Gradual Nash Bargaining with Endogenous Agenda: A Path-Dependent Model

Julián J. Arévalo*
jarevalo@uexternado.edu.co

February 2004

Abstract

This article proposes a method for considering the bargaining agenda as an endogenous phenomenon in gradual bargaining games, understood as being path-dependent processes. Some short, medium and long-term results for bargaining are presented, as well as a possible application for the model.

JEL classification: C60, C71, C78.
Keywords: bargaining games, path-dependent processes.

1 Introduction

Bargaining games refer to situations where two or more players must reach agreement regarding how to distribute an object or monetary amount. Each player prefers to reach an agreement in these games, rather than abstain from doing so; however, each prefers that agreement which most favours his interests. Examples of such situations would be the bargaining involved in a labour union and the directors of a company negotiating wage increases, the dispute between two communities about the distribution of a common territory or the conditions under which two countries can start a programme of nuclear disarmament. Analyzing these kinds of problem looks for a solution specifying which component in dispute will correspond to each party involved.

Players in a bargaining problem can bargain for the objective as a whole at a precise moment in time. The problem can also be divided so that parts of the whole objective become subject to bargaining during different stages.

Unlike the classical one-stage approach towards developing bargaining problems, this article explores a procedure subdividing the objective bargained for. Each part of the agreement then becomes individually and sequentially negotiated. Partial agreements

*This article, which belongs to a larger study, was done under the guidance of Professor Sergio Monsalve, to whom I am very thankful for all his support. I also thank Mauricio Pérez for the revision of the english version of a preliminary draft, and for all his comments.

leave the remaining points of disagreement for later stages. The process continues until the whole of the objective being negotiated has been bargained for. In other words, the procedure consists of defining a bargaining agenda covering all points to be bargained for, reaching separate agreement on each one. If, at any given stage, agreement is not reached, the result of the process is that agreement reached in the previous stage.

The main advantage of subdividing a bargaining problem in this way is the reduction of the risk that the process may break down. In the words of Robert Axelrod:

> "...for example, a treaty of armaments control or disarmament could be divided in many intermediate stages; it would allow the two negotiating parts to advance with relatively small steps instead of having to give one or two great steps... if both parts knew that to an improper step of the other it is possible to respond with the reciprocal decision in the following phase, both parts would have more confidence that the process will work as it was predicted. (1984, p. 128-129)"

More specifically, he added that "having to take many small steps will help to promote the cooperation, more than having to give just one or two very important ones." Schelling wrote about the importance of partial agreements in generating confidence, more than forty years ago :

> "If one is able to conclude certain number of agreements, each one of the parts can be arranged to risk a small investment with the purpose of creating a confidence tradition. The persecuted purpose is to allow that each part demonstrates that understands the confidence necessity and that it knows that the other also understands it. Thus, if it must bargain on an important subject, it can be necessary to look for and to bargain other secondary questions for "practicing"; in order to establish the necessary confidence of each one of the parts in which the other understands the long term value of the good faith. Even though the situation in the future is not going to repeat itself, fits the possibility of creating a situation equivalent dividing the subject question to bargaining in consecutive parts." (1960, p. 62)

Nevertheless, this approach has received little attention in the literature. Wiener and Winter (1999) and O´Neill, Samet, Wiener, and Winter (2003) remark that whilst discussion on static (classic) bargaining solutions is prolific, in contrast there is little analysis of bargaining problems in which bargaining set expands gradually (i.e. bargaining problems in which an agenda has been previously set). They suggest a problem solution path for problems of gradual bargaining, in which the "bargaining agenda" is considered exogenous. They further argue that, "This intuition raises, of course, important questions regarding the agenda. The ultimate outcome of the bargaining depends on the way the large pie is broken into small pieces."

This is to say that the results of the process could change, depending on the way in which the agenda is structured.

When decomposing a bargaining problem into several stages, it usually turns out that the agenda cannot be fully set *a priori*, or even if it is, that such an agenda is susceptible of modification as the bargaining process advances. It is natural to think that the offers, requests and agreements that may take place in later stages of bargaining will be strongly related to those reached during previous stages. It may happen that as the process advances one of the parties becomes stronger than the other and demands more in each successive stage. It is also possible that the process will tend to equalize the parts, so that the party that received greater benefits in the early stages of the bargaining process will make greater concessions in the later stages. Returns may thus increase, decrease or remain constant during the negotiation. It should be recognised when designing a model of the bargaining agenda that the process is one in which the parties face changing conditions, partly determined by chance and partly by the process's history where certain of the parties'positions can gain strength as the process advances.[1].

Arthur, Ermoliev, and Kaniovski (1983, 1987) methodology will be taken as a first approach to the problem of endogenously modelling the bargaining agenda for studying path-dependent processes. The same analysis is valid not only for increasing returns on scale but also for other types of returns to scale.

This paper aims to implement a mechanism making a bargaining agenda endogenous and thus analyse possible differences in the results of bargaining situations, depending on how an agenda has been set.

Some elements of classical bargaining theory are reviewed in the second section. The third section presents the main aspects of Wiener and Winter's model of gradual bargaining (1999). Section 4 is the most important part of the article; it develops a bargaining model having an endogenous agenda. A possible application of this model to the negotiations which took place in 1978 between Egypt and Israel is dealt with in section 5. Section 6 contains the general conclusions.

2 Classical Bargaining Theory

In a classical bargaining problem the result is an agreement reached between all interested parties, or the status quo of the problem. It is clear that studying how individual parties make their decisions is insufficient for predicting what agreement will be reached. However, classical bargaining theory assumes that each participant in a bargaining process will choose between possible agreements, following the conduct predicted by the rational choice model. It is particularly assumed that each player's preferences regarding the possible agreements can be represented by a von Neumann-Morgenstern utility function.[2].

[1]For example, Schelling (1960) emphasizes the importance in the inicial agreements in a bargaining process with a phrase common in this type of situation:"If I yield now, you will review your opinion about me for our future bargainings; in order to defend my reputation, I must stay right". p.45

[2]I.e, it is assumed that the preferences of each agent satisfy the axioms of completeness, transitivity, independence, and continuity, in the conventional form (Mas-Colell, Whinston, and Green (1995)).

Nash (1950) defines a classical bargaining problem as being a set of joint allocations of utility, some of which will correspond to that the players would obtain if they reach an agreement, and another which represents what they would get if they failed to do so.

In what follows we assume a bargaining problem just between two players ($n = 2$)[3]. We can, then, formally define a bargaining game as:

Definition 1. (Bargaining Game)
A *bargaining game* for two players is defined as a pair (F, d) where $F \subset \mathbb{R}^2_+$ is the set of possible joint utility allocations (possible agreements), and $d \in F$ is the disagreement point[4]. We asume F closed, bounded, convex and comprenhensive.[5]

Definition 2. (Bargaining Solution)
If Λ denotes the set of *all* the bargaining problems in $\mathbb{R}^2$, a *bargaining solution* is a rule $\phi : \Lambda \to \mathrm{R}^2_+$ which assigns, to every bargaining problem (F, d), utility vectors $\phi(F, d) = (\phi_1(F, d), \phi_2(F, d)) \in F$ (one for each agent), which corresponds to the evaluation of his utility function in the prescribed solution.

For the definition of a specific bargaining solution is usual to follow Nash's proposal, setting out the axioms this solution should satisfy. Some of the most frequent axioms used in the building of bargaining solutions are efficiency, symmetry, independence of irrelevant alternatives, scalar invariance, monotonicity, etc. An exhaustive list appears in Thomson (1994). Three of the most important solutions to classic bargaining problems are:

Nash Bargaining Solution (Nash (1950))
The *Nash bargaining solution* $\phi_N(F, d)$ is the bargaining solution which *maximizes the product of agent´s utilities* on the bargaining set:

$$\phi_N(F, d) \in \text{argmax}(x - d_1)(y - d_2)$$
$$\text{s.t:}$$
$$x, y \in F$$
$$x, y \geq d_i$$

It is easy to prove that if the Pareto frontier of F is smooth and is determined by an equation of the form $H(x, y) = 0$, this frontier and the curve $(x - d_1)(y - d_2) = t$ for

[3]There is no loss of generality in this assumption if we let as the only choices for the players the unanimity or the full disagreement.
[4]$\mathbb{R}^2_+ = \{(x, y) \in \mathbb{R}^2 | x \geq 0, y \geq 0\}$.
[5]I.e,:

i. F is convex if for all $x, y \in F \subset \mathbb{R}^2$, y $\alpha \in [0, 1], \alpha x + (1 - \alpha)y \in F$.
ii. F is bounded if exists a $K > 0$ such that for all $x \in F, ||x|| \leq K$.
iii. F is closed if $\{x_n\}_n \subseteq F$ and $x_n \to x$, then $x \in F$
iv. F is comprenhensive if $x \in F$ and $x \geq y \geq 0$, implies $y \in F$.

a t, have a common tangent. This is, in the Nash bargaining solution, the gradient[6] of the function which determines the Pareto frontier $(H_x(x,y), H_y(x,y))$, and the gradient of the "Nash product" (y,x) are relate in this way:

$$\frac{H_x(x,y)}{H_y(x,y)} = \frac{y}{x} \tag{1}$$

Note 1.
For situations where the players present different "*bargaining powers*"[7], it has been used *the generalized Nash product*, where the solution maximizes the expression $(x_1 - d_1)^\alpha (x_2 - d_2)^\beta$, and the parameters $\alpha, \beta > 0$ are indicators of the bargaining power of players 1 and 2 respectively.

Kalai - Smorodinsky Solution (Kalai and Smorodinski (1975))
Defining the "ideal point" $I(F,d)$ of a bargaining problem (F,d) where $I_i(F,d) = \max\{x_i | x \in F, \text{ for all } i\}$, the Kalai - Smorodinsky solution $\phi_K(F,d)$ is defined as the point in the frontier of the feasible set which connects the disagreement point to the ideal point.

Thus, the Kalai-Smorodinsky solution is the only point in the efficient frontier where

$$\frac{(y - d_2)}{(x - d_1)} = \frac{(I_2(F,d) - d_2)}{(I_1(F,d) - d_1)} \tag{2}$$

***Egalitarian Solution* (Kalai (1977))**
The egalitarian solution $\phi_I(F,d)$, divides the gains of the bargaining by equal parts between the players. Hence, for every bargaining problem, (F,d), $\phi_I(F,d)$ is the vector in the frontier of F whose coordinates are equal. In general, it maximizes the social welfare function $\min\{x,y\}$ on F, which implies that always $y - d_2 = x - d_1$.

3 Gradual Bargaining Model [Wiener and Winter (1999); O´Neill, Samet, Wiener, and Winter (2003)]

A gradual bargaining problem is a process where players' bargaining possibilities gradually expand or shrink, determining what will henceforth be called a bargaining agenda, formally defined as:

Definition 3. (Bargaining Agenda)
A *bargaining agenda* is a pair of functions (H,F),

$$H : \mathbb{R}^2_+ \to \mathbb{R}, \qquad F : \mathbb{R}_+ \to \mathbb{R}^2_+$$

[6]The gradient of a function $H : \mathbb{R}^2 \to \mathbb{R}$ is the vector of partial derivatives $\left(\frac{\partial H}{\partial x}, \frac{\partial H}{\partial y}\right)$, which is also written as (H_x, H_y).

[7]Because of their risk attitudes, disposable information, etc.

such that, for all $t \in \mathbb{R}_+, F_t$ is the bargaining set

$$F_t = \{(x, y) \in \mathbb{R}^2_+ | H(x, y) \leq t\}$$

We will assume that:

i. H is continuous, differentiable with continuity, increasing in x, y and convex[8]

ii. F is monotone in t: if $t \leq t'$ then $F_t \subseteq F_{t'}$

Definition 4. (Gradual Bargaining Problem)
A *gradual bargaining problem* is a triplet (H, F, d) consisting of a bargaining agenda (H, F) and a point of disagreement $d \in \mathbb{R}^2_+$.

Once a gradual bargaining problem has been specified, then a *gradual bargaining solution becomes a rule specifying an allocation of utilities for each player at each point in time* (i.e. as bargaining set grows, a solution will consist of a path determining players' payoffs at any given moment).

Definition 5. (Gradual Bargaining Solution)
A *gradual bargaining solution* of the bargaining gradual problem (H, F, D) is a differentiable path

$$\begin{aligned} \phi : \mathbb{R}_+ \to & \mathbb{R}^2_+ \\ t \to & \phi(t) = (x(t), y(t)) \end{aligned}$$

such that $\phi(t) \in F(t)$ for all t.

To illustrate the point, consider the case where a set of values t is enumerable, meaning that the bargaining set expands discretely. A classical bargaining problem thus becomes established during each period where the point of disagreement lies in the allocation settled during the preceding period. Several "arbitration schemes" can be resorted to for distributing the new objective bargained for. Two of them would be:

"Nash for Each Crumb" Arbitration Scheme
Each additional crumb in the bargaining process is distributed according to the Nash-bargaining solution (see figure 1).

"Alternating Turns" Arbitration Scheme
A single, previously chosen, player, gets the whole crumb at each stage. The other player gets the new crumb at next stage and they continue in this fashion *ad infinitum.* Figure 2 depicts an example, where the whole crumb corresponds to player 1 in stage $(t + 1)$ and corresponds to player 2 in $(t + 2)$.

Definition 6. (Gradual Nash Solution)
A *Gradual Nash Solution* (GNS) is a gradual bargaining solution $\phi(t) = (x(t), y(t))$ assigning a solution for the differential equation for every gradual bargaining problem (H, F, d):

$$\frac{dy}{dx} = \frac{H_x(x, y)}{H_y(x, y)} \tag{3}$$

[8] H is convex in $\mathbb{R}^2_+$ if for all $x, y \in \mathbb{R}^2_+$ y $\lambda \in [0, 1], H(\lambda x + (1 - \lambda)y) \leq \lambda H(x) + (1 - \lambda)H(y)$

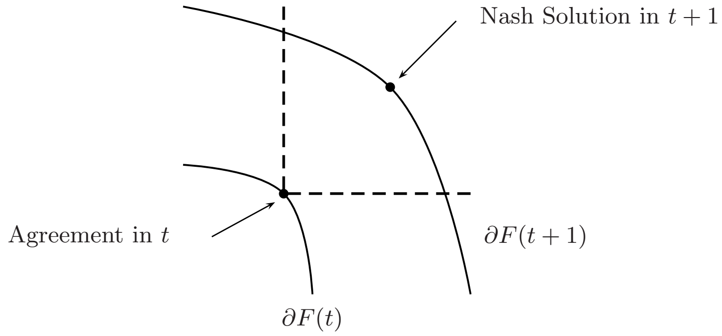

Figure 1: "Nash on each crumb" Arbitration Scheme

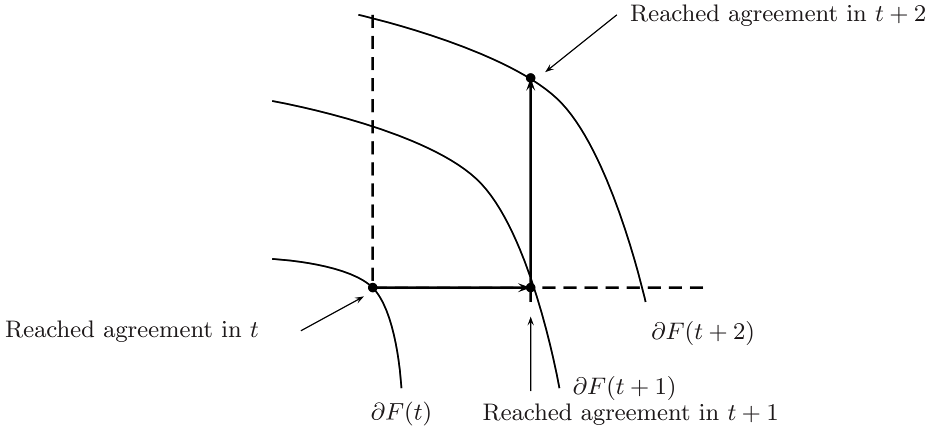

Figure 2: "Alternating Turns" Arbitration Scheme

Note that no matter how gains originating from expanding the negotiation set are distributed, all gradual solutions satisfy the previous condition *as long as each expansion is small enough*. This solution can thus be given for small expansions of the bargaining set, independently of the arbitration scheme used. For a formal exposition, see Wiener and Winter (1999).

According to this gradual bargaining solution, the crumb favours the needier player at each stage of distribution where "neediness" is determined by the marginal rate of substitution between the players' welfare (i.e. the number of units of utility player 2 must forgo to increase player 1's welfare in 1 unit of utility while remaining within the same set of possible agreements).

The difference between the Nash bargaining solution and the gradual Nash solution should be emphasized. The latter favours the needier player whilst the former favours the one less averse to risk..

Since the frontier of H is smooth and strictly increasing in x and y, the GNS will set a sole solution path. The solution is assumed to be on the frontier of the feasible set at each moment to determine the exact value of the function during each period.

4 Endogenous Bargaining Agenda

The gradual bargaining solution developed by Wiener and Winter (1999) supposes a *previously settled* agenda; however, the topics normally included in bargaining at each stage depend on previous agreements and on random events. *Initial bargaining conditions, the process's history and unpredictable events occurring during it can affect determining the bargaining agenda.* Analyzing these kinds of dynamic problems means that one can focus on distribution or the model's steady states as well as the paths through the model to reach such states.

Only two players are assumed as a first approach, both of whom are risk-neutral.[9].

The following bargaining dynamic is proposed:

i. Players must decide about distributing one part of an object during the first stage of the bargaining process. This fraction is set exogenously.

ii. Once players have reached an agreement, they evaluate their gains in bargaining respecting what they could have won in their individual ideal state.

iii. A player's evaluation will determine each player's dominant or unfavourable position during the following stage. This situation will condition the set of possible results during the next stages.

iv. Once this new bargaining set has been determined, the process repeats itself from stage ii until the bargaining object has been fully negotiated.

The part of the object that will be bargained for during each stage (as well as the number of bargaining process stages) are endogenous to the model.

4.1 The Bargaining Agenda as a Path-Dependent Process

Since this article concerns a bargaining problem where the feasible set expands gradually, but such expansion is conditional on the process's history, a first approach will be made considering such expansion (the bargaining agenda) as being a path-dependent process expanding through Nash bargaining solutions.

The bargaining sets in each stage can be defined as follows as long as two risk-neutral agents are being dealt with. In the first stage:

$$F_1 = \{(x, y) \in \mathbb{R}^2_+ | ax + by \leq \eta\} \tag{4}$$

where η, a and b have been determined beforehand, and $a, b, \eta > 0$. Once we have reach an agreement, the bargaining set expands *in a direction such that only one*

[9] A player is risk-neutral if his utility function is of the form $u(w) = \alpha w + \beta, \alpha > 0$.

of the players is benefit. The size of the expansion will be δ^z where z is the times that player who benefits from the expansion, has been benefited in the past, and $0 < \delta < 1$[10]. Thus, in the second stage the bargaining set will be:

$$F_2 = \{(x, y) \in \mathbb{R}^2_+ | ax + by \leq \eta + \delta x\} \quad (5)$$

if the expansion favored player 1; or

$$F_{2'} = \{(x, y) \in \mathbb{R}^2_+ | ax + by \leq \eta + \delta y\} \quad (6)$$

if the expansion favored player 2. This is shown in figure 3, where the point ϕ_1 indicates the agreement reached at the first stage.

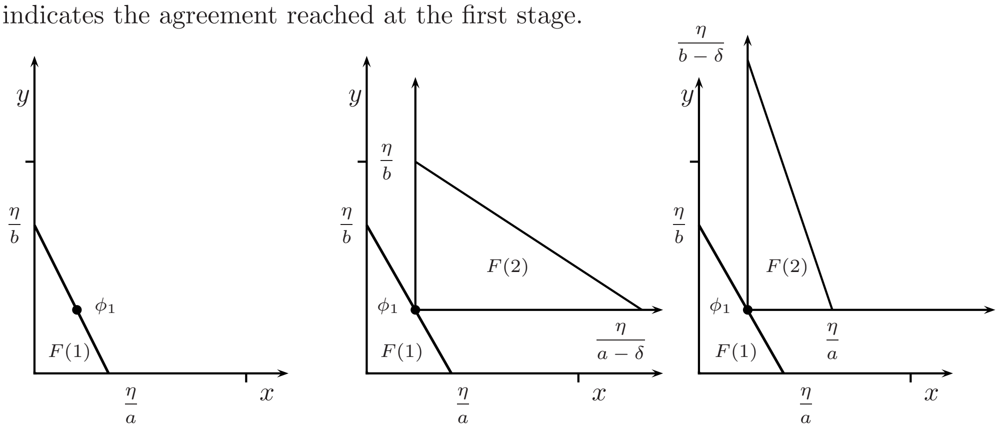

Figure 3: Expansion of the Bargaining Set

Generalising previous expressions, the bargaining set is given by the following for any stage t:

$$F_t = \left\{ (x, y) \in \mathbb{R}^2_+ | ax + by \leq \eta + x \left(\sum_{i=0}^{r} \delta^i - 1 \right) + y \left(\sum_{j=0}^{t-r} \delta^j - 1 \right) \right\}$$

where $r \leq t$ is the number of times (until stage t) the bargaining set has expanded in the direction which favors player 1, and $t - r$ is the number of times the bargaining set has expanded in the direction favouring player 2.

Once a bargaining set is known at each stage, a solution must be sought for each of them. A *Nash for each crumb* arbitration scheme has been assumed.[11]. As long as the frontier of the bargaining set is a straight line, then the slope of the line describing the frontier and the slope of curve xy are equal in a Nash bargaining solution.

Thus, in the first stage the equation describing the frontier of the bargaining set is $ax + by = \eta$, so we get

$$y = \frac{\eta}{b} - \frac{a}{b}x; \quad (7)$$

[10]Notice that, characterizing the expansions in this way, we are assuming each player values nearer expansions more than the distant ones.

[11]Since both players are risk-neutral, the Nash bargaining solution and the Kalai-Smorodinski one are the same in every period.

then, in the Nash solution we have $\dfrac{a}{b} = \dfrac{y}{x}$, with payoffs

$$x = \frac{\eta}{2a}, \quad y = \frac{\eta}{2b} \tag{8}$$

for players 1 and 2 respectively. This corresponds to the utilities in the first stage of the bargaining process.

The bargaining set will be $F(2)$ o $F(2')$ in the second stage once this agreement has been reached. Payoffs will be $\phi_2 = \left(\dfrac{\eta}{2(a-\delta)}, \dfrac{\eta}{2b}\right)$ if the set expands favoring player 1; if the set expands favoring player 2, these payoffs will be $\phi_2' = \left(\dfrac{\eta}{2a}, \dfrac{\eta}{2(b-\delta)}\right)$.

Repeating the same argument for the next stages we get the payoffs series corresponding to the possible bargaining paths (see figure 4).

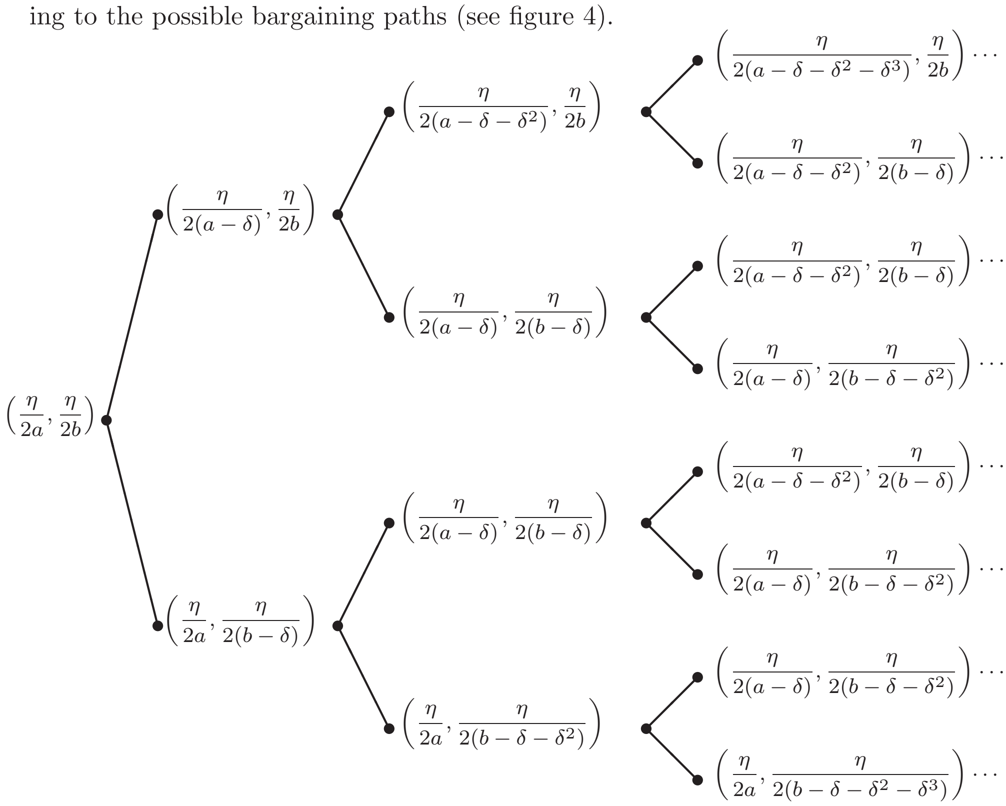

Figure 4: Bargaining Paths

Generalizing these results, we find:

$$x_t^1 = \begin{cases} \dfrac{\eta}{2(a+1-\sum_{j=0}^{r}\delta^j)} & \text{if the last expansion favored player 1,} \\ x_{t-1}^1 & \text{if the last expansion favored player 2.} \end{cases} \tag{9}$$

$$x_t^2 = \begin{cases} \dfrac{\eta}{2(b+1-\sum_{j=0}^{t-r}\delta^j)} & \text{if the last expansion favored player 2,} \\ x_{t-1}^2 & \text{if the last expansion favored player 1.} \end{cases}$$

Is easy to check that the feasible maximum earns for players 1 and 2, respectively, *in stage* t are given by:

$$x_t^{1^*} = \frac{\eta}{a+1-\sum_{k=1}^{t}\delta^{k-1}}, \qquad x_t^{2^*} = \frac{\eta}{b+1-\sum_{k=1}^{t}\delta^{k-1}} \tag{10}$$

Making t large enough, the non-negativity condition for these expressions implies $\delta < \dfrac{a}{1+a}, \delta < \dfrac{b}{1+b}$; with a similar analysis to the one developed to find the payoffs in each period, is easy to show that the maximum payoffs *until the stage* t, are given by: $y_t^{i^*} \equiv \sum_{j=1}^{t} x_j^{i^*}$, i.e.,

$$y_t^{1^*} = \sum_{j=1}^{t} \frac{\eta}{\left(a+1-\sum_{k=1}^{j}\delta^{k-1}\right)}, \qquad y_t^{2^*} = \sum_{j=1}^{t} \frac{\eta}{\left(b+1-\sum_{k=1}^{j}\delta^{k-1}\right)} \tag{11}$$

It may be observed that each player's effective gains, as well as their maximum gains, are given by the values of η, a and b and by δ, (representing the magnitude of the set's expansion during each stage). The question, as in any path dependent process, is whether it will stabilize in a fixed path of "bargaining gains". That is, whether the process leads to the *emergence* of a macrostructure and, if so, what its structure and emergence path will be. Modelling the bargaining agenda as a path-dependent process tries to capture the idea that the probability of the bargaining set expanding during a given period in the direction favouring one of the players is a given function of the relationship between the same player's effective gains and potential gains during previous stages. The following definition has been included to this effect:

Definition 7. (Advantage on Bargaining)
In a gradual bargaining problem (H, F, d), let's call $x_t^{i^*}$ the maximum possible earn (in utility) for player i on stage t (ex-ante), and x_t^i the effective earn of player i on stage t (ex-post); *the (absolute) advantage on bargaining* for player i on stage t, α_t^i is defined for $i = 1, 2$, as

$$\alpha_t^i \equiv \frac{\sum_{k=1}^{t-1} x_k^i}{\sum_{k=1}^{t-1} x_k^{i^*}} \qquad \text{si } t > 1 \tag{12}$$

$\alpha_1^1 = \alpha_1^2 = 0.5$. Note that $0 \leq \alpha_t^i \leq 1$ for all $t \geq 1, i = 1, 2$

For example, if two bargaining stages have taken place and player 1 could have obtained a maximum of 0.4 during the first and managed to obtain it, while his potential gains were 0.5 during the second stage but he only obtained 0.3, then his bargaining gain is (0.4+0.3)/(0.4+0.5) =0.777. As stated above, each player evaluates his gains regarding the ideal case up to the immediately preceding agreement (see *ii*).

To simplify notation in the following analysis and considering that:

$$y_t^i \equiv \sum_{k=1}^{t} x_k^i, \qquad i = 1, 2 \tag{13}$$

and,

$$y_t^{i^*} \equiv \sum_{k=1}^{t} x_k^{i^*}, \qquad i = 1, 2; \tag{14}$$

then,

$$\alpha_t^i = \frac{y_{t-1}^i}{y_{t-1}^{i^*}} \qquad i = 1, 2; \tag{15}$$

The superscript denoting a particular player is suppressed during what follows since analysis is valid for both of them. Note that evolution of accumulated gains by a player in stages up to period t is given by the following expression from equation (13):

$$y_t = y_{t-1} + x_t; \tag{16}$$

Likewise, the evolution of absolute gains from bargaining may be obtained by dividing the previous expression by y_t^*:

$$\frac{y_t}{y_t^*} = \frac{y_{t-1}}{y_t^*} + \frac{x_t}{y_t^*} \tag{17}$$

Replacing the values of α_{t+1} and y_t^* in (17) we get

$$\alpha_{t+1} = \frac{y_{t-1} + x_t}{y_{t-1}^* + x_t^*} \tag{18}$$

Adding and reducing α_t at the right side of (18) we get:

$$\alpha_{t+1} = \alpha_t + \frac{y_{t-1} + x_t}{y_t^*} - \frac{(y_{t-1}^* + x_t^*)\alpha_t}{y_t^*} \tag{19}$$

and, then,

$$\alpha_{t+1} = \alpha_t + \frac{1}{y_t^*}(x_t - \alpha_t x_t^*) \tag{20}$$

Equation (20) describes the system's dynamics. The conditional expected value on α_t should be calculated to observe its evolution:

$$E[\alpha_{t+1}|\alpha_t] = \alpha_t + \frac{1}{y_t^*}[E[x_t|\alpha_t] - x_t^*\alpha_t] \tag{21}$$

The first two terms on the equation's right side are invariable, while the last term's numerator corresponds to the effective gains of the player in stage t, given by equation (9). Effective gain in t depends on the number of times the bargaining set has expanded up to stage t in a way favouring that player. The expected value of such gain will be given by the expression

$$E[x_t|\alpha_t] = \sum_{h=0}^{t-1} \frac{\eta}{2\left(a+1-\sum_{j=0}^{h}\delta^j\right)} \binom{t-1}{h} p_t^h (1-p_t)^{t-1-h} \tag{22}$$

where p_t is the probability, on stage t, that the bargaining set will expand in the direction favouring the chosen player.

The evolution of the gains from bargaining is then given by the dynamic discrete system of equation (23)

$$\alpha_{t+1} = \alpha_t + \frac{1}{y_t^*}\left[\sum_{h=0}^{t-1} \frac{\eta}{2\left(a+1-\sum_{j=0}^{h}\delta^j\right)} \binom{t-1}{h} p_t^h (1-p_t)^{t-1-h} - x_t^*\alpha_t\right] \tag{23}$$

where x_t^* and y_t^* are given by equations (10) and (11), respectively.

Once the equation determining the evolution of gains from bargaining has been specified, its behaviour is then examined in the short, medium and long-term of negotiation for a given set of parameters. An attempt is made to establish whether a long-term structure of negotiation "emerges" and, if so, by which path it has been reached. This is, precisely, the main result of this paper.

Theorem 1. *Let (H, F, d) a gradual bargaining problem between two risk-neutral players whose [effective earnings/potential earnings] ratios are given by the discrete system (23). Then, their long term behaviour is the egalitarian treatment, (in this case coinciding with the Nash solution and also that of Kalai-Smorodinski); i.e.,* $\lim_{t\to\infty} \alpha_t^i = 0.5; i = 1, 2$. *Moreover, long term behavior depends neither on probability p_t of the set to expand in favor of one or another player, nor in a, b or δ. However, these parameters may determine bargaining behaviour in the short and medium-term.*

Proof.
It can be easily seen that succession $\{\alpha_t\}$ is strictly increasing for a large enough t, and, since $\{\alpha_t\}$ is bounded, then it's convergent.

Replacing the α values for $\alpha_{\lim} = \lim_{t\to\infty} \alpha_t$ in the equation (23)

$$\alpha_{\lim} = \lim_{t\to\infty} \frac{\sum_{h=0}^{t-1} \frac{\eta}{2\left(a+1-\sum_{j=0}^{h}\delta^j\right)} \binom{t-1}{h} p_t^h (1-p_t)^{t-1-h}}{\frac{\eta}{a+1-\sum_{k=1}^{t}\delta^{k-1}}}$$

which can also be expressed as

$$\alpha_{\lim} = \frac{1}{2} \lim_{t\to\infty} \frac{\sum_{h=0}^{t-1} \dfrac{\eta(1-\delta)}{a-\delta(a+1)+\delta^{h+1}} \binom{t-1}{h} p_t^h (1-p_t)^{t-1-h}}{\dfrac{\eta(1-\delta)}{a-\delta(a+1)+\delta^{t+1}}}$$

or, which is the same,

$$\alpha_{\lim} = \frac{1}{2} \lim_{t\to\infty} \sum_{h=0}^{t-1} \frac{a-\delta(a+1)+\delta^{t+1}}{a-\delta(a+1)+\delta^{h+1}} \binom{t-1}{h} p_t^h (1-p_t)^{t-1-h} \tag{24}$$

Now: if we define, for $h \leq t-1$, $z_h \equiv \left[\dfrac{a-\delta(a+1)+\delta^{t+1}}{a-\delta(a+1)+\delta^{h+1}}\right]^{\frac{1}{h}}$
then equation (24) can be written as

$$\alpha_{\lim} = \frac{1}{2} \lim_{t\to\infty} \sum_{h=0}^{t-1} \binom{t-1}{h} (p_t z_h)^h (1-p_t)^{t-1-h} \tag{25}$$

But, since $t > h$, we have

$$1 \geq z_h \geq z_t;$$

where $z_t \equiv \left[\dfrac{a-\delta(a+1)}{a-\delta(a+1)+\delta^{t+1}}\right]^{\frac{1}{t}}$ and, using the binomial theorem,

$$1 \geq \sum_{h=0}^{t-1} \binom{t-1}{h} (p_t z_h)^h (1-p_t)^{t-1-h} \geq \sum_{h=0}^{t-1} \binom{t-1}{h} (p_t z_t)^h (1-p_t)^{t-1-h} = [p_t z_t + (1-p_t)]^{t-1} \tag{26}$$

so, to complete our proof, is enough to see that $\lim_{t\to\infty} z_t = 1$. But this is evident, as

$$\ln(z_t) = \frac{1}{t} \ln[a-\delta(a+1)] - \frac{1}{t} \ln[a-\delta(a+1)+\delta^{t+1}] \to 0 \text{ when } t \to \infty$$

Finally, in the following discussion we will show how the parameters a, b, δ, and the probability p_t, do determine behaviour in the short and medium term. □

4.2 Discussion

Once it has been established that bargaining leads to a 1/2 [effective earnings/potential earnings] ratio for each player in the long-term, then the short and medium-term path of such advantages can be explored, depending on the model's parameters.

The conventional distribution function for problems having binary choices will be used for determining p_t, i.e. the logistic distribution function:

$$p_t = p(\alpha_t) = \frac{1}{1 + e^{-\beta \alpha_t}} \tag{27}$$

where $\beta \in \mathbb{R}$ will be an exogenous factor called "external influence bargaining factor". Note that, if this parameter β is equal to zero (impartiality in bargaining), then there will be an equal probability that the set will expand in favour of either of the two players at each stage. Similarly, a higher value of β, makes expansion favouring player 1 more probable for any given value of t.

An $a = b = 1$ seting was used with *Mathematica* software for evaluating possible absolute advantage paths in bargaining for different values of δ (size of the set expansion) and β (external influence factor on bargaining). Thus, several values for β were set, aimed at establishing the α depending on the values for δ. Figures 5, 6 and 7 show some of the results.

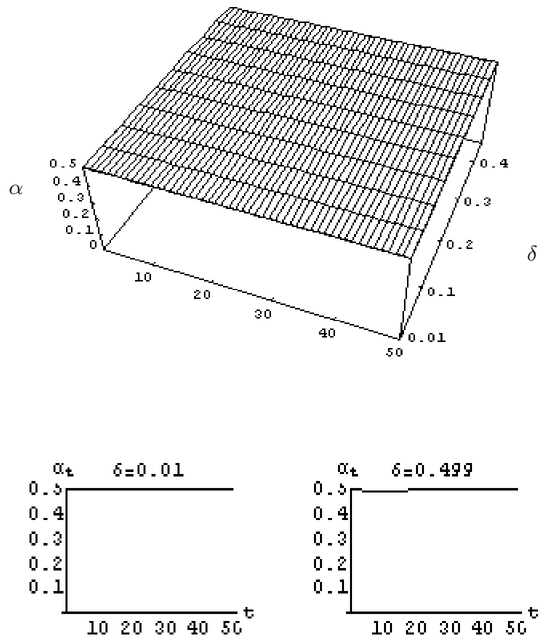

Figure 5: Evolution of α when $\beta = 10$

i. It can be seen from figure 5 that, when the external influence on bargaining factor is high enough ($\beta = 10$, for example), the α value for the favoured player remains almost unaffected during bargaining, showing only a very small reduction during the first stages of bargaining in cases where the set expansion is high enough (δ close 0.499, in this case) but quickly stabilizing at 0.5. The bottom panel of Figure 5 shows the evolution of α for extreme values of δ.

ii. A different result is obtained when the external influence on bargaining factor is modified (β). In particular, the path of α is represented in Figure 6 under a condition of impartiality ($\beta = 0$).

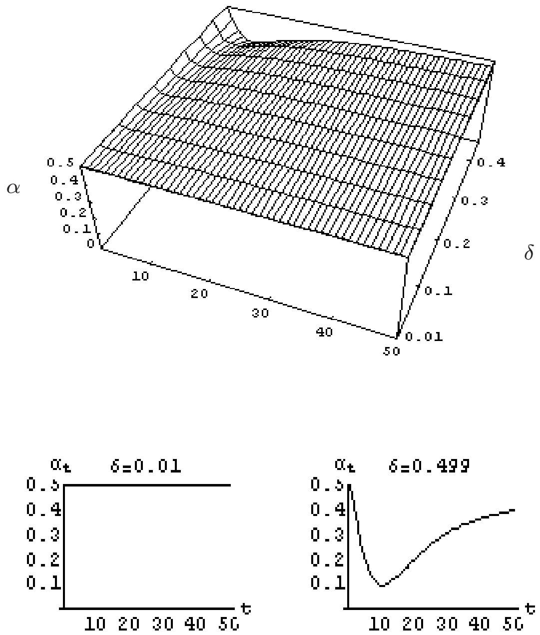

Figure 6: Evolution of α_t when $\beta = 0$

Note that the value of α decreases during the early stages of bargaining for all values of δ, especially when there are large bargaining set expansions. However, also note that α is sufficiently near its long term value, towards the end of the period modelled ($t = 50$), *independently of the magnitude of expansion of the set* δ.

iii. Finally, it may be observed that when the external influence on bargaining factor is sufficiently large but adverse to the player in question ($\beta = -10$, in this case), the value of α falls substantially during the early stages of bargaining. It only reaches its long term value during the time period being analyzed if bargaining set expansion is sufficiently small (δ close to 0, as in Figure 7). Given enough time and even with high values of δ, α it will eventually reaches its long term value according with Theorem 1. A period of 1000 stages was calculated to observe this, maintaining δ at 0.499; (Figure 8).

Two conclusions may be drawn from the above results. The larger the bargaining set expansion, the larger short and medium-term absolute bargaining advantage fluctuation will be. This is to say that as the value of the parameter of expansion of the

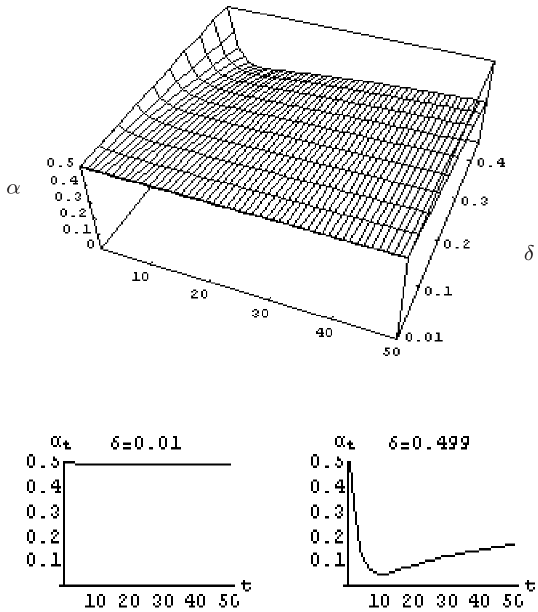

Figure 7: Evolution of α_t when $\beta = -10$

bargaining set, δ, increases, the more time is required to reach the long term gain ratio. Thus, a "stable" behaviour of the bargaining advantage can only be achieved with small values of δ. Additionally, the effect of the external influence on bargaining factor (β) is to induce fluctuations in the absolute bargaining advantage during early stages. Its influence thus practically disappears during sufficiently distant stages of sufficiently lengthy negotiations. This leads to affirm that $\beta = 0$ *is the long term influence factor*.

It should be emphasized that egalitarian treatment in the long term predicted by Theorem 1 is only reached if bargaining proceeds indefinitely. However it is probable that if one of the players a sufficiently low value of α_t or, likewise, low values of α_t during several stages (as in the case of the combined effect of an adverse the external influence on bargaining factor and large expansions of the bargaining set) then negotiation could be broken-off in the short or medium-term.

A second estimation of absolute bargaining advantage paths was undertaken for observing whether short and medium-term results were determined by choice of distribution function. The probability of bargaining set expansion was thus specified as follows:

$$p(\alpha_t) = \epsilon + \alpha_t(1 - 2\epsilon) \tag{28}$$

where the role of ϵ was to equalize the bargaining advantage between the parties. Note that if ϵ is 0.5, the probability that the bargaining set will expand to the benefit

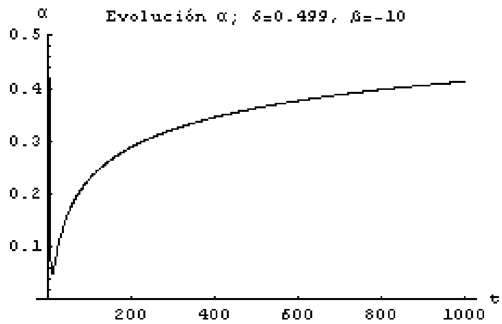

Figure 8: Long term evolution of α; $\beta = -10$

of either player is also equal to 0.5. Likewise, if ϵ is greater than 0.5, the probability of a favourable expansion of the bargaining set will be less than 0.5. The opposite happens if ϵ is less than 0.5.

The paths of the absolute bargaining advantages with $p(\alpha_t)$, given by equation 28, can be observed in Figures 9, 10 and 11 for values of ϵ equal to 0, 0.5 y 1, respectively.

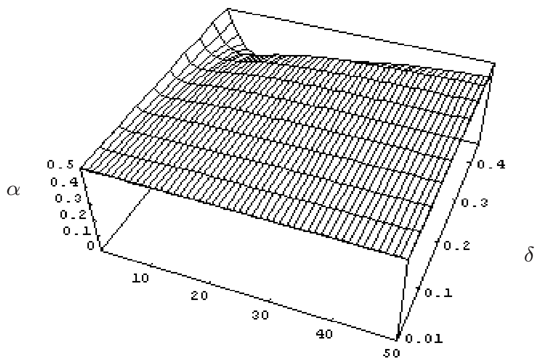

Figure 9: α_t paths when $\epsilon = 0$

Note that behaviour in the three scenarios is similar. α tends to decrease more when the value of ϵ is lower, especially when the probability that the set will expand in his favour coincides in each stage with his absolute bargaining advantage. These results do not change substantially with respect to the case where p is assumed to proceed from a logistic distribution.

The results of this paper (particularly the convenience of bargaining small crumbs at each stage) contrast with those of Flamini (2002) whose description of each player's preferences regarding possible agendas suggested that a player will prefer to leave her opponent's most important issues at the end of the list so that matters she considers

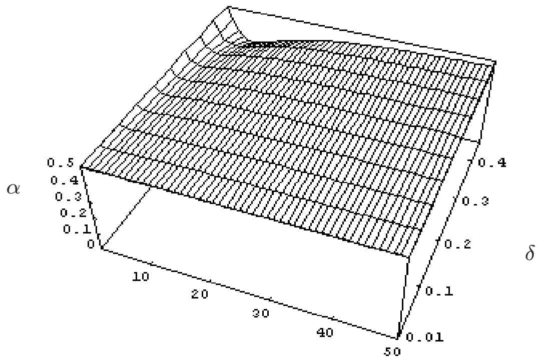

Figure 10: α_t paths when $\epsilon = 0.5$

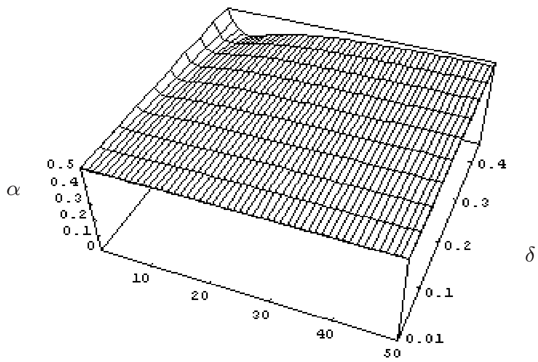

Figure 11: α_t paths when $\epsilon = 1$

of greatest interest to herself may be discussed first. These results do agree in that a disagreement can put the bargaining process in jeopardy in the case of difficult issues. Flamini has demonstrated that postponing negotiation is Pareto efficient in these cases.

A possible application of the model to the negotiations that took place between Egypt and Israel in 1978 is now given.

5 On the Bargaining Process between Egypt, Israel and the Arab League

The idea of unity became very important for the Arab world following United Nations recognition of the Jewish state in Palestine in 1948. In the words of Hourani, it implied "that the newly independent Arab states had enough in common, in shared culture and historical experience as well as shared interests, to make it possible for them to come into close union with each other, and such a union would not only give them greater collective power but would bring about that moral unity between people and government which would make government legitimate and stable" (1991, p. 401). The Panarabist project promoted creating a country for the Palestinian people and non-recognition of the Israeli state. The Arab League (established in 1945) became the vehicle for Arab states' international coordination.

When Nasser came to power and applied measures like nationalising the Suez Canal and introduced reforms aimed at promoting redistribution of income and public ownership of factors of production then Egypt became the natural leader of the Arab League. However, representing Arab interests became too heavy a burden for Egypt. Part of the cost lay in defeats at the hands of Israel in 1956 and 1967 and occupation of the Sinai Peninsula and the Gaza Strip after the second of these wars.

Nasser's successor Sadat (who suffered his own defeat at the hands of Israel in 1973) clearly saw that the end of the conflict was necessary for Egypt and that it could not be achieved by military means.

Sadat travelled to Jerusalem in November 1977 to begin bilateral talks with Israel. This meant the dissolution of the Arab block in practice, not just because Egypt abdicated its position as the head of Pan-Arabism but because negotiating with Israel meant recognising the existence of the Jewish state. Sadat's bargaining strategy was built on gradualism: negotiating small pieces, or crumbs, at each stage of the process. There had been no negotiation of any kind between the Arabs and the Israelis before Sadat's visit, since starting any peace process meant bargaining with the whole Arab world for Israel; this would have been tantamount to making important concessions regarding territories occupied during wars since 1948. Instead, negotiating bilaterally with Egypt implied that Israel would only have to return just a part of the occupied territories (in such context). This also opened up the possibility of starting future negotiations with Syria and Jordan. Breaking up the bargaining object was a successful strategy. Three years later Israel was at peace with Egypt and could begin negotiations with its other Arab neighbours.

The peace process between Israel and Egypt (based on Kissinger's concept of land for peace) was also carefully divided into stages. Israel promised (in the Camp David agreements) to evacuate the occupied territories in a first transitory nine-month stage after signing the peace treaty; this was to be followed by a permanent evacuation over a lapse of three more. Each step was subdivided into a series of small retreats whose dates had been previously accorded (the Egypt-Israel treaty, 1979).

The idea of negotiating in stages and bargaining a small fragment of the whole objective during each stage can ease conflicts resolution. The United States' role in the negotiation was also noteworthy. While the US position has always been biased towards Israel, it seemed to have been even-handed in this case, because U.S. interest in gaining Egypt as an ally made its stance relatively neutral.

6 Conclusions

It is recognised that offers, demands and agreements reached during later stages are influenced by initial negotiating conditions, previous agreements (process history) and chance in a gradual bargaining problem. Because of this, it is thus believed that the dynamics of bargaining can be considered as being a path-dependent process. The results of bargaining from this perspective, and the relative position of each player as the process advances indicate that negotiators jointly create bargaining conditions for future stages. Some aspects of this process recall aspects of what has been named the new "complexity economy" (Arthur (1999)).

Modelling the bargaining process as path-dependent might be questionable, given that it does not necessarily have economies of scale (i.e. achieving superior gains in early stages does not make superior gains more probable in later ones). However, assuming that the objective being bargained for is distributed in accordance with a Nash solution in each new stage involves significant, albeit implicit, normative criteria. Seeking resolution in each stage "compensates" the path generated by the process's long-term dynamics.

A natural question arising from these dynamics is what types of states emerge when the elements of a negotiation create themselves (more particularly, which bargaining solutions arise). This paper has demonstrated that egalitarian treatment will emerge in the long-term under certain conditions, proper to both Nash and Kalai-Smorodinski solutions. These results coincide with those of Binmore, Samuelson, and Young (2003) regarding evolutionary games, where under certain conditions players will coordinate with the Nash or Kalai-Smorodinski solutions.

Once bargainers' long-term behaviour has been established, an attempt must be made to learn which kinds of path lead to such states, given possible values for the model's parameters. It was found that small expansions in bargaining set as well as impartiality (absence of external influences on bargaining) would more rapidly lead to a long-term [effective earnings/potential earnings] ratio for both bargainers. It has been demonstrated that a large expansion of bargaining set, as well as an absence of external influence on bargaining factor substantially different to 0, can lead to bargaining becoming broken-off through reducing the [effective earnings/potential earnings] ratio which at least one of the parties would experience during the early stages of the process.

This is a very important topic for setting initial conditions for a bargaining process. For example, analyzing the factors that lead to a break-up of the bargaining process between the Colombian Government and FARC guerrillas during the Pastrana Administration, it is probable that the decision to bargain large pieces of the bargained object in each stage (i.e., making important concessions in the earlier stages of the process) contributed to the failure of the process. Some suggest that in the future a "step-by-step" strategy should be followed, as for example, specific engagements on regions, sub-units of armed groups, etc. These positions agree with the results presented in this article.

It must be emphasised that the model must be improved to make it more flexible and general for improving its applicability to current political situations. Further research is needed into generalising the model to cases where the bargainers have different attitudes towards risk and analysing situations where more than two players are bargaining for a determined objective.

References

Arthur, B. (1999): "Complexity and the Economy," *Science*, 284, 107–109.

Arthur, B., Y. Ermoliev, and Y. Kaniovski (1983): "The Generalized Urn Problem and Its Application," *Cybernetics*, 19, 61–71.

——— (1987): "Path Dependent Processes and the Emergence of Macrostructure," *European Journal of Operations Research*, 30, 394–303.

Axelrod, R. (1984): *La Evolución de la Cooperación*. Alianza Editorial.

Binmore, K., L. Samuelson, and P. Young (2003): "Equilibrium Selection in Bargaining Models," *Games and Economic Behavior*, 45, 296–328.

Flamini, F. (2002): "First things first? The Agenda Formation Problem for Multi-issue Committees," Department of Economics, University of Glasgow.

Hourani, A. (1991): *La Historia de los Arabes*, Bs. As. Javier Vergara Editores.

Kalai, E. (1977): "Proportional Solution to Bargaining Problems: Interpersonal Utility Comparisons," *Econometrica*, 45, 1023–1030.

Kalai, E., and M. Smorodinski (1975): "Other Solutions to Nash´s Barganing Problem," *Econometrica*, 43, 513–518.

La Casa Blanca (1979): "El Tratado Egipcio-Israelí," *Washington, Marzo.*

Mas-Colell, A., M. Whinston, and J. Green (1995): *Microeconomic Theory.* Oxford university Press.

Nash, J. F. (1950): "The Bargaining Problem," *Econometrica*, 18, 155–162.

O´Neill, B., D. Samet, Z. Wiener, and E. Winter (2003): "Bargaining with an Agenda," *Games and Economic Behavior, Forthcoming.*

Schelling, T. (1960): *The Strategy of Conflict*. Harvard University Press.

Thomson, W. (1994): "Cooperative Models of Bargaining," *Handbook of Game Theory*, II.

Wiener, Z., and E. Winter (1999): "Gradual Nash Bargaining," *Forthcoming.*