

Electrodynamical model of quasi-efficient financial market

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Abstract

The modelling of financial markets presents a problem which is both theoretically challenging and practically important. The theoretical aspects concern the issue of market efficiency which may even have political implications [1], whilst the practical side of the problem has clear relevance to portfolio management [2] and derivative pricing [3]. Up till now all market models contain “smart money” traders and “noise” traders whose joint activity constitutes the market [4, 5]. On a short time scale this traditional separation does not seem to be realistic, and is hardly acceptable since all high-frequency market participants are professional traders and cannot be separated into “smart” and “noisy”. In this paper we present a “microscopic” model with homogenous quasi-rational behaviour of traders, aiming to describe short time market behaviour. To construct the model we use an analogy between “screening” in quantum electrodynamics and an equilibration process in a market with temporal mispricing [6, 7]. As a result, we obtain the time-dependent distribution function of the returns which is in quantitative agreement with real market data and obeys the anomalous scaling relations recently reported for both high-frequency exchange rates [8], S&P500 [9] and other stock market indices [10, 11].

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When a mispricing appears in a market, market speculators and arbitrageurs rectify the mistake by obtaining a profit from it. In the case of profitable fluctuations they move into profitable assets, leaving comparably less profitable ones. This affects prices in such a way that all assets of similar risk become equally attractive, i.e. the speculators restore the equilibrium. If this process occurs infinitely rapidly, then the market corrects the mispricing instantly and current prices fully reflect all relevant information. In this case one says that the market is efficient. However, clearly it is an idealization and does not hold for small enough times [12]. Here, we propose a “microscopic” model to describe the money flows, the equilibration and the corresponding statistical dynamics of prices.

The general picture, sketched above, of the restoration of equilibrium in financial markets resembles screening in electrodynamics. Indeed, in the case of electrodynamics, negative charges move into the region of the positive electric field, positive charges get out of the region and thus screen the field. Comparing this with the financial market we can say that a local virtual arbitrage opportunity with a positive excess return plays a role of the positive electric field, speculators in the long position behave as negative charges, whilst the speculators in the short position behave as positive ones. Movements of positive and negative charges screen out a profitable fluctuation and restore the equilibrium so that there is no arbitrage opportunity any more, i.e. the speculators have eliminated the arbitrage opportunity.

The analogy is apparently superficial, but it is not. It was shown in [6] that the analogy emerges naturally in the framework of the Gauge Theory of Arbitrage (GTA). The theory treats a calculation of net present values and asset buying and selling as a parallel transport of money in some curved space, and interpret the interest rate, exchange rates and prices of asset as proper connection components. This structure is exactly equivalent to the geometrical structure underlying the electrodynamics where the components of the vector-potential are connection components responsible for the parallel transport of the charges. The components of the corresponding curvature tensors are the electromagnetic field in the case of electrodynamics and the excess rate of return in case of GTA. The presence of uncertainty is equivalent to the introduction of noise in the electrodynamics, i.e. quantization of the theory. It allows one to map the theory of the capital market onto the theory of quantized gauge field interacting with matter (money flow) fields. The gauge transformations of the matter field correspond to a change of the

par value of the asset units which effect is eliminated by a gauge tuning of the prices and rates. Free quantum gauge field dynamics (in the absence of money flows) is described by a geometrical random walk for the assets prices with the log-normal probability distribution. In general case the consideration maps the capital market onto Quantum Electrodynamics where the price walks are affected by money flows.

To drop technicalities and put it in simple terms, we consider a composite system of price and money flows. In this model "money" represents high frequency traders with a short characteristic trading time (investment horizon) Δ (for the case of S&P500 below we use 0.5 min as the smallest horizon). The participants trade with each other and investors with longer time horizons. This system is characterized by the joint probability distribution of money allocation and price. If we neglect the money, the price obeys the geometrical random walk which is due to incoming information and longer time horizons traders. The trader's behaviour on time step Δ at price S is described by the decision matrix of non-normalized transition probabilities [6]:

$$\pi(\Delta) = \begin{pmatrix} 1 & S^{\beta(\Delta)} \\ S^{-\beta(\Delta)} & 1 \end{pmatrix} \quad (1)$$

where the upper row corresponds to a transition to cash from cash and shares and lower row gives corresponding probabilities for a transition to shares. The parameter β is a fitting parameter playing the role of the effective temperature. At this stage different traders are independent of each other. We introduce an interaction by making hopping elements depending additionally on change in traders configuration. This interaction models the "herd" behaviour for large changes and mean-reversion anticipation for small changes. Each trader possesses lot of shares or the equivalent cash amount. The formulation of the model is completed by saying that the transition probability for the market is a product of the geometrical random walk weight for price and the matrices (1) for each participant.

The matrix $\pi(\Delta)$ has exactly the same form as the hopping matrix for charged particles in Quantum Electrodynamics. This form can also be derived from the assumption that traders want to maximize their profit [6]. This seems to be realistic for the small times that we are interested in.

It can be shown that a model with just one time horizon cannot correctly describe effects which have characteristic times of more than 5 minutes. To

improve the model we have to include traders with other characteristic time horizons. It is known that there are conventional intra-day time horizons like 1, 10, 30, 60, 480 mins which, however, have certain measure of idealization. We use a continuous set of time horizons between 30 seconds and 480 minutes to describe the spread and uncertainty in the time horizons definition. It means that the model contains a set of money flows defined by the matrices (1) with the corresponding parameters Δ and β . The suggested model thereby consists of the Fractional Market Hypothesis (FMH) [13] (which states that a stable market consists of traders with different time horizons but with identical dynamics) and the “microscopical” electrodynamical model for the dynamics. Hence the FMH substitutes in our approach the information cascade suggested recently to explain the scaling properties of the \$/DM exchange rate [8].

The use of the FMH is not the only feature of this model which differs from the ones proposed earlier. The other feature is the homogeneity of the traders set. In earlier models traders have always been divided into “smart” (who trade rationally) and “noisy” (who follow a fad) [4, 5]. We believe that for the consideration of short times trades this differentiation is not appropriate. Indeed, all high-frequency market participants are professional traders with years of experience. Unsuccessful traders quickly leave the market and do not affect the dynamics. At the same time, each of the traders has their own view on the market and their own anticipations. That is why their particular decision can be only modeled in a probabilistic way. In this sense the traders are not strictly rational but “quasi-rational” and the corresponding market where the *quasi*-rational investors deal, can be called a quasi-effective market.

Let us turn to the results. First of all, the constructed model allows us to explain quantitatively the observed high-frequency return data. In Ref [9] Figs.1,2 show the form of the distribution function for changes in the S&P500 market index, which is a price of the portfolio consisting of the main 500 stocks traded on the New York Stock Exchange. The changes in price have been normalized by the standard deviation. In the approximation that the changes are much smaller than the index itself, which is obeyed with very high accuracy, the distribution function of the normalized changes can be considered as the distribution function of the return on the portfolio, normalized by the standard deviation of the return. The return on the portfolio during the period Δ is defined as $r(\Delta) = (S(t + \Delta) - S(t))/S(t)$. In Ref [9] it was also shown that the distribution function obeys the scaling property and

that this property is reflected in the dependence on time of the probability to return to the origin. It was demonstrated that for a time period between 1 min and 1000 min (two trading days) the probability decrease as $t^{-\alpha}$ with the exponent $\alpha = 0.712 \pm 0.25$ (see Fig.1). Similar results have been obtained in Ref. [8] for the high-frequency return for the \$/DM exchange rate with slightly different values of the exponent. We choose parameters of our model to get the correct scaling behaviour and define $\beta(\Delta)$ as $\beta(\Delta) = 30/\Delta^{0.71}$. We also take the number of traded lots infinite. Fig.1 clearly demonstrates the correct scaling property of the model constructed above and gives the same scaling exponent $\alpha = 0.71$ which is technically due to the scaling form of $\beta(\Delta)$. Now we can plot also the probability distribution function of returns for S&P500 as depicted on Fig.2. The same analysis leads to similar results for the \$/DM exchange rate [8] with slightly different values of the parameters. It is easy to see that the theoretical and observed distribution functions coincide exactly with the observed data accuracy.

The shape of the distribution function does not characterize it completely. Indeed, a similar form can be obtained using the GARCH/ARCH models [14, 15], which still are phenomenological rather than microscopic. However, those models cannot explain the scaling properties obeyed by the real data [8, 9]. In the proposed model both the shape of the distribution function and scaling properties are presented. It is interesting to add that the swings of the real data on Fig.1 can be interpreted as a sign of the inhomogeneity of the distribution of traders across time horizons, with a larger number of traders on 15 mins, 60 mins and a day investment horizons.

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Figure caption

FIG.1 Theoretical (solid line) and experimental (squares) probability of return to the origin (to get zero return) $P(0)$ as a function of time. The slope of the best-fit straight line is -0.712 ± 0.025 [9]. The theoretical curve converges to the Brownian value 0.5 as time tends to one month.

FIG.2 Comparison of the $\Delta = 1$ min theoretical (solid line) and observed [9] (squares) probability distribution of the return $P(r)$. The dashed line (long dashes) shows the gaussian distribution with the standard deviation σ equal to the experimental value 0.0508. Values of the return are normalized to σ . The dashed line (short dashes) is the best fitted symmetrical Levy stable distribution [9].



