IMPLICIT COLLUSION IN DEALER MARKETS WITH DIFFERENT COSTS OF MARKET MAKING

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Abstract. This paper introduces different costs into the Dutta-Madhavan model of implicit collusion between market makers. It will be shown how different costs of market making influence the possibility of implicit collusion and the price setting. The derived model will then be applied to the case of discrete prices. We will see how the tick size and the location of the reservation prices relative to the price grid affect the price setting. It will be shown which problems arise when examining the developed model in an empirical investigation. It is shown that the empirical evidence may only be very poor, even under ideal circumstances. Finally policy implications are considered.

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In their articles *W. Christie*, and *P. Schultz* (1994) and *W. Christie* et al. (1994) documented that NASDAQ market makers quoted bid-ask spreads above competitive levels by avoiding odd-eighth quotes. After their findings had become public, market makers began also to quote prices at odd-eighths, thereby reducing the spread significantly. Implicit collusion among market makers to quote only even-eighths was presented as an explanation of this behavior. Quoting only even-eighths has been interpreted as a coordination device to obtain larger spreads and hence larger profits. Fearing regulatory interventions, so the argumentation, market makers changed their behavior after these results had become public and quoted competitive prices.

P. K. Dutta, and *A. Madhavan* (1997) presented a model and derived conditions under which market makers are able to quote prices such that the spread is above competitive levels. Market makers do not have to agree to fix prices above competitive levels. The chosen prices maximize individual profits of the market makers (*implicit collusion*), no explicit agreement is needed to obtain this result. A defecting market maker has not to be sanctioned, it is even not needed to identify him. In deriving their results Dutta and Madhavan have not to include informational asymmetries, they hold under very general conditions.

One condition, nevertheless, is central for the argumentation of Dutta and Madhavan, the assumption that all market makers face zero costs of market

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making. Although this assumption can easily be generalized to non-zero costs without changing the results, for the argumentation it is essential that all market makers have equal costs. This assumption does not capture the nature of market making as known from market microstructure theory. These theories state that the costs of market makers will be different and change over time. It therefore is not reasonable to assume equal costs for all market makers.

This paper develops a model of implicit collusion when market makers face different costs.

The plan of the paper is as follows: The first section reviews some results from market microstructure theory that will be useful in the development of the results in this article and motivates the introduction of different costs; according to these results, the model of Dutta and Madhavan is modified and some simple results are shown in the second section; the third section shows which prices are sustainable under implicit collusion and the fourth section determines the prices that will be quoted; in the fifth section we introduce discrete prices; empirical problems that may arise, are treated in the sixth section; policy implications to reduce the effects of implicit collusion are considered in section seven; section eight concludes the findings.

1. Results from Market Microstructure Theory

From market microstructure theory it is well known that the costs of market making consist of the following components:

Result 1. ***(Costs of market making)***

- ***Order processing costs***
- ***Inventory costs***[1]
  - *increasing in the variance of the true value of the asset*
  - *increasing in the trade size*
  - *increasing in the absolute risk aversion of the market maker*
  - *increasing in the inventory of the market maker for a trade at the ask*
  - *decreasing in the inventory of the market maker for a trade at the bid*
- ***Asymmetric information costs***[2]
  - *increasing in the share of informed trades*
  - *increasing in the quality of information insiders have relative to the information of the market maker*

With this result assuming equal costs of market making requires the above components to sum up to the same costs. While order processing costs can reasonably be assumed to be almost equal, it is well known that inventories

[1] See *H. Stoll* (1978).
[2] See *L. R. Glosten*, and *P. R. Milgrom* (1985).

differ among market makers. By executing an order the market maker's inventory and hence his costs change. So inventory costs will not only differ among market makers but also a market maker will face different inventory costs over time. It is common to assume the other factors influencing inventory costs to be equal for all market makers and therefore to impose the same costs.

In most cases costs of asymmetric information are assumed to be equal for all market makers by stating that they have equal information and face the same composition of the order flow. However, it would be more reasonable to assume that the composition of the order flow depends on the main source a market maker receives the orders from. These sources will vary among market makers, e.g. as a result of preferencing arrangements. Consequently also asymmetric information costs will differ between market makers. Therewith it is not reasonable to assume equal costs of market making.

A market maker will not set a price below his reservation ask price, $p^i_{R,a}$, or above his reservation bid price, $p^i_{R,b}$, as he is not willing to make a loss by trading. If we assume market makers to act competitively, standard neoclassical theory suggests that they will quote their reservation prices. But if we assume that market makers know the reservation prices of each other, or can infer them from observing the quoted prices, it has been shown by *T. Ho*, and *H. R. Stoll* (1980) that this will not be true in general.

Result 2. ***(Competitive Equilibrium)***

Let the market maker with the lowest reservation ask price be called the "best" market maker, the market maker with the second lowest reservation ask price the "second-best" market maker, and so forth. We further assume strict price priority, i.e. only the market maker with the lowest quoted ask price and the market maker with the highest quoted bid price executes an arriving order and when quoting the same prices the trade is assigned to one of the market makers according to an exogenously determined probability law. Prices can be set continuously.[3]

The best market maker will not quote his reservation ask price, but only undercut the reservation ask price of the second-best market maker by a fraction. With this strategy he can make a profit (the difference between his reservation price and the reservation price of the second best market maker) that no other market maker can prevent by undercutting his price. Only in the case that the best and the second-best market maker have the same reservation ask prices, they will quote their reservation prices.

The quotes and costs of all other market makers are of no interest for the market, decisions are only based on the best quotes with the assumption of

[3]A definition of a competitive equilibrium in the case of discrete prices is given in proposition 5.

strict price priority. The best price quoted is the reservation price of the second best market maker.

An equal mechanism can be applied for a trade at the bid price.

We can generalize this result and state that a market maker who wants to exclude another market maker from the order flow, will do so not by quoting his own reservation price, but by undercutting his competitor's reservation price by a fraction. As this fraction can be infinitesimal small in the case of continuous prices, we will neglect it for simplicity and say that he has to quote the reservation price of his competitor.[4]

Having in mind these results we now can introduce different costs into the model of *P. K. Dutta*, and *A. Madhavan* (1997).

2. A model of implicit collusion

In this article we make the same assumptions on the demand for the service of market makers as in *P. K. Dutta*, and *A. Madhavan* (1997). Investors face a random liquidity event $L > 0$,[5] whose realization for this period is known to all market participants before market makers quote their prices and trade occurs. Future realizations are not known to any market participant, but the distribution is known. The liquidity event forces investors to change their portfolio compositions, hence they have to trade.

The demand for the service of a particular market maker i, provided that he quotes the best price, is denoted d_a^i for the demand of submitting a buy order and d_b^i for the demand of submitting a sell order. They depend on the quoted prices (p_a^i and p_b^i, respectively) and the size of the liquidity event L. If the market maker does not quote the best price, the demand for his service is zero by the assumption of strict price priority.

Assumption 1. ***(Monotonicity)***

The demands are assumed to be monotone increasing in the liquidity event. The demand of submitting a buy order is monotone decreasing in the ask price and the demand for submitting a sell order is monotone increasing in the bid

[4]It will be of importance that prices have to be undercut, when introducing price discreteness in section 5.

[5]A liquidity event has not to be interpreted as noise trading, it also can encompass informed trades. This term is only used to point out that the reason for trading is exogenously given.

price:

$$\frac{\partial d_a^i(p_a^i, L)}{\partial L} \geq 0, \qquad \frac{\partial d_b^i(p_b^i, L)}{\partial L} \geq 0,$$
$$\frac{\partial d_a^i(p_a^i, L)}{\partial p_a^i} \leq 0, \qquad \frac{\partial d_b^i(p_b^i, L)}{\partial p_b^i} \geq 0$$

for every $i = 1, \ldots, N$, where N is the number of market makers.

We further assume the increase in the profits to be strictly diminishing the larger the ask, the smaller the bid price and the larger L is:

Assumption 2. ***(Concavity)***

The profit function

$$\pi^i(p_a^i, p_b^i, p_{R,a}^i, p_{R,b}^i, L) = (p_a^i - p_{R,a}^i)d_a^i(p_a^i, L) + (p_b^i - p_{R,b}^i)d_b^i(p_b^i, L)$$

is strictly concave in the bid and ask prices for every L and in L for every bid and ask price and for every $i = 1, \ldots, N$, where $p_{R,a}^i$ and $p_{R,b}^i$ are the reservation prices of the market maker for a trade at the ask and at the bid, respectively.

Another useful assumption is the monotone demand ratio:

Assumption 3. ***(Monotone demand ratio)***

Let $p_a' \leq p_a$, $p_b' \geq p_b$ and $L' \leq L$, then

$$\frac{d_a^i(p_a', L')}{d_a^i(p_a, L')} \leq \frac{d_a^i(p_a', L)}{d_a^i(p_a, L)}$$

and

$$\frac{d_b^i(p_b', L')}{d_a^i(p_b, L')} \leq \frac{d_b^i(p_b', L)}{d_b^i(p_b, L)}$$

for every $i = 1, \ldots, N$.

With these assumptions we now can describe two benchmark equilibria: the competitive and the cooperative equilibrium. The competitive equilibrium has been described in result 2. The cooperative equilibrium has been proofed to exist and to be unique in *P. K. Dutta*, and *A. Madhavan* (1997), this result does not depend on the assumption of equal costs.

Result 3. ***(Cooperative Equilibrium)***

Denote $p_{R,a} = (p_{R,a}^1, \ldots, p_{R,a}^N)$ and $p_{R,b} = (p_{R,b}^1, \ldots, p_{R,b}^N)$ the vector of reservation prices.

There exist unique prices $p_a^c(p_{R,a}, L)$ and $p_b^c(p_{R,b}, L)$ that maximize the joint profits of all market makers.

This equilibrium has the following properties:

$$\frac{\partial p_a^c(p_{R,a},L)}{\partial L} \geq 0, \qquad \frac{\partial p_b^c(p_{R,b},L)}{\partial L} \leq 0,$$

$$\frac{\pi^i(p_a^c,p_b^c,p_{R,a}^i,p_{R,b}^i,L)}{\partial L} \geq 0$$

for every $L > 0$ *and* $i = 1,\ldots,N$.

We will see that it is of special importance that the ask price is increasing and the bid price decreasing in the liquidity event, independent of the value of L.

If we do not allow market makers to cooperate on the price setting, every market maker has, independently of the other, to maximize his present value of future profits, taking into account the behavior of his competitors.

Denote the performance of market maker i by J^i and the discount factor for his future profits by ρ^i. By using the concept of dynamic programing we can write J^i as

$$(1) \qquad J^i(L,p_{R,a}^i,p_{R,b}^i) = \max_{p_a^i,p_b^i}(\pi^i(p_a^i,p_b^i,p_{R,a}^i,p_{R,b}^i,L) + \rho^i E_L[J^i(L,p_{R,a}^i+\Delta p_{R,a}^i,p_{R,b}^i+\Delta p_{R,b}^i)]).$$

The solution to this equation is denoted $p_a^{c,i}$ and $p_b^{c,i}$.

If market makers collude implicitely, i.e. quote equal prices resulting in spreads above competitive levels,[6] it has to be more profitable to follow this strategy than to quote prices competitively. If we for now assume that, if market makers quote the same price, every market maker receives the same share of the order flow, for every $i = 1,\ldots,N$ the following constraint has to hold:

$$(2) \frac{1}{N}(\pi^i(p_a^{c,i},p_b^{c,i},p_{R,a}^i,p_{R,b}^i,L) + \rho^i E_L[J^i(L,p_{R,a}^i+\Delta p_{R,a}^i,p_{R,b}^i+\Delta p_{R,b}^i)]) \geq \pi^i(p_a^{c,i},p_b^{c,i},p_{R,a}^i,p_{R,b}^i,L) + \frac{1}{N}\rho^i E_L[J_c^i(L,p_{R,a}^i+\Delta p_{R,a}^i,p_{R,b}^i+\Delta p_{R,b}^i)].$$

The left side represents the expected present value of the profits when applying the collusive pricing strategy. It consists of the total expected profits from the next order arriving at the market and the present value of the total expected future profits, multiplied by the fraction of the order flow a market maker receives. This has to be larger than the expected present value of the profits from defecting on the right side. The first term represents the profits that a defecting market maker can earn in this period by undercutting the collusive

[6] If the spread has to be above competitive levels, either the ask price has to be above or the bid price below the competitive price. We will concentrate here on showing that the ask price can be above the competitive ask price and analyze his behaviour, taking the bid price for simplicity as given. The bid price can either be below the competitive bid price or equal it. Similar considerations have to be made for the bid price.

price by a fraction. The second term is the present value of the expected future profits when applying competitive pricing afterwards (denoted J_c^i).[7]

This constraint can be transformed into

$$\text{(3)} \qquad \frac{K^i}{N-1} \geq \pi^i(p_a^{c,i}, p_b^{c,i}, p_{R,a}^i, p_{R,b}^i, L),$$

where $K^i = \rho^i E_L[J^i(L, p_{R,a}^i + \Delta p_{R,a}^i, p_{R,b}^i + \Delta p_{R,b}^i) - J_c^i(L, p_{R,a}^i + \Delta p_{R,a}^i, p_{R,b}^i + \Delta p_{R,b}^i)]$ is the difference of the joint expected profits of the collusive and the competitive pricing strategy.

This restriction is similar to the restriction obtained by *P. K. Dutta*, and *A. Madhavan* (1997). The prices that actually can be quoted have to be smaller than the lowest ask price and larger than the highest bid price fulfilling (3), i.e.

$$p_a^c = \min_{i=1,\ldots,N} p_a^{c,i},$$

$$p_b^c = \max_{i=1,\ldots,N} p_b^{c,i}.$$

For all prices below p_a^c and above p_b^c profits are smaller than at p_a^c and p_b^c, hence (3) is fulfilled. With prices above p_a^c or below p_b^c there exists at least one market maker for which (3) is not fulfilled. He would defect and the pricing strategy of implicit collusion would no longer be an equilibrium.

The results obtained by *P. K. Dutta*, and *A. Madhavan* (1997) remain valid in this new setting:

Result 4. ***(Implicit collusion)***

If $\rho^i > \rho_0 = 1 - \frac{1}{N}$ for all $i = 1, \ldots, N$ then there exists a collusive equilibrium.

We define a critical value $L^c = L^c(\min_i \rho^i)$ of L such that

- *If $L < L^c$ the optimal prices are equal to the prices under explicit collusion.*
- *If $L > L^c$ the optimal ask prices are below the ask prices of explicit collusion, but above competitive ask prices. The optimal bid prices are below competitive prices, but above collusive prices.*
- *The ask price is increasing and the bid price decreasing in L for $L < L^c$, for $L > L^c$ the ask price is decreasing and the bid price is increasing in L. Hence the spread is increasing for $L < L^c$ and decreasing for $L > L^c$.*

[7]The assumption that, if a market maker defects, the competitive pricing rule will be applied to punish the defective market maker is due to the fallback rule, see e.g. *D. Abreu* (1988). Other more realistic forms of punishing a defecting market maker is e.g. to refuse any interdealer trading with him. This would impose only small costs to the other market makers. For the further argumentation is not important by which means future profits are reduced after defection.

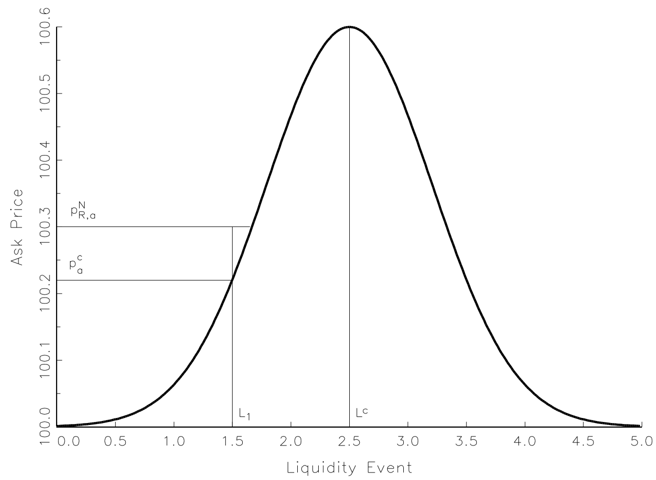

FIGURE 1. Sustainable prices with equal costs

If $\rho^i < \rho_0$ *for any market maker, the only equilibrium is to quote competitive prices.*

This result shows that the ask price is increasing in the liquidity event only if $L < L^c$, whereas with explicit collusion the ask price is increasing for every liquidity event. This behaviour of the ask price can be used to distinguish explicit from implicit collusion.[8]

If market makers are sufficient patient, they can earn higher profits than by quoting competitive prices, although they do not collude in the common sense. The prices are the outcome of independent maximization of each market maker. To maintain this equilibrium, no measures have to be imposed on a defecting market maker if afterwards competitive pricing is known to be applied, even his identity has not to be made public to the other market makers in this case. All market makers will follow this rule as the result of pure self-interest. Figure 1 visualizes the results as presented by *P. K. Dutta*, and *A. Madhavan* (1997) for the ask price, where reservation prices equal 100. All prices below the bold line (including this line) fulfill constraint (3) and can therefore be quoted. These prices we call "sustainable".

Before demonstrating the effect different costs have on the sustainability of prices and the choice of quoted prices, we will show two simple results for a varying number of market makers that will be useful in the next section.

[8]See *P. K. Dutta*, and *A. Madhavan* (1997).

Lemma 1. *If N decreases $\rho_0 = 1 - \frac{1}{N}$ decreases, i.e. the less market makers are present in the market, the more impatient the market makers can be to maintain collusive prices.*

Proof. The proof is straightforward: As N decreases, $1 - \frac{1}{N}$ decreases. □

The intuition behind this lemma is that the less market makers compete, the larger the share of the order flow is for every market maker if he sets collusive prices. This results in larger profits from collusive pricing, the profits from undercutting the price in this period are less important.

Lemma 2. *The sustainable ask prices increase and the sustainable bid prices decrease for any L if N decreases.*

Proof. The prices have to fulfill constraint (3). If N decreases the left side increases. Holding L constant we can increase the ask and decrease the bid price at the left side, thereby increasing profits, until the two sides equal. □

3. Determination of sustainable prices

If we return to figure 1, we see that with a liquidity event of L_1 the price p_a^c could be quoted. But now suppose that the N^{th} best market maker (the market maker with the highest reservation price) has a reservation price higher than p_a^c, say $p_{R,a}^N$. He will not quote the price p_a^c as he would make a loss, but it is also not possible for the other market makers to raise the price above p_a^c to enable him to participate in the order flow, because this would lead to defection of some market makers and competitive pricing as constraint (3) is not longer fulfilled for all market makers.

The best strategy for all other market makers is to quote the collusive price and therewith exclude the market maker with higher costs.[9] The number of market makers colluding is reduced to $N - 1$.

In general we have $M \leq N$ market makers that are able to collude, the other market makers having reservation prices above sustainable prices. Constraint (3) changes to

$$(4) \qquad \frac{K^i}{M-1} \geq \pi^i(p_a^{c,i}, p_b^{c,i}, p_{R,a}^i, p_{R,b}^i, L).$$

Let us now order the reservation prices such that $p_{R,a}^1 \leq p_{R,a}^2 \leq \cdots \leq p_{R,a}^M \leq \cdots \leq p_{R,a}^N$. As shown in Lemma 2 the sustainable prices will increase with less market makers. But if we increase the price above the reservation price of the

[9]He also has to quote a price, his reservation price or a higher price, but as this price is higher than the collusive price quoted by the other market makers, he will not receive a trade by the assumption of strict price priority and we can neglect them.

$(M+1)^{th}$ market maker, we have to consider his decisions and include him, thus reducing the sustainable prices under his reservation price. So we have to add another constraint on the prices:

$$p_{R,a}^{M} \leq p_a^c < p_{R,a}^{M+1}. \tag{5}$$

The following proposition shows how sustainable prices behave:

Proposition 1. *Let L^0 be a liquidity event such that $p_a^c(L_0^M) = p_{R,a}^{M+1}$. If the sustainable prices resulting from constraint (4) are lower than the reservation price $p_{R,a}^{M+1}$ of some market makers the price has the following properties if $\rho^i > \rho_0$ for all $i = 1, \dots, M$:*

1. *$L > L^c$: There exists a liquidity event $L = L^M \geq L^c$ such that $p_a^c = p_{R,a}^{M+1}$ for $L_0^M < L < L^M$. For $L > L^M$ p_a^c is decreasing in L.*
2. *$L < L^c$: There exists a liquidity event $L = L^M \leq L^c$ such that $p_a^c = p_{R,a}^{M+1}$ for $L_0^M > L > L^M$. For $L < L_0^M$ p_a^c is increasing in L.*

If $\rho^i < \rho_0$ for any $i = 1, \dots, M$, the only equilibrium is to price competitively.

Proof. If the highest sustainable price is lower than the reservation price of a market maker, he will be excluded from the order flow as he quotes a too high price (at least his reservation price), M in constraint (4) is reduced. We can now increase the collusive price p_a^c, while (4) still is fulfilled, the increase is limited by $p_{R,a}^{M+1}$ due to constraint (5).

Suppose now we have a liquidity event such that the collusive price with $M+1$ colluding market makers is not much lower than $p_{R,a}^{M+1}$. An increase in the price is limited by constraint (5). The more L reduces (increases) if $L < L^c$ ($L > L^c$), i.e. the smaller the collusive price with M colluding market makers is compared to $p_{R,a}^{M+1}$, the more we can increase the right site of constraint (4). There will now exist a liquidity event L^M such that both sides equal. For $L > L^M$ ($L < L^M$) we have $p_a^c = p_{R,a}^{M+1}$.

With a further reduction (increase) in L it will not be possible to quote $p_{R,a}^{M+1}$, a smaller price has to be quoted to fulfill this constraint. This price decreases the more the liquidity event decreases (increases) according to result 4). □

Figure 2 visualizes this finding. In this figure (and the following illustrations) we assume $N = 5$ market makers. Their reservation prices are $p_{R,a}^1 = 100$, $p_{R,a}^2 = 100.055$, $p_{R,a}^3 = 100.1$, $p_{R,a}^4 = 100.2$ and $p_{R,a}^5 = 100.25$, unless otherwise stated.

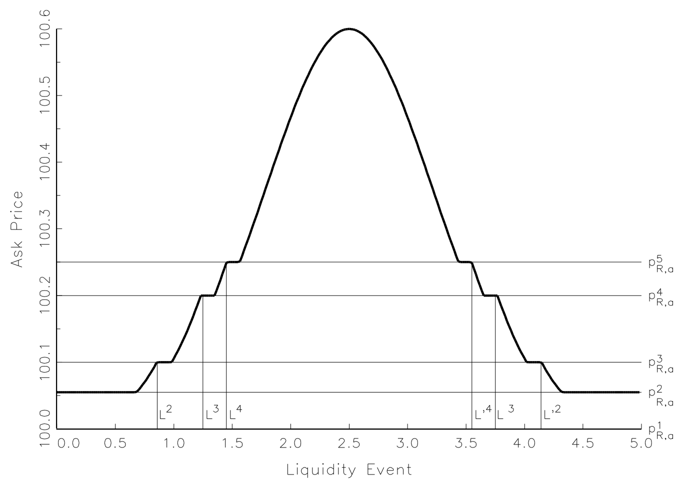

FIGURE 2. Sustainable prices with different costs

4. DETERMINATION OF THE MOST PROFITABLE PRICES

Thus far we have determined sustainable prices fulfilling constraints (4) and (5). Nothing has been said about which price the market makers will actually quote. This will depend on the performance every price gives the market makers.

Let H^i denote the expected performance of market maker i if the prices p_a and p_b are quoted:[10]

$$(6)\quad H^i(p_a, p_b, p^i_{R,a}, p^i_{R,b}, L, M) = \frac{1}{M}(\pi^i(p_a, p_b, p^i_{R,a}, p^i_{R,b}, L) + \rho^i E_L[J(p^i_{R,a} + \Delta p^i_{R,a}, p^i_{R,b} + \Delta p^i_{R,b}, L)]).$$

If we would hold M fixed to the number of market makers that are able to collude, we could easily maximize (6), obtaining the same prices as in (1) by replacing N with M. But it is a possible strategy to quote a lower price and therewith exclude more market makers with higher costs, i.e. reducing M further. H^i may increase although the numerator in (6) will decrease, because the denominator (M) also decreases.

A market maker will lower his prices from p_a to p'_a if

$$(7)\quad H^i(p_a, p_b, p^i_{R,a}, p^i_{R,b}, L, M) \leq H^i(p'_a, p_b, p^i_{R,a}, p^i_{R,b}, L, M'),$$

[10] We assume that the future expected performance $E_L[J(p^i_{R,a} + \Delta p^i_{R,a}, p^i_{R,b} + \Delta p^i_{R,b}, L)]$ is not affected by the current prices, although with the prices the probability of a trade and hence of a change in the inventory is affected. These affects are assumed to have on average no influence.

where $M' \leq M$ denotes the number of market makers having a lower reservation price than p'_a. This condition states that it may be profitable to reduce the price if we can exclude many market makers with a small decrease in the price. Whether it is profitable to reduce the price depends on the slope of H^i and the distribution of the reservation prices. The market makers will quote an ask price (taking p_b as given) such that

$$J^i(p_a, p_b, p^i_{R,a}, p^i_{R,b}, L) = \max_{p_a} H^i(p_a, p_b, p^i_{R,a}, p^i_{R,b}, L, M). \tag{8}$$

Proposition 2. *Market makers with high costs may be excluded from the order flow by other market makers with lower reservation prices through lowering their quoted prices. The exclusion is facilitated if*

- *the slope of H^i is small,*
- *the reservation prices are bundled.*

The quoted prices will be the reservation price of the last market maker excluded if H^i is increasing at this price. If H^i is decreasing at the reservation price of the last excluded market maker, the quoted price will be at the price, where the first derivative is zero.

Proof. With a small slope of H^i the performance will decrease only by a small amount with a decrease in prices.

If the reservation prices are bundled only a small reduction in the price and hence in H^i is needed to reduce the number of market makers significantly, i.e. the right side in (7) is relatively large.

At the reservation price of the market maker just excluded H^i increases by a discrete amount as M decreases at this point. If H^i is increasing, any lower price than the reservation price of the last excluded market maker will have a lower value of H^i. Whereas when H^i is decreasing, the most profitable price is at the point where the first derivative of H^i is zero, which is at a lower price because of the assumption of strict concavity (assumption 2) □

The bold line in figure 3 shows a function H^i with different costs, where the reservation prices are the same as in figure 2. The thin line shows H^i when all market makers have equal reservation prices of 100.[11]

With propositions 1 and 2 the number of market makers colluding can be viewed as endogenously determined. The sustainability of prices reduces the number of market makers that can collude. Further market makers can be

[11] We assume in the illustrations that the maximum of H^i does not depend on L. We only model the increase in the level of H^i with L. Making the more realistic assumption that the maximum of H^i in increasing in L would not change the main line of argumentation.

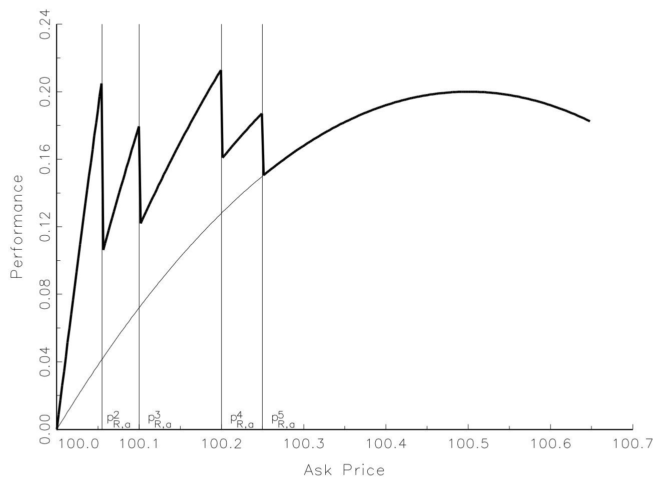

FIGURE 3. Performance with different costs

excluded by other market makers through quoting a lower price in order to raise the profits of the remaining market makers, i.e. we find that $M = M(L, p_a)$.

To determine the effectively quoted prices in our model we have to determine for every L the sustainable price that has the highest performance. Figure 4 shows the performance the sustainable prices have for every L.

The bold line in figure 5 shows the prices with the highest performance that are actually chosen by market makers. The thin line represents the competitive price and the dashed line the prices that would be quoted if all market makers had equal reservation prices of 100.

It can be seen that, as long as the liquidity event is not too small or too large, the market makers will quote prices above competitive levels. We also can see that it is not always most profitable to quote the highest sustainable price. In this example the quoted prices are below this price for most liquidity events.

In general this function will be a step function, with a jump from one reservation price of the market makers to the next higher.

The result of *P. K. Dutta*, and *A. Madhavan* (1997) that the quoted ask price is increasing for $L < L^c$ and decreasing for $L > L^c$ still remains valid with different costs. But this effect is much more difficult to observe as the quoted price can remain constant for a large range of liquidity events.

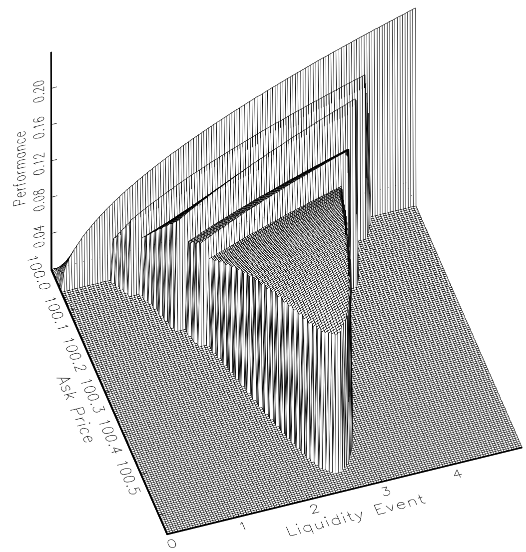

FIGURE 4. Performance of sustainable prices

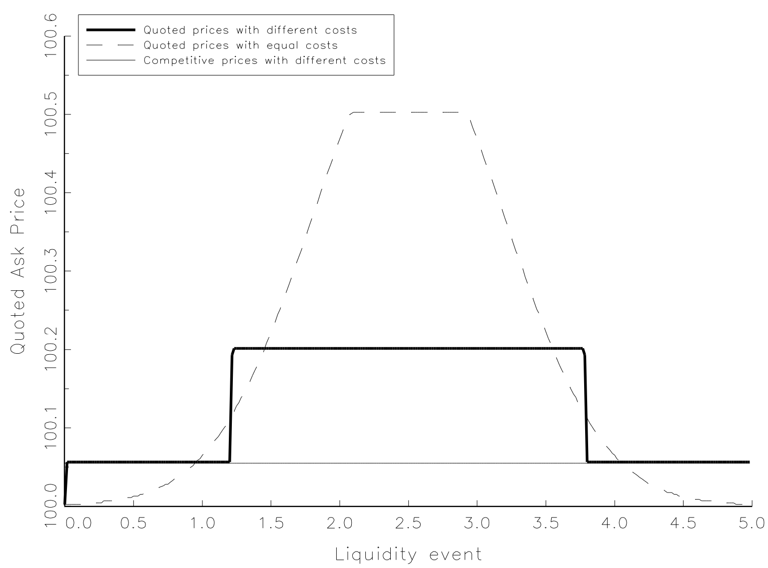

FIGURE 5. Quoted prices

We will now add two more findings showing that it is easier to achieve implicit collusion of at least some market makers if costs are different, although it is not possible to achieve implicit collusion of all market makers.

Proposition 3. *If there exist market makers such that for at least one of them* $\rho^i < \rho_0 = 1 - \frac{1}{M}$*, it is still possible to maintain collusive prices:*

- *These market makers can be excluded by quoting prices below their reservation prices.*
- *By quoting lower prices some market makers can be excluded from collusion (not necessarily those who are too impatient), thereby reducing* M *and including more impatient market makers into implicit collusion.*

Proof. The first part is obvious. If we can exclude impatient market makers by lowering prices, their decisions do not have to be considered. This will always be a profitable strategy as long as the quoted prices are above competitive levels.

The second part can easily be seen from lemma 1. If we exclude some market makers ρ_0 decreases. Again this is a profitable strategy as long as the prices are above competitive levels. □

Proposition 4. *Assume that the market makers do not share the order flow equally.*

- *If the market maker with the smallest market share colludes, all other market makers also will collude.*
- *If the market makers with the smallest market shares are not willing to collude, it may be possible to maintain collusive pricing by lowering the quoted prices and thereby exclude market makers with small market shares if they have high reservation prices. It is also possible to exclude other market makers, increasing the market share of the remaining market makers until they are willing to collude.*

Proof. The first part has already been proofed in *P. K. Dutta*, and *A. Madhavan* (1997) as proposition 4.

The second part is straightforward to proof: To maintain higher prices and thus higher profits, the market makers with higher market shares can lower the price under the reservation prices of the market makers with low market shares, thereby avoiding competitive pricing. Another strategy is to exclude other market makers, the market share of all remaining market makers is increased (as they have to serve the same order flow with less market makers), until their share is large enough and they are willing to collude. □

5. Price discreteness

Thus far we have assumed that prices can be quoted continuously, but in real financial markets prices can only be quoted in fixed intervals. In U.S. stock markets ticks of 1/8 and 1/16 are most commonly found.[12]

We first have to define the competitive equilibrium with discrete prices. Let t_k denote the ticks and $d > 0$ the interval between two ticks (the tick size), such that $t_{k+1} = t_k + d$. We have to distinguish two cases: $t_0 \leq p^1_{R,a} \leq p^2_{R,a} \leq t_1$ and $t_0 \leq p^1_{R,a} < t_1 < \cdots < t_k \leq p^2_{R,a} \leq t_{k+1}$. If we define the competitive equilibrium as the price at which no market maker wants to change his decision and the price cannot be undercut by a competitor, in the first scenario the only equilibrium is to quote the price t_1. At t_0 the best market maker would make no profits and all other market makers would make a loss. But at t_1 he would make a profit, although he has to share the order flow with the second best market maker. At a price of t_2 the second best market maker would be able to undercut this price. The competitive price is higher than with continuous prices, where it would be $p^2_{R,a}$.

In the second case two equilibria are possible. The best market maker could quote t_k, no other market maker would be able to undercut this quote. But he could also quote t_{k+1}, again no other market maker would be able to undercut this price, but the best market maker has to share the order flow with the second best market maker. Both quotes can be an equilibrium, which price will be selected by the best market maker, depends on the profits he can make with each strategy.

We can summarize these findings in the following proposition:

Proposition 5. *With discrete prices the competitive equilibrium is no longer the reservation price of the second best market maker:*

- *If* $t_0 \leq p^1_{R,a} \leq p^2_{R,a} \leq t_1$ *the only competitive equilibrium is* t_1.
- *If* $t_0 \leq p^1_{R,a} < t_1 < \cdots < t_k \leq p^2_{R,a} \leq t_{k+1}$ *there exist two possible competitive prices,* t_k *and* t_{k+1}. *Which price is chosen depends on the profits the best market maker can make in each situation.*

In the first case the competitive price is higher with discrete than with continuous prices, in the second case it is lower in one and higher in the other case.

With discrete prices it is no longer possible to undercut prices by just a fraction. The difference has to be one tick. With a tick size of d we have to rewrite

[12] In 1997 the NASDAQ changed the tick size for most stocks from 1/8 to 1/16.

constraint (2) as

$$(9)\frac{1}{N}(\pi^i(p_a^{c,i}, p_{R,a}^i, p_{R,b}^i, L) + \rho^i E_L[J^i(L, p_{R,a}^i + \Delta p_{R,a}^i, p_{R,b}^i + \Delta p_{R,b}^i)])$$
$$\geq \pi^i(p_a^{c,i} - d, p_{R,a}^i, p_{R,b}^i, L) + \frac{1}{N}\rho^i E_L[J_c^i(L, p_{R,a}^i + \Delta p_{R,a}^i, p_{R,b}^i + \Delta p_{R,b}^i)].$$

If we approximate $\pi^i(p_a^{c,i} - d, p_{R,a}^i, p_{R,b}^i, L)$ by a first order Taylor series around $(p_a^{c,i}, p_{R,a}^i, p_{R,b}^i, L)$ we get

$$(10) \quad \pi^i(p_a^{c,i} - d, p_{R,a}^i, p_{R,b}^i, L) = \pi^i(p_a^{c,i}, p_{R,a}^i, p_{R,b}^i, L) - \frac{\partial \pi^i(p_a^{c,i}, p_{R,a}^i, p_{R,b}^i, L)}{\partial p_a^{c,i}} d.$$

Defining $D = \frac{\partial \pi^i(p_a^{c,i}, p_{R,a}^i, p_{R,b}^i, L)}{\partial p_a^{c,i}} d$ constraint (9) can be transformed into

$$(11) \qquad \frac{K^i}{M-1} + \frac{M}{M-1} D \geq \pi^i(p_a^{c,i}, p_{R,a}^i, p_{R,b}^i, L),$$

where K^i is defined as in (3). The larger the tick size, the larger the left side of this constraint compared to continuous pricing.[13] So the profits, and therewith the collusive prices can be higher the larger the tick size is. Holding L constant the collusive ask prices can increase, still fulfilling (11). In the case of discrete prices, higher prices are sustainable than with continuous prices. This result is equal to that found by *P. K. Dutta*, and *A. Madhavan* (1997). We herewith have proofed the following proposition:

Proposition 6. *The larger the tick size the higher are the sustainable ask prices for any given $L > 0$.*

The bold line in figure 6 shows the sustainable prices with a tick size of 1/8, whereas the thin line shows the sustainable prices with continuous prices.

To choose the prices that are actually quoted we have to find the sustainable price with the highest performance function for every $L > 0$ as in the case of continuous prices. Additionally the quoted prices have to lie on the discrete price grid. As the performance function is discontinuous at the reservation prices, the quoted prices are sometimes lower and sometimes higher compared to the case of continuous prices. Which case applies, depends on the value of the performance function for the different prices.

As figure 7 shows, the outcome depends on how the reservation prices fit into the price grid. Panel (a) shows the result for a tick size of 1/8 with the same reservation prices as used in figure 2. For the subsequent panels all reservation prices are shifted by 1/32.

The quoted prices also depend on the tick size. The smaller the tick size, the more the quoted prices converge to the prices quoted under continuous pricing. As it is possible that some prices under discrete pricing are below the prices at

[13] For continuous prices ($d = 0$) this constraint is equal to (3).

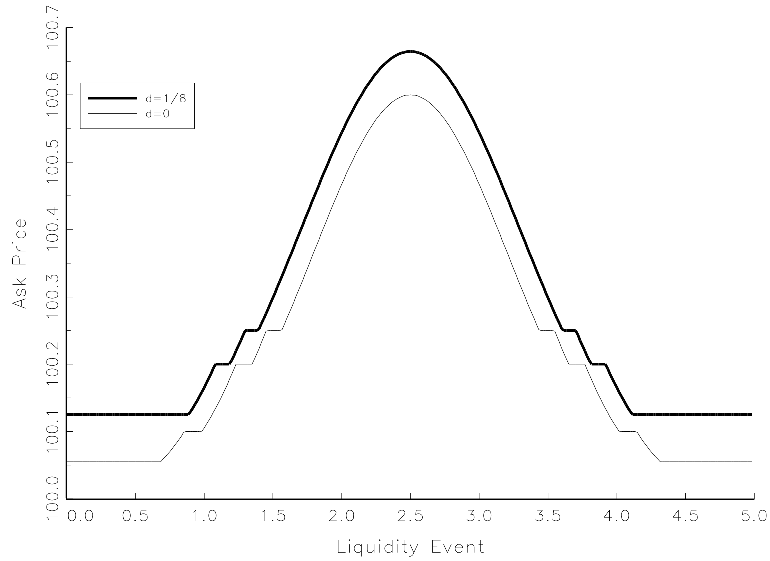

FIGURE 6. Comparison of sustainable prices with discrete and continuous pricing

continuous pricing, a smaller tick size does not necessarily mean lower prices, it can also happen that prices increase by reducing the tick size.

Figure 8 shows the quoted prices for tick sizes of 1/16, 1/8, 1/4 and 1/2. For comparison the thin line shows the prices quoted with continuous prices.

6. Empirical Implications

P. K. Dutta, and *A. Madhavan* (1997) predicted a decreasing spread for times of large trading volume. To overcome difficulties with other factors affecting the spread, they suggested to form a control sample with similar characteristics to isolate these effects. A major problem is that in times of large trade volume the adverse selection costs are likely to increase, this increases the spread.[14] The decrease in the spread due to implicit collusion is overlapped by this increase.

Even if we know the effect adverse selection has on the spread, the negative relation between volume and spread for $L > L^c$ is less visible in the model presented here than predicted by the Dutta-Madhavan model, although in the

[14]See *M. O'Hara* (1995) for a more detailed analysis of the influence of adverse selection on the spread.

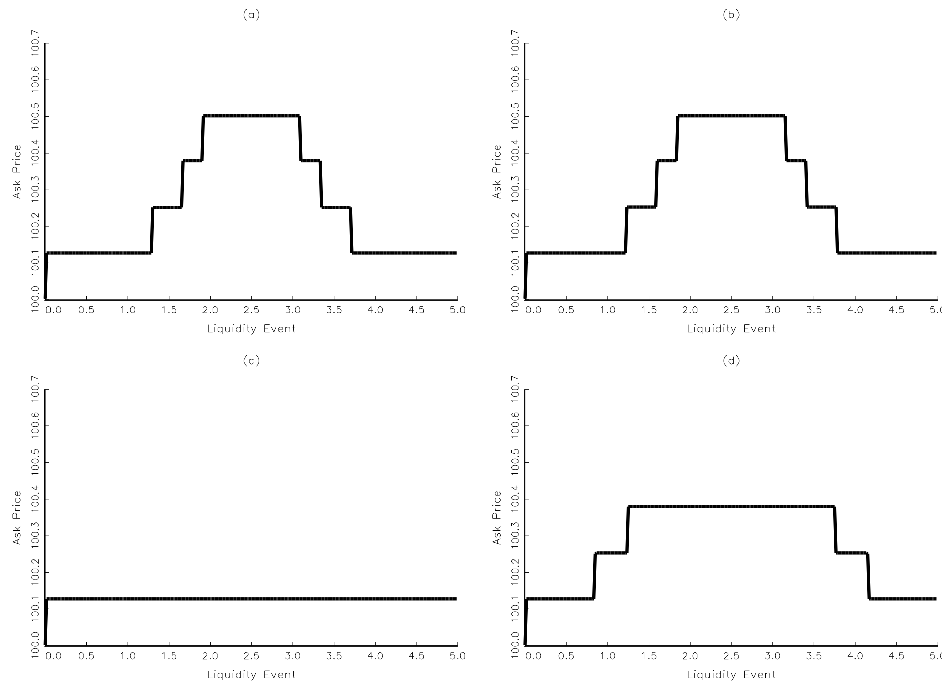

FIGURE 7. Quoted prices with a tick size of 1/8

case of discrete prices the negative relation between volume and spread can be more visible than with continuous prices as can be seen by comparing the quoted prices in figure 8.

But the reservation prices do not remain constant. They change after every trade. With changing prices the considerations of the market makers concerning their price quotes change. If we turn to figure 9 we see that if the reservation prices change, an increase in volume may result in a higher price.

We constructed figure 9 by using four different reservation price schemes, where all reservation prices are between 100 and 100.4.[15]

If we assume that the reservation prices schemes change only between the four possibilities shown in figure 9, assigning equal probability to every reservation price scheme, we can estimate the correlations between the volume (L) and the quoted prices for $L > L^c$. Table 1 compares the R^2 for our model and the

[15]The price schemes are obtained following no specific rule. The reservation prices are: (a) 100, 100.055, 100.1, 100.2, 100.25; (b) 100.05, 100.1, 100.2, 100.25, 100.3; (c) 100.075, 100.1, 100.25, 100.3, 100.4; (d) 100.1, 100.15, 100.2, 100.3, 100.4.

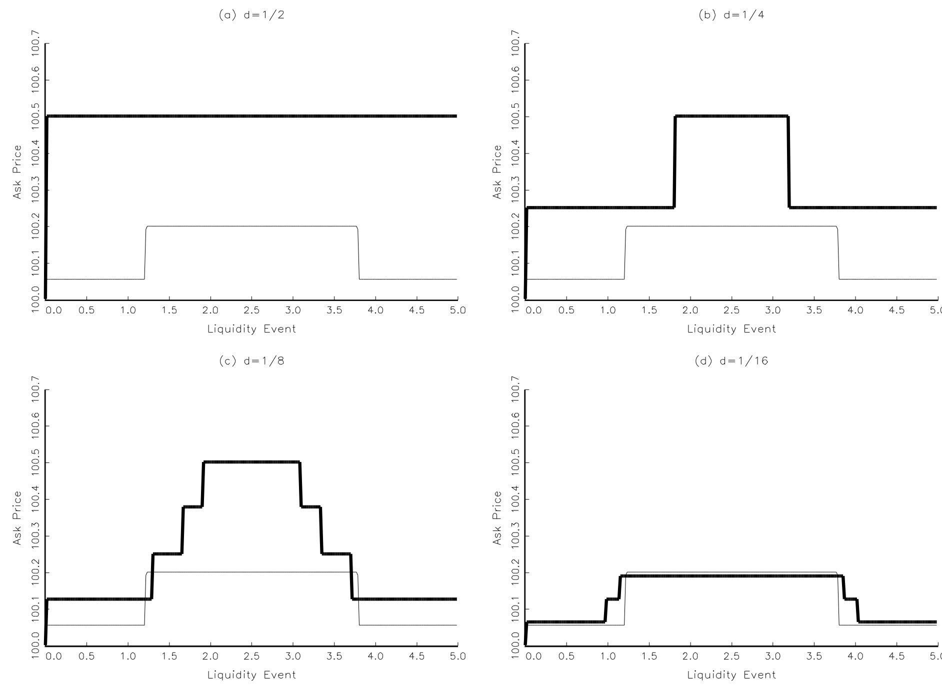

FIGURE 8. Quoted prices with different tick sizes

	equal costs	different costs
d=1/2	.00	.00
d=1/4	.66	.25
d=1/8	.82	.48
d=1/16	.90	.26
d=1/32	.92	.24
d=0	.89	.02

TABLE 1. R^2 for a regression of the quoted prices on L

Dutta-Madhavan model[16] for continuous and some discrete prices. The results are derived by a simulation of 50000 uniformely distributed random liquidity events $L > L^c$, each is randomly assigned to one of the four price schemes with equal probability. The R^2 is computed by an OLS regression of the quoted prices on the liquidity event.

[16]We assume the reservation prices for all market makers to equal (a) 100; (b) 100.05; (c)100.075; (d) 100.1.

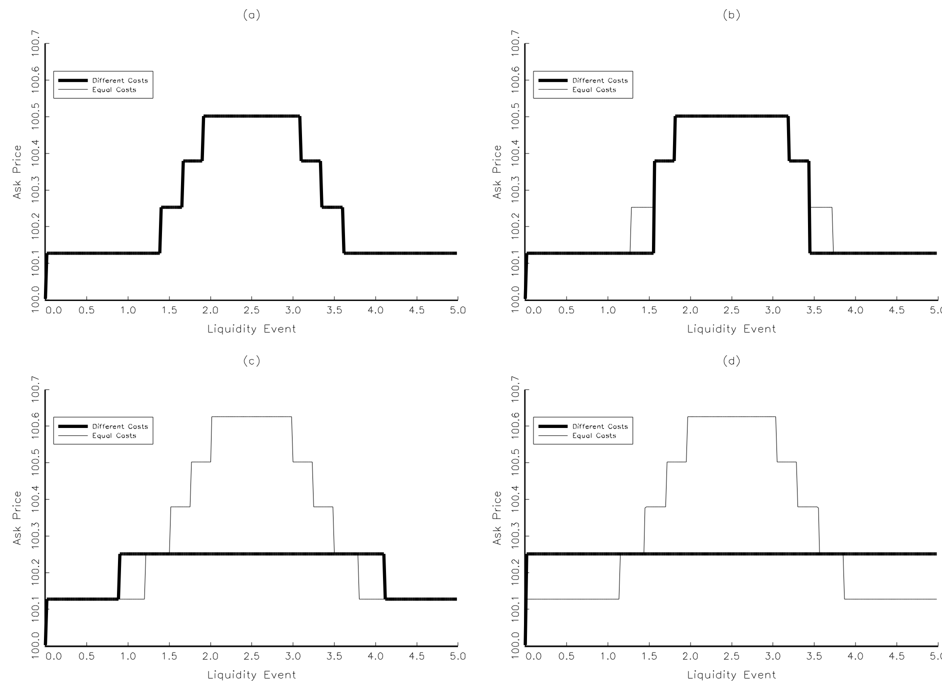

FIGURE 9. Quoted prices with different reservation price schemes

In the case of discrete prices the R^2 for our model is highest with a tick size of 1/8, as with this tick size the prices decrease with increasing volume quite well (see figure 8). Whereas for large tick sizes the price does nearly not change, the same is true for very small tick sizes, where the quoted prices are near continuous prices, which in our case do not vary much with L.

The correlation in the Dutta-Madhavan model with equal costs is increasing with a decreasing tick size.[17] The smaller the tick size the better the quoted prices are decreasing with increasing volume and fulfill the prediction of the model.

We used a linear regression to derive the results, but as is visible from figure 1, the price is not a linear function of L. But applying a non-linear regression would not change the results significantly as the non-linearities are not very pronounced for $L > L^c$.

[17]The decrease for the change from d=1/32 to d=0 is due to the fact that in our example the maximal performance can be found at 100.5 and does not depend on L, which reduces the R^2.

With a tick size of 1/8 the possible correlation of volume and quoted prices is much smaller than in the Dutta-Madhavan model. Even if in an empirical investigation the data would behave as in the model presented here, we only would find a R^2 of .48. For a tick size of 1/16, as now applied by NASDAQ, the evidence would even be poorer. When we take into account the problems of identifying the influence of adverse selection costs on the spread and other factors not included in this model, it will be very difficult to verify the existence of implicit collusion in an empirical investigation. The evidence from a regression of the quoted prices on volume will be poor.

It has to be noticed that the results are only derived from the analysis of particular situation as described above. A generalization of this finding to other situations cannot be made without further analysis. In a complete Monte-Carlo study we should therefore simulate the order flow and the assigned reservation prices. This will give us a more realistic picture of what we can expect to find in an empirical investigation.

Nevertheless this result shows the problems we might face in an empirical investigation associated with the introduction of different costs into the model of implicit collusion.

7. Policy Implications

The previous analysis suggested that the possibility of implicit collusion is extended, but on the other hand the introduction of different costs also implied that there exist incentives to quote not too high ask prices in order to exclude market makers with high costs. The effect of implicit collusion on prices is much less pronounced than in the model of Dutta-Madhavan with equal costs of market making. But also an increase in the spread of only a single tick increases trading costs significantly and measures should be taken to avoid such effects.

The problems associated with proving the existence of implicit collusion make it very difficult to avoid implicit collusion by regulation of the price setting behavior. A simple rule prohibiting implicit collusion promises not much success as any supervisory body will have great difficulties in proving the existence of implicit collusion. The permanent debate over this issue that would consequently arise with any actions taken by supervisory institutions, will probably harm the exchange more than it will benefit from lower trading costs.

Another option would be to impose restrictions on the spread size. Such a maximum spread has to be large enough to cover the costs even in extraordinary times, otherwise the market makers would close the market to avoid losses, a very undesirable effect. If market makers have to hold the market open, they will quote larger spreads even in ordinary times to compensate them for possible losses in exceptional times. The result would very likely be larger average spreads than those observed with implicit collusion.

It could also be considered to impose no fixed restriction on the size of the spread but to allow only for a "justifiable" spread depending on market conditions. The supervisory authority has to intervene if it finds spreads to be too large. Such a regulation would impose uncertainties on market makers whether their current spread can be justified. Problems to determine the costs of market makers will make the interpretation of such a rule very difficult. This approach faces the same problems as the prohibition of implicit collusion by permanent conflicts about implicit collusion.

Another way to reduce the effect of implicit collusion is to enhance competition. One possibility is to allow market makers more easily to enter. If large spreads allow market makers to make extraordinary profits they face the threat of new market makers entering. In the absence of entry barriers this potential competition will give incentives to quote prices more competitively.[18]

Another way to enhance competition is to allow limit orders competing directly with the quotes of market makers. This enlarges possible competitors for market makers to all investors in the market. In the aftermath of the Christie-Schultz debate this is the way NASDAQ has reacted to the threat of implicit collusion.

8. Conclusions

We developed a model of implicit collusion in dealer markets, where market makers face different costs. Compared to the Dutta-Madhavan model, which assumes equal costs for all market makers, we showed that the possibility of implicit collusion is extended. The prices sustainable under implicit collusion are higher or equal than those in the Dutta-Madhavan model. But the profit maximization of market makers prevents in many cases from quoting the highest possible prices, the quotes are in most cases lower than with equal costs.

To distinguish between explicit and implicit collusion by analyzing the behavior of prices for volumes above the critical value, as suggested by the Dutta-Madhavan model, was shown to be difficult. The poor evidence of a decrease

[18]In industrial economics this concept is known as *contestable markets*. This theory suggests that with free market entry prices will be quoted competitively. Of course, there cannot be complete free entry as obviously market makers have to fulfill certain requirements to ensure an orderly trading process. The approach of the NASDAQ to allow all registered market makers to choose the assets they want to make a market in freely and allowing them to change the assets without any meaningful restrictions is very close to the concept of contestable markets.

in the ask price (and an increase in the bid price) for volumes above the critical value will be problematic as we have to exclude other factors (e.g. adverse selection) from the observed data. The absence of a decreasing spread with high volumes in an empirical investigation may therefore not be an evidence for missing implicit collusion or a misspecification of the model.

To reduce the impact of implicit collusion on the quoted prices the best measure would be to enhance competition by allowing market makers to enter the market more freely and by allowing limit orders to compete directly with the quotes of market makers.

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