Looking Behind the U.K. Term Structure: Were there Peso Problems in Inflation?

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Abstract

This paper develops and estimates a general equilibrium model for the term structures of nominal and real interest rates that incorporates regime-switching into the dynamics of the state variables. The model generates time-varying risk premia via changes in the covariance structure of the state variables and Peso problems through regime-switching. When the model is estimated using real and nominal yields from the U.K., I find that Peso problems emanating from instability in inflation have a significant impact on the nominal term structure. Peso problems affect (i) the sample predictability of excess returns, (ii) nominal term premia, and (iii) the inflation risk premia linking real and nominal yields with expected inflation.

I. Introduction

The behavior of the term structure continues to puzzle researchers. After more than a decade of regression-based tests rejecting forms of the expectations hypothesis (EH), a consensus has yet to develop around an alternative model. One strand of the literature has focused on general equilibrium bond pricing models which generate time-varying risk premia. These models ascribe the rejections of the EH to time-varying risk premia but generally have not been very successful empirically. Another strand of the literature considers statistical problems with the regression-based tests of the EH. Here the focus has been on the inference problems caused by small samples. In particular, Evans and Lewis (1994) and Bekaert, Hodrick and Marshall (1997) examine how changes in the time-series behavior of interest rates during the sample could affect the sample properties of standard tests. Such changes have been widely observed in the U.S. and often appear closely linked to changes in the monetary policy regime. When their presence is characterized by regime-switching models, the evidence against the EH is considerably weakened, but it is not entirely eliminated.

In this paper, I attempt to synthesize both strands of the literature. General equilibrium and regime-switching models represent alternative rather than competing views of the term structure. If anything, recent simulation results in Bekaert, Hodrick and Marshall (1997) suggest that both regime-switching and time-varying risk premia may have a role to play in explaining the behavior of interest rates. My aim here is to empirically investigate this possibility.

The paper examines a general equilibrium model with regime-switching that aims to explain the behavior of the nominal and real term structures in the United Kingdom. My focus on the U.K. data is motivated by two main considerations. First, there have been a series of widely documented changes in U.K. monetary policy over the past two decades. For example, the U.K.'s departure from the EMS in 1992 represented a significant change in policy regime. Thus, there is a premia facie case that regime-switching may be present in the U.K. data. Second, there has been a well-established market for both conventional and index-linked debt in the U.K. for the past fifteen years. In Evans (1998), I showed how prices from this market can be used to construct nominal and real yields curves. These data allows us to distinguish between the real and nominal factors affecting the nominal term structure with a good deal of precision. And, as a result, we can focus on whether instability in the U.K. inflation process significantly affected the term structure.

The model I develop has its antecedents in the model of Cox Ingersoll and Ross (1985) (CIR) and is related to the Affine Class of general equilibrium models that been recently used by Backus, Foresi, Mozummdar and Wu (1997), Fisher and Gilles (1996), and Roberds and Whiteman (1996) to study the U.S. term structure.[1] These models all generate time-varying risk premia from changes in the covariance structure of shocks to the state variables that follow stable time series processes. The key difference in my

[1] Campbell, Lo and MacKinlay (1997) summarize earlier emprical research based on Affine models of the term structure.

model is that I allow the state variables to follow switching processes. This innovation formalizes the idea that the term structure is affected by instability in state variables generally, and inflation in particular. In this respect, the model builds on Hamilton (1988), Sola and Driffie (1992) and Naik and Lee (1994) who examine term structure models where short rates follow switching processes.

The introduction of regime-switching complicates the model considerably. Technically speaking, it now falls outside the class of Affine models where analytically solutions for equilibrium bond prices are readily calculated. In principle it is possible to solve for equilibrium bond prices numerically. However, in practice, the computation burden is too large for this approach to be used as part of an estimation procedure. I therefore approximate the equilibrium solution when estimating the model and demonstrate that the approximation error so introduced is very small. Beyond tractability, this approach has the benefit of allowing us to study the impact of switching in some detail.

In common with other regime-switching models, this model generates Peso problems; situations where the potential for discrete shifts in the distribution of key decision variables affects the rational expectations held by investors. It is widely recognized that the presence of Peso problems can distort econometric inferences in small samples.[2] One type of distortion occurs in the behavior of forecast errors. When a Peso problem is present, the forecast errors made by investors will generally appear biased and serially correlated within the sample. While this contradicts the predictions of standard rational expectations models, it is perfectly consistent with rational investor behavior in samples where the distribution of regime switches differs from the underlying distribution used by investors. To date, most research on Peso problems has focussed on this distortion. In particular, Evans and Lewis (1994) and Bekaert, Hodrick and Marshall (1997) consider how regression tests of the EH are affected by the sampling properties of the forecast errors. Here I use the model estimates to calculate how these distortions affect the small sample predictability of excess returns.

Peso problems also distort small sample inferences about risk. Rational investors will take account of possible future regime switches when evaluating risk through their calculation of (conditional) second moments. Estimates of these moments derived from realized values in a small samples will generally be biased. As a consequence, the risk premia consistent with rational investors' view of future regime switches, can be very different than the small sample estimates. This type of distortion has not been the focus of Peso problem research to date. Here we can use the model to calculate the contribution of regime-switching to the estimated risk premia.

The importance of these small sample effects depends upon the degree of instability in the state variables and the extent to which rational investors account for switches when forecasting. Evaluating this empirically is difficult. Clearly, it is always possible to construct a switching model which perfectly explains the observed data if we are willing to posit that investors were anticipating a switch to a (appropriately configured) regime

[2] For a recent survey of the literature of Peso problems, see Evans (1995).

that never took place during the sample. Such pathological examples of Peso problems are observationally equivalent to irrational expectations and will not be considered in the analysis below. Rather I shall confine my attention to cases where all the possible regimes are observed. This means that the small sample effects I identify only arise because there is a difference between the empirical distribution of observed regimes and underlying distribution used by investors to forecast. In this sense, my analysis down-plays the role of Peso effects in the term structure.

Another key feature of the model is that it is estimated using both time-series and cross-section data. In particular, I utilize monthly yields on four real and four nominal bonds. If Peso effects are indeed present, one should be able to obtain much more precise estimates of the parameters governing regime-switching from successive term structures than from the dynamics of a single variable alone. Intuitively, each term structure contains precise information about investors' perceptions of future regime switches. The theoretical model "decodes" this information and makes sure it is consistent with the dynamics of the state variables that drive the observed yields through time.

I begin the empirical analysis by considering the affects of regime-switching on the behavior of the real term structure. For this purpose, I compare estimates of a one factor CIR model with and without regime-switching. Although there are statistically significant differences across regimes in the state variable process, the predictions both models make for the behavior of yields are highly correlated with each other. Moreover, both models explain a very high fraction of both the time series and cross-sectional behavior of real yields. These findings indicate that we can accurately describe the behavior of the U.K.'s real term structure without resort to regime-switching.

Based on these results, I next consider models for the nominal and real term structures where regime-switching is confined to the behavior of inflation and only affects nominal yields. Comparing estimates with and without regime-switching, I find that the switching model is better able to explain both the average level and volatility of the nominal yield curve. Although the model does not predict the movements in nominal yields as accurately as it does real yields, I argue that this could be due to a small sample problem. The model can accurately match the nominal term structure if we allow for very small changes in the transition probabilities governing regime switches over the sample period. Although such changes are difficult to identify directly from the time series behavior of yields, they are plausible given the significant economic events that took place over the sample.

I then use to the switching model estimates to quantify the impact of Peso problems. The results from Monte Carlo experiments show that there is a high probability that Peso problems will significantly contributing to the predictability of excess returns in a single sample. Estimates of the inflation risk premia are very volatile and, to a large degree, reflect the presence of Peso problems. Taken together, these findings support the idea that Peso problems originating from instability in the inflation process have significantly contributed to the behavior of the U.K. term structure.

The paper is organized as follows: I begin in Section 2 with a description of the U.K. data. Here I document how both the real and nominal term structures differ from the (large sample) predictions of the EH. Section 3 develops the model. The estimates are presented in Section 4, and the impact of Peso problems are analyzed in Section 5. Section 6 concludes and outlines how the results may apply to the behavior of the U.S. term structure.

II. The Real and Nominal Term Structures

This section describes the data and presents some statistics on the behavior of the U.K. term structures. These statistics highlight the similarities between the U.K. and U.S., data. They also provide information that is useful in formulating the theoretical model.

A. Definitions and Notation

Let $Q^n_{t,k}$ denote the nominal price of a zero coupon bond at period t paying £1 at period $t+k$. (Time periods are assumed to be discrete.) The continuously compounded yield on a bond of maturity k, is

$$y^n_{t,k} \equiv -\frac{1}{k} \ln Q^n_{t,k}. \qquad (1)$$

I shall also be interested in the behavior of the holding returns. The log one-period holding return on an k-period bond realized at $t+1$ is

$$h^n_{t+1,k} \equiv \ln Q^n_{t+1,h-1} - \ln Q^n_{t,k}. \qquad (2)$$

Similar relationships exist between the prices, yields and holding returns on real bonds. Let $Q^r_{t,k}$ denote the nominal price of a zero coupon bond at time t paying £(P_{t+k}/P_t) at period $t+k$, where P_t is the (known) price level at t. $Q^r_{t,k}$ also defines the real price of a claim to one unit of consumption at $t+k$. The k-period real yield is

$$y^r_{t,k} \equiv -\frac{1}{k} \ln Q^r_{t,k}, \qquad (3)$$

while the one-period holding return is defined by

$$h^r_{t+1,k} \equiv \ln Q^r_{t+1,k-1} - \ln Q^r_{t,k}. \qquad (4)$$

The analysis below will examine the behavior of expected excess returns. Of particular interest will be the expected excess return on an h-period bond relative to the one period rate:

$$\mu^j_{t,k} \equiv E[h^j_{t+1,k}|F_t] - y^j_t \qquad j = \{n, r\}; \tag{5}$$

where F_t represents investors information at period t, and $y^j_t \equiv y^j_{t,1}$ is the one period or short rate. For convenience, I shall refer to $\mu^j_{t,k}$ as the term premia on nominal ($j = n$) or real bonds ($j = r$). I shall also examine the behavior of the inflation risk premia which is defined as

$$\varphi_t \equiv E[y^n_t - \Delta p_{t+1}|F_t] - y^r_t. \tag{6}$$

This is the expected excess real return on nominal bonds relative to the real short rate. Using these definitions, we can describe all the risk premia in both the nominal and real term structures.

B. Data

The analysis in this paper uses data on nominal and real yield curves derived from the secondary market prices of nominal and indexed-linked bonds that trade in the U.K. The nominal yields come from The Bank of England and are constructed using the method described in Deacon and Derry (1994). Briefly, a no-arbitrage condition is use to link the prices of discount bonds, $Q_{t,h}$, to the prices of coupon-paying bonds seen in the market. Then, at each date, the parameters of a discount function are chosen to match observed prices against their theoretical values implied by the no- arbitrage condition. The yield curve for each period are then constructed from the estimated discount function.

The construction of the real yield curve is complicated by two factors. First, indexed-link bonds issued by the U.K. government only provide incomplete indexation for the principle and coupon payments because there is an eight month indexation lag built into the payoff structure of the bonds. Second, there is a two week reporting lag in the price index. As a result, uncertainty about the current and future prices has some effect on the prices of index-linked bonds. Both these facts make it impossible to derive the real term structure directly from the observed prices of index-linked bonds. However, in Evans (1998) I show how real yields can be constructed using a two-step procedure. First, the index-linked yield curve is calculated from market prices using a no-arbitrage technique like the one used to find the nominal term structure. Second, the effects of inflation uncertainty (arising from the indexation and reporting lags) are purged from the index linked yields to derive estimates of the real yield curve. The analysis below utilizes these estimates.

Table 1 reports summary statistics on the estimated log yields for nominal and real bonds estimated from the U.K. term structures on the last business day of the month from January 1983 until November 1995. The yields are expressed in annual per cent calculated as $y^r_k \equiv -\frac{1200}{k}\ln Q^r_k$ and $y^n_k \equiv -\frac{1200}{k}\ln Q^n_k$

Table 1: Summary Statistics							
k: months Nominal yields: y^n_k	mean	std.	skewness	kurtosis	ρ_1	ρ_2	ρ_3
12	9.468	2.292	-0.265	2.479	0.958	0.912	0.874
24	9.464	1.923	-0.476	2.657	0.951	0.893	0.845
36	9.548	1.742	-0.541	2.712	0.945	0.877	0.821
48	9.626	1.636	-0.525	2.687	0.941	0.869	0.809
60	9.680	1.559	-0.481	2.637	0.939	0.867	0.805
120	9.665	1.252	-0.314	2.645	0.929	0.864	0.799
Real yields: y^r_k	mean	std.	skewness	kurtosis	ρ_1	ρ_2	ρ_3
12	5.031	2.992	1.242	4.859	0.491	0.443	0.441
24	4.426	1.465	0.926	4.441	0.547	0.480	0.455
36	4.246	0.996	0.583	3.975	0.610	0.519	0.472
60	4.122	0.660	0.068	3.436	0.718	0.589	0.510
84	4.067	0.538	-0.175	3.287	0.791	0.646	0.552
120	4.009	0.465	-0.287	3.191	0.856	0.713	0.615
Monthly Inflation: Δp	mean	std.	skewness	kurtosis	ρ_1	ρ_2	ρ_3
	4.675	5.863	1.486	8.513	0.210	0.072	-0.230

Notes:
The yields are calculated as $y^n_{t;h} \equiv -\frac{1200}{h} \ln Q^n_{t;h}$ and $y_{t;h} \equiv -\frac{1200}{h} \ln Q_{t;h}$: Δp is the monthly difference in the log of the Retail Price Index. The asymptotic standard errors for the skewness and kurtosis statistics are 0.197, and 0.395.

where k is measured in months. The upper panel of the table shows that the nominal yield curve was on average mildly upward sloping while the real yield curve was downward sloping. Short-term yields are much more volatile than long-term yields in both term structures but volatility falls more quickly along the real term structure. From the skewness and kurtosis statistics, the unconditional distributions for both sets of yields appear non-normal. The autocorrelations, ρ_i; reported in right hand columns of the table indicate that movements in nominal yields are highly persistent. The persistence of real yields are generally lower, particularly at the shorter maturities.

The lower panel reports statistics for monthly inflation (measured by the retail price index). At this high frequency inflation is extremely volatile and not very persistent. These statistics imply that forecasts of inflation over a year or more derived from univariate (ARMA) models would not be very variable. If changes in long-term nominal yields primarily reflect changing inflation expectations, investors must be using information beyond the history of actual inflation to forecast. Under these circumstances, rational investors' inflation forecasts derived from the term structure should be superior to those derived form univariate time series models.

C. Preliminary Analysis

The model developed below is based on the idea that both regime-switching and time-varying risk premia affect the behavior of the U.K. term structure. Before turning to the model, it is useful to review some evidence supporting this premise.

Consider the relation between long and short term yields. From the definition of the term premium we have the identity $\theta^j_{t,k} \equiv k y^j_{t,k} - (k-1)E[y^j_{t+1,k-1}|F_t] - y^j_t$. Iterating this expression forward gives

$$y^j_{t,k} = \frac{1}{k}\sum_{i=0}^{k-1} E[y^j_{t+i}|F_t] + \frac{1}{k}\sum_{i=1}^{k} \theta^j_{t+i,k-i} \qquad j = \{r, n\}. \tag{7}$$

According to this equation, yields move either because investors revise their forecasts of future short rates or because the term premia change. Because (7) follows directly from the definition of the term premia, it can be used to empirically decompose the volatility of yields without reference to a specific theoretical model of the term structure. Below I compare estimated decompositions against the predictions of one version of the EH, the Log Expectations Hypothesis. Under LEH $\theta^j_{t,k}$ are assumed constant so all the movements in yields are attributed to changing short rate forecasts.

The relation between nominal and real yields depends on both the term and inflation risk premia. From the definition of the inflation risk premia in (5) we have

$$y^n_t = y^r_t + E[\Delta p_{t+1}|F_t] + \psi_t; \tag{8}$$

which is an augmented version of the Fisher Equation. While it is theoretically possible for ψ_t to equal zero, as the standard Fisher Equation implicitly assumes, in general the inflation risk premium will differ from zero and may be time-varying. Combining (7) and (8) we obtain the following equation for the yield spread

$$\begin{aligned} y^n_{t,k} - y^r_{t,k} &= \frac{1}{k}\sum_{i=1}^{k} E[\Delta p_{t+i}|F_t] \\ &\quad + \frac{1}{k}\sum_{i=1}^{k}\{\theta^n_{t+i,k-i} - \theta^r_{t+i,k-i}\} + \frac{1}{k}\sum_{i=0}^{k-1} E[\psi_{t+i}|F_t]. \end{aligned} \tag{9}$$

Here we see that volatility in the yield spread can generally be attributed to three factors: (i) changing forecasts of future inflation, (ii) variations in the real and nominal term premia and (iii) movements in the expected inflation risk premia. Once again, notice that (9) follows from the definitions of the risk premia and so can be used to empirically decompose the volatility of the spread without reference to a specific term structure model. Clearly, if both the LEH and the standard Fisher Equation hold, changing forecasts of inflation will be the only factor contributing to the spread's volatility.

Table 2 reports decompositions of the yields and spread based on forecasts of future short rates and inflation derive from a Vector Autoregression (VAR). These are based on the following ratios:

$$\mathcal{R}_1 \equiv \frac{Cov\left(\frac{1}{k}\sum_{i=0}^{k-1} E[y^n_{t+i}|\Phi_t], y^n_{t,k}\right)}{Var(y^n_{t,k})} \qquad \mathcal{R}_2 \equiv \frac{Cov\left(\frac{1}{k}\sum_{i=0}^{k-1} E[y^r_{t+i}|\Phi_t], y^n_{t,k}\right)}{Var(y^n_{t,k})}$$

$$\mathcal{R}_3 \equiv \frac{Cov\left(\frac{1}{k}\sum_{i=1}^{k} E[\Delta p_{t+i}|\Phi_t], y^n_{t,k}\right)}{Var(y^n_{t,k})} \qquad \mathcal{R}_4 \equiv \frac{Cov\left(\frac{1}{k}\sum_{i=0}^{k-1} E[y^r_{t+i}|\Phi_t], y^r_{t,k}\right)}{Var(y^r_{t,k})}$$

$$\mathcal{R}_5 \equiv \frac{Cov\left(\frac{1}{k}\sum_{i=1}^{k} E[\Delta p_{t+i}|\Phi_t], y^n_{t,k} - y^r_{t,k}\right)}{Var(y^n_{t,k} - y^r_{t,k})}$$

where $\Phi_t \subset F_t$ is the information set comprising current and lagged values of the variables in the VAR. In each case, the present value term is calculated from estimates of a third-order VAR that contains the long and short-term yields on nominal and real bonds as well as monthly inflation. The reported ratios are the slope coefficients from the regression of the estimated present value on the yield or spread. The table also reports standard errors corrected for conditional heteroskedasticity.

The left hand column shows estimates of the ratio $\mathcal{R}_1$ which measures the contribution of changing short rate forecasts to the volatility of nominal yields.[3] These ratios fall between 0.63 and 0.34, well below the value of unity implied by equation (7) under LEH. Thus variations in expected excess holding returns appear to contribute significantly of the variability of nominal yields. The ratios $\mathcal{R}_2$ and $\mathcal{R}_3$ allow us to decompose the variability of nominal yields further. The estimates of $\mathcal{R}_2$ measure the contribution of changing real rate forecasts. These estimates contribute about 30% to the volatility of nominal yields. The contribution of inflation forecasts, measured by the estimates of $\mathcal{R}_3$, is higher, at about 50%. These results are consistent with the findings reported by Barr and Pesaran (1995) in their analysis of U.K. nominal yields. They are also broadly similar to the results in Campbell and Ammer (1993) based on U.S. data.

The estimates of the ratio $\mathcal{R}_4$ measure the contribution of changing real rate forecasts to the volatility of real yields. These ratios are much higher than their nominal counterparts. For maturities of two to four years, the ratios are close to unity, the value implied by the LEH. The estimates of $\mathcal{R}_5$ in the right hand column measure the contribution of changing inflation forecasts to the volatility of the yield spread. Again the estimates are well below unity, the value implied by LEH and Fisher Equation [see (9)]. Changing inflation expectations contribute at most between 50% to 60% to the volatility of the spread. Both of these

[3] To see this formally, take expectations on both sides of (7) conditional on Φ_t, and multiply the result by $y^j_{t,k}$. Taking expectations once again gives a decomposition of $Var(y^n_t)$ as the sum of $Cov\left(\frac{1}{k}\sum_{i=0}^{k-1} E[y^n_{t+i}|\Phi_t], y^n_{t,k}\right)$ and $Cov\left(\frac{1}{k}\sum_{i=1}^{k} E[\theta^n_{t+i,k-i}|\Phi_t], y^n_{t,k}\right)$. Similar calculations applied to real yields and the spread lead to the ratios $\mathcal{R}_4$ and $\mathcal{R}_5$. The ratios $\mathcal{R}_2$ and $\mathcal{R}_3$ are derived by applying the same procedure to (7) combined with (8).

Table 2: Yield Volatility Decompositions					
$R_z = Cov(\frac{1}{h}\sum_{i=0}^{h-1} E[z_{t+i}\|\Omega_t]; x_t)/Var(x_t)$					
x_t	Nominal Yield: $y^n_{t;k}$			Real Yield: $y^r_{t;k}$	Yield Spread: $y^n_{t;k} - y^r_{t;k}$
z_t	y^n_t	y^r_t	Δp_{t+1}	y^r_t	Δp_{t+1}
k	R_1	R_2	R_3	R_4	R_5
24	0.586	0.311	0.559	(0.991)	0.510
	(0.010)	(0.040)	(0.022)	(0.021)	(0.055)
36	0.631	0.313	0.537	0.985	0.623
	(0.019)	(0.033)	(0.027)	(0.048)	(0.050)
48	0.610	0.302	0.488	0.873	0.590
	(0.023)	(0.029)	(0.028)	(0.070)	(0.041)
60	0.565	0.290	0.433	0.727	0.510
	(0.024)	(0.026)	(0.027)	(0.082)	(0.035)
120	0.340	0.245	0.207	0.140	0.204
	(0.020)	(0.018)	(0.019)	(0.078)	(0.020)

Notes:
Expectations are calculated from a third order VAR for that contains the long and short-term yields on nominal and real bonds as well as monthly inflation. Each ratio is computed as the slope coefficient from the regression of the expected present value on the yield/yield spread. Asymptotic standard errors corrected for conational heteroskedasticity are reported in parenthesis.

results are consistent with the regression tests in Evans (1998).

One way to interpret of these findings is to attribute the results to the presence of time-varying term premia and inflation risk premia. According to this view, the VAR forecasts are unbiased estimates of investors' forecasts so the low values of R_1 and R_4 reflect the fact that both nominal and real yields move in response to changes in the term premia. Similarly, the low values for R_5 reflect the presence of time-varying inflation risk premia. Thus, according to this view, the results in Table 2 constitute evidence against both the LEH and the Fisher equation. Alternatively, the VAR forecasts may provide rather poor estimates of investors' forecasts. According to this interpretation, if rational investors' anticipated switches in the behavior of short rates and/or inflation, Peso problems will induce small sample bias in their forecasts. VAR forecasts cannot exhibit this property by their very construction. Thus, the low ratios reported in Table 2 reflect insufficient sample variability in the VAR forecasts due to Peso problems.

The analysis in this paper takes both of these interpretations seriously. My aim is to develop a model that will allow us to examine how Peso problems and time-varying risk premia contribute to the behavior of the U.K. term structure.

III. Bond Pricing

This section presents the model used to analyze the U.K. term structures. The model gives rise to variations in expected excess holding returns on both nominal and real bonds as well as changes in the expected excess real return on nominal bonds. It also allows for discrete changes in the time series behavior of inflation and the covariance structure between inflation and real rates. As a result, equilibrium yields are affected by the presence of Peso problems through both their impact on expectations and risk premia. First, I describe the equilibrium pricing equations that lie at the heart of the model. I then present a Baseline specification that generates time-varying risk premia without regime-switching. The model with regime-switching and time-varying risk premia is developed from this specification.

Let $\mathcal{M}_{t+1}$ be a random variable that prices one-period state-contingent claims. If the economy admits no pure arbitrage opportunities, it can be shown that the one-period real returns on all traded assets, i, must satisfy

$$E_t[\mathcal{M}_{t+1} R^i_{t+1} | \mathcal{F}_t] = 1; \tag{10}$$

where R^i_{t+1} is the gross real return on asset i between t and t+1. I shall refer to $\mathcal{M}_t$ as the real pricing kernel. In economies where there is a complete set of markets for state-contingent claims, there is a unique random variable $\mathcal{M}_t > 0$ satisfying (10). Under other circumstances, this no-arbitrage condition still holds but for a range of $\mathcal{M}_t$s [see Duffie (1992)]. In economies with a representative agent, $\mathcal{M}_{t+1}$ is the intertemporal marginal rate of substitution so that (10) also represents a first-order condition.

We can use (10) to find equations that price both real and nominal bonds. In the case of a nominal bond with k periods to maturity, the one period real return is $(Q^n_{t+1,k-1}/Q^n_{t,k})(P_t/P_{t+1})$. Substituting this for R^i_{t+1} in (10) and rearranging, gives (for k > 0),

$$Q^n_{t,k} = E\left[\frac{\mathcal{M}_{t+1} P_t}{P_{t+1}} Q^n_{t+1,k-1} \middle| \mathcal{F}_t\right]. \tag{11}$$

For the case of real bonds, recall that $Q^r_{t,k}$ is the nominal price of a claim at t to £(P_{t+k}/P_t) paid at t+k. Consider the real return from holding this k-period claim for one period. In t+1 the nominal price of a claim to £(P_{t+k}/P_{t+1}) is $Q^r_{t+1,k-1}$ so the price of a claim to £(P_{t+k}/P_t) must be $Q^r_{t+1,k-1}(P_{t+1}/P_t)$. The real return on holding the k-period claim is therefore $Q^r_{t+1,k-1}/Q^r_{t,k}$. Substituting this for $R_{t+1,i}$ in (10) gives (for k > 0);

$$Q^r_{t,k} = E\left[\mathcal{M}_{t+1} Q^r_{t+1,k-1} \middle| \mathcal{F}_t\right]. \tag{12}$$

Equations (11) and (12) determine the complete set of real and nominal bond prices in the economy in terms of the dynamics of the pricing kernel, M_t, and aggregate price level, P_t. Since $Q^r_{t,0}$ and $Q^n_{t,0}$ must equal unity, we can use (11) and (12) to solve recursively for bonds prices given these dynamics. To facilitate these calculations, I will work with the log linearized versions of (11) and (12):

$$q^n_{t,k} = E\left[m_{t+1} - \Delta p_{t+1} + q^n_{t+1,k-1} | F_t\right] + \frac{1}{2} Var\left(m_{t+1} - \Delta p_{t+1} + q^n_{t+1,k-1} | F_t\right); \quad (13)$$

$$q^r_{t,k} = E\left[m_{t+1} + q^r_{t+1,k-1} | F_t\right] + \frac{1}{2} Var\left(m_{t+1} + q^r_{t+1,k-1} | F_t\right); \quad (14)$$

where lowercase letters denote the logs of the corresponding uppercase letters. These equations hold exactly if the joint conditional distribution of m_{t+1}, and Δp_{t+1} are normal. This will be the case in the Baseline model presented below. When the model is extended to allow for regime-switching, (13) and (14) contain approximation errors that will need to be quantified.

A. The Baseline Model

The Baseline model does not allow for regime switching and falls into the class of Duffie and Kan's (1996) Affine term structure models. These models have been used extensively to study the behavior of the nominal term structure [see, for example, Backus, Foresi, Mozumdar and Wu (1997), Fisher and Gilles (1996), and Roberds and Whiteman (1996)]. The model I develop focuses on the joint behavior of inflation and bond yields. In this respect it builds on Pearson and Sun (1991), Pennacchi (1991), Sun (1992), Foresi, Penati and Pennacchi (1996) and Gong and Remolona (1996) who model the joint behavior of inflation and nominal yields. To take full advantage of the U.K. data, like Remolona, Wickens and Gong (1996) the model developed below focuses on the behavior of both nominal and real yields and their interaction with inflation.

The Baseline model contains two state variables; μ_t that governs the expected path of the real pricing kernel, and π_t that governs the dynamics of expected inflation. The Baseline model assumes that the state variables follow

$$\mu_{t+1} = \mu + \alpha_m \mu_t + \sigma_u \mu_t^{1/2} u_{t+1}; \quad (15)$$

$$\pi_{t+1} = \pi + \alpha_\pi \pi_t + \alpha_{\pi m} \mu_t + \rho\sigma_u \mu_t^{1/2} u_{t+1} + \sigma_{\pi\pi} \pi_t^{1/2} v_{t+1}; \quad (16)$$

where u_t and v_t are $i.i.d.N(0,1)$ shocks and $\sigma_\mu, \sigma_{\pi\pi} > 0$. Equations (15) and (16) form a recursive system. The state variable μ_t affects the conditional mean and variance of π_t but not vise versa. With this structure, μ_t turns out to be the only state variable governing the real term structure and so can be viewed as summarizing information in real yields. Notice too that (15) and (16) include a time-varying covariance structure. This feature introduces time-varying risk premia into the model.[4]

Bond prices are governed by the joint behavior of the log real pricing kernel, m_t; and inflation, Δp_t. These variables are related to the state variables by

$$-m_{t+1} = \kappa_m + \mu_t + \lambda_m \sigma_u \mu_t^{1/2} u_{t+1} \tag{17}$$

$$\Delta p_{t+1} = \kappa_p + \pi_t + \lambda_p \rho \sigma_u \mu_t^{1/2} u_{t+1} + \lambda_p \sigma_\pi \pi_t^{1/2} v_{t+1}. \tag{18}$$

The parameters λ_m and λ_p determine the extent to which innovations in the state variables affect the real pricing kernel and inflation within the period. As we shall see, these parameters determine the equilibrium market price of risk and play an important role in the determination of the term premia in each term structure. Notice too that the covariance between the real pricing kernel and inflation varies with the state variable μ_t. This time-varying covariance gives rise to changes in the inflation risk premium.

The dynamics of the real pricing kernel in (15) and (17) represent a discrete time version of the one factor CIR model. Thus the Baseline model assumes that the dynamics of the real term structure can be captured by a single factor. The dynamics of the nominal term structure are determined by two factors. Notice that the second factor, π_t, is not independent of the first unless $\alpha_{\pi m} = \rho = 0$. Thus, the model does not make the strong money neutrality assumption used in the models of Pearson and Sun (1991), Gong and Remolona (1996) and Remolona, Wickens and Gong (1996). This is important feature of the model because it generates time-varying inflation risk premia

To solve for the equilibrium real and nominal bond prices, first write (15) - (18) in vector form as

$$x_{t+1} = \kappa + z_t + \Lambda e_{t+1}$$
$$z_{t+1} = z + \alpha z_t + e_{t+1}$$

where $x_t' \equiv [-m_t, \Delta p_t]$; $z_t' \equiv [\mu_t, \pi_t]$; $e_t' \equiv [u_t, v_t]$ and $E\left[e_{t+1} e_{t+1}' | F_t\right] = \Omega(z_t)$. With this structure, real and nominal bond prices satisfy

$$-q_{t,k}^j = A_k^j + B_k^j z_t \qquad j = \{n, r\} \qquad k = 0, 1, \ldots \tag{19}$$

for some parameters A_k^j and vectors B_k^j. These parameters are derived from the pricing equations (13) and

[4] One theoretical drawback of the discrete time framework adopted here is that it is always possible to obtain negative realizations of μ_t and π_t that are inconsistent with the square root terms multiplying the shocks. I insure that the sample paths of u_t and π_t remain in the positive orthant when estimating the models.

(14). Both sets of the parameters follow the general recursions

$$A^j_k = A^j_{k-1} + d^j\kappa + B^j_{k-1} z;$$

$$B^j_k = d^j + B^j_{k-1}\alpha - \frac{1}{2}\left[d^j \ast + B^j_{k-1}\right]\Omega\left[d^j \ast + B^j_{k-1}\right]'; \qquad j = \{n, r\};\ k = 1, 2 \ldots$$

where $\mathrm{vec}[\Omega(z_t)] = -z_t$; $d^r = [1, 0]$ and $d^n = [1, 1]$ with $A^j_0 = 0$ and $B^j_0 = [0, 0]$.

The Baseline model generates time varying risk premia through the mechanism found in other Affine models. Movements in the state variables, z_t, alter the covariance structure between equilibrium bond prices and the relevant pricing kernel. These changes, in turn, affect the relative riskiness of investing in long rather than short term bonds, or nominal rather than real bonds, and so equilibrium expected excess returns have to adjust to compensate. This can be seen more clearly if we combine the premia definitions in (4) and (5) with the pricing equations in (13) and (14) to give

$$\theta^j_{t,k} = -\frac{1}{2}\mathrm{Var}\left(q^j_{t+1,k-1}\,\middle|\,\mathcal{F}_t\right) + \mathrm{Cov}\left(q^j_{t+1,k-1}, d^j x_{t+1}\,\middle|\,\mathcal{F}_t\right) \qquad j = \{r, n\} \tag{20}$$

$$\psi_t = -\frac{1}{2}\mathrm{Var}\left(\Delta p_{t+1}\,\middle|\,\mathcal{F}_t\right) + \mathrm{Cov}\left(\Delta p_{t+1}, m_{t+1} \mid \mathcal{F}_t\right). \tag{21}$$

Using (15) - (19) is straightforward to verify that all the variance and covariance terms above vary with at least one of the state variables (μ_t and π_t). Section 5 provides a detailed examination of these effects. The point to emphasize here is that the Baseline model allows for quite general variation in risk premia throughout the nominal and real term structures. In principle, therefore, it is capable of explaining the results in Table 2. The Baseline model also provides us with a benchmark general equilibrium model against which we can judge the importance of Peso effects induced by the introduction of regime-switching.

B. The Peso Model

Like other general equilibrium models of the term structure, the Baseline model assumes that the state variables follow stable time series processes.[5] I will now extend the model to allow for instability by introducing regime-switching in these processes. This is a natural extension of the Baseline model given the sample period covered by the U.K. data. As noted in the introduction, there is little economic reason to believe that U.K. inflation followed a stable process during the period. There were several changes in policy regime which were accompanied by vigorous debate and considerable variation in inflation. There were also numer-

[5] One exception is the model of Naik and Lee (1994) who extend the Vasicek (1977) model to allow for switches in the mean and variance of the short rate.

ous structural/institution changes in the economy during the period that may have resulted in instability in the dynamics of real variables. Consequently, its seems quite likely that rational investors took account of possible future changes in the behavior of inflation and real variables or were learning about past changes during the sample. Under these circumstances, both the real and nominal term structures will be affected by the presence of Peso problems.

To examine these effects, I modify the dynamics of the state variables, inflation and the pricing kernel. In particular, I shall assume that $x_t' \equiv [-m_t; \Delta p_t]$ and $z_t' \equiv [\mu_t; \pi_t]$ now follow Markov switching processes:

$$
\begin{aligned}
x_{t+1} &= \kappa(s_{t+1}) + z_t + \Lambda(s_{t+1}) e_{t+1}; \qquad (22)\\
z_{t+1} &= z(s_{t+1}) + \alpha(s_{t+1}) z_t + e_{t+1};
\end{aligned}
$$

with $E\left[e_{t+1}e_{t+1}' | F_t\right] = \Omega(s_{t+1}; z_t)$. Here the elements of $\kappa(s); \Lambda(s); z(s)$ and $\alpha(s)$ vary according to the regime which is determined by the discrete-valued variable s_t that follows an independent Markov process with constant probabilities. I also allow the coefficients in the covariance matrix, $\Omega(s; z)$ [i.e., σ_u, σ_v and ρ] to vary with the regime as well as the state variables. This means that both the first and second moments of x_t and z_t are regime-dependent. Although s could take on a large number of values, for computational tractability, I only estimate models with two regimes. I will therefore confine my analysis here to the case where $s = \{1; 0\}$.

To solve the model, I proceed much as before. I posit that log bond prices bonds satisfy

$$
-q^j_{t;k} = A^j_k(s_t) + B^j_k(s_t) z_t \qquad j = n; r \qquad k = 0; 1; \ldots \qquad (23)
$$

and verify that some regime dependent parameters $A^j_k(s)$ and vectors $B^j_k(s)$ exist satisfying the log pricing equations in (13) and (14) given the dynamics in (22). Appendix A shows that this gives the following recursions for the parameters:

$$
\begin{aligned}
A^j_k(s) &= -\frac{1}{2}\left\{A^j_{k-1}(1) - A^j_{k-1}(0) + d^j(\kappa(1) - \kappa(0))\right\} Var(s_{t+1}|s_t = s) \qquad (24)\\
&\quad + E_s\left\{d^j\kappa(s) + A^j_{k-1}(s) + B^j_{k-1}(s) z(s)\right\} \qquad j = \{n; r\} \qquad k = 1; 2 \ldots \\
B^j_k(s) &= d^j + E_s\left\{B^j_{k-1}(s)\alpha(s) - \frac{1}{2}\left[d^j\Lambda(s) + B^j_{k-1}(s)\right] \otimes \left[d^j\Lambda(s) + B^j_{k-1}(s)\right]\Gamma(s)\right\}
\end{aligned}
$$

where $E_s\{f(s)\} = \sum_{s=0}^{1} f(s) Pr(s_{t+1} = s | s_t = s)$ and $vec[\Omega(s_{t+1} z_t)] = \Gamma(s_{t+1}) z_t$ with $A^j_0(s) = 0$ and $B^j_0(s) = [0; 0]$ for $s = \{1; 0\}$.

Although (23) and (24) exactly solve the log linear bond pricing equations given the switching dynamics, they only provide an approximate solution to the non-linear pricing equations in (11) and (12). The reason is that the log linear pricing equations (13) and (14) only hold exactly when the joint distribution of $m_{t+1}, \Delta p_{t+1}$ and $q^j_{t+1,k}$ conditional on information F_t is normal. While this holds true in the Baseline model, it does not in the switching model because the conditional distribution of m_{t+1} and Δp_{t+1} implied by the switching dynamics (22) is a convolution of normals. Equations (13) and (14) therefore contain an approximation error that affects the pricing solution in (23) Fortunately, these errors turn out to be very small in practice. Appendix B presents simulation results based on the model estimates to show exact how small. I find, for example, that the pricing errors for 12 month real and nominal yields (expressed in annual per cent) have sample means of 1 36 and -0 2593 basis points and standard deviations of 1 57 and 0 007 basis points. The pricing errors for longer maturity bonds used to estimate the models are even smaller.

IV. Model Estimates

I shall begin by examining alternative model estimates of the real term structure. This allows us to pin down an appropriate specification for the dynamics of the real pricing kernel. Based on these results, I will then examine models for both the real and nominal term structures.

All the models are estimated by maximum likelihood using the yields on bonds of 1, 3, 5 and 7 year maturities. In common with previous studies, I introduce a pricing error into the equation for equilibrium yields when estimating each model. Specifically, I assume that the observed yields, $y^j_{t,k}$; are related to the theoretically determined bond prices, $q^j_{t,k}$; by

$$y^j_{t,k} = -\frac{1}{k} q^j_{t,k} + \xi^j_t \qquad k = 12, 36, 60, 84 \qquad j = \{n, r\};$$

where ξ^j_t is an i.i.d mean zero normal variable (with a variance specific to each yield). Initially, I allowed for pricing errors in all the equations and estimated the Baseline model with a Kalman Filter technique. In this case I found that the estimated error variances for the 3 year bonds were extremely small. To obtain greater precision, I therefore re-estimated the models without pricing errors in these equations. Under these circumstances, there is no need to use the Kalman Filter and it is possible to calculate the exact likelihood function for the Peso models.[6] The estimates reported below are based on this procedure.

[6]The Kalman Filter is needed to form the sample likelihood because the state variables z_t cannot be inferred directly from observed yields. Pennachi (1991) first used this technique to estimate a homoskedastic Affine model based on Vasicek (1977). To obtain my initial estimates of the baseline model I extended the technique to allow for conditional heteroskedasticity. When there are no pricing errors in the equations for 3 year nominal and real yields, the state variables can be recovered directly from these yields. This simplifies calculations of the baseline sample likelihood. It also means that the sample likelihood for the Peso model can be calculated using Hamilton's (1988) filtering algorithm. Kim (1993) explains why this is not possible when the state variables cannot be recovered directly.

Table 3: Real Term Structure Models						
parmeters	Baseline Model		Peso Model			
			$s_t = 1$		$s_t = 0$	
κ_m	-35.190	(0.310)	-52.055	(0.290)	-49.035	(0.363) ***
α_m	0.692	(0.002)	0.547	(0.004)	0.560	(0.003) **
μ	26.804	(0.203)	38.206	(0.527)	38.233	(0.003)
σ_u^2	2.763	(0.007)	1.657	(0.028)	2.177	(0.025) ***
λ_m	-0.824	(0.001)	-0.926	(0.014)	-0.803	(0.009) ***
$Pr(s_{t+1} = s_t)$			0.974	(0.001)	0.989	(0.001)

Notes:
The structure of the models is given by (14) (15) and (17). In the baseline model all the parameters are constant. In the Peso model $\kappa_m; \alpha_m; \mu; \sigma_u^2$ and λ_m vary across regimes according to the value of s_{t+1} that follows an independent first order Markov Switching process. Asymptotic standard errors are reported in parenthesis. Both models are estimated using the yields on 1,3,5 and 7 year real bonds from the U.K. over the period January 1983 to December 1995. The symbols "**" and "***" denote a rejection of the null hypothesis that the parameter pairs are same across regimes.

A. The Real Term Structure

Table 3 reports estimates of two models for the real term structure: The left hand column shows estimates of the Baseline model where the single state variable follows an AR(1) process. These estimates imply that variations in the real pricing kernel are quite persistent and display considerable conditional heteroskedasticity. The autocorrelation coefficient is approximately equal to 0.7. Interestingly, the estimate of λ_m is significantly negative. As we shall see below, this implies that all the term premia are negative. The reason is that realized holding return on long term bonds covaries positively with the real pricing kernel so that they provide a hedge against future states where the representative investors' marginal utility is high. The value of this hedge lowers the equilibrium expected excess holding return.

The right hand columns show parameter estimates from the Peso model. This model has the same structure as the Baseline except that the parameters of the state process vary across two regimes. As the table shows, the parameter estimates in each regime are generally quite similar to their counterparts in the Baseline model. However, using a series of Wald tests, we can reject hypothesis of parameter constancy across regimes for $\kappa_m; \alpha_m; \sigma_u^2$ and λ_m: By this measure, there appears to be a significant degree of instability in the dynamics of the real pricing kernel. The estimated Markov transition probabilities are very close to one indicating that there is a very small probability of a change in regime over a single month. Using a Wald test, we can also easily reject the hypothesis that Markov transition probabilities sum to unity. This implies that there is a good deal of serial correlation in s_t:

k	Data		Baseline Model			Peso Model			Cross-Model
	Mean	Std	Mean	Std.	Corr.	Mean	Std	Corr	Correlations
12	4.987	3.002	5.022	2.943	0.956	5.045	3.036	0.925	0.959
36	4.284	0.989	4.284	0.989	1.000	4.284	0.989	1.000	0.989
60	4.173	0.632	4.135	0.609	0.970	4.203	0.599	0.956	0.992
84	4.122	0.499	4.069	0.455	0.892	4.045	0.428	0.860	0.992

Table 4: Model Comparisons

Notes:
The table reports sample statistics for real yields calculated from the data, the Baseline model, and the Peso Model using the parameter estimates in Table 3. Under Cross-Model Correlations the table reports the sample correlation between the predicted yields from each model.

Although there are statistically significant differences between the regime-dependent parameters, in itself this should not be viewed as evidence favoring the Peso model. Formal testing for the presence of regime-switching requires non-standard testing procedures that are extremely computationally intensive.[7] As an alternative, Table 4 reports comparative statistics based on the two models. Here we see that the average yield curve estimated by both models is within 10 basis points of the data and the estimates of the standard deviations are even closer. Both sets of estimated yields are highly correlated with the data and with one another. Based on these statistics, there are no economically significant grounds to choose between the models. Both do an extremely good job of describing the dynamics of the real term structure. From this perspective, therefore, the question of whether there are truly switches in the dynamics of the real pricing kernel is largely mute.

B. The Nominal Term Structure

I now turn to consider Baseline and Peso models for both the real and nominal term structures. The structure of the Baseline model is given by equations (15)-(18) where all the variables follow stable time series processes. In the Peso model, regime-switching is confined to the equations for Δp_t and π_t that determine the dynamics of inflation. Since the presence of switching in the real price kernel did not improve the performance of the real term structure model, this seems to be a judicious restriction It means that instability in the inflation process is the sole source of Peso problems in the nominal term structure.

The Baseline model estimates are reported in the left hand column of Table 5. Here we see that the parameters determining real rates are almost identical to the estimates in Table 3. They imply that the real

[7] Common hypothesis tests cannot be used to test for the number of regimes because unidentified nuisance parameters are present under the null of no switching that invalidate the use of standard asymptotic theory. Hansen (1992) has developed a test that circumvents this problem but it is very computationally intensive to apply to even univariate models. It is simply impractical to apply the technique to the multivariate model studies here.

Table 5: Real and Nominal Term Structure Models						
parmeters	Baseline Model		Peso Model			
			$s_t = 1$		$s_t = 0$	
κ_m	-35.322	(0.510)	-63.392	(1.795)	-63.392	(1.795)
α_m	0.691	(0.002)	0.624	(0.002)	0.624	(0.002
μ	26.969	(0.338)	51.591	(1.255)	51.591	(1.255)
σ_u^2	2.762	(0.001)	3.079	(0.020)	3.079	(0.020)
λ_m	-0.824	(0.001)	-0.7776	(0.001)	-0.7776	(0.001)
κ_p	-10.011	(1.376)	-13.991	(7.227)	-5.597	(17.053)
α_π	0.977	(0.003)	0.989	(0.005)	1.000	- **
$\alpha_{\pi 1}$	-0.104	(0.018)	-0.030	(0.004)	-0.056	(0.016) *
π	8.833	(1.651)	6.571	(1.650)	14.285	(2.265) ***
σ_π^2	0.688	(0.082)	0.168	(0.020)	0.913	(0.087) ***
λ_π	-0.137	(0.124)	0.113	(0.094)	-1.280	(0.048) ***
ρ	-0.026	(0.020)	-0.044	(0.008)	-0.027	(0.020)
$Pr(s_{t+1} = s_t)$			0.990	(0.001)	0.965	(0.004)

Notes:
The structure of the model is given by equations (13) (14) and (15) - (18). In the Baseline model all the parameters are constant. In the Peso model the parameters κ_p π; α_π; $\alpha_{\pi 1}$; σ_π; ρ and λ_p all vary according the value of s_{t+1} that follows an independent first order Markov Switching process. Asymptotic standard errors are reported in parenthesis. Both models are estimated using the yields on 1,3,5 and 7 year real and nominal bonds from the U.K. over the period January 1983 to December 1995. The symbols "**" and "***" denote a rejection of the null hypothesis that the parameter pairs are same across regimes.

pricing kernel displays a good deal of persistence and conditional heteroskedasticity. The state variable, π_t; displays even greater persistent. The model estimates imply that π_t; follows a univariate AR(2) process with roots equal to 0.977 and 0.691.[8] The negative estimates of ρ and λ_π imply that innovations in π_t covary negatively with shocks to the real pricing kernel and inflation. The parameter estimates also imply that inflation covaries positively with the real pricing kernel. This means that the inflation premium implied by the model will be positive for large values of μ_t (see below).

The parameter estimates from the Peso model determining real rates are generally similar to the Baseline model except for κ_m and μ: However, since the A_k^j coefficients are determined by the estimates of $\kappa_m + \mu$ which are similar, there is little difference between behavior of real rates implied by the models (see Table 6 below). This similarity does not carry over to the nominal term structure because the parameter estimates of the inflation process differ across regimes. As the right hand column shows, that we can reject the null

[8] To see this, substitute for μ_t with (15) in the equation for π_{t+1} to obtain a univariate AR(2) representation. The roots are calculated from this representation evaluated at the parameter estimates.

Table 6: Model Comparisons

	k	Data		Baseline Model			Peso Model		
		Mean	Std	Mean	Std.	Corr.	Mean	Std.	Corr.
real									
	12	4.987	3.002	4.787	2.914	0.913	4.943	2.946	0.925
	36	4.284	0.989	4.284	0.989	1.000	4.284	0.989	1.000
	60	4.173	0.632	4.167	0.618	0.950	4.133	0.591	0.956
	84	4.122	0.499	4.033	0.453	0.849	4.068	0.422	0.860
nominal									
	12	9.429	2.372	9.614	4.344	0.872	9.190	3.816	0.864
	36	9.474	1.781	9.474	1.781	1.000	9.474	1.781	1.000
	60	9.581	1.572	9.003	1.048	0.987	9.619	1.071	0.989
	84	9.605	1.420	8.769	0.749	0.966	9.674	0.765	0.972

Notes:
The table reports sample statistics for real and nominal yields calculated from the data, the Baseline model, and the Peso Model using the parameter estimates in Table 5. For each model the table reports the sample mean and standard deviation of the predicted yields and their correlation with the actual yields.

hypothesis of constant coefficients across regimes for the parameters, $\kappa_p, \alpha_\pi, \sigma_\pi$ and λ_π at the 5% level. In regime one, the estimates imply that π_t follows a highly persistent stationary process with the largest root equal to 0.989. Changes in π_t have a moderate effect on the variance of innovations, and the covariance between π_{t+1} and Δp_{t+1} is positive. In regime zero, the process for π_t contains a unit root[9] Here changes in π_t have a larger impact in the innovation variance and the covariance between Δp_{t+1} and π_{t+1} is strongly negative. These estimates imply significant differences in the dynamics of inflation across regimes that will manifested in the behavior of nominal yields. In particular, the estimates imply that nominal yields contain a unit root in regime 0 and are mean reverting in regime 1. This implication of the model accords well with the results from estimating univariate switching models in Ang and Bekaert (1998). These authors are also able to formally reject the null hypothesis of no regime-switching in the behavior of U.K. nominal interest rates.

Table 6 compares the predictions of both models against the data. The statistics on real yields in the upper panel are very similar to those in Table 4. The Baseline model underpredicts short term yields by 20 basis points while the largest error made by the Peso model is 5 basis points. Both models underpredict the standard deviations of three yields by similar amounts. Despite these discrepancies, the general impression portrayed by these statistics is that both models accurately describe the dynamics of the real term structure.

The lower panel of the table compares the behavior of nominal yields. Here there are some more significant differences between the models. In particular, while the data and Peso model show an upward sloping

[9] The reported estimates are based on a model with the parameter $\alpha_\pi(0)$ restricted to equal one. Estimates from an unconstrained version of the model gave almost identical estimates.

average yield curve, the Baseline model predicts an inverted curve. As a consequence, the Baseline model underpredicts the average 7 year yield by 94 basis points. The largest error made by the Peso model is on the one year yield which is underpredicted by 24 basis points. Both models underpredict the volatility of long-term yields and overpredict the volatility of short-term yields. This bias in the volatility term structure is somewhat less in the Peso Model. As in the case of the real term structure, the correlations between the actual and estimated nominal yields are high for both models.

Figure 1 plots the estimated probability of being in regime one, $\Pr(s_t = 1 | F_t)$; from the Peso Model. As the plot shows, inflation followed the regime one process from 1983 to 1986, and from 1988 until the end of 1992. According to the model estimates, inflation and nominal interest rates were mean-reverting during these episodes. In the interim, the inflation process contained a unit root. Comparing these regimes against path of nominal interest rates [shown in Figure 2 below], we see that nominal interest rates rose to and fell from sharp peaks during each occurrence of regime one. Thus, the model estimates imply that nominal rates displayed more mean-reversion when they were at historically high levels. A similar relationship between the level and degree of mean reversion has been found in the U.S. term structure by Bekaert, Hodrick and Marshall (1998) and Gray (1996).

Figure 1 also hints at one potential problem with the Peso model estimates; small sample bias. According to the model estimates, the behavior of the nominal interest rates implies that there were relatively few regime shifts during the sample period. This means that the estimated transition probabilities are based on relatively few observations and may consequently suffer from small sample bias. In particular, these estimated probabilities may have differed from the probabilities rational investors used at the time. For example, investors' views about the prospects for future inflation prior to the U.K.'s exit from the E.M.S. might quite reasonably have been based on different probabilities than were consistent with the incidence of regime shifts during the previous decades. The Peso model makes no allowance for such differences. It is therefore possible that the performance of the model could be significantly improved if we allowed the transition probabilities governing investors' expectations to be similar but not identical to those estimated from the data.

To investigate this issue, I found the transition probabilities determining investors' forecasts that minimized the sum of squared differences between the actual and predicted yields from the Peso model. Surprisingly this produce probabilities that were nearly identical to the estimates in Table 5. I then repeated this procedure using only the 18 months of data following October 1992, the date the U.K. left the E.M.S. In this case the probabilities of remaining in regime one and zero were 0.999 and 0.972. While these values are very close to the estimates of 0.999 and 0.965 in Table 5, their impact on the estimated yields is quite dramatic. This can be seen from the plots of the one, five and seven year nominal yields shown in Figure 2. Comparing the actual data (plotted with the solid line) against the estimated yields from the Peso model (plotted with dashed line), we can see that the model severely over-estimates the slope of the yield curve immediately after the U.K.'s exit from the EMS; estimated short-term yields are much too low, while long-term estimates are much too high. By contrast, estimates implied by the probabilities of 0.999 and 0.972 (shown as a dotted line) are very close to the actual data. A small increase in the probability of remaining in regime zero flattens the estimated term structure considerably.

The point I wish to make here is not that the transition probabilities changed in exactly this way. Rather, it is to illustrate how sensitive the yield curve estimates are to small variations in the transition probabilities used to value bonds. Given the relatively low occurrence of actual regime shifts, it is hard to argue that rational investors could not have used slightly different probabilities to value bonds than we can estimate from the dynamics of actual yields over the whole sample. In which case, the differences between the Peso model estimates and actual yields are probably not as economically significant as they might have appeared at first sight. Moreover, our results suggests that Peso problems in inflation could have significantly affected the behavior of nominal yields. The next section examines this hypothesis in detail.

V. Peso Effects

I now turn to examine the implications of the Peso Model in detail. Specifically, I shall use the estimation results from Table 5 to examine the behavior of the real and nominal term premia and the inflation risk premia. Of particular interest will be the role played by Peso problems in the determination of these risk premia. Below I begin by examining the general impact of Peso problems. I then apply this analysis to quantify their effects on the behavior of returns and the risk premia.

A. Identification

Peso problems manifest themselves in several ways in the model. First, they affect the relationship between realized and expected returns. These effects arise from the behavior of investor forecast errors and are common to other models. Peso problems also give rise to a new source of risk that affects the behavior of both the term and inflation risk premia.

To identify these effects, let γ_t denote a generic variable in the model (i.e., an element of x_t, z_t or a log bond price). We can decompose realizations of γ_{t+1} as

$$\gamma_{t+1} = \gamma^e_t(0) + [\gamma^e_t(1) - \gamma^e_t(0)]s_{t+1} + w_{t+1}; \qquad (25)$$

where $\gamma^e_t(s) \equiv E[\gamma_{t+1}|F_t; s_{t+1} = s]$ is the expectation conditional on the regime in $t+1$; the within-regime forecast. Notice that it is always possible to decompose future realizations in this way irrespective of the process they follow in each regime or the specification of information F_t. Rational investors' expectations, $E[\gamma_{t+1}|F_t]$, coincide with the mathematical expectation of γ_{t+1}. Taking expectations on both sides of (25) conditioned on F_t, for $s_{t+1} = \{0,1\}$, we find that $E[w_{t+1}|F_t] = 0$. Thus w_{t+1} inherits the properties of conventional rational expectations forecast errors and represents the error investors would make if the future regime were known, i.e., the within-regime forecast error.

Investors' actual forecasts are only conditioned on F_t. Taking the difference between γ_{t+1} and $E[\gamma_{t+1}|F_t]$ calculated from (25), we can write investors' forecast errors as

$$\gamma_{t+1} - E[\gamma_{t+1}|F_t] = \nabla\gamma_t^e(s_{t+1} - E[s_{t+1}|F_t]) + w_{t+1}. \tag{26}$$

The first term on the right is equal to the difference between the within-regime forecasts, $\nabla\gamma_t^e \equiv \gamma_t^e(1) - \gamma_t^e(0)$, multiplied by the error investors make in forecasting next period's regime. In large samples where the frequency of regime shifts is representative of the underlying distribution of regime changes, $s_{t+1} - E[s_{t+1}|F_t]$ will have a mean zero and will be uncorrelated with the elements of F_t. In small samples, by contrast, $s_{t+1} - E[s_{t+1}|F_t]$ may have a mean different from zero and be autocorrelated with elements of F_t. In this case, the behavior of the forecast errors will appear inconsistent with standard rational expectations assumptions.

Next, consider how Peso problems may affect risk premia. Since the decomposition in (25) holds for any two variables in the model, say $\gamma_{i,t}$ and $\gamma_{j,t}$, it is easy to show that

$$Cov(\gamma_{i,t+1}, \gamma_{j,t+1}, |F_t) = Cov(w_{i,t+1}, w_{j,t+1}, |F_t) + \nabla\gamma_{i,t}^e \nabla\gamma_{j,t}^e Var(s_{t+1}|F_t) \tag{27}$$

In the absence of Peso problems, $\nabla\gamma_{i,t}^e = \nabla\gamma_{j,t}^e = 0$ so the second term vanishes and the covariance is solely determined by $Cov(w_{i,t+1}, w_{j,t+1}, |F_t)$, the within-regime covariance. When Peso problems are present, the covariance between the forecasts, $E[\gamma_{i,t+1}|F_t, s_{t+1} = s]$ and $E[\gamma_{j,t+1}|F_t, s_{t+1} = s]$, identified by the second term on the right affects $Cov(\gamma_{i,t+1}, \gamma_{j,t+1}, |F_t)$. This cross-regime covariance term accounts for the forecast uncertainty investors face across regimes.

B. Returns

We now use the results above to study how Peso problems affect the behavior of returns. In particular, consider the behavior of excess holding returns, $eh^j_{t+1,k} \equiv h^j_{t+1,k} - y^j_t$, and the excess real returns on nominal bonds, $er_{t+1} \equiv y^n_t - \Delta p_{t+1} - y^r_t$. By definition both sets of returns can be written as the sum of a risk premium and a forecast error. Rewriting these errors using (26), (23) and (24), we have

$$er_{t+1} = \varphi_t - w^p_{t+1} - \nabla\beta^p_t(s_{t+1} - E[s_{t+1}|F_t])$$

$$\tilde{h}^j_{t+1,k} = \theta^j_{t,k} + w^j_{t+1} + \bar{\nabla}^j_t (s_{t+1} - E[s_{t+1}|F_t]) \tag{28}$$

where

$$\bar{\nabla}^p_t \equiv \kappa_p(1) - \kappa_p(0)$$

$$\bar{\nabla}^j_t \equiv A^j_{k-1}(0) - A^j_{k-1}(1) + B^j_{k-1}(0)z(0) - B^j_{k-1}(1)z(1) + \left[B^j_{k-1}(0)\alpha(0) - B^j_{k-1}(1)\alpha(1)\right] z_t.$$

Here we see that both sets of excess returns contain a Peso term that may affect their small sample properties.

Let $E_T[\gamma_{t+1}|\Omega_t]$ denote the predicted value of γ_{t+1} from the regression of γ_{t+1} on $\Omega_t = \{\pi_t \mu_t, s_t, \pi_t s_t, \mu_t s_t\}$ in a sample of length T. Multiplying both sides of (28) by the predicted value of excess returns and taking expectations of the result, we obtain.

$$\begin{aligned} Cov_T\left(\tilde{h}^j_{t+1,k}, E_T[\tilde{h}^j_{t+1,k}|\Omega_t]\right) &= Cov_T\left(\theta^j_{t,k}, E_T[\tilde{h}^j_{t+1,k}|\Omega_t]\right) \\ &\quad + Cov_T\left(\bar{\nabla}^j_t (s_{t+1} - E[s_{t+1}|F_t]), E_T[\tilde{h}^j_{t+1,k}|\Omega_t]\right) \\ Cov_T\left(\tilde{r}_{t+1}, E_T[\tilde{r}_{t+1}|\Omega_t]\right) &= Cov_T\left(\varphi_t, E_T[\tilde{r}_{t+1}|\Omega_t]\right) \\ &\quad - Cov_T\left(\bar{\nabla}^p_t (s_{t+1} - E[s_{t+1}|F_t]), E_T[\tilde{r}_{t+1}|\Omega_t]\right) \end{aligned} \tag{29}$$

where $Cov_T(.)$ denotes the sample covariance based on T observations. By least squares theory, the covariance terms on the left equal the sample variance of the predicted values. Thus, (29) provides a decomposition of these variances that holds for all sample sizes. In large samples, the second covariance term in each equation disappears because the sample moments approach their population counterparts and $\Omega_t \subset F_t$. Here all of the volatility in predictable excess returns is attributable to their covariance with the risk premia. In small samples, by contrast, both covariance terms will generally contribute to the volatility in predictable excess returns.

Table 7 reports the empirical distribution of the covariance between the risk premium and predictable excess returns generated from 1000 Monte Carlo experiments. In each experiment I used the estimates of the Peso Model to generate a sample of $\Delta p_t, \pi_t \mu_t$ and s_t containing 155 observations (the length of the U.K. data set). Excess returns and risk premia were calculated from these simulated data using the pricing equations of the Peso model and the returns regressed on Ω_t to form predictable returns. The variance of predicable returns and their covariance with the risk premia are then calculated for each sample. The table reports results based on returns and risk premia calculated for holding periods of twelve rather than one

Table 7: Peso Effects					
$Cov_T(v; E_T[w\|\Omega_t])/Var_T(E_T[w\|\Omega_t])$					
	Excess Holding returns: $w = eh^n_{t+12;k-12}$ $v = \mu^n_{t;k-12}$				Excess Real Returns $w = er_{t+12}$ $v = \varphi_{t;12}$
k	24	36	60	84	
5 %	0.860	0.835	0.834	0.834	-0.325
10 %	0.890	0.871	0.870	0.870	-0.306
25 %	0.950	0.933	0.932	0.932	-0.243
50 %	1.027	1.005	1.005	1.005	-0.184
75 %	1.132	1.106	1.104	1.104	-0.142
90 %	1.307	1.266	1.266	1.266	-0.094
95 %	1.410	1.380	1.378	1.378	-0.072

Notes:
$eh^n_{t+12;k-12} \equiv q^n_{t+12;k-12} - q^n_{t;k} + q^n_{t;12}$ is the excess return on holding a nominal k period bond for 12 months relative to the 12 month nominal yield. $er_{t+12} \equiv y^n_t - \Delta^{12} p_{t+12} - y^r_t$ is the excess real return from holding a 12 month nominal bond to maturity relative to a 12 month real bond. $\mu^n_{t;k-12} \equiv E\left[eh^n_{t+12;k}|F_t\right]$ and $\varphi_{t;12} \equiv E\left[er_{t+12}|F_t\right]$ are the comparable risk premia. The table reports percentiles of the distributions derived from 1000 Monte Carlo experiments [see Appendix C for details].

month as shown above.[10] Although the choice of a longer holding period complicates the calculation of risk premia [see Appendix C], the same logic applies. In large samples, the variance of predictable returns should closely approximate their covariance with the risk premia.

The left hand columns report the distributions for annual excess holding returns on nominal bonds.[11] The median values of the distribution are close to unity, which is the value we would see in a large sample. Thus, Peso effects do not appear to exert a significant influence on the predictability of excess holding returns in a "typical" small sample. This is not to say that they are always insignificant. On the contrary, from the distributions we see that in 35% of the samples the covariance differs in absolute terms from the variance by a least 10%. Hence there is quite a high probability that Peso effects will significantly contribute to the predictability of excess returns in any single sample.

The right hand column shows the distribution for annual real excess returns. Here the results are quite dramatic. They show that the covariance between the inflation risk premium and predictable excess returns is negative in almost all samples. According to these statistics, the predictable variations in excess real

[10] I consider the 12 month holding periods because the accruacy of the Peso model trails off at the short end of the term structure [see Appendix B]. Also, estimates of short term U.K. yeilds are based on the prices of relatively few actual bonds and so are prone to error.

[11] Since there is no regime switching in the process for the real pricing kernel, Peso effects are absent from real holding returns so these returns are not examined.

returns are overwhelmingly due to Peso effects.

These results indicate that there is a high probability that Peso problems significantly affect the predictability of excess returns in typical samples. Next, I consider in turn the impact of Peso problems on the term premium and inflation risk premium.

C. Term Premia

Consider the term premia identified by

$$\theta^j_{t,k} = -\frac{1}{2}Var\left(q^j_{t+1,k-1}\Big|F_t\right) + Cov\left(q^j_{t+1,k-1}, d^j x_{t+1}\Big|F_t\right) \qquad j = \{n, r\}. \tag{29}$$

The first term on the left arises from Jensen's inequality; greater variability in $q^j_{t+1,k-1}$ lowers the expected holding return on long bonds but not the current short rate. The second is equal to the covariance between future bond prices and the relevant pricing kernel. To interpret this term, recall that in representative agent models, m_{t+1} is equal to the real intertemporal marginal rate of substitution. Thus, in the case of real bonds where $d^r x_{t+1} = -m_{t+1}$, the covariance term is equal to $Cov\left(q^r_{t+1,k-1}, -m_{t+1}\Big|F_t\right)$. When this term is negative, long term real bonds provide a hedge against states where marginal utility is high so the equilibrium expected return is lower to compensate. In the case of nominal bonds, $d^n x_{t+1} = -(m_{t+1} - \Delta p_{t+1})$, so the covariance term in (29) comprises the sum of $Cov\left(q^n_{t+1,k-1}, -m_{t+1}\Big|F_t\right)$ and $Cov\left(q^n_{t+1,k-1}, \Delta p_{t+1}\Big|F_t\right)$. The second term identifies a determinant of the nominal term premia absent from the real term structure. When bond prices and inflation covary positively, the expected real return on nominal bonds falls. In these circumstances, (29) shows that the equilibrium the nominal term premium rises to compensate.

To see how regime-switching affects the premium, I apply the decomposition in (27) to the terms in (29). After some rearrangement, this gives

$$\theta^j_{t,k} = E_s\left\{\theta^j_{t,k}(s)\right\} - \frac{1}{2}\left(\nabla\beta^j_t\right)^2 Var(s_{t+1}|F_t) + \nabla\left[d^j\kappa\right]\nabla\beta^j_t Var(s_{t+1}|F_t) \tag{30}$$

$$\begin{aligned}\theta^j_{t,k}(s) &\equiv -\frac{1}{2}Var\left(q^j_{t+1,k-1}\Big|F_t, s_{t+1} = s\right) + Cov\left(q^j_{t+1,k-1}, d^j x_{t+1}\Big|F_t, s_{t+1} = s\right)\\ &= -\frac{1}{2}\left[B^j_{k-1}(s) \odot B^j_{k-1}(s)\right]\Gamma(s)z_t + \left[B^j_{k-1}(s) \odot d^j\kappa(s)\right]\Gamma(s)z_t.\end{aligned}$$

$\theta^j_{t,k}(s)$ is the premium that would prevail if the future regime, s_{t+1}, were known to investors at t. Thus, the first term in (30) is the expected value of $\theta^j_{t,k}(s)$ and so represents the certainty equivalent component of the term premium. The remaining two terms show the direct effects of investor uncertainty about the future regime. They identify the cross-regime term in $-\frac{1}{2}Var\left(q^j_{t+1,k-1}\Big|F_t\right)$ and $Cov\left(q^j_{t+1,k-1}, d^j x_{t+1}\Big|F_t\right)$. Together, they represent the regime-uncertainty component of the premium.

Table 8 : Term Premia

k	Real Yields		Nominal Yields					
	Mean	Std.	Mean	Std.	Mean	Mean	$Cov(\cdot)$	$Var(\cdot)$
	(x100)	(x100)	Sample	Sample	s = 1	s = 0		
24	0.116	0.001	0.055	0.097	-0.031	0.222	-0.005	0.111
36	0.204	0.001	0.048	0.087	-0.029	0.198	-0.037	0.077
60	0.199	0.003	0.049	0.087	-0.027	0.199	-0.087	0.076
84	0.193	0.004	0.049	0.087	-0.027	0.199	-0.142	0.076

Notes:
The table reports sample statistics for estimates of holding return premia derived from the estimates of the Peso model shown in Table 5. Holding returns on the k-month bond are calculated over a 12 month horizon. The fraction of the covariance and variance terms in the premia due to the cross-regime components are shown in the columns headed $Cov(\cdot)$ and $Var(\cdot)$. Appendix C describes details of the calculations.

Table 8 reports statistics on the term premia in the real and nominal term structures implied by the estimates of the Peso model. [As above, the premia are calculated for twelve month holding periods.] The left hand columns report statistics on the real term structure. As the statistics indicate, these premia are small and not very variable. This finding is consistent with the results in Table 2. There we saw that most of the variations in real yields could be attributed to changing forecasts of future short rates.

The term premia in the nominal term structure behave quite differently. As the left hand columns of the table show, over the whole sample, the premia average about 5 basis points with the standard deviation of 9 basis points. Thus, the nominal premia are larger and considerably more variable than their real counterparts. There are also significant differences in the behavior of the premia across regimes. In regime one, the average premia equal -3 basis points, while in regime zero the average is 20 basis points. The (unreported) standard deviation of the premia in each regime is approximately 4 basis points. Under $Cov(\cdot)$ and $Var(\cdot)$ the table reports the fraction of the covariance and variance terms due to the cross-regime components. Together, these components identify the direct effects of regime-uncertainty. The small numbers in the table imply that the premia are primarily determined by certainty equivalent component.

We can now explain the cross-regime differences in the premia. From the parameter estimates in Table 5 we see that λ_π is weakly positive in regime one and strongly negative in regime zero. As a consequence, the covariance between inflation and nominal bond prices will be negative in regime one and positive in regime zero. If there was no uncertainty about the future regime, this difference would make the term premia much higher in regime zero. Because the transition probabilities are so high, this difference is almost perfectly mirrored in the certainty-equivalent component, and hence accounts for the cross-regime differences in the premium.

D. Inflation Risk Premia

Finally, we turn to examine the behavior of the inflation risk premium:

$$\varphi_t = -\frac{1}{2}Var\left(\Delta p_{t+1}\middle|\mathcal{F}_t\right) + Cov(\Delta p_{t+1}, m_{t+1}|\mathcal{F}_t). \tag{21}$$

The first term arises because greater variability in prices raises the expected future purchasing power of money, thereby making nominal bonds relatively more attractive. The equilibrium yield on nominal bonds, and hence the inflation risk premium, must therefore fall to compensate. The second term, the covariance between inflation and the real pricing kernel, identifies the real hedging value of nominal bonds. This is most easily understood with reference to a representative agent model. When the covariance is positive, the realized real return on nominal bonds will be unexpectedly low in states where marginal utility is high thereby lowering their values as a real hedge. Equation (21) shows that the equilibrium inflation risk premium has to rise to compensate. Notice that there is no general presumption about the sign of the inflation risk premium. It all depends upon the degree to which future inflation can be predicted and the extent to which unexpectedly high future inflation occurs in states with high marginal utility.

We examine the impact of regime-switching by applying the decomposition in (27) to the terms in (21):

$$\varphi_t = E_s\{\varphi_t(s)\} - \nabla k_p \nabla k_m Var(s_{t+1}|\mathcal{F}_t) - \frac{1}{2}(\nabla k_p)^2 Var(s_{t+1}|\mathcal{F}_t) \tag{31}$$

$$\begin{aligned}\varphi_t(s) &\equiv -\frac{1}{2}Var\left(\Delta p_{t+1}\middle|\mathcal{F}_t, s_{t+1} = s\right) + Cov(\Delta p_{t+1}, m_{t+1}|\mathcal{F}_t, s_{t+1} = s)\\ &= -\frac{1}{2}\lambda_p(s)^2\left(\rho(s)^2\sigma(s)_m^2\nu_t + \sigma(s)_\pi^2\pi_t\right) - \lambda_m(s)\lambda_p(s)\rho(s)\sigma_m(s)^2\nu_t\end{aligned}$$

$\varphi_t(s)$ identifies the value of the inflation risk premium is the future regime, s_{t+1}, were known. As in the case of the term premium, we can therefore identify certainty equivalent and regime uncertainty components in the inflation risk premium. The former component is identified by the first term in (31), while the latter is equal to the second and third terms which equal cross-regime components of $Cov(\Delta p_{t+k}, m_{t+1}|\mathcal{F}_t)$ and $\frac{1}{2}Var(\Delta p_{t+1}|\mathcal{F}_t)$.

Table 9 reports statistics derived form the Peso model estimates on the spread between nominal and real one year bonds, $y^n_{t,12} - y^r_{t,12}$; the expected rate of annual inflation, $\frac{1}{12}E[\Delta^{12}p_{t+12}|\mathcal{F}_t]$; and the annual inflation risk premia, $\varphi_{t,12} \equiv y^n_{t,12} - y^r_{t,12} - \frac{1}{12}E[\Delta^{12}p_{t+12}|\mathcal{F}_t]$. Over the whole sample that spread averages 4.25% of which 4.68% represents expected inflation and -0.43% the inflation risk premium.[12] There is also considerable

[12] Because the Peso model estimates are based on bond yields alone, we cannot identify the average rate of expected inflation or the average inflation risk premium without a further restriction. This identification problem is a common feature of Affine models and is discussed in Appendix D. Here I solve the problem by imposing a parameter restriction on the inflation process to insure that the sample averages of expected inflation and actual inflation (measured by the R.P.I.) are equal. This is a

Table 9: Inflation Risk Premia

	Spread	Expected Inflation			Inflation Risk Premium, $\varphi_{t,12}$					
	$y^n_{t,12} - y^r_{t,12}$	$\frac{1}{12}E[\Delta^{12}p_{t+12} \mid F_t]$						Cross-Regime %		
		Sample	$s=1$	$s=0$	Sample	$s=1$	$s=0$	Total	$Cov(.)$	$Var(.)$
Mean	4.247	4.680	5.347	3.493	-0.433	-1.142	0.623	0.996	0.297	0.995
Std.	3.487	1.341	1.006	1.076	2.510	2.805	2.180	0.002	0.046	0.002

Notes:
The table reports sample statistics for estimates of the inflation risk premia derived from estimates of the Peso model in Table 5. The premia are calculated for a 12 month holding period. The fraction of the covariance and variance terms in the premia due to the cross-regime components are shown in the columns headed $Cov(.)$ and $Var(.)$. Appendix C describes details of the calculations.

variability in the spread, as measured by the sample standard deviation. Interestingly, most the variability originates from changes in the risk premium, which is almost twice as variable as expected inflation. These statistics are obviously a sharp deviation from the prediction of the standard Fisher Equation. The estimates imply the presence of a significant and highly variable inflation risk premium. The table also shows that the behavior of expected inflation and the inflation risk premia differ significantly across regimes. In regime one, expected inflation averages over 5% while the mean risk premium is -1.1%. In regime zero, the average risk premium is much higher at 0.6% and the mean of expect inflation is lower at 3.5%. Notice too that there is considerable variation in risk premium within each regime. It would be an oversimplification to think of regime zero as high inflation risk regime.

The rightmost columns report the contribution of the cross-regime components to the inflation risk premium. In the one month case these components are identified by the last two terms in (31). Appendix C describes the calculations behind the statistics in the table. Unlike the case of the term premia, here the cross-regime components contribute significantly. Regime-uncertainty accounts for approximately 30% of the covariance term and almost 100% of the variance term. The reason is that inflation contains a unit root in one regime but not the other. As a result, there are large differences between forecasts conditioned on different future regimes.

Figure 3 allows us to examine the time series behavior of these variables more closely. The upper panel plots actual and expected rate of annual inflation (solid and dashed lines respectively). It is important to remember that the Peso model only uses data from the real and nominal term structure so the estimates of expected inflation are not directly derived from actual inflation. Rather they represent the set of expectations consistent with the dynamics of interest rates. As the plot shows, there are considerable and persistence

minimal rational expectations assumption. Importantly, it has no impact on the changes in expected inflation or the inflation risk premium which are identified from the model estimates. There is also no affect on the differences in expected inflation and the risk premium across regimes.

differences between the two series on occasion. For example, actual inflation is persistently higher that expected inflation between 1989 and 1991. Between 1992 and 1995, by contrast, expected inflation was persistently higher.

Such a pattern in forecast errors is hard to explain within the context of a standard rational expectations model. There rational investors would immediately adjust their forecasts to eliminate any persistent patterns. This need not happen when instability in the inflation process induces Peso problems. In a Peso model such patterns can emerge during periods where rational investors are anticipating a change in the inflation process that does not materialize. The forecasts of inflation shown in the plot contain such expectations to the extent they are embodied in the term structures.

With this perspective, the expectational errors have a rather natural interpretation. Between 1989 and 1991 investors expected a switch from the current inflation process generating high rates, to a low inflation process. Since the U.K. was experiencing a severe recession during this period, it is hard to say that these expectations were unwarranted. Similarly, the high rates of expected inflation between 1992 and 1995 coincide with a surge in GDP growth that could have led rational investors to fear that there might have been a return to a high inflation process. The departure of the U.K. from the EMS may have also compounded such fears.

The central panel of Figure 3 shows the yield spread, $y^n_{t,12} - y^r_{t,12}$ (solid line) and expected inflation (dashed line): Here we see that many of the largest movements in spread are not accompanied by similar changes in expected inflation. One notable example occurs between 1990 and 1993 were the spread falls much further than expected inflation. During this period, the inflation risk premium (represented by the vertical distance between the two lines and plotted in the lower panel) fell from approximately 2% to -6%. According to the statistics in Table 9, most of this change can be attributed to Peso effects. Although inflation was falling, the model estimates imply that investors were increasingly uncertain about its future course. This raised the variance term in (21) and lowered the inflation risk premium. Interestingly, as the lower panel shows the inflation risk premium quickly returned towards zero after the U.K. left the EMS.

VI. Conclusion

This paper has argued that instability in the inflation process lead to Peso problems that significantly affected the behavior of the U.K. term structure. First, I showed that we can well-describe the behavior of the real term structure without the resort to regime-switching. A single factor CIR model explains a high fraction of both the time series and cross section behavior of real yields. Next, I compared models for the nominal and real term structures with and without switching in the inflation process. Here we saw that the switching model was better able to explain both the average level and volatility of the nominal yield curve. Moreover, this model could accurately match the time series behavior of the nominal term structure once we allowed for very small changes in the probabilities governing regime switches over the sample period.

Based on these findings, I then used the switching model estimates to quantify the impact of Peso problems. Monte Carlo experiments showed they were the primary factor affecting predictable excess real returns on nominal bonds in small samples. The experiments also indicated that Peso problems would significantly affect the small sample predictability of excess holding returns with a high probability. These findings undermine the standard rational expectations' view that changing risk premia are solely responsible for the observed predictability in excess returns. I also identified and evaluated the impact of Peso problems on the risk premia. Here we found them to have a small effect on the term premia and a large effect on the inflation risk premium.

Overall, these results show that both regime-switching and time-varying risk premia play important roles in explaining the behavior of U.K. interest rates. Could this also be true in the U.S.? There are several reasons for optimism. First, the behavior of nominal yields in the U.K. and U.S. are quite similar. Second, as Bekaert, Hodrick and Marshall (1998) and Gray (1996) have shown, U.S. yields display greater mean reversion when rates are high than when they are low. Nominal yields display this same feature in the Peso model. Third, changes in U.S. monetary policy over the past two decades have been just as pronounced as they have in the U.K.. Consequently, it would be remarkable if investors viewed U.S. inflation as a stable process throughout this period. Indeed, Lewis (1991) examined this hypothesis around the 1979-82 period. Furthermore, Evans and Lewis (1995) argue that Peso problems resulting from instability in the inflation process were responsible for the anomalous long run relationship between nominal rates and inflation observed in U.S. data. These observations suggest that developing of models for the U.S. term structure that combine regime-switching and time-varying risk premia may well prove successful.

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Appendix A

This appendix derives the solution for log bond prices in the Peso Model presented in equations (23) and (24). To begin, note that the log linear pricing equations (13) and (14) can be written as

$$-q^j_{t,k} = E[d^j x_{t+1}|F_t] - E[q^j_{t+1,k-1}|F_t] - \frac{1}{2}Var\left(-d^j x_{t+1} + q^j_{t+1,k-1}|F_t\right) \qquad j = \{n, r\} \tag{A1}$$

where $x_t = [-m_t, \Delta p_t]'$, $d^n = [1, 1]$, and $d^r = [1, 0]$. Substituting for the posited pricing solution (23) on both sides, I obtain

$$\begin{aligned} A^j_k(s_t) + B^j_k(s_t)z_t &= E\left[d^j\kappa(s_{t+1}) + A^j_{k-1}(s_{t+1}) + B^j_{k-1}(s_{t+1})z(s_{t+1})|F_t\right] \\ &\quad + E\left[d^j + B^j_{k-1}(s_{t+1})\alpha(s_{t+1})|F_t\right] z_t \\ &\quad - \frac{1}{2}Var\left(\left[d^j\Lambda(s_{t+1}) + B^j_{h-1}(s_{t+1})\right]e_{t+1}|F_t\right). \end{aligned} \tag{A2}$$

Since s_t follows an independent Markov process, we can apply the law of iterated expectations to the first two terms on the right to obtain

$$E_s\left\{d^j\kappa(s) + A^j_{k-1}(s) + B^j_{k-1}(s)z(s)\right\} \quad \text{and} \quad E_s\left\{d^j + B^j_{k-1}(s)\alpha(s)\right\} z_t, \tag{A3}$$

where $E_s\{f(s)\} \equiv \sum_{s=0}^{1} f(s)\Pr(s_{t+1} = s|s_t)$. To evaluate the variance term, note that $Var(\gamma_{t+1}|F_t) = E[Var(\gamma_{t+1}|F_t, s_t)|F_t] + Var(E[\gamma_{t+1}|F_t, s_{t+1}]|F_t)$. Since $E[e_{t+1}|F_t, s_{t+1}] = 0$, we can write the variance as

$$\begin{aligned} & E\left[Var\left([d^j\Lambda(s_{t+1}) + B^j_{h-1}(s_{t+1})]e_{t+1}\Big|F_t, s_{t+1}\right)\Big|F_t\right] \\ = \; & E\left[\left[d^j\Lambda(s_{t+1}) + B^j_{h-1}(s_{t+1})\right]\Omega(s_{t+1}; z_t)\left[d^j\Lambda(s_{t+1}) + B^j_{h-1}(s_{t+1})\right]'\Big|F_t\right] \\ = \; & E_s\left\{\left[d^j\Lambda(s) + B^j_{k-1}(s)\right]\Omega\left[d^j\Lambda(s) + B^j_{k-1}(s)\right]\Gamma(s)\right\} z_t \end{aligned} \tag{A4}$$

where $vec[\Omega(s_{t+1}; z_t)] = \Gamma(s_{t+1})z_t$. Substituting (A2), (A3) and (A4) into (A1) produces

$$\begin{aligned} A^j_k(s_t) + B^j_k(s_t)z_t &= E_s\left\{d^j\kappa(s) + A^j_{k-1}(s) + B^j_{k-1}(s)z(s)\right\} \\ &\quad + E_s\left(d^j + B^j_{k-1}(s)\alpha(s) - \frac{1}{2}\left[d^j\Lambda(s) + B^j_{k-1}(s)\right]\Omega\left[d^j\Lambda(s) + B^j_{k-1}(s)\right]\Gamma(s)\right) z_t, \end{aligned} \tag{A5}$$

an equation that must hold for all values of z_t and $s_t = \{0, 1\}$. Equating coefficients gives the recursive equations for $A^j_k(s)$ and $B^j_k(s)$ shown in (24). Notice also that if there is no switching, $E_s\{f(s)\} = f(1) =$

$f(0)$. Applying this simplification to (A5) and equating coefficients, gives the solution to the Baseline Model in (19).

Appendix B

This appendix examines the approximation error introduced in the Peso model by using the log linear pricing equations in (13) and (14) rather than the nonlinear equations in (11) and (12). In principle the exact solution for bond prices can be calculated from the nonlinear equations by solving them forward as

$$\exp(\tilde{q}^j_{t,k}) = E\left[\prod_{i=1}^{k} \exp(-d^j x_{t+i}) \middle| F_t\right] \tag{B1}$$

and substituting the switching forecasts of x_{t+i}. However, this is not a practical proposition once k becomes large. With 2 states, there are 2^k forecasts conditional on future realizations of s_t that need to be calculated in order to evaluate (B1). So, since the estimated model considers seven year bonds, we would have to calculated $2^{84} = 1.9343 \times 10^{25}$ conditional forecasts and the probabilities associated with each possible realization of path for $\{s_{t+1}, s_{t+2} \ldots s_{t+k}\}$. To get around this problem, I adopted the procedure described below.

Let $\tilde{q}^j_{t,k}$ denote the true log bond price, satisfying (11) or (12) given the dynamics of the state variables. Our interest is in the behavior of the approximation error $\ln\xi^j_{t,k} \equiv \tilde{q}^j_{t,k} - q^j_{t,k}$ where $q^j_{t,k}$ is the approximate solution for the log bond price defined in (23). According to the nonlinear pricing equations, $\tilde{q}^j_{t,k}$ satisfies

$$\exp(\tilde{q}^j_{t,k}) = E\left[\exp\left(d^j x_{t+1} + \tilde{q}^j_{t+1,k-1}\right)\middle| F_t\right] \qquad k = 1, 2, \ldots .$$

Substituting for $\tilde{q}^j_{t,k}$ with (B1), and rearranging gives

$$\xi^j_{t,k} = E\left[\exp\left(d^j x_{t+1} + q^j_{t+1,k-1} - q^j_{t,k} + \ln\xi^j_{t+1,k-1}\right)\middle| F_t\right] \tag{B2}$$

Since both $\tilde{q}^j_{t,0}$ and $q^j_{t,0}$ equal zero by definition, $\xi^j_{t,0} = 0$. Hence, (B1) provides us with a nonlinear recursive equation for the approximation errors.

To evaluate the size of the approximation errors, I proceed as follows. Since $\xi^j_{t,0} = 0$, I first calculate $\xi^j_{t,1}$ directly from (B2) using the Peso model estimates to evaluate the expectation for each value of the state variables, z_t, in the sample. This give me two sets of errors, one for each value of s_t, $\ln\xi^j_{t,1}(s)$. Next, I regress $\ln\xi^j_{t,1}(s)$ on the values of z_t. [While in principle $\xi^j_{t,1}$ can vary nonlinearly with the elements of z_t, in practice I found no significant evidence of nonlinearites.] To calculate the next set of approximation errors,

Appendix Table								
	Real Yeild Errors (basis points)				Nominal Yield Errors (basis points)			
k	s = 1		s = 2		s = 1		s = 2	
	Mean	Sdt.	Mean	Sdt.	Mean	Sdt.	Mean	Sdt.
1	-2.769	0.092	4.011	3.110	-1.309	0.034	-0.322	0.004
2	-2.004	0.096	4.907	2.538	-0.914	0.020	-0.432	0.006
3	-1.530	0.061	3.279	1.806	-0.788	0.018	-0.626	0.005
4	-1.216	0.055	2.990	1.502	-0.687	0.016	-0.672	0.004
5	-1.019	0.043	2.386	1.219	-0.628	0.014	-0.729	0.003
6	-0.873	0.039	2.210	1.066	-0.474	0.012	-0.409	0.003
7	-0.766	0.033	1.916	0.919	-0.480	0.011	-0.572	0.003
8	-0.682	0.030	1.793	0.826	-0.365	0.010	-0.302	0.002
9	-0.615	0.027	1.625	0.738	-0.387	0.009	-0.467	0.002
10	-0.560	0.025	1.534	0.674	-0.296	0.008	-0.241	0.002
11	-0.514	0.023	1.428	0.616	-0.324	0.007	-0.393	0.002
12	-0.476	0.021	1.358	0.570	-0.250	0.007	-0.201	0.001
18	-0.328	0.015	1.064	0.390	-0.170	0.005	-0.135	0.001
24	-0.249	0.012	0.929	0.296	-0.129	0.003	-0.102	0.001
36	-0.175	0.009	0.850	0.200	-0.087	0.002	-0.069	0.001
48	-0.143	0.007	0.802	0.151	-0.066	0.002	-0.052	0.000
60	-0.120	0.006	0.763	0.122	-0.053	0.001	-0.042	0.000
72	-0.105	0.006	0.737	0.102	-0.044	0.001	-0.035	0.000
84	-0.095	0.005	0.718	0.087	-0.038	0.001	-0.030	0.000

I substitute these regression results into (B2):

$$\xi^j_{t;k} = E\left[\exp\left(d^j x_{t+1} + q^j_{t+1;k-1} - q^j_{t;k} + \beta^j_{k-1}(s) z_{t+1} + \varepsilon^j_{t+1;k-1}(s)\right) \Big| \mathcal{F}_t\right] \quad (B3)$$

where $\beta^j_{k-1}(s)$ is the vector of coefficients from the regression of $\ln \xi^j_{t;k-1}(s)$ on z_t. By construction the regression residuals $\varepsilon^j_t(s)$ are uncorrelated with $z_t; x_{t+1}$ and $q^j_{t+1;k-1}$; so we can rewrite (B3) as.

$$\xi^j_{t;k} = E\left[\exp\left(d^j x_{t+1} + q^j_{t+1;k-1} - q^j_{t;k} + \beta^j_{k-1}(s) z_{t+1}\right) \Big| \mathcal{F}_t\right] E\left[\exp\left(\varepsilon^j_{t+1;k-1}(s)\right) \Big| \mathcal{F}_t\right]: \quad (B4)$$

The first term on the right can be evaluated analytically using the parameters of the model while the second term is evaluated as $\frac{1}{T}\sum_{t=1}^{T} \exp\left(\varepsilon^j_{t;k-1}(s)\right)^2$: (B4) can therefore be used to calculate the next set of approximation errors, for $s_t = \{0;1\}$; i.e., $\xi^j_{t;2}(s)$: This whole procedure is then repeated for $k = 2;3 \ldots$

Clearly the accuracy of this procedure depends on precision with which the regression of $\ln \xi^j_{t;1}(s)$ on z_t represents the true (but unknown) nonlinear relationship between the variables. One way to check this, is to consider the fit of the regressions. Based on the Peso model estimates, R^2 statistics from the regressions are all greater than 0.996. (In fact the majority are greater that 0.999.). Moreover, there is no sign of serial correlation or heteroskedasticity in the estimated residuals. These statistics indicate that the regressions are indeed highly accurate. They are also consistent with the i.i.d. assumption that lies behind the use of $\frac{1}{T}\sum_{t=1}^{T}\exp\left(\xi^{j}_{t;k-1}(s)\right)^{2}$ to evaluate $E\left[\exp\left(\xi^{j}_{t+1;k-1}(s)\right)\Big|F_t\right]$.

The Appendix table above reports the sample average standard deviation of $\frac{k}{12}\xi^{j}_{t;k}(s)$ which measures the impact on the yields in terms of basis points. The left hand columns of the table show the errors in real yields based on the Peso model estimates from Table 3. The errors in nominal yields based on the Peso estimates in Table 5 are shown in the right hand columns. [Real yields do not contain an approximation error in this model because regime-switching is confined to the inflation process.] From the statistics in the table, is clear that the difference between the approximate and exact solution to the Peso models is extremely small for the bonds studied [$k \geq 12$].

Appendix C

This appendix describes how the Peso model estimates are used to calculate excess holding returns and excess real returns over 12 month horizons. Let $\mu^{j}_{t;k,12} \equiv \frac{1}{12}E\left[q^{j}_{t+12;k-12} - q^{j}_{t;k} + q^{j}_{t,12}|F_t\right]$ be the 12 month term premium and $\varphi_{t,12} \equiv y^{n}_{t,12} - \frac{1}{12}E[\Delta^{12}p_{t+12}|F_t] - y^{r}_{t,12}$ the 12 month inflation risk premium. Substituting for log prices and yields from the log linear pricing equations, we have that

$$\mu^{j}_{t;k,12} = \frac{1}{12}Cov\left(q^{j}_{t+12;k-12}, d^{j}x^{12}_{t+12}\Big|F_t\right) - \frac{1}{2\cdot 12}Var\left(q^{j}_{t+12;k-12}\Big|F_t\right) \qquad \text{(C1)}$$

$$\varphi_{t,12} = \frac{1}{k}Cov\left[\Delta^{k}p_{t+k}, m_{t+k;k}\Big|F_t\right] - \frac{1}{2k}Var\left(\Delta^{k}p_{t+k}\Big|F_t\right) \qquad \text{(C2)}$$

for $j = \{r, n\}$ where $x^{12}_{t+12} \equiv [-m^{12}_{t+12}, \Delta^{12}p_{t+12}]$ and $m^{12}_{t+12} = \sum_{i=1}^{12} m_{t+i}$.

To study the behavior of these risk premia, consider the vector $Y^{j}_{t;k} \equiv [q^{j}_{t;k}, z'_t, x'_t, x'_{t-1}, x'_{t-2}, \ldots, x'_{t-11}, 1]'$. According to the Peso Model, $Y^{j}_{t;k}$ follows a switching VAR:

$$Y^{j}_{t+1;k} = A^{j}_{k}(s_{t+1})Y^{j}_{t;k} + C^{j}_{k}(s_{t+1})e_{t+1} \qquad \text{(C3)}$$

where

$$A^{j}_{k}(s) = \begin{bmatrix} 0 & B^{j}_{k}(s)\alpha(s) & 0 & A^{j}_{k}(s_{t+1}) + B^{j}_{k}(s)z(s) \\ 0 & \alpha(s) & 0 & z(s) \\ 0 & \iota & 0 & \kappa(s) \\ 0 & 0 & \iota_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad C^{j}_{k}(s) = \begin{bmatrix} B^{j}_{k}(s) \\ \iota \\ \sigma(s) \\ 0 \\ 0 \end{bmatrix}$$

(C3) can now be used to calculate multi-period forecasts as follows:

Let $S^{\tau}_{t+1} = \{s_{t+1}, s_{t+2}, \ldots, s_{t+\tau}\}$ be the set of future regimes over the next τ periods. From (C3) it follows that

$$E\left[Y^{j}_{t+\tau;k} \middle| \mathcal{F}_t, S^{\tau}_{t+1} \right] = \prod_{i=1}^{\tau} A^{j}_{k}(s_{t+i}) Y^{j}_{t;k} \tag{C4}$$

Taking expectations on both sides conditioned on $\mathcal{F}_t$, gives

$$\begin{aligned} E\left[Y^{j}_{t+\tau;k} \middle| \mathcal{F}_t \right] &= \sum_{s_{t+1}} \sum_{s_{t+2}} \cdots \sum_{s_{t+\tau}} \left[\prod_{i=1}^{\tau} A^{j}_{k}(s_{t+i}) \right] \Pr\left(S^{\tau}_{t+1} | s_t \right) Y^{j}_{t;k} \\ &= B^{j}_{k,\tau}(s_t) Y^{j}_{t;k} \end{aligned} \tag{C5}$$

Notice that $\Delta^{12} p_{t+12} = \iota_p Y_{t+12}$ and $m^{12}_{t+12} = \iota_m Y_{t+12}$ where $\iota_p \equiv [0,0,0,0,1,0,1,\ldots,0,1,0]$ and $\iota_m \equiv [0,0,0,1,0,1,0,\ldots,1,0,0]$. We can therefore use the equation above to calculate the risk premia in (C1) and (C2) as

$$\begin{aligned} \phi_{t,12} &= y^{n}_{t,12} - \frac{1}{12} \iota_p B^{j}_{k-12,12}(s_t) Y^{j}_{t;k-12} - y^{r}_{t,12} \\ \mu^{j}_{t;k,12} &= \frac{1}{12} \left[\iota_q B^{j}_{k-12,12}(s_t) Y^{j}_{t;k-12} - q^{j}_{t;k} + q^{j}_{t,12} \right] \qquad j = \{r, n\} \end{aligned}$$

where ι_q is the vector picking out the first element of $Y^{j}_{t;k}$; i.e. $q^{j}_{t;k} = \iota_q Y^{j}_{t;k}$. Table 7 reports results of Monte Carlo experiments where risk premia are calculated from these equations using the Peso Model parameter estimates and simulated data on $y^{j}_{t,12}$, $Y^{j}_{t;k-12}$, $q^{j}_{t;k}$ and $q^{j}_{t,12}$.

Tables 8 and 9 report the fraction of the term and inflation risk premia due to regime uncertainty. To calculate these components, I apply the covariance decomposition

$$\mathrm{Var}\left(Y^{j}_{t+\tau;k} | \mathcal{F}_t \right) = E\left[\mathrm{Var}\left(Y^{j}_{t+\tau;k} | \mathcal{F}_t, S^{\tau}_{t+1} \right) \middle| \mathcal{F}_t \right] + \mathrm{Var}\left(E\left[Y^{j}_{t+\tau;k} \middle| \mathcal{F}_t, S^{\tau}_{t+1} \right] \middle| \mathcal{F}_t \right) \tag{C6}$$

to the terms in (C1) and (C2). To calculate the first term on the right, let $G(s_{t+1}) Y^{j}_{t;k} \equiv \mathrm{vec}[\Omega(s_{t+1}; z_t)]$, so that

$$\mathrm{Var}\left(e_{t+\tau} | \mathcal{F}_t, S^{\tau}_{t+1} \right) = \mathrm{vec}^{-1} \left\{ G(s_{t+\tau}) E\left[Y^{j}_{t+\tau-1;k} \middle| \mathcal{F}_t, S^{\tau-1}_{t+1} \right] \right\}.$$

From (C3), we now have

$$\mathrm{Var}\left(Y^{j}_{t+\tau;k} | \mathcal{F}_t, S^{\tau}_{t+1} \right) = A^{j}_{k}(s_{t+\tau}) \mathrm{Var}\left(Y^{j}_{t+\tau-1;k} | \mathcal{F}_t, S^{\tau-1}_{t+1} \right) A^{j}_{k}(s_{t+\tau})' + C^{j}_{k}(s_{t+\tau}) \mathrm{Var}\left(e_{t+\tau} | \mathcal{F}_t, S^{\tau}_{t+1} \right) C^{j}_{k}(s_{t+\tau})'$$

Using this recursion, we can calculate

$$E\left[Var\left(Y^j_{t+\tau,k}|F_t,S^\tau_t\right)\Big|F_t\right]=\sum_{s_{t+1}}\sum_{s_{t+2}}\ldots\sum_{s_{t+\tau}}Var\left(Y^j_{t+\tau,k}|F_t,S^\tau_t\right)\Pr\left[S^\tau_{t+1}|s_t\right]Y^j_{t,k}=D^j_{k,\tau}(s_t)Y^j_{t,k}. \quad (C7)$$

The second term in (C6) is by definition equal to

$$E\left[\left(E\left[Y^j_{t+\tau,k}\big|F_t,S^\tau_{t+1}\right]-E\left[Y^j_{t+\tau,k}\big|F_t\right]\right)\left(E\left[Y^j_{t+\tau,k}\big|F_t,S^\tau_{t+1}\right]-E\left[Y^j_{t+\tau,k}\big|F_t\right]\right)'\Big|F_t\right].$$

Substituting for the expectations, we can therefore write

$$Var\left(E\left[Y^j_{t+\tau,k}\big|F_t,S^\tau_{t+1}\right]\Big|F_t\right)=\sum_{s_{t+1}}\sum_{s_{t+2}}\ldots\sum_{s_{t+\tau}}H(S^\tau_{t+1},Y^j_{t,k})\Pr\left[S^\tau_{t+1}|s_t\right] \quad (C8)$$

where

$$H(S^\tau_{t+1},Y^j_{t,k})=\left\{\prod_{i=1}^{\tau}A^j_k(s_{t+i})-B^j_{k,\tau}(s_t)\right\}Y^j_{t,k}\left(Y^j_{t,k}\right)'\left\{\prod_{i=1}^{\tau}A^j_k(s_{t+i})-B^j_{k,\tau}(s_t)\right\}'.$$

Tables 8 and 9 reports the sample average of cross-regime components of the covariance and variance shown in (C1) and (C2). Let ϕ^{ce}_t and ϕ^{ru}_t be the appropriate elements of $E\left[Var\left(Y^j_{t+\tau,k}|F_t,S^\tau_{t+1}\right)\Big|F_t\right]$ and $Var\left(E\left[Y^j_{t+\tau,k}\big|F_t,S^\tau_{t+1}\right]\Big|F_t\right)$ calculated from (C7) and (C8). The tables report the sample average of $\phi^{ru}_t/(\phi^{ce}_t+\phi^{ru}_t)$.

Appendix D

This appendix explains why the average rate of expected inflation cannot be identified from the model estimates. Suppose inflation followed

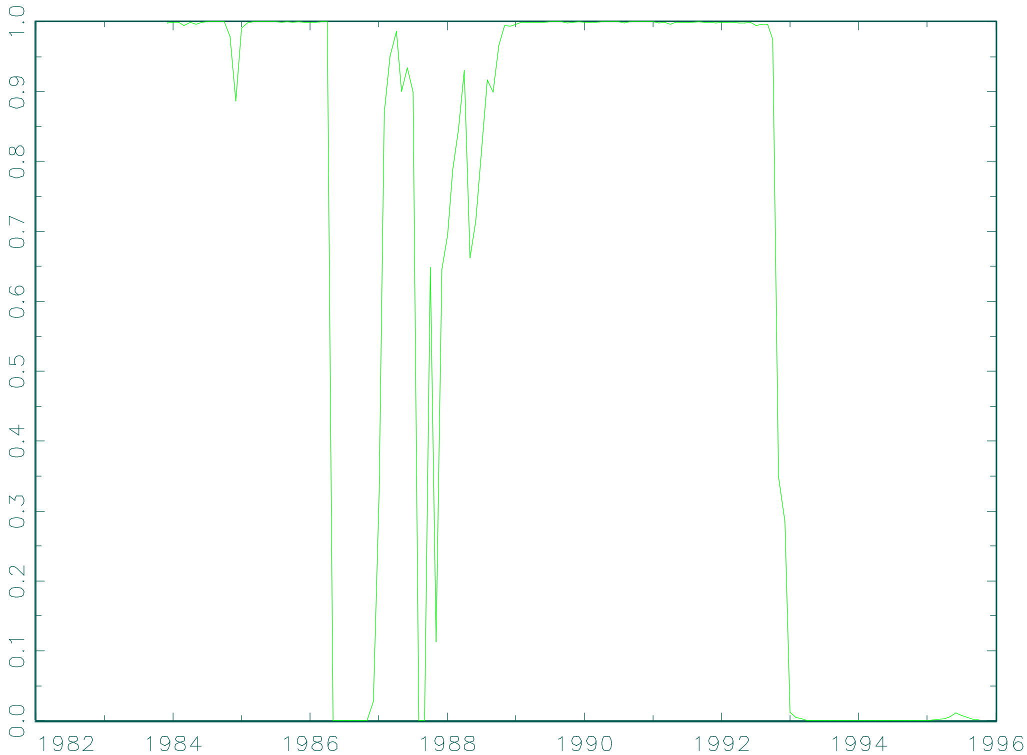
$$\Delta p_{t+1}=\kappa_p+\pi_t+\lambda_{p\rho\sigma_u}\mu_t^{1/2}u_{t+1}+\lambda_{p\sigma_\pi\pi}\pi_t^{1/2}v_{t+1}+\sigma_\xi\xi_{t+1} \quad (D1)$$

where ξ_{t+1} is an i.i.d. $N(0,1)$ shock. With this specification, the coefficients in equation for nominal bond prices in the Baseline model become

$$A^n_k = A^n_{k-1}+B^n_{k-1}z+\kappa_m+\kappa_p-\frac{1}{2}\sigma^2_\xi \qquad k=1,2\ldots \quad (D2)$$

$$B^n_k = d^n+B^n_{k-1}\alpha-\frac{1}{2}\left[d^n\Lambda+B^n_{k-1}\right]\Omega\left[d^n\Lambda+B^n_{k-1}\right]; \quad (D3)$$

Here we see that he recursion for A^n_k in (D2) depends on $\kappa_p-\frac{1}{2}\sigma^2_\xi$ rather than just κ_p as in (19). Thus, if we only use bond prices to estimate the model's parameters, $\kappa_p-\frac{1}{2}\sigma^2_\xi$ rather than κ_p will be identified. This means that the average rate of inflation, equal to $\kappa_p+[\pi+\alpha_{\pi,1}/(1-\alpha_1)]/(1-\alpha_\pi)$; cannot be identified from the model estimates. To generate the statistics in Tables 7 and 9, I set the value of σ_ξ^2 to make average rate of expected inflation and actual inflation equal consistent with the estimated value of the intercept term in the inflation equation, i.e., $\kappa_p - \frac{1}{2}\sigma_\xi^2$ above.



1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
1982
1984
1986
1988
1990
1992
1994
1996

Figure 1

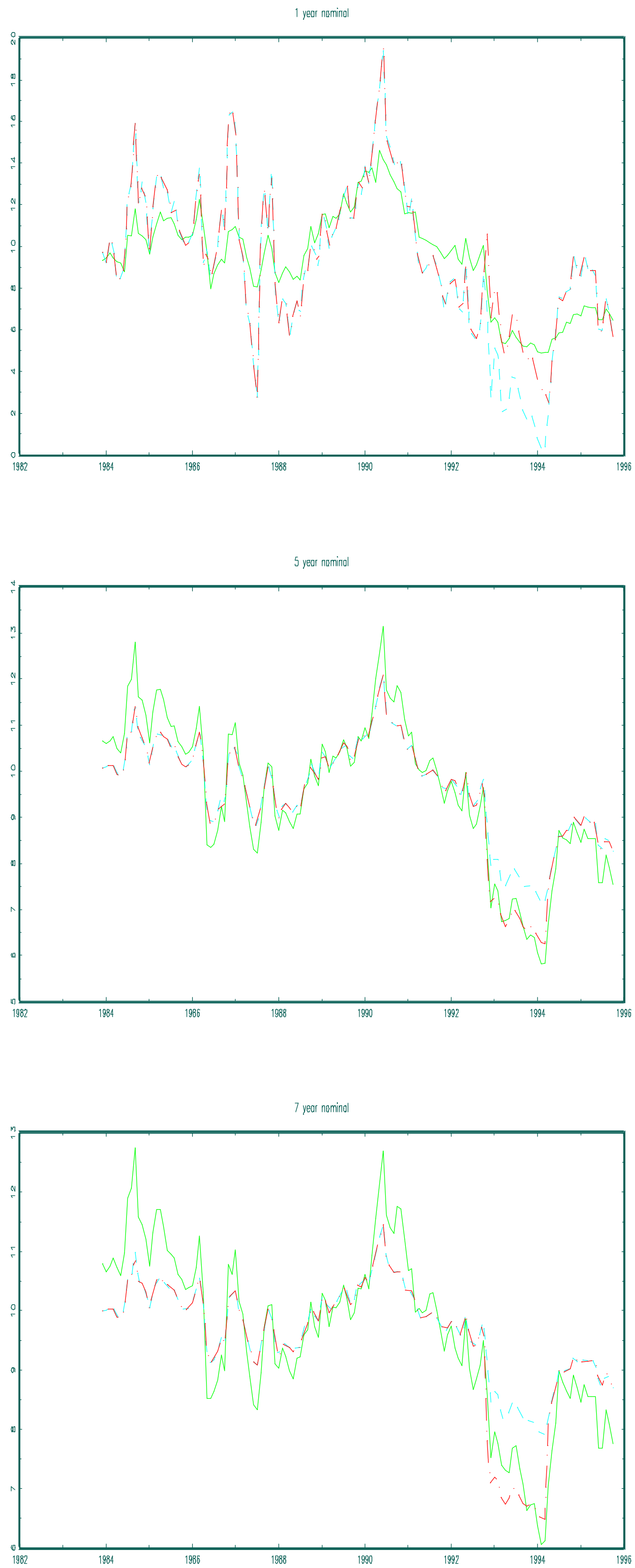

1 year nominal
5 year nominal
7 year nominal
1982
1984
1986
1988
1990
1992
1994
1996

Figure 2


Actual and Expected Inflation
0
2
4
6
8
10
12
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
Spread and Expected Inflation
-12
-8
-4
0
4
8
12
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
Inflation Risk Premium
-12
-8
-4
0
4
8
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996

Figure 3