Joint Bidding in Common Value Auctions: Theory and Evidence*

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Abstract

We examine theoretically and experimentally two countervailing effects of collusion and symmetric mergers among bidders. On one hand, the pooling of information within bidding rings increases the precision of competing estimates. We demonstrate that, in average value auctions, this leads to more aggressive bidding. On the other hand, since collusion decreases the number of active bidders, competition is lessened, reducing the price paid at auction. We demonstrate that the reduction in competition dominates the informational effects, resulting in lower prices. We examine these hypothesized effects experimentally by conducting a series of auctions with constant informational content but a varying number of bidders among whom this information is distributed. The experimental results are consistent with our theoretical predictions for different value and auction mechanism specifications.

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1 Introduction

Antitrust policy is driven by an almost universally accepted maxim among economists: in the absence of offsetting efficiency gains, collusion and mergers decrease competition, leading to higher market prices. This traditional industrial organization viewpoint is directly applicable to private-value auction markets – mergers among parties are privately profitable (Mailath and Zemsky 1991) and reduce the number of active bidders, leading to diminished revenue for the seller (Waehrer and Perry 2003, Froeb, Tschantz, and Crooke 1999).

However, pure common value auctions, in which buyers would agree on the object's value *ex-post*, present a challenge to this traditional viewpoint. The post-merger decrease in competition is inexorably linked to an increase in the precision of value-estimates as merged (or colluding) parties share their information. On one hand, we will still have a reduction in competition which tends to lower the auction price. On the other hand, several effects can lead to more aggressive bidding as a result of collusion or mergers. First, by diminishing the number of active bidders, mergers mitigate the effects of the winner's curse (Pinkse and Tan 2000, Bulow and Klemperer 2002, Hendricks, Pinkse, and Porter 2003). Since the winner of an auction likely has the most optimistic estimate, outbidding fewer competitors implies less chance that one's estimate is overly optimistic, decreasing the likelihood of overbidding. Second, a merged or colluding entity is likely more informed about the value of the underlying asset than individual bidders. This "information pooling" alleviates the winner's curse and could also lead to more aggressive bidding (DeBrock and Smith 1983, Hendricks and Porter 1992).

While mergers in traditional markets can be analyzed by holding sources of potential efficiency gains constant and later determining what efficiency gains can offset the loss of competition, these effects are harder to isolate in common value auction settings. While the above studies, among others, have contributed to our understanding of joint bidding, none resolve definitively the impact on revenue when the greater aggressiveness of bidding is tempered by the fewer bids tendered to the auctioneer. The goal of this paper is to disentangle these two effects and suggest what role joint bidding plays on the hammer price in particular types of common value auctions. Further, we take a view of joint bidding as the sharing of individual bidders' estimates within a group, whether by collusion or by mergers. Most studies envision information as a notion of precision – e.g., a group of two bidders receives a single estimate of the object's value which is more precise than that obtainable by either bidder individually. Yet, if two bidders merge, each with an independent signal of an object's value, the new entity now has two signals.

In our model, a merger entails decreasing the number of bidders while simultaneously providing each bidder with additional signals, or estimates of the object's value. Hence, we envision a merger not as changing the *structure* of information in an industry, but the *concentration* of that information among bidders. We espouse a broad view of "mergers," including the acquisition of one firm by another and the formation of temporary, single-purpose bidding consortia, (Hendricks, Pinkse, and Porter 2003, for example). We develop a theory of symmetric mergers in average value auctions, in which the object's value is equal to the mean of all of the signals. To the existing literature on this model (Krishna and Morgan 1997, Mares 2001, Bulow and Klemperer 2002), we add a comprehensive description of post-merger equilibrium bidding in both first-price and second-price mechanisms and a general result on the price impact of mergers. We demonstrate that since merged bidders are better informed, they do bid more aggressively. However, since mergers also entail a reduction in the number of bidders, the impact of reduced competition outweighs the effects of better information, resulting in lower prices.

Laboratory experiments were run to verify these theoretical predictions and evaluate the sensitivity of prices to information concentration. The growing body of literature on common value auctions in experimental settings shows that people often fail to bid in accordance with equilibrium predictions and often fall prey to the winner's curse.[1] We wish to test the qualitative implications of our theoretical results, i.e., that greater industry concentration, holding information constant, leads to more aggressive bidding and declines in expected price. The experimental data show marginally more aggressive bidding post-merger and a decline in revenues. For robustness, we also examined experimentally an affiliated signals model in which signals are independent draws from a distribution centered on the object's true (unknown) value.[2] In these auctions, bidding is again more aggressive as information is concentrated, and the decline in revenues as a result of the merger is more dramatic.

In the next section, we present our theoretical results for mergers in average value auctions. Section three describes the experimental methodology and design considerations. Section four contains experimental results and general observations. Section five concludes.

2 Mergers and Collusion in Auctions

Our model of common value auctions is a special case of the general symmetric model developed by Milgrom and Weber (1982). This modeling choice offers the benefit of relatively clear and well-understood baseline comparisons. The modeling challenge in this line of research on information pooling and joint bidding arises from the multidimensional nature of the signals obtained by bidders.

We can conceive of better information in two ways: either the better-informed party receives a more precise estimate of the object's value than other bidders, or simply receives more estimates. In the first case, more informed bidders may draw their signals from a distribution more concentrated about the true value.[3] In the second, collusion or joint bidding is represented by a greater concentration of signals among fewer bidders.[4] While the multiple signals approach is a more intuitive notion for mergers – the *amount* of information does not change following a merger, but simply the *allocation* of it – the framework presents a challenge for analysis. In fact, Milgrom and Weber implicitly avoid the complications of bidding with multidimensional signals in their treatment of the general symmetric model:

> To represent a bidder's information by a single real-valued signal is to make two substantive assumptions. Not only must his signal be a sufficient statistic for all the information he possesses concerning the value of the object to him, it must also adequately summarize his information concerning the signals received by other bidders. The derivation of such a statistic from several separate pieces is in general a difficult task. (Milgrom and Weber 1982, p.1097 fn.14).

Most inroads in this field concentrate on cases where the information conveyed by a vector of signals can be summarized by a scalar statistic. This has a clear advantage as it reduces the

[1]See Kagel and Levin (2002) for a recent review.

[2]This formulation of a common affiliated value has received much experimental attention, and is analyzed in Dyer, Kagel, and Levin (1989) and Kagel and Levin (1986), and in the context of private value auctions by Kagel, Harstad, and Levin (1987) and Harstad (2000), among others.

[3]This is the approach in Schweizer and von Ungern-Sternberg (1983) and in the information acquisition models of Matthews (1984) and Persico (2000).

[4]See, for example, Mares (2001), Krishna and Morgan (1997), and DeBrock and Smith (1983).

problem to one within the general Milgrom and Weber (1982) framework. Krishna and Morgan (1997) and Mares (2001) analyze second-price average value auctions by recognizing the average of a bidding ring's signals as a scalar summary statistic for the group's information. Within the context of an average value auction, this is both an intuitive notion and a sufficient statistic. However, in most auctions, the reduction of a multidimensional signal to a scalar statistic is less obvious. Mares (2001) derives a general expression for the reduction of multidimensional signals into a scalar summary statistic, but notes that such a scalarization does not exist in general.[5] Furthermore, examples by Jackson (1999) and Mares (2001) suggest that in general the problem of bidding with multidimensional signals might not admit an equilibrium.

In the rest of this section and in the subsequent experiments, we examine two models of pure common value auctions. First, the average value model has been used extensively in the literature and has the advantage that it allows for closed-form solutions of the bidding function for most merger scenarios.[6] To derive the equilibrium bid functions for this model, we rely crucially on the possibility of aggregating the informational vector of each bidding ring (group) into a scalar – a single number which is a sufficient statistic for the informational content of the group's signals. Second, in order to check the robustness of our results, we also conduct experiments in an affiliated signals model that does not admit scalar sufficient statistics for a multidimensional vector of signals. In this auction, signals are drawn from some distribution centered on the object's true but unknown value.

2.1 The average value model

Consider first an average value auction, in which the object's value, V, is equal to the mean of all the signals received by all participants:

$$V = \frac{\sum_{i=1}^{n} X_i}{n}$$

Furthermore assume that the private signals are *i.i.d.* with a common distribution function F and a density f. Denote by

$$\overline{X}_k = \frac{\sum_{i=1}^{k} X_i}{k}$$

the random variable that is the average of k independent signals distributed as X_i. Denote the distribution function of $\overline{X}_k$ with G_k and its density with g_k. We will limit ourselves in this section to situations where the support of the distribution is bounded, and for computational simplicity, we assume that the support of X_i is $[0, 1]$, although most results can be extended easily to non-compact supports. Throughout this paper we will make use of the following notation for order statistics: let $f^X_{k:n}, F^X_{k:n}$ and $\mu^X_{k:n}$ denote the density, distribution and expected value of the k-th lowest order statistic out of a sample of n i.i.d. random variables distributed as X.[7] If there is no danger of confusion we drop the superscript in this notation. We also employ the notation $\beta_n^{FP,Y}, \beta_n^{SP,Y}$ to

[5] Other scalar summary statistics have been adopted for specific models. DeBrock and Smith (1983) use the geometric mean of signals when the signals and value are distributed lognormal. Goeree and Offerman (2003) propose using the expected surplus in a model with additively separable private and common value components.

[6] See Krishna and Morgan (1997) and Mares (2001) for a full description. Additive specifications are common in the literature (Albers and Harstad 1991, Bikhchandani and Riley 1991, Bulow and Klemperer 2002, Campbell and Levin 2002).

[7] Thus, we adopt the convention that the 1^{st} order statistic is the smallest and the n^{th} (in a sample of n) is the largest.

denote the symmetric equilibrium bids in first-price and second-price average value auctions with n bidders where signals are distributed *i.i.d.* as Y.

In this paper, we consider only symmetric mergers, i.e., mergers where each of the bidding rings sees the same number of signals. This makes the determination of the equilibrium bids relatively straightforward since the merged problem is another instance of the general symmetric model where the private information of a bidder is $\overline{X}_k$ rather than X. Thus a merger is described as m agents receiving k signals, where $n = km$. In other words the total amount of information in the economy, across merger profiles is constant.

A number of studies have identified certain effects of mergers on bidding in *second* price auctions described by this model in which a collection of bidders share their information truthfully and submit a joint bid. First, the aggregation of independent signals allows a bidding consortium to pool its information, deriving more precise estimates of the object's value and potentially implying a smaller winner's curse correction (DeBrock and Smith 1983). Second, an "inference effect" captures the notion that one bidder's better information may lead others to bid more aggressively (Krishna and Morgan 1997). Particularly i n a second-price auction, a less-informed bidder is not as fearful of overbidding since a better-informed bidder would effectively set the price. In effect, outbidding a well-informed party carries less risk in a second-price mechanism than in a first-price auction. Third in asymmetric settings, a "confidence effect" drives more informed bidders to form optimistic estimates of other bidders' information (Mares 2001). Fourth, decreasing the number of bidders, even without an offsetting increase in information for each bidder, may lead to more aggressive bidding. To avoid the winner's curse, subjects bid more cautiously in auctions with more bidders.[8] All of these identified effects tend to work in the same direction – mergers reduce the number of bidders and correspondingly increase the concentration of information, all leading to higher bids, on average.

In first-price auctions the effects of a merger on bidding behavior may be decoupled into two effects. The first isolates the role of the number of bidders without an offsetting increase in informational precision. The second considers the role of more precise information without a change in the number of bidders. In the following lemmas, we determine the role of each of these effects independently, and then show the overall effect of mergers on bidding behavior.

Competition Effect In a second-price average value auction, the reduction in the number of active bidders has a positive effect on the aggressiveness of bidding. The symmetric first-price and second-price equilibrium bids, computed using the formulation in Milgrom and Weber (1982), are given by

$$\beta_n^{FP,X}(x) = \frac{(n-1)}{n} E[X|X \leq x] + \frac{1}{n} E[\max(X_1, \ldots, X_{n-1})| \max(X_1, \ldots, X_{n-1}) < x], \tag{1}$$

$$\beta_n^{SP,X}(x) = \frac{2}{n} x + \frac{n-2}{n} E[X|X \leq x]. \tag{2}$$

[8]In first-price auctions, this effect is ambiguous. Actually, a winner's curse effect dictates a reduction in bids as the number of bidders increases, but the competition effect suggests that one needs to bid higher when competing against more bidders to maintain a chance of winning. Pinkse and Tan (2000) and Bulow and Klemperer (2002) demonstrate that bids may decline with more bidders. Hendricks, Pinkse, and Porter (2003) demonstrate this effect in OCS wildcat auctions. The opposite effect has also been obtained both experimentally (Kagel and Levin 1986) and in econometric analysis of field data (Hong and Shum 2002).

from which we deduce that

$$\beta_{n-1}^{SP,X}(x) - \beta_n^{SP,X}(x) = \frac{2}{n(n-1)}\left(x - E[X|X \leq x]\right) \geq 0.$$

That is, decreasing the number of bidders increases the bid in a second-price auction. Intuitively, the more people I outbid, the more likely that I have overestimated the object's value and fallen prey to the winner's curse. Thus, more bidders implies a greater adjustment for the winner's curse. Analysis of bidding in a first-price auction is less straightforward since bidders shade from the expected value not only to account for the winner's curse but also to balance the price paid with the odds of winning. While we still expect that reducing the number of bidders should encourage a smaller winner's curse adjustment, less bidders also implies more bid shading since any bid is more likely to win with less opponents. Since these effects work in opposite directions, the effect of reducing the number of competitors is ambiguous. However, the next result shows that for large enough n we find the same result as in the second-price auction – namely that the reduction in the number of participants produces more aggressive bidding.

Lemma 1 ***(Competition effect)*** *For every $x \in [0,1]$,*
(i) $\beta_n^{FP,X}(x)$ is unimodal in n, and
(ii) $\beta_n^{SP,X}(x)$ is decreasing in n.

The proof of this and all other results are relegated to the appendix. Note that varying n in the firstprice auction does not generate the same clean comparative results as in a second-price auction. The difference follows from the fact that a decrease in the number of bidders reduces both competition and the winner's curse but increases the relative importance of one own's signal. In this model in particular, fewer bidders are associated with more informational content of one's private signals. Since, in this construction, the value is a deterministic function of private information, a decrease in the number of players is also an increase in the informational content of each private signal.

Information Pooling In this section, we consider the effect of increasing the precision of bidders' signals while holding the number of bidders constant. We borrow from information acquisition models the intuition that a "more accurate" signal is one drawn from a distribution more concentrated about the true value (Persico 2000, Athey and Levin 1998). It is well established that $\overline{X}_2$ is more precise than X in the sense of a lower variance.[9] We adopt a stronger notion of concentration than the one implied above. Following Whitt (1985), consider two random variables X and Y with equal supports and identical first moments. Y is more precise than X in the sense of the log-concave order, denoted by $Y \preceq_{lc} X$, if $\frac{f_Y}{f_X}$ is log-concave. The advantage of this ordering of information is that it offers a natural condition for the comparison of truncated means since it can be established that $Y \preceq_{lc} X$ implies that

$$E[X|X \leq x] \leq E[Y|Y \leq x]. \tag{3}$$

for arbitrary x (see appendix for a formal proof). Note that the log-concave order implies the usual convex order between random variables. For our purposes, the salient feature is that if X has a log-concave density then $\overline{X}_2$ is more precise than X in the log-concave order. We use this order to evaluate the effect of more precise information on equilibrium bidding in first-price auctions.

[9]See, for example, Shaked and Shanthikumar (1994).

Lemma 2 ***(Information pooling)*** *If Y is more precise than X in the sense of the log-concave order then*

(i) there exist a $t_{X,Y,n}$ and $t'_{X,Y,n}$ such that $0 < t_{X,Y,n} \leq t'_{X,Y,n} < 1$ and $\beta_n^{FP,Y}(x) \geq \beta_n^{FP,X}(x)$ for x in $[0, t_{X,Y,n}]$ and $\beta_n^{FP,Y}(x) \leq \beta_n^{FP,X}(x)$ for x in $[t'_{X,Y,n}, 1]$, and

(ii) $\beta_n^{SP,Y}(x) \geq \beta_n^{SP,X}(x)$ for all x.

Increasing the precision of the signal has an ambiguous role in first-price auctions. It tends to decrease the variability of the equilibrium bid without moving the entire bid function in one direction. For low values of the signal, the increase in precision generates more aggressive bidding through the effect of (3). For high values of the signal, this effect diminishes since $E[X] = E[Y]$, that is, the unconditional expectations are equal. Consider a bidder with a signal equal to 1, the upper support. If signals are more concentrated, having a competitor with a signal close to 1 is less likely than if the distribution were more dispersed. Hence, a bidder with a signal of 1 can profitably shade his bid more when he faces opponents with more concentrated information, leading to lower bidding.

On the other hand, it is also easy to establish that in second-price auctions the effect of more precise information is clear cut. In the equilibrium bidding function (2), the term $E[X|X \leq x]$ increases if X is replaced by a more precise distribution (in the sense of the log-concave order). Thus, we will see higher bids when the precision of information increases.

Equilibrium Bidding In considering a merger, both the competition and information pooling effects are present since we are reducing the number of bidders while simultaneously increasing the informational precision. In second-price auctions, both effects work to make bids more aggressive, while first-price auctions present some subtleties. The following result characterizes the overall impact of symmetric mergers on bidding in first-price average value auctions. Consider an average value model with $n = k \times m$ signals where we allow mergers into m groups of k bidders each. The merged situation gives rise to an auction in which each bidding ring receives an *i.i.d.* signal $\overline{X}_k$. Denote by $\beta_{km}^{FP,X}$ and $\beta_m^{FP,\overline{X}_k}$ the respective symmetric equilibrium bid functions before and after a merger for first-price auctions. Analogously, denote by $\beta_{km}^{SP,X}$ and $\beta_m^{SP,\overline{X}_k}$ as the corresponding bid functions in the second-price auction.

Theorem 1 *If the distribution of signals, F, is log-concave then*

(i) there exists a $s_{k,n,F} > 0$ such that $\beta_{km}^{FP,X} \leq \beta_m^{FP,\overline{X}_k}$ for x in $[0, s_{k,n,F}]$. For fixed k and high enough n there exists a $s'_{k,n,F} \geq s_{k,n,F}$ such that $\beta_{km}^{FP,X} \leq \beta_m^{FP,\overline{X}_k}$ for x in $[s'_{k,n,F}, 1]$, and

(ii) $\beta_{km}^{SP,X} \leq \beta_m^{SP,\overline{X}_k}$ for all x.

In essence, the theorem states that post-merger equilibrium bidding in first-price auctions is more aggressive for low values of the signal. For high signals, the comparison depends on n. When the number of bidders is small, it is possible that merged bidding is less aggressive than "solo" bidding. As we increase the amount of information in the economy, however, this comparison will reverse. In particular, the lemma implies that, for any k, $\beta_{km}^{FP,X}(1) < \beta_m^{FP,\overline{X}_k}(1)$ for high enough n. For high values of the signal, a large amount of information in the economy (large n) implies that the precision of information effect (information pooling) outweighs the reduction in the number of bidders. Note that the above lemma leaves the door open for the possibility of a crossing between $\beta_{km}^{FP,X}$ and $\beta_m^{FP,\overline{X}_k}$, at least for low values of m. The example in our experimental set-up, in which signals

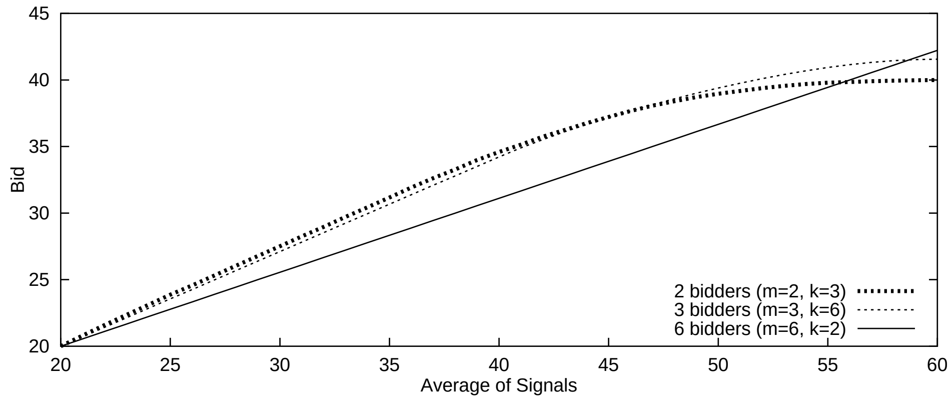

Figure 1: Equilibrium bidding functions when $X \sim U[20, 60]$ and $N = 6$, for $k \in \{1, 2, 3\}$. Greater concentration leads to more aggressive bidding for low signals, and less aggressive bidding for signals close to the upper bound of the support.

are distributed uniform, exhibits this property (See Figure 1). Mergers in second-price auctions unambiguously increase the aggressiveness of bidding.

Equilibrium Prices Having established that post-merger bidding is by and large more aggressive, we turn to the analysis of prices. From the standpoint of the auctioneer, these potentially higher bids come at the expense of the number of bidders. By construction, our model exhibits independence of signals and symmetry both before and after a merger, which implies revenue equivalence of standard auction formats.[10] In a second-price average value auction, Krishna and Morgan (1997) and Mares (2001) prove that log-concavity of the signals is sufficient to guarantee increased aggressiveness of bidding in symmetric mergers. Krishna and Morgan also conjecture that revenue may increase with concentration since the increased aggressiveness might outweigh the reduction in the number of bidders.

Aggregating all the forces that affect equilibrium bidding, we find that the overall reduction in the number of bidders outweighs in equilibrium all gains that can be expected from more aggressive bidding. The result below holds for both first-price and second-price auctions since, by construction, we have focused only on situations where revenue equivalence between auction types prevails.

Theorem 2 *Symmetric mergers reduce expected revenues in average value auctions.*

2.2 An Affiliated Signals Model

While in the average value auction, the mean of a group's signals is a natural sufficient statistic, the existence of symmetric bidding equilibria in auctions with multidimensional types is not a foregone conclusion (Jackson 1999). We present a simple conditionally-independent affiliated signals model which exemplifies the impossibility of information aggregation.

[10]Independence and symmetry amongst bidder types is a sufficient condition for revenue equivalence. See, for example, Waehrer, Harstad, and Rothkopf (1998).

Consider an auction in which each participant receives a signal drawn independently from a uniform distribution on $[V-a, V+a]$, centered at the object's (unknown) value, V, which is itself drawn from some known distribution. In effect, nature draws a value and then provides noisy signals from a distribution centered on that value.

It can be established that, in this situation,

$$f_{V|(x_1,\ldots,x_k)}\left(v|(x_1,\ldots,x_k)\right) \sim \Phi\left(\max\{x_1,\ldots,x_k\},\min\{x_1,\ldots,x_k\}\right)\Psi(v),$$

or that the minimum and the maximum constitute a minimal sufficient statistic for V.[11]

Since intermediary signals do not contribute to an estimate of the object's value, at most two signals carry informational content. It also follows that no uni-dimensional sufficient statistic exists. In the above example, if two of a bidder's signals are a and $3a$, the value of the object is known with certainty to be $2a$ and no adjustment for the winner's curse is warranted.[12]

2.3 Summary

Our modeling choice is motivated by two factors. First, both auction formats are rather intuitive for experimental subjects. The average value auction can be motivated by the familiar wallet game (Klemperer 1998) in which each person bids for an amount of money equal to the average of the participants' wallets' contents, based solely on knowledge of his own net worth. The affiliated signals auction represents a standard story of bidding for drilling rights by bidders who have collectively conducted exploratory drilling over the field. Exploration, by its very nature, provides a noisy signal of the value of oil contained in the site.

A second motivation for selecting these two cases is the significant difference between the two mechanisms in the way multiple signals enter into one's expectation of the object's value. In the average value auction, multidimensional signals can be easily aggregated into a scalar sufficient statistic. In our affiliated signals model, such a reduction is not possible. At the interim stage, these two models also have contrasting implications. In the average value auction, a bidder draws the same inference from any two signals, regardless of their respective realizations, as long as their sum is constant. In the affiliated signals case we study, a bidder facing more dispersed signals can make tighter predictions about the true value or at least its support. As mentioned above, in the extreme case in which two signals are $2a$ apart, a bidder knows the value for sure. This also implies that at the interim stage a bidder with two signals can be better informed than a bidder with three signals with positive probability. It is only at the *ex ante* stage that more signals provide better information in both models. Finally, in the average value auction, the value is a deterministic function of the information sample – i.e., the grand coalition knows the value of the asset with certainty. The affiliated signals model exhibits persistent residual uncertainty – no matter how large a finite sample of draws we consider, we will find, with probability 1, that the information contained in the sample will not allow us to predict the value with certainty.[13] Thus, we envision these two auctions to be quite different, qualitatively. This gives an additional degree of robustness to any experimental findings that are constant across the two treatments.

[11] See, for example, Migliorati (1998).

[12] Kagel and Levin (1986) provide a characterization of the equilibrium bid in a first-price auction for this model assuming uniform priors.

[13] This is true for a continuous uniform distribution. As we utilize a discrete distribution in the experiments, there is always a small chance of receiving two signals that are fully informative.

Table 1: Experimental Treatments

Treatment	Auction Type	Auction Format	m	k
I a			6	1
I b	Average Value	1st Price	3	2
I c			2	3
II a			6	1
II b	Average Value	2nd Price	3	2
II c			2	3
III a			6	1
III b	Affiliated Signals	1st Price	3	2
III c			2	3
IV a			6	1
IV b	Affiliated Signals	2nd Price	3	2
IV c			2	3

3 Experimental Design

In this section, we describe an experiment intended to test our theoretical conclusions. Since we can expect subjects to deviate from equilibrium bidding behavior, the experiments serve as a robustness check allowing us to verify if the comparative statics results still obtain.

Experimental subjects participated in a series of three auctions. In each auction, a total of six "signals" were distributed among the bidders. However, the number of bidders varied across periods. In the first period, each of six bidders received a signal, and placed a bid. In the second period, subjects competed against two other bidders (three total), with each participant receiving two signals. Lastly, period three saw two bidders competing with three signals each.

Each subject was randomly assigned to one of four treatments, combining an auction type (average value or affiliated signals) with and auction format (first-price or second-price sealed bid). These treatments are summarized in Table 1. Subjects submitted bids independently over the Internet and were not informed of the outcome of any auction until after the conclusion of the experiment, at which point earnings were tabulated.[14]

A total of 204 subjects participated, 60 in each of the first-price treatments (I and III), and 42 in each of the second-price treatments (II and IV). For each auction type, a total of ten first-price and seven second-price auctions were run with groups of six bidders, each receiving one signal. Twenty (fourteen) and thirty (twenty-one) first-price (second-price) auctions were held with three and two bidders, respectively, for both the average value and affiliated signals auctions.

Subjects were MBA students and all had some classroom exposure to common value auctions, some in multiple courses. Further, in informal interviews after the experiment, most subjects indicated some experience with participating in auctions, ranging from low value online purchases to formulating bidding "strategies" for mid-size businesses in procurement auctions. Notably, none admitted to any experience with collusion in auctions.

In the average value auction, six signals were drawn from a discrete uniform distribution over

[14]Subjects received a $5 participation fee and participated in other experiments not reported here in which losses were not possible, to make up for the potential losses from the winner's curse.

the integers $\{20, 21, \ldots, 60\}$. The value of the object was known to be equal to the average of these signals. The affiliated signals auction followed a two-step procedure. A value was selected from a known uniform distribution over the integers $\{0, 1, \ldots, 200\}$. Then, each signal was drawn independently from the uniform distribution $\{V - 20, \ldots, V + 20\}$. All numbers represent actual dollar amounts.

The experimental design abstracts from the real issue of collusion and mergers in three important ways. First, each bidder receives multiple signals representing the additional information obtained by a ring of bidders who pool their information. However, this assumes that the consortium's representative has access to all of the members' signals and ignores the very real issue of truthful revelation of signals in collusive settings. Collusion involves consideration of revenue-sharing issues and incentive-compatible mechanisms for truthful signal revelation.[15] Hence, this experiment is more akin to a takeover scenario since the acquiring firm has an incentive to aggregate all available information.

Second, a single decision-maker obtains all of the signals, and captures all of the potential revenue from the auction. In some sense, collusion should involve interaction among participants, perhaps in the sense of joining subjects into groups, rather than signals. However, this would impose a less-tractable social-psychology dimension to the experiment, and it is unlikely that the dynamics that would be observed in the laboratory represent inter- and intra-firm relationships.

Lastly, the partition of signals is exogenous, enforced by the experimenter. Issues of incentives to merge and collude are therefore avoided. In this sense, this experiment answers the "what if" effect of mergers, rather than the "why." Alternately, it may be seen as mergers among multi-product firms for whom these auctions represent minor portions of revenue, and hence the effects of merging in this auction market are negligible relative to other concerns.

4 Experimental Results

All four treatments demonstrate both an upward trend in bids and a downward trend in price as the number of bidders is decreased (Figure 2). Specifically, the bidding functions increase with concentration (Figure 3), yet this is not enough to offset the loss of competition. However, the results of these experimental sessions are mostly anecdotal. A different random pairing of subjects would potentially produce different results. For example, two of the four highest bids in the first-price average value auction were matched into the same group of six bidders. If these bidders were to find themselves in different sessions, both would likely win, significantly increasing the average winning price. In the affiliated signals auction, two of the three "most conservative" bidders, bidding the minimum possible value given their signals, were randomly grouped, leading to unusually high surplus for the winner.

Our interest, however, is not in the specific results of these sessions, but in the effect of collusion and mergers on *expected* bids and revenue. Effectively, auction participants map signals into bids, and the auctioneer selects the highest or second highest such signal as the winning price. In the remainder of this section, we formulate an analogous approach. First, we compute the empirical distribution of bids in each auction and in each treatment. This distribution will allow us to consider whether bidding is more aggressive with fewer bidders. Then, we use that distribution to arrive at an empirical distribution of resulting prices based on the observed bidding strategies.

[15] Mailath and Zemsky (1991) characterize a mechanism for collusive rings in a private-value auction to both elicit truthful revelation of signals and allocate the object efficiently within the ring. Also see McAfee and McMillan (1992).

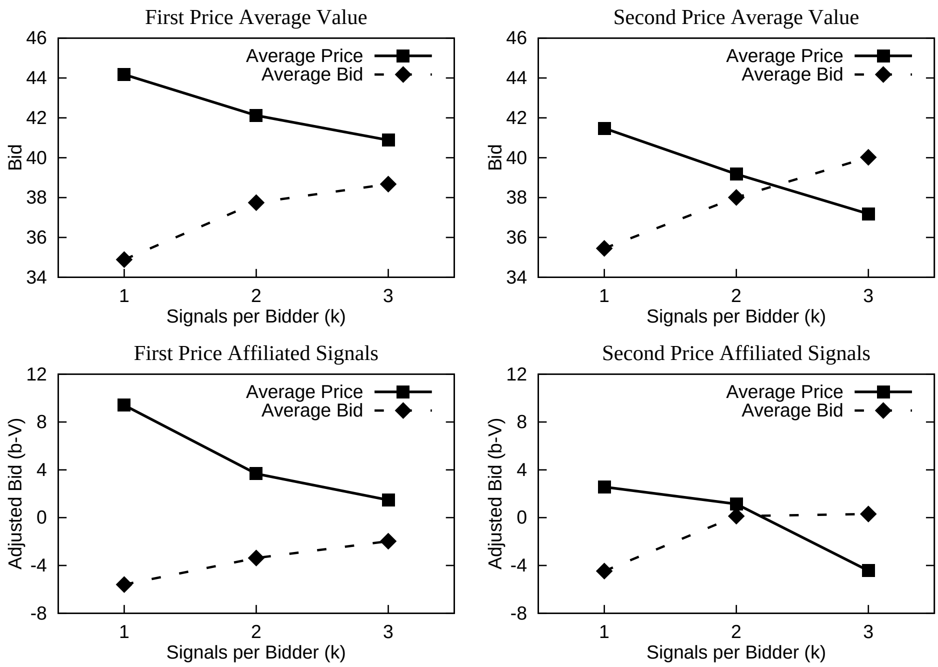

Figure 2: Bids and prices in experimental average value and affiliated signals auctions. Decreasing the number of bidders while simultaneously increasing the number of signals per bidder leads to higher bids on average but decreases the winning price.

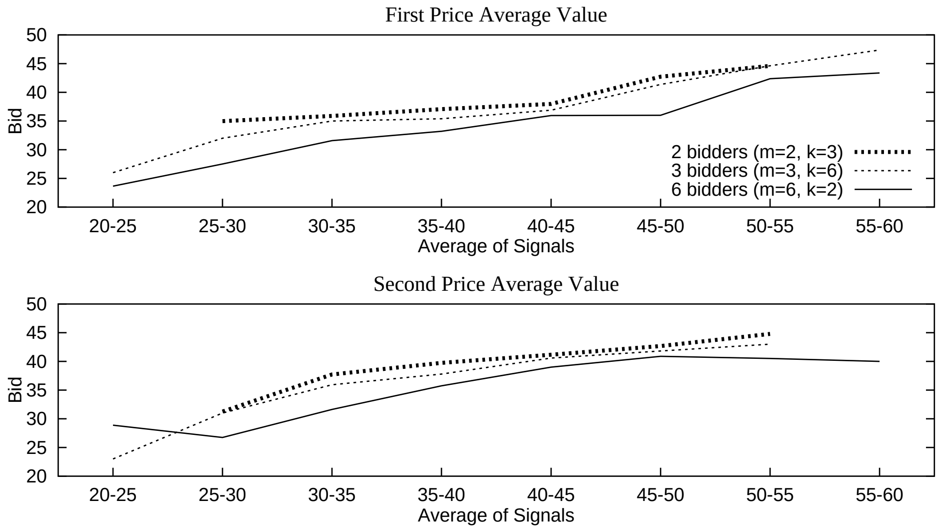

Figure 3: Empirical bidding functions in average value auctions. In both first-price and second-price average value auctions, bidding becomes more aggressive with increased concentration, holding the total number of signals constant.

As a preliminary note, it is important to verify that, for each auction type, the distribution of signals is similar across the three treatments. In effect, we wish to confirm that enough sessions were run to justify this simulation approach and rule out treatment effects resulting from distinct empirical distributions of signals. The empirical distribution of signals (Figure 4) should closely approximate the uniform distribution, from which signals were drawn. In comparison with the uniform distribution, each of the twelve empirical distributions yields a Kolmogorov-Smirnov (K-S) test statistic of between 0.596 and 1.166 (p-values between 0.132 and 0.869). In the twelve pair-wise tests between empirical distributions of signals (three within each of four treatments), the K-S test for equivalence of distributions resulted in p-values between 0.492 and 0.999. In all cases, we cannot reject that signals are distributed uniform and similarly across the treatments.

If similar signals are received in each treatment, on average, then systematic differences in bids across treatments may be attributed to the effects of mergers and information concentration. We can consider the bids of subjects independent of their signals to determine, on average, what bidding patterns emerge. In all four auction formats, bidding is more concentrated about the mean bid as information is dispersed among fewer bidders (Figure 5). This is hardly surprising since a more precise estimate of the object is obtained. More interesting results are obtained with respect to bidder aggressiveness.

Result 1 *Bidding becomes more aggressive in the average value auction as information is concentrated from six bidders to two. The effect of intermediary concentration is ambiguous.*

The empirical distribution of bids in the average value auction suggests that bids issued by participants with three signals (and one competitor) stochastically dominate the bids issued by

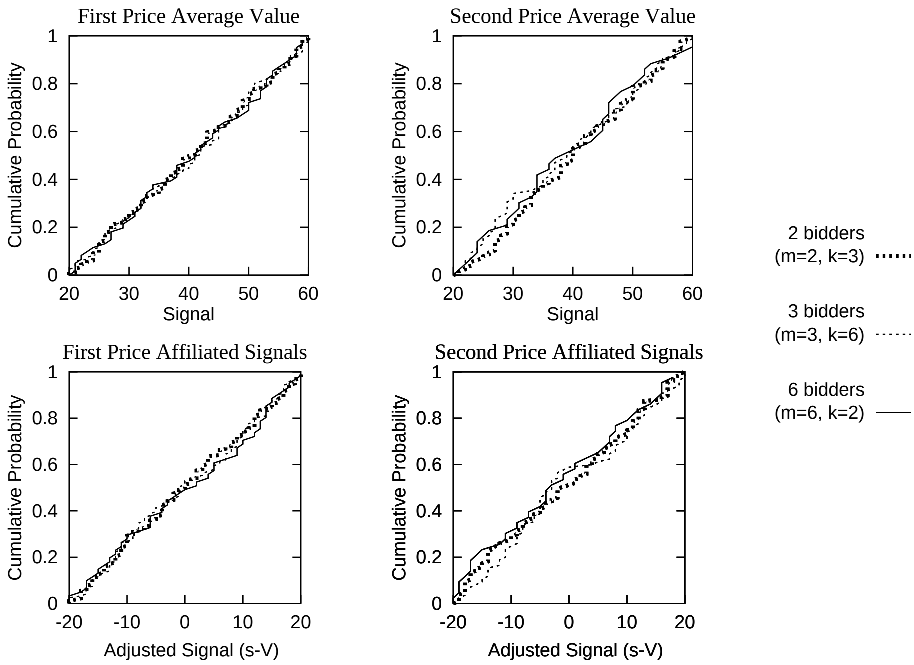

Figure 4: Cumulative distribution of signals for the average value auctions and affiliated signals auction. For the affiliated signals auctions, signals are adjusted $(s - V)$. The distribution of signals does not vary significantly across treatments.

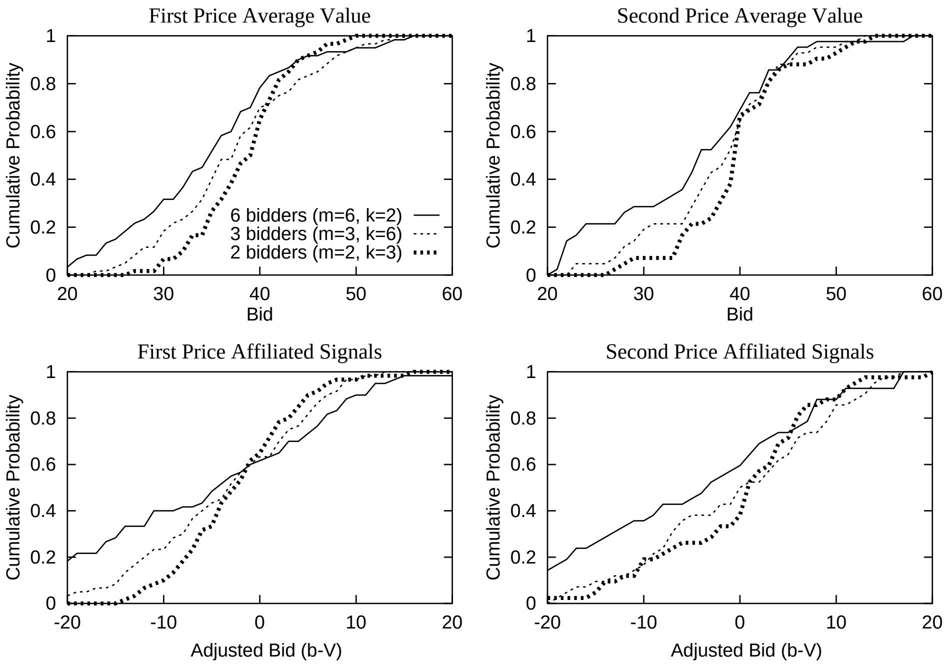

Figure 5: Empirical cumulative distributions of bids for the average value and affiliated signals auctions. For the affiliated signals auction, bids are adjusted $(b - V)$.

holders of only one signal in the six bidder case. Further, the distribution of bids in the six-bidder treatments are significantly different from the distribution in the two bidder case (K-S p=0.013 for first-price and p=0.028 for second-price). However, the effect on bidding of incremental concentration – from six bidders to three, and from three to two – is unclear (K-S p ranges from 0.345 to .564). A symmetric merger to two firms leads each party to bid more aggressively in a stochastic-dominance sense – for most bids (those below the expected value of the object conditioned on the signal), the probability that the merged group bids higher than some given amount is larger than the probability that an unmerged party will bid as aggressively.

Likely, the number of sessions does not allow for greater statistical precision in comparison of the intermediary case. Graphically, the three-bidder case appears to represent more aggressive bidding than the six-bidder treatments. However, these treatments cannot provide enough data to test this proposition. Yet, in the affiliated signals auction, we can confirm a continuous trend with greater concentration.

Result 2 *Bidding becomes more aggressive with increased concentration of information in the affiliated signals auction. Further, bidding behavior incorporates the (ex-post) accuracy of a bidder's signals.*

Visual inspection of the empirical distribution of bids in the affiliated signals auction only serves to reinforce the finding that bids become more concentrated about the mean as a result of mergers. Little can be concluded about the aggressiveness of bids by examining the empirical distributions. However, in the first-price auction, all three distributions seem to suggest an equal probability of placing a bid above (below) the object's actual value. Those bidding above (to the right of the intersection near 0) are generally falling prey to the winner's curse. The bids of the remaining bidders demonstrate significant treatment effects – there is much greater discounting, on average, in the case of a single signal for each bidder than the discounting after a merger. Nevertheless, in comparisons of the three empirical distributions, the distribution of bids with six bidders is significantly different from that with three bidders (K-S p=0.012), and the move from six to two bidders is more dramatic than in the average value auction (K-S p<0.001).

This affiliated signals auction differs from the common value auction in two important respects. First, not each signal is equally valuable, since an additional signal contributes to my estimate of the object's value only if it lies outside the convex hull of my current information. Second, my expected value for the object is independent of the number of bidders or signals. In the average value auction, the total number of signals distributed among bidders determines what proportion of information a given bidder has. Increasing the total number of signals leads to less weight placed on one's own signals, as they are less informative. In the affiliated signals auction, the estimation of the object's value depends only on one's lowest and highest signals. The more signals I have, for example, the better chance that two will highlight the range of the uniform distribution from which they are drawn. Hence, the *concentration* of information within a merged party is more important than the *distribution* of information among bidders. This allows us to better isolate the effects of information.

We posit a simple bidding heuristic in which a bidder shades an amount, α, from the expected value of the object given the bidder's signals, (incorrectly) accounting for the winner's curse effect. Further, the bidder raises the bid in response to better information. A simple measure of the precision of information in this context is:

$$\text{measure of certainty} = \max\left[s_1, \ldots, s_k\right] - \min\left[s_1, \ldots, s_k\right]$$

Table 2: Regression results for bidding behavior. p-values for the t-test statistic are in parentheses.

	Auction	
Parameter	First Price	Second Price
α	6.0711	2.3063
	(< 0.001)	(0.002)
β	0.1914	0.1890
	(< 0.001)	(< 0.001)
F statistic	24.52	17.55
	(< 0.001)	(< 0.001)

Consider the following regression equation:

$$bid = E\left[V|s_1,\ldots,s_k\right] - \alpha + \beta(\text{measure of certainty})$$

The results of this regression are presented in Table 2.

Two inferences may be drawn from this exercise. First, β, the parameter representing the precision of information, is highly significant and does not vary significantly across treatments. Second, the regression results are highly significant, suggesting that the simple model is a reasonable approximation of bidding behavior. Perhaps more interesting is that participants appear to bid without consideration for the number of competitors.[16] In short, subjects appear to follow a bidding heuristic based only on their signals, since neither the number nor the allocation of signals enters into $E[V|s_1,\ldots,s_k]$ or into the regression. Yet, despite the use of simple bidding heuristics, more aggressive bidding and lower prices still obtain.[17]

The effect of mergers on bidding appears in line with Theorem 1. A greater concentration of signals among fewer bidders increases the aggressiveness of bids. However, this effect is tempered by the loss of competition.

Result 3 *The expected auction price decreases with greater concentration for both auction types.*

This inference could be drawn from the observed winning bids (Figure 2). However, a more general question is whether bidding behavior observed in this experiment implies higher bids *on average* rather than as the result of one pattern of random pairing of subjects. To answer this question, the empirical bid distributions (Figure 5) were used to calculate the probability of a given auction price. For example, for the six-bidder (one signal each) first-price average value auction, the

[16] Additional regressions included combinations of the number of active bidders and the number of signals per bidder. None of these treatments induced any qualitative change in the regression results. None of the newly introduced variables were significant at the 10% level.

[17] A methodological question remains: can bidding behavior be explained if we restrict subjects to a single, scalar variable as a descriptive statistic of the whole of a bidder's information. A number of regression equations were estimated, incorporating the number of bidders (m), the number of signals per bidder(k), along with various transformations of the signals into scalar values. Specifically, individual regressions were run using the maximum, minimum, and median signal, as well as various linear and nonlinear transformations (mean, geometric mean, etc.). Further, both equilibrium and heuristic bidding behaviors were modeled. None of the eighteen functional forms tried produced significant fits at the 1% level, compared to the highly-significant fit of the simple model. This supports our intuition that subjects' behavior cannot be explained by a scalar summary statistic. Further, the addition of parameters m and k to the base model considered here did not increase the R^2 by more than 0.2%

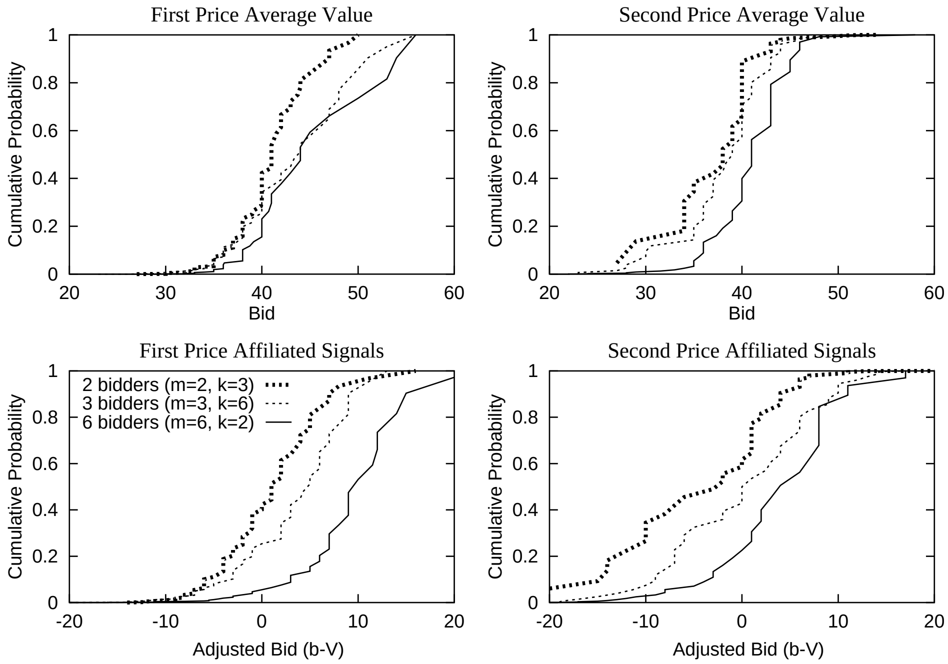

Figure 6: Empirical cumulative distributions of winning bids for the average value and affiliated signals auctions. For the affiliated signals auctions, bids are adjusted $(b - V)$.

probability of a given price is equivalent to the probability that one bidder "draws" this value from the appropriate bid distribution and the remaining bidders draw smaller values.

This exercise likens an auction to a lottery drawing. The seller draws six balls from an urn containing numbered balls (with replacement). The seller's lottery winnings are equal to the highest number drawn (or second highest in a second-price auction). The effect of a merger, in the most optimistic case, is that each ball is relabeled with a higher number. Hence, the new distribution of balls stochastically dominates the former, and a given draw from the new urn will produce a higher number on average. However, if the inflation of winnings comes at the expense of the number of draws, then the lottery participant is drawing higher numbers on average, but less of them. Thus, the effect on the auction price is ambiguous, and depends on the increase in an average draw relative to the decrease in the number of draws. The derived distributions of prices are presented in Figure 6.

In both auction models considered, the distribution of prices in more information-concentrated auctions dominate those of less-concentrated treatments. For almost any target price, the post-merger bidders are less likely to meet the target than the pre-merged firms. This effect is not subtle. The K-S test statistic in all twelve pair-wise comparisons of distributions (three per treatment) yields p-values of less than 0.001.

5 Conclusion

In this paper we address two simple questions related to mergers in common value auctions. We first ask whether mergers increase the aggressiveness of bidding, reflecting the effects of more concentrated information on the winner's curse correction. Second, we ask what impact such aggressiveness has on the final auction price. We find that bids do increase as a result of mergers and do so in correlation with concentration, although the latter can be confirmed statistically only for the affiliated signals case. However, this more aggressive bidding does not offset the downward price pressures of diminished competition.

A first extension is to consider asymmetric mergers. Such analysis could have a prescriptive purpose, aiding our understanding of the incentives to merge and of merger "waves," as firms not yet part of a bidding ring may perceive their inferior information as a competitive disadvantage. A second extension may focus on multi-unit auctions in which the informational effects must also be balanced against demand reduction.

Traditional policy analysis exploits the robust relationship between market concentration and industry performance. In the case of private-value auctions, traditional industrial organization models prove fruitful if capacities and market shares are replaced by budget constraints and distributions of valuations. The growth of auctions as allocation tools has called into question the causal relationship between market structure and efficiency, driven by a number of interdependent effects which make the consequences of mergers and collusion unclear. While we highlight experimental results in line with the traditional thinking about the effects of mergers on prices, much more exploration is required.

It is difficult to provide comparisons across auction types. Ideally, a unified theory would provide the conditions under which post-merger bidding is more aggressive, and under which the final auction price is expected to be higher. However, we know that such results rely critically on the specification of the value function, distribution of bids, and the partition of information among bidders. Even subtle changes in any of these assumptions may put the analytical computation of the equilibrium bids out of reach. Yet, in two common value auctions with very different properties, we observe similar responses to mergers both in aggressiveness of bidding and final price. While the magnitude of these adjustments varies, the commonality of the patterns implies (limited) hope that generalizations are possible.

Appendix A: Proof of Lemma 1

To establish the proof of the first lemma, we require two technical results.

Claim 1 *The equilibrium bid in a first-price average value auction with n bidders can be written as*

$$\beta_n^{FP,X}(x) = \frac{(n-1)}{n} E[X|X \leq x] + \frac{1}{n} E[\max(X_1, \ldots, X_{n-1})| \max(X_1, \ldots, X_{n-1}) < x],$$

while for the same model the second-price symmetric equilibrium bid is

$$\beta_n^{SP,X}(x) = \frac{2}{n} x + \frac{n-2}{n} E[X|X \leq x].$$

Proof. Following Milgrom and Weber (1982), define

$$v(x, y) = E[V|X_1 = x, Y^1 = y]$$

where Y^1 is the highest amongst all signals other than X_1. In first-price auctions with independent signals the symmetric equilibrium bid is given by

$$\beta_n^{FP,X}(x) = \frac{\int_0^x v(s, s) dF^{n-1}(s)}{F^{n-1}(x)},$$

while in second-price auctions

$$\beta_n^{SP,X}(x) = v(x, x).$$

Recomputing, we get

$$\beta_n^{FP,X}(x) = \frac{1}{F^{n-1}(x)} \int_0^x \left(s - \frac{n-2}{n} \left(\frac{\int_0^s F(t)dt}{F(s)} \right) \right) dF^{n-1}(s) =$$

$$E[\max(X_1, \ldots, X_{n-1})| \max(X_1, \ldots, X_{n-1}) < x] - \left(\frac{n-2}{n} \right) \frac{1}{F^{n-1}(x)} \left(\int_0^x \left(\frac{\int_0^s F(t)dt}{F(s)} \right) dF^{n-1}(s) \right)$$

where n is the number of active bidders and $X, X_1, \ldots, X_{n-1}$ are independent draws from the same distribution as the private information. The second term can be rewritten as

$$\left(\frac{n-2}{nF^{n-1}(x)} \right) \left(\int_0^x \left(\frac{\int_0^s F(t)dt}{F(s)} \right) dF^{n-1}(s) \right) =$$

$$\frac{(n-2)(n-1)}{nF^{n-1}(x)} \int_0^x f(s)F^{n-3}(\int_0^s F(t)dt)ds =$$

$$\frac{(n-1)}{nF^{n-1}(x)} \left(F^{n-2}(x) \int_0^x F(t)dt - \int_0^x F^{n-1}(t)dt \right) =$$

$$\frac{(n-1)}{n} \left(\frac{\int_0^x F(t)dt}{F(x)} - \frac{\int_0^x F^{n-1}(t)dt}{F^{n-1}(x)} \right) =$$

$$\frac{(n-1)}{n} \left(E[\max(X_1, \ldots, X_{n-1})| \max(X_1, \ldots, X_{n-1}) < x] - E[X|X \leq x] \right).$$

The bidding function may be rewritten as

$$\beta_n^{FP,X}(x) = \frac{(n-1)}{n} E[X|X \le x] + \frac{1}{n} E[\max(X_1, \ldots, X_{n-1})| \max(X_1, \ldots, X_{n-1}) < x]$$

Note that in the case where $n = 2$, this expression is simply $\beta_2(x) = E[X|X \le x]$.

For the second-price auction in our model we have, without loss of generality,

$$\begin{aligned} \beta_n^{SP,X}(x) &= E\left[\frac{\sum X_i}{n} | X_1 = x, X_2 = x, X_3 \le x, \ldots, X_n \le x\right] \\ &= \frac{2}{n}x + \frac{n-2}{n} E[X|X \le x]. \end{aligned}$$

■

Claim 2 *If F and G are positive continuous functions such that $[F(x)/G(x)]$ is decreasing (increasing) in x, then*

$$\frac{\int_0^x F(s)ds}{\int_0^x G(s)ds} \ge \frac{F(x)}{G(x)},$$

and $\frac{\int_0^x F(s)ds}{\int_0^x G(s)ds}$ is decreasing (increasing) in x.

Proof. We will provide a proof only for the decreasing case. The statement for increasing can be obtained analogously. By Lagrange's theorem there exists $\xi \in [0, x]$ such that

$$\frac{\int_0^x F(s)ds}{\int_0^x G(s)ds} = \frac{F(\xi)}{G(\xi)} \ge \frac{F(x)}{G(x)}.$$

This property implies that $\frac{\int_0^x F(s)ds}{\int_0^x G(s)ds}$ is decreasing in x, since there exists an $\eta \in [x, x+y]$

$$\frac{\int_0^x F(s)ds}{\int_0^x G(s)ds} \ge \frac{F(x)}{G(x)} \ge \frac{\int_x^{x+y} F(s)ds}{\int_x^{x+y} G(s)ds} = \frac{F(\eta)}{G(\eta)},$$

and hence

$$\frac{\int_0^{x+y} F(s)ds}{\int_0^{x+y} G(s)ds} = \frac{\int_0^x F(s)ds + \int_x^{x+y} F(s)ds}{\int_0^x G(s)ds + \int_x^{x+y} G(s)ds} \le \frac{\int_0^x F(s)ds}{\int_0^x G(s)ds}.$$

■

Finally, we can provide a proof of the first lemma.

Proof of Lemma 1. We proceed in two steps. First, we establish that if $X_1, \ldots, X_m$ are i.i.d. variates, then

$$E[\max(X_1, \ldots, X_k)| \max(X_1, \ldots, X_k) \le x] \le E[\max(X_1, \ldots, X_m)| \max(X_1, \ldots, X_m) \le x]$$

for all x if $k \le m$. Note that $\max(X_1, \ldots, X_k) \sim F^k$ and $\max(X_1, \ldots, X_m) \sim F^m$ and by the above claim, this means that $\frac{\int_0^x F^k(s)ds}{\int_0^x F^m(s)ds}$ is decreasing. Taking the derivative of the logarithm of the last expression we observe that

$$\frac{\int_0^x F^m(s)ds}{F^m(x)} \leq \frac{\int_0^x F^k(s)ds}{F^k(x)}$$
$$x - \frac{\int_0^x F^k(s)ds}{F^k(x)} \leq x - \frac{\int_0^x F^m(s)ds}{F^m(x)}$$

which establishes the first step. In the second step, we show that $\beta_n^{FP,X}(x) - \beta_{n-1}^{FP,X}(x)$ crosses 0 only once. Note that

$$\begin{aligned}
\beta_n^{FP,X}(x) - \beta_{n-1}^{FP,X}(x) &= \frac{1}{n(n-1)}E[X|X \leq x] \\
&+ \frac{1}{n}E[\max(X_1, \ldots, X_{n-1})|\max(X_1, \ldots, X_{n-1}) \leq x] \\
&- \frac{1}{n-1}E[\max(X_1, \ldots, X_{n-2})|\max(X_1, \ldots, X_{n-2}) \leq x] \\
&= \frac{1}{n(n-1)}\left(n\frac{\int_0^x F^{n-2}(s)ds}{F^{n-2}(s)} - (n-1)\frac{\int_0^x F^{n-1}(s)ds}{F^{n-1}(s)} - \frac{\int_0^x F(s)ds}{F(s)}\right).
\end{aligned}$$

Define the random variable

$$Z = X|X \leq x$$

Then the above may be rewritten as

$$\beta_n^{FP,X}(x) - \beta_{n-1}^{FP,X}(x) = \frac{1}{n(n-1)}\left((n-1)\mu_{n-1:n-1}^Z - n\mu_{n-2:n-2}^Z + \mu_{1:1}^Z\right).$$

Using the recurrence relationship for the moments of order statistics we have that

$$(n-1)\mu_{n-1:n-1}^Z - n\mu_{n-2:n-2}^Z + \mu_{1:1}^Z = \left(\mu_{n-1:n-1}^Z - \mu_{n-2:n-2}^Z\right) + \left(\mu_{1:1}^Z - \mu_{n-2:n-1}^Z\right).$$

Note that both $\mu_{n:n}^Z$ and $\mu_{n-1:n}^Z$ are increasing sequences that tend to x. Furthermore, it is well established that $\mu_{n:n}^Z - \mu_{n-1:n-1}^Z$ is decreasing in n. Combining these facts we have that $\left(\mu_{n-1:n-1}^Z - \mu_{n-2:n-2}^Z\right) + \left(\mu_{1:1}^Z - \mu_{n-2:n-1}^Z\right)$ crosses 0 only once for each Z (and implicitly for each x) giving us the desired result. ■

Appendix B: Proof of Lemma 2

In the next lemma, we use the log-concave order ($\preceq_{lc}$) introduced by Whitt (1985). Considering the *i.i.d.* random variables $Y_1, \ldots, Y_n$ and the *i.i.d.* random variables $X_1, \ldots, X_n$, $Y_i \preceq_{lc} X_i$ indicates that the Y's are more concentrated (less dispersed) than the X's. One way to obtain the Y's is by considering mean-preserving spreads that are symmetric around the mean.[18] This corresponds to

[18]The definition is restricted to cases where X and Y are incomparable in the sense of first order stochastic dominance. See Shaked and Shanthikumar (1994). All we need for the result is the log-concavity of the ratio F_Y/F_X.

the notion that bidders are better informed upon receiving a signal Y rather than X. Note that if X is distributed with a log-concave density then it can be easily established that

$$\frac{\sum_{i=1}^{k} X_i}{k} \preceq_{lc} X.$$

We make use of the following claim from (3). Assume that the support of X and Y is $[0, 1]$.

Claim 3 *If $X \preceq_{lc} Y$ and $E[X] = E[Y]$, then $E[X|X \leq x] \geq E[Y|Y \leq x]$ for all x.*

Proof. $X \preceq_{lc} Y$ means by definition that $\frac{f_X}{f_Y}$ is unimodal on the support of Y, implying that

$$\frac{F_X}{F_Y} \text{ and } \frac{\int_0^x F_X(s)ds}{\int_0^x F_Y(s)ds}$$

are both unimodal (Whitt 1985). The equality of means, $E[X] = E[Y]$, implies that $\frac{\int_0^1 F_X(s)ds}{\int_0^1 F_Y(s)ds} = 1$. Hence, we need to demonstrate that $\frac{\int_0^x F_X(s)ds}{\int_0^x F_Y(s)ds}$ has a mode at 1 to establish our result. In other words, we need to show that $\frac{\int_0^x F_X(s)ds}{\int_0^x F_Y(s)ds} \leq 1$ for all x. We will proceed by contradiction. Assume that $x^* < 1$ is the mode of $\frac{\int_0^x F_X(s)ds}{\int_0^x F_Y(s)ds}$. Also denote by $y^* < 1$, the mode of $\frac{F_X}{F_Y}$. The existence of y^* is guaranteed by the definition of the unimodal order. We also know by Whitt (1985) that $y^* \leq x^*$. Then it should be true that for every $s > y^*$ we have that

$$\frac{F_X(s)}{F_Y(s)} > 1$$

which implies that

$$\frac{\int_x^1 F_X(s)ds}{\int_x^1 F_Y(s)ds} > 1$$

for every $x > y^*$. In particular it means that

$$\frac{\int_{x^*}^1 F_X(s)ds}{\int_{x^*}^1 F_Y(s)ds} > 1$$

but

$$\frac{\int_0^{x^*} F_X(s)ds + \int_{x^*}^1 F_X(s)ds}{\int_0^{x^*} F_Y(s)ds + \int_{x^*}^1 F_Y(s)ds} = 1$$

this means

$$\frac{\int_0^{x^*} F_X(s)ds}{\int_0^{x^*} F_Y(s)ds} < 1$$

in contradiction with the assumption that x^* is a mode. Finally, note that $\frac{\int_0^x F_X(s)ds}{\int_0^x F_Y(s)ds}$ is increasing in x, implying $E[X|X \leq x] \geq E[Y|Y \leq x]$ for all x, since

$$E[X|X \leq x] = x - \frac{\int_0^x F_X(s)ds}{F_X(x)}.$$

■

Define $\mu^X_{k:n}$ as the expectation of the k^{th} order statistic out of a sample of n independent draws from a distribution X. Let X and Y have common support given by $[0,1]$ and let $E[X] = E[Y]$. Denote by $\beta^{FP,X}_n(x)$ and $\beta^{FP,Y}_n(x) : [0,1] \to \mathbb{R}$ the symmetric equilibrium bid functions in first-price average value auctions with n bidders who receive private signals drawn from X and Y, respectively.

Proof of Lemma 2. Denote by F and G the cumulative distribution functions of X and Y, respectively. The log-concave order implies that $\frac{G}{F}$ is unimodal. Let $t \in (0,1)$ be the mode. Then t is also the mode for $\frac{G^{n-1}}{F^{n-1}}$. Following a reasoning similar to that in claim 2 we can infer that $\frac{\int_0^x G^{n-1}(s)ds}{\int_0^x F^{n-1}(s)ds}$ is also unimodal with a mode $\hat{t} \geq t$. In other words $\frac{\int_0^x G^{n-1}(s)ds}{\int_0^x F^{n-1}(s)ds}$ is increasing on $[0,\hat{t}]$. This implies that

$$E[\max(Y_1,\ldots,Y_{n-1})|\max(Y_1,\ldots,Y_{n-1}) \leq x] \geq E[\max(X_1,\ldots,X_m)|\max(X_1,\ldots,X_m) \leq x]$$

for all x in $[0,\hat{t}]$, since $\max(Y_1,\ldots,Y_{n-1}) \sim G^{n-1}$ and $\max(X_1,\ldots,X_{n-1}) \sim F^{n-1}$. Claim 3 also guarantees that $E[Y|Y \leq x] \geq E[X|X \leq x]$ for all x. This establishes the existence of $t_{X,Y,n} \geq \hat{t}$ with the desired property.

To demonstrate the existence of t', we observe that the function $max : [0,1]^n \to \mathbb{R}$ is convex in every component and since $Y_i \preceq_{lc} X_i$ we have that

$$E[\max(Y_1,\ldots,Y_n)] < E[\max(X_1,\ldots,X_n)]$$

for all $n > 2$.[19] Using (1), we have

$$\begin{aligned}\beta^{FP,Y}_n(1) &= \frac{n-1}{n}E[Y] + \frac{1}{n}E[\max(Y_1,\ldots,Y_n)] \\ &< \frac{n-1}{n}E[X] + \frac{1}{n}E[\max(X_1,\ldots,X_n)] = \beta^{FP,X}_n(1)\end{aligned}$$

establishing the existence of $t'_{X,Y,n}$ with the desired properties. ■

Observe that in our case, $t_{X,Y,n} = t'_{X,Y,n}$, which is a special case of the theorem above. Further, in our formulation, as in any other example based on symmetric distributions, one can easily show that $t_{X,Y,n} \geq E[X]$. The lemma states that better informed bidders will increase their bids for low-values while decreasing their bid for high signals. Mares (2001) demonstrates that this effect is even stronger in second-price auctions since (for equivalent signals and inferences) the bidder does not have to shade his equilibrium bid. In that case, better information leads to more aggressive bidding for every signal.

Appendix C: Proof of Theorem 1

A merger or collusive bidding ring involves the effects described in both of the above lemmas. A decrease in the number of bidders is associated with a reduction in the winner's correction. We can also note that (at least for log-concave signals) the merged groups are better informed in the sense

[19] See Shaked and Shanthikumar (1994) for details.

of the previous lemma than individual bidders. As such, this should add to their level of bidding for low signal values while tempering the bids for high signals. The combination of these effects is characterized in the following result. Consider $n = km$ and denote as before by $\beta_{km}^{FP,X}$ the equilibrium bid function followed by individual bidders in a first-price average value auction. Let $\frac{\sum_{i=1}^{k} X_i}{k} \sim \overline{X}_k$ and denote by $\beta_m^{FP,\overline{X}_k}$ the equilibrium bid function of the m bidding rings each receiving k signals. Note that the theorem below leaves the door open for the possibility of a crossing between the two bidding functions, at least for low values of m.

Proof of Theorem 1. The first part of (i) is a direct consequence of the previous two lemmas. For the second part, observe that the equilibrium bids are

$$\begin{aligned} \beta_{km}^{FP,X}(1) &= \frac{km-1}{km} E[X] + \frac{1}{km} E[\max(X_1, \dots, X_{km})], \text{ and} \\ \beta_m^{FP,\overline{X}_k}(1) &= \frac{m-1}{m} E[\overline{X}_k] + \frac{1}{m} E[\max(\overline{X}_{k,\,1}, \dots, \overline{X}_{k,\,m})]. \end{aligned}$$

By definition, $E[X] = E[\overline{X}_k]$. Note that for large m we have the following relationships

$$\left| E[\max(\overline{X}_{k,\,1}, \dots, \overline{X}_{k,\,m})] - E[\max(X_1, \dots, X_{km})] \right| \leq \varepsilon$$

for ε arbitrarily small since

$$\lim_{m \to \infty} E[\max(\overline{X}_{k,\,1}, \dots, \overline{X}_{k,\,m})] = \lim_{m \to \infty} E[\max(X_1, \dots, X_{km})] = 1.$$

This means that we can find m_k such that for $m > m_k$

$$E[\max(\overline{X}_{k,\,1}, \dots, \overline{X}_{k,\,m})] > \frac{(k-1)}{k} E[X] + \frac{1}{k} E[\max(X_1, \dots, X_{km})]$$

since as before we have that

$$E[\max(X_1, \dots, X_{km})] > E[\max(\overline{X}_{k,\,1}, \dots, \overline{X}_{k,\,m})] > E[X].$$

But this also establishes that for $m > m_k$

$$\begin{aligned} \beta_{km}^{FP,X}(1) &= \frac{m-1}{m} E[X] + \frac{1}{m} \left(\frac{(k-1)}{k} E[X] + \frac{1}{k} E[\max(X_1, \dots, X_{km})] \right) \\ &< \frac{m-1}{m} E[X] + \frac{1}{m} E[\max(\overline{X}_{k,\,1}, \dots, \overline{X}_{k,\,m})] \\ &= \beta_m^{FP,\overline{X}_k}(1). \end{aligned}$$

This gives us the second part of the theorem. ■

Appendix D: Proof of Theorem 2

In the proof of the revenue theorem we make use of the following claim. Let private signals be distributed according to X, and let F represent the cumulative distribution.

Claim 4 *The revenue from the symmetric equilibrium in an average value auction with n bidders may be expressed as*

$$R^F(n) = \mu^F - (\mu^F_{n:n} - \mu^F_{n-1:n-1}).$$

Proof. We will present a proof for second-price auctions. Since we are in symmetric environments with independent signals, revenue equivalence applies, so our results carry over to first-price auctions.[20] The symmetric equilibrium bid in a second-price average value auction is

$$\beta_n^{SP,X}(x) = \frac{2}{n}x + \frac{n-2}{n}E[X|X \le x] = x - \frac{n-2}{n}\frac{\int_0^x F(s)ds}{F(x)}.$$

The revenue, given distribution F, can be computed as

$$R^F(n) = \int_0^\infty \beta_n^{SP,X}(x) f_{n-1:n}(x)dx.$$

We will make use of the following recurrence relationship between the expectations for order statistics

$$n\mu^F_{n-1:n-1} = \mu^F_{n-1:n} + (n-1)\mu^F_{n:n}.$$[21]

Also note that the density of the second highest order statistic out of a draw of n *i.i.d.* variates is

$$f_{n-1:n}(x) = n(n-1)f(x)F^{n-2}(x)(1-F(x)).$$

Using the above relationships we can establish using integration by parts

$$\begin{aligned}
R^F(n) &= \mu^F_{n-1:n} - \int_0^\infty (n-1)(n-2)\left(\int_0^x F(s)ds\right) f(x)F^{n-3}(x)(1-F(x))dx \\
&= \mu^F_{n-1:n} - \int_0^\infty \left(\int_0^x F(s)ds\right) f_{n-2:n-1}(x)dx \\
&= \mu^F_{n-1:n} - \int_0^\infty F(x)\left(1 - F_{n-2:n-1}(x)\right)dx \\
&= \mu^F_{n-1:n} - \int_0^\infty F(x) - (n-1)F^{n-1} + (n-2)F^n dx \\
&= \mu^F_{n-1:n} + \mu_F - (n-1)\mu^F_{n-1:n-1} + (n-2)\mu^F_{n:n} \\
&= \mu^F - (\mu^F_{n:n} - \mu^F_{n-1:n-1}).
\end{aligned}$$

■

Proof of Theorem 2. Consider for simplicity the case where $n = 2m$ bidders participate in the auction and merge into m groups, each of size 2. Let G be the distribution of the average of two private signals. According to the previous claim we have that the revenue following a merger is given by

$$\begin{aligned}
R^G(m) &= \mu^G - (\mu^G_{m:m} - \mu^G_{m-1:m-1}) \\
&= \int_0^\infty \left(1 - G(s) - (1 - G^m(s)) + 1 - G^{m-1}(s)\right) ds \\
&= \int_0^\infty (1 - G(s))(1 - G^{m-1}(s))ds
\end{aligned}$$

[20] See for example Waehrer, Harstad, and Rothkopf (1998).

[21] See, for example, Arnold, Balakrishnan, and Nagaraja (1992).

By a similar argument the revenue in the absence of a merger is

$$R^F(2m) = \int_0^\infty \big(1 - F(s))(1 - F^{2m-1}(s)\big)\, ds.$$

Note also that by standard stochastic variability order arguments,[22] we can establish that

$$\int_0^x (1 - F(s))ds \geq \int_0^x (1 - G(s))ds \tag{4}$$

for every x. Since, by definition, we have that $\mu^F = \mu^G$, we can conclude that

$$R^F(2m) \geq R^G(m) \Leftrightarrow \int_0^\infty F^{2m-1}(s)(1 - F(s))ds \leq \int_0^\infty G^{m-1}(s)(1 - G(s))ds.$$

We will establish the latter inequality in two steps

$$\int_0^\infty F^{2m-1}(s)(1 - F(s))ds \leq \int_0^\infty F^{2m-1}(s)(1 - G(s))ds$$

and

$$\int_0^\infty F^{2m-1}(s)(1 - G(s))ds \leq \int_0^\infty G^{m-1}(s)(1 - G(s))ds.$$

The last step is apparent once we observe that $\max(X_1, X_2) \geq \frac{X_1+X_2}{2}$, and since $\max(X_1, X_2) \sim F^2$ and $\frac{X_1+X_2}{2} \sim G$ we have by stochastic dominance that $F^2 \leq G$ and hence

$$F^{2m-1} \leq G^{m-1}$$

which gives us the desired second inequality.

For the first step, we can treat $1 - F$ and $1 - G$ (if necessarily normalized by μ^F and μ^G) as probability densities. The inequality in (4) then establishes the stochastic dominance relationship between these two densities. Since F^{2m-1} is increasing, we obtain the desired inequality in step 1. Similar arguments establish the result for mergers with group size of $k \geq 2$. ■

[22] See for example Shaked and Shanthikumar (1994).

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