Market Power and Information Revelation in Dynamic Trading*

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Abstract

We study a strategic model of dynamic trading where agents are asymmetrically informed over common value sources of uncertainty. There is a continuum of uninformed buyers and a finite number of sellers, some of them informed. When there is only one seller, full information revelation never occurs in equilibrium and the only information transmission happens in the first period. The outcome with n sellers depends both on the structure of sellers' information and the intensity of competition among them allowed by the market rules. We show that the latter plays an even more important role. With intense competition (absence of clienteles), information is fully and immediately revealed to the buyers in every equilibrium for n large enough, both when all sellers are informed and when only one seller is informed. On the other hand, with a less intense form of competition (presence of clienteles), collusive equilibria, where information is never revealed, also exist, whatever the number of sellers. Moreover, when

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only one seller is informed, for many parameter configurations there are no equilibria with full information revelation, for any n.

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1. Introduction

This paper studies a strategic model of dynamic trading where: (1) non-negligible agents interact with negligible ones –a finite number of sellers serve a continuum of buyers, and (2) there is asymmetric information about the quality of the good being traded, an instance of common values uncertainty. In this situation the sources of a seller's market power are two-fold: his size relative to the market allows him to manipulate the terms of trade, and his private information serves him to manipulate the other traders' beliefs. Furthermore, since trade extends over infinitely-many periods, current actions have important effects on the agent's market power in the future (if today's action were to completely reveal the trader's information, his informational advantage would be null in the future; similarly, a gain in today's market share may only come at the expense of future trades).

We are interested in identifying how the agents' market power depends on their size and the nature of their information, and the implications of this for the properties of the trading process. In the dynamic set-up we consider, we can then analyze the dynamics of competition and how market power evolves over time. We shall identify the conditions under which the equilibria with strategic trading "converge" to the "competitive" equilibria.

With complete information, perfect competition arises as a fairly robust limit of game theoretic models (describing both strategic and cooperative interaction among traders) when the number of competitors increases.[1]

Even with asymmetric information, if this is of the private values type, the competitive limit result seems to be the norm (see, e.g., Rustichini, Satterthwaite and Williams (1994) in a double auction context). This is true also in decentralized dynamic market structures, where agents' market power is further constrained by competition over time (Gale (1987), Serrano (2000)). In a dynamic set-up market power may vanish even in the case of monopoly: the Coase conjecture

[1] See Mas-Colell, Whinston and Green (1995, Chapter 18).

for a durable goods monopolist, uninformed about the demand curve he faces, shows in fact that monopoly pricing, under reasonable assumptions, converges to the competitive price as the frequency of trades increases (see, for example, Gul, Sonnenschein and Wilson (1986)).

In contrast to these encouraging results, when asymmetric information is of the common values type, various complications arise.[2] A first issue concerns the notion of competitive equilibrium. The most frequently used Walrasian equilibrium notion in economies with common values uncertainty and asymmetric information is given by rational expectations equilibria (REE).[3] These are (generically) characterized by full and immediate information revelation. This notion is more demanding than usual Walrasian equilibria, since it requires both that agents have no market power (act as price-takers) and that all private information is fully disclosed. The strategic foundations of this concept require then the investigation of trading procedures where the transmission of information among the agents in the market is explicitly modelled.

Decentralized trading procedures were investigated in one line of literature, initiated by Wolinsky (1990) and continued by Blouin and Serrano (2001), among others. In these papers there are infinitely many trading dates. Each period agents chosen from a continuum are randomly matched in pairs; within each pair they bargain over the terms of trade (rejection of all offers and hence refusal to trade is always an available option). Two aspects of this trading procedure are worth stressing. First, information is heavily restricted; agents have no access to any public market signal, nor do they have any information over the terms at which trade occurred in the past in the market. And second, each pairwise meeting (which can be thought of as a buyer visiting a particular store) represents a "local monopoly" in its own right, in the sense that in that period the buyer can buy the good only from that store. Of course, he can walk out and visit a different store, but this comes at a cost, as captured by discounting. The findings of these papers are very different from the fully revealing REE: even in the limit when discounting is zero and there are many traders, a sizeable fraction of uninformed traders transact at prices that are *ex post* not individually rational.

[2] Non-convergence results can also be found outside of this case; the question of whether there is a general competitive limit in economies with asymmetric information over states affecting the allocation rule remains far from settled. For example, as shown in Serrano, Vohra and Volij (2001), even in the presence of private information over sunspot uncertainty, there are serious obstacles to the core convergence theorem, a cornerstone of the competitive limit in economies with complete information.

[3] See Radner (1982) for a survey.

Fully centralized trading mechanisms were also analyzed in various other papers. Dubey, Geanakoplos and Shubik (1987) and more recently Forges and Minelli (1997) studied the case where trading takes place via market games a la Shapley-Shubik, in the presence of a continuum of agents of finitely many types. They showed that, when the trading game is repeated, the Nash equilibria of the trading game are such that the first stage is used to exchange information among traders (equilibrium prices act as public signals), and in the subsequent stages the outcome coincides, under appropriate conditions, with the fully revealing REE.

The case of a single informed trader acting strategically on the basis of his information was considered both in a one-period trading model by Grinblatt and Ross (1985), Laffont and Maskin (1990), and in a dynamic trading model by Kyle (1985): in both cases it was shown that the monopolist may choose not to completely reveal his private information and that equilibria differ from REE. This is more surprising in a dynamic set-up because hiding the information over time is a harder task. Some qualifications are needed here. Kyle in fact considers a model where noise traders are also present and prices are determined each period by competitive market-makers on the basis of the observation of aggregate trades. Thus, aggregate trades and, a fortiori prices, cannot fully reveal the private information of the informed trader. Even if he were to act non strategically, full revelation could only be achieved in the limit, after infinitely many rounds of trade. When the informed acts strategically, convergence of prices to their full information value also occurs, but it is slower since the informed "hides" his trades from the market. Hence, the relevant issue here is that of the speed of convergence to the full information prices.

Kyle's work was later extended in various directions in a number of papers. In particular, Vives (1995) examines the case of a large number of risk averse informed traders acting myopically, finding that convergence is rather fast. On the other hand, more recently, Berg et al. (2001) find that information aggregation increases with the "number of agents-number of states of the world" ratio, thereby uncovering a threshold for complete learning. The case of finitely many traders with private information was analyzed by Kyle (1989) in the framework of a static model, again with noise traders: he showed the existence of equilibria where only part of the information is revealed and studies how the amount of information revelation varies with the share of informed traders relative to both uninformed and noise traders.[4]

[4] Analyzing a static oligopoly model with asymmetric information about firms' costs, Vives (1999b) points out that information rents, while disappearing as the number of firms $n \to \infty$,

In this paper we attempt to reconcile these seemingly contradictory results. Can one expect information revelation even in a large decentralized common values market? To what extent informed traders can successfully hide their information in the absence of noise traders? Under what conditions, in terms of number of traders and proportion of informed agents, do we obtain convergence to REE?

We will study a highly stylized model of dynamic trading, where all traders are fully rational (there are no noise traders) and sellers compete by posting prices at which they are willing to sell. In order to better focus on informational issues, price competition is suitably simplified. We analyze the model under various market and information structures, to identify the conditions under which information revelation occurs. Finding the optimal pricing policy over time will allow us to determine how much information is revealed in each period.

One criticism of the pairwise meetings papers mentioned above is that it has gone too far in limiting the possibilities of information transmission among traders. It is plausible to model a situation in which traders receive some market signal, though not necessarily the price function called out by the auctioneer. Along these lines, in this paper we shall assume that each period buyers get to observe *all* prices posted by sellers before they agree to buy the good.[5] The specifics of our model follow.

Consider a market for an indivisible good of uncertain quality. All the units of the good in all periods are of the same quality, either high or low. Thus, there are two states of the world: H and L. There is a continuum of buyers, all of them uninformed about the state. There is a finite number n of sellers, at least one of them being informed (in fact, we will study the two extreme benchmarks – all sellers being informed and exactly one seller being informed[6]). If all sellers are informed, the economy is one of non-exclusive information and all of them are "informationally small."[7] If only one of them is informed, this seller is "informationally big" relative to the other agents in the model. Every period, each seller simultaneously chooses whether to post a high or a low price for the good. The restriction of having only two possible prices allows us to limit the role of the beliefs of the uninformed and to characterize all the equilibrium outcomes

do so at a lower pace than the strategic rents of a standard oligopoly. Vives (1999a, Chapter 8) surveys other models of oligopolistic and monopolistic competition with asymmetric information.

[5]See Peters (1991) for a related model, where agents' matching is non-random because it is affected by the posted prices.

[6]It follows from our analysis that information structures between these two yield similar results.

[7]See Gul and Postlewaite (1992).

of the model[8]. Upon observing the prices posted in a period, buyers can either refuse to buy at any of these prices (in which case they will continue to be in the market the next period), or accept to buy from one of the sellers at the price he announced (in which case they will leave the market forever). Sellers remain in the market until all buyers are served, which may never happen. There are no new entrants in the market after the first period. All traders discount future payoffs at the same rate, and all our results hold for small but positive discounting.

Given our simple information structures and the two-price assumption, it follows that an informed seller in state H will always price high, while an informed seller in state L may or may not price low. To find his optimal pricing policy in state L, he needs to weight the gains from hiding the information and selling at a high price against the potential losses that may be created by losing market share if his competitors undercut.

We begin by considering the case of an informed monopolist. We find that a monopolist reveals only part of his information, only in the first period and when the buyers' prior belief is too pessimistic about the state being H. Not threatened by the presence of any competitor, a monopolist has no incentive to reveal the information once the buyers are convinced that consuming the good brings them a non-negative expected utility; no further information is thus released at any later date. In contrast to Kyle's results, in the absence of noise traders it is both possible and optimal for the single informed trader to permanently hide his information from the rest of the market; continued randomization, which progressively reveals the information, turns out to be not optimal. This section serves as an important benchmark for the analysis in the following sections.

The paper proceeds to analyze the case of oligopoly under the two extreme information structures mentioned above. It turns out that the intensity of competition among sellers plays a more important role than the structure of the information for the revelation of information and the convergence to REE. To see this, we will distinguish two cases characterized by the difference in the intensity of price competition. In the first one - denoted as the model without clienteles - competition is quite intense, as in the classic Bertrand model: there are no frictions to price competition within each period, since buyers are free to buy the good from the seller that is offering it at the lowest price. The model with clienteles imposes instead the restriction that within a period each of the n sellers is "assigned" to $1/n-th$ of the market, which cannot be accessed in that

[8]Moreover, as argued in the final section, the substance of our results remains valid even if this restriction is dropped.

period by the other sellers. In future periods, though, the buyers remaining in the market are "reassigned", randomly, to all the sellers; hence whatever information was transmitted in the offers made by the informed sellers can be profitably used in the subsequent periods by the uninformed sellers and buyers. Thus, price competition is mitigated - but not eliminated - by the presence of clienteles (as in Cournot, or Bertrand with capacity constraints).

We show that in the model without clienteles, information is fully and immediately revealed to buyers in every equilibrium for large enough n, no matter what is the structure of the information. The intensity of price competition and the strong incentive to undercut in order to expand the own market share imply that any equilibrium with many informed sellers where they "collude" to hide their information is very fragile and will not survive, when the number of sellers is large enough. But even with only one informed seller, who has then exclusive information, the opportunity to expand his market today by posting a low price proves too strong, when n is sufficiently high, even though this means to dissipate any current and future informational rent. When the prior belief of the other uninformed sellers about high quality is sufficiently low, there is an equilibrium where all the uninformed immediately post a low price; all trade in the high state takes place at the low price. The information is then not revealed to the uninformed sellers, who end up trading at a non-individually rational price in state H, while it is revealed to buyers since the informed still follows a fully revealing strategy. All other equilibria where information is hidden to the buyers to some degree cease to exist when n is large enough.

The situation is rather different in the model with clienteles. When all sellers are informed there is again an equilibrium with full and immediate information revelation; however, a collusive equilibrium where (at least part of the) information is not revealed at any date also exists, whatever the number of sellers. Moreover, when only one seller is informed, for many parameter values there are no separating equilibria, all the equilibria are such that at least part of the information is never revealed, even for n large.

The lack of information revelation in the model with clienteles resembles the one created by the "local monopoly" of the pairwise meetings technology. It shows that even allowing for the presence of market signals observed by all traders and the non-exclusivity of information, the full revelation of information is not guaranteed. Our results suggest that the intensity of competition among sellers proves more effective than the non exclusivity of information in inducing full and immediate disclosure of the agents' private information. These insights shed some light on the negative results found in the pairwise meetings models and show that some qualification is needed for the convergence results obtained for centralized markets.

2. The Model

The economy. Agents are of two types: buyers and sellers. There are two commodities, an indivisible consumption good, initially owned by the sellers, and a perfectly divisible commodity ('money'), initially owned by the buyers. There is a continuum of buyers, whose measure is normalized to 1. All buyers are identical: each of them is willing to buy at most one unit of the consumption good. There are n sellers, $1 \leq n < \infty$, and each of them can sell an arbitrarily large number of units of the consumption good. Except for the information that each seller has, on which various assumptions will be made in the following sections, all sellers are identical.

There is uncertainty over the quality of the commodity (in particular over the buyers' and the sellers' valuations for it). There are two (aggregate) states of the world, H and L. The buyers' valuation for (one unit of) the commodity is u_H in state H and u_L in state L. Similarly, the sellers' valuation for each unit of the good is constant and equal to c_H in state H and c_L in state L. Note that the valuation of the commodity is perfectly correlated across agents; we are in a situation of common values uncertainty. We assume that: $u_H > c_H > u_L > c_L \geq 0$. Thus in each state there are gains from trade.

Let us denote by $\alpha_0 \in (0, 1)$ the prior belief, common to all agents in the economy, that the state of the world is H. We will always assume that buyers are uninformed over the realization of the state of the world; hence their belief, when trading begins, is given by α_0. On the other hand, different cases will be considered over the information sellers have over the realization of the uncertainty.

Trading rules. Trades are organized as follows. There are infinitely many discrete trading periods. At each date t, $t = 1, \ldots$, every seller simultaneously posts a price at which he is willing to sell the consumption good (to any number of buyers). After observing all the quoted prices, each buyer chooses whether or not to trade in that period. If the buyer accepts an offer made in the period, he buys a unit and then exits the market. If the buyer rejects, he remains in the market and can trade in a future.

When there is more than one seller, $n > 1$, we shall distinguish two cases in our analysis:

1. *The model without clienteles.* Each period t, following the announcement of the n posted prices by sellers, every buyer is free to choose to trade with any of the sellers. In this situation sellers compete in prices, under no capacity constraints; competition among sellers is quite extreme (as in the classical model of Bertrand competition).

2. *The model with clienteles.* Each period t, a fraction of size $1/n$ of the buyers who are still in the market is randomly assigned to every seller. Each buyer can only buy in that period from the seller he is assigned to. Since buyers observe all n posted prices, and always have the option to refuse to trade, the (temporary) segmentation of the market introduced here does not eliminate price competition among sellers. It only mitigates its intensity, since undercutting prices has a less dramatic effect on the demand for each seller (as in models of Bertrand competition where firms face capacity constraints). Furthermore, information leakages from one seller to other sellers or to the customers assigned to other sellers can still happen, as the information which may be revealed by his trading strategy is valuable not only for his own clientele, but for all agents in the economy.

For simplicity, we will restrict the set of prices that the sellers can propose to only two possible values, p_H and p_L. This parsimonious formulation allows us to provide a complete characterization of all the equilibria of the trading game, and to better focus on the issues concerning information revelation we are primarily concerned with. The two prices are such that $p_H \in [c_H, u_H]$ and $p_L \in [c_L, u_L]$. Thus, if the state was observable by all traders (with full and symmetric information), all possible trades would take place at the initial date, at a price p_H in state H and p_L in state L.

Both buyers and sellers evaluate payoffs from future trades according to the common discount factor $\delta \in (0, 1)$. In this paper we are more interested in the limiting results as $n \to \infty$ rather than as $\delta \to 1$. However, all our results hold for any $\delta < 1$ sufficiently large. The reader is invited to fix δ at this arbitrarily large value.

Various cases with regard to the number of sellers in the economy and their information will be considered. This will allow us to disentangle in the revelation of information the role of market power given by the sellers' size from that given by their private information. In each case we characterize the perfect Bayesian equilibria of the trading game described above. From now on we refer to a perfect Bayesian equilibrium simply as an equilibrium.

As a benchmark case, we begin by analyzing the model with an informed monopolist in Section 3. In Section 4 we examine the model without clienteles and in Section 5 the model with clienteles. In both cases, we analyze the properties of equilibria as the number of sellers goes to infinity and perform our study under the following two extreme information structures. First, one where all sellers are informed. In this case, as n grows large sellers are both "informationally small" and their market power as determined by their individual size becomes negligible. Second, we analyze the polar opposite case: there are various sellers, but only one of them is informed. Now, as n gets large, this seller remains "informationally large", while his size relative to the market decreases (as that of all sellers), so that information is the only possible source of market power. The welfare properties of the equilibria are then discussed and compared in Section 6. We conclude by arguing, in the last section, that most of the qualitative results obtained on the characterization of the equilibria are robust to the generalization of the model along several dimensions (in particular, when we have finitely many rather than infinitely many trading dates, and when sellers are free to announce any price, rather than only one of two possible prices).

3. Monopolist

Consider the case where there is only one seller ($n = 1$), who is fully informed of the realization of the uncertainty.

The following observations will allow us to simplify the definition of an equilibrium. Note first that when the seller observes state H (i.e., the seller is of 'type H') he will always propose p_H at any trading date. Also, whenever a buyer is proposed a price p_L he will always accept, no matter what his belief is over the realization of the uncertainty.

On the other hand, the 'type L' seller faces a non-trivial choice between offering p_H and p_L; similarly the buyers, when they are proposed p_H, have to decide whether to accept or reject. In both cases we will allow for the possibility that the agents may randomize in their choice. Let then $q_S(t)$ denote the probability that the seller in state L proposes p_H at date t (given histories according to which in all previous periods the seller always proposed p_H and the buyers always rejected); similarly, let $q_B(t)$ be the probability that a typical buyer accepts, at date t, if the seller proposes p_H, given histories in which the seller always proposed p_H in the past.[9]

Each period t, after observing the price proposed by the seller, the buyers will also update their belief over the state of the world. If the seller proposes p_L the buyers inference is irrelevant since, as already argued above, their optimal action is always to accept. Let α_t denote then the buyers' belief at date t that the state of the world is H if the seller proposed p_H at t and in all past periods; such belief is updated every period, using Bayes' rule and taking into account the strategy of the type L seller:

$$\alpha_t = \frac{\alpha_{t-1}}{\alpha_{t-1} + (1 - \alpha_{t-1})q_S(t)} \text{ for all } t \geqslant 1 \tag{3.1}$$

Note that α_t is always weakly increasing with t, and is strictly increasing as long as $q_S(t) < 1$.

An *equilibrium* of the trading game with an informed monopolist is then described by the sequences $\{q_B(t)\}_{t\geq 1}$, $\{q_S(t)\}_{t\geq 1}$, $\{\alpha_t\}_{t\geq 1}$ such that:

(i) $\{\alpha_t\}_{t\geq 1}$ satisfies (3.1);

(ii) at each t, after every partial history in which not all buyers accepted the seller's offer in one of the previous periods, $\{q_B(\tau)\}_{\tau\geq t}$ maximizes the buyers' discounted (to that date) payoff, given $\{q_S(\tau), \alpha_\tau\}_{\tau\geq t}$, and $\{q_S(\tau)\}_{\tau\geq t}$ maximizes the type L seller's discounted payoff, given $\{q_B(\tau)\}_{\tau\geq t}$.

We will provide a complete characterization of the equilibria of this game. It will be shown that we never have complete revelation of the seller's information. In particular, if the prior belief α_0 that the true state of the world is H is sufficiently high, no information is ever revealed in the trading process: all equilibria exhibit perfect pooling (of the two types of the seller) and no delay (all trades take place at the initial date). On the other hand, if the agents' prior belief over H is not high enough, some information gets revealed in the first trading date (as, with

[9] Additional equilibria can be found in which all buyers play pure strategies, and a proportion $q_B(t)$ accepts the high price at date t, while the rest reject it. These equilibria are outcome equivalent to the ones we study. This is the sense in which there is no loss of generality in restricting attention to symmetric equilibria, as we shall do.

some probability, the type L seller will propose a revealing price, p_L). But this information is revealed only to convince the buyer that the quality of the good is high enough to be able to sustain a high price with probability 1. After that is accomplished with the randomization of the first date, no further information is ever revealed. In this situation we have 'partial pooling'. In addition there is delay: while p_L is always immediately accepted by buyers, when p_H is proposed the buyers will reject, in all periods, with some positive probability. Delay can even be infinite.

Define $\bar{\alpha}$ to be such that $\bar{\alpha}u_H + (1-\bar{\alpha})u_L = p_H$. That is, $\bar{\alpha}$ is the belief that makes buyers exactly indifferent between trading at p_H with probability 1 and not trading at all. Formally, we have the following:

Proposition 1. *In the model with an informed monopolist the following equilibria obtain:*
(i) ***No information revelation****: when $\alpha_0 \geq \bar{\alpha}$, in any equilibrium, we have $q_S(t) = 1$ for all t. In particular, if $\alpha_0 > \bar{\alpha}$, the equilibrium is unique with $q_B(t) = 1$ for every t. On the other hand, if $\alpha_0 = \bar{\alpha}$, there are also equilibria where $q_B(t) \in (0, 1]$ for all $t \geq 1$.*
(ii) ***Partial and immediate revelation****: when $\alpha_0 < \bar{\alpha}$, in all equilibria, we have $q_S(1) \in (0, 1)$ so that $\alpha_1 = \bar{\alpha}$, $q_S(t) = 1$ for all $t > 1$, $q_B(t) \in (0, 1]$ for all $t \geq 1$.*

To gain some intuition on the result, notice that in our set-up the profits a monopolist seller can get from his private information and his market power lie in the possibility of manipulating buyers' beliefs and induce buyers to agree to trade at a high price, with minimal delay. Our result shows that the seller will always succeed in generating such beliefs, by revealing the minimal amount of information (possibly zero) which is necessary, and all in the first period. There is however a cost, given by the fact that there may be (possibly considerable) delay in trade; the need for such cost comes from the fact that the seller is otherwise unable to credibly commit to reveal his information.

To prove the proposition we will establish first some preliminary results.

A) Full and immediate information revelation never occurs (in finite time); at no date the seller in state L proposes p_L with probability 1 :

Lemma 1. *At any equilibrium, $q_S(t) > 0$ for every t.*

Proof of Lemma 1: Suppose not: there exists a period t such that $q_S(t) = 0$. Since, if the state is H, the seller proposes p_H for certain, it follows that there is full separation at t. Thus we have $\alpha_t = 1$. Upon observing p_H at t the buyers can infer the state is H for certain and so their optimal strategy must always be to accept p_H with probability 1 at date t ($q_B(t) = 1$). But then the optimal strategy of the seller in state L at t would be to propose p_H rather than p_L, a contradiction. ∎

B) If, at some t, the buyers' strategy is to accept p_H for sure, then in that period the type L seller will propose p_H for certain. This in turn implies, when the buyers' belief α_t is sufficiently close to 1, that at each earlier date the buyers should also prefer to accept p_H for sure; hence the seller must prefer to propose p_H for certain. Hence we have:

Lemma 2. *If, at an equilibrium, $q_B(t) = 1$ for some t, then we must also have $q_S(t) = 1$. If, in addition, $\alpha_t > \bar{\alpha}$ we obtain that, for every $t' < t$, $q_B(t') = q_S(t') = 1$.*

Proof of Lemma 2: This is easily established by backwards induction. If in period t, $q_B(t) = 1$, the seller's best response in that period is clearly $q_S(t) = 1$. Since $q_S(t) = 1$, $\alpha_{t-1} = \alpha_t$; thus, if $\alpha_t > \bar{\alpha}$ the buyers' payoff from accepting at $t-1$ is positive and strictly higher than the payoff from accepting at t, for all $\delta < 1$. The buyers' best response in period $t-1$ must then be $q_B(t-1) = 1$. Iterating the argument, we find that the same must be true at all previous dates $t' < t$. ∎

C) If in equilibrium buyers randomize for infinitely many periods, their belief α_t must jump to a sufficiently high level at the initial date and stay constant at that level forever after:

Lemma 3. *If, at every date t, $q_B(t) \in (0, 1)$, then we must have $\alpha_t = \bar{\alpha}$ for all t.*

Proof of Lemma 3: Note that the evolution of posterior beliefs α_t is determined from Bayes' rule using the seller's strategy and, accordingly, follows equation (3.1). As we already noticed, this sequence of posterior beliefs is non-decreasing in t.

In addition, to sustain the buyers' randomization in every period, we need the following condition to hold at all t:[10]

$$\begin{aligned} &\alpha_t u_H + (1-\alpha_t)u_L - p_H = \delta[(1-\alpha_t)(1-q_S(t+1))(u_L - p_L)+ \\ &+(1-(1-\alpha_t)(1-q_S(t+1)))(\alpha_{t+1}u_H + (1-\alpha_{t+1})u_L - p_H)] \end{aligned} \tag{3.2}$$

where on the left hand side we have the payoff from accepting p_H and, on the right hand side, the payoff from rejecting it at t and accepting it at $t+1$.

Because by hypothesis, the randomization involves infinitely many periods, the infinite sequence of posteriors $\{\alpha_t\}$, which is monotone and bounded, has a limit. We will show that $\lim_{t\to\infty}\alpha_t = \bar{\alpha}$.

Suppose $1 > \lim_{t\to\infty}\alpha_t > \bar{\alpha}$; in this case, by equation (3.1), $\lim_{t\to\infty} q_S(t) = 1$. But if we consider (3.2) and take the limit as $t \to \infty$, plugging $\lim_{t\to\infty} q_S(t) = 1$ in and using the fact that $\lim_{t\to\infty}(\alpha_t - \alpha_{t+1}) = 0$, we reach a contradiction, because for $\delta < 1$ the term on the right hand side of (3.2) will be strictly lower than the one on the left hand side, so buyers strictly prefer to accept.

Similarly, if $\lim_{t\to\infty}\alpha_t = 1$: the term on the left of (3.2) tends to $u_H - p_H$ while the term on the right tends to $\delta(u_H - p_H)$, which is strictly lower, thus again we get a contradiction.

It remains then to show that we cannot have $\lim_{t\to\infty}\alpha_t < \bar{\alpha}$ either. But this is immediate, since the expected payoff from accepting a p_H offer, $p_H - \alpha_t u_H + (1-\alpha_t)u_L$, is negative whenever $\alpha_t < \bar{\alpha}$. Thus buyers would be better off by refusing to trade altogether at t, instead of randomizing between accepting and rejecting.

Finally, note that, in order to sustain the randomization of the buyers at any date t, it must be $\alpha_t \geqslant \bar{\alpha}$. This fact, together with the property $\lim_{t\to\infty}\alpha_t = \bar{\alpha}$ established above, implies that infinite randomization of the buyers requires $\alpha_t = \bar{\alpha}$ for all t. ∎

We are now ready to prove the main result of the section:

Proof of Proposition 1: By Lemma 1, pure strategies where $q_S(t) = 0$ for some t can never be part of an equilibrium. Consider then the only other possible pure strategy of the type L seller, $q_S(t) = 1$ for all t.

If $\alpha_0 \geq \bar{\alpha}$, a best reply for the buyers to this strategy is $q_B(t) = 1$ for all t: their expected payoff is in fact non-negative and any other strategy would only induce delay and still result in either no trade or trade at the same price, p_H, yielding

[10] The same arguments apply if the randomization does not involve two consecutive periods. The right hand side of (3.2) is then more involved, but the essence of the argument is identical.

so a lower (weakly if $\alpha_0 = \bar{\alpha}$) payoff. The strategy $q_S(t) = 1$ for all t is then also the seller's best reply to $q_B(t) = 1$ for all t since the payoff obtained by the type L seller is $p_H - c_L$, the highest possible. This establishes that $q_B(t) = q_S(t) = 1$ for all t is an equilibrium if $\alpha_0 \geq \bar{\alpha}$, as claimed in part (i) of the statement of the proposition.

On the other hand, if $\alpha_0 < \bar{\alpha}$, the buyers' best reply to $q_S(t) = 1$ for all t is $q_B(t) = 0$ for all t; but then the seller's best reply is $q_S(t) = 0$ for all t, so that we do not have a pure strategy equilibrium in this case.

Consider next the candidate equilibria where buyers randomize for infinitely many periods. From Lemma 3, such equilibria require the seller to - possibly - randomize at the initial date, so as to induce the posterior belief $\alpha_1 = \bar{\alpha}$, and to propose p_H with probability 1 at all later dates $t > 1$. This is clearly possible only if $\alpha_0 \leq \bar{\alpha}$ (hence, when $\alpha_0 > \bar{\alpha}$ these equilibria never exist).

Let us denote by $V_L(t)$ the present value, at t, of the discounted expected flow of payoffs of the type L seller, given that he always proposed p_H in the past (including the current period t), and got always rejected; $V_L(t)$ satisfies then the following:

$$V_L(t) = q_B(t)(p_H - c_L) + \delta(1 - q_B(t))V_L(t+1) \tag{3.3}$$

Any sequence of values $q_B(t) \in (0, 1)$ satisfying the conditions:

$$\begin{aligned} p_L - c_L &= q_B(1)(p_H - c_L) + \delta(1 - q_B(1))V_L(2) \\ p_L - c_L &\leq q_B(t)(p_H - c_L) + \delta(1 - q_B(t))V_L(t+1) \text{ for all } t > 1 \end{aligned} \tag{3.4}$$

supports the strategy $q_S(1) \in (0, 1), q_S(t) = 1$ for all $t > 1$ as the seller's best response to $\{q_B(t)\}_t$. It is immediate to verify that we can always find, in fact many, sequences with this property. Furthermore, since $\alpha_t = \bar{\alpha}$ for all $t \geqslant 1$, any sequence of values $q_B(t) \in (0, 1)$ is a best reply for the buyers. Thus, as stated in part (ii) of the statement of the proposition, such equilibria always exist if $\alpha_0 < \bar{\alpha}$. By a similar argument, any sequence of values $q_B(t) \in (0, 1)$ satisfying the inequality in the second equation of (3.4) for all $t \geq 1$ and the seller's strategy $q_S(t) = 1$ for all $t \geq 1$ are a best response to each other (and hence constitute an equilibrium) when $\alpha_0 = \bar{\alpha}$.

To complete the proof of the Proposition, it remains to consider the possibility of an equilibrium where buyers randomize for a positive but finite number of periods.[11] Suppose that there is an equilibrium where $q_B(t) = 0$ for all t greater

[11] It is immediate to see that we can only have an equilibrium where buyers never randomize if the seller also never randomizes, the case already considered at the beginning of the proof.

or equal than some date $T \geqslant 2$. Then the seller's best response, as we already argued, would be $q_S(t) = 0$ for all $t \geqslant T$, which by Lemma 1 cannot be part of an equilibrium.

On the other hand, if $q_B(t) = 1$ for all $t \geqslant T \geqslant 2$ is part of an equilibrium strategy, we must have $\alpha_T \geqslant \bar{\alpha}$. If $\alpha_T > \bar{\alpha}$ we reach again a contradiction, by Lemma 2. If $\alpha_T = \bar{\alpha}$ and $\alpha_t < \bar{\alpha}$ for $t < T$, we must have $q_S(T) \in (0, 1)$; for this choice of the seller to be optimal we need, from (3.4), taking into account that $q_B(T) = 1$, the following equality to hold, $p_L - c_L = p_H - c_L$, which is impossible. We are then left with the case where $\alpha_T = \bar{\alpha}$ and, for some $\bar{t} < T$, $\alpha_t = \bar{\alpha}$ for $\bar{t} < t < T$, so that $q_S(t) = 1$ for all $t > \bar{t}$, $q_S(\bar{t}) \in (0, 1)$. Note first that this can only be part of an equilibrium if $\bar{t} = 1$ and hence $\alpha_t = \bar{\alpha}$ for all $t \geq 1$.[12] The conditions for the optimality of the seller's strategy are again given by (3.4), where $V_L(2)$ is defined recursively by (3.3) together with the equality $V_L(T-1) = p_H - c_L$. It is easy to check that, for any $\delta < 1$ we can find T, sufficiently high, such that these conditions are satisfied for some sequence $\{q_B(t)\}_{t \geq 1}$ exhibiting the property $q_B(t) = 1$ for all $t \geqslant T$. The closer is δ to 1, the larger is the minimal number of periods of randomization T required.

We conclude that, when $\alpha_0 \leq \bar{\alpha}$, there exist equilibria where buyers randomize both for an infinite and a finite number T of periods, with T larger the closer is δ to 1. On the other hand, if $\alpha_0 > \bar{\alpha}$, there are no equilibria where buyers randomize. ∎

4. Oligopoly without Clienteles

In this section we examine the case where there are $n > 1$ sellers, who compete in prices among them, in the absence of clienteles. As described in Section 2, this means that each period sellers simultaneously announce a price in the set $\{p_H, p_L\}$, buyers get to observe the list of announced prices and the identity of the seller behind each price and can freely choose whom to trade with. Hence, if both prices are called on by firms, those who announce p_L split the entire market equally among them, while those announcing p_H sell no units. In this situation, competition is quite fierce since each seller, by undercutting, can steal immediately its competitors' market share.

[12] If $\bar{t} > 1$, so that $\alpha_t < \bar{\alpha}$ and hence $q_B(t) = 0$ for $t < \bar{t}$, from the inequality in the second equation of (3.4) we obtain $p_L - c_L \leq \delta V_L(t+1)$ for $t < \bar{t}$. But this is impossible since $q_S(\bar{t}) \in (0, 1)$ implies that $V_L(\bar{t}) = p_L - c_L$.

We will explore this model, as well as its counterpart with clienteles in the next section, under two extreme information structures: one in which all sellers are informed regarding the true state of the world and the other where only one of them is informed.

4.1. All Informed Oligopolists

As in the case of monopoly, state H sellers will always charge p_H and, as soon as the price p_L is in the list of announced prices, all buyers remaining in the market will always buy at this price. If, on the other hand, all sellers announced p_H in the first t periods, the buyers' choice at t depends on their belief α_t that the state of the world is H. Let $q_B(t)$ denote then the probability that a buyer accepts, at t, if all sellers propose p_H, given histories where all sellers offered p_H in the past. The formal definition of the strategies of buyers and sellers - and hence of an equilibrium - is otherwise the same as in the previous section (we denote by $q_S(t)$ the probability that each state L seller charges p_H in period t); similarly, α_t is the buyers' belief at t that the state is H if all sellers proposed p_H at t and in each prior period.[13]

The nonexclusivity of the sellers' information, as well as the limits on their market power given by the presence of various sellers competing among them, impose severe constraints on the sellers' ability to hide their information and manipulate buyers' beliefs as in the case of monopoly. To hide the information would in fact require to repeatedly announce a high price even when the state is L. However, doing so now would give any other seller a strong incentive to undercut. Even though by undercutting the seller would reveal its information, the benefits from expanding its market share would be higher the larger is the number n of sellers; hence, for n sufficiently large they will outweigh the costs of revealing the information.

We proceed to characterize the equilibria of the model in this case. We will show that, due to the non-exclusivity of the information that each seller holds, there is an equilibrium, where all the information is immediately revealed: each seller in period 1 announces p_H in state H and p_L in state L with probability 1, while buyers always accept both p_L and p_H (if all sellers offered p_H). When the number of sellers n is small enough, we can also have 'collusive' equilibria,

[13] Again we can restrict our attention, without loss of generality, to symmetric equilibria. The purification argument outlined above (in footnote 8) continues to apply to the buyers, and by the structure of the model, it is easy to see that there are no asymmetric equilibria among sellers. Hence the notation $q_S(t)$ just presented.

where sellers behave as in the equilibrium with monopoly obtained in the previous section, or where they randomize in state L for the first T periods and then always propose p_L. However, as n grows, all these 'collusive' strategies cease to be part of equilibria, and there is no other equilibrium than full and immediate information revelation. Formally,

Proposition 2. *In the model with n informed sellers without clienteles:*
(i) ***Full and immediate information revelation****: for any $n \geqslant 2$, there always exists an equilibrium where $q_S(1) = 0$ and $q_B(1) = 1$.*
(ii) For n small enough, the following equilibria also exist:

(ii.a) ***No revelation or partial immediate revelation****: all sellers (and hence buyers) behave as in the monopoly equilibrium.*

(ii.b) ***Full revelation but with delay****: $q_S(t) \in (0, 1)$ and $q_B(t) \in (0, 1)$ for $1 \leq t < T$, and $q_S(t) = 0$, $q_B(t) = 1$ for $t \geq T$).*
(iii) ***Asymptotically, full and immediate revelation in all equilibria****: for n sufficiently large all the equilibria in (ii) vanish and the unique equilibrium is the one described in (i).*

Proof of Proposition 2: Claim (i) follows immediately from the non-exclusivity of each seller's information. Effectively, if a seller anticipates that his competitors offer price p_L in state L (i.e. choose to fully reveal their information), the unique best response for him is to do the same. The alternative is in fact losing entirely his market share.

Consider next the seller's strategy in the monopoly equilibrium. The best possible deviation for a seller, when every other seller follows this strategy, is to announce p_L at a node where the strategy prescribes to offer p_H with probability 1. If the seller undercuts and offers p_L, he sells to the whole market at the price p_L, so that his profits (starting from that node) are $p_L - c_L$. On the other hand, the profits obtained by adhering to the collusive strategy are $\frac{p_H - c_L}{n}$ (i.e. the seller's share of the monopoly profits)[14]. Hence, the choice of the monopoly strategy for all sellers remains optimal if and only if $\frac{p_H - c_L}{n} \geq p_L - c_L$, which holds for small enough n; this proves claim (ii.a).

Turning to (ii.b), the optimality of the prescribed strategy for sellers require them to be indifferent, in each of the first T periods, between offering p_H or p_L. In particular, for $t = T$, we must have:

[14]Strictly speaking, in the special case where $\alpha_0 = \bar{\alpha}$ the profits are less or equal this level.

$$q_S^{n-1}(T)[q_B(T)\frac{p_H - c_L}{n} + (1 - q_B(T))\delta\frac{p_L - c_L}{n}] = q_S^{n-1}(T)(p_L - c_L) + R(T), \quad (4.1)$$

where on the left hand side is the payoff from offering p_H (at T), on the right hand side the payoff from p_L and $R(T)$ denotes the expected payoff to charging p_L in the event that some of the competitors also charge p_L. Since $R(T) \geq 0$, when n is sufficiently large (4.1) cannot hold, for any $q_S(T), q_B(T) \in [0, 1]$, thus showing that even this strategy cannot be part of an equilibrium. On the other hand, for n small (but greater than 2) we may be able to find $q_S(t), q_B(t) \in [0, 1]$ satisfying (4.1) and the analogous equalities for $t = 1, .., T - 1$, thus showing that temporary collusion may be an equilibrium in that case.

To complete the proof of the Proposition, we have to show that no other equilibrium exists. Claims (i) and (ii.a) characterize the equilibria where sellers follow pure strategies or randomize for finitely many periods and switch then to p_H; claim (ii.b) describes the possible equilibria where sellers randomize for finitely many periods and then switch to p_L. It remains to consider the case where sellers in state L randomize for infinitely many periods:

Lemma 4. *There is no equilibrium where the sellers in state L randomize for infinitely many periods.*

Proof of Lemma 4: Note that for sellers to randomize at any period t ($0 < q_S(t) < 1$), buyers have to accept p_H with positive probability: $q_B(t) > 0$. Moreover, for the game not to end with probability 1 in finite time, it must be $q_B(t) < 1$ for all t, i.e. buyers have also to randomize for infinitely many periods. Recall then Lemma 3: this result is still valid and implies that $\lim_{t\to\infty} \alpha_t = \bar{\alpha}$. But, if sellers randomize during infinitely-many periods, the sequence $\{\alpha_t\}_t$ is strictly increasing so that, for any t, we have $\alpha_t < \bar{\alpha}$, which contradicts the fact that $q_B(t) > 0$. ∎

The above provides a complete characterization of the symmetric (among sellers) equilibria. There are no asymmetric equilibria. so the proof of the Proposition is complete. ∎

4.2. One Informed Oligopolist

We explore a different information structure, in which there is only one informed seller, whose information is then exclusive. As a consequence, the informed seller has the same ability as the monopolist to hide his information. However, to be able to profit from this in the presence of other, uninformed sellers, the informed seller must be able to successfully manipulate both the buyers' and the other sellers' beliefs to induce all of them to trade at a high price, and this may not always be possible, as we will see. Furthermore, even if it were possible we will show that it is not optimal, when the number of sellers in the market is sufficiently large: in that case in fact the incentives to undercut and steal all his competitors' market share prove too strong, even though by so doing all information would be revealed.

More specifically, we will show that, for n small, we have equilibria where no information is revealed (and there may even be no equilibrium with full revelation): the informed seller hides his information, partially or completely, and with some positive probability trade takes place at p_H in both states. However, for n sufficiently large, the equilibrium is unique and reveals immediately the information to the buyers, as in the case where all sellers are equally informed.

Combining the results of both subsections it becomes apparent that the large number of sellers competing in the market (and the absence of clienteles), but not the differences in the information structure seem to be the key feature to allow information to be fully and immediately revealed to the consumers.

To characterize the equilibria formally, note that we need now to describe separately both the strategy of the informed seller, in states H and L, and the uninformed sellers (and buyers). As before, the informed seller in state H will always charge p_H. We denote then by $q_I(t)$ and $q_U(t)$ the probability that the state L informed seller and each uninformed seller, respectively, charge p_H at date t. Let $q_B(t)$ be the probability that buyers accept p_H in period t, following a history where the only price announced by sellers has been p_H.

Some further notation is also needed to identify the relevant cutoff values in the beliefs of uninformed sellers. Let:

$\widetilde{\alpha}$ be the belief that makes uninformed sellers indifferent between trading at p_L and not trading at all: $\widetilde{\alpha}(p_L - c_H) + (1 - \widetilde{\alpha})(p_L - c_L) = 0$. Thus for $\alpha < \widetilde{\alpha}$ an uninformed seller strictly prefers trade at p_L to no trade.

$\hat{\alpha}$ be the belief that makes uninformed sellers indifferent between not trading at all and announcing p_L, when every other uninformed seller proposes p_H

while the informed state L seller proposes p_L: $\hat{\alpha}(p_L - c_H) + (1-\hat{\alpha})\frac{p_L - c_L}{2} = 0$. For $\alpha > \hat{\alpha}$ no uninformed seller would offer p_L in this situation.

It can be easily verified that $\widetilde{\alpha} > \hat{\alpha}$.

Proposition 3. *In the model with n sellers, only one of whom is informed, without clienteles:*
(i) For n large enough, there is always an equilibrium with full and immediate information revelation. Specifically,

(i.a) ***Full information revelation to all uninformed traders****: for large enough n and for all $\alpha_0 \geqslant \hat{\alpha}$, there exists an equilibrium where $q_U(1) = 1$, $q_I(1) = 0$ and $q_B(1) = 1$ (that is, at $t = 1$ the uninformed sellers charge p_H, the informed seller charges p_H in state H and p_L in state L, and buyers accept both p_L and p_H, provided all sellers offered p_H).*

(i.b) ***Full information revelation to buyers****: for large enough n and for all $\alpha_0 < \widetilde{\alpha}$, there exists an equilibrium where $q_U(1) = q_I(1) = q_B(1) = 0$ (at $t = 1$ the uninformed sellers charge p_L, the informed seller charges p_H in state H and p_L in state L, and buyers accept only p_L at $t = 1$).*
(ii) ***No full revelation****: for n sufficiently small there are equilibria where $q_I(t), q_U(t) \in (0,1]$ and $q_B(t) \in (0,1]$ for all $t \leq T$, for some finite $T \geq 1$.*
(iii) ***Asymptotically, full and immediate revelation in all equilibria****: as $n \to \infty$, the only equilibria are the ones in (i).*

Proof of Proposition 3: (i.a) Given the strategies for the other players described in the statement of (i.a), when $\alpha_0 \geqslant \hat{\alpha}$ the expected profit for an uninformed seller from offering p_L is non-positive. Hence in this situation an optimal choice for any uninformed seller is indeed to offer p_H, which yields a positive expected profit, $\frac{1}{n}(p_H - c_H)\alpha_0$. The informed seller in state L then strictly prefers to charge price p_L if:

$$p_L - c_L > \frac{1}{n}(p_H - c_L)$$

always satisfied for n sufficiently high.

(i.b) When the other uninformed sellers, as well as the informed one in state L, offer p_L, an uninformed seller also prefers to charge p_L if:

$$\alpha_0(p_L - c_H)\frac{1}{n-1} + (1-\alpha_0)(p_L - c_L)\frac{1}{n} \geq 0.$$

If $\alpha_0 < \tilde{\alpha}$ this inequality is always satisfied for n large enough. It is then immediate to see that the informed seller's choice of offering p_L (in state L) is an optimal response to the uninformed's strategy, since $\frac{p_L - c_L}{n} \geq 0$.

(ii) Let n be sufficiently small so that $p_L - c_L \leq \frac{1}{n}(p_H - c_L)$. Note that $p_L - c_L \geq (p_L - c_H)\alpha_0 + (1-\alpha_0)(p_L - c_L)$ for all α_0; the unit expected payoff of trading at p_L is always higher for the informed than for the uninformed seller since the first one can choose to trade at this price only in state L.

First, we have equilibria that resemble the ones found in the monopoly section. That is, if $\alpha_0 \geq \bar{\alpha}$, offering p_H every period both for the uninformed sellers and the informed seller in the H and L states constitutes an equilibrium.[15]

In addition, there are equilibria that involve randomization on the part of the informed seller in state L for $T > 1$ periods: in period $T+1$, the informed seller in state L must charge p_L (this low price in the final trading date is needed to sustain the randomization of buyers, which in turn is required for the informed seller to be willing to randomize). In the first T periods, in some of these equilibria the uninformed sellers charge p_H while in others they randomize between p_H and p_L; in period $T+1$ they will charge p_L (if $\alpha_{T+1} < \widetilde{\alpha}$) or randomize.

(iii) We show finally that as $n \to \infty$ the only equilibria are those with full and immediate separation described in (i). Evidently, there is no equilibrium where the informed seller in state L charges p_H with probability 1 (as the first of the equilibria described in (ii)): with n large undercutting is always preferred. By an argument similar to the one in the proof of Lemma 4, sellers cannot randomize for infinite periods. We are left then with examining the other equilibria described in (ii), where the type L seller randomizes for T periods before choosing p_L with probability 1.

Suppose first that the uninformed sellers choose p_H at $T+1$. Then, the indifference condition for the informed seller in period T between p_L and p_H is:

$$q_U^{n-1}(T)(p_L - c_L) + \sum_{r=1}^{n-1}(1 - q_U(T))^r q_U^{n-1-r}(T)\frac{(n-1)!}{r!(n-1-r)!}\frac{p_L - c_L}{r+1} =$$
$$q_U^{n-1}(T)[\frac{p_H - c_L}{n}q_B(T) + \delta(p_L - c_L)].$$

For n sufficiently large, the term on the right hand side is approximately $q_U^{n-1}(T)\delta(p_L - c_L)$. This is smaller than the first term on the left hand side because $\delta < 1$, and in addition, the rest of terms on the left hand side are not negligible. Thus, the expression on the left hand side exceeds the one on the right hand side, which is a contradiction.

[15] If, on the other hand $\alpha_0 < \bar{\alpha}$, there is an equilibrium where the informed seller in state L randomizes in the initial period to induce the belief $\alpha_1 = \bar{\alpha}$.

The same is true, a fortiori, if the uninformed choose p_L with positive probability at $T+1$. So we conclude that these also cease to be equilibria for n large. ∎

The uninformed sellers' behavior also depends on their beliefs about the state. However, in the situation considered here there is still no need to specify off-equilibrium path beliefs after unilateral deviations (of the informed seller), as in the case of monopoly. The inference both of buyers and uninformed sellers is in fact irrelevant when the informed seller (or, for that matter, any seller) announces p_L since the optimal response of buyers is always to accept in this case and hence the game ends immediately. On the other hand, there is no off-equilibrium deviation to p_H.[16]

In this model information is monopolized by one seller. However, Proposition 3 makes it clear that, as n grows large, the intensity of price competition among sellers in the model with no clienteles gives too strong an incentive to undercut and hence all information is revealed to the buyers right away. The equilibrium in (i.b), that also survives for all n, is also characterized by the fact that all information is immediately revealed to the buyers, but there is no revelation to the uninformed sellers, who end up trading at p_L in state H. Their prior belief giving low probability to the state being high leads them to bear a payoff in that state which is not ex post individually rational.

5. Oligopoly with Clienteles

We analyze here to what extent the results obtained in the previous section (in particular, the fact that all information, whether exclusive or not, is immediately revealed to the buyers when there are sufficiently many sellers), remain valid when less extreme forms of price competition are considered. To this end, as anticipated in Section 2, we introduce the model with clienteles, whose main difference from the previous one is that each period a buyer can only buy from his designated seller; hence the term clientele. This association only lasts one period, as each time t the buyers remaining in the market are randomly reassigned to sellers. By undercutting each seller can now steal only a limited fraction of his competitors' market share, as in models of Bertrand competition with capacity constraints or models of Cournot competition.

[16] With regard to deviations by uninformed sellers, when $n > 2$ there will always be at least one uninformed offering p_L, thus leading to an immediate termination of the game.

Again we proceed to study the model when there are n sellers under the two extreme information structures.

5.1. All Informed Oligopolists

If all other informed sellers choose p_L, i.e. to reveal their information, and this is commonly observed by all buyers, the best reply of a seller is clearly to do the same, as long as there is some, even very weak, competition among sellers. Thus full and immediate revelation of the information remains an equilibrium in the model with clienteles, because of the non-exclusivity of the information.

On the other hand, as already mentioned, the gains from undercutting are now much more limited. To see this more precisely, consider the situation where, at some date t in state L all sellers announce p_H. We can construct now different equilibria supporting this outcome using different off-equilibrium beliefs when one seller deviates to p_L. If these beliefs are that the state is L with probability 1, buyers will reject all offers of p_H. Thus only the seller who announced p_L will sell to the $1/n-th$ of the market constituting his clientele for the period. The remaining $(n-1)/n-th$ of the buyers on the market at t will then still be on the market at $t+1$ and will be equally split among the n sellers. At $t+1$ all sellers will offer p_L if the buyers' strategy is to keep rejecting all offers of p_H. By undercutting, an informed seller can then only increase his market share from $\frac{1}{n}-th$ to $\frac{1+(n-1)/n}{n}-th$ of the market; moreover, his increase in market share will take one period to materialize. As a consequence, collusive behavior among sellers, and in particular to hide the information and profitably manipulate buyers' beliefs, is now easier. This shows that if the collusive payoff in state L, $(p_H - c_L)$, is tempting enough, the "local monopoly" power created by the presence of clienteles suffices for the existence of another (collusive) equilibrium regardless of the number of sellers: each seller behaves as in the monopoly equilibrium and no information, or only the minimal amount which is necessary to trade at the high price, is revealed. Furthermore, with other off-equilibrium beliefs, there is a collusive equilibrium that does not necessitate the assumption of the collusive payoff being large enough. We give its details in part (ii) of the next proposition.

This stands in clear contrast with our findings for the model without clienteles. It reveals that, even in the absence of exclusivity of information, no revelation may occur at equilibrium, whatever the number of sellers, when the intensity of competition among sellers is not too strong. It also shows that the nature of competition among sellers appears to play a more important role than the exclusivity of information in determining whether or not information is fully revealed to the buyers, when there are many sellers.

Formally, we have[17]:

Proposition 4. *In the model with n informed sellers with clienteles:*
(i) ***Full and immediate information revelation****: for all $n \geq 2$ and all α_0 there is always an equilibrium where $q_S(1) = 0$ and buyers accept p_H in period 1 when all prices announced in that period are p_H.*
ii) For any $n \geq 2$, the following collusive equilibria (where sellers behave as in the case of monopoly) also exist:

(ii.a) ***No revelation****: when $\alpha_0 \geqslant \bar{\alpha}$, $q_S(t) = 1$ for all t and buyers immediately accept p_H;*

(ii.b) ***Partial revelation****: when $\alpha_0 < \bar{\alpha}$ and*

$$p_H - c_L \geqslant (p_L - c_L)[\delta + \frac{(1-\alpha_0)\bar{\alpha}}{\alpha_0(1-\bar{\alpha})}],$$

$q_S(1) \in (0,1)$ so that $\alpha_1 \geq \bar{\alpha}$ and buyers accept with probability 1 if all sellers announce p_H; for $t > 1$ we have $q_S(t) = 1$ and all buyers accept p_H.

Proof of Proposition 4: (i) We omit this proof, as it is similar to that of claim (i) of Proposition 2.

(ii.a) When the strategy of the buyers is to accept p_H if all sellers propose p_H, and that of all the other sellers is to propose p_H in state L at date 1, the profit for a seller if he does the same (offer p_H) is $\frac{p_H - c_L}{n}$. On the other hand, if he were to undercut and charge p_L, he would sell immediately to the $\frac{1}{n}$ share of buyers constituting his clientele at $t = 1$. Assigning off-equilibrium path beliefs after this deviation equal to the beliefs on the equilibrium path and assuming that the sellers' strategy is still to offer p_H at any later date (i.e., the same equilibrium behavior as in period 1), buyers will continue to accept all offers of p_H at $t = 1$. The payoff for undercutting is then only $\frac{p_L - c_L}{n}$, so that charging p_H is clearly the sellers' best response[18]. Given the sellers' strategy, since $\alpha_0 \geqslant \bar{\alpha}$, all buyers then prefer to immediately accept both p_L and p_H.

[17] We use here again $q_S(t)$ to denote the probability that a seller proposes p_H in state L at date t.

[18] On the other hand, when the off-equilibrium path beliefs following a deviation to p_L are such that the probability of L is 1 (as discussed earlier, in the second paragraph of this section), the same conclusion holds under the condition $p_H - c_L \geqslant (p_L - c_L)(1 + \delta)$.

(ii.b) Taking as given the strategies of the buyers and the other sellers as described in part (ii.b) of the statement, the payoff to a seller in period 1 in the event that $r > 0$ sellers other than himself announce p_L is:

$$\delta(p_L - c_L)\frac{n-r}{n^2} \tag{5.1}$$

if he charges p_H, and

$$\frac{p_L - c_L}{n} + \delta(p_L - c_L)\frac{n-1-r}{n^2} \tag{5.2}$$

if he charges p_L. The probability of this event is then $(q_S(1))^{n-1-r}\,(1-q_S(1))^r \frac{(n-1)!}{(r!(n-1-r)!)}$.

On the other hand, if $r = 0$ sellers other than him charge p_L, his payoff is

$$\frac{p_H - c_L}{n}$$

if he charges p_H,

$$\frac{p_L - c_L}{n} + \delta(p_L - c_L)\frac{n-1}{n^2}$$

if he charges p_L and the probability of this event is $(q_S(1))^{n-1}$.

To sustain indifference, we need that the expected payoff of p_H equals the expected payoff of p_L, i.e. that the sum of the above terms describing the payoff associated to p_H, weighted by their respective probabilities, over all r running between 0 and $n-1$, equals the sum of the corresponding terms describing the payoff associated to p_L. Noting that, for all $r > 0$ the difference between (5.1) and (5.2) equals $\frac{p_L - c_L}{n}[\frac{\delta}{n} - 1]$, simplifying terms we obtain the following equality:

$$(1-(q_S(1))^{n-1})\frac{p_L - c_L}{n}[\frac{\delta}{n}-1]+(q_S(1))^{n-1}\,\frac{1}{n}[(p_H-c_L)-(p_L-c_L)(1+\delta\frac{n-1}{n})] = 0,$$

which can be simplified to:

$$\frac{p_L - c_L}{n}[\frac{\delta}{n} - 1 - (q_S(1))^{n-1}\,\delta] + \frac{p_H - c_L}{n}\,(q_S(1))^{n-1} = 0. \tag{5.3}$$

Next, let $\bar{q}_S$ be the value of $q_S(1)$ that generates an updated belief of the buyers, after observing all sellers announcing p_H, of $\alpha_1 = \bar{\alpha}$:

$$\bar{q}_S^n = \frac{\alpha_0(1-\bar{\alpha})}{(1-\alpha_0)\bar{\alpha}}.$$

Observe that $q_S(1)$ can be set to take any value between $\bar{q}_S$ and 0, thus inducing a belief $\alpha_1 \geq \bar{\alpha}$, and hence supporting the buyers' choice to immediately accept both p_H and p_L. When $q_S(1) = 0$, the term on the left hand side of (5.3) is clearly negative. On the other hand, when $q_S(1)$ is such that $q_S(1) = \bar{q}_S$, the sign of this term is equal to the sign of:

$$(p_H - c_L)\left(\frac{\alpha_0(1-\bar{\alpha})}{(1-\alpha_0)\bar{\alpha}}\right)^{(n-1)/n} + (p_L - c_L)\left[\frac{\delta}{n} - 1 - \left(\frac{\alpha_0(1-\bar{\alpha})}{(1-\alpha_0)\bar{\alpha}}\right)^{(n-1)/n}\delta\right] \quad (5.4)$$

which is positive, for all n, under the condition in the statement of part (ii.b)[19]. Therefore, it is always possible to find a value of $q_S(1) \in [0, \bar{q}_S]$ so that (5.3) is satisfied. ∎

5.2. One Informed Oligopolist

As we saw in the previous section, in the presence of clienteles hiding the information can be profitable for an informed seller, whatever the number of sellers (this is true in particular when the profits per unit sold at a high price are sufficiently higher than the profits from a sale at a low price). However, with non-exclusive information a fully revealing equilibrium always exists, together with collusive equilibria, for any n. When there is only one informed seller, information is exclusive and, as already argued in Section 4.2, the informed seller can always hide it, as in the case of monopoly. We should expect then that in this situation full revelation is difficult to achieve. Indeed, we will show that, for many configurations of parameters describing an economy, separation is impossible at equilibrium and an equilibrium exists where at least part of the information is never revealed. The only case in which hiding the information can prove too costly for the informed seller is when both the gain in the per unit profit obtained by selling at p_H in state L is sufficiently low and the prior belief of the uninformed sellers sufficiently optimistic so they may be willing to offer a low price.

The results are again in clear contrast with what we found in the model without clienteles. If information is exclusive, we should not expect it to be revealed at

[19]The term on the left hand side of (5.4) is positive if $(p_H - c_L) > (p_L - c_L)\left[\delta + (1 - \frac{\delta}{n})\left(\frac{(1-\alpha_0)\bar{\alpha}}{\alpha_0(1-\bar{\alpha})}\right)^{(n-1)/n}\delta\right]$. Noting that $\left(\frac{(1-\alpha_0)\bar{\alpha}}{\alpha_0(1-\bar{\alpha})}\right)$ is the reciprocal of a probability, and hence is greater than 1, we have $\left(\frac{(1-\alpha_0)\bar{\alpha}}{\alpha_0(1-\bar{\alpha})}\right) > \left(\frac{(1-\alpha_0)\bar{\alpha}}{\alpha_0(1-\bar{\alpha})}\right)^{(n-1)/n}$ so that the condition in the statement of (ii.b) can be used in the above inequality to get the result.

equilibrium, when the intensity of the competition among sellers is not too strong.

Proposition 5. *In the model with n sellers, only one of whom is informed, and with clienteles:*
(i.a) ***Full revelation is impossible for many parameter configurations****: for any $n \geq 2$, if $p_H - c_L > (p_L - c_L)(1+\delta)$ (the collusive payoff is not too small), or $\alpha_t < (p_L - c_L) / (p_H - c_L)$ for all t (beliefs are sufficiently pessimistic), in equilibrium we have $q_I(t) > 0$ for all t;*
(i.b) ***Full revelation only occurs for some parameter values****: there exists n large enough such that for $p_H - c_L < (p_L - c_L)(1+\delta)$, and $\alpha_0 > (p_L - c_L) / (p_H - c_L)$, there is an equilibrium with $q_I(1) = 0$.*
(ii) For any $n \geq 2$, the following additional equilibria exist:

(ii.a) ***No revelation****: when $\alpha_0 \geq \bar{\alpha}$, $q_U(t) = q_I(t) = 1$ for all t (both the uninformed sellers and the informed seller in both states charge p_H) and buyers immediately accept;*

(ii.b) ***Partial revelation****: when $\alpha_0 < \bar{\alpha}$ and $(p_H - c_L) > (p_L - c_L)(1+\delta)$, $q_I(1) \in (0,1)$ and $q_U(1) = 1$ (the informed seller randomizes in state L in the initial period, while the uninformed charge p_H); for all $t > 1$ we have $q_I(t) = q_U(t) = 1$. In every period, buyers randomize between accepting and rejecting when all announced prices are p_H.*

Proof of Proposition 5: (i.a) Suppose full separation occurs in equilibrium in some period t. Then, the state H informed seller charges p_H in that period, and the state L informed seller charges p_L. By Bayes' rule, the buyers and the uninformed sellers must update their beliefs at t to $\alpha_t = 1$, upon observing the informed seller charging p_H, and to $\alpha_t = 0$ upon observing the informed seller charging p_L. If p_L is observed, the clientele of the informed seller must clearly accept. But so must the clientele of the informed seller if he charges p_H, because the only reason to reject would be to expect p_L in the future. However, the low price in the future could only come from the uninformed sellers, who believe now that the state is H with probability 1, and hence will never charge the low price. Therefore, the clientele of the informed seller must accept both p_H and p_L. It follows that in state H all units are sold in period t because all buyers accept even p_H. Then, the informed seller's profits from announcing p_H in state L are then $\frac{p_H - c_L}{n}$ while the profits from announcing p_L are $\frac{p_L - c_L}{n}$ when all the uninformed announce p_L and $\frac{p_L - c_L}{n} + \delta(p_L - c_L)\frac{n-1}{n^2}$ when they all announce p_H (which never occurs when

$\alpha_t < (p_L - c_L) / (p_H - c_L))$. Thus, under the conditions stated in (i), the informed seller in state L has an incentive to deviate and charge p_H.

(i.b) The strategies supporting separation are as follows. The informed seller charges p_L in state L and p_H in state H in every period. The uninformed sellers also charge p_H in period 1, but switch to p_L if they observe the informed seller charging p_L. Buyers accept both prices if they observe the informed seller charging p_H. They accept only p_L if they observe the informed seller charging p_L. The optimality of the buyers' strategy easily follow from the degeneracy of their beliefs (obtained by using Bayes' rule at all information nodes where the informed charge p_H or p_L, thus assigning off-equilibrium beliefs when only uninformed sellers deviate equal to the ones on the equilibrium path). Given that the collusive payoff is not too attractive (the first condition imposed on the parameter values), there exists n large enough such that p_L is a best response for the informed seller in state L. That is,

$$\frac{p_L - c_L}{n} + \delta(p_L - c_L)\frac{n-1}{n^2} > \frac{p_H - c_L}{n}.$$

Finally, the uninformed sellers prefer to charge p_H at $t = 1$, given the assumption on their prior belief, because

$$\alpha_0 \frac{p_H - c_H}{n} + \delta(1-\alpha_0)\frac{(p_L - c_L)(n-1)}{n^2} > \alpha_0 \frac{p_L - c_H}{n} + (1-\alpha_0)(p_L - c_L)(\frac{1}{n} + \frac{\delta(n-2)}{n^2}),$$

which simplifies to:

$$\alpha_0(p_H - p_L) > (1 - \alpha_0)(p_L - c_L)(1 - \delta/n).$$

The validity of this condition is ensured by the assumption made on α_0 for large enough n.

(ii.a) We construct here an equilibrium where p_H is announced by all sellers in period 1. Therefore, the belief held by uninformed traders is equal to their prior α_0. Because $\alpha_0 \geq \bar{\alpha}$, buyers are at a best response accepting p_H. Furthermore, we consider the case where the off-equilibrium belief, following a deviation to p_L in period 1, remains α_0; hence p_H continues to be accepted by buyers. In this situation, charging p_H is clearly the best response for the informed seller in state L as the effect of charging p_L would be to lower the profits from $\frac{p_H - c_L}{n}$ to $\frac{p_L - c_L}{n}$ (because of the clientele friction, given the assigned off-equilibrium beliefs, undercutting would only have the effect of lowering revenues over the same market share[20]).

By essentially he same argument, charging p_H is also a best response for any uninformed seller. If he were to deviate to charging p_L, the beliefs held by uninformed traders are unchanged and hence buyers continue to accept p_H from all the other sellers. Thus his expected profits would be $\alpha_0 \frac{p_L - c_H}{n} + (1-\alpha_0)\frac{p_L - c_L}{n}$, lower than the ones obtained by charging p_H, given by $\alpha_0 \frac{p_H - c_H}{n} + (1-\alpha_0)\frac{p_H - c_L}{n}$.

(ii.b) Let q_B be the probability with which buyers accept p_H, in any period t, when all prices announced (at t and any earlier date) are p_H. By Bayes' rule, upon observing p_L charged by the informed seller and p_H by all uninformed, the posterior belief is that the state is L with probability 1.

We begin by analyzing the incentives of the informed seller in state L. For him to be willing to randomize in period 1 the following equality must hold:

$$\frac{p_L - c_L}{n}[1 + \delta\frac{n-1}{n}] = \frac{p_H - c_L}{n}[q_B + \frac{\delta(1-q_B)q_B}{1-\delta(1-q_B)}]$$

Under the condition we imposed that the collusion payoff is attractive enough, it is easy to see that, for fixed δ and n, there exists a unique value of q_B that makes the equality hold. This will be the equilibrium value of q_B. At any later date $t > 1$ the payoff from announcing p_H and p_L are again the same (when the off-equilibrium beliefs, following a deviation to p_L, are that the state is L with probability 1); hence charging p_H is (weakly) a best response.

Consider next the uninformed sellers. If one of them deviates to p_L we assign off-equilibrium beliefs equal to the ones on the equilibrium path. Clearly the payoff associated to this deviation is only positive provided $p_L > \alpha_0 c_H + (1-\alpha_0)c_L$. In that case, if the strategy of all traders following this deviation prescribes offering p_L, it is easily verified that the expected payoff for deviating to p_L is $\frac{p_L - (\alpha_0 c_H + (1-\alpha_0)c_L)}{n}[1 + \delta\frac{n-1}{n}]$. This expression, taking into account the above condition stating the equality of the payoffs associated to p_H and p_L for the informed seller in state L, is strictly smaller than the expected payoff from p_H, $\frac{p_H - (\alpha_0 c_H + (1-\alpha_0)c_L)}{n}[q_B + \frac{\delta(1-q_B)q_B}{1-\delta(1-q_B)}]$. This shows that the deviation considered is unprofitable.

Note, finally, that the probability that the informed seller charges p_H at $t = 1$ is chosen so as to yield $\alpha_1 = \bar{\alpha}$. Thus, buyers are willing to randomize at $t = 1$

[20] As in Proposition 4, when the off-equilibrium path beliefs following a deviation to p_L are, on the other hand, such that the probability of L is 1, a limited increase in the seller's market share is obtained if he undercuts . The deviation is again non profitable under the condition $p_H - c_L \geqslant (p_L - c_L)(1+\delta)$.

between accepting and rejecting upon the observation of all prices being p_H. Given that pooling on p_H takes place from period 2 on, buyers are best-responding by continuing their randomization in any period $t > 1$. ∎

Thus, for a large subset of the parameter region full separation never occurs and only collusive equilibria exist, whatever the number of sellers n. For the complementary region, separation can be supported, but collusive equilibria are also found. Thus, the message of the model with clienteles is somewhere between the positive findings - with regard to information revelation and hence convergence to REE - of the model without clienteles, where competition is quite intense, and the negative results of the pairwise meetings literature, where the local monopoly power of each seller in each meeting is reinforced by the lack of observability of public signals that could help reveal the information (see Wolinsky (1990), Blouin and Serrano (2001)).

6. Welfare

We conclude by discussing the welfare properties of the equilibria we found. We will show that, in our model, interim incentive inefficiencies are identified with the presence of delay in trading. This is true even if information is exclusive, when interim improvements are harder to construct because incentive compatibility imposes non-trivial restrictions.

6.1. monopoly

The equilibria with perfect pooling and immediate acceptance by the buyers, obtained when $\alpha_0 \geq \bar{\alpha}$, are clearly interim Pareto efficient: all gains from trade are exhausted, with no delay.[21] The equilibrium allocation yields in fact the highest possible payoff the seller can attain in the model; any interim improvement for the seller would have to be based on trades at a price higher than p_H, which would make buyers worse off.

Moreover, at this equilibrium, both types of the seller and the buyers attain a strictly positive payoff. Let U_H, U_L, U_B be the present value, at $t = 1$, of the

[21] As with any other equilibrium, the allocation obtained is clearly incentive compatible (this is even clearer in this case, since the allocation is constant); hence, a fortiori, it is also interim incentive efficient.

discounted expected flow of payoffs of the two types of seller and the buyers; we have $U_H = p_H - c_H$, $U_L = p_H - c_L$, $U_B = \alpha_0 u_H + (1-\alpha_0)u_L - p_H$.

On the other hand, the equilibria with delay (obtained when $\alpha_0 \leq \bar{\alpha}$) are always interim incentive inefficient. Since delay is costly, they are obviously interim inefficient. But we will show that we can always construct an incentive compatible mechanism that interim dominates the equilibrium. This is true even though - when $\alpha_0 < \bar{\alpha}$ - some information is revealed in the first trading date (there is partial pooling). To see this, consider the following: each period t the seller always charges p_H in state H and p_L in state L; all buyers immediately accept when p_L is offered and reject with probability $q_B(t)$ at each period t when p_H is proposed, where $\{q_B(t)\}_{t\geq 1}$ is the same as in the equilibrium. The specification of the buyers' behavior ensures the incentive compatibility of the prescribed choice of the seller and hence of the mechanism proposed; we immediately see that such mechanism leaves the two types of monopolist indifferent, but strictly improves the buyers.

>From the equality condition in (3.4), needed to sustain the type L seller' s randomization in $t = 1$, it follows that his payoff at this other equilibrium is $U_L = p_L - c_L$, which must equal the payoff obtained by choosing p_H at $t = 1$. The latter is obtained by iterating the other expressions in (3.4), from which we obtain that the type L seller's payoff U_L must also be equal to

$$\sum_{\tau=1}^{\infty}\left[\delta^{\tau-1}q_B(\tau)\ \left(\Pi_{l=1}^{\tau-1}(1-q_B(l))\right)\right](p_H - c_L).$$

The type H seller's payoff is then

$$U_H = \sum_{\tau=1}^{\infty}\left[\delta^{\tau-1}q_B(\tau)\ \left(\Pi_{l=1}^{\tau-1}(1-q_B(l))\right)\right](p_H - c_H).$$

Hence $U_H = U_L\left(\frac{p_H - c_H}{p_H - c_L}\right) < U_L$. Since the probability of being able to trade at a price p_H is the same for the H and the L type of seller and the valuation of type H is higher, it follows that his payoff at equilibrium is strictly lower than the payoff of the type L seller. Note that the same is also true in the perfect pooling equilibria. Finally, the buyers' payoff is

$$\begin{array}{l} U_B = \alpha_0 \sum_{\tau=1}^{\infty}\left[\delta^{\tau-1}q_B(\tau)\ \left(\Pi_{l=1}^{\tau-1}(1-q_B(l))\right)\right](u_H - p_H) \\ +(1-\alpha_0)q_S(1)\sum_{\tau=1}^{\infty}\left[\delta^{\tau-1}q_B(\tau)\ \left(\Pi_{l=1}^{\tau-1}(1-q_B(l))\right)\right](u_L - p_H)+ \\ (1-\alpha_0)(1-q_S(1))(u_L - p_L) = (1-\alpha_0)(1-q_S(1))(u_L - p_L) \end{array}$$

where the last equality follows from the specification of $q_S(1)$ at the equilibrium considered.

Although the comparison among the equilibrium payoffs of buyers and sellers in the two types of equilibria (with and without delay) involves agents in different economies, characterized by different values of the prior belief α_0, it is still interesting to note that both types of the seller have a strictly lower payoff in the second equilibrium than in the first. Thus, information revelation comes at a cost to the seller.

6.2. oligopoly without clienteles

When all sellers are informed, the economy is one of non-exclusive information; in particular, every interim individually rational allocation can be made incentive compatible by punishing contradictory reports about the state with the "no trade" outcome. In this version of the model, the only equilibrium that survives for any n is the one with full information revelation. This is not only ex post but also interim Pareto efficient because of the absence of delay. Recall that traders are risk neutral, which implies that the interim gains from trade are just the appropriately weighted averages of ex post surpluses. In this equilibrium each seller in state H receives a payoff of $(p_H - c_H)/n$, each seller in state L a payoff of $(p_L - c_L)/n$, and the buyers get an expected payoff of $\alpha_0(u_H - p_H) + (1 - \alpha_0)(u_L - p_L)$.[22]

When only one seller is informed, information is exclusive. Two types of equilibria exist for all n. Both are separating equilibria and the information, privately held by the only informed seller is fully revealed to the buyers. In the equilibrium described in part (i.a) of Proposition 3, the informed seller in state H receives a payoff of $(p_H - c_H)/n$, while its counterpart in state L manages to steal all the market from the other sellers and receives a payoff of $p_L - c_L$. Uninformed sellers receive an expected payoff of $\alpha_0(p_H - c_H)/n$. Each buyer continues to receive the same expected payoff as in the presence of all informed sellers, $\alpha_0(u_H - p_H) + (1 - \alpha_0)(u_L - p_L)$. This equilibrium is also interim efficient and the total payoff of sellers and buyers is the same as when all sellers are informed. The only difference one should stress is the transfer of surplus from the uninformed sellers to the informed seller in state L.

While the equilibrium described in part (i.b) of Proposition 3 is also interim incentive efficient because of the absence of delay in trading, its features are some-

[22] When compared to the pooling equilibrium obtained under monopoly, the sum of the payoffs of sellers in state L is now lower while buyers' payoffs are higher.

what distinct. In this equilibrium the more optimistic beliefs of the uninformed traders crowd out the informed seller in state H, who receives a payoff of 0. The informed seller in state L receives a payoff of $(p_L - c_L)/n$, and the uninformed sellers get an expected payoff equal to $\alpha_0(p_L - c_H)/(n-1) + (1-\alpha_0)(p_L - c_L)/n$. Buyers are of course the clear winners, receiving an expected payoff of $\alpha_0 u_H + (1-\alpha_0)u_L - p_L$. Recall that even though this equilibrium is interim efficient, information is not revealed to the uninformed sellers, all trade occurs at p_L in state H and uninformed sellers are making ex post losses.

6.3. oligopoly with clienteles

The allocation at the separating equilibrium of Proposition 4, part (i), is the same as the one in the equilibrium found in Proposition 2, for n large. The other collusive equilibria obtained in Proposition 4 are very close to the ones found in Proposition 1 for the case of the monopolist. The reader is referred to the previous subsections for discussions of their welfare properties.

With regard to the equilibria obtained for the case in which there is only one informed seller, when separation is sustained in equilibrium all trade takes place at p_L in state L and at p_H in state H. The welfare properties of this equilibrium are similar to the ones of the separating equilibrium described in (ia) of Proposition 3 (with two differences: (i) there is delay, though this should now be viewed as a trade friction generated by the presence of clienteles; (ii) the distribution between uninformed and informed of the sellers' profits in state L is now more equal).

In contrast, at the equilibria where the informed hides his information, the total payoff of buyers and sellers is the same as at the equilibria obtained under monopoly. In some of these equilibria, delay occurs due to the buyers' randomization and this causes inefficiencies. The same incentive compatible mechanism proposed above in the subsection on monopoly allows to find an improvement in this case: the informed seller in state L, rather than randomizing is asked to charge p_L with probability 1, while buyers, when all prices announced are p_H, keep randomizing according to the same probabilities as in the equilibrium. Using the same arguments as above, one can see that this mechanism, which is incentive compatible, interim dominates the equilibrium. Note also that the expected payoff for the informed and uninformed sellers is the same in this case (unlike in the equilibria of the model without clienteles); thus the existence of clienteles exerts a positive externality on the uninformed sellers and, clearly a negative externality on buyers, whose payoff is lower.

6.4. asymptotic inefficiency

As in the literature on pairwise meetings (see Blouin and Serrano (2001)), interim incentive inefficiencies are associated with equilibria where information revelation lacks, while equilibria with full (and immediate) revelation are interim incentive efficient. However, the identification between welfare and information revelation properties found in that literature is not perfect here, a new feature of our model.

As we stated earlier, all our results remain valid for any large enough value of δ. In particular, inefficiencies may persist even as $\delta \to 1$. To illustrate this claim, we will show that in our set-up, by letting δ vary, we can find equilibrium sequences where for each δ trade takes place at infinitely many trading dates (some trades are infinitely delayed) and, as $\delta \to 1$, the sum of interim payoffs is smaller than the total interim surplus that is available; i.e., these sequences of equilibria are asymptotically interim incentive inefficient.

Consider the mixed strategy equilibrium under monopoly. Notice that the available interim surplus is

$$W = \alpha_0(u_H - c_H) + (1 - \alpha_0)(u_L - c_L).$$

Recall from the above subsection on monopoly that the payoff of the seller in state L equals:

$$U_L = p_L - c_L,$$

the one in state H is

$$U_H = (p_H - c_H)\frac{p_L - c_L}{p_H - c_L},$$

and both are independent of δ. The buyers' expected payoff is then

$$U_B = (1 - \alpha_0)(1 - q_S(1))(u_L - p_L)$$

which, using the fact that $\alpha_1 = \bar{\alpha}$ implies $q_S(1) = \frac{\alpha_0(1-\alpha_1)}{\alpha_1(1-\alpha_0)}$, simplifies as follows, $U_B = (\bar{\alpha} - \alpha_0)(u_L - p_L)/\bar{\alpha}$, that is also independent of δ.

It is then straightforward to check that $\alpha_0 U_H + (1 - \alpha_0)U_L + U_B < W$. The source of the inefficiency is the fact that, as $\delta \to 1$, a significant fraction of trades takes place with considerable delay. To see this, consider the case where buyers randomize every period according to a constant probability, i.e., $q_B(t) = q_B$ for all t; q_B is then pinned down by the condition needed to sustain the randomization of the seller in state L: $p_L - c_L = (p_H - c_L)\frac{q_B}{1-\delta(1-q_B)}$, yielding

$$q_B = \frac{1-\delta}{1-\delta\frac{p_L - c_L}{p_H - c_L}}$$

which clearly tends to 0 as $\delta \to 1$.

The only exception to the identification of interim incentive efficient equilibria and equilibria where information revelation takes place is the equilibrium in Proposition 3, part (i.b). In it, uninformed sellers, driven by their optimistic belief, end up transacting at a price that is not ex post individually rational in the high state. However, this equilibrium is interim efficient because all trades take place in the first period.

7. Final Comments

We argue here that the results obtained on the characterization of equilibria are robust to the extension of the model along several dimensions.

Finite horizon. All the equilibria we obtained for the case of infinitely many trading dates remain (or are approximated by) equilibria when there is only a finite number of trading dates ($\mathbf{T} \geq 1$). This is clear when the equilibrium in question involves a finite number of periods of trade; but even when there are infinite periods of trade, for example with infinite randomization on the part of buyers, for any $\mathbf{T}$ there is an analogous equilibrium where buyers randomize at each trading date.

To illustrate, consider the one-period ($\mathbf{T} = 1$) version of the monopolist model. In this case, rejection of a price leads to "no trade", thus to a zero payoff. It is easy to see that if $\alpha_0 > \bar{\alpha}$, the unique equilibrium continues to yield trade at p_H in both states. If $\alpha_0 < \bar{\alpha}$, the unique equilibrium prescribes that the seller in state L randomizes between p_H and p_L so as to induce the belief $\alpha_1 = \bar{\alpha}$, while the buyers must accept the high price with probability $q_B(1)$ (where $q_B(1)$ is set at a level such that the seller in L is exactly indifferent between charging p_L and p_H).

In the two-period monopoly model, the equilibrium when $\alpha_0 > \bar{\alpha}$ continues to have pooling on p_H and immediate trade, with no delay. When $\alpha_0 < \bar{\alpha}$, with $\mathbf{T} = 2$ we continue to have as the unique equilibrium the one in which $q_S(1)$ is chosen to yield $\alpha_1 = \bar{\alpha}$, while $q_B(1)$ and $q_B(2)$ are chosen to make the seller in state L indifferent between charging p_L and p_H at $t = 1$ and to (weakly) prefer p_H at $t = 2$; the information is then partially revealed only in period 1.

We can also show that no other equilibria exist, in particular there is no equilibrium where both buyers and the seller in state L randomize at each trading date: in that case in fact we must have $\alpha_t > \bar{\alpha}$ for all t, thus buyers at the terminal date accept p_H for sure, and so the seller always proposes p_H at $\mathbf{T}$, but then buyers cannot be indifferent between accepting and rejecting p_H at $\mathbf{T}-1$.

We thus conclude that the equilibria are essentially the same when $\mathbf{T} < \infty$ and $\mathbf{T} = \infty$, our findings are not generated by a possible discontinuity at $\mathbf{T} = \infty$.

Continuum of possible prices. We have performed the analysis under the simplifying assumption that sellers can only propose one of two possible prices, p_H and p_L, which are exogenously given. We discuss briefly here what one finds in the model when sellers are free to propose any price in the interval $[c_L, u_H]$ and show that the main qualitative properties remain essentially the same. Thus, for the issues we have studied, our simplifying assumption is without essential loss of generality.

Consider again the monopolist model. Note first that we can never have an equilibrium with separation[23], i.e., the seller charging different prices in state H and state L. Evidently, in state H the seller will always offer a price $\bar{p} \geq c_H$ (sales at any price below c_H would in fact result in losses). At a separating equilibrium buyers must accept this price, but then the seller in state L would prefer to also charge $\bar{p}$, rather than the separating price, which is some price $\underline{p}$ less or equal than u_L.

If the prior belief α_0 is high enough, so that we can find prices (typically an interval I) in $[c_H, u_H]$ at which both types of the seller and the buyers are willing to trade, we have again pooling equilibria with immediate trading, but now there is a continuum of such equilibria. For any $p \in I$, there is in fact an equilibrium where the seller charges p both in state H and L and buyers accept; if the seller deviates to $p' \neq p$, off-equilibrium beliefs are such that $\alpha_1 = 0$, thus buyers accept p' if and only if $p' \leq u_L$, which ensures that no profitable deviation exists for any type of seller. Having this continuum of equilibria, the complicated issue of equilibrium coordination arises, which we avoided in our simpler setup.

On the other hand, if α_0 is small enough so that there is no high price at which buyers are willing to trade with the seller in both states, equilibria are always characterized by the fact that the state L seller randomizes in the initial period between a low price (always given by u_L), that only the state L seller is willing to charge, and some high price (above c_H). The latter price is also charged by the seller in state H. Buyers randomize each period between accepting and

[23] This stands in contrast with the result in Laffont and Maskin (1990) who show that a separating equilibrium always exists, in the set-up of a one period trading model where the uninformed's demand is strictly decreasing in the price.

rejecting while the seller randomizes only at the initial period. Such equilibrium has then the same features as the equilibrium with partial information revelation we obtained for the case of low α_0. Again there is a continuum of such equilibria, for the different values that the high price can take.

Can there be other equilibria, where the seller (in state L) randomizes for more than one period? We show next that this is possible, thus we can have information being revealed now for an arbitrary, possibly infinite, number of periods; however, it is important to notice that the payoffs of buyers and the seller, at all such equilibria, are very similar to the ones obtained in the equilibria described above, where the seller in state L randomizes for only one period.

By essentially the same argument as in the proof of Proposition 1, for the state L seller to be willing to randomize, buyers have to randomize for infinitely many periods. This will in turn be optimal, for t large, provided the following condition holds:

$$\alpha_t u_H + (1-\alpha_t)u_L - p_H(t) \simeq \delta\left(\alpha_{t+1}u_H + (1-\alpha_{t+1})u_L - p_H(t+1)\right), \qquad (7.1)$$

where $p_H(t)$ denotes the price charged by the seller in state H with probability 1 and in state L with positive probability[24], which may now vary with t. Since both the sequence of posterior beliefs $\{\alpha_t\}$ and prices $\{p_H(t)\}$ must converge, as t tends to infinity, condition (7.1) can only hold if, for all t, we have

$$\alpha_t u_H + (1-\alpha_t)u_L - p_H(t) = 0 = \alpha_{t+1}u_H + (1-\alpha_{t+1})u_L - p_H(t+1)$$

i.e., if buyers are always indifferent between trading and not trading at $p_H(t)$, as in the equilibrium with one period randomization we found above. The present value of the discounted expected flow of payoffs is then $u_L - c_L$ for the seller in state L and 0 for buyers (since, as we argued, $p_L = u_L$), thus for both it is the same as in that equilibrium. The payoff for the state H seller has then also the same expression as in such equilibrium[25], $p_L - c_L - \sum_{\tau=1}^{\infty}\left[\delta^{\tau-1}q_B(\tau)\ \left(\Pi_{l=1}^{\tau-1}(1-q_B(l))\right)\right](c_H - c_L)$, though its value may be different (since buyers may randomize with different probabilities).

[24] It is possible that the seller randomizes in state H too; a similar condition to (7.1) must then hold and the argument in the text easily extends to this case.

[25] It is immediate to verify that the expression for U_H obtained in section 6.1 can be equivalently written in this form.

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