

# A Game Theoretical Model of Land Contract Choice<sup>1</sup>

**Américo M. S. Carvalho Mendes**

**Portuguese Catholic University  
Faculty of Economics and Management**

Rua Diogo Botelho, 1327

4169-005 Porto - Portugal

[amendes@porto.ucp.pt](mailto:amendes@porto.ucp.pt)

[carvalho.mendes@sapo.pt](mailto:carvalho.mendes@sapo.pt)

## **ABSTRACT**

In most of the land tenancy literature the type of contract is exogenous. Also even though these contracts vary a lot among farms, between regions and over time, the theoretical literature has not always acknowledged this idiosyncrasy. Building on the strategic bargaining theory initiated by Rubinstein, this model not only makes the type of contract endogenous, but also provides the surplus sharing rules and the conditions giving rise to each type of contract, showing how the type and terms of the contract are tailored to fit the characteristics of the parties and their economic environment.

Pairwise bargaining is embedded into a market context by putting “competitive pressure” on the players through the opportunity they have to break up bargaining and look for alternative partners. Because of this threat of opting out, the outcome of the bargaining process depends not only on the characteristics of the players, but also on events outside their match and the information they have about them.

The model departs from price-taking assumptions. Type and terms of the contract result from negotiation and are shaped by the “relative bargaining powers” of the players whose relevant components are identified in a precise way in the model.

**KEYWORDS:** land tenancy, sharecropping, land contract choice, game theory.

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## 1. Main features of the model

Bernat, Jr. (1987), in conclusion to his survey of the land tenancy literature, points out that one of the most obvious deficiencies of these theories “*is the absence of explicit consideration of the bargaining process. Bargaining plays a major role in only one study. Bell and Zusman (1976) modelled the landlord-tenant bargaining process as a Nash cooperative game under the assumption that renters outnumber landlords. The situation in the United States is the reverse: landlords outnumber renters roughly two to one. Hence, if either party has a bargaining advantage, it is likely to be the renter. Recent work on non-cooperative bargaining may provide some useful insights.*” Roumasset (1979) also pointed out that “*many of the rich variations of the real world contracts will require a theory which incorporates transaction costs and bargaining power into the explanatory framework.*”

Cooperative game theory like the one used by Bell and Zusman cannot model the structure of the bargaining process and the influence it might have on the types of contracts. Only extensive form games can do it. So we appeal to non-cooperative bargaining theory (Rubinstein, 1982; Sutton, 1986; Osborne and Rubinstein, 1990) and set up an extension of Rubinstein’s original model which describes the structure of the bargaining process. The specification of this structure enables to show, in detail, the contents of the “relative bargaining powers” of the parties and how they determine the contract choices and the surplus sharing rules in each type of contract.

The model embeds pairwise bargaining into a market context by putting “competitive pressure” on the players through the opportunity they have to break up bargaining and look for alternative partners. Because of this threat of opting out the outcome of any pairwise bargaining process may depend not only on the characteristics of the two players in the bargaining process, but also on events outside their match and the information they have about those events (intercontractual externalities).

## 2. Assumptions

The model is based on the following set of assumptions.

### *Assumption A1*

There are two types of agents in the market: the farmland suppliers labelled as “landlords” and the farmland demanders labelled as “tenants”.

### *Assumption A2*

Agents from opposite sides of the market may join to form two person coalitions with some common goal that none can completely attain alone.

### *Assumption A3*

The value of each two-person coalition in the land tenancy market depends on the “identities” of its members (e. g. location and quality of the land, professional qualification and trustworthiness of the players).

***Assumption A4***

The partners in a coalition have conflicting interests about the division of the value of the coalition and no agreement may be imposed on any partner without his approval.

***Assumption A5<sup>2</sup>***

The value of a coalition can be divided among its members in infinitely many ways (the set of alternatives is a continuum).

***Assumption A6***

Each player has to incur costs to gather information about the identities of the other players. However, once a partner is located, the two parties in the match become immediately and perfectly informed about the characteristics of each other including their reservation incomes and the surplus that can be generated by their partnership.

***Assumption A7***

Partner substitutability is possible, that is, no party in a coalition is an essential member of that coalition. So there is a possibility to switch partners.

These assumptions characterize the farm tenancy market as involving a search problem, a bargaining problem and matching problem.

The search problem results from the fact that each agent in the market is not completely informed about the “identity” of potential partners. Therefore there are “search costs” to locate them. The number of contacts made in a given time interval depends on the resources spent on search. So there is an economic decision to be taken about the optimal level of search intensity.

The matching problem results from the fact that forming a coalition depends on whether it can generate a surplus relative to the players’ reservation incomes which depend on the “identities” of the partners. Therefore not all the contacts resulting from an agent’s search activity need to result in a match. To be so the match has to be “admissible”, that is, it has to generate a positive surplus.

The bargaining problem results from the fact that there is, a priori, no unique way to distribute the surplus. That has to be determined by negotiation between the partners. Assumption A2 limits the model to the case of bilateral bargaining.

This paper presents only the bargaining model. For the search and the matching models the interested reader should refer to the dissertation from which this work comes from. So we will proceed with the list of assumptions keeping only those which are relevant for the bargaining model.

***Assumption A8***

The two parties in the negotiation have common expectations about the value of the forthcoming surplus.

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<sup>2</sup> Van Damme *et al.* (1990) showed that this assumption is needed to obtain a unique perfect equilibrium for the bargaining game.

***Assumption A9***

The bargaining process is divided into discrete bargaining time periods. There is no end point time constraint for the negotiation period, that is, two agents can keep on negotiating as long as it is in their interest to do so. However, extension of the negotiation period will make the production period shrink.

***Assumption A10***

The value of the expected surplus shrinks as the negotiation process goes on<sup>3</sup>. It is assumed that it shrinks at a constant positive rate  $\delta$ . To simplify things further, we consider that both players estimate this rate at the same value and we will denote the corresponding discount factor by  $r^t = [1 / (1 + \delta)^t]$ .

***Assumption A11***

This negotiation is assumed to evolve as a bargaining game with alternating offers: in each time period no agent can make an offer at the same time as the other agent and the offers are ephemeral<sup>4</sup>, so that an agent does not have to commit to an offer for more than one bargaining period.

***Assumption A12***

The agent who makes the first offer comes from the short side of the market.

***Assumption A13***

The bargaining process terminates when an offer is accepted and the accepted offer is the one that will actually be implemented<sup>5</sup>.

***Assumption A14***

The players can take outside options, that is, a player facing an outstanding offer can reject it, quit the current partner and look for an alternative partner.

***Assumption A15***

The players' strategies are semi-stationary in the following sense: there is no recall between market periods so that a player's strategy in the current negotiation is not conditional on the history of previous matches where the player was involved. However, within the same market period and within the same match there is perfect recall, that is, the actions prescribed by a player's strategy at each stage of the current bargaining process depend on what has happened in the previous stages.

***Assumption A16***


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<sup>3</sup> This shrinkage can be interpreted as denoting the fact that an extension of the negotiation reduces the production period with a consequent decline in the expected output.

<sup>4</sup> Stahl II (1990) showed that this assumption is needed to obtain a unique perfect equilibrium for the bargaining game.

<sup>5</sup> Muthoo (1990) showed that this assumption is needed to obtain a unique perfect equilibrium for the bargaining game.

The players have perfect information about the structure of the game.

Given these assumptions, player  $i$ 's strategy is the function that assigns to every possible stage of the bargaining process one of the four possible actions:

- an offer to make to his partner;
- an acceptance of an outstanding offer made by his partner;
- a rejection of an outstanding offer, without threatening to opt out;
- a rejection of an outstanding offer along with the threat to opt out.

The bargaining process starts with an agent, say agent  $j$ , making an offer to the partner, say agent  $i$ , about the distribution of the expected surplus. The bargaining process then evolves as follows:

- agent  $i$  can either accept or reject  $j$ 's offer;
- if  $i$  accepts the game is over with surplus distributed according to  $j$ 's offer;
- if  $i$  rejects,  $j$  can search among the pool of unmatched agents in the opposite side of the market and eventually find a more profitable partner;
- if  $i$  does not take an outside option,  $j$  can make a counter offer to  $i$  and then the process repeats itself with  $i$  and  $j$  switching roles.

Agents  $i$  and  $j$  can keep the negotiation going, but they have two economic motivations not to go on forever: one is the shrinkage in the value of the expected surplus as the negotiation goes on; the other is the possibility that the partner may quit the negotiation and take an outside option.

Both agents have perfect information about the structure of the game, that is, their information sets are singletons.

Since there is no recall between market periods, the analysis can be limited to a single market period without having to take into account the histories and outcomes of previous market periods or the expectations about future market periods.

### 3. Characterization of the market equilibrium

#### Proposition

There is a unique market equilibrium characterized as follows: all agents who are able to match for negotiation reach an agreement.

To specify their equilibrium strategies we need the following notation:

- $e$  is the surplus generated by the match defined as follows:

$$(1) e = y - \alpha_i - \alpha_j$$

where  $y$  is the joint income of the match and  $\alpha_k$  is the reservation income of player  $k$ ;

- $p_k$  is the probability player  $k$  has of finding an alternative partner;
- $\beta_k$  is the expected value of player's  $k$  outside option, that is, the income he is expected to get in case he breaks up the negotiation with the current partner and looks for an alternative one.

In the dissertation where this work comes from we derive the expressions relating  $p_k$  and  $\beta_k$  with the parameters and the technologies of the search and matching

processes (number of landlords/number of tenants, search intensities, etc.). The interested reader should look there for these results.

Let  $j$  denote the agent who makes the first offer and  $i$  the agent who moves after. Their equilibrium strategies are then characterized as follows.

***Case I: Both players prefer to continue bargaining rather than threatening to take up their outside options***

If  $p_i = p_j = 0$ ,  $j$  offers at the beginning of the negotiation period the following division of the surplus

$$(2) e_j = \frac{e}{1+r}$$

and  $i$  immediately accepts.

***Case II: Player  $j$  threatens to take up his outside option but player  $i$  doesn't***

If  $p_j \neq p_i = 0$ ,  $j$  offers at the beginning of the negotiation period the following division of the surplus

$$(3a) e_j = \frac{1-r}{1-r^2(1-p_j)} e + \frac{rp_j\beta_j}{1-r^2(1-p_j)} \text{ and } e_i = e - e_j$$

if  $e > (1-p_j)re_j + p_j\beta_j$

$$(3b) e_j = e \text{ and } e_i = 0 \text{ if } e \leq (1-p_j)re_j + p_j\beta_j$$

and  $i$  immediately accepts the offer.

***Case III: Player  $j$  does not threaten to take up his outside option but player  $i$  does***

If  $p_i \neq p_j = 0$ ,  $j$  offers at the beginning of the negotiation period the following division of the surplus

$$(4a) e_j = \frac{1-r(1-p_i)}{1-r^2(1-p_i)} e + \frac{p_i\beta_i}{1-r^2(1-p_i)} \text{ and } e_i = e - e_j$$

if  $e > p_i\beta_i + (1-p_i)r(e - re_j)$

$$(4b) e_j = 0 \text{ and } e_i = e$$

and  $i$  immediately accepts the offer.

***Case VI: Both players threaten to take up their outside options***

If  $p_i \neq 0$  and  $p_j \neq 0$ ,  $j$  offers at the beginning of the negotiation period the following division of the surplus

$$(5a) e_j = \frac{1-r(1-p_i)}{1-r^2(1-p_i)(1-p_j)} e + \frac{p_i\beta_i - p_j(1-p_i)\beta_j r}{1-r^2(1-p_i)(1-p_j)} \text{ and } e_i = e - e_j$$

if  $e > p_i \beta_i + (1 - p_i)r[e - p_j \beta_j - (1 - p_j)re_j]$   
 (5b)  $e_j = e - p_i \beta_i$  and  $e_i = p_i \beta_i$  if  $e \leq p_j \beta_j + (1 - p_j)re_j$   
 and  $i$  immediately accepts the offer.

The proof of this proposition following the method used by Shaked and Sutton (1984) is contained in our dissertation and will be omitted here.

## 4. Types of contract choices

From the results in the previous section it is possible to typify 68 possible contract choices, depending on the combinations of the following elements which are the ingredients of the players' bargaining powers:

1. probabilities of getting outside options  $(p_i, p_j)$ ;
2. player who is the first to make an offer in the bargaining process;
3. player's impatience;
4. surplus of the match ("high surplus" match or "low surplus" match).

**Table 1: Structure of the bargaining process and types of contract choices**

Probabilities of finding an outside option	Landlord is the first mover				Tenant is the first mover			
	High surplus		Low surplus		High surplus		Low surplus	
	$r < 1$	$r = 1$	$r < 1$	$r = 1$	$r < 1$	$r = 1$	$r < 1$	$r = 1$
$p_T = 0, p_L = 0$	S1	S(50:50)	---	---	S2	S(50:50)	---	---
$p_T = 0, 0 < p_L < 1$	S3	FR1	HM3	HM3	S3	FR1	HM3	HM3
$p_T = 0, p_L = 1$	S3	FR1	HM3	HM3	S3	FR1	HM3	HM3
$0 < p_T < 1, p_L = 0$	S4	HM1	FR3	FR3	S4	HM1	FR3	FR3
$p_T = 1, p_L = 0$	HM1	HM1	FR3	FR3	S4	HM1	FR3	FR3
$0 < p_T < 1, 0 < p_L < 1$	S3, S4	S3, S4	HM2	HM2	S3, S4	S3, S4	FR2	FR2
$0 < p_T < 1, p_L = 1$	S4	S3, S4	HM2	HM2	S3	FR1	FR1	FR1
$p_T = 1, 0 < p_L < 1$	S4	HM1	HM1	HM1	S3	S3, S4	FR2	FR2
$p_T = 1, p_L = 1$	S4	HM1	HM1	HM1	S3	FR1	FR1	FR1

"High surplus" matches are those whose surplus is such that it is possible for a player to make an acceptable offer to his partner and still capture part of the surplus. When this is not the case it is a "low surplus" match. "High surplus" matches correspond to equations (2), (3a), (4a) and (5a). "Low surplus" matches correspond to equations (3b), (4b) and (5b).

### **Share contracts without fixed payments (50:50, S1 and S2)**

The model predicts that a share contract without fixed payments will be chosen in situations meeting the following conditions simultaneously:

1. the match has a "high surplus";
2. all land put up for tenancy is taken and there are no tenants willing to take more land ( $p_L = 0$ ) nor landlords willing to lease out more land ( $p_T = 0$ ).

The surplus earned by each player is a linear function of the joint surplus with no fixed payment component. There are three different situations to distinguish:

**1. The players are “patient” ( $r=1$ )**

This is the case of the 50:50 share tenancy contract<sup>6</sup>.

**2. The players are “impatient” ( $r<1$ ) and the landlord is the first mover (contract type S1)**

In this case the landlord gets a better share than the tenant. The surplus earned by the landlord is  $e_L = \frac{1}{1+r} e$  and the tenant earns  $e_T = \frac{r}{1+r} e$ .

**3. The players are “impatient” ( $r<1$ ) and the tenant is the first mover (contract type S2)**

In this case the tenant gets a better share than the landlord. The surplus earned by the landlord is  $e_L = \frac{r}{1+r} e$  and the tenant earns  $e_T = \frac{1}{1+r} e$ .

This type of contracts corresponds to situations where the bargaining powers of the tenants and landlords are balanced: none of them has a credible threat to opt out and joining efforts yields a higher income than operating independently. Both parties share the risks of production uncertainty equally (50:50 contract) if they are both “patient”, or with some advantage for the first mover if they are “impatient”. Since none of the parties has a credible threat to opt out, intercontractual externalities have no role to play here. The surplus shares are determined only by the characteristics of the parties, without interference from their outside options.

**Share contracts with fixed payments (S3 and S4)**

In this type of contracts the parties receive a variable payment proportional to the joint surplus of the match and a fixed fee paid by one party to the other. The model predicts that this type of share tenancy will be chosen in situations which meet simultaneously the following conditions:

1. the match has a “high surplus”;
2. at least one of the parties has a probability to get an outside option;
3. the players are “impatient” and if the landlord is the “first mover” and the tenant can get an outside option for sure the landlord should also have some probability to get an outside option, or
4. the players are “patient”, both have some probability to get an outside option, but the “second mover” cannot get it for sure.

There are 19 possible contract choices of this kind listed in table 1 which we can classify in two types of contracts:

1. Type S3 contracts: the fixed fee is paid by the tenant to the landlord;
2. Type S4 contract: the fixed fee is paid by the landlord to the tenant.

Table 1 defines conditions on the values of the rate of time impatience, the values of the probabilities of getting an outside option and the identity of the “first mover”

<sup>6</sup> With a different model Allen (1985) gets the same kind of contract for  $r=1$ .

determining which of the 19 possible contract choices will arise. The corresponding surplus sharing rules can be obtained by evaluating equations (3a), (4a) and (5a) for those sets of conditions.

These share contracts differ from the previous ones (without fixed payments) by the fact that the bargaining powers of the parties are less balanced: partnership still yields more income than independent operation, but one or both parties have a credible threat to opt out. So intercontractual externalities are bound to play a role. More precisely, they are taken into account in the fixed payment received by the player who has a credible threat to opt out.

### **Fixed rent contracts (FR1, FR2 and FR3)**

The fixed rent contract will be chosen in the following situations:

1. tenants have stronger bargaining power than the landlords (the match has a “low surplus”, matched tenants can find alternative partners, either tenants or landlords can make the first offer, but when the landlord moves first he has no chance to find alternative partners);
2. landlords have stronger bargaining power than tenants (the match has a “high surplus”, the parties are “patient”, matched landlords can find alternative partners, but matched tenants can’t);
3. the match has a “high surplus”, the parties are “patient”, tenants make the first offer and have some chance to find alternative partners, but matched landlords can get alternative partners for sure.

In the “high surplus” matches the rent is equal to the full expected value of the landlord’s outside option (contract type FR1):  $e_L = \beta_L$ .

In “low surplus” matches the rent is equal to the following values:

- i) the landlord’s reservation income, when the landlord does not have any chance to get an alternative partner (contract type FR3):  $e_L = \alpha_L$ ;
- ii) a fraction of the expected value of the landlord’s outside option, when the landlord has some chances to find an alternative partner but doesn’t get it for sure (contract type FR2):  $e_L = p_L \beta_L$ ;
- iii) the full expected value of the landlord’s outside option, when the landlord is sure of finding an alternative partner (contract type FR1):  $e_L = \beta_L$ .

When the tenant has a very strong bargaining power the landlord’s rent is such that all the surplus of the match goes to the tenant (contract type FR3).

If the landlord’s bargaining power is not weak (he owns a good quality holding, or has a credible threat to opt out), intercontractual externalities play a role and the landlord gets a better share which takes into account the expected value of the outside options (contract type FR1 or FR2).

### **Hired management contracts (HM1, HM2 and HM3)**

We call “hired management contracts” the ones where the tenant gets a fixed payment and the landlord is the residual claimant. In the land tenancy literature these are called “wage contracts”. We don’t think this is an appropriate designation. The land

contracts are deals between suppliers of two basic inputs: one party supplies essentially land and the other supplies essentially managerial skills and labour monitoring ability, including monitoring of the tenant's own family labour. So these are basically managerial contracts which should be distinguished from labour contracts (fixed wage contracts or others).

The hired management contracts will be chosen in the following situations:

1. the match has a "low surplus", matched landlords can find alternative partners, either landlords or tenants can make the first offer, but when the tenant moves first he has no chances to find alternative partners;
2. the match has a "high surplus", the parties are "patient", matched tenants can find alternative partners, but matched landlords can't;
3. the match has a "high surplus", the parties are "patient", landlords make the first offer and have some chances to find alternative partners, but matched tenants can get alternative partners for sure;
4. the match has a "high surplus", the parties are "impatient", matched tenants can find alternative partners for sure, landlords make the first offer and have no chance to find alternative partners.

In "high surplus" matches the tenant earns the full expected value of his outside options (contract type HM1):  $e_T = \beta_T$ .

In "low surplus" matches the tenant's surplus has the following values:

- i) the tenant's reservation income, when there is no chance to find an alternative partner (contract type HM3):  $e_T = \alpha_T$ ;
- ii) a fraction of the expected value of the tenant's outside options, when there is some chance to find an alternative partner but the tenant cannot get it for sure (contract type HM2):  $e_T = p_T \beta_T$ ;
- iii) the full expected value of the tenant's outside options, when he is sure to find an alternative partner (contract type HM1):  $e_T = \beta_T$ .

When the landlord has a very strong bargaining power, no surplus goes to the tenant and so all the surplus goes to the landlord (contract type HM3).

If the bargaining power of the tenant is not weak (he has a good entrepreneurial ability, or a credible threat to opt out), intercontractual externalities play a role and the tenant gets a better surplus share which takes into account the expected value of the outside options (contract type HM1 or HM2).

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