

Reinterpreting the meaning of breakdown*

Juan J. Vidal-Puga[†]
vidalpuga@uvigo.es

January 27, 2005

Abstract

A typical assumption of the standard alternating-offers model under risk is that the breakdown event means a complete and irrevocable halt in negotiations. We reinterpret the meaning of breakdown as the imposition to finish negotiations immediately. Specifically, after breakdown the last offer becomes definitive. A full characterization of the set of subgame perfect equilibrium payoffs is provided. We show that Rubinstein's allocation $(1/(1+\delta), \delta/(1+\delta))$ is attainable under non-stationary strategies. Moreover, the payoffs in delayed equilibria are potentially better for the proposer than those in which agreement is immediately reached.

Keywords: breakdown, bargaining.

JEL codes: C72, C78

*Latest version at: <http://webs.uvigo.es/vidalpuga/>. A previous version of this paper has benefited from helpful comments from Clara Ponsatí. Usual disclaimer applies. Financial support by the Spanish Ministerio de Ciencia y Tecnología and FEDER through grant BEC2002-04102-C02-01 and Xunta de Galicia through grant PGIDIT03PXIC30002PN is gratefully acknowledged.

[†]Departamento de Estatística e IO. Universidade de Vigo. 36200 Vigo (Pontevedra), Spain.

1 Introduction

Many economic situations can be modelled as two agents with independent interests who may benefit from cooperation. However, no agreement can be imposed without the approval of both individuals. Assuming the agents are rational, it seems natural to predict that cooperation will be carried out, and the question which arises is how the benefits from cooperation are bound to be distributed between them.

In his seminal paper, Rubinstein [4] addressed the above question. He studied a simple bargaining protocol of alternating-offers between two agents who want to share a pie of size 1. The agents alternatively propose a share of the pie until one of the proposals is accepted. No prior deadline is imposed on the number of offers that can be made. The driving force of the negotiation is that agents are impatient. The pie shrinks as time passes by. Binmore, Rubinstein and Wolinsky [1] reformulated the model in a way that the driving force is the possibility of breakdown after a rejection, in which case both agents get nothing. In both models, the only subgame perfect (SP) equilibrium has two main properties: First, it is stationary. Second, the advantage of being the first proposer disappears as the cost of delay (Rubinstein [4]) or the probability of breakdown (Binmore et al. [1]) vanishes¹.

In this paper, we reinterpret the meaning of breakdown. In the classical literature of bargaining, breakdown means a complete and irrevocable halt in negotiations, so that no agreement is possible afterwards. Typically, two reasons are given of why the agents reach such a radical dead-end. First, the agents can get fed up as negotiations get protracted. Second, an unpredictable event or external intervention can eliminate the benefit of cooperation.

In the first case, it is certainly plausible that an agent gets fed up and wants to finish negotiation immediately. In this case, however, we claim that the rational thing to do is to *accept the last offer on the table* (as long as it

¹The resemblance between the results in [4] and [1] has led to the interpretation of the cost of delay as an exogenous probability of breakdown.

is better than nothing). In the second case, the external intervention may become so imminent that agents are forced to finish negotiations immediately. Even though there is no room for making a new proposal, there is still possible to accept the last offer.

We model this situation as follows: Two agents bargain over the share of a pie of size 1 by alternating offers. Each time an offer is rejected, there is a (small) probability $1 - \rho$ that the last proposal becomes a ‘take-it-or-leave-it’ offer (breakdown). The responder can then reconsider to accept this last offer. There is a discounting factor $\delta < 1$, so that agents are impatient in the sense that their utility function for a piece of size u at time t is given by $\delta^t u$. If the last offer is rejected after breakdown, the utility is zero for both agents.

Many situations may be interpreted under this assumption. First, there can be an external authority that may force the agents to finish negotiations immediately (for example, a superior authority to which they are delegating). Second, there may exist a small probability that the negotiation proceeds are made public and one of the parties cannot change his offer in order to keep its credibility. Third, one of the parties makes a mistake with a small probability (following the ideas presented in [5]) and accepts an offer that could have been improved had he been more insistent.

Hence, after an offer is made, the proposer may become committed to it in such a way that further renegotiation is not possible. This leads to an endogenous payoff allocation after breakdown, as opposed to the standard alternating-offers model under risk².

In real-life negotiations, small delays may happen because the agents begin with proposals that are more favorable to them than the ones they actually consider as the most reasonable (*cheap talk*). The driving force is that there is a small probability that the other party considers the offer as

²A recent model with complete information that makes endogenous the level of surplus destruction after the deadline is [2]. In their model, however, the deadline is also endogenously imposed.

final.

Our model combines both the discounting factor of [4] and the risk of breakdown of [1]. The driving force that induces the agents to reach an agreement is their impatience, i.e. the presence of a discounting factor, and not the risk of breakdown (as opposed to [1], where breakdown has a different meaning).

Our results may be summarized as follows: If the agents are patient (i.e. they do not care when agreement is reached) then their optimal strategy will be to permanently ask for the whole pie. As the agents become more impatient, delay may occur, but an agent eventually presents a reasonable offer and agreement may be reached without breakdown. If agents are impatient enough, we recover Rubinstein's result of a unique stationary SP equilibrium with immediate agreement.

We maintain the original assumptions of Rubinstein's, i.e. unbounded rationality, complete information and no outside options, as well as the most tractable concept of subgame perfect equilibrium. Our model generalizes [4] in the sense that both models coincide when $\rho = 1$. However, this model could easily be adapted to generalize [1] by allowing both kinds of breakdown: the original of [1] with no possibility of agreement, and ours with a last 'take-it-or-leave-it' offer.

For arbitrary values of ρ and δ , we can distinguish three regions in the set $\{(\rho, \delta)\}_{\rho, \delta \in]0, 1[}$. A first region *IA* where "Immediate Agreement" is reached, i.e. the responder agrees on the offer at time 0 (like in [4]). A second region *DA* ("Delayed Agreement") where agreement is reached, but with a possible delay. Finally, a third region *PD* ("Perpetual Disagreement") where breakdown always occurs. The three regions are represented in Figure 1.

We are interested in studying the agreements that arise when ρ and δ are close to 1. A value δ close to 1 can be interpreted as either that agents are patient (see [3, p. 52]) or that the interval between offers and counteroffers is short. A value ρ close to 1 means that the probability of breakdown is small. Under these assumptions, region *PD* vanishes (Figure 2). Thus, we

can consider that the only significant regions are those in which the agents make reasonable offers.

The SP equilibrium payoffs for the proposer as a function of δ when $\rho = 0.5$ are depicted in Figure 3. The figures for other values of ρ are similar. In *IA* there exists a unique SP equilibrium payoff allocation $(\frac{1}{1+\delta}, \frac{\delta}{1+\delta})$ which coincides with Rubinstein's. Moreover, SP equilibria are stationary and efficient (Theorem 3.1). The dotted line in Figure 3 represents the security level payoff $\frac{1-\rho}{1-\rho^2\delta^2}$, which corresponds to the strategy of permanently asking for the whole pie, waiting for breakdown to occur (Remark 2.1).

In *PD*, there does not exist any SP equilibrium with immediate agreement, and thus the dotted line in Figure 3 coincides with the unique SP equilibrium payoff (Proposition 2.1).

In *DA* there does not exist any stationary SP equilibria, and there exists a continuum of SP equilibrium payoffs (Theorem 4.1). Remarkably, Rubinstein's allocation $(\frac{1}{1+\delta}, \frac{\delta}{1+\delta})$ is always an attainable outcome in *DA* under non-stationary strategies for any value of ρ (Corollary 4.1). Hence, even though Rubinstein's stationary equilibrium is not robust to reinterpretations of the meaning of breakdown, the allocation $(\frac{1}{1+\delta}, \frac{\delta}{1+\delta})$ is. An immediate consequence is that the classical assumption of the meaning of breakdown can be made without loss of generality, as long the final payoff allocation is concerned.

Another remarkable feature in *DA* is that, in general, the payoffs attainable when agreement is immediate (below M^a in Figure 3) are potentially worse for the proposer than those attainable when agreement is delayed (below M^r in Figure 3). The interpretation of this is as follows: The most unfavorable SP equilibria for the proposer are those in which her initial proposal is accepted. Hence, in general it is beneficial to begin with an unacceptable offer. Do bargainers behave like this real-life negotiations? It is obvious that they do. For example, in the Spanish house market, the consultants Re/Max and Look&Find have recently pointed out that sellers put a high price to their houses, planning to decrease it during the bargaining. Claiming high

prices gives somehow a favorable status quo in case negotiations should finish immediately.

It should be stressed that we characterize all the SP equilibria payoffs, and not only the stationary ones. There do not exist stationary SP equilibria in DA , but only non-stationary ones. The implication on the other direction is also true: if there exist non-stationary SP equilibria, then there do not exist stationary ones. In IA , the only stationary SP equilibrium is efficient (i.e. agreement is reached immediately). In PD , the only stationary SP equilibrium is inefficient (agreement is only reached after breakdown).

In Section 2 we formally present the model and characterize the payoffs in PD . In Section 3 we study the stationary SP equilibria and characterize the payoffs in IA . In Section 4 we study the non-stationary SP equilibria and characterize the payoffs in DA . In Appendix (Section 5) we prove the results.

2 The model

Two agents, A and B , bargain over the share of a pie of size 1 following a protocol of alternating offers: At time $t = 0, 2, 4, \dots$, agent A is the proposer. At time $t = 1, 3, 5, \dots$, agent B is the proposer. At time $t = 0$, A proposes a share $x = (x_A, x_B)$ of the pie, where $x_A + x_B \leq 1$. B should then accept or reject the offer. If he accepts, the game finishes with this share at time $t = 0$. If he rejects, then with probability ρ , the process is repeated at time $t = 1$ with B being the new proposer. With probability $1 - \rho$, the proposal made by A becomes final (breakdown), and B should decide if he accepts or rejects it. If he accepts, the game finishes with this share at time $t = 0$. If he rejects, the game finishes in disagreement and each agent receives zero (see Figure 4). Each time an offer is rejected, the value of the pie decreases by a discounting factor of $\delta \in [0, 1[$ for each agent. This means that a share of x_i of the cake at time t yields a utility of $\delta^t x_i$ for agent i .

The factor $1 - \rho$ can be considered as some kind of *external* factor which

represents the belief that the proposal on the table will become a ‘take-it-or-leave-it’ offer. On the other hand, the discounting factor δ is an *internal* factor that represents the impatience of the agents.

For simplicity, we assume that agents propose admissible shares of the pie (i.e. $x_i \in [0, 1]$ for $i \in \{A, B\}$ besides $x_A + x_B \leq 1$). We say that bargaining finishes in agreement when the agents follow strategies in which an offer is accepted (without breakdown). Notice that by breakdown we mean that the last offer is made final, and not necessarily that each agent receives zero.

The structure of the game is stationary, and so all the subgames that begin when agent i makes an offer are equivalent. Let U be the set of possible SP equilibrium payoffs for the proposer in these subgames (by symmetry, it is the same for both agents). If U is nonempty, we denote the supremum of U as M , and the infimum of U as m .

There are two kinds of SP equilibria: SP equilibria in which the first offer of the proposer is accepted and SP equilibria in which the first offer of the proposer is rejected. Let U^a and U^r be their respective sets of possible payoffs for the proposer.

Remark 2.1 *Notice that, given this model, A can assure herself a positive expected final payoff by always offering $(1 - \varepsilon, \varepsilon)$. If her offer becomes final (i.e. breakdown occurs), the perfectness of the equilibrium will imply that B accepts. Since this is true for any $\varepsilon > 0$, we will assume without loss of generality that an agent always accepts a payoff of at least 0 when the proposal is final. Hence, the SP equilibrium payoff³ for the proposer is at least*

$$\sum_{t=0}^{\infty} (\rho\delta)^{2t} (1 - \rho) = \frac{1 - \rho}{1 - \rho^2\delta^2}.$$

This is the final payoff for the proposer when both agents always ask for the whole pie. It represents the security level payoff for the proposer. Hence, if $U \neq \emptyset$, we have

$$\frac{1 - \rho}{1 - \rho^2\delta^2} \leq m. \tag{1}$$

³From now on, we omit the term ‘expected’ when referring to final payoffs.

Intuitively, it seems clear that if the agents are sufficiently patient (δ large with respect to ρ) then they will keep asking for the whole pie until a proposal is made final (breakdown). We will see that the set of pairs (ρ, δ) where this always happens is given by

$$PD := \{(\rho, \delta) : \delta(1 - \rho) > \rho(1 - \rho\delta^2)\}$$

where PD stands for “*Perpetual Disagreement*”.

Proposition 2.1 *If $(\rho, \delta) \in PD$, then there exists a unique SP equilibrium in which the proposer always claims the whole pie, and the responder rejects when this proposal is not final. Moreover,*

$$U = U^r = \left\{ \frac{1 - \rho}{1 - \rho^2\delta^2} \right\}.$$

Proof. Assume we are in a SP equilibrium and the proposer, say A , offers (x_A, x_B) and B accepts. By rejecting, B can ensure himself a final payoff of at least $\rho\delta m + (1 - \rho)x_B$. Thus, $\rho\delta m + (1 - \rho)x_B \leq x_B$, i.e. $\delta m \leq x_B$. Since $x_A + x_B \leq 1$, we have $x_A \leq 1 - \delta m$. On the other hand, A can assure herself a payoff of at least m . Thus, $m \leq x_A$. Under (1),

$$\begin{aligned} m &\leq x_A \leq 1 - \delta m \\ \frac{1 - \rho}{1 - \rho^2\delta^2} &\leq x_A \leq 1 - \delta \frac{1 - \rho}{1 - \rho^2\delta^2} \end{aligned}$$

which implies

$$1 - \rho \leq 1 - \rho^2\delta^2 - \delta(1 - \rho)$$

thus

$$\delta(1 - \rho) \leq \rho(1 - \rho\delta^2)$$

and hence $(\rho, \delta) \notin PD$. This means that in PD there cannot be an accepted proposal in SP equilibrium. From this, it is clear that the only optimal strategy is to ask for the whole pie. In particular, consider the following strategy for A : if she is the proposer, offer $(1, 0)$; if she is the responder and the offer is (x_A, x_B) , accept iff

$$x_A > \frac{\delta(1 - \rho)}{1 - \rho^2\delta^2}.$$

In case of breakdown, accept any offer. B follows an analogous strategy.

These strategies constitute a SP equilibrium when $(\rho, \delta) \in PD$.

If A deviates when she is the proposer, then she will receive at most

$$1 - \frac{\delta(1-\rho)}{1-\rho^2\delta^2}$$

which is strictly less than $\frac{1-\rho}{1-\rho^2\delta^2}$ in PD .

If A deviates when she is the responder and accepts $x_A \leq \frac{\delta(1-\rho)}{1-\rho^2\delta^2}$, she will receive no more than

$$\rho\delta\frac{1-\rho}{1-\rho^2\delta^2} + (1-\rho)x_A \tag{2}$$

which is the payoff she would receive by rejecting. Finally, if A deviates when she is the responder and rejects $x_A > \frac{\delta(1-\rho)}{1-\rho^2\delta^2}$, she will get (2), which is strictly less than x_A . ■

The set PD is stretched in Figure 1. Notice that, even though the SP equilibria given by Proposition 2.1 are "bad", they only survive when δ approaches 1 more quickly than ρ (Figure 2).

3 Stationary SP equilibria

We study the SP equilibria which satisfy the following properties:

Efficiency Whenever a proposer makes an offer, this proposal is accepted by the responder. Moreover, the proposals always satisfy $x_A + x_B = 1$.

Stationarity The proposer always makes the same offer.

Given stationarity, we denote the offer made by agent i in SP equilibrium as x_i^* . Given efficiency, when A proposes $(x_A^*, 1 - x_A^*)$, B accepts. By perfectness, the offer made by A should leave B indifferent to accepting or rejecting. If B rejects and there is no breakdown (probability ρ), B will propose $(1 - x_B^*, x_B^*)$ and this proposal is due to be accepted by A (by efficiency). Thus,

$$1 - x_A^* = \rho\delta x_B^* + (1 - \rho)(1 - x_A^*)$$

analogously

$$1 - x_B^* = \rho \delta x_A^* + (1 - \rho)(1 - x_B^*)$$

which yields as unique solution

$$x_A^* = x_B^* = \frac{1}{1 + \delta}. \quad (3)$$

Thus, any SP equilibrium payoff satisfying efficiency and stationarity should be characterized by this offer. Notice that this payoff coincides with Rubinstein's [4]. Such a strategy profile is a SP equilibrium in the following region:

$$IA := \{(\rho, \delta) : \delta(1 - \rho) \leq \rho(1 - \delta^2)\}$$

where IA stands for “*Immediate Agreement*”.

Theorem 3.1 *a) There exists an efficient and stationary SP equilibrium if and only if $\delta(1 - \rho) \leq \rho(1 - \delta^2)$.*

b) if $\rho(1 - \delta^2) > \delta(1 - \rho)$, then the above SP equilibrium is unique; and

c) if $\delta(1 - \rho) = \rho(1 - \delta^2)$, then there exists one inefficient SP equilibrium. In this SP equilibrium, A asks for the whole pie in the first round, B rejects and proposes $(\frac{\delta}{1+\delta}, \frac{1}{1+\delta})$ in the second round, and A accepts.

Moreover, $\delta(1 - \rho) \leq \rho(1 - \delta^2)$ implies

$$U = U^a = \left\{ \frac{1}{1 + \delta} \right\}.$$

Proof. See Appendix. ■

We represent the set IA in Figure 1. The set IA includes the case $\rho = 1$, where this model coincides with Rubinstein's [4].

4 Non-stationary SP equilibria

Since we have characterized the SP equilibria in IA and PD , we focus our attention on the remaining values of (ρ, δ) . We define

$$DA := \{(\rho, \delta) : (\rho, \delta) \notin IA \cup PD\}$$

where DA stands for “*Delayed Agreement*”.

Let

$$\tilde{m} := \max \left\{ \frac{(1-\delta)(1+\rho\delta)}{1-\rho\delta^2}, \frac{1-\rho}{1-\rho^2\delta^2} \right\};$$

$$\widetilde{M}^r := \rho\delta(1-\tilde{m}) + 1 - \rho;$$

and

$$\widetilde{M}^a := 1 - \delta\tilde{m}.$$

Theorem 4.1 *If $(\rho, \delta) \in DA$, then there exist non-stationary SP equilibria. Furthermore, the set of SP equilibrium payoffs for the proposer is given by*

$$U = U^r = [\tilde{m}, \widetilde{M}^r]$$

and

$$U^a = [\tilde{m}, \widetilde{M}^a].$$

Proof. See Appendix. ■

These intervals are represented in Figure 3 for $\rho = 0.5$. Notice that when $(\rho, \delta) \in DA$, we have $\widetilde{M}^a < \widetilde{M}^r$.

The strategy profiles which yield these intervals are described in Table I. We consider the language of automata (see [3, p. 39-40]). Acceptance applies when the offer is not final. If the offer is final, the responder always accepts (as explained in Remark 2.1). The strategies are the most favorable to A in *Favor A*; the strategies are the most favorable to B in *Favor B*; the initial offer is accepted in *Agreement* when $u < \widetilde{M}^a$; and the initial offer is rejected in *Delay*.

TABLE I: SP equilibrium strategies in DA

	<i>Favor A</i>	<i>Favor B</i>	<i>Agreement</i>	<i>Delay</i>
<i>A offers</i>	$(1, 0)$	$(\tilde{m}, 1 - \tilde{m})$	$(u, 1 - u)$	$\left(\frac{u - \rho\delta(1-\tilde{m})}{1-\rho}, 0\right)$
<i>A accepts</i>	$x_A \geq 1 - \tilde{m}$	$x_A > \delta\tilde{m}$	—	—
<i>B offers</i>	$(1 - \tilde{m}, \tilde{m})$	$(0, 1)$	—	—
<i>B accepts</i>	$x_B > \delta\tilde{m}$	$x_B \geq 1 - \tilde{m}$	—	—
<i>Transitions</i>	if A deviates, go to <i>Favor B</i>	if B deviates, go to <i>Favor A</i>	if A deviates, Otherwise,	go to <i>Favor B</i> go to <i>Favor A</i> .

The following corollary is an important consequence of Theorem 4.1.

Corollary 4.1 *Given $(\rho, \delta) \in DA$,*

$$\frac{1}{1+\delta} \in U^a.$$

Proof. From Theorem 4.1, it is enough to prove that $\tilde{m} \leq \frac{1}{1+\delta} \leq \widetilde{M}^a$.

Assume first $\tilde{m} = \frac{(1-\delta)(1+\rho\delta)}{1-\rho\delta^2}$. We have to prove

$$\frac{(1-\delta)(1+\rho\delta)}{1-\rho\delta^2} \stackrel{?}{\leq} \frac{1}{1+\delta} \stackrel{?}{\leq} 1 - \delta \frac{(1-\delta)(1+\rho\delta)}{1-\rho\delta^2}$$

The first inequality is equivalent to

$$\rho\delta(1-\delta^2) \stackrel{?}{\leq} \delta^2(1-\rho)$$

which holds because $(\delta, \rho) \notin IA$.

The second inequality is equivalent to

$$\rho\delta^2(1-\delta^2) \stackrel{?}{\leq} \delta^3(1-\rho)$$

which holds because $(\delta, \rho) \notin IA$.

Assume now $\tilde{m} = \frac{1-\rho}{1-\rho^2\delta^2}$. We have to prove

$$\frac{1-\rho}{1-\rho^2\delta^2} \stackrel{?}{\leq} \frac{1}{1+\delta} \stackrel{?}{\leq} 1 - \delta \frac{1-\rho}{1-\rho^2\delta^2}.$$

The first inequality is equivalent to

$$\delta(1-\rho) \stackrel{?}{\leq} \rho(1-\rho\delta^2)$$

which holds because $(\delta, \rho) \notin PD$.

The second inequality is equivalent to

$$\delta^2(1-\rho) \stackrel{?}{\leq} \rho\delta(1-\rho\delta^2)$$

which holds because $(\delta, \rho) \notin PD$. ■

5 Appendix

Lemma 5.1 $1 - \delta M \leq m$.

Proof. Clearly, B would not reject any offer (x_A, x_B) with x_B satisfying

$$x_B > \rho\delta M + (1 - \rho)x_B$$

i.e.

$$x_B > \delta M$$

because $\rho\delta M + (1 - \rho)x_B$ would be his maximum payoff after rejection. Thus, there does not exist $u \in U$ such that

$$1 - \delta M > u.$$

Hence,

$$1 - \delta M \leq m.$$

■

5.1 Proof of Theorem 3.1

a) We have previously proved that any stationary and efficient SP equilibrium should be characterized by (3). We now check that the strategy profile "offer $\frac{1}{1+\delta}$ and accept at least $\frac{\delta}{1+\delta}$ " constitutes a SP equilibrium in IA . Assume the proposer A deviates and makes an unacceptable proposal (x_A, x_B) . Thus, B rejects and A 's final payoff is

$$\rho\delta \frac{\delta}{1+\delta} + (1 - \rho)x_A$$

which should be no more than $\frac{1}{1+\delta}$, i.e.

$$\rho\delta \frac{\delta}{1+\delta} + (1 - \rho)x_A \stackrel{?}{\leq} \frac{1}{1+\delta}.$$

Since $x_A \leq 1$, it is enough to prove that

$$\rho\delta \frac{\delta}{1+\delta} + 1 - \rho \stackrel{?}{\leq} \frac{1}{1+\delta}$$

which holds because $(\rho, \delta) \in IA$.

Assume the responder B receives an offer (x_A, x_B) . By rejecting, he gets in expected terms

$$\rho\delta\frac{1}{1+\delta} + (1-\rho)x_B.$$

Thus, it is optimal to accept iff $\rho\delta\frac{1}{1+\delta} + (1-\rho)x_B \leq x_B$, i.e. $\frac{\delta}{1+\delta} \leq x_B$. Hence the strategy for B is optimal. This proves that there exists a stationary SP equilibrium when $(\rho, \delta) \in IA$.

Assume now $(\rho, \delta) \notin IA$. Assume the proposer A deviates and proposes $(1, 0)$. Her final payoff will be

$$\rho\delta\frac{\delta}{1+\delta} + 1 - \rho.$$

Since $(\rho, \delta) \notin IA$, the above payoff is strictly more than $\frac{1}{1+\delta}$. Hence, A improves. This shows that there does not exist efficient and stationary SP equilibria when $(\rho, \delta) \notin IA$.

We now prove that $U = \{\frac{1}{1+\delta}\}$. Since the above strategies constitute a SP equilibrium in IA , we know that $U \neq \emptyset$ in IA .

We will prove that $M = m = \frac{1}{1+\delta}$. We proceed by a series of Claims.

Claim 1 $M \leq 1 - \delta m$.

Proof. Let (x_A, x_B) be A 's proposal. In SP equilibrium, B would reject any offer such that

$$\rho\delta m + (1-\rho)x_B > x_B$$

i.e. $\delta m > x_B$. Since $x_A + x_B \leq 1$, this means that B would reject any offer such that

$$x_A > 1 - \delta m.$$

There exist two types of SP equilibria: (I) SP equilibria in which B accepts A 's initial offer, and (II) SP equilibria in which B rejects A 's initial offer.

In SP equilibria of type (I), it is clear that $x_A \leq 1 - \delta m$.

In SP equilibria of type (II), let u be A 's final payoff and let v be B 's final payoff. Assuming B becomes the proposer (probability ρ), B gets $u' \in U$ and A gets v' . Since $u' + v' \leq 1$, we deduce that $v' \leq 1 - u' \leq 1 - m$. Moreover, $x_A \leq 1$. Thus,

$$u = \rho\delta v' + (1 - \rho)x_A \leq \rho\delta(1 - m) + 1 - \rho.$$

In summary, $u \leq \max\{1 - \delta m, \rho\delta(1 - m) + 1 - \rho\}$. We will prove that this maximum is $1 - \delta m$ and thus $M \leq 1 - \delta m$. We want to prove that

$$\rho\delta(1 - m) + 1 - \rho \stackrel{?}{\leq} 1 - \delta m$$

i.e.

$$\delta(1 - \rho)m \stackrel{?}{\leq} \rho(1 - \delta).$$

Since $m \leq \frac{1}{1+\delta}$ in IA , it is enough to prove that

$$\delta \frac{1 - \rho}{1 + \delta} \stackrel{?}{\leq} \rho(1 - \delta)$$

which holds because $(\rho, \delta) \in IA$. ■

Claim 2 $M \leq \frac{1}{1+\delta}$.

Proof. Under Claim 1 and Lemma 5.1,

$$M \leq 1 - \delta m \leq 1 - \delta(1 - \delta M)$$

and thus

$$(1 - \delta^2)M \leq 1 - \delta$$

hence

$$M \leq \frac{1}{1 + \delta}.$$

■

Claim 3 $1 - \delta u \in U$ for all $u \in U$.

Proof. Let $u \in U$, and let v be A 's final payoff in a subgame in which B is the proposer and he gets u . Hence, $v \leq 1 - u$. We consider the following strategies: A proposes $(1 - \delta u, \delta u)$ and B accepts an offer (x_A, x_B) iff $\delta u \leq x_B$. After this, agents play the strategies that yield u to B and v to A . We will check that these strategies constitute a SP equilibrium. It is clear that these strategies are SP equilibrium after the first offer has been rejected.

If B rejects a proposal (x_A, x_B) with $\delta u \leq x_B$, he gets

$$\rho \delta u + (1 - \rho) x_B \leq \rho x_B + (1 - \rho) x_B = x_B$$

and he does not improve. Analogously, B does not improve by accepting a proposal (x_A, x_B) with $\delta u > x_B$.

If A deviates and proposes $x_A > 1 - \delta u$, he gets $\rho \delta v + (1 - \rho) x_A$. We need to prove that A does not improve, i.e.

$$\rho \delta v + (1 - \rho) x_A \stackrel{?}{\leq} 1 - \delta u.$$

Since $v \leq 1 - u$ and $x_A \leq 1$, it is enough to prove that

$$\rho \delta (1 - u) + 1 - \rho \stackrel{?}{\leq} 1 - \delta u$$

i.e.

$$u \leq \frac{\rho(1 - \delta)}{\delta(1 - \rho)}.$$

Since $u \leq M$, under Claim 2 it is enough to prove that

$$\frac{1}{1 + \delta} \stackrel{?}{\leq} \frac{\rho(1 - \delta)}{\delta(1 - \rho)}$$

which holds because $(\rho, \delta) \in IA$. ■

Claim 4 $m \leq 1 - \delta M$ and $1 - \delta m \leq M$.

Proof. Assume $m > 1 - \delta M$. Thus, there exists $u \in U$ such that $m > 1 - \delta u$. But this is not possible under Claim 3.

Assume $1 - \delta m > M$. Thus, there exists $u \in U$ such that $1 - \delta u > M$. But this is not possible under Claim 3. ■

From Lemma 5.1, Claim 1 and Claim 4, we deduce that

$$\begin{aligned} m &= 1 - \delta M \\ M &= 1 - \delta m \end{aligned}$$

which yield a unique solution

$$M = m = \frac{1}{1 + \delta}.$$

This proves that $U = \left\{ \frac{1}{1 + \delta} \right\}$.

b) Assume $\rho(1 - \delta^2) > \delta(1 - \rho)$. It is enough to prove that all SP equilibria are efficient. Assume, on the contrary, that there exists a SP equilibrium where A proposes (x_A, x_B) and B rejects. Let v be A 's payoff in the subgame which begins when B is the proposer. Thus, A 's final payoff is

$$\rho\delta v + (1 - \rho)x_A \leq \rho\delta \frac{\delta}{1 + \delta} + 1 - \rho.$$

Since we know that A 's final payoff is $\frac{1}{1 + \delta}$, we deduce that

$$\frac{1}{1 + \delta} \leq \rho\delta \frac{\delta}{1 + \delta} + 1 - \rho$$

which is not possible when $\rho(1 - \delta^2) > \delta(1 - \rho)$.

c) Assume $\delta(1 - \rho) = \rho(1 - \delta^2)$. Assume A proposes (x_A, x_B) and B rejects. Let v be A 's payoff in the subgame which begins when B is the proposer. By an analogous argument as in the proof of part b), we prove that

$$\rho\delta v + (1 - \rho)x_A = \rho\delta \frac{\delta}{1 + \delta} + 1 - \rho$$

and thus $v = \frac{\delta}{1 + \delta}$ and $x_A = 1$. This implies that A asks for the whole pie in the first round (because $x_A = 1$) and that the second round is efficient (because $v = \frac{\delta}{1 + \delta}$).

5.2 Proof of Theorem 4.1

Let $u \in [\tilde{m}, \tilde{M}^r]$. We describe the strategies that yield u as final payoff for the proposer in Table I. Acceptance applies when the offer is not final. If the offer is final, the responder always accepts.

We will prove that the strategies are the most favorable to A (resp. B) in *Favor A* (resp. *Favor B*); the initial offer is accepted in *Agreement* when $u < \tilde{M}^a$; and the initial offer is rejected in *Delay*.

In order to check that these strategies are well-defined, we prove that $\frac{u - \rho\delta(1 - \tilde{m})}{1 - \rho} \leq 1$, or equivalently,

$$\tilde{m} \stackrel{?}{\leq} \frac{1 - \rho + \rho\delta - u}{\rho\delta}.$$

Since $u \leq \tilde{M}^r$, it is enough to prove that

$$\tilde{m} \stackrel{?}{\leq} \frac{1 - \rho + \rho\delta - \tilde{M}^r}{\rho\delta}$$

which holds because $\tilde{M}^r = \rho\delta(1 - \tilde{m}) + 1 - \rho$.

We now check that these strategies are a SP equilibrium.

Favor A/B: Assume the state is *Favor A*. The proof for *Favor B* is analogous (note that both states are symmetric). If the proposer is B , then his offer is accepted and the final payoff is $1 - \tilde{m}$ for A and \tilde{m} for B . If the proposer is A , then her offer is rejected and the final payoffs are $\rho\delta(1 - \tilde{m}) + 1 - \rho = \tilde{M}^r$ for A and $\rho\delta\tilde{m}$ for B .

We prove that these strategies are optimal.

A's strategy is optimal when she is the responder. If A rejects (x_A, x_B) with $1 - \tilde{m} \leq x_A$, we will go to *Favor B* and B will only accept $x_B \geq 1 - \tilde{m}$ and propose $(0, 1)$. Thus, A 's final payoff will be at most $\rho\delta\tilde{m} + (1 - \rho)x_A$ (if A offers $x_B \geq 1 - \tilde{m}$ afterwards) or $\frac{\rho\delta(1 - \rho)}{1 - \rho^2\delta^2} + (1 - \rho)x_A$ (if A permanently makes unacceptable offers). We have to prove that x_A is not less than these payoffs.

1. First,

$$\rho\delta\tilde{m} + (1 - \rho)x_A \stackrel{?}{\leq} x_A$$

is equivalent to

$$\tilde{m} \stackrel{?}{\leq} \frac{x_A}{\delta}$$

since $1 - \tilde{m} \leq x_A$, it is enough to prove

$$\tilde{m} \stackrel{?}{\leq} \frac{1 - \tilde{m}}{\delta}$$

i.e.

$$\tilde{m} \stackrel{?}{\leq} \frac{1}{1 + \delta}.$$

(a) Assume first $\tilde{m} = \frac{(1-\delta)(1+\rho\delta)}{1-\rho\delta^2}$. We have to prove

$$\frac{(1 - \delta)(1 + \rho\delta)}{1 - \rho\delta^2} \stackrel{?}{\leq} \frac{1}{1 + \delta}$$

which holds because $(\rho, \delta) \notin IA$.

(b) Assume now $\tilde{m} = \frac{1-\rho}{1-\rho^2\delta^2}$. We have to prove

$$\frac{1 - \rho}{1 - \rho^2\delta^2} \stackrel{?}{\leq} \frac{1}{1 + \delta}$$

which holds because $(\rho, \delta) \notin PD$.

2. Second,

$$\frac{\rho\delta(1 - \rho)}{1 - \rho^2\delta^2} + (1 - \rho)x_A \stackrel{?}{\leq} x_A$$

is equivalent to

$$\frac{\delta(1 - \rho)}{1 - \rho^2\delta^2} \stackrel{?}{\leq} x_A$$

since $1 - \tilde{m} \leq x_A$, it is enough to prove

$$\frac{\delta(1 - \rho)}{1 - \rho^2\delta^2} \stackrel{?}{\leq} 1 - \tilde{m}.$$

(a) Assume first $\tilde{m} = \frac{(1-\delta)(1+\rho\delta)}{1-\rho\delta^2}$. We have to prove

$$\frac{\delta(1-\rho)}{1-\rho^2\delta^2} \stackrel{?}{\leq} 1 - \frac{(1-\delta)(1+\rho\delta)}{1-\rho\delta^2}$$

which holds trivially.

(b) Assume now $\tilde{m} = \frac{1-\rho}{1-\rho^2\delta^2}$. We have to prove

$$\frac{\delta(1-\rho)}{1-\rho^2\delta^2} \stackrel{?}{\leq} 1 - \frac{1-\rho}{1-\rho^2\delta^2}$$

which holds because $(\rho, \delta) \notin PD$.

B's strategy is optimal when he is the proposer. If B offers (x_A, x_B) with $x_A > 1 - \tilde{m}$, then his offer is accepted, but his final payoff decreases. If B proposes (x_A, x_B) with $x_A < 1 - \tilde{m}$, then his offer is rejected and his final payoff is at most $\rho^2\delta^2\tilde{m} + 1 - \rho$. This deviation is not profitable when

$$\rho^2\delta^2\tilde{m} + 1 - \rho \stackrel{?}{\leq} \tilde{m}$$

i.e.

$$\frac{1-\rho}{1-\rho^2\delta^2} \stackrel{?}{\leq} \tilde{m}$$

which holds by definition.

B's strategy is optimal when he is the responder. Assume B is the responder and the offer is (x_A, x_B) . Thus, B 's final payoff after rejection is $\rho\delta\tilde{m} + (1-\rho)x_B$. Perfectness implies that B must reject if $x_B < \rho\delta\tilde{m} + (1-\rho)x_B$ and accept if $x_B > \rho\delta\tilde{m} + (1-\rho)x_B$. However, it is equivalent to say that B must reject when $x_B < \delta\tilde{m}$ and accept when $x_B > \delta\tilde{m}$. This completes the proof that B 's strategy is optimal (when the state is *Favor A*).

A's strategy is optimal when she is the proposer. Assume A is the proposer. She may deviate in two possible ways: 1. by doing an acceptable offer (by the transition rule this means $x_B \geq 1 - \tilde{m}$ and thus $x_A \leq \tilde{m}$) or 2. by doing an unacceptable offer ($x_B < 1 - \tilde{m}$).

1. The deviation is not profitable when

$$x_A \stackrel{?}{\leq} \widetilde{M}^r$$

and since $x_A \leq \widetilde{m}$, it is enough to prove that

$$\widetilde{m} \stackrel{?}{\leq} \widetilde{M}^r,$$

i.e.

$$\widetilde{m} \stackrel{?}{\leq} \rho\delta(1 - \widetilde{m}) + 1 - \rho$$

equivalently,

$$\widetilde{m} \stackrel{?}{\leq} 1 - \frac{\rho}{1 + \rho\delta}.$$

(a) Assume first $\widetilde{m} = \frac{(1-\delta)(1+\rho\delta)}{1-\rho\delta^2}$. We have to prove

$$\frac{(1-\delta)(1+\rho\delta)}{1-\rho\delta^2} \stackrel{?}{\leq} 1 - \frac{\rho}{1+\rho\delta}$$

which holds because $(\rho, \delta) \notin IA$.

(b) Assume now $\widetilde{m} = \frac{1-\rho}{1-\rho^2\delta^2}$. We have to prove

$$\frac{1-\rho}{1-\rho^2\delta^2} \stackrel{?}{\leq} 1 - \frac{\rho}{1+\rho\delta}$$

which holds because $\delta < 1$.

2. The deviation is not profitable when

$$\rho^2\delta^2\widetilde{m} + (1-\rho)x_A \stackrel{?}{\leq} \widetilde{M}^r.$$

Since $x_A \leq 1$, it is enough to prove that

$$\rho^2\delta^2\widetilde{m} + 1 - \rho \stackrel{?}{\leq} \widetilde{M}^r$$

i.e.

$$\rho^2\delta^2\widetilde{m} + 1 - \rho \stackrel{?}{\leq} \rho\delta(1 - \widetilde{m}) + 1 - \rho$$

equivalently,

$$\widetilde{m} \stackrel{?}{\leq} \frac{1}{1 + \rho\delta}.$$

(a) Assume first $\tilde{m} = \frac{(1-\delta)(1+\rho\delta)}{1-\rho\delta^2}$. We have to prove

$$\frac{(1-\delta)(1+\rho\delta)}{1-\rho\delta^2} \stackrel{?}{\leq} \frac{1}{1+\rho\delta}$$

i.e.

$$1 - 2(1-\rho) \stackrel{?}{\leq} \rho^2\delta^2 + \rho\delta(1-\rho)$$

since $(1-\rho)^2 > 0$, it is enough to prove that

$$1 - 2(1-\rho) + (1-\rho)^2 \stackrel{?}{\leq} \rho^2\delta^2 + \rho\delta(1-\rho)$$

i.e.

$$\rho^2 \stackrel{?}{\leq} \rho^2\delta^2 + \rho\delta(1-\rho)$$

which holds because $(\rho, \delta) \notin IA$.

(b) Assume now $\tilde{m} = \frac{1-\rho}{1-\rho^2\delta^2}$. We have to prove

$$\frac{1-\rho}{1-\rho^2\delta^2} \stackrel{?}{\leq} \frac{1}{1+\rho\delta}$$

which holds trivially.

Agreement: Assume now we are in *Agreement*. Following these strategies, A proposes $(u, 1-u)$ and B accepts if $1-u > \delta\tilde{m}$. Thus, the proposal is accepted when $u < \widetilde{M}^a$.

1. If A changes her strategy and proposes an unacceptable offer (i.e. (x_A, x_B) with $x_B < 1-\tilde{m}$), she gets no more than $\rho^2\delta^2\tilde{m} + (1-\rho)x_A$. Thus, we need to prove that

$$\rho^2\delta^2\tilde{m} + (1-\rho)x_A \stackrel{?}{\leq} u.$$

Since $x_A \leq 1$ and $\tilde{m} \leq u$, it is enough to prove that

$$\rho^2\delta^2\tilde{m} + 1 - \rho \stackrel{?}{\leq} \tilde{m}$$

i.e.

$$\frac{1-\rho}{1-\rho^2\delta^2} \stackrel{?}{\leq} \tilde{m}$$

which holds by definition.

2. If A changes her strategy and proposes an acceptable offer (i.e. (x_A, x_B) with $x_B \geq 1 - \tilde{m}$), she gets no more than \tilde{m} . Since $u \geq \tilde{m}$, this deviation is not profitable.

Hence, $[\tilde{m}, \widetilde{M}^a] \subset U^a$. By a slight change of B 's strategy in *Favor A* when he is the responder, it is analogous to check that $\widetilde{M}^a \in U^a$. Hence, $[\tilde{m}, \widetilde{M}^a] \subset U^a$.

Delay: Assume now we are in *Delay*. Following these strategies, A proposes $(\frac{u - \rho\delta(1 - \tilde{m})}{1 - \rho}, 0)$, and B rejects the offer. The final payoff for A is

$$\rho\delta(1 - \tilde{m}) + (1 - \rho) \frac{u - \rho\delta(1 - \tilde{m})}{1 - \rho} = u.$$

By an analogous reasoning as in *Agreement*, we deduce that no deviation is profitable as long as $u \in [\tilde{m}, \widetilde{M}^r]$. Hence, $[\tilde{m}, \widetilde{M}^r] \subset U^r$.

We have then

$$[\tilde{m}, \widetilde{M}^a] \subset U^a \tag{4}$$

and

$$[\tilde{m}, \widetilde{M}^r] \subset U^r \tag{5}$$

from where we conclude that $[\tilde{m}, \widetilde{M}^r] \subset U$.

We now prove that $u \in [\tilde{m}, \widetilde{M}^r]$ for all $u \in U$.

Let M^a be the supremum of the SP equilibrium payoffs in which the proposer's initial offer is accepted. Let M^r be the supremum of the SP equilibrium payoffs in which the proposer's initial offer is rejected. Clearly, $M = \max\{M^a, M^r\}$. Notice that both M^a and M^r are well-defined because U^a and U^r are nonempty.

Claim 5 $M^a \leq \min\left\{1 - \delta + \delta^2 M, 1 - \frac{\delta(1 - \rho)}{1 - \rho^2 \delta^2}\right\}$.

Proof. Assume we are in a SP equilibrium in which the first offer of the proposer, say A , is accepted. Thus, A proposes (x_A, x_B) with $x_A = u \in U^a$ and B agrees. If B disagreed, then he would get at least

$$\rho\delta m + (1 - \rho)x_B.$$

Thus, the fact that B agrees with x_B implies that $\rho\delta m + (1 - \rho)x_B \leq x_B$. This means that $\delta m \leq x_B$. Since $u \leq 1 - x_B$, we deduce

$$M^a \leq 1 - \delta m. \quad (6)$$

Under (6) and Lemma 5.1,

$$M^a \leq 1 - \delta(1 - \delta M) = 1 - \delta + \delta^2 M.$$

Under (6) and (1),

$$M^a \leq 1 - \frac{\delta(1 - \rho)}{1 - \rho^2\delta^2}.$$

■

Claim 6 $M^r \leq \min \left\{ \rho\delta^2 M + 1 - \rho, \rho^2\delta \frac{1 - \rho\delta^2}{1 - \rho^2\delta^2} + 1 - \rho \right\}.$

Proof. Assume we are in a SP equilibrium in which the offer of the proposer, say A , is rejected. Thus, A proposes (x_A, x_B) and B disagrees. Let $u \in U^r$ be A 's final payoff. Thus, $u = \rho\delta v + (1 - \rho)x_A$ with $v \leq 1 - u'$ such that $u' \in U$. Since $1 - u' \leq 1 - m$ and $x_A \leq 1$, we have

$$u \leq \rho\delta(1 - m) + 1 - \rho$$

and thus

$$M^r \leq \rho\delta(1 - m) + 1 - \rho. \quad (7)$$

Under (7) and Lemma 5.1,

$$M^r \leq \rho\delta^2 M + 1 - \rho.$$

Under (7) and (1),

$$M^r \leq \rho\delta \left(1 - \frac{1 - \rho}{1 - \rho^2\delta^2} \right) + 1 - \rho = \rho^2\delta \frac{1 - \rho\delta^2}{1 - \rho^2\delta^2} + 1 - \rho.$$

■

We use Claim 5 and Claim 6 in order to prove $M = \widetilde{M}^r$. We have two cases:

1. $M = M^a$.

(a) Assume first $\widetilde{m} = \frac{(1-\delta)(1+\rho\delta)}{1-\rho\delta^2}$. Under Claim 5,

$$M \leq 1 - \delta + \delta^2 M$$

equivalently,

$$M \leq \frac{1}{1 + \delta}.$$

Thus, it is enough to prove

$$\frac{1}{1 + \delta} \stackrel{?}{\leq} \widetilde{M}^r = \rho\delta(1 - \widetilde{m}) + 1 - \rho$$

i.e.

$$\frac{1}{1 + \delta} \stackrel{?}{\leq} \rho\delta \left(1 - \frac{(1 - \delta)(1 + \rho\delta)}{1 - \rho\delta^2} \right) + 1 - \rho$$

which holds because $(\rho, \delta) \notin IA$.

(b) Assume now $\widetilde{m} = \frac{1-\rho}{1-\rho^2\delta^2}$. Under Claim 5,

$$M \leq 1 - \frac{\delta(1 - \rho)}{1 - \rho^2\delta^2}.$$

Thus, it is enough to prove

$$1 - \frac{\delta(1 - \rho)}{1 - \rho^2\delta^2} \stackrel{?}{\leq} \widetilde{M}^r = \rho\delta(1 - \widetilde{m}) + 1 - \rho$$

i.e.

$$\rho - \delta + \rho\delta \stackrel{?}{\leq} \rho^3\delta^2 + \rho^2\delta - \rho^3\delta^3.$$

Since $\widetilde{m} = \max \left\{ \frac{(1-\delta)(1+\rho\delta)}{1-\rho\delta^2}, \frac{1-\rho}{1-\rho^2\delta^2} \right\}$, we have

$$\frac{(1 - \delta)(1 + \rho\delta)}{1 - \rho\delta^2} \leq \frac{1 - \rho}{1 - \rho^2\delta^2}$$

equivalently,

$$\rho - \delta + \rho\delta \leq 2\rho^2\delta^2 + \rho^3\delta^3 - \rho^2\delta^3 - \rho^3\delta^4.$$

Thus, it is enough to prove

$$2\rho^2\delta^2 + \rho^3\delta^3 - \rho^2\delta^3 - \rho^3\delta^4 \stackrel{?}{\leq} \rho^3\delta^2 + \rho^2\delta - \rho^3\delta^3$$

i.e.

$$-\rho^3\delta^2(1-\delta)^2 \stackrel{?}{\leq} \rho^2\delta(1-\delta)^2$$

which holds trivially.

2. $M = M^r$.

(a) Assume first $\tilde{m} = \frac{(1-\delta)(1+\rho\delta)}{1-\rho\delta^2}$. Under Claim 6,

$$M \leq \rho\delta^2 M + 1 - \rho$$

equivalently,

$$M \leq \frac{1-\rho}{1-\rho\delta^2}.$$

Thus, it is enough to prove

$$\frac{1-\rho}{1-\rho\delta^2} \stackrel{?}{\leq} \tilde{M}^r = \rho\delta(1-\tilde{m}) + 1 - \rho = \rho\delta \left(1 - \frac{(1-\delta)(1+\rho\delta)}{1-\rho\delta^2}\right) + 1 - \rho$$

i.e.

$$\frac{1-\rho}{1-\rho\delta^2} \stackrel{?}{\leq} \frac{\rho\delta^2(1-\rho)}{1-\rho\delta^2} + 1 - \rho$$

which holds trivially.

(b) Assume now $\tilde{m} = \frac{1-\rho}{1-\rho^2\delta^2}$. Under Claim 6,

$$M \leq \rho^2\delta \frac{1-\rho\delta^2}{1-\rho^2\delta^2} + 1 - \rho.$$

Thus, it is enough to prove

$$\rho^2\delta \frac{1-\rho\delta^2}{1-\rho^2\delta^2} + 1 - \rho \stackrel{?}{\leq} \tilde{M}^r = \rho\delta(1-\tilde{m}) + 1 - \rho = \rho\delta \left(1 - \frac{1-\rho}{1-\rho^2\delta^2}\right) + 1 - \rho,$$

which holds trivially.

This proves that $M = \widetilde{M}^r$. We now check that $m = \widetilde{m}$. Assume first $\widetilde{m} = \frac{(1-\delta)(1+\rho\delta)}{1-\rho\delta^2}$. Under Lemma 5.1,

$$\begin{aligned}
m &\geq 1 - \delta M = 1 - \delta \widetilde{M}^r \\
&= 1 - \delta (\rho\delta (1 - \widetilde{m}) + 1 - \rho) \\
&= 1 - \delta \left(\rho\delta^2 \frac{1 - \rho}{1 - \rho\delta^2} + 1 - \rho \right) \\
&= \frac{(1 - \delta)(1 + \rho\delta)}{1 - \rho\delta^2} = \widetilde{m}.
\end{aligned}$$

Assume now $\widetilde{m} = \frac{1-\rho}{1-\rho^2\delta^2}$. Under (1),

$$m \geq \frac{1 - \rho}{1 - \rho^2\delta^2} = \widetilde{m}.$$

Hence, $U = [\widetilde{m}, \widetilde{M}^r]$.

The last step is to prove that $M^a = \widetilde{M}^a$ and $m^r = m^a = m$, where m^r (resp. m^a) is the minimum of the SP equilibrium payoffs in which the proposer's initial offer is rejected (resp. accepted).

Under (4), we know that $m^a \leq \widetilde{m} = m$. Thus, $m^a = m$.

Under (5), we know that $m^r \leq \widetilde{m} = m$. Thus, $m^r = m$.

Under (4), we know that $\widetilde{M}^a \leq M^a$. Assume first $\widetilde{m} = \frac{(1-\delta)(1+\rho\delta)}{1-\rho\delta^2}$. Under Claim 5,

$$M^a \leq 1 - \delta + \delta^2 M$$

since $M = \rho\delta (1 - \widetilde{m}) + 1 - \rho$,

$$\begin{aligned}
M^a &\leq 1 - \delta + \delta^2 (\rho\delta (1 - \widetilde{m}) + 1 - \rho) \\
&= 1 - \delta + \delta^2 \left(\rho\delta^2 \frac{1 - \rho}{1 - \rho\delta^2} + 1 - \rho \right) \\
&= 1 - \delta \frac{(1 - \delta)(1 + \rho\delta)}{1 - \rho\delta^2} \\
&= 1 - \delta \widetilde{m} = \widetilde{M}^a.
\end{aligned}$$

Assume now $\tilde{m} = \frac{1-\rho}{1-\rho^2\delta^2}$. Under Claim 5,

$$\begin{aligned} M^a &\leq 1 - \frac{\delta(1-\rho)}{1-\rho^2\delta^2} \\ &= 1 - \delta\tilde{m} = \widetilde{M}^a. \end{aligned}$$

Thus, $M^a = \widetilde{M}^a$.

References

- [1] K. G. Binmore, A. Rubinstein, A. Wolinsky, The Nash bargaining solution in economic modelling, *Rand J. Econ.* **17** (1986), 176-188.
- [2] A. Mauleon, V. J. Vannetelbosch, Bargaining with endogenous deadlines, *J. Econ. Behav. Organ.* **54** (2004), 321-335.
- [3] M. J. Osborne, A. Rubinstein, Bargaining and markets, Academic Press, San Diego, 1990.
- [4] A. Rubinstein, Perfect SP equilibrium in a bargaining model, *Econometrica* **50** (1982), 97-109.
- [5] R. Selten, Reexamination of the perfectness concept for SP equilibrium points in extensive games, *Int. J. Game Theory* **4** (1975), 25-55.

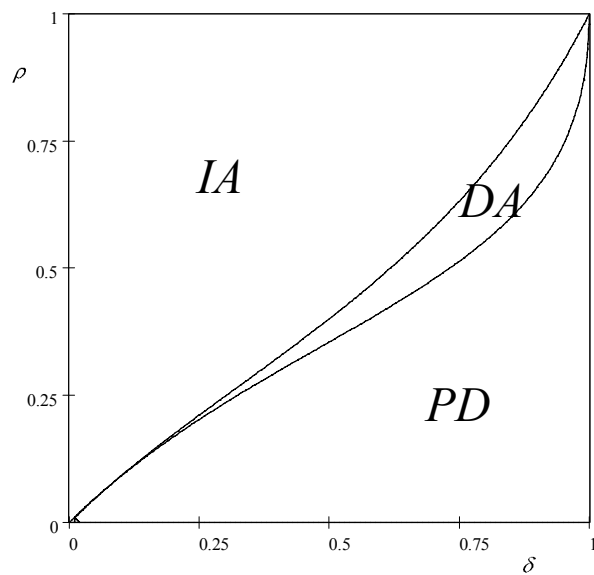


Figure 1: Regions IA , DA and PD .

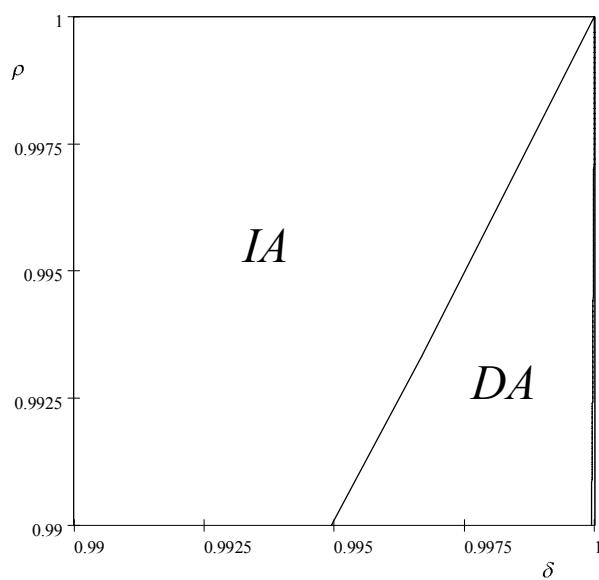


Figure 2: Regions when ρ and δ are close to 1.

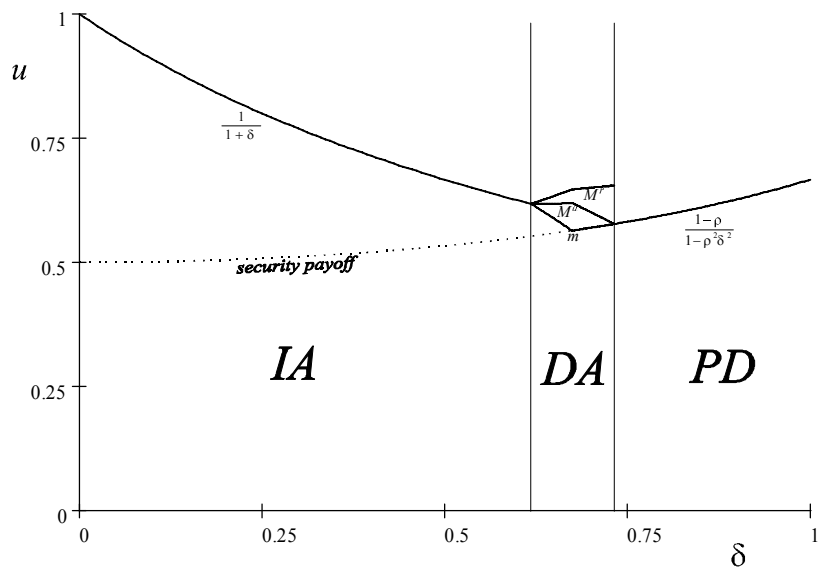


Figure 3: The SP equilibrium payoffs for the proposer as a function of δ when $\rho = 0.5$. The dotted line represents the security level payoff.

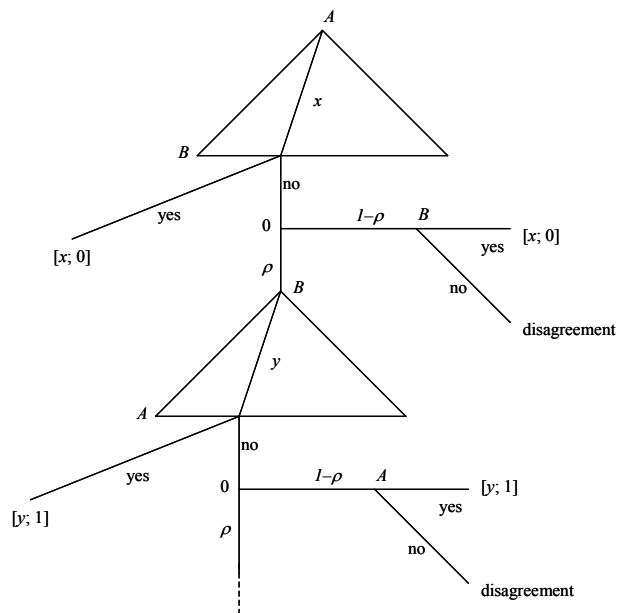


Figure 4: The first two periods of the bargaining model.