

Log-periodic power law bubbles in Latin-American and Asian markets and correlated anti-bubbles in Western stock markets: An empirical study

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Abstract

Twenty-two significant bubbles followed by large crashes or by severe corrections in the Argentinian, Brazilian, Chilean, Mexican, Peruvian, Venezuelan, Hong-Kong, Indonesian, Korean, Malaysian, Philippine and Thai stock markets indices are identified and analysed for log-periodic signatures decorating an average power law acceleration. We find that log-periodic power laws adequately describe speculative bubbles on these emerging markets with very few exceptions and thus extend considerably the applicability of the proposed rational expectation model of bubbles and crashes which has previously been developed for the major financial markets in the world. This model is essentially controlled by a crash hazard rate becoming critical due to a collective imitative/herding behavior of traders. Furthermore, three of the bubbles are followed by a log-periodic “anti-bubble” previously documented for the decay of the Japanese Nikkei starting in Jan. 1990 and the price of Gold starting in Sept. 1980 thus rendering a qualitative symmetry of bubble and anti-bubble around the date of the peak of the market. A set of secondary western stock market indices (London, Sydney, Auckland, Paris, Madrid, Milan, Zurich) as well as the Hong-Kong stock market are also shown to exhibit well-correlated log-periodic power law anti-bubbles over a period 6-15 months triggered by a rash of crises on emerging markets in the early 1994. As the US market declined by no more than 10% during the beginning of that period and quickly recovered, this suggests that these smaller stock western markets can “phase lock” (in a weak sense) not only because of the over-arching influence of Wall Street but also independently of the current trends on Wall Street due to other influences.

1 Introduction

A series of works have documented a robust and universal signature preceding large crashes occurring in major financial stock markets, namely accelerated price increase decorated by large scale log-periodic oscillations culminating close to a critical point [Sornette *et al.*, 1996, Feigenbaum and Freund, 1996, Sornette and Johansen, 1997, Johansen and Sornette, 1998, Johansen, 1997, Sornette and Johansen, 1998, Feigenbaum and Freund, 1998, Gluzman and Yukalov, 1998, Vandewalle *et al.*, 1998a, Vandewalle *et al.*, 1998b, Johansen and Sornette, 1999a, Johansen and Sornette, 1999b, Johansen *et al.*, 2000, Johansen *et al.*, 1999c, Drozd *et al.*, 1999]. Specifically, in the simplest form, the index $I(t)$ can be represented by the following time dependence

$$I(t) = A + B(t_c - t)^z + C(t - t_c)^z \cos(\omega \log(t_c - t) - \phi) , \quad (1)$$

where A is the terminal price at the critical time t_c , the exponent $0 < z < 1$ describes an acceleration and ω and ϕ are respectively the angular frequency of the log-periodic oscillations and their phase (or time unit). The log-periodic oscillations, *i.e.*, periodic in the variable $\log(t_c - t)$, are the hallmark of a discrete scale invariance [Sornette, 1998] since the argument of the cosine is reproduced at each time t_n converging to t_c according to a geometrical time series $t_c - t_n \propto \lambda^{-n}$ where $\lambda = e^{2\pi/\omega}$.

The previously reported cases well-described by eq.(1) comprise the Oct. 1929 US crash, the Oct. 1987 world market crash, the Oct. 1997 Hong-Kong crash, the Aug. 1998 global market events, the 1985 Forex event on the US dollar, the correction on the US dollar against the Canadian dollar and the Japanese Yen starting in Aug. 1998, as well as the bubble on the Russian market and its ensuing collapse in June 1997 [Johansen *et al.*, 1999c]. Symmetrically, “anti-bubbles” with decelerating market devaluations following all-time highs have also been found to carry strong log-periodic structures [Johansen and Sornette, 1999d], represented by eq.(1) with $t_c - t$ changed into $t - t_c$,

$$\log(I(t)) = A + B(t - t_c)^z + C(t - t_c)^z \cos(\omega \log(t - t_c) - \phi) . \quad (2)$$

The use of the logarithm of the index instead of the index as in (1) has been discussed in [Sornette and Johansen, 1997, Johansen and Sornette, 1999a, Johansen *et al.*, 1999c] and is related to the duration of the bubble/anti-bubble as well as to the dynamics controlling the amplitude of the collapse. A quite remarkable strong example of such an anti-bubble is given by the Japanese Nikkei stock index from 1990 to present as well as by the Gold future prices after 1980, both after their all-time highs. For the Nikkei, a theoretical formulation [Johansen and Sornette, 1999d] allowed us to issue a quantitative prediction in early Jan.1999 (when the Nikkei was at its low), that the index will exhibit a recovery over 1999 and 2000 [Johansen and Sornette, 1999d, Johansen and Sornette, 1999e].

The hypothesis to rationalize these empirical facts first proposed in [Sornette *et al.*, 1996] is that stock market crashes are caused by the slow buildup of long-range correlations between traders leading to the expansion of a speculative bubble that may become unstable at a critical time and lead to a crash or to a drastic change of market regime. Bubbles are considered to be natural occurrences of the dynamics of stock markets, as argued persuasively by Keynes [Keynes, 1964] and illustrated intuitively in classroom experiments [Ball and Holt, 1998]. It is possible to be more quantitative and construct a rational expectation model of bubbles and crashes based on Blanchard’s model [Blanchard, 1979], which has two main components: (1) we assume that a crash may be caused by *local* self-reinforcing imitation processes between noise traders which can be quantified within the frame-work of critical phenomena developed in the Physical Sciences and (2) we allow for a remuneration of the risk of a crash by a higher rate of growth of the bubble, which reflects that the crash is not a certain deterministic outcome of the bubble and, as a consequence, it remains rational for traders to remain invested provided they are suitably compensated. The bubble price is then completely controlled by the crash hazard rate and we have proposed that its acceleration and its log-periodic structures are the hallmark of a discrete scale invariance, appearing as a result of self-organising interactions between traders [Sornette *et al.*, 1996, Johansen *et al.*, 2000, Johansen *et al.*, 1999c].

These empirical facts have until now been restricted to the major financial markets of the world (WMFMs), *i.e.*, the stock markets on Wall Street, Tokyo and Hong Kong as well as the foreign exchange market (FOREX) and the Gold market in the seventies and early eighties. Recently, it was established [Johansen *et al.*, 1999c] that the Russian stock market in $\approx [1996.2 - 1997.6]$ exhibited an extended bubble followed by a (relatively slow but) large crash, which had strong characteristics of log-periodicity decorating a power law acceleration of the index, similar to those found in the WMFMs [Johansen and Sornette, 1999a]. This raises the question whether such behavior may be found in emerging markets in general or if the Russian case is unique due to its rather special characteristics [Intriligator, 1998].

The purpose of the present analysis is twofold. The main objective is to answer this question concerning whether log-periodic power laws can be as successfully applied to speculative bubbles on emerging markets as it has been done on the WMFMs. This is done by analyzing a range of emerging stock markets using the same tools as in the previous analysis of the WMFMs, as well as comparable time scales. The second objective is to illustrate on a qualitative level using log-periodic signatures that the smaller Western stock markets are strongly influenced by the leading trends on Wall Street. Furthermore, we will show quantitatively that these smaller stock markets can “phase lock” (in a weak sense) not only because of the over-all influence of Wall Street but also independently of the current trends on Wall Street.

The methodology we adopt is the one used in our previous works on the WMFMs, which consists in a combination of parametric fits with formulas like (1) and of non-parametric log-frequency analysis [Johansen and Sornette, 1999a, Johansen and Sornette, 1999b, Johansen *et al.*, 2000, Johansen *et al.*, 1999c]. We have established the reliability of this approach by extensive numerical tests on synthetic data. The use of the same method will allow us to test the hypothesis that emerging markets exhibit bubbles and crashes with similar log-periodic signatures as in the WMFMs. We stress from the beginning that the results obtained on the emerging markets analysed here does not carry the same robustness as obtained for the WMFMs with respect to identification of the bubble and the values obtained for the exponent z and the frequency ω of the log-periodic oscillations. We expect in part the technical difficulties in maintaining a high-quality stock markets index on these smaller emerging markets to be responsible for this. More important is presumably the fact that these emergent markets are strongly influenced by events not directly related to the economy and stock market of that particular country due to their smaller size. This also explains the fact that the life-time of the bubbles identified on these emergent markets are in general of somewhat shorter time-span compared to the bubbles and anti-bubbles previously identified on the WMFMs. Fundamentally, this is related to the question over which time-scales can a given market be regarded as a closed system with a good approximation and connects to the question of finite-size effects, a question that has been much studied in relation to critical phenomena in physical systems [Cardy, 1998].

2 Emerging Markets

2.1 Speculative bubbles

Emerging markets are often the focus of interest and also often exhibit large financial crises [Lowell *et al.*, 1998]. The story of financial bubbles and crashes has repeated itself over the centuries and in many different locations since the famous tulip bubble of 1636 in Amsterdam, almost without any alteration in its main global characteristics [Galbraith, 1997, Montroll and Badger, 1974].

1. The bubble starts smoothly with some increasing production and sales (or demand for some commodity), in an otherwise relatively optimistic market.
2. The interest for investments with good potential gains then leads to increasing investments possibly with leverage coming from novel sources, often from international investors. This leads to price appreciation.
3. This in turn attracts less sophisticated investors and, in addition, leveraging is further developed with small

down payment (small margins or binders), which lead to a demand for stock rising faster than the rate at which real money is put in the market.

4. At this stage, the behavior of the market becomes weakly coupled or practically uncoupled from real wealth (industrial and service) production.
5. As the price skyrocket, the number of new investors entering the speculative market decreases and the market enters a phase of larger nervousness, until a point when the instability is revealed and the market collapses.

This scenario applies essentially to all market crashes, including old ones such as Oct. 1929 on Wall Street, for which the US market was considered to be at that time an interesting “emerging” market with good investment potentialities for national as well as international investors. The robustness of this scenario is presumably deeply rooted in investors psychology and involves a combination of imitative/herding behavior and greediness (for the development of the speculative bubble) and over-reaction to bad news in period of instabilities.

2.2 Classification of markets

The commonalities recalled above do not imply that different markets exhibit the same price trajectories. There can be strong difference due to local constraints, such as cash flow restrictions, government control and so on. In our analysis of several emerging markets, we find three main classes.

2.2.1 Latin-American markets

The Latin-American stock markets, that we analyze in details below, seems to display features reminiscent of the largest financial markets, however, with much larger fluctuations in the values obtained for the exponent z and log-angular frequency ω . In the next sections, we will see to what extent this similarity can be quantified. Specifically, the accelerating log-periodic power law (1) will be fitted to the various stock market data preceding large crashes as well as large decreases. We do not posit that a crash has to occur suddenly, only that it marks the end of an accelerating bullish period and the beginning of a bearish regime cumulating in a significant drop.

2.2.2 Asian tigers

The stock markets of Asian Tigers, specifically Korea, Malaysia and Thailand as well as that of the Philippines and Indonesia also display approximate accelerating power law bubbles and subsequent crashes. However, the acceleration accompanying the observed bubbles in these markets often seem incompatible with the requirement of either $0 < z < 1$ or, more important, that of a *real* power law with $t < t_c$ in eq. (1). This since the optimisation algorithm kept on insisting on a t_c smaller than the last data point, thus causing a floating point error. For the smaller Asian stock markets studied here this problem could be cured working on the *logarithm* stock market index instead with the exception of the Korean stock market and the 1997 crash in Indonesia. Again we mention that depending on the price dynamics ending the bubble, the index or the logarithm of the index turns out to be the relevant observable quantifying the acceleration of the bubble [Johansen and Sornette, 1999a, Johansen *et al.*, 1999c]. Naturally, the nature of the log-periodic oscillations do not depend of which observable is used.

As an additional example of the difference between the WMFMs and the stock markets of Indonesia, Korea, Malaysia, Philippines and Thailand, we find that similarly to what is seen for the Latin-American markets, the values obtained for the exponent z and log-angular frequency ω from the fitting with eq. (1) fluctuates considerably compared to the WMFMs, as reported in table 4 and [Johansen and Sornette, 1999a]. A recent analysis shows that the Asian stock returns exhibit characteristics of bubbles, which are however incompatible in details with the prediction of the model of rational speculative bubbles [Chan *et al.*, 1998]. This suggests that a different

formulation than simply using eq. (1) is needed in order to capture the trends displayed by these South-East and East Asian stock markets prior to large corrections and crashes. In this respect, we note that the Korean stock market could not be shown to display bubbles following eq. (1) nor the Indonesian crash of July 1997.

2.2.3 East-European stock markets

The East-European stock markets seems to be following a completely different logic than their larger Western counterparts and their indices do not resemble those of the other markets. In particular, we find that they do not follow neither power law accelerations nor log-periodic patterns though large crashes certainly occurs.

2.3 Latin-American markets

2.3.1 Identification of bubbles

In figure 1 to 6, the evolution of six Latin-American stock market indices (Argentina, Brazil, Chile, Mexico, Peru and Venezuela) is shown as a function of time from early in this decade to Feb. 1999.

We first define a bubble as a period of time going from a pronounced minimum to a large maximum by a prolonged price acceleration, followed by a crash or a large decrease represented by a bear-market. As for the WMFMs, such a bubble is defined unambiguously by identifying its end with the date t_{max} where the highest value of the index is reached prior to the crash/decrease. For the bubbles prior to the largest crashes on the WMFMs, the beginning of a bubble is clearly identified as coinciding always with the date of the lowest value of the index prior to the change in trend. However, this identification is not as straightforward for the Latin-American and smaller Asian indices analyzed here. Hence, in approximately half the cases, the date of the first data point used in defining the beginning of the bubble had to be moved up and the bubble had to be truncated in order to obtain fits with non-pathological values for z and ω . This may well be an artifact stemming from the restrictions in the fitting imposed by using a single cosine as the periodic function in eq. (1). We recall that the exponent z is expected to lie between zero and one and it should be not too close to both zero and one: too small a z implies a flat bubble with a very sudden acceleration at the end. Too large a z corresponds to an almost linear non-accelerating bubble. The angular frequency ω of the log-periodic oscillations must also not be too small or too large. If it is too small, less than one oscillation occurs over the whole interval and the log-periodic oscillation has little meaning. If it is too large, the oscillations are too numerous and they start to fit the high-frequency noise.

From the six stock market indices, we have identified by eye four Argentinian bubbles, one Brazilian bubble, two Chilean bubbles, two Mexican bubbles, two Peruvian bubbles and a single Venezuelan bubble, with a subsequent large crash/decrease, as shown in figures 1 to 6. Before the reader starts to argue that our procedure is rather arbitrary and that many other bubbles can be seen on the figures, we stress that times scales considered should be comparable with those of the larger crashes analyzed in [Johansen and Sornette, 1999a, Johansen *et al.*, 2000, Johansen *et al.*, 1999c] and not considerably less than one year. This has been achieved in most cases for the bubbles, whereas the life-time of the anti-bubbles seems to be shorter as a rule. Exceptions are the first and second Argentinian bubbles, the second Chilean bubble and the first Mexican, as shown in figures 7, 8, 13 and 15, where the fitted interval is ≈ 0.7 years except for the second Argentinian bubble, where only ≈ 0.4 years could be fitted. On purpose, we have restrained from analyzing log-periodic structures on smaller scales in order to obtain a good comparison with our previous analysis on WMFMs. That the time-scales on which the bubbles has been identified in general are shorter than for the WMFMs is as mentioned not very surprising.

2.3.2 Results

In figures 7 to 19, we see the fits of the bubbles indicated in figures 1 to 6 as well as the spectral Lomb periodogram [Flannery *et al.*, 1992] of the difference between the indices and the pure power law defined as

$$I(t) \rightarrow \frac{I(t) - [A - B(t_c - t)^z]}{C(t_c - t)^z}. \quad (3)$$

One exception is the second Peruvian bubble of which a numerically stable fit could not be obtained due to an almost vertical raise at the very end of the bubble. Using the logarithm of the index instead did only in part solve this problem and we have not included this fit in the present paper.

If log-periodicity is present in the data as quantified by eq. (1), the residue defined by eq. (3) should be a pure cosine of $\omega \ln(t_c - t)$ and a spectral analysis of this variable should give a strong peak around ω . For this, we use the Lomb spectral analysis, which corresponds to a harmonic analysis using a series of local fits of a cosine (with a phase) with some user chosen range of frequencies. The advantage of the Lomb periodogram over a Fast Fourier transform is that the points do not have to be equidistantly sampled, which is the generic case when dealing with power laws. For unevenly sampled data, the Lomb method is superior to FFT-methods because it weights data on a “per point” basis instead of “per time interval” basis. Furthermore, the significance level of any frequency component can be estimated quite accurately if the nature of the noise is known.

It is clear from simply looking at the figures, that the overall quality of these fits is rather good and both the acceleration and the accelerating oscillations are rather well captured by eq. (1). We let the reader directly appreciate the quality of the fits on the figures. We notice that, notwithstanding their value, the fits do not have the same excellent over-all quality as for those obtained for the WMFMs as well as for the Russian stock market [Johansen *et al.*, 1999c]. A plausible interpretation is that we deal here with relatively small markets in terms of capitalization and number of investors, for which *finite size effects*, in the technical sense given in Statistical Physics [Cardy, 1998], are expected and thus may blur out the signal with systematic distortions and unwanted fluctuations. In this vein, numerical simulations of all (with one single exception [Stauffer and Sornette, 1994]) available microscopic stock market models have shown that simple regular deterministic dynamics is obtained when the limit of a large effective number N of traders is taken while the stock market behavior seems realistically random and complex when only a few hundred traders are simulated [Busshaus and Rieger, 1999, Egenter *et al.*, 1999, Hellthaler, 1995, Kohl, 1997].

In tables 1 and 2, the parameters of the various fits are given as well as the beginning and ending dates of the bubble and the size of the crash/correction, defined as

$$\text{drop \%} = \frac{I(t_{max}) - I(t_{min})}{I(t_{max})}. \quad (4)$$

Here, t_{min} is defined as the date after the crash/correction where the index achieves its lowest value before a clear novel market regime is observed. The duration $t_{max} - t_{min}$ of the crash/correction is found to range from a few days (a crash) to a few months (a change of regime).

From table 1, we observe that the fluctuations in the parameters values z and ω obtained for the 11 Latin-American crashes are considerable. The lower and upper values for the exponent z are 0.12 and 0.62, respectively. For ω , the lower and upper values are 2.9 and 11.4 corresponding to a range of λ 's in the interval 1.8 – 8.8. Removing the two largest values for λ reduces the fluctuations to 2.8 ± 1.1 , which is still much larger than the 2.5 ± 0.3 previously seen on WMFMs [Johansen and Sornette, 1999a]. Again, we attribute these larger fluctuations to finite-size effects.

Last, we note that three cases of anti-bubbles could be identified for the Latin-American markets analysed here, see figures 8, 14 and 19 and table 3. Quite remarkably, the first and the last are preceded by a bubble thus exhibiting a qualitative symmetry around comparable t_c 's as defined in eq.'s (1) and (2).

2.4 Asian markets

2.4.1 Identification of bubbles

In figures 20 to 25, the evolution of six Asian stock market indices (Hong-Kong, Indonesia, Korea, Malaysia, Philippines and Thailand) is shown as a function of time from 1990 to Feb. 1999 except for Hong-Kong, which goes back to 1980.

From the six stock market indices, we have identified three bubbles on the Hong-Kong stock market, two on the Indonesian, two on the Korean and one on the Malaysian, Philippine and Thai stock markets, respectively, with subsequent crashes/decreases that could be identified by eye, as indicated in figures 20 to 25. Of these, the two Korean bubbles and the second Indonesian could not be quantified using eq. (1). Of the remaining seven which could, all except the Hong-Kong crashes of Oct. 1987 and Oct. 1997 belonged to the same period ending in Jan. 1994 as also found for the Latin-American markets analysed in section 2.3.2 with the exception of Venezuela. As we shall see in section 3, this globally coordinated crash on emerging markets triggered a correlated anti-bubble on the smaller Western stock markets.

2.4.2 Results

In figures 26 to 32, we see the fits of the bubbles indicated in figures 20 to 25 as well as the spectral Lomb periodogram of the difference between the indices and the pure power law with the exceptions of the Korean stock market and the second Indonesian bubble, which could not be quantified by eq. (1).

In tables 4 and 5, the parameters of the various fits are given as well as the beginning and ending dates of the bubble and the size of the crash/correction. We again see somewhat larger fluctuations in the values for the exponent z and the log-angular frequency ω compared to the WMFMs as for the Latin-American markets. However, except for the Indonesian and Korean bubbles, the results are surprisingly consistent with what has been obtained for the WMFMs as well as for the Latin-American markets.

3 Correlations across Markets

It is well-known that the Oct. 1987 crash was an international event, occurring within a few days in all major stock markets [Barro *et al.*, 1989]. It is also often noted that smaller West-European stock markets as well as other markets around the world are influenced by dominating trends on Wall Street. This correlation seems to have increased over the years as can be seen with the naked eye by comparing the start and the end intervals of figure 33 (showing several market indices prior and after the Aug. 1998 turmoil). We observe a clear qualitative strengthening of the correlations between Wall Street and the smaller Western stock markets in this decade. Specifically, identifying “spikes” in either direction for the two end intervals of figure 33, a stronger correspondence between changes in the various indices is clearly observed in the later period compared to the former. We stress that the suggested dependence between these markets would not necessarily be detected by standard correlation measures, which are averages over long periods of time and detect only a part of possible dependence structures. What we unravel here corresponds to “phasing-up” between markets at special times of large moves and/or large volatilities.

An example of a decoupling between the West-European stock markets and Wall Street in the first part of this decade comes from the period following the crashes/corrections on most emerging stock markets in early 1994. This rash of crises occurred from January to June 1994 and concerned the currency markets (Mexico, South Africa, Turkey, Venezuela) and the stock markets (Chile, Hungary, India, Indonesia, Malaysia, Philippines, Poland, South Africa, Turkey, Venezuela, Germany, Hong-Kong, Singapore, U.K.) [Lowell *et al.*, 1998]. The period of time is associated to sharply rising U.S. interest rates. Whereas the S&P500 dipped less than 10% and recovered within a few months, see figure 34, the effect was much more profound on smaller Western stock markets worldwide. Surprisingly, the toll on a range of western countries resembled that of a mini-recession with decreases between

18% (London) and 31% (Hong Kong) over a period from ≈ 5 months (London) to ≈ 13 months (Madrid), as summarized in table 6. For each stock market, the decline in the logarithm of the index has been fitted with eq. (2). In figures 35 to 41, we see that the decreases in all the stock markets analyzed can be quantified by eq. (2) as *log-periodic anti-bubbles* [Johansen and Sornette, 1999d].

Using the second best fit of the CAC40 and the Swiss indices, we see that the dates of the start of the decline is well-estimated by the value of t_c obtained from the fit. Furthermore, from table 6, we observe that the value of the preferred scaling ratio $\lambda = e^{2\pi/\omega}$ is remarkable consistent $\lambda \approx 2.0 \pm 0.3$. This comes as a good surprise, considering that the stock markets that have been analyzed belong to three very different geographical regions of the world (Europe, Asia and Pacific). With respect to the value of the exponent z , the fluctuations are as usual much larger. However, excluding New Zealand and Hong-Kong*, we obtain $z \approx 0.4 \pm 0.1$, which again is quite reasonable compared to WMFMs [Johansen *et al.*, 2000, Johansen *et al.*, 1999c]. Furthermore, the amplitudes C of the log-periodic oscillations are remarkable similar with $C \approx 0.3 - 0.4$, except for London (≈ 0.02) and Milan (≈ 0.05), as shown in table 7.

4 Conclusions

Log-periodic bubbles followed by large crashes/corrections seem to be a statistical significant feature of Latin-American and Asian stock markets. Indeed, it seems quite improbable to attribute the results obtained for the Latin-American and smaller Asian stock markets to pure noise-fitting because of the relatively large number of successful cases (18) compared to the number of unsuccessful cases (4) as well as the objective criteria used in identifying them. Furthermore, removing the extreme value of $\lambda = 8.8$ for one of the Chilean bubbles gives an average of $\langle \lambda \rangle \approx 2.6$ for the remaining 17 cases, which is very close to the average value found for the worlds major financial markets [Johansen and Sornette, 1999a, Johansen and Sornette, 1999b, Johansen *et al.*, 2000, Johansen *et al.*, 1999c]. However, the results obtained for the Latin-American and smaller Asian markets are as expected less striking on a one-to-one basis than those obtained on the major financial markets of the world (WMFMs) that we analyzed previously with exactly the same methodology [Johansen and Sornette, 1999a, Johansen and Sornette, 1999b, Johansen *et al.*, 2000, Johansen *et al.*, 1999c]. In this respect, it is quite remarkable that the bubbles prior to the 3 largest crashes on the Hong-Kong stock market have the same log-frequency within $\pm 15\%$ and quite similar to what has been found for bubbles on Wall Street and the FOREX.

One important difference lies in the identification of a bubble. For the WMFMs, the identification of the first and last data point to be used in the fitting to our formulas was straightforward: The last point was chosen as the highest value of the index prior to the crash and the first point was the lowest value prior to the bubble. The results using these criteria have always been conclusive and a re-run of the fitting algorithm on a different interval was never necessary. This was not the case for the Latin-American and smaller Asian stock markets, where the first point had to be changed in approximately half the cases. This ambiguity is also reflected in the large fluctuations seen in the parameter values obtained for the meaningful variables z and ω (or equivalently $\lambda = e^{2\pi/\omega}$). Weaker signatures naturally gives larger fluctuations as well as additional sensitivity to truncation of the fitted interval. The cause for the weaker signatures can be (at least) three-fold. The signatures can be truly weaker, or they appear weaker due to the poorer quality of these smaller indices compared to those the major stock markets. Another hypothesis is the “finite-size effect” already mentioned according to which the smallest market size entails larger fluctuations and possible systematic bias. It seems at present difficult to distinguish between these different hypotheses.

With respect to the values obtained for the frequency for the best fits of the 18 Latin-American and Asian bubbles that could be quantified using eq. (1), it is rather interesting to see that the fit falls in two rather distinct

*A possible explanation for the very low value $z \approx 0.03$ may be the under-representation of trading days in the first part of the data interval due to holidays. Hence, the last part of the data, where the deceleration is weaker, is allowed to dominate thus underestimating the trend. A somewhat less severe under-sampling was also present in the New Zealand index compared to, *e.g.*, the Australian index.

clusters one around $\omega \approx 6$ and another around $\omega \approx 11$ with few values in between as shown in figure 42. It looks like a frequency-doubling, which correspond to squaring λ , as allowed by the theory of critical phenomena [Sornette, 1998, Saleur and Sornette, 1996].

In the second part of this research, we have tried to argue that, in bullish times, the leading trends on Wall Street will tend to dominate the smaller Western stock markets. However, it was shown by a quantitative analysis that these smaller stock markets can collectively decouple their dynamics from Wall Street. The case we document corresponds to a surprisingly correlated anti-bubble with log-periodic signatures and power law decay similar to what has been found on longer times scales for the Nikkei and Gold decays [Johansen and Sornette, 1999d]. In fact, the results obtained for the majority of these smaller anti-bubbles, *i.e.*, excluding the small values of the exponent z obtained for the New Zealand and Hong Kong stock markets, are quite compatible with what was obtained for the Gold decay both with respect to the values for the exponent z and preferred scaling ratio λ . This supports the notion that the higher values obtained for ω is presumably due to a more rapid dynamics present in smaller market as proposed for the Gold decay in the early eighties [Johansen and Sornette, 1999d]. With respect to the identification of the data intervals used for the smaller anti-bubbles, we stress that it did not suffer from the same problems as the Latin-American and Asian bubbles and could be directly identified unambiguously prior to and independently from the fitting procedure.

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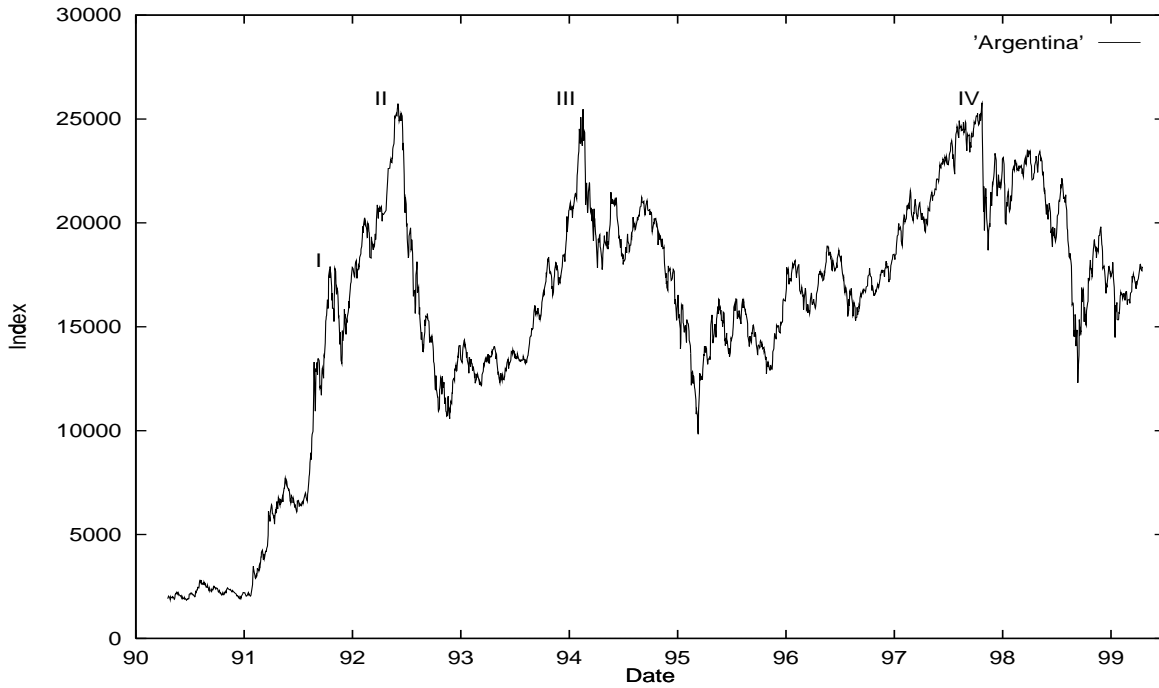


Figure 1: The Argentinian stock market index as a function of date. 4 bubbles with a subsequent very large draw down can be identified. The approximate dates are in chronological order mid-91 (I), early 93 (II), early 94 (III) and late 97 (IV).

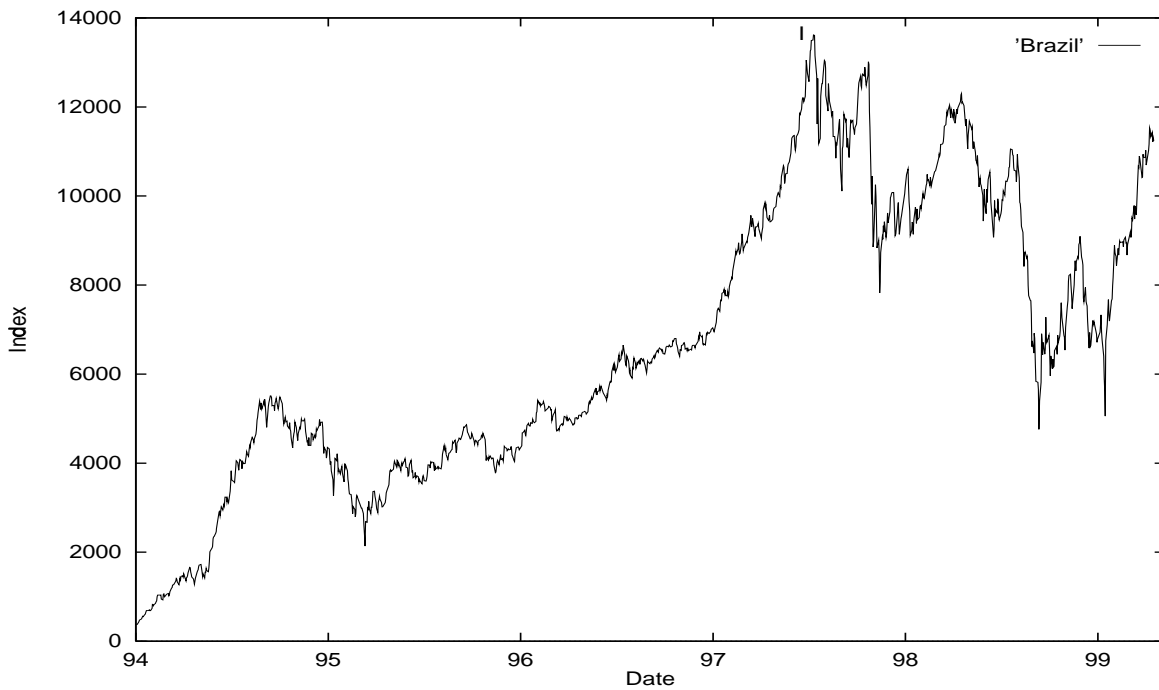


Figure 2: The Brazilian stock market index as a function of date. 1 bubble with a subsequent very large draw down can be identified. The approximate date is mid-97 (I).

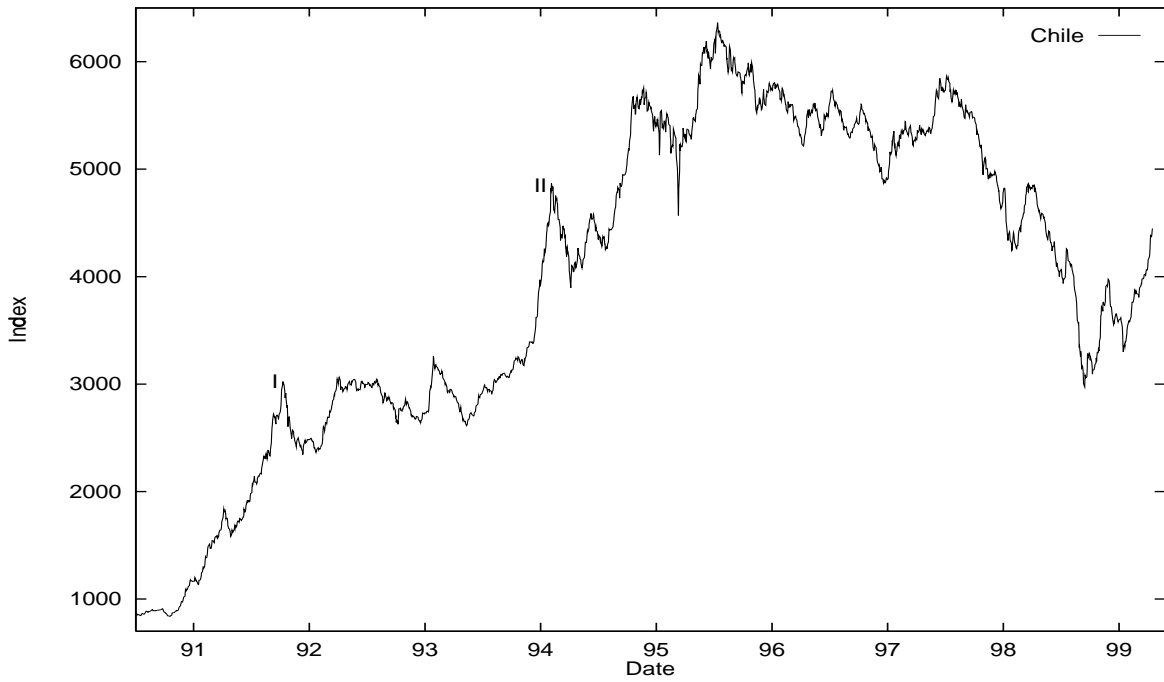


Figure 3: The Chilean stock market index as a function of date. 2 bubbles with a subsequent very large draw down can be identified. The approximate dates are in chronological order mid-91 (I) and early 94 (II).

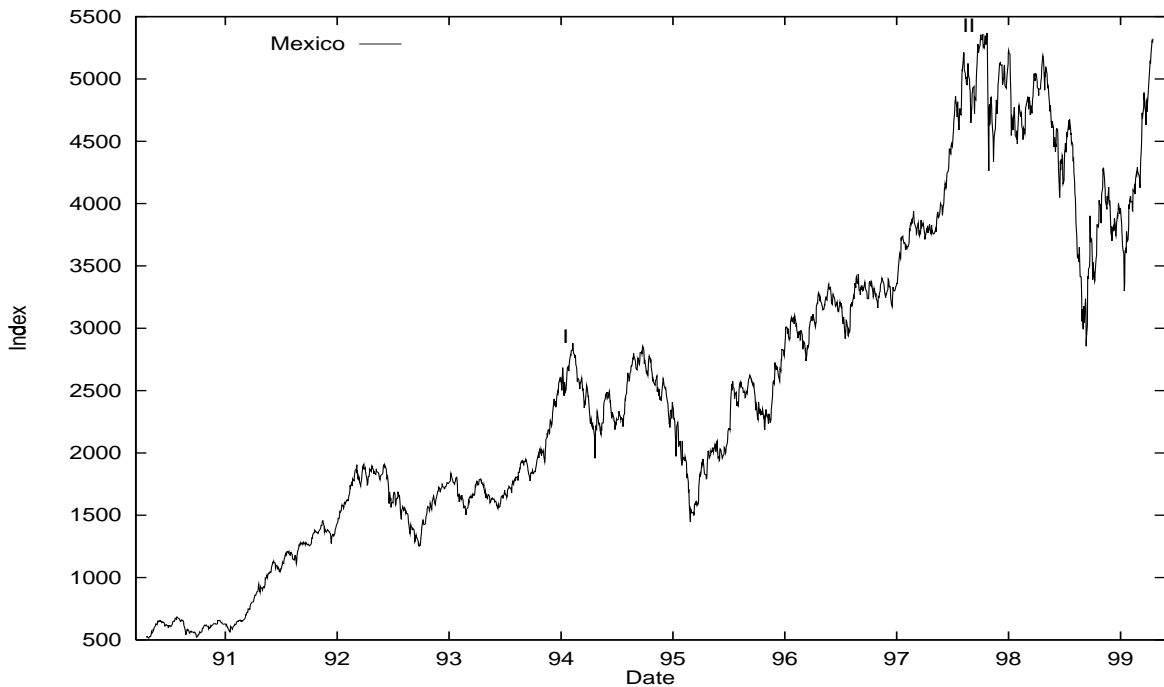


Figure 4: The Mexican stock market index as a function of date. 2 bubbles with a subsequent very large draw down can be identified. The approximate dates are in chronological order early 94 (I) and mid-97 (II).

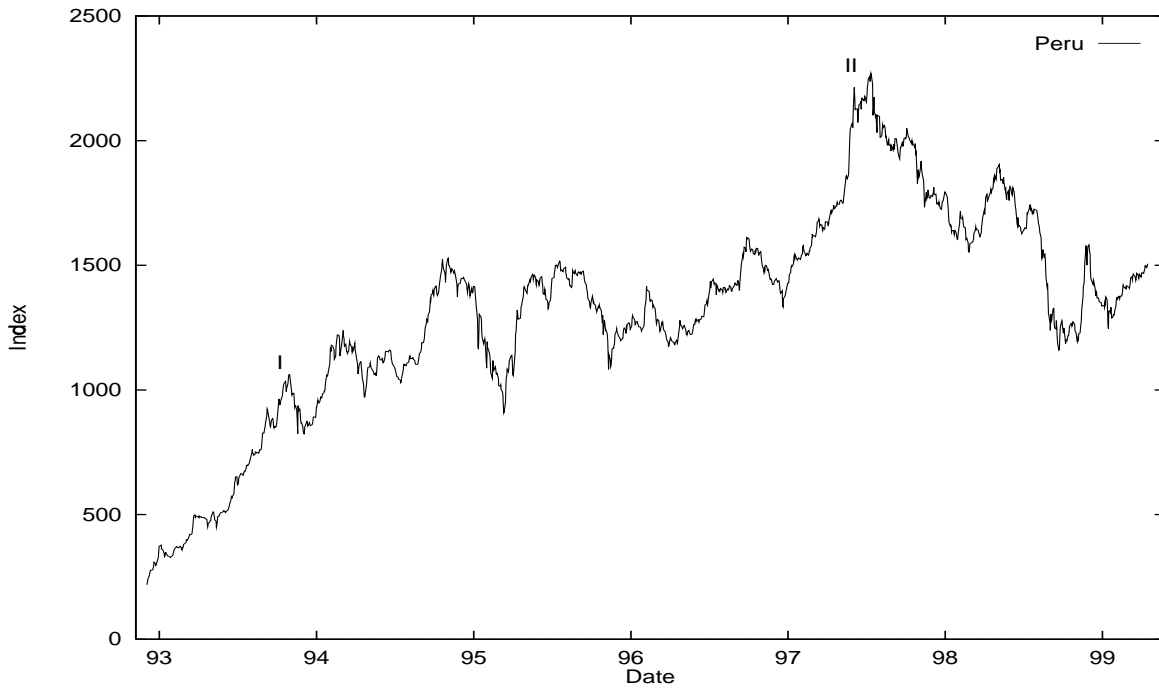


Figure 5: The Peruvian stock market index as a function of date. 2 bubbles with a subsequent very large draw down can be identified. The approximate dates are in chronological order late 93 (I) and mid-97 (II).

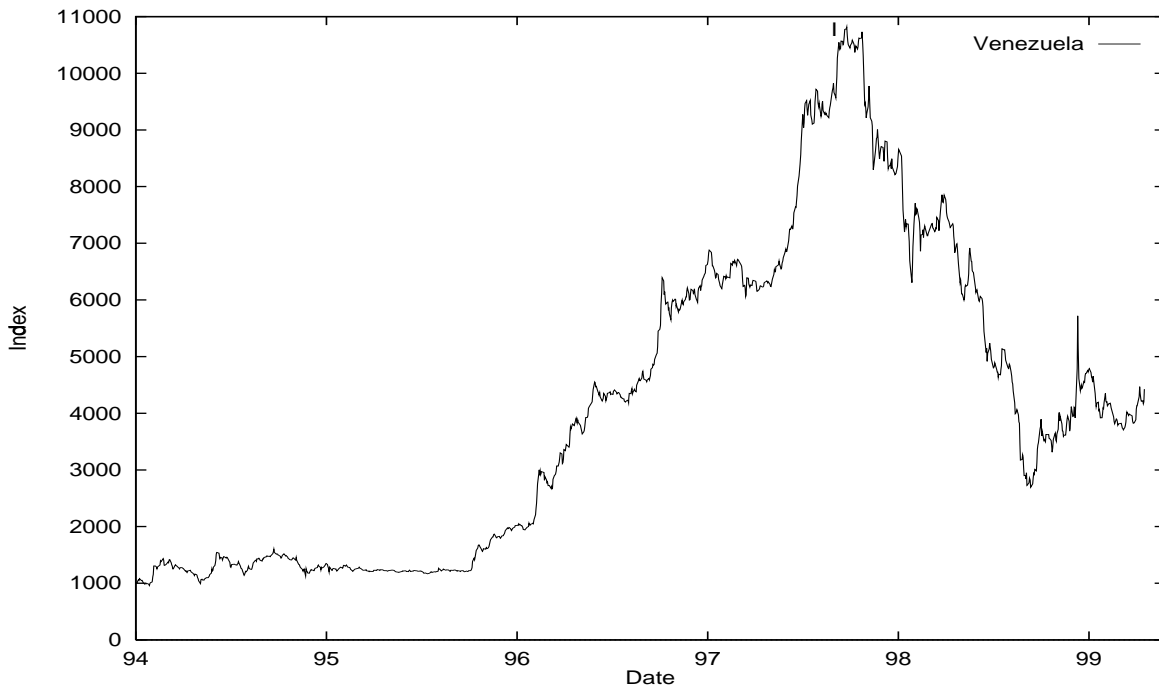


Figure 6: The Venezuelan stock market index as a function of date. 1 bubble with a subsequent very large draw down can be identified. The approximate date is mid-97 (I).

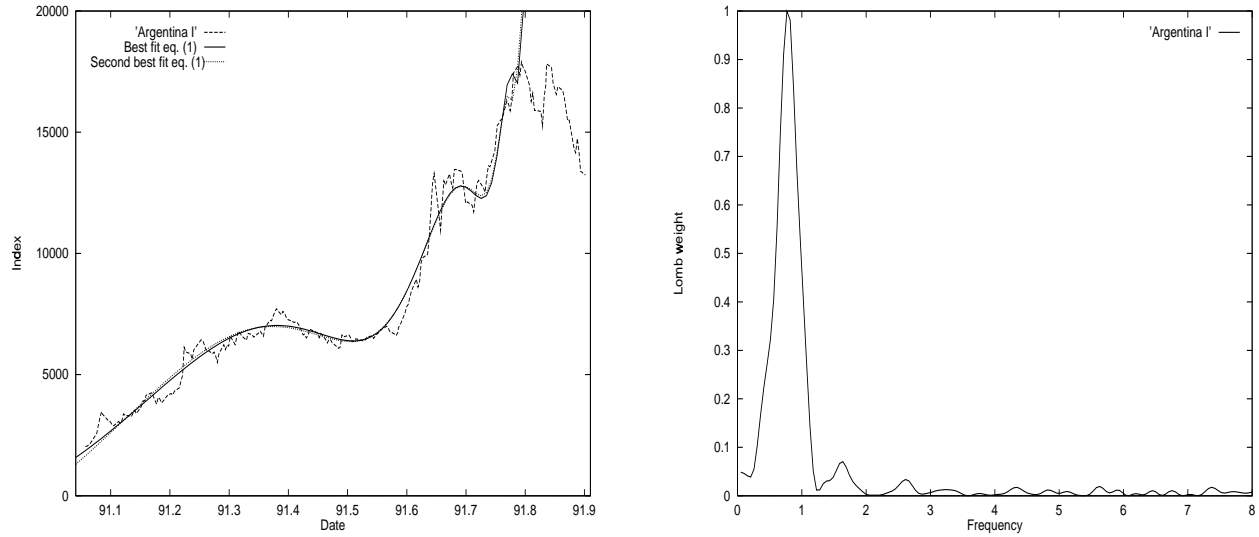


Figure 7: The Argentinian stock market bubble of 1991. See table 2 for the parameter values of the fits with eq. (1). Only the best fit is used in the Lomb periodogram.

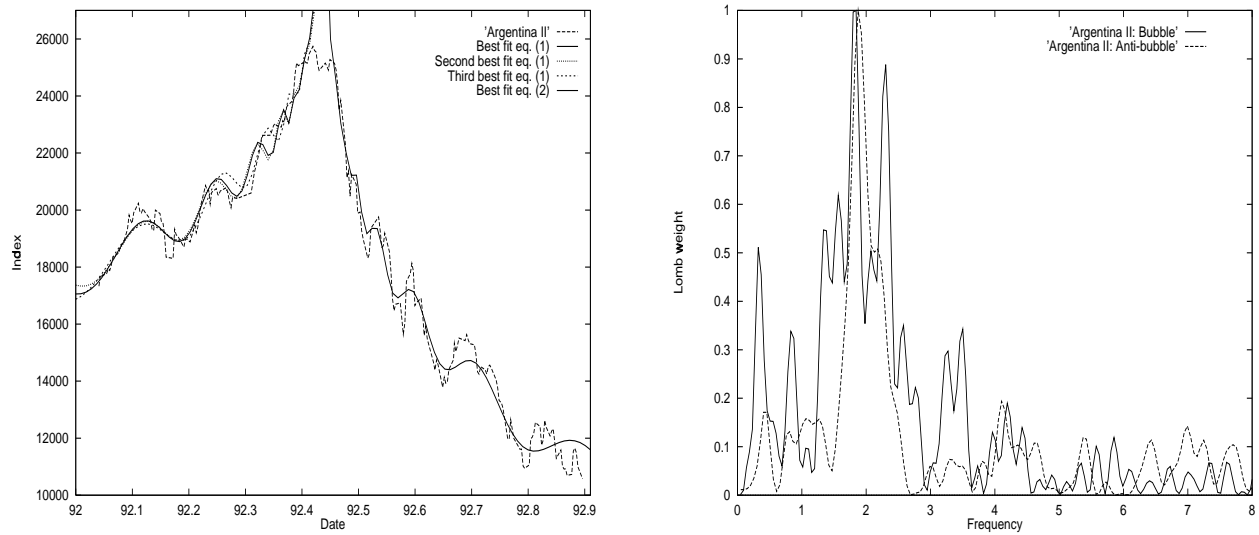


Figure 8: The Argentinian stock market bubble and anti-bubble of 1992. See table 2 for the parameter values of the fit with eq. 1 and table 3 for eq. (2). Only the best fit is used in the Lomb periodograms.

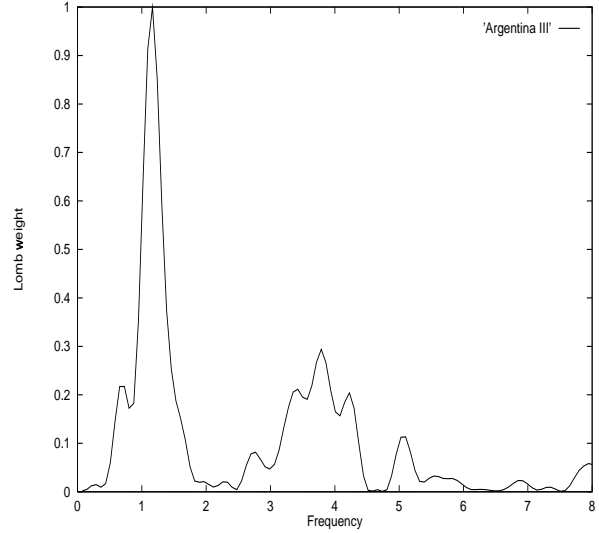
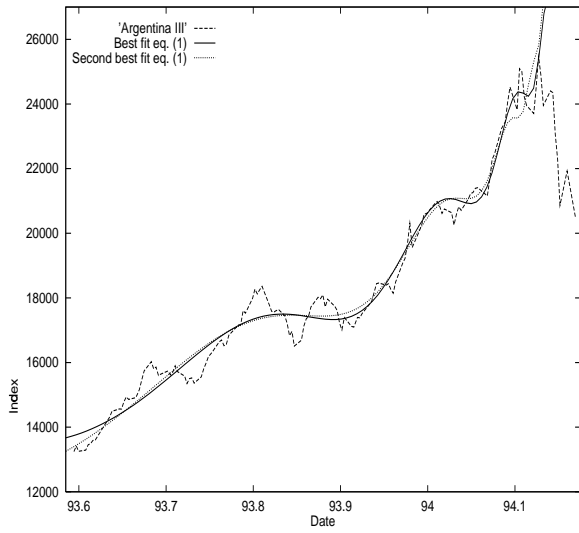


Figure 9: The Argentinian stock market bubble ending in 1994. See table 2 for the parameter values of the fit with eq. (1). Only the best fit is used in the Lomb periodogram.

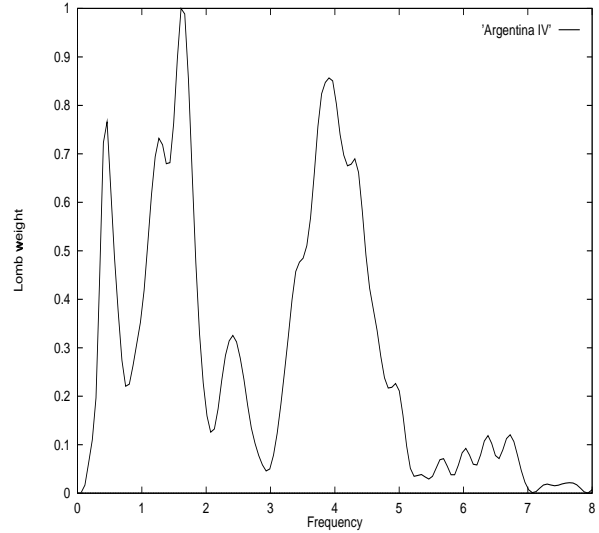
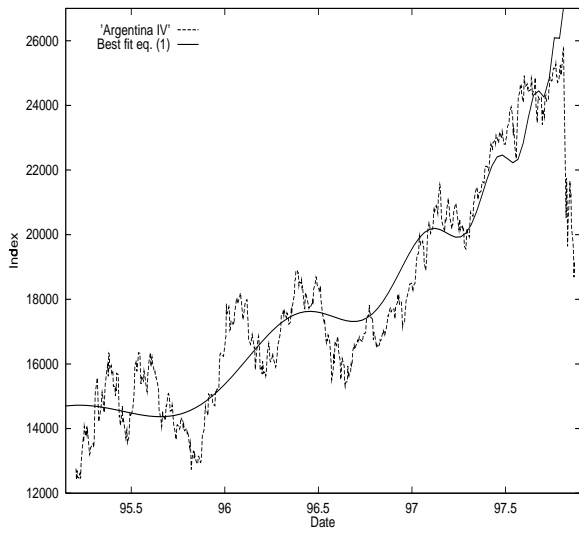


Figure 10: The Argentinian stock market bubble ending in 1997. See table 2 for the parameter values of the fit with eq. (1). Only the best fit is used in the Lomb periodogram.

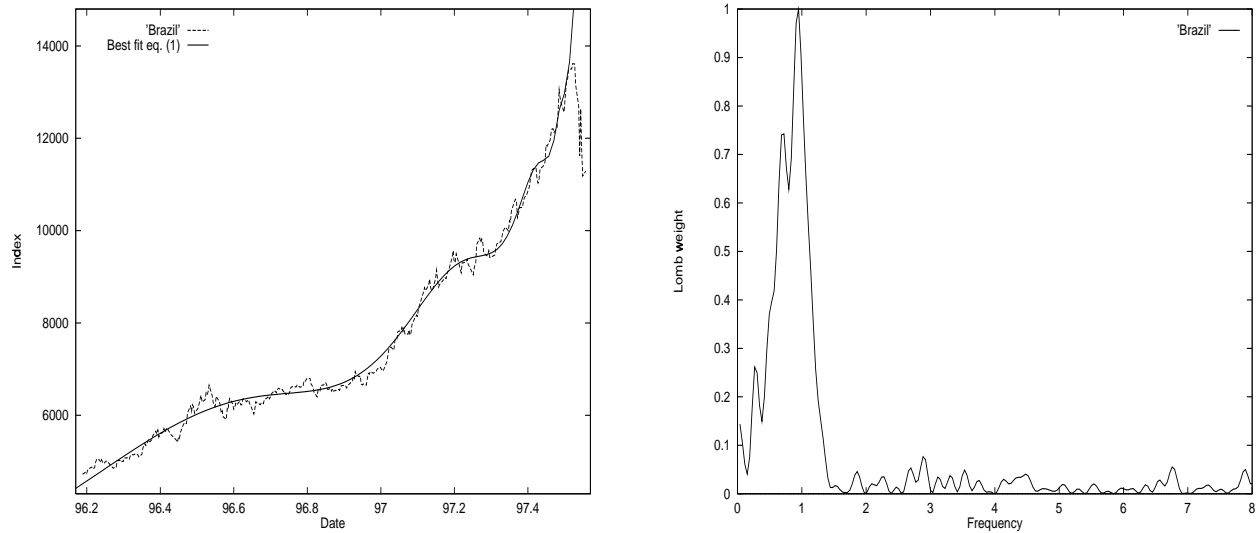


Figure 11: The Brazilian stock market bubble ending in 1997. See table 2 for the parameter values of the fit with eq. (1). Only the best fit is used in the Lomb periodogram.

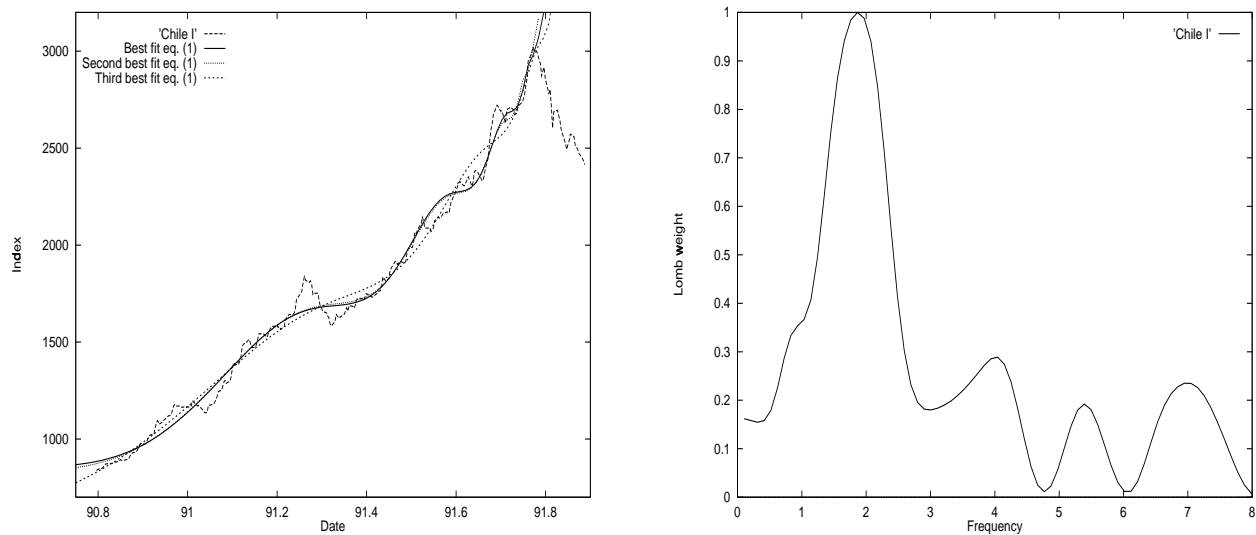


Figure 12: The Chilean bubble ending in 1991. See table 2 for the parameter values of the fit with eq. (1). Only the best fit is used in the Lomb periodogram.

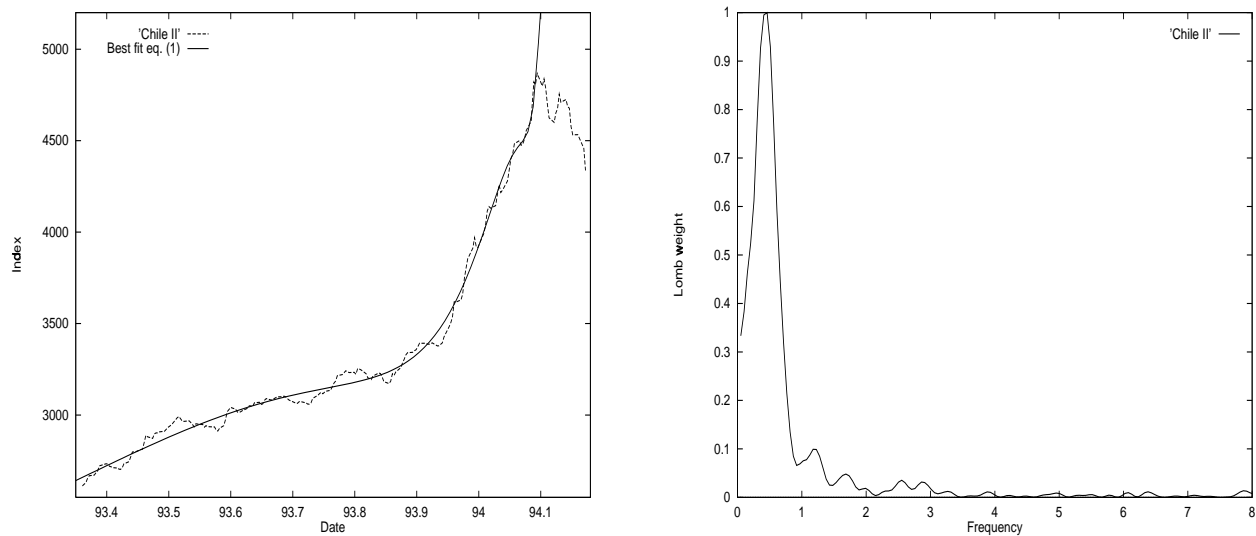


Figure 13: The Chilean bubble of 1993. See table 2 for the parameter values of the fit with eq. (1).

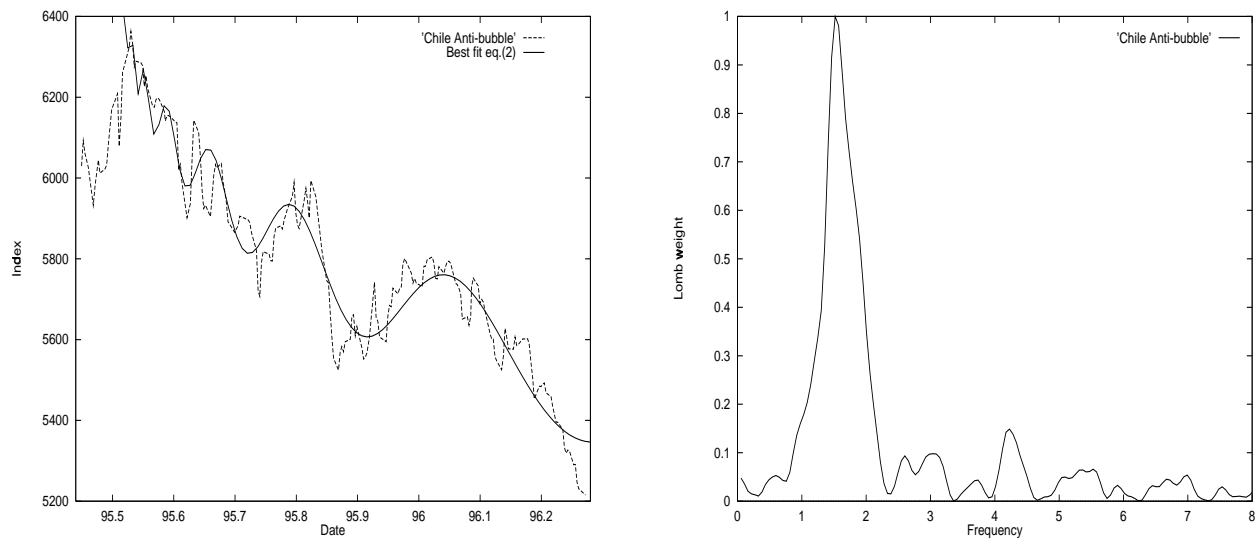


Figure 14: The Chilean anti-bubble beginning in 1995. See table 3 for the parameter values of the fit with eq. (2).

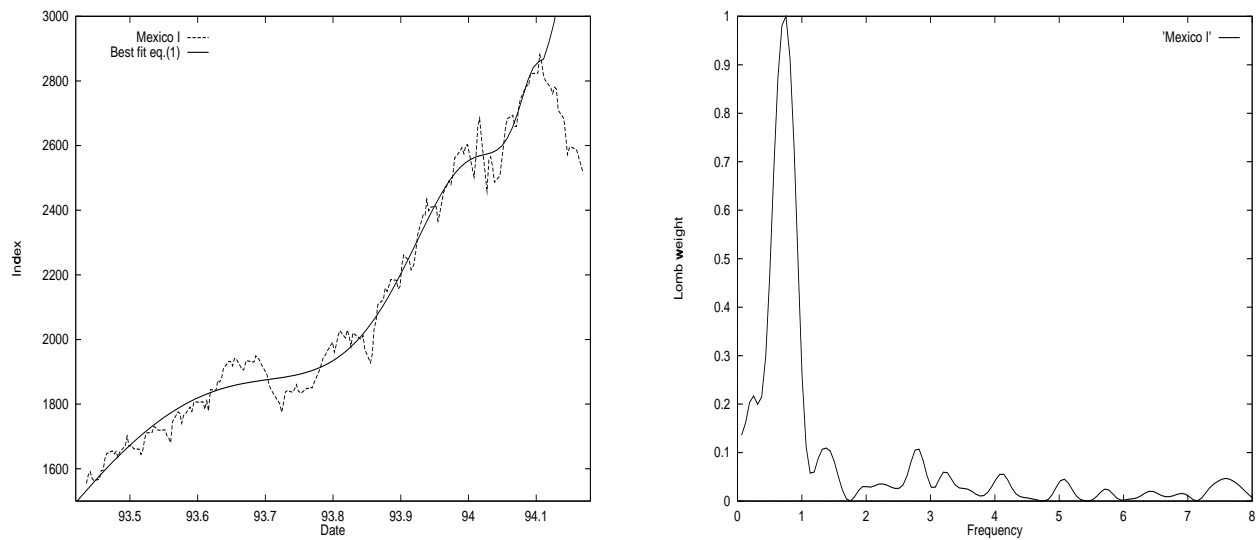


Figure 15: The Mexican bubble ending 1994. See table 2 for the parameter values of the fit with eq. (1).

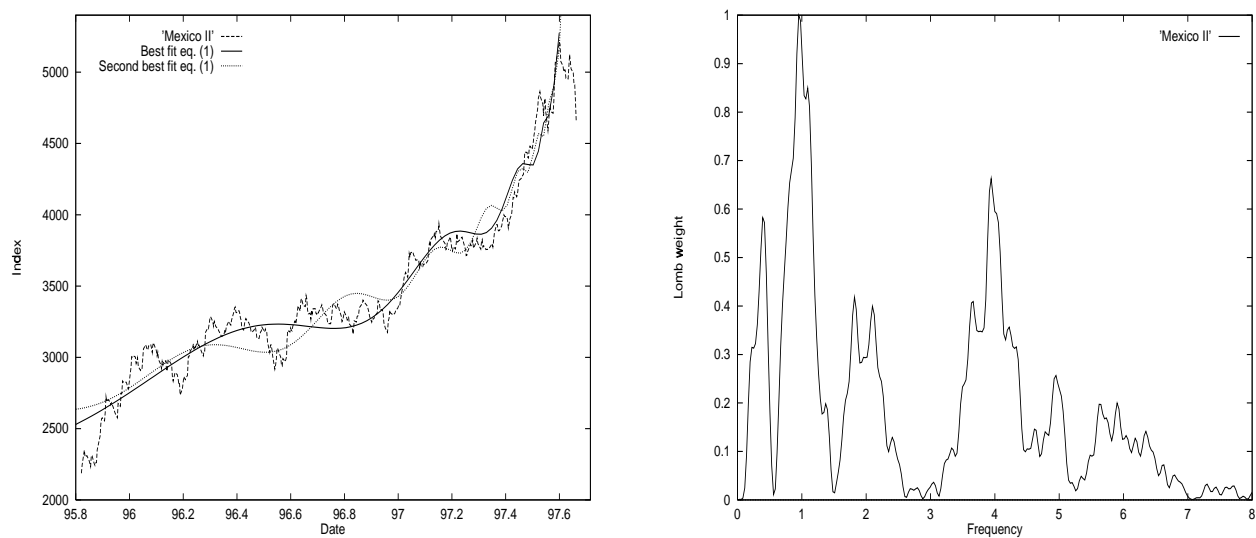


Figure 16: The Mexican bubble ending in 1997. See table 2 for the parameter values of the fit with eq. (1). Only the best fit is used in the Lomb periodogram.

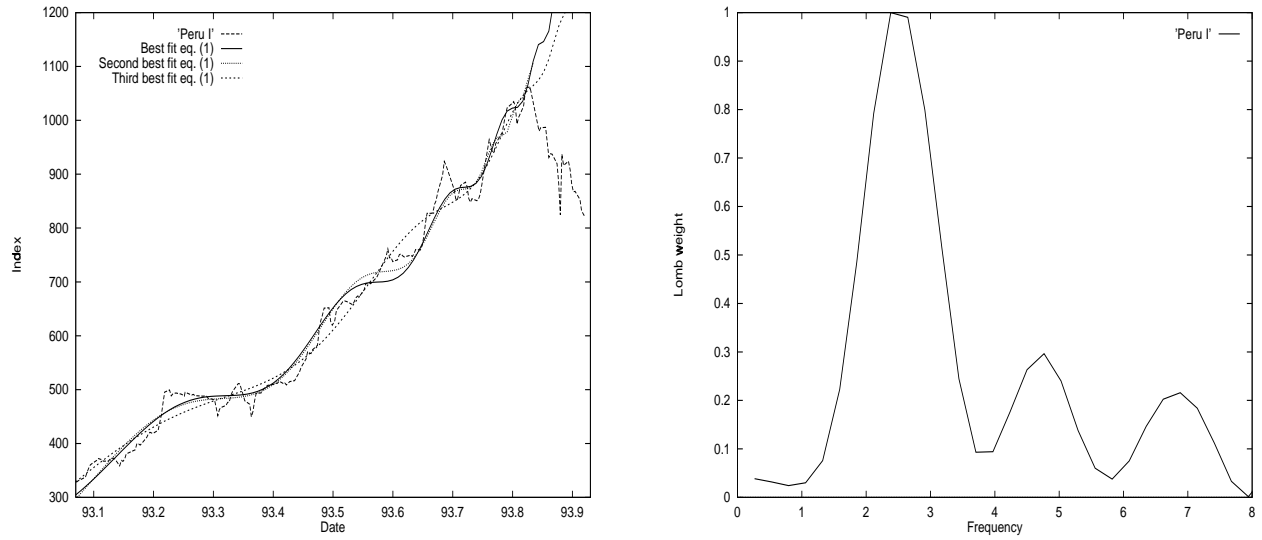


Figure 17: The Peruvian bubble of 1993. See table 2 for the parameter values of the fit with eq. (1).

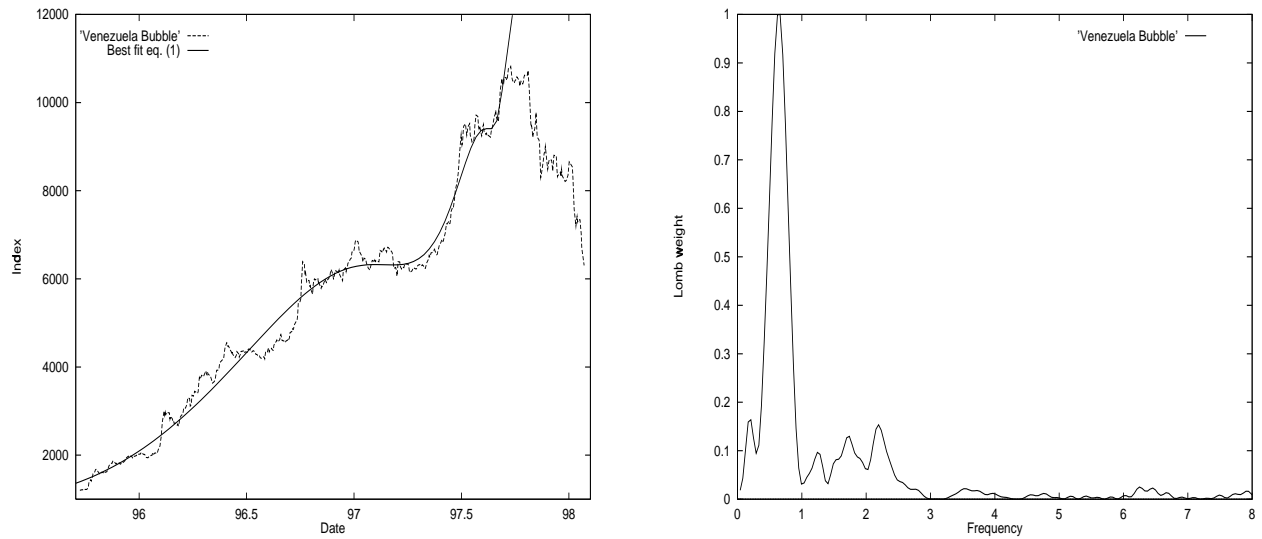


Figure 18: The Venezuelan bubble ending in 1997. See table 2 for the parameter values of the fit with eq. (1).

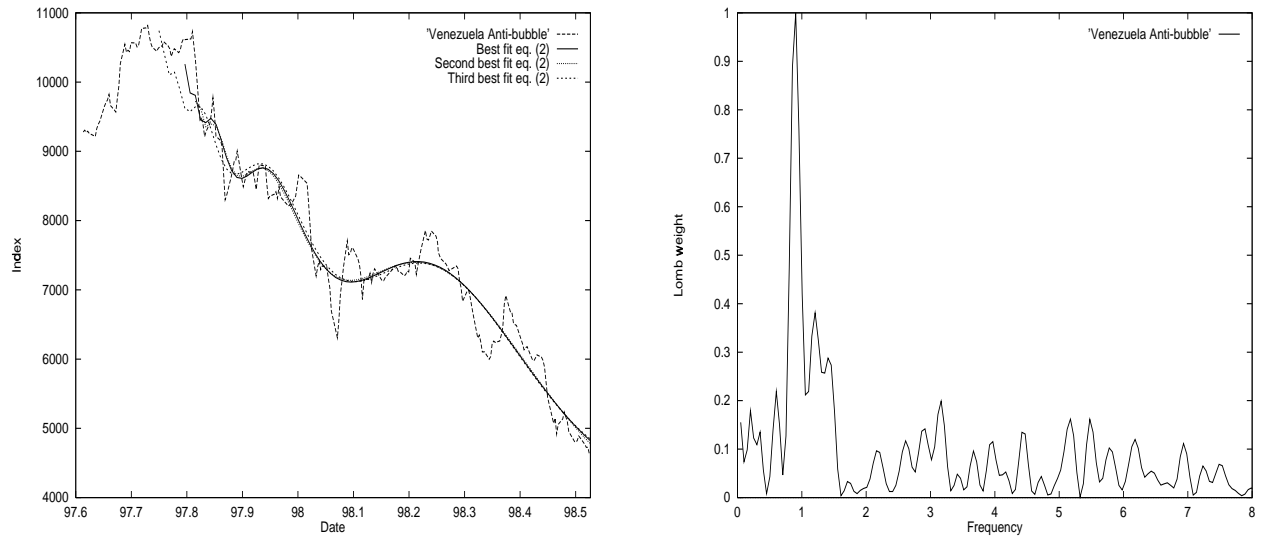


Figure 19: The Venezuelan anti-bubble starting in 1997. See table 3 for the parameter values of the fit with eq. (2). Only the best fit is used in the Lomb periodogram.

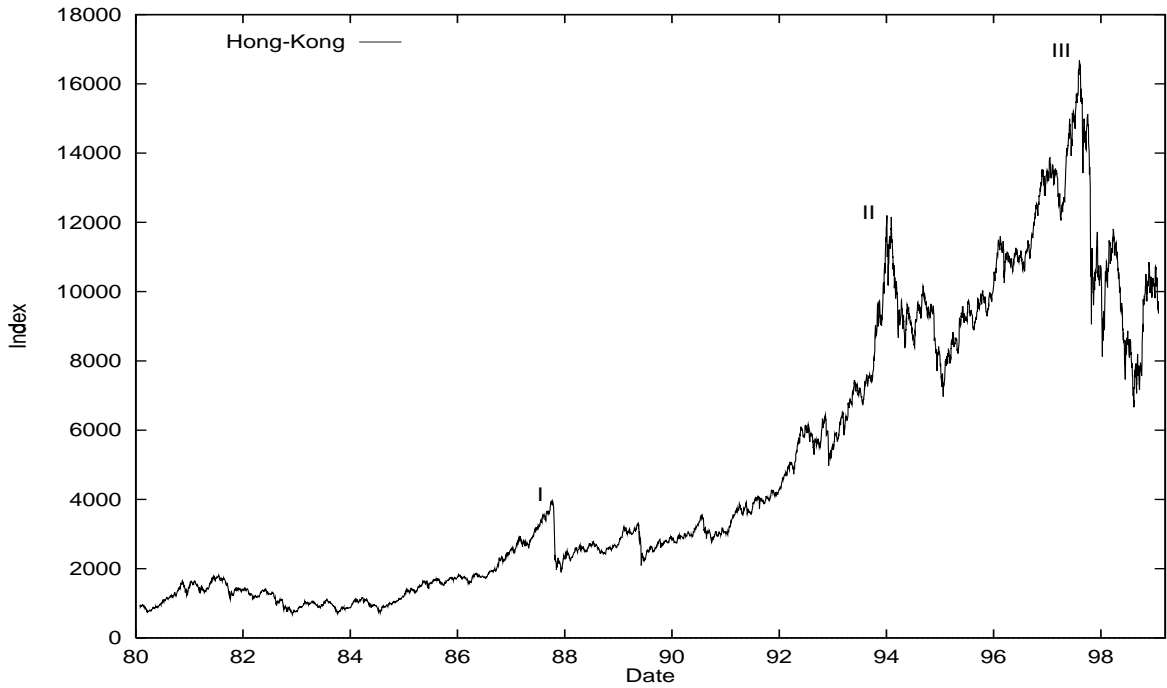


Figure 20: The Hong-Kong stock market index as a function of date. 3 extended bubbles followed by large draw downs can be identified. The approximate dates of the crashes are Oct. 87 (I), Jan 94 (II) and Oct 97 (III).

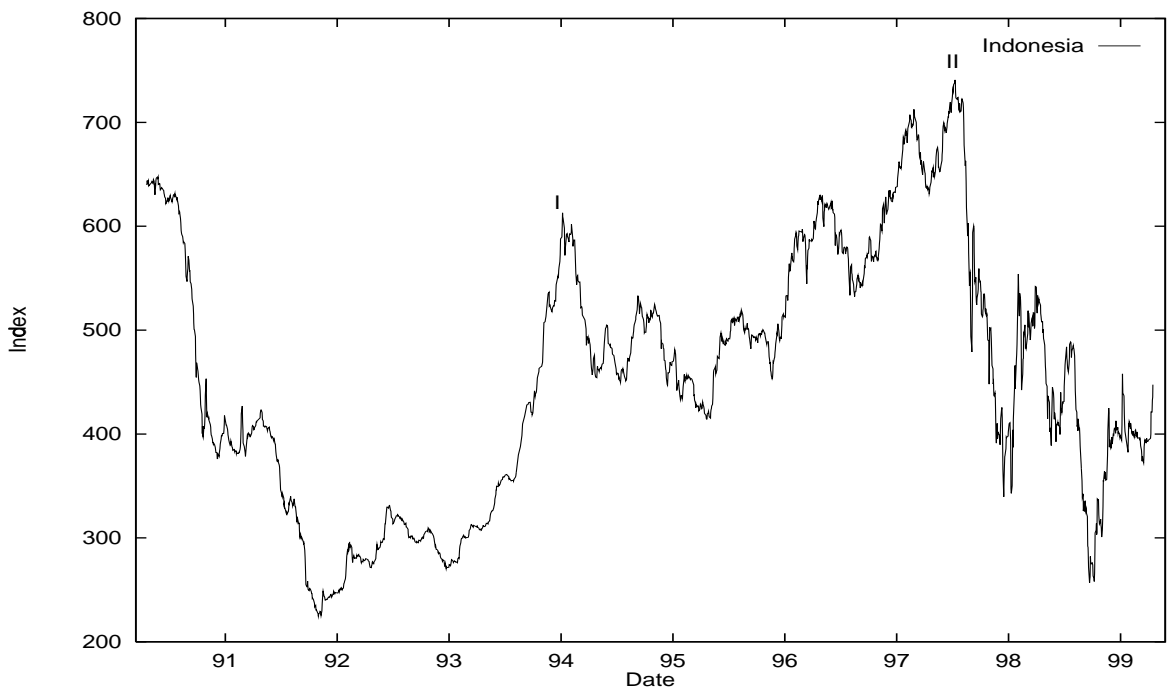


Figure 21: The Indonesian stock market index as a function of date. 2 bubbles with a subsequent very large draw down can be identified. The approximate dates for the draw downs are early 94 (I) and mid-97 (II), see figure. Note that only the first bubble gives a reasonable fit with eq. (1).

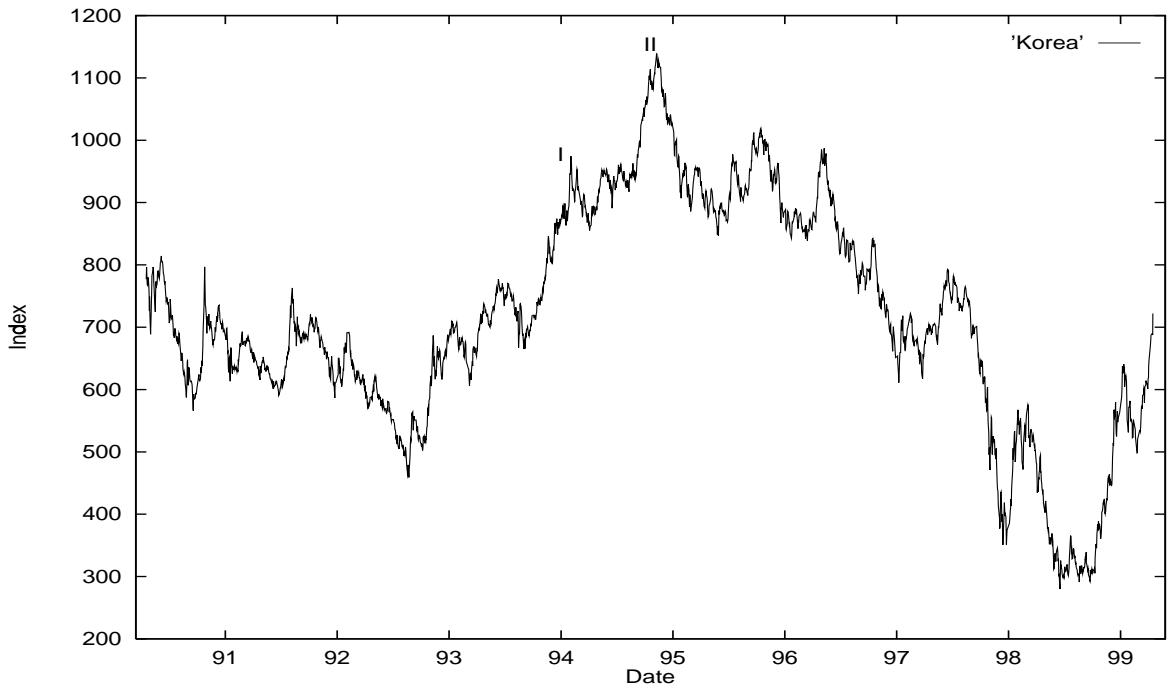


Figure 22: The Korean stock market index as a function of date. 2 bubbles with a subsequent very large draw down can be identified. The approximate dates are early 94 (I) and late 94 (II). None of the two bubbles gave a reasonable fit with eq. (1).

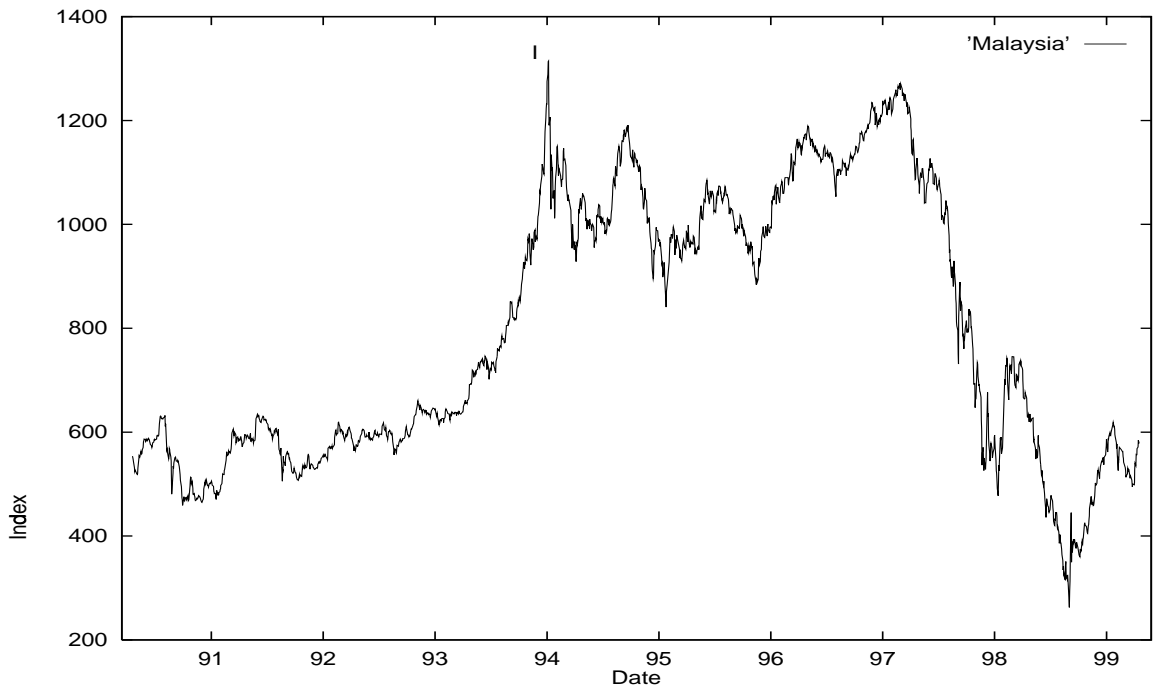


Figure 23: The Malaysian stock market index as a function of date. 1 extended bubble with a subsequent very large draw down can be identified. The approximate date for the draw down is early 94, see figure.

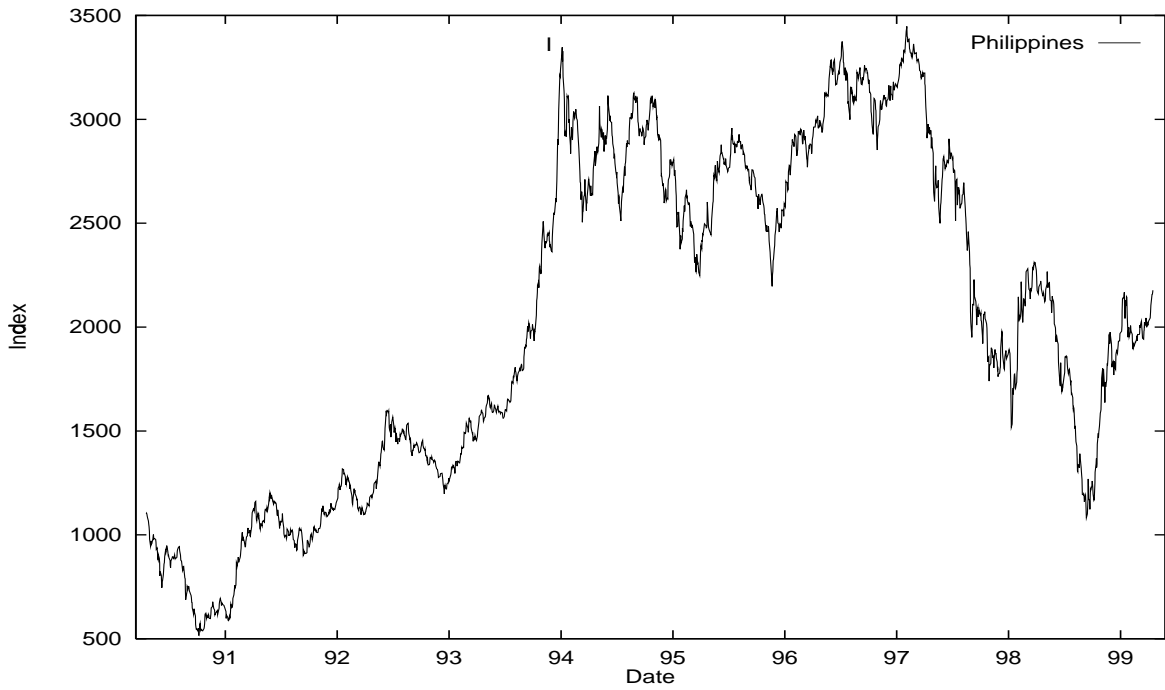


Figure 24: The Philippines stock market index as a function of date. 1 bubble with a subsequent very large draw down can be identified. The approximate date for the draw down is early 94, see figure.

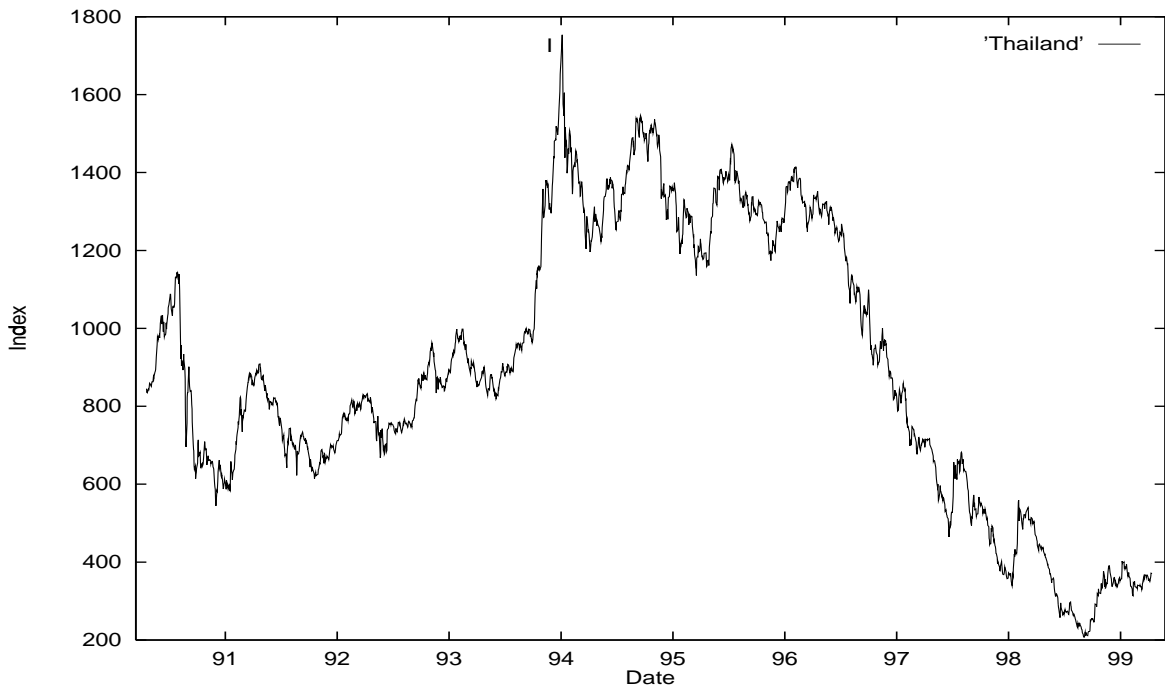


Figure 25: The Thai stock market index as a function of date. 1 bubble with a subsequent very large draw down can be identified. The approximate date for the draw down is early 94, see figure.

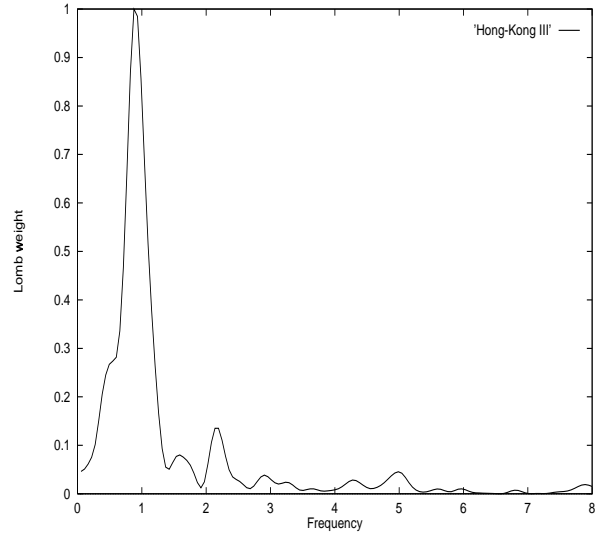
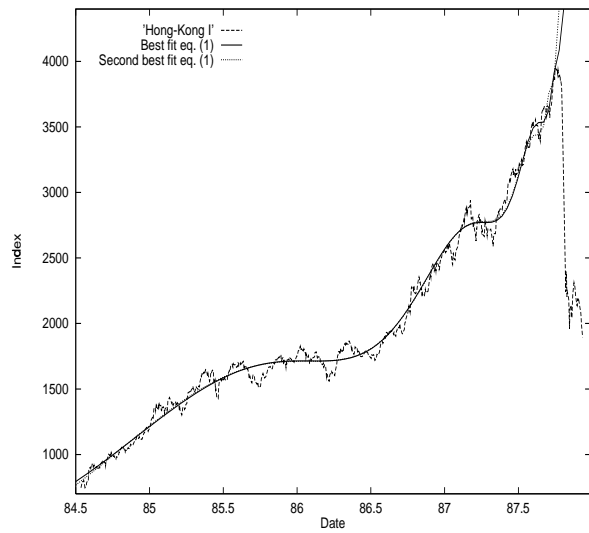


Figure 26: Hong Kong stock market bubble ending with the crash of Oct. 87. See table 5 for the parameter values of the fit with eq. (1). Only the best fit is used in the Lomb periodogram.

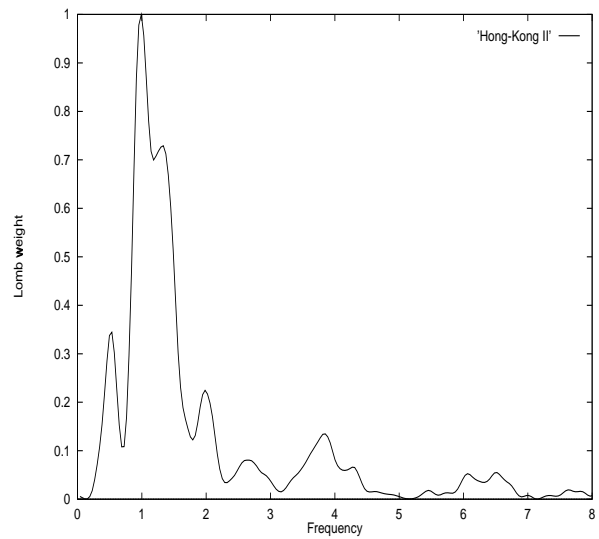
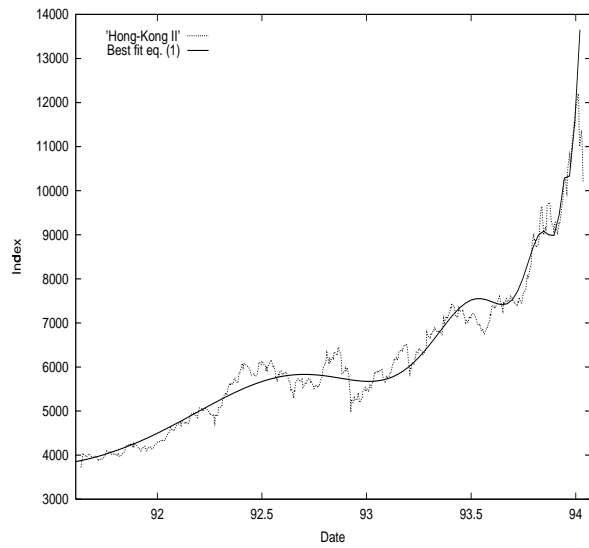


Figure 27: Hong Kong stock market bubble ending with the crash of Jan. 94. See table 5 for the parameter values of the fit with eq. (1).

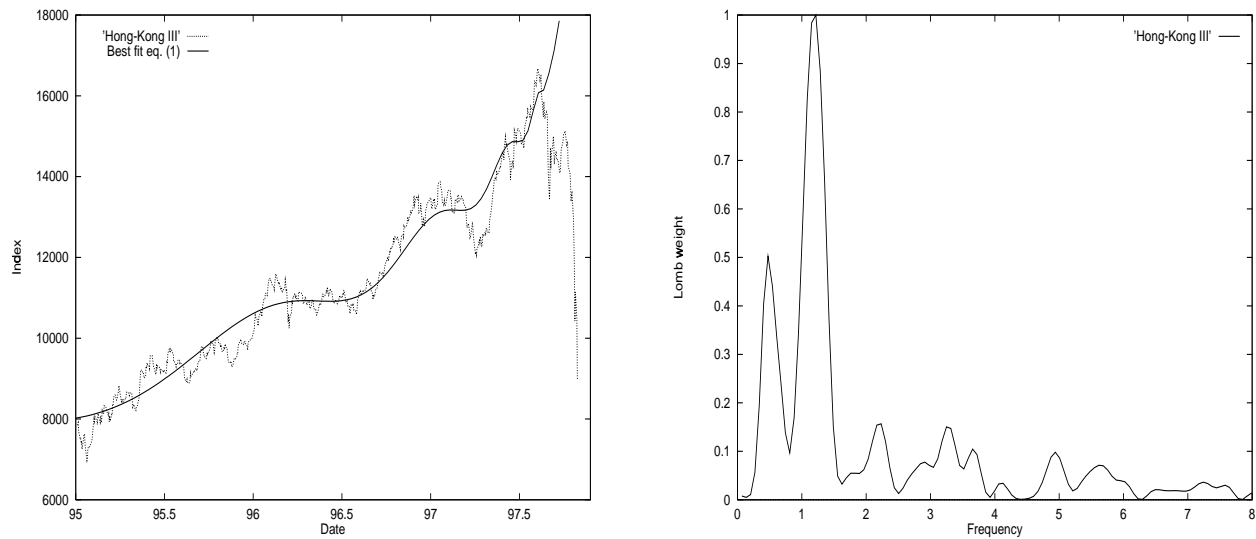


Figure 28: Hong Kong stock market bubble ending with the crash of Oct. 97. See table 5 for the parameter values of the fit with eq. (1).

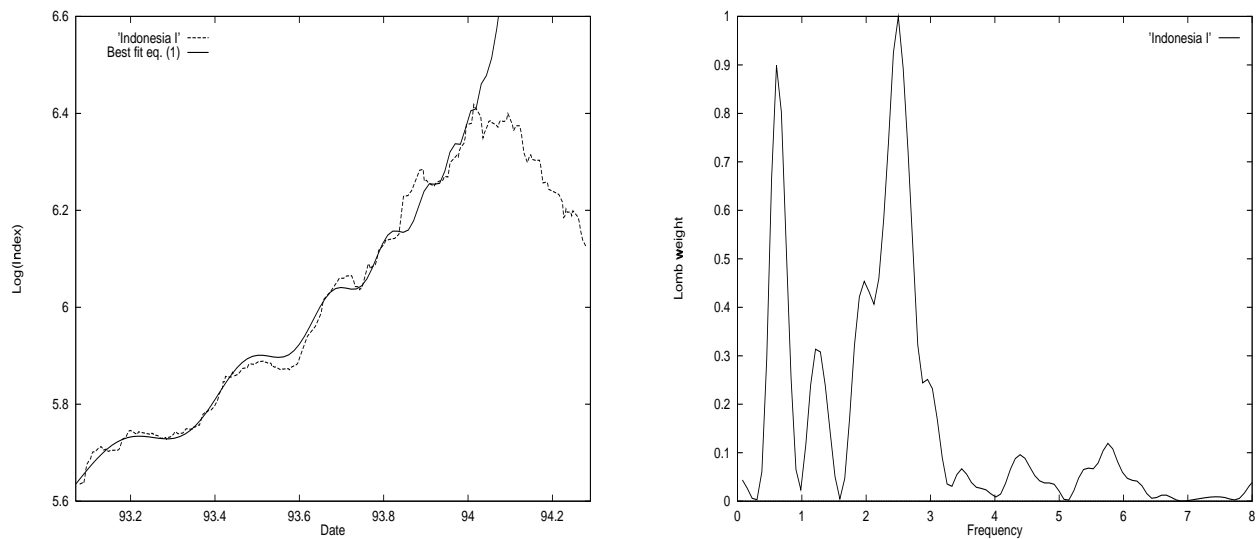


Figure 29: Indonesian stock market bubble ending in Jan. 1994. See table 5 for the parameter values of the fit with eq. (1).

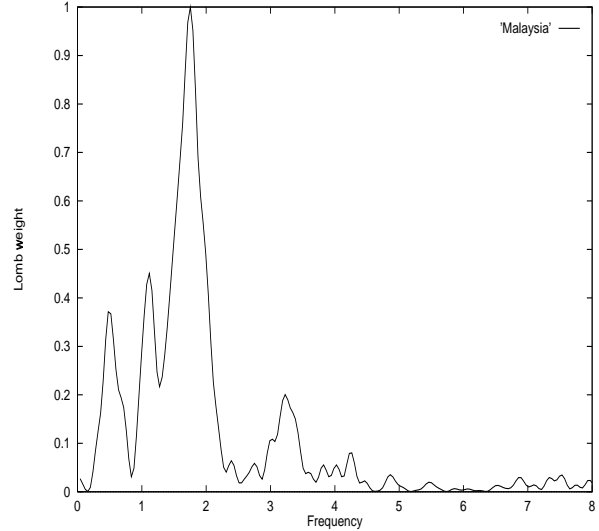
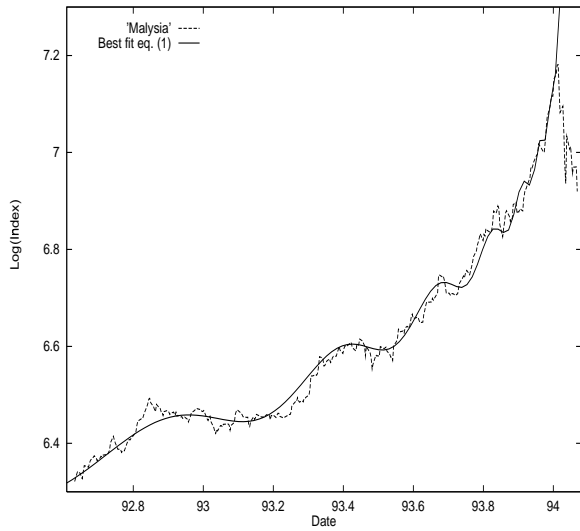


Figure 30: Malaysian stock market bubble ending with the crash of Jan. 94. See table 5 for the parameter values of the fit with eq. (1).

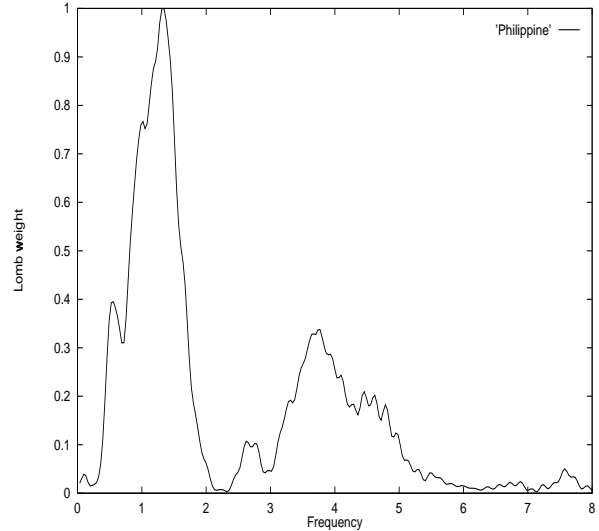
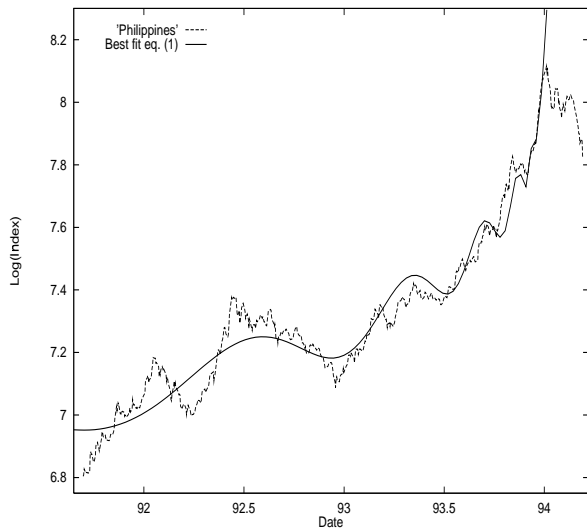


Figure 31: Philippine stock market bubble ending in Jan. 1994. See table 5 for the parameter values of the fit with eq. (1).

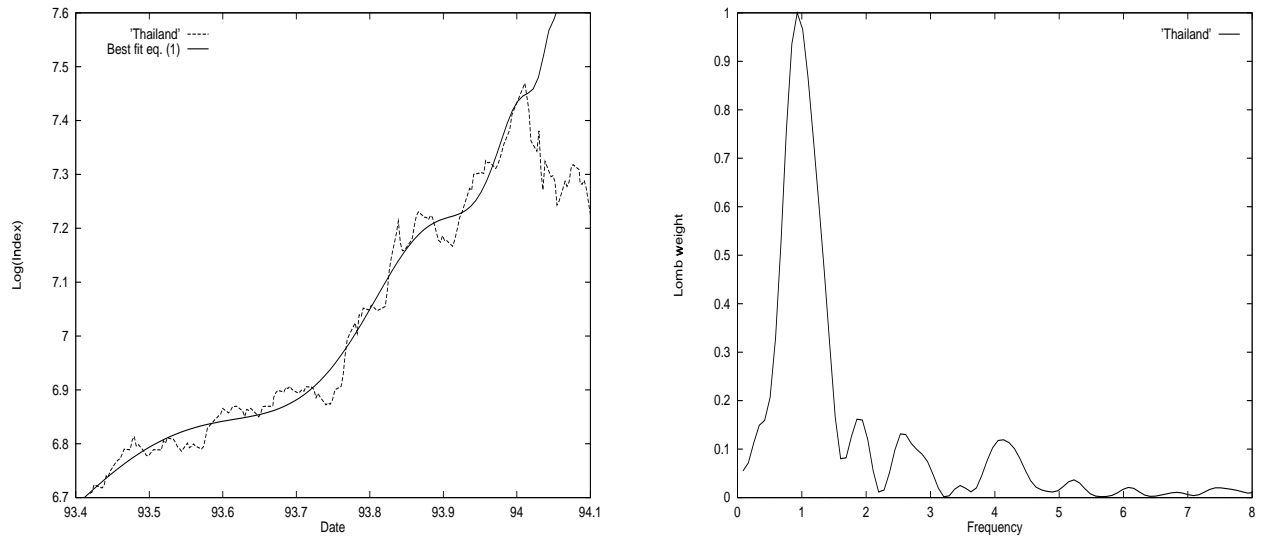


Figure 32: Thai stock market bubble ending with the crash of Jan. 94. See table 5 for the parameter values of the fit with eq. (1).

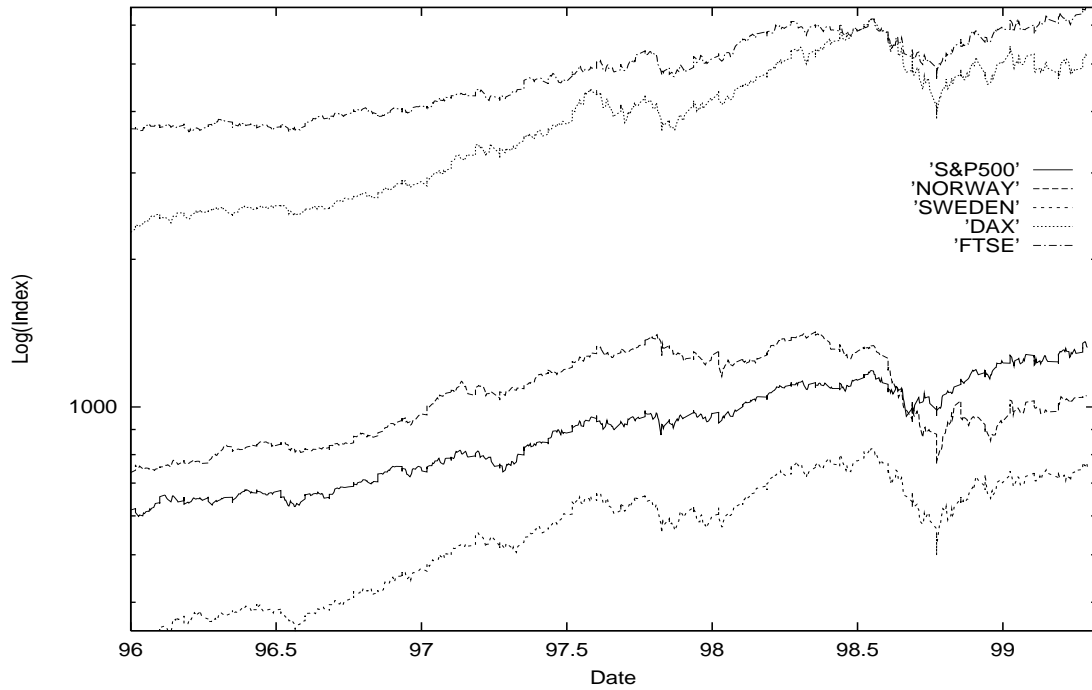


Figure 33: The US (S&P500), DAX (Frankfurt), FTSE (London), Norwegian and Swedish stock market indices as a function of time. The “dips and peaks” shown for the smaller markets seems to be better correlated temporally with the S&P500 in the end of the time interval than in the beginning.

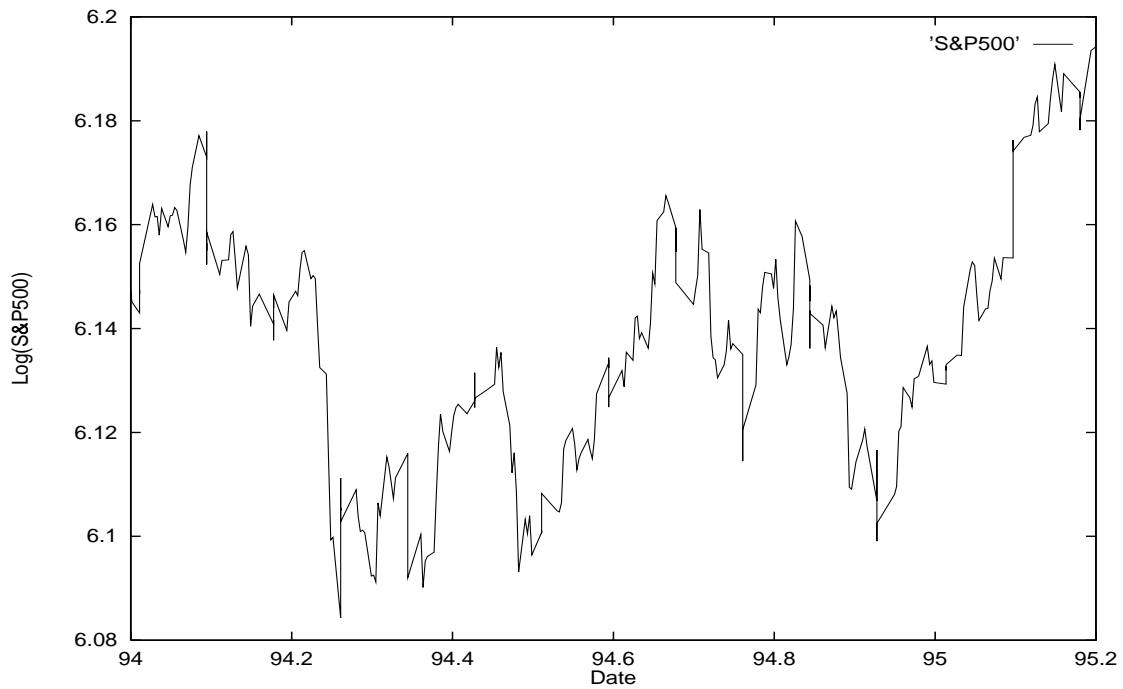


Figure 34: The S&P500 just before and ≈ 1 year after the early 1994 financial crises on emerging markets.

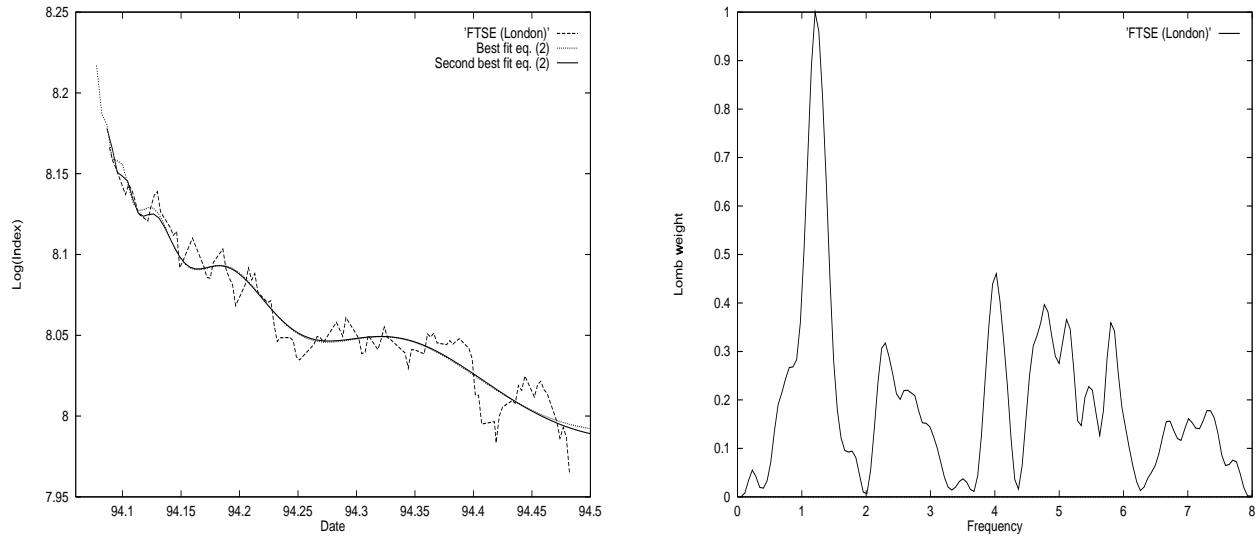


Figure 35: FTSE (London). The 2 lines are the best and the second best fit with eq. (2). See table 7 for the parameter values of the fits. Only the best fit is used in the Lomb periodogram.

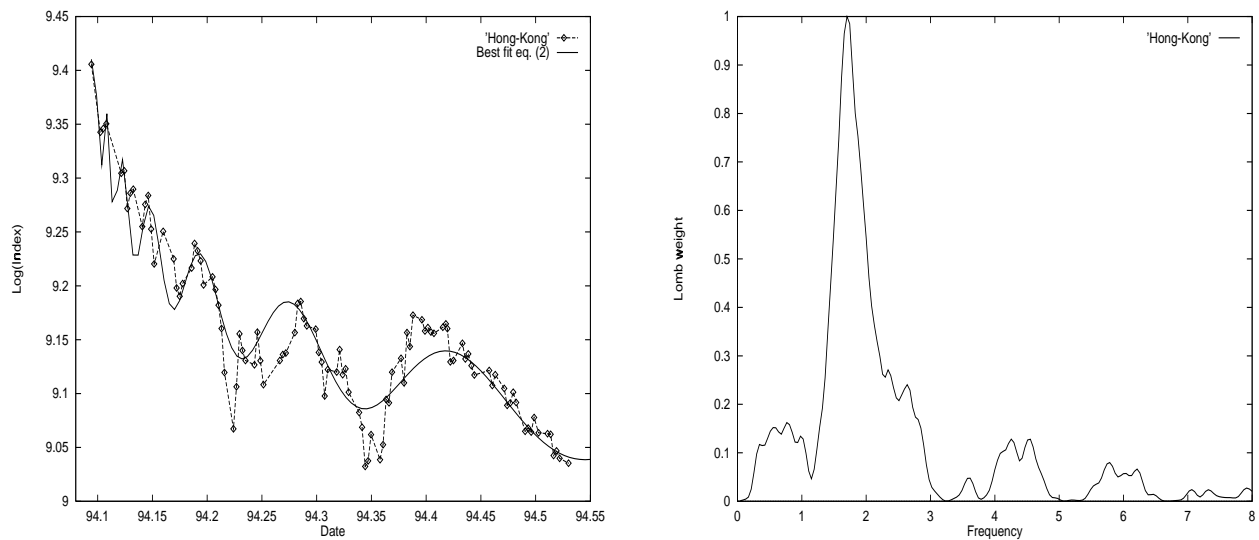


Figure 36: Hong-Kong. The line is the best fit with eq. (2). See table 6 for the parameter values of the fits. Note the small value for the exponent z . This is presumably due to the under-sampling of the data in the very first part of the data set.

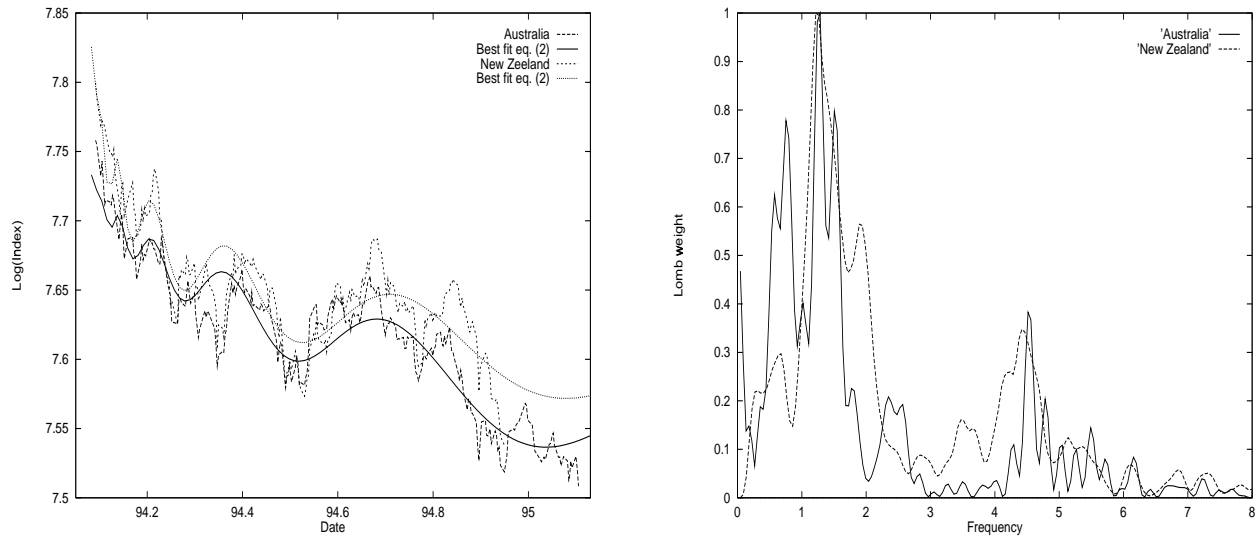


Figure 37: The Australian and New Zealand stock market indices. The line is the best fit with eq. (2). See table 7 for the parameter values of the fits.

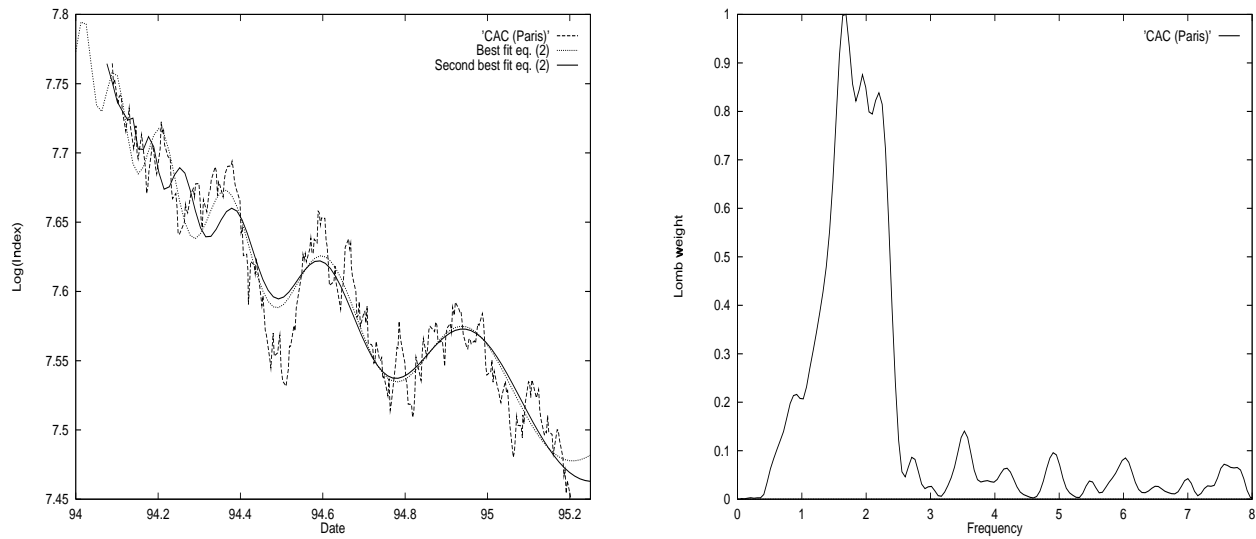


Figure 38: The French CAC40. The 2 lines are the best and the second best fit with eq. (2). See table 7 for the parameter values of the fits. Only the second best fit is used in the Lomb periodogram.

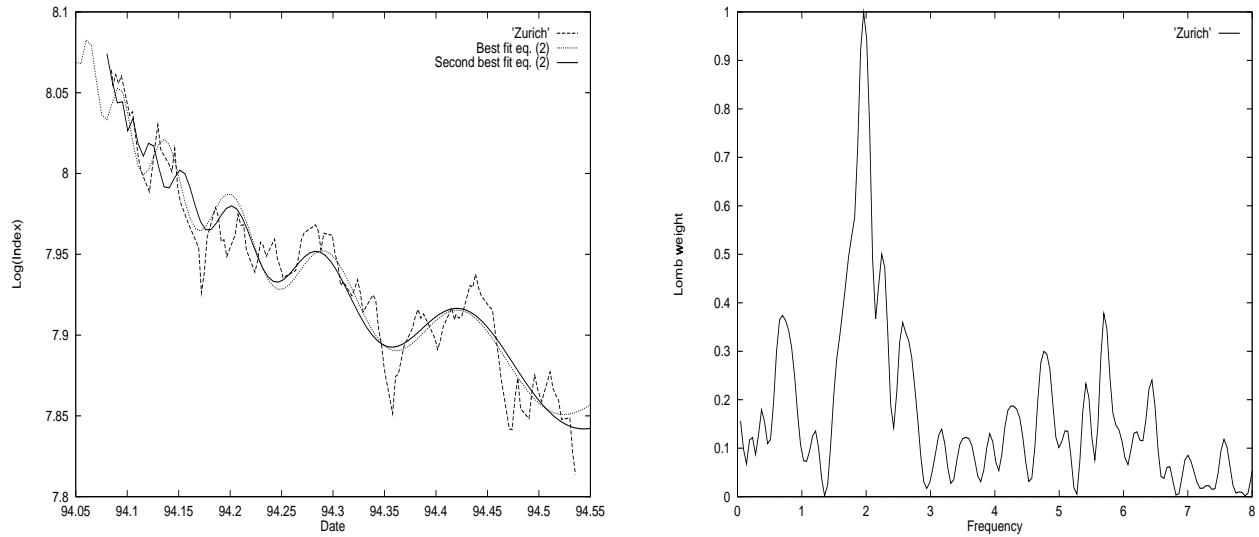


Figure 39: The Swiss stock market index. The lines are the two best fit with eq. (2). See table 7 for the parameter values of the fits. Only the second best fit is used in the Lomb periodogram.

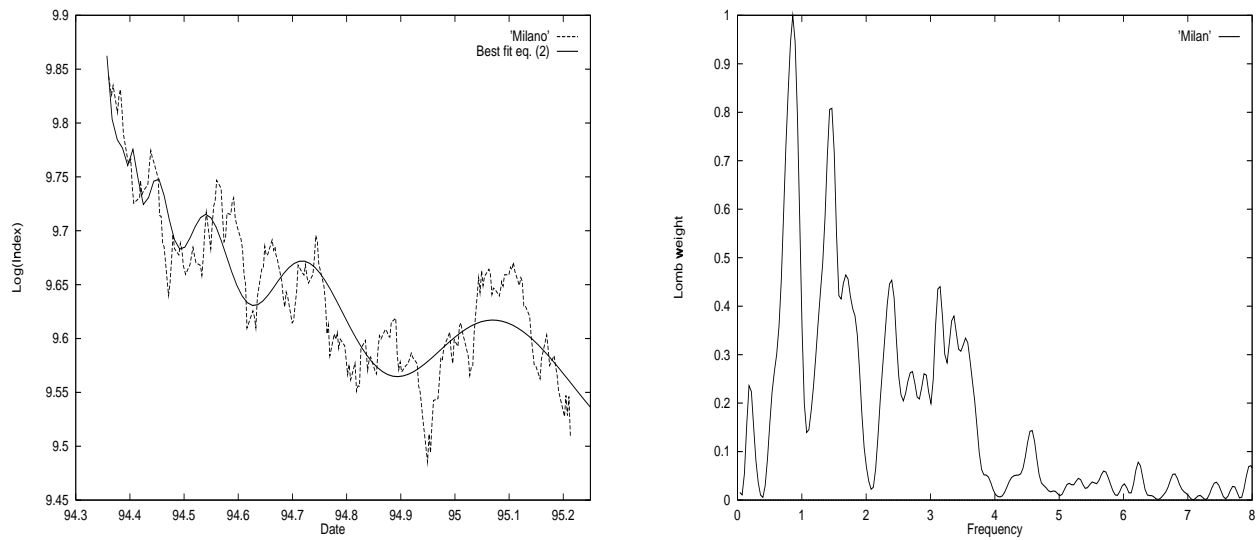


Figure 40: Italian stock market index. The line is the best fit with eq. (2). See table 7 for the parameter values of the fits.

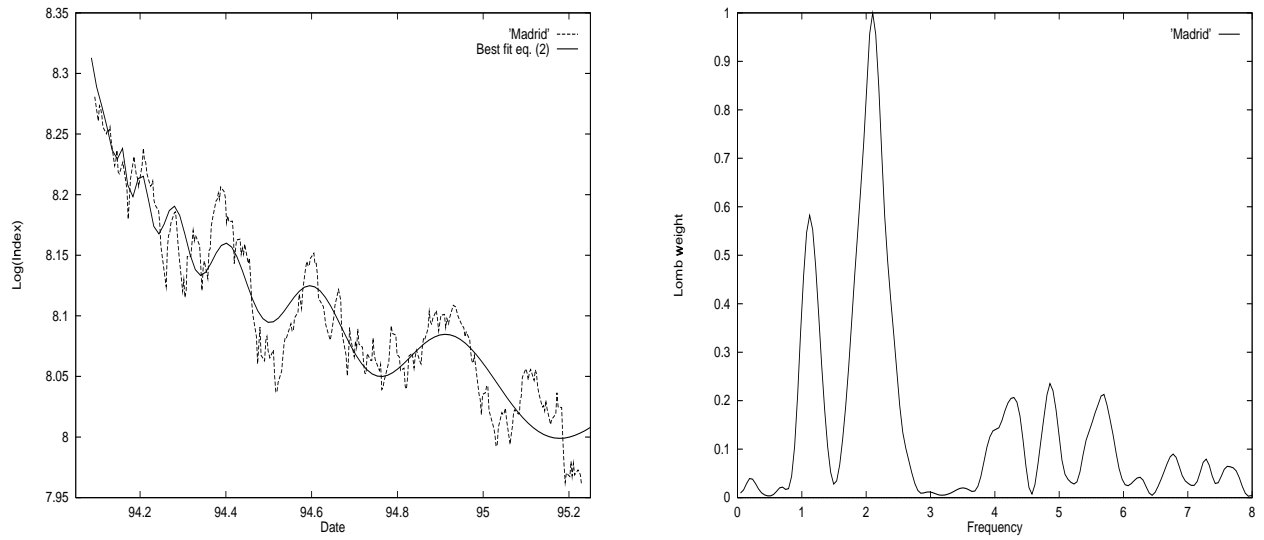


Figure 41: The Spanish (Madrid) stock market index. The line is the best fit with eq. (2). See table 7 for the parameter values of the fits.

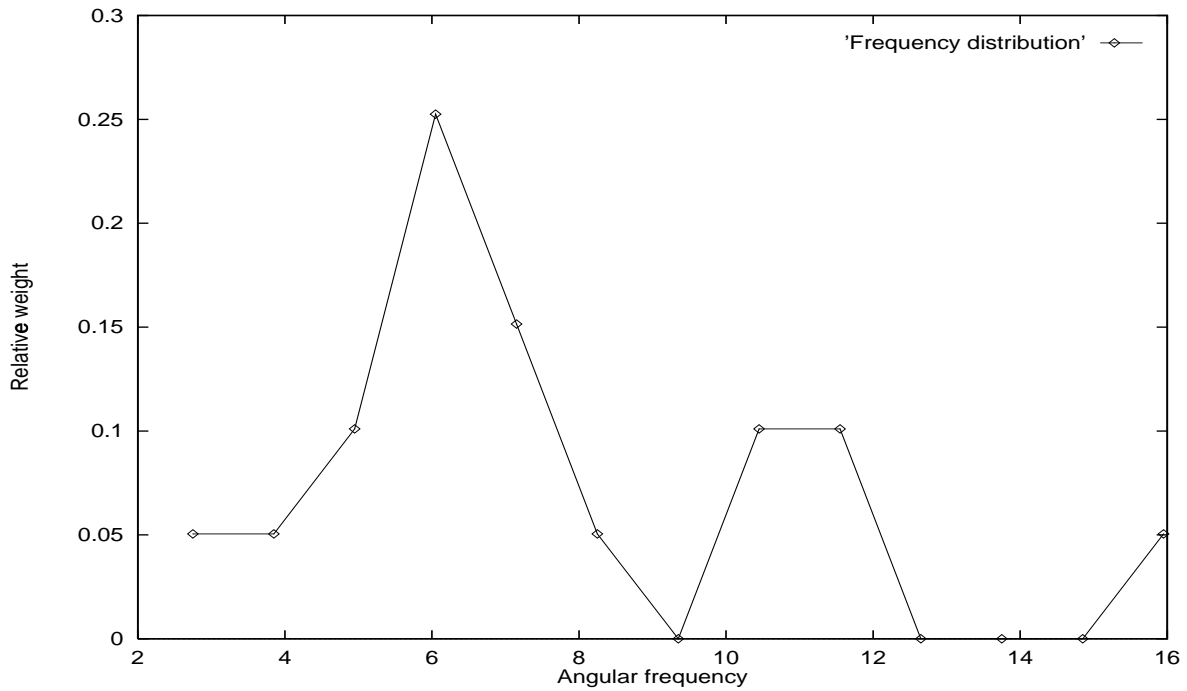


Figure 42: Distribution of ω 's for the bubble-fits given in tables 2 and 5.

Stock market	t_c	t_{max}	t_{min}	% drop	z	ω	λ
Argentina I	91.80	91.80	91.90	26%	0.37	4.8	3.7
Argentina II	92.43	92.42	92.90	59%	0.22	11.4	1.7
Argentina III	94.13	94.13	94.30	30%	0.19	7.2	2.4
Argentina IV	97.89	97.81	97.87	27%	0.20	10.1	1.9
Brazil	97.58	97.52	97.55	18%	0.49	5.7	3.0
Chile I	91.77	91.75	91.94	22%	0.50	7.2	2.4
Chile II	94.10	94.09	94.26	20%	0.30	2.9	8.8
Mexico I	94.10	94.09	94.30	32%	0.12	4.6	3.9
Mexico II	97.93	97.80	97.82	21%	0.50	6.1	2.8
Peru I	93.84	93.83	93.88	22%	0.62	11.2	1.8
Venezuela	97.75	97.73	98.07	42%	0.35	3.9	5.0

Table 1: Crash and fit characteristics of the various speculative bubbles on the Latin-American market leading to a large draw down in this decade. t_c is the critical time predicted from the fit of the market index to eq. (1). When multiple fits exist the fit with the smallest difference between t_c and t_{max} is chosen. Typically, this will be the best fit, but occasionally it is the second best fit. The other parameters z , ω and λ of the fit are also shown. The fit is performed up to the time t_{max} , at which the market index achieved its highest maximum before the crash. The percentage drop is calculated from the total loss from t_{max} to t_{min} , where the market index achieved its lowest value as a consequence of the crash.

Stock market	A	B	C	z	t_c	ω	ϕ
Argentina I	36960; 23439	-35902; -23319	-18839; -23905	0.16; 0.31	91.81; 91.80	4.8; 4.5	-0.3; 0.0
Argentina II	30737; 36858	-16199; -21792	1034; -834	0.26; 0.17	92.42; 92.43	11.4; 12.1	-0.8; 1.6
Argentina II	34211	-19832	950	0.22	92.43	10.4	0.7
Argentina III	65798; 29095	-53867; -18531	863; 1183	0.09; 0.35	94.16; 94.13	7.2; 6.0	-0.9; 0.0
Argentina IV	39906	-21267	-666	0.20	97.89	10.1	1.2
Brazil I	16327	-10717	512	0.33	97.53	5.7	0.1
Chile I	36785; 33727	-26853; -24040	-131; -135	0.40; 0.45	91.81; 91.79	7.2; 6.8	-0.7; -0.7
Chile I	40660	-30726	106	0.38	91.85	5.8	0.4
Chile II	79138	-56312	-203	0.15	94.11	2.9	1.4
Mexico I	3065	-2097	-244	0.65	94.13	4.6	0.6
Mexico II	5637; 6764	-2475; -3571	159; -100	0.31; 0.20	97.60; 97.62	6.1; 12.1	1.1; 0.6
Peru I	1770; 1151	-1516; -986	-41.5; -62.7	0.32; 0.59	93.91; 93.84	11.2; 8.6	-1.1; -0.9
Peru I	1435	-1226	47.7	0.50	93.93	6.7	-1.9
Venezuela I	13460	-8613	801	0.35	97.75	3.9	-0.5

Table 2: Fit parameters of the various speculative bubbles on the Latin-American financial markets leading to a crash in this decade. Multiple entries correspond to the two best fits.

Stock market	A	B	C	z	t_c	ω	ϕ
Argentina	37072	-31073	-971	0.22	92.44	11.8	0.2
Chile	6599	-1216	166	0.36	95.51	9.7	0.3
Venezuela	10273; 9997	-5650; -5539	1002; -1076	0.59; 0.63	97.80; 97.81	5.7; 5.3	1, 8; -1.2
Venezuela	10819; 13460	-6064; -8613	884; 801	0.58; 0.35	97.75; 97.75	6.7; 3.9	1.8; -0.5

Table 3: Fit parameters of the anti-bubbles on the Latin-American financial markets.

Stock market	t_c	t_{max}	t_{min}	% drop	z	ω	λ
Hong-Kong I	87.84	87.78	87.85	50%	0.29	5.6	3.1
Hong-Kong II	94.02	94.01	94.04	17%	0.12	6.3	2.7
Hong-Kong III	97.74	97.60	97.82	42%	0.34	7.5	2.3
Indonesia I	94.09	94.01	94.32	26%	0.44	15.5	1.5
Malaysia I	94.02	94.01	94.04	22%	0.24	10.9	1.8
Philippines I	94.02	94.01	94.19	25%	0.16	8.2	2.2
Thailand I	94.07	94.01	94.05	20%	0.48	6.1	2.8

Table 4: Crash and fit characteristics of the various speculative bubbles on the Asian market leading to a large draw down in this decade. t_c is the critical time predicted from the fit of the market index to eq. (1). When multiple fits exist the fit with the smallest difference between t_c and t_{max} is chosen. Typically, this will be the best fit, but occasionally it is the second best fit. The other parameters z , ω and λ of the fit are also shown. The fit is performed up to the time t_{max} , at which the market index achieved its highest maximum before the crash. The percentage drop is calculated from the total loss from t_{max} to t_{min} , where the market index achieved its lowest value as a consequence of the crash.

Stock market	A	B	C	z	t_c	ω	ϕ
Hong-Kong I	5523; 4533	-3247; -2304	171; -174	0.29; 0.39	87.84; 87.78	5.6; 5.2	-1.6; 1.1
Hong-Kong II	21121	-15113	-429	0.12	94.02	6.3	-0.6
Hong-Kong III	20077	-8241	-397	0.34	97.74	7.5	0.8
Indonesia I	6.76	-1.11	0.039	0.44	94.09	15.6	-1.3
Malaysia I	7.61	-1.16	0.038	0.24	94.02	10.9	1.4
Philippines I	9.00	-1.74	-0.078	0.16	94.02	8.2	0.2
Thailand I	7.81	-1.41	-0.086	0.48	94.07	6.1	-0.2

Table 5: Fit parameters of the various speculative bubbles on the South-East and East Asian financial markets leading to a large draw down in this decade. For Hong-Kong, the crash of 87 has been included for completeness. Multiple entries correspond to the two best fits.

Stock market	t_c	t_{max}	t_{min}	% drop	z	ω	λ
London	94.08	94.09	94.48	18%	0.25	7.6	2.3
Hong Kong	94.09	94.09	94.53	31%	0.03	11	1.8
Australia	94.08	94.09	95.11	22%	0.46	8.0	2.2
New Zealand	94.08	94.09	94.95	23%	0.09	7.7	2.3
France	94.06	94.09	95.20	27%	0.51	12	1.7
Spain	94.08	94.09	95.23	27%	0.28	13	1.6
Italy	94.36	94.36	95.21	28%	0.35	9.2	2.0
Switzerland	94.08	94.08	94.54	22%	0.45	12	1.7

Table 6: Fit characteristics of the 1994 anti-bubble on the Western financial markets plus Hong Kong following the emerging markets collapse in early 1994. t_c is the critical time predicted from the fit of the market index to eq. (2). When multiple fits exists the fit with the smallest difference between t_c and t_{max} is chosen. Typically, this will be the best fit, but occasionally it is the second best fit. The other parameters z , ω and λ of the fit are also shown. The fit is performed from the time t_{max} , at which the market index achieved its highest maximum before the decrease, to the time t_{min} , which is the time of the lowest point of the market before a shift in the trend. The percentage drop is calculated from the total loss from t_{max} to t_{min} using eq. (4).

Stock market	A	B	C	z	t_c	ω	ϕ
Australia	7.74	-0.177	-0.035	0.46	94.08	8.0	-0.7
France	8.36; 7.78	-0.810; 0.265	0.032; 0.036	0.18; 0.51	93.86; 94.07	16.5; 12.2	1.6; -1.4
Hong-Kong	12.15	-3.14	0.039	0.03	94.09	10.8	0.72
Italy	9.88	-0.340	-0.050	0.35	94.36	9.2	0.2
London	8.29; 8.21	-0.35; -0.29	-0.016; -0.019	0.25; 0.35	94.08; 94.08	7.6; 7.1	-0.6; -0.1
New Zealand	8.15	-0.551	-0.028	0.09	94.08	7.8	-0.3
Spain	8.41	-0.370	-0.030	0.28	94.08	13.2	1
Switzerland	8.79; 8.09	-0.986; -0.313	0.024; 0.036	0.12; 0.45	93.99; 94.08	17.1; 12.2	-1.7; -0.3

Table 7: Fit parameters of the various anti-bubbles on the western markets and Hong-Kong