A microsimulation of traders activity in the stock market: the role of heterogeneity, agents' interactions and trade frictions.

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We propose a model with heterogeneous interacting traders which can explain some of the stylized facts of stock market returns. In the model synchronization effects, which generate large fluctuations in returns, can arise either from an aggregate exogenous shock or, even in its absence, purely from communication and imitation among traders. A trade friction is introduced which, by responding to price movements, creates a feedback mechanism on future trading and generates volatility clustering.

I. INTRODUCTION

Stock prices occasionally have violent fluctuations which follow empirically established scaling laws. As pointed out by several authors [1–4], the distribution of returns is leptokurtic and the returns of many market indices and currencies, over different but relatively short time intervals, can be described by a Lévy stable distribution, except for tails which are approximately exponential. The estimation is that the shape of a Gaussian is recovered only on longer scales of typically one month.

Moreover, while stock market returns are uncorrelated on lags larger than a single day, the autocorrelation function of the volatility (the daily volatility is not directly observable but it is indirectly defined by the absolute value of stock returns or by the stock squared returns) is positive and slowly decaying, indicating long memory effects. This phenomenon is known in the literature as volatility clustering [5–10]. Scaling analysis on market indexes and exchange rates shows that the volatility autocorrelations are power-laws over a large range of time lags (from one day to one year), in contrast with ARCH-GARCH models [6,11] where they are supposed to decay exponentially. Multiscaling behaviour of volatility autocorrelations has also been detected [12–14].

Daily financial time series also provide empirical evidence [15,16] of a positive autocorrelation, with slowly decaying tails, for the trading volume, and positive cross correlation between volatility of returns and trading volume.

It is not settled yet whether the emergence of these power law fluctations is due to external factors or to the inherent interaction among market players and the trading process itself. For example, herd behavior [17–22] has been proposed as a possible form of interaction which could explain the observed statistical outcomes in financial markets. Asymmetric information models have also been introduced [23,24].

In a noise traders model, if the agents do not exchange information and are not coupled by any external signal their individual actions would be independent of each other. Consequently the aggregate demand and supply would be the sum of random i.i.d. variables and, in the limit of a large number of traders, market returns would be Gaussian distributed.

The assumption that decisions of individual agents may be represented as independent random variables ignores an essential ingredient of market organization, namely the interaction and communication among agents. Stock markets are highly vulnerable to collective behaviour which manifests itself in large price fluctations, and eventually crashes when a large group of agents place the same order simultaneously. This macro-level organization can emerge from the micro-level communication among traders even in the absence of an external field, for example the arrival of aggregate news.

To model how the decisions of agents are influenced by their mutual interaction, the way agents communicate with each other becomes an important issue. The communication structure can be modeled as a graph where the nodes correspond to the agents and the arcs to the links between pair of agents [25–27]. An important question is how connected the graph is. If it may be decomposed into disconnected subgraphs with no links between them, then the inability to transmit information can have important consequences in the process of decision making.

Another important element in modelling trading activity is the heterogeneous character of the agents. For example, if aggregate news could be symmetrically accessed and quickly transmitted, communication would be superfluous unless traders reacted differently to its arrival.

Our aim is to understand which mechanisms in the process of trading can generate the statistical features observed in the financial data. We believe that the interactions among traders and their heterogeneous nature by themselves, independently of other details of the microscopic environment, might be responible for many of these features and for the large scale behaviour of aggregate economic variables.

The crucial point in our model is not the exact description of individual behaviour but the inter-relation between individuals and the statistical properties of traders' characteristics. Consequently, our model of decision making will be based on noise trading expanded to allow for heterogeneity and communication effects across traders.

II. THE MODEL

A modified version of the random field Ising model (RFIM) [28,29] is employed to describe the trading behaviour of agents in a stock market. We consider an $L \times L$ square lattice with periodic boundary conditions. Each node i represents an agent and the links represent the connections among agents.

We start with each agent initially owning the same amount of capital consisting of two assets: cash $M_i(0)$ and $N_i(0)$ units of a single stock. At any time step t the capital of trader i is given by $K_i(t) = M_i(t) + p(t)N_i(t)$, where $p(t)$ is the current price of the stock. At each time step t a given trader, i, chooses an action $S_i(t)$ which can take one of three values: $+1$ if he buys one unit of the stock, -1 if he sells one unit of the stock, or 0 if he remains inactive. The trades undertaken by each player are bounded by his resources plus the contraint that he can buy or sell only one indivisible unit at a time.

The agents' decision making is driven by idiosyncratic noise and the influence of their nearest neighbours. At each time t, each agent i receives a signal $Y_i(t)$:

$$Y_i(t) = \sum_{<i,j>} J_{ij} S_j(t) + A\nu_i(t) + B\epsilon(t) \tag{1}$$

$< i, j >$ denotes that the sum is taken over the set of nearest neighbours of agent i. On a square lattice every agent has four nearest neighbours. J_{ij} measures the influence that is exercised on agent i by the action S_j of his neighbour j; J_{ij} are assumed to be symmetrical in the present case but asymmetric J_{ij} could also be considered. Agents do not anticipate other agents future actions but respond to the aggregate of the other agents' current actions. Idiosyncratic noise ν_i represents a uniformily distributed shock to the agent's personal preference, while $\epsilon(t)$ represents an aggregate signal, uniformly accessible to all traders, following the arrival of news.

Under frictionless trading each agent would buy at the slightest positive signal and sell at the slightest negative one. We depart from this benchmark by assuming a trade friction which leads a fraction of the agents to being inactive in any time period. This friction can be interpreted, for example, as a transaction cost which is specific to each agent. Alternatively it could be interpreted as an imperfect capacity to access information. Formally we model this friction as an individual threshold which each agent's signal must exceed to induce him to trade.

Each agent compares the signal he receives with his individual thresholds, $\xi_i^+(t), \xi_i^-(t)$, and undertakes the decision:

$$\begin{array}{rrll} S_i(t) = & 1 & if & Y_i(t) \geq \xi_i^+(t) \\ S_i(t) = & 0 & if & \xi_i^-(t) < Y_i(t) < \xi_i^+(t) \\ S_i(t) = & -1 & if & Y_i(t) \leq -\xi_i^-(t) \end{array} \tag{2}$$

The $\xi_i(t)$ are chosen from a gaussian distribution, with initial variance $\sigma_\xi(0)$ and mean $\mu_\xi(0)$, and are adjusted over time proportionally with movements in the stock price. We will consider the case $\mu_\xi = 0$ and $\xi_i^-(t) = -\xi_i^+(t)$. Agents' heterogeneity enters through the distribution of thresholds. The homogeneus traders scenario can be recoverd in the limit when $\sigma_\xi = 0$.

As is well known in statistical physics, the behaviour of the system is affected by the different choices for the J_{ij} in eq.(1). For example taking all $J_{ij} = 1$ in the limit of zero thresholds, the decision making problem would become analogous to the study of the paramagnetic/ferromagnetic transition in the RFIM in an external random field. Decreasing the noise level, the system eventually magnetizes, i.e. the traders would reach the same selling/buying decisions. Alternatively, taking $J_{ij} = 1$ with probability p and $J_{ij} = 0$ with probability $1 - p$ we would be in the framework of the bond percolation model [30]. For a square lattice, it is well known that there is a critical value of p below which the system will decompose into disconnected clusters. In this situation we expect that different groups of traders will take their decision independently from the others, and eventually agree within the same group. In this situation large price fluctuations and crashes can be avoided. If J_{ij} are positive and negative variables gaussian distributed we would have an analogy with spin-glasses [31]. In this system the structure of the phase space is extremely complicated with many possible stable and metastable states hierarchically organized. Eventually if the $J_{ij} = 0$ the traders actions become uncorrelated with each other. In the model we shall consider these alternatives.

A consultation round to make decisions is allowed before trading takes place. Initially agents whose idiosyncratic signal exceeds their individual thresholds make a decision to buy or sell and subsequently influence their neighbours' according to eq. 2. Traders decide sequentially and can revise past decisions on the basis of signals received from their neighbours. This process converges when no agent changes his decision. Once the decision making process is complete traders place their orders simultaneously.

Traders buy from or sell to a market maker who, at the end of every trading period, adjusts the stock prices according to the relative demand and supply and the overall trading volume.

The demand, $D(t)$, and supply, $Z(t)$, of stocks at time t are

$$D(t) = \sum_{i:S_i(t)>0} S_i(t) \qquad Z(t) = -\sum_{i:S_i(t)<0} S_i(t).$$

The trading volume is $V(t) = Z(t) + D(t)$. After the transactions are complete the market-maker increments the stock price according to the rule

$$P(t+1) = P(t)\left(\frac{D(t)}{Z(t)}\right)^{\alpha} \tag{3}$$

where

$$\alpha = a\frac{V(t)}{L^2} \tag{4}$$

L^2 is the number of traders and represents the maximum number of stocks that can be traded in any time step. This rule describes the asymmetric reaction of market makers to imbalanced orders placed in periods of high versus low activity in the market and is consistent with the empirically observed positive correlation between absolute price returns and trading volume. For comparison, we will also consider the case where the exponent α in eq.(3) is independent of the trading volume.

After the price has been updated the market volatility can be estimated as the absolute value of relative returns:

$$\sigma(t) = |log \frac{P(t+1)}{P(t)}| \tag{5}$$

Price changes lead to an adjustment of next period's thresholds, $\xi_i(t+1)$:

$$\xi_i(t+1) = \frac{P(t+1)}{P(t)} \xi_i(t) \tag{6}$$

This can be interpreted as an adaptive process such that the thresholds follow the local price trend. It is equivalent to rescaling σ_ξ, which in turn will affect the subsequent volume of trade while conserving the symmetry in the probability of buying versus selling. If ξ_i arise from transaction costs, such as brokerage commissions, the adjustment process reflects a positive dependence of such costs on stock prices. Note that there is a memory effect in the readjustment mechanism for thresholds, i.e. next period's threshold is proportional to last period's one and not to the initial one. A different scenario could be considered where the mean $\mu_\xi(0)$ of the thresholds distribution follows the price trend creating an asymmetry in ξ_i^+ and ξ_i^- and, consequently, in the buying and selling probablities. This last case could be considered as a framework to study the emergence of bubbles and crashes.

III. SIMULATIONS AND RESULTS

Through simulations we shall study scenarios with no thresholds and compare the results with alternative scenarios with thresholds that remain constant over time and thresholds which are adjusted according to the feedback mechanism in eq.(6).

The outcomes of the model for different values of the parameters are simulated numerically. We center our analysis on the statistical properties of the probability distribution of stock returns and on the autocorrelation of market volatity.

The dimension of the lattice is set at $L = 100$. Each agent is initially given the same amount of stocks $N_i(0) = 100$ and of cash $M_i(0) = 100P(0)$, where $P(0) = 1$. The market maker is given a number of stocks, N_m, which is a multiple m of the number of traders (L^2) and an infinite amount of money.

The initial value of the thresholds' variance $\sigma_\xi(0) = 1$ and $\mu_\xi(0) = 0$. The coefficients in eq.(1) are $A = 0.2$ and initially we will neglect the effect of news arrival which correspond to choosing $B = 0$. Different choices for J_{ij} are considered. In any trading round, $S_i(t)$ are initially set to zero. Then each agent observes his individual $\nu_i(t)$ following which consultation with other agents takes place. The decision of each trader is updated sequentially following the rule in eqs.(1) and (2). Holding the value of $\nu_i(t)$ fixed, $Y_i(t)$ and $S_i(t)$ are iterated until they converge for each trader. At this point each trader places her order simultaneously determining the values of $D(t), Z(t), V(t)$. Prices are then adjusted by the market maker accoring to eq.(3), feed back into thresholds according to eq.(6) and a new trading round begins.

It is interesting to analyze first the case without thresholds. As we said in the introduction, we expect that large crashes will occur when the probability, p, of having points on the lattice connected by a link increases above a critical value. As suggested by [3] if we choose p close to this critical value we should be able to generate price fluctuations of any size, power-law distributed over different orders of magnitude (a cut-off is imposed by the finite size of the system). Fig.(1) shows how the nature of fluctuations changes when the value of p is increased. At larger p price jumps of different sizes are observed which can eventually drive the system to a collapse.

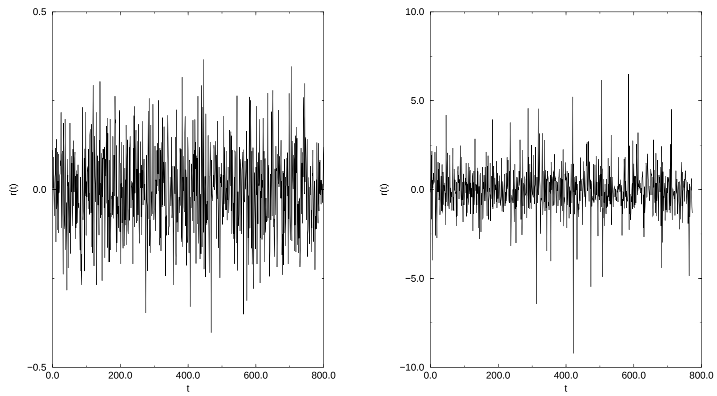

FIG. 1. Price returns for $\sigma_\xi = 0$ when increasing p from $p = 0.4$ to $p = 0.8$

When thresholds are introduced the system stabilizes and large crashes can be avoided even in the limit $p = 1$. In fig.(2) we plot the returns $r(t)$ for the two scenarios: (a) constant thresholds and (b) adjusting thresholds, each case with and without interactions among agents.

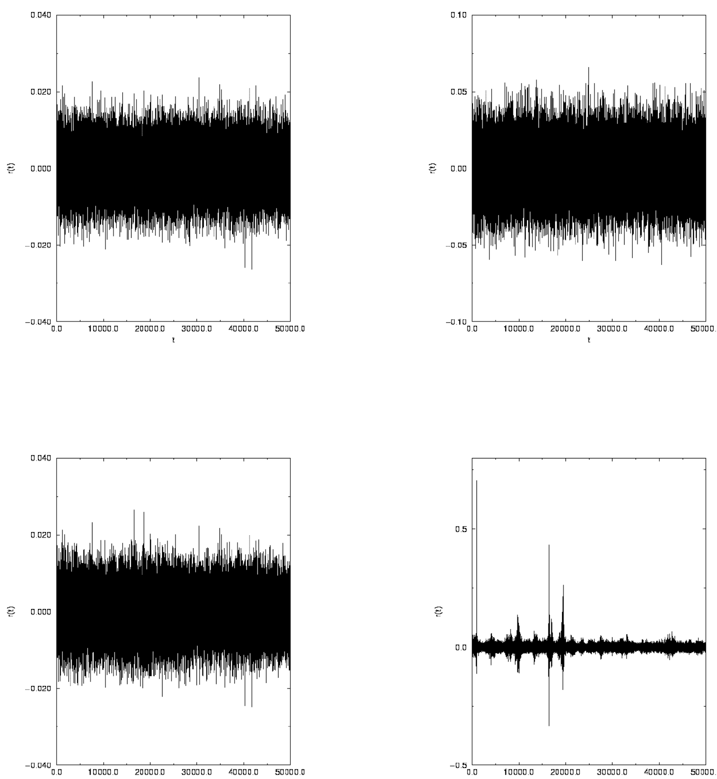

FIG. 2. Returns $r(t)$ for, above, (a1) $\sigma_{\xi(t)} = 1$, $J_{ij} = 0$, (a2) $\sigma_{\xi(t)} = 1$, $J_{ij} = 1$; below, (b1) adjusting $\sigma_{\xi(t)}$, $J_{ij} = 0$, (b2) $\sigma_{\xi(t)}$, $J_{ij} = 1$

It appears that both adjusting thresholds and communication between neighbours are essential for generating volatility clustering. Communication between traders helps synchronize their decisions, making the model capable of generating both large gaps between demand and supply and large overall trading volume. Indeed even if the idiosyncratic signal of an agent is below her threshold, the effect of imitating her neighbours' actions can nonetheless induce her to trade in the market. Furthermore an agent could be induced to act in a direction opposite to his own idiosyncratic noise signal. This imitative behaviour can spread through the system from one consultation round to the next, generating avalanches of different sizes in supply, demand and overall trading volume. The effect of larger trading volume and imbalance in supply and demand would be to increase volatility at one point in time. However without the feedback on thresholds there would be no clustering effect. Volatility increases with both positive and negative large price fluctuation. Thresholds, on the other hand, increase when prices go up and decrease when prices go down. Through the effect of thresholds on trading volume and on the exponent α, subsequent volatility would increase (decrease) following an initially negative (positive) price change. If subsequent volatility were to increase following each direction of price change the system would become unstable while if it were to decrease in each case, any initial shock would be immediately dampened. A direct interpretation of such an asymmetric change in trading volume to the direction of price change is also possible: when prices fall by a large amount agents are more likely to become aware and to react to subsequent news than when prices rise or stay constant. Casual empiricism suggests that news of a collapse in stock prices is given disproportionate prominence in the media. Orosel [32] analyzed an overlapping generation model where market participation covaries positively with share prices. This situation could also be considered in our model and we would still be able to generate the results on volatility clustering. Empirically [15] however, a larger response of volatility to negative as against positive price changes has been observed. Our model accords better with this observation.

Budget constraints can affect the propagation of avalanches and possibly have an important role in generating a disequilibrium in the demand and supply. Agents can be prevented from buying or selling by, respectively, a shortage of money or stocks. This could reduce the influence of their neighbours. Nonetheless our results do not require a fine tuning of the agents' initial wealth or, indeed, other parameters. The clustering effect can be reproduced for a wide range of the model's specification.

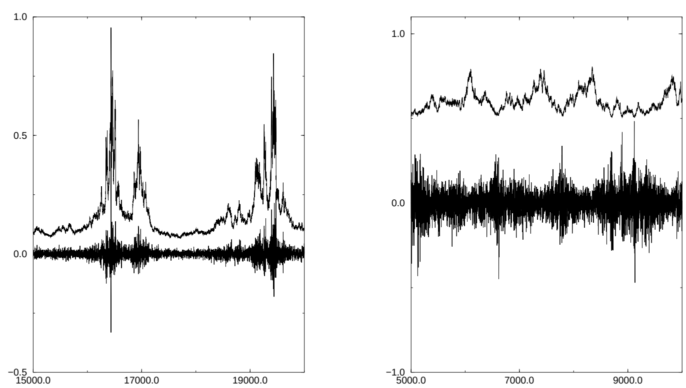

FIG. 3. Price return (below) and percentual trading volume (above) for (3a) $\alpha = V(t)/L^2$ and $J_{ij} = 1$, (3b) $\alpha = 1$ and $J_{ij} = 0$ (the trading volume has been shifted upward by 0.5 for a better inspection).

It is interesting to analyze what happens when the exponent α in eq.(6) is kept constant. The role of a variable α is that of stabilizing the system, reducing the consequences of a large imbalance in demand and supply if this is generated by the activity of only a small fraction traders. Through this mechanism we are able to generate a positive correlation between price volatility and trading volume as shown in fig.(3a). Choosing α constant and large enough, asymmetries in demand and supply could produce large price fluctuations and consequently significant changes in the thresholds even when the fraction of active traders is very small. This could generate volatility clustering even when communications among agents is not allowed. This happen for example, see fig.(3b), when

$\alpha = 1$. In this case though, a negative correlation between trading volume and price volatility emerges, in contradiction with the empirical observation. On the other hand, by choosing $\alpha = 0.1$ any imbalance in demand and supply is dampened away and clustering does not emerge even when a high level of communication among the agents is allowed.

We have seen that syncronized action by traders can be produced through communication and imitation in the absence of aggregate news. By the same token the arrival of aggregate news can also lead to syncronized action even in the absence of communication. We studied cases where $J_{ij} = 0$ but news arrived randomly with a given probability. These examples show that news of equal amplitude can generate price fluctuations of different sizes (fig.(4a) and fig.(4b)) and, if combined with adjusting thresholds, can also generate volatility clustering fig.(4c). When the intensity of the news, measured by the parameter B in eq.(1), increases above a certain level, the effect would be that of producing large price jumps of almost the same size and the probability distribution of returns would be, in this case, bimodal. Eventually, when both news and communications among agents are introduced in the model, the price fluctuations show a pattern as the one in fig.(4d).

Previous studies [23,24,33,34] on the effect of news on volatility autocorrelation have relied on a mechanism of sequential information arrival. This is not the case in our model where news are uncorrelated over time. Hence, communication among traders and aggregate news serve as complementary channels through which large fluctuations in stock prices and volatility clustering in a real market may be explained.

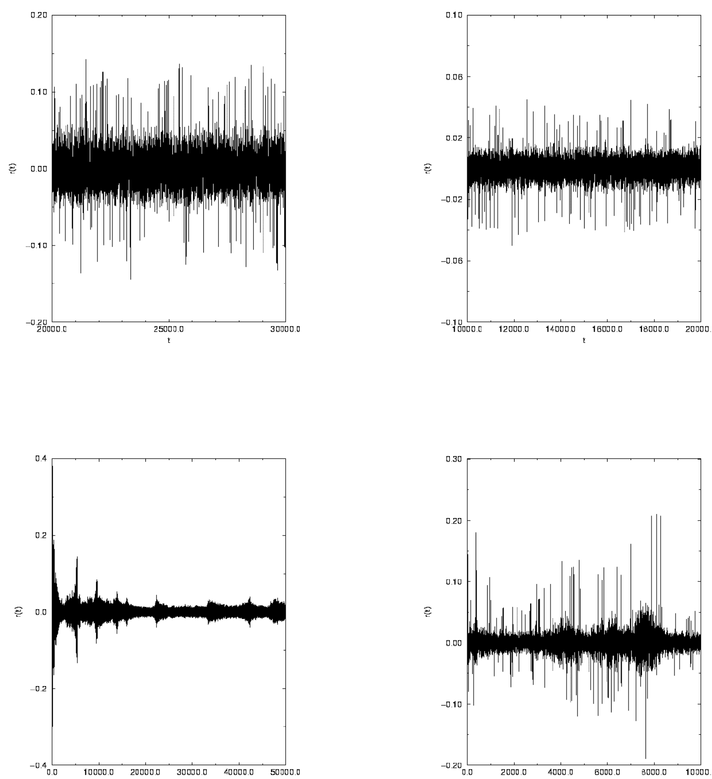

FIG. 4. (a)Returns in the model with news arrival (a) no thresholds, $J_{ij} = 0, B = 0.01$; (b) fixed thresholds $J_{ij} = 0, B = 0.02$; (c) adjusting thresholds, $J_{ij} = 0, B = 0.05$; (d) adjusting thresholds, communications and $B = 0.03$. News arrive with probability $p_n = 0.01$ in cases (a),(b) and (d) and with probability $p_n = 1$ in case (c)

We will focus in the following on the statistical properties of price fluctuation in the case where thresholds adjust, communication among agents is allowed and there are no aggregate exogenous news.

It is commonly accepted that the returns evaluated at different time lags,

$$r^{\tau}(t) = \ln \frac{P(t+\tau)}{P(t).} \tag{7}$$

do not behave according to a gaussian at small τ, and the gaussian behavior is recovered only for large τ. In fig.(5) we plot the probability distribution $\sigma(\tau)P(r^{\tau})$ for the rescaled returns $r^{\tau}/\sigma(\tau)$. Fat tails can be observed at $\tau = 1$ while at $\tau = 2^{11}$ the probability distribution of returns converges, as expected to a Gaussian.

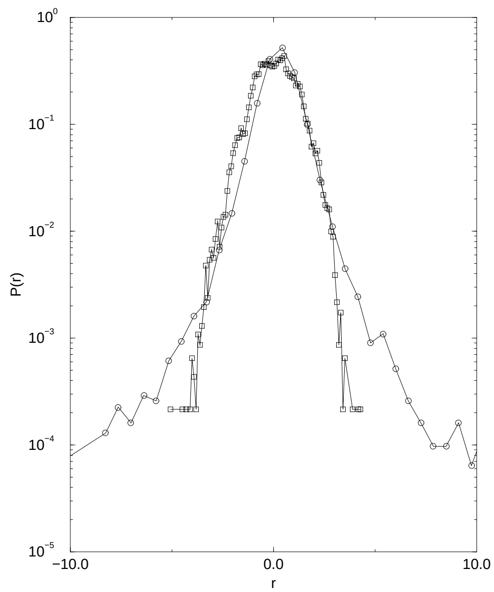

FIG. 5. Rescaled probability distribution of cumulative returns $P(r^{\tau})$ at different time lag, $\tau = 1$ (circle), $\tau = 2^{11}$ (squares)

In fig.(6) we plot the autocorrelations function of return C_r and absolute return $C_{|r|}$

$$\begin{aligned} C_r(L) &= \; < r(t)r(t+L) > - < r(t) >< r(t+L) > \\ C_{|r|}(L) &= \; < |r(t)||r(t+L)| > - < |r(t)| >< |r(t+L)| > \end{aligned} \tag{8}$$

as a function of the time lag L.

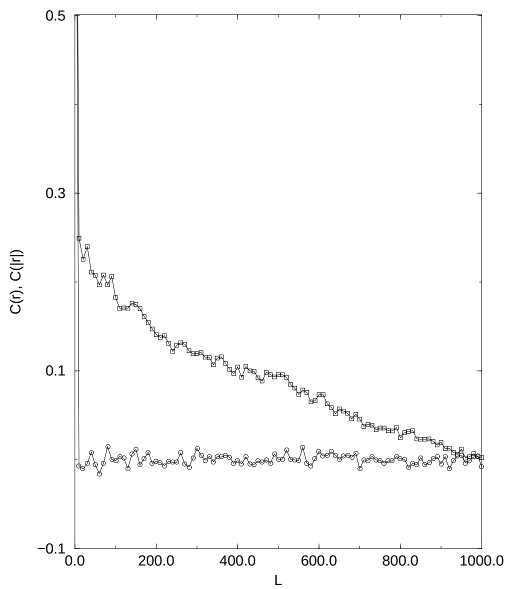

FIG. 6. Autocorrelations function of returns (circles) and absolute returns (squares).

Fig.(6) show that while returns are not correlated, the autocorrelation functions of absolute returns has a slowly decaying tail revealing the presence of long term memory. Inferring whether the decay of the autocorrelation function is exponential or power-law is nonetheless difficult from fig.(6). The nature of the long term correlations can be better investigated through the analysis of the variance of the cumulative returns and absolute returns. We construct the variables $\tilde{r}(t,L)$ and $|\tilde{r}(t,L)|$:

$$\begin{aligned} \tilde{r}(t,L) &= \tfrac{1}{L}\textstyle\sum_{i=1}^{L} r(t+i) \\ |\tilde{r}(t,L)| &= \tfrac{1}{L}\textstyle\sum_{i=1}^{L} |r(t+i)| \end{aligned} \tag{9}$$

where the sum is taken over non overlapping intervals. The quantity we are interested is the variance these new variables as a function of L. It can be shown [13,12] that if the correlation functions of absolute returns has a power-law decay $C_{|r|}(L) \sim L^{-\beta}$ with an exponent $\beta < 1$ then the $var(|\tilde{r}(t,L)|)$ is a power-law with the same exponent:

$$var(|\tilde{r}(t,L)|) \sim L^{-\beta} \tag{10}$$

On the contrary if the $|r(t)|$ are uncorrelated or short term correlated we would find

$$var(|\tilde{r}(t,L)|) \sim L^{-1}. \tag{11}$$

In other words the hypothesis of long range memory for absolute returns can be checked via the numerical analysis of the variance of the absolute cumulative returns.

In fig.(7) we compare the volatility of cumulative returns and absolute cumulative returns. The value of the exponent β is $\beta \simeq 1.$ for cumulative returns and $\beta \simeq 0.2$ for absolute cumulative returns. Empirical studies [12,13] have estimated $\beta \simeq 0.38$ for the absolute returns autocorrelation in the NYSE index and $\beta \simeq 0.39$ for the USD-DM exchange rate.

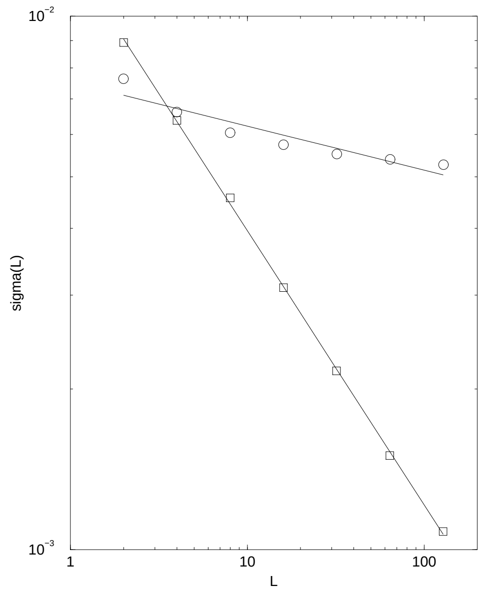

FIG. 7. Volatility of cumulative return (squares) and cumulative absolute return (circles)

IV. CONCLUSIONS

This paper has outlined a mechanism which can explain certain stylized facts of stock market returns. According to our model synchronization effects, which generate large fluctuations in returns, can arise purely from communication and imitation among traders, even in the absence of an aggregate exogenous shock. While aggregate news, i.e. information on past price changes, plays a role in our model via the feedback effect on thresholds, this news is determined endogenously in the model.

Many interesting questions which have arisen in other studies could also be addressed in the context of our model. One of these is how the trading mechanism affects the distribution of wealth among the traders. Under what conditions is the trading mechanism capable of increasing the average level of wealth? Also, given an initial flat wealth distribution, how does it change with time as a result of trading mechanisms? Previous studies suggest [35] that, in a stationary state, the distribution of wealth follows a well defined power law in accord with the Zipf law [36].

Recent studies [37] suggest that crashes have a characteristic log-periodic signature in analogy with earthquakes and other self-organizing cooperative system [38]. Another interesting question is whether precursory patterns and after shock signatures of financial crashes can be identified in the simulated price histories of our model.

If the model is expanded to allow for intra-period trading, the arrival of news, simultaneously observed by all agents, could lead to agents who have lower thresholds trading first and influencing their neighbours with higher thresholds. Hence, a first-mover advantage could arise for to agents who have lower thresholds, in that they could benefit from trading at prices which do not fully incorporate the news. The intra period trading mechanism could serve to explain the occurrence of the short-term correlations observed in stock returns. Memory effects in market returns could also be generated if the agents actions $S_i(t)$ were not reset to zero at the beginning of each trading period.

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