$, without changing the equilibrium payoffs.*[16]

The optimal collusion-proof contract entails no rent for a high-cost agent 2 and the minimum informational rent for a low-cost agent 2. Therefore, nothing is changed for agent 2 with respect to the second-best contract in terms of transfer functions (of course, the effective transfers are different because the quantities are distorted). So, the whole loss coming from the possibility of collusion is transferred to agent 1, in order to prevent him from offering a collusion contract that could undo the principal's offer. Indeed, agent 1 receives a rent *even when he is inefficient.* The rent is equal to 0 when both agents are inefficient, but it is strictly positive whenever agent 1 is inefficient and agent 2 is efficient if $q^c_{12} > q^c_{22}$. The reason is that the principal has to guarantee a non-negative rent in all states of nature and, at the same time, avoid collective misreport. The principal wants to discourage a coalition of (θ_1, θ_2) from pretending to be (θ_2, θ_2) (that was the problem with the second-best contract) by increasing agent 1's rent in state (θ_1, θ_2). This implies, then, that a rent has to be given to agent 1 to prevent a coalition of (θ_2, θ_1) from pretending to be (θ_1, θ_2), which is now more attractive.

This additional rent is proportional to $(q_{12} - q_{22})$, so the principal has incentives to decrease production in state (θ_1, θ_2) and increase it in state (θ_2, θ_2), compared to the second-best quantities. In any case, however, production is downward distorted compared to the first-best levels. The distortions with respect to the second-best increase in ν_1. For ν_1 large enough the monotonicity condition $q_{12} \geq q_{22}$ becomes binding and the optimal collusion-proof mechanism entails bunching. When ν_1 is large, it becomes too costly for the principal to deter collusion with a separating contract. The principal would like to offer a contract with $q_{12} < q_{22}$, but this goes against collusion-proofness. Therefore, she destroys the stakes for collusion by choosing $q_{12} = q_{22}$.

One interesting result of Proposition 9 is that the principal offers an asymmetric contract, both in transfers and in quantities ($q^c_{21} > q^c_{12}$). The asymmetry in transfers comes from the fact that agents actually are asymmetric with respect to the bargaining power at the collusion stage, so the principal has to give special incentives to agent 1 in order to prevent

[16] Indeed, with such a punishment strategy, it is easy to show that the null contract is the RSW allocation and it is interim efficient relative to the prior beliefs, so it is the unique equilibrium of the collusion game.

collusion. The fact that collusion happens before the agents decide whether to accept or reject the grand contract introduces one additional asymmetry, now in quantities, aiming at reducing informational rents in an efficient way. Distorting q_{21} is useful to reduce the rent in state (θ_1, θ_1), while distorting q_{12} helps reducing rents in all states of nature. So the principal is willing to reduce more production in state (θ_1, θ_2) than in state (θ_2, θ_1), even though the total marginal cost is the same in both cases. Indeed, the virtual marginal cost in state (θ_1, θ_2) is $\theta_1 + \theta_2 + \frac{\nu_1}{\nu_2}\Delta\theta$, larger than in state (θ_2, θ_1). The virtual marginal cost includes the cost of the informational rent that has to be given by agent 1 to agent 2 in order to obtain truthful revelation at the collusion stage.

5 Discussion

We considered a very simple model of collusion under asymmetric information. However, many results may be extended to more general cases. According to the informed principal literature, the results concerning the collusion game are also valid when agents have n types. The methodology of analysis can, therefore, be used to look at cases with more than 2 types.

All along this paper, we have looked at a very particular production function that implies strong complementarities between the two agents. In particular, the optimal punishment strategy is highly associated to this property. Indeed, the harshest punishment we can think of when goods are perfect complements is the rejection of the grand contract, because the production of one agent is completely useless without the production of the other. Of course, this is not the rule. Take, for instance the other extreme of perfect substitutes. Then, the rejection of the grand contract more than a punishment becomes a reward. On the contrary, the harshest punishment is to announce a low cost in order to exclude the other agent from the market. Nevertheless, the methodology developed here can also be used to analyze other cases, with the corresponding adjustment in the optimal punishment strategy.

One point that deserves some words concerns the enforcement of collusion. In order to be able to apply the tools available in contract theory, we have to assume that there is a third party able to enforce collusion. However, collusion is meant to be a secret (and often illegal) agreement between the parties, so it is difficult to think that a court of justice will play this role. A more complete analysis would incorporate collusion as part of a repeated game in which enforcement comes either from reputation effects or from the threat of punishment in posterior stages of the game.

The last important issue is the allocation of the bargaining power at the collusion stage. We considered here the simplest possible case in which one party has all the bargaining power at the collusion stage and the identity of this party is common knowledge. Another possibility, certainly more realistic, is that the allocation of the bargaining power is private information of the colluding parties. It turns out that this model can handle this situation. Suppose that the bargaining power is represented by a parameter, $\alpha \in \{1, 2\}$, where $\alpha = i$ means that all the bargaining power belongs to agent i. Suppose that α is unknown by the principal. However, both agents know who has the bargaining power, and this is common knowledge between them. The principal can then construct a mechanism in which she asks the agents to announce, not only their marginal costs, but also the value of α. If they both announce $\alpha = i$, then the optimal collusion-proof contract of Proposition 9 (corresponding to i having all the bargaining power) is implemented. If both agents announce a different value for α, the principal implements the null grand contract, a contract with no production and no transfers. The agent who has the bargaining power has always incentives to tell the truth about the value of α. Moreover, he can convince the other agent to also announce the truth even if the other agent rejects the collusion contract, by committing to announce the truth in any case. The other agent will then announce the truth to avoid the null grand contract.

6 Conclusions

In this paper we analyzed the problem of collusion under asymmetric information in mechanisms with multi-agents. The main contribution with respect to the existing literature is that we explicitly model collusion as an offer from one informed party to the other one. This introduces more realism in the analysis of coalition formation, at the cost of adding technical complexities in solving the collusion problem. However, we believe that these technical difficulties are worth being looked at, in order to understand the real constraints that the principal has to consider when collusion is an issue.

The results in this kind of models are usually very sensitive to the timing of the game. For this reason, we look here at two alternative timings. First, we assume that agents decide whether to accept or reject the principal's offer before collusion occurs. If there is no collusion, the agents play non-cooperatively the grand mechanism. In this context, the relevant framework to analyze the collusion problem is to look at an informed sub-principal with common values, because the status quo utility is determined by the expected utility in the grand contract, which depends, in turn, on the private information of all the agents. We show that the principal can implement the optimal contract without collusion.

In the second part of the paper, we assume that agents accept or reject the principal's offer after colluding. Then, the principal loses some degrees of freedom because she has to satisfy the ex post participation constraints of the agents. This, in turn, gives more power to the coalition since agents are implicitly able to collude on their participation decisions. Moreover, we assume that agent 1 commits to punish agent 2 in the grand mechanism if agent 2 rejects the side contract. Then, if the punishment strategy is independent of agent 1's private information, collusion has to be analyzed using the private value framework, which has, in general, a unique equilibrium. We show that it is ex ante optimal for agent 1 to commit to a type independent punishment strategy and that the second-best contract is no longer implementable. Therefore, collusion becomes costly, and the principal will distort production with respect to the second-best contract in order to trade-off efficiency and rents. The optimal collusion-proof contract is asymmetric both in transfers and in production, and the cost of collusion is transferred to the agent who has the bargaining power at the collusion stage.

In terms of comparisons with the previous literature and, in particular with the third party methodology, our analysis shows that, if agents have differences in their bargaining power, a third party who maximizes joint utility is unable to reproduce the equilibrium of a more realistic collusion game. Indeed, with ex ante collusion and feasible punishment the principal cannot implement the second-best contract when agent 1 has all the bargaining power, while she could have done it if collusion were organized by an uninformed third party maximizing joint utility. The reason is that the principal prefers to contract with a coalition which is internally more efficient to avoid double marginalization of rents. In order to obtain a better approximation using the third party approach the third party must be allowed to maximize a weighted sum of the agents' utilities in which the weights would depend on the differences in bargaining power. As a sub-product, we show that, at least under quasi linear preferences, the outcome of collusion as an informed principal problem can be reproduced by a third party that gives all the weight to one agent. However, this result is not likely to extend to more general preferences.

A Proofs

Proof of Proposition 1. The proof follows the lines of Laffont and Martimort (1997). We will prove that any distribution of outcomes obtained as an equilibrium of the overall game of grand contract offer followed by collusion can be obtained as an equilibrium of the collusion game that takes place after P offers a direct revealing weakly collusion-proof grand contract. First, suppose that there is no punishment strategy. Any equilibrium of the full game consists of 3 elements. A grand contract, $GC^* \in \mathcal{GC}$, out of equilibrium beliefs about A^2's type if he rejects the side contract, ν^* and a side contract, $SC^* \in \mathcal{SC}_{GC^*}$. SC^* is an equilibrium side contract, so it is a direct mechanism, interim incentive compatible for the two agents and gives agent two at least the utility he would get by playing GC^* non-cooperatively with beliefs ν^*.

Suppose now that the principal offers contract $GC^{**} = GC^* \circ SC^*$. We claim that the null collusion contract is an equilibrium side contract following this offer with out of equilibrium beliefs following a rejection equal to the priors, i.e. $\nu^{**} = \nu$. Suppose not. Then, there is another collusion contract, $SC^{**} \in \mathcal{SC}_{GC^{**}}$ such that, for any out of equilibrium beliefs about A^1's type and for any corresponding out of equilibrium payoffs, at least one type of agent 1 gets more utility than under the null contract. SC^{**} is interim incentive compatible and gives agent 2 at least the utility he would get by playing GC^{**} with beliefs ν^{**}. But then SC^* could not have been an equilibrium in the first place, since the contract $SC' = SC^* \circ SC^{**}$ is interim incentive compatible and gives agent 2 at least the same utility as contract GC^* with beliefs ν^* and gives at least one type of agent 1 more than SC^*, *whatever the out of equilibrium beliefs about agent 1's type.*

Next, consider the case in which the side contract includes a punishment strategy. Then, the equilibrium does not need to specify out of equilibrium beliefs about A^2's type because the out of equilibrium move of agent 1 is predetermined by the punishment. Now with the same argument, we show that the null collusion contract together with punishment strategy $x^{**}(\theta^1) = \theta^1$ is an equilibrium side contract following GC^{**}. ■

Proof of Proposition 3. By definition, the dominant strategy implementation satisfies A^i's incentive constraints type by type, therefore, it is weakly collusion-proof if and only if at the collusion stage, the null contract is the RSW allocation and it is interim efficient for some strictly positive beliefs (this last point is proved in Proposition 5). By definition, the null contract is RSW relative to itself if it solves

$$\begin{aligned}
&\max_{(\phi,y)} \textstyle\sum_k \sum_\ell \nu_\ell \left(t^1(\phi_{k\ell}) - y_{k\ell} - \theta_k q(\phi_{k\ell})\right) \\
&\text{subject to} \\
&t^2(\phi_{ij}) + y_{ij} - \theta_j q(\phi_{ij}) \geq t^2(\phi_{ik}) + y_{ik} - \theta_j q(\phi_{ik}) && \forall i,j,k \quad (\lambda_{ij}) \\
&t^2(\phi_{i1}) + y_{i1} - \theta_1 q(\phi_{i1}) \geq \Delta\theta q_{i2}^{sb} && \forall i \quad (\mu_{i1}) \\
&t^2(\phi_{i2}) + y_{i2} - \theta_2 q(\phi_{i2}) \geq 0 && \forall i \quad (\mu_{i2}) \\
&\textstyle\sum_\ell \nu_\ell \left(t^1(\phi_{i\ell}) - y_{i\ell} - \theta_i q(\phi_{i\ell})\right) \geq \sum_\ell \nu_\ell \left(t^1(\phi_{k\ell}) - y_{k\ell} - \theta_i q(\phi_{k\ell})\right) && \forall i,k \quad (\gamma_i).
\end{aligned}$$

At the null contract, we have $\phi_{k\ell} = (\theta_k, \theta_\ell)\ \forall k, \ell$, $y_{k\ell} = 0\ \forall k, \forall \ell$ and $\lambda_{12} = \lambda_{22} = \gamma_2 = 0$, because the incentive constraints for a high-cost agent i are slack. Moreover, it satisfies all the constraints of the problem above, so we just need to show that it maximizes the objective function. Taking derivatives with respect to $y_{k\ell}$, we obtain that an RSW allocation satisfies the first order conditions:

$$\begin{aligned}
\mu_{11} &= \nu_1(1+\gamma_1) - \lambda_{11} \geq 0, \\
\mu_{21} &= \nu_1(1-\gamma_1) - \lambda_{21} \geq 0, \\
\mu_{12} &= \nu_2(1+\gamma_1) + \lambda_{11} \geq 0, \\
\mu_{22} &= \nu_2(1-\gamma_1) + \lambda_{21} \geq 0. \qquad (6)
\end{aligned}$$

Optimizing with respect to ϕ_{ij} and using (6), we obtain that the null contract is RSW if, for some $\mu_{ij} \geq 0$, $\lambda_{11} \geq 0$, $\lambda_{21} \geq 0$ and $\gamma_1 \geq 0$, and $\forall i, j$,

$$(\theta_i, \theta_j) \in \arg\max_{\tilde{\phi}} \left\{ t^1\left(\tilde{\phi}\right) + t^2\left(\tilde{\phi}\right) - (\theta_i + \theta_j + \xi_{ij}\Delta\theta) q\left(\tilde{\phi}\right) \right\}, \qquad (7)$$

where $\xi_{11} = 0$, $\xi_{12} = \frac{\lambda_{11}}{\nu_2(1+\gamma_1)}$, $\xi_{21} = \frac{\gamma_1}{1-\gamma_1}$ and $\xi_{22} = \frac{\lambda_{21}+\nu_2\gamma_1}{\nu_2(1-\gamma_1)}$.

Using (1), we have that the dominant-strategy implementation of the second-best satisfies all the collusion constraints for $\lambda_{11} = \lambda_{21} = \gamma_1 = 0$. The assumption that $S'''(\cdot) \geq 0$ guarantees that a coalition of (θ_1, θ_2) does not want to mimic (θ_1, θ_1). Therefore, the null contract is RSW relative to itself and the second-best contract is weakly collusion-proof, according to Proposition 2. ■

Proof of Proposition 5. Given Proposition 4, the second-best contract is strongly collusion-proof if it is weakly collusion proof and the null contract is interim efficient relative to the prior beliefs. By definition, the null contract is interim efficient relative to beliefs $\hat{\nu}$ if, for some vector of strictly positive weights (w_1, w_2) it solves

$$\max_{(\phi_{k\ell}, y_{k\ell})} \sum_k \sum_\ell w_k \nu_\ell \left(t^1(\phi_{k\ell}) - y_{k\ell} - \theta_k q(\phi_{k\ell})\right)$$

subject to

$$\sum_k \hat{\nu}_k \left(t^2(\phi_{kj}) + y_{kj} - \theta_j q(\phi_{kj})\right) \geq \sum_k \hat{\nu}_k \left(t^2(\phi_{k\ell}) + y_{k\ell} - \theta_1 q(\phi_{k\ell})\right) \quad \forall j, \ell \quad \left(\hat{\lambda}_j\right)$$

$$\sum_k \hat{\nu}_k \left(t^2(\phi_{k1}) + y_{k1} - \theta_1 q(\phi_{k1})\right) \geq \sum_k \hat{\nu}_k \Delta\theta q^{sb}_{k2} \quad (\hat{\mu}_1)$$

$$\sum_k \hat{\nu}_k \left(t^2(\phi_{k2}) + y_{k2} - \theta_2 q(\phi_{k2})\right) \geq 0 \quad (\hat{\mu}_2)$$

$$\sum_\ell \nu_\ell \left(t^1(\phi_{i\ell}) - y_{i\ell} - \theta_i q(\phi_{i\ell})\right) \geq \sum_\ell \nu_\ell \left(t^1(\phi_{k\ell}) - y_{k\ell} - \theta_i q(\phi_{k\ell})\right) \quad \forall i, k \quad (\hat{\gamma}_i).$$

Keeping the assumption that agent 1 is truthful also out of the equilibrium path (which is true for the dominant-strategy implementation), we know that the null contract is RSW relative to itself, so, it satisfies all the constraints for any vector $\hat{\nu}$. Moreover, $\hat{\lambda}_2 = \hat{\gamma}_2 = 0$ in the null contract. So we only need to check that it maximizes the objective function for the prior beliefs.

Suppose without loss of generality that $w_1 + w_2 = 1$. From the first order conditions with respect to $y_{k\ell}$, the null contract is interim efficient relative to beliefs $\hat{\nu}$ if

$$\begin{aligned} \hat{\gamma}_1 &= \hat{\nu}_1 - w_1 \geq 0, \\ \hat{\mu}_2 &= 1 - \hat{\mu}_1 \geq 0, \\ \hat{\lambda}_1 &= \nu_1 - \hat{\mu}_1 \geq 0. \end{aligned} \tag{8}$$

So, it is interim efficient relative to the prior beliefs if the conditions are satisfied for $\hat{\nu}_i = \nu_i$.

Optimizing with respect to ϕ_{ij} and using (8), the following conditions have to be satisfied for some $\hat{\mu}_1 \in [0, \nu_1]$, $w_1 \in (0, \nu_1]$ and $\forall i, j$

$$(\theta_i, \theta_j) \in \arg\max_{\tilde{\phi}} \left\{ t^1\left(\tilde{\phi}\right) + t^2\left(\tilde{\phi}\right) - (\theta_i + \theta_j + \hat{\xi}_{ij}\Delta\theta) q\left(\tilde{\phi}\right) \right\}, \tag{9}$$

where $\hat{\xi}_{11} = 0$, $\hat{\xi}_{12} = \frac{\nu_1 - \hat{\mu}_1}{\nu_2}$, $\hat{\xi}_{21} = \frac{\nu_1 - w_1}{\nu_2}$ and $\hat{\xi}_{22} = \frac{2\nu_1 - \hat{\mu}_1 - w_1}{\nu_2}$.

By assumption, the null contract is RSW relative to itself, so conditions (7) are satisfied. Moreover, (7) imply (9) whenever

$$\begin{aligned} \hat{\mu}_1 &= \frac{\nu_1(1+\gamma_1) - \lambda_{11}}{1+\gamma_1} \in [0, \nu_1], \\ w_1 &= \frac{\nu_1 - \gamma_1}{1 - \gamma_1} \in (0, \nu_1] \text{ for } \gamma_1 < \nu_1, \\ \lambda_{21} &= \lambda_{11} \frac{1 - \gamma_1}{1 + \gamma_1} \geq 0. \end{aligned}$$

The last condition can be satisfied, because there are enough degrees of freedom in choosing the values of the multipliers in the RSW problem. In particular, all the conditions are satisfied for the dominant-strategy implementation, for which $\gamma_1 = \lambda_{11} = \lambda_{21} = 0$. ■

Proof of Proposition 6. Take any offer by the principal: $\left(t_{ij}^1, t_{ij}^2, q_{ij}\right)$. Suppose that the punishment strategy is to announce θ_1 with probability x_i and θ_2 with probability $1 - x_i$ if agent 1 is of type i. Since the punishment strategy is type dependent, the problem has to be analyzed under the common value context. In general, there are many equilibria, but the best possible equilibrium for a type i agent 1 is the allocation that solves

$$\max_{(\phi_{kj}, y_{kj})_{(k,j)\in\{1,2\}^2}} \sum_j \nu_j \left(t^1(\phi_{ij}) - y_{ij} - \theta_i q(\phi_{ij})\right)$$

$$\sum_k \nu_k \left(t^2(\phi_{kj}) + y_{kj} - \theta_j q(\phi_{kj})\right) \geq \sum_k \nu_k \left(t^2(\phi_{k\ell}) + y_{k\ell} - \theta_j q(\phi_{k\ell})\right) \quad \forall (j,\ell) \in \{1,2\}^2, \quad \left(\lambda_j^i\right)$$

$$\sum_k \nu_k \left(t^2(\phi_{kj}) + y_{kj} - \theta_j q(\phi_{kj})\right) \geq \tilde{u}^2(\theta_j) \quad \forall j \in \{1,2\}, \quad \left(\mu_j^i\right)$$

$$\sum_j \nu_j \left(t^1(\phi_{kj}) - y_{kj} - \theta_k q(\phi_{kj})\right) \geq \sum_j \nu_j \left(t^1(\phi_{mj}) - y_{mj} - \theta_k q(\phi_{mj})\right) \quad \forall (m,k) \in \{1,2\}^2, \quad \left(\gamma_k^i\right)$$

$$\sum_j \nu_j \left(t^1(\phi_{hj}) - y_{hj} - \theta_h q(\phi_{hj})\right) \geq \hat{u}^1(\theta_h) \quad \forall h \in \{1,2\}, h \neq i, \quad \left(w_k^i\right)$$

with

$$\tilde{u}^2(\theta_j) = \max_m \left[(\nu_1 x_1 + \nu_2 x_2)\left(t^2(\theta_1, m) - \theta_j q(\theta_1, m)\right) + (1 - \nu_1 x_1 - \nu_2 x_2)\left(t^2(\theta_2, m) - \theta_j q(\theta_2, m)\right) \right]$$

and $\hat{u}^1(\theta_k)$ the RSW payoff of type k agent 1.

Taking derivatives with respect to y_{ij}, we obtain for type i agent 1, for $k \neq i$:

$$\begin{aligned}
\nu_k \left(1 + \gamma_i^i - \gamma_k^i\right) &= \nu_2 \left(\lambda_1^i - \lambda_2^i + \mu_1^i\right), \\
\nu_k \left(1 + \gamma_i^i - \gamma_k^i\right) &= \nu_1 \left(-\lambda_1^i + \lambda_2^i + \mu_2^i\right), \\
\nu_i \left(w_k^i + 1 - \gamma_i^i + \gamma_k^i\right) &= \nu_2 \left(\lambda_1^i - \lambda_2^i + \mu_1^i\right), \\
\nu_i \left(w_k^i + 1 - \gamma_i^i + \gamma_k^i\right) &= \nu_1 \left(-\lambda_1^i + \lambda_2^i + \mu_2^i\right),
\end{aligned} \tag{10}$$

which in turn implies

$$w_k^i = \mu_1^i + \mu_2^i - 2 \geq 0.$$

Therefore, the best possible equilibrium for type i agent 1 can be obtained by solving the following problem CV^i:

$$\max_{(\phi_{kj}, y_{kj})_{(k,j)\in\{1,2\}^2}} \sum_k \sum_j w_k^i \nu_j \left(t^1(\phi_{kj}) - y_{kj} - \theta_k q(\phi_{kj})\right)$$

$$\sum_k \nu_k \left(t^2(\phi_{kj}) + y_{kj} - \theta_j q(\phi_{kj})\right) \geq \sum_k \nu_k \left(t^2(\phi_{k\ell}) + y_{k\ell} - \theta_j q(\phi_{k\ell})\right) \quad \forall (j,\ell) \in \{1,2\}^2, \quad \left(\lambda_j^i\right)$$

$$\sum_k \nu_k \left(t^2(\phi_{kj}) + y_{kj} - \theta_j q(\phi_{kj})\right) \geq \tilde{u}^2(\theta_j) \quad \forall j \in \{1,2\}, \quad \left(\mu_j^i\right)$$

$$\sum_j \nu_j \left(t^1(\phi_{kj}) - y_{kj} - \theta_k q(\phi_{kj})\right) \geq \sum_j \nu_j \left(t^1(\phi_{mj}) - y_{mj} - \theta_k q(\phi_{mj})\right) \quad \forall (k,m) \in \{1,2\}^2, \quad \left(\gamma_k^i\right)$$

with $\left(w_1^1, w_2^1\right) = K^1\left(1, \mu_1^1+\mu_2^1-2\right)$ and $\left(w_1^2, w_2^2\right) = K^2\left(\mu_1^2+\mu_2^2-2, 1\right)$, for any strictly positive constants K^1 and K^2.

On the other hand, when the punishment strategy is type independent, the relevant framework to analyze collusion is private values. Suppose that the punishment strategy is to announce θ_1 with probability $x = \nu_1 x_1 + \nu_2 x_2$ and θ_2 with probability $1 - \nu_1 x_1 - \nu_2 x_2$. Then, given that preferences are quasilinear, an equilibrium collusion contract solves, for a vector of positive weights $(w_1, w_2) = K(1, \frac{\nu_1}{\nu_2})$, problem PV:[17]

$$\begin{array}{lll}
\max\limits_{(\phi_{ij}, y_{ij})_{(i,j)\in\{1,2\}^2}} \sum_i \sum_j w_i \nu_j \left(t^1\left(\phi_{ij}\right) - y_{ij} - \theta_i q\left(\phi_{ij}\right)\right) & & \\
\sum_i \nu_i \left(t^2\left(\phi_{ij}\right) + y_{ij} - \theta_j q\left(\phi_{ij}\right)\right) \geq \sum_i \nu_i \left(t^2\left(\phi_{i\ell}\right) + y_{i\ell} - \theta_j q\left(\phi_{i\ell}\right)\right) & \forall (j,\ell) \in \{1,2\}^2, & (\lambda_j) \\
\sum_i \nu_i \left(t^2\left(\phi_{ij}\right) + y_{ij} - \theta_j q\left(\phi_{ij}\right)\right) \geq \overline{u}^2\left(\theta_j\right) & \forall j \in \{1,2\}, & (\mu_j) \\
\sum_j \nu_j \left(t^1\left(\phi_{ij}\right) - y_{ij} - \theta_i q\left(\phi_{ij}\right)\right) \geq \sum_j \nu_j \left(t^1\left(\phi_{kj}\right) - y_{kj} - \theta_i q\left(\phi_{kj}\right)\right) & \forall (i,k) \in \{1,2\}^2, & (\gamma_i)
\end{array}$$

with $\gamma_1 = \gamma_2 = 0$, because the principal's incentive constraints are not binding in the private value case, and

$$\overline{u}^2\left(\theta_j\right) = \max_m \left[x\left(t^2\left(\theta_1, m\right) - \theta_j q\left(\theta_1, m\right)\right) + (1-x)\left(t^2\left(\theta_2, m\right) - \theta_j q\left(\theta_2, m\right)\right)\right] = \tilde{u}^2\left(\theta_j\right).$$

Therefore, by setting $\mu_1^i + \mu_2^i = \frac{1+\nu_i}{\nu_i}$ problems CV^i and PV are equivalent, so type i agent 1 can always reproduce the best possible outcome of a type dependent punishment strategy x_i with a type independent one with $x = \nu_1 x_1 + \nu_2 x_2$. ■

We first prove the following lemma on monotonicity of the quantity profile, and we prove then Proposition 7.

Lemma 1 *If the punishment strategy is type-independent, any collusion-proof mechanism satisfies the monotonicity condition* $q_{11} \geq q_{21} \geq q_{12} \geq q_{22}$.

Proof of Lemma 1. A grand contract $\left(t_{ij}^1, t_{ij}^2, q_{ij}\right)$ is collusion-proof if the null contract is a solution of the collusion game. Given that the collusion game fits the private value framework, and because agents are risk neutral, we can analyze this game as if agent 2 knew agent 1's marginal cost.

[17] See Proposition 2 in Quesada (2003) for the quasilinear case.

If agent 1 is type i, the best contract he could offer to agent 2 is the contract that solves

$$\max_{(\phi_j, y_j)_{j=1,2}} \sum_j \nu_j \left[t^1(\phi_{ij}) - \theta_i q(\phi_{ij}) - y_{ij}\right]$$
subject to
$$t^2(\phi_{ij}) - \theta_j q(\phi_{ij}) + y_{ij} \geq t^2(\phi_{ik}) - \theta_j q(\phi_{ik}) + y_{ik} \quad \forall j,k \quad (\lambda_{ik})$$
$$t^2(\phi_{ij}) - \theta_j q(\phi_{ij}) + y_{ij} \geq \max\left\{0, u_0^2(\theta_j /x)\right\} \quad \forall j \quad (\mu_{ij}).$$

If the null contract is a solution to this problem, then the grand contract satisfies the following conditions:

$$t_{ij}^2 - \theta_j q_{ij} \geq t_{ik}^2 - \theta_j q_{ik}, \quad \forall i,j,k \quad \mathrm{IC}_{ij}^2$$
$$t_{ij}^2 - \theta_j q_{ij} \geq \max\left\{0, u_0^2(\theta_j /x)\right\}, \quad \forall i,j \quad \mathrm{IR}_{ij}^2.$$

and the incentive compatibility constraints of agent 1 (with private values, the incentive constraints of the party who offers the contract are never binding). Then, from (IC_{11}^2) and (IC_{12}^2), and from (IC_{21}^2) and (IC_{22}^2) we obtain that $q_{11} \geq q_{12}$ and $q_{21} \geq q_{22}$.
By definition,

$$u_0^2(\theta_j /x) = \max\{0; t^2(x,\theta_1) - \theta_j q(x,\theta_1); t^2(x,\theta_2) - \theta_j q(x,\theta_2)\}.$$

Moreover, given conditions (IC_{ij}^2) and (IC_{ij}^2), we have that

$$u_0^2(\theta_1 /x) = t^2(x,\theta_1) - \theta_1 q(x,\theta_1) \geq 0, \tag{11}$$
$$u_0^2(\theta_2 /x) = t^2(x,\theta_2) - \theta_2 q(x,\theta_2) \geq 0. \tag{12}$$

Optimizing with respect to y_{ij} we obtain the first order conditions:

$$\lambda_{11} - \lambda_{12} + \mu_{11} = \nu_1,$$
$$\lambda_{12} - \lambda_{11} + \mu_{12} = \nu_2,$$
$$\lambda_{21} - \lambda_{22} + \mu_{21} = \nu_1,$$
$$\lambda_{22} - \lambda_{21} + \mu_{22} = \nu_2.$$

Optimizing with respect to ϕ_{ij} and using the conditions above we obtain:

$$(\theta_i, \theta_j) \in \arg\max_{\tilde{\phi}} \left\{t^1\left(\tilde{\phi}\right) + t^2\left(\tilde{\phi}\right) - \left(\theta_i + \theta_j + \tilde{\xi}_{ij}\Delta\theta\right) q\left(\tilde{\phi}\right)\right\}, \tag{13}$$

with $\tilde{\xi}_{11} = -\frac{\lambda_{12}}{\nu_1}$, $\tilde{\xi}_{12} = \frac{\lambda_{11}}{\nu_2}$, $\tilde{\xi}_{21} = -\frac{\lambda_{22}}{\nu_1}$ and $\tilde{\xi}_{22} = \frac{\lambda_{21}}{\nu_2}$.
Case 1: the punishment strategy is $x = \theta_2$.

Suppose that at the collusion stage the participation constraint of the high-cost agent 2 and the incentive constraints of the low-cost agent 2 are binding (we will verify ex post that the other constraints are satisfied too). Then, transfers are given by

$$\begin{aligned} t_{11}^2 &= \theta_1 q_{11} + \Delta\theta q_{12} + u_0^2(\theta_2/\theta_2), \\ t_{12}^2 &= \theta_2 q_{12} + u_0^2(\theta_2/\theta_2), \\ t_{21}^2 &= \theta_1 q_{21} + \Delta\theta q_{22} + u_0^2(\theta_2/\theta_2), \\ t_{22}^2 &= \theta_2 q_{22} + u_0^2(\theta_2/\theta_2), \end{aligned} \tag{14}$$

and multipliers are $\lambda_{12} = \lambda_{22} = \mu_{11} = \mu_{21} = 0$, $\lambda_{11} = \lambda_{21} = \nu_1$, $\mu_{12} = \mu_{22} = 1$, implying that $\tilde{\xi}_{11} = \tilde{\xi}_{21} = 0$ and $\tilde{\xi}_{12} = \tilde{\xi}_{22} = \frac{\nu_1}{\nu_2}$.

Replacing in (13), we obtain the collusion constraints when the punishment strategy is $x = \theta_2$. Moreover, these conditions imply the monotonicity constraints

$$q_{11} \geq q_{21} \geq q_{12} \geq q_{22}. \tag{15}$$

We verify now that the other constraints are satisfied. For that we need

$$\begin{aligned} u_0^2(\theta_1/\theta_2) - u_0^2(\theta_2/\theta_2) &\leq \Delta\theta q_{12}, \\ u_0^2(\theta_1/\theta_2) - u_0^2(\theta_2/\theta_2) &\leq \Delta\theta q_{22}, \\ q_{11} &\geq q_{12}, \\ q_{21} &\geq q_{22}. \end{aligned}$$

The last two constraints come immediately from (15). According to (11) and (12), we have that

$$\begin{aligned} u_0^2(\theta_1/\theta_2) &= t_{21}^2 - \theta_1 q_{21} = \overline{u}_{22}^2 + \Delta\theta q_{22}, \\ u_0^2(\theta_2/\theta_2) &= t_{22}^2 - \theta_2 q_{22} = \overline{u}_{22}^2, \end{aligned}$$

and therefore

$$u_0^2(\theta_1/\theta_2) - u_0^2(\theta_2/\theta_2) = \Delta\theta q_{22} \leq \Delta\theta q_{12}.$$

Case 2: the punishment strategy is $x = \theta_1$.

In this case, the previous solution does not satisfy condition (IR_{21}^2) if $q_{12} > q_{22}$. So this constraint has to be binding at the collusion stage: $\mu_{21} > 0$. Suppose that the multipliers are $\lambda_{12} = \lambda_{22} = \mu_{11} = 0$, $\lambda_{11} = \nu_1$, $\mu_{12} = 1$, $\mu_{21} = \nu_1 - \lambda_{21}$, $\mu_{22} = \nu_2 + \lambda_{21}$ and $\nu_1 \geq \lambda_{21} \geq \max\{0, \nu_1 - \nu_2\}$ and $\lambda_{21} > 0 \Rightarrow q_{12} = q_{22}$. This implies also that $(q_{12} - q_{22})(\nu_2 - \nu_1) \geq 0$ and $\tilde{\xi}_{11} = \tilde{\xi}_{21} = 0$, $\tilde{\xi}_{12} = \frac{\nu_1}{\nu_2}$ and $\tilde{\xi}_{22} = \frac{\lambda_{21}}{\nu_2}$.

We will check that indeed the other constraints are satisfied at the optimal solution.
Replacing in 13, we obtain the optimal manipulation of reports function for $x = \theta_1$.
Moreover, given that $\lambda_{21} \geq \nu_1 - \nu_2$, these conditions imply the monotonicity constraints

$$q_{11} \geq q_{21} \geq q_{12} \geq q_{22}. \tag{16}$$

We verify now that the other constraints are satisfied. For that we need

$$\begin{aligned} u_0^2(\theta_1/\theta_1) - u_0^2(\theta_2/\theta_1) &\leq \Delta\theta q_{12}, \\ u_0^2(\theta_1/\theta_1) - u_0^2(\theta_2/\theta_1) &\geq \Delta\theta q_{22}, \\ q_{11} &\geq q_{12}, \\ q_{21} &\geq q_{22}. \end{aligned}$$

The last two constraints come immediately from (16). According to (11) and (12), we have that

$$\begin{aligned} u_0^2(\theta_1/\theta_1) &= t_{11}^2 - \theta_1 q_{11} = \overline{u}_{12}^2 + \Delta\theta q_{12}, \\ u_0^2(\theta_2/\theta_1) &= t_{12}^2 - \theta_2 q_{12} = \overline{u}_{12}^2, \end{aligned}$$

and therefore

$$u_0^2(\theta_1/\theta_1) - u_0^2(\theta_2/\theta_1) = \Delta\theta q_{12} \geq \Delta\theta q_{22}.$$

■

Proof of Proposition 7. a) The proof follows immediately from the proof of Lemma 1, case 1 and from the fact that a collusion-proof grand contract has to satisfy the participation constraints of agent 1.
b) Following the proof of Lemma 1, if the punishment strategy is $x = \theta_1$, a collusion-proof grand contract has to satisfy:

$$\begin{aligned} t_{11}^2 - \theta_1 q_{11} &= t_{12}^2 - \theta_1 q_{12}, \\ t_{21}^2 - \theta_1 q_{21} &= u_0^2(\theta_1/\theta_1) \geq t_{22}^2 - \theta_1 q_{22}, \\ t_{12}^2 - \theta_2 q_{12} &= u_0^2(\theta_2/\theta_1), \\ t_{22}^2 - \theta_2 q_{22} &= u_0^2(\theta_2/\theta_1). \end{aligned} \tag{17}$$

Given that the contract has to satisfy the type by type incentive and participation constraints for agent 2, we have that, for any punishment strategy x, we have $u_0^2(\theta_j/x) = t_{xj}^2 - \theta_j q_{xj}$.

Take any collusion-proof contract with some $u_{22}^2 = t_{22}^2 - \theta_2 q_{22}$. Using (17) and (14) we can determine all the other transfers to agent 2, for a given punishment strategy.

Since $q_{12} \geq q_{22}$, it follows that

$$\begin{aligned} u_0^2(\theta_2/\theta_1) &= u_0^2(\theta_2/\theta_2) = u_{22}^2, \\ u_0^2(\theta_1/\theta_1) &= u_{22}^2 + \Delta\theta q_{12} \geq u_0^2(\theta_1/\theta_2) = u_{22}^2 + \Delta\theta q_{22}. \end{aligned}$$

The outside option of agent 2 at the collusion stage is weakly smaller when $x = \theta_2$. Agent 1's payoff is decreasing in agent 2's outside option, therefore, agent 1 weakly prefers $x = \theta_2$. ■

Proof of Proposition 9. The optimal collusion-proof mechanism solves

$$\max_{(t_{ij}^1, t_{ij}^2, q_{ij})} \sum_i \sum_j \nu_i \nu_j \left[S(q_{ij}) - t_{ij}^1 - t_{ij}^2 \right]$$
$$\text{subject to } (2), (3), (4).$$

We do not need to include agent 1's incentive compatibility constraints, because they are implied by (2) and (4). The monotonicity conditions $q_{11} \geq q_{21} \geq q_{12} \geq q_{22}$ have to be satisfied and the binding constraints are

$$\begin{aligned} t_{i1}^2 - \theta_1 q_{i1} &\geq t_{i2}^2 - \theta_1 q_{i2}, \ i = 1, 2, \\ t_{i2}^2 - \theta_2 q_{i2} &\geq 0, i = 1, 2, \\ t_{22}^1 - \theta_2 q_{22} &\geq 0, \\ t_{11}^1 + t_{11}^2 - 2\theta_1 q_{11} &\geq t_{21}^1 + t_{21}^2 - 2\theta_1 q_{21}, \\ t_{21}^1 + t_{21}^2 - (\theta_1 + \theta_2) q_{21} &\geq t_{12}^1 + t_{12}^2 - (\theta_1 + \theta_2) q_{12}, \\ t_{12}^1 + t_{12}^2 - (\theta_1 + \theta_2) q_{12} - \frac{\nu_1}{\nu_2} \Delta\theta q_{12} &\geq t_{22}^1 + t_{22}^2 (\theta_1 + \theta_2) q_{22} - \frac{\nu_1}{\nu_2} \Delta\theta q_{22}. \end{aligned}$$

Indeed, if the monotonicity condition is satisfied, all the other constraints are satisfied at the optimal contract.

Solving the system gives the corresponding transfers. Replacing in the principal's objective function and maximizing with respect to q_{ij} gives the first part of (5). This is indeed a solution if the monotonicity condition is satisfied. It is easy to see that $q_{11}^c > q_{21}^c > q_{12}^c$. However, $q_{12}^c \geq q_{22}^c$ if and only if $\nu_2 \geq \frac{\sqrt{5}-1}{2}$:

$$S'(q_{22}^c) - S'(q_{12}^c) = \frac{\nu_2^2 - \nu_1}{\nu_2^3} \Delta\theta.$$

So, if $\nu_2 < \frac{\sqrt{5}-1}{2}$, the monotonicity constraint is binding: $q_{12}^c = q_{22}^c$ and solving the principal's problem with the constraint $q_{12} = q_{22}$ gives the second part of (5). ■

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