

# How do Firms Choose their Lenders? An Empirical Investigation

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June 5, 1997

## Abstract

This article investigates which firms borrow directly from the capital markets and which raise funds through intermediaries. Our empirical results show that large companies with abundant cash and collateral tap the credit markets directly. These markets cater to safe and profitable industries, and are most active when riskless rates or intermediary earnings are low. We show that determinants of lender selection sharpen during investment downturns and that there are substantial asymmetries in the way firms enter and exit capital markets. These results support a theoretical framework where intermediaries have better reorganizational skills but a higher cost of capital than bondholders.

(JEL E44, G20, G31, G32)

**Keywords:** Financial Intermediation, Bank Loans, Bonds, Commercial Paper, Bankruptcy, Reorganizations, Costly State Verification

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## How do Firms Choose their Lenders? An Empirical Investigation

Debt, the most common source of external funds for American corporations, is classified in many ways: short versus long term, secured versus unsecured, or publicly traded versus privately held. This article explores the relevance of this last distinction: do firms borrow from bondholders or intermediaries in a systematic way? We suspect that this is an important question, given how extensively these obligations are used: at the end of 1992 corporate commercial paper and publicly traded bonds stood at about \$1 trillion, while bank loans, private placements, business mortgages, and other intermediated debt represented over \$2.4 trillion. Wright (1995) has also shown that bank loans and private placements are highly procyclical, while bond and commercial paper issues are quite countercyclical; understanding how firms choose their lenders may then unveil the mechanisms through which recessions and booms propagate and persist in the economy.

Despite the topic's appeal, there are very few studies that analyze how companies choose their lenders; this is no doubt related to the absence of corporate databases that classify debt into publicly traded or privately held. For this study we gathered information about the way in which 291 mature firms issued different types of obligations between 1974 and 1992. We have less extensive data on another panel of 5554 corporations that we track from 1985 to 1992. The article examines how these firms choose their lenders in light of a simple theoretical model.

Our model, which is a variation of Cantillo (1996), assumes that intermediaries can reorganize firms faster than bondholders, but that bondholders have a lower cost of capital than intermediaries. These particular attributes will dominate in different circumstances: safe companies prefer to tap the bondmarket directly since they are unlikely to default, and only want to bypass a costly middleman. Firms with more uncertain prospects, on the other hand, need the intermediary's reorganization skills more frequently and for this reason borrow from

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banks. The model predicts that large, profitable firms, with a high proportion of tangible assets, and high or stable cash flows borrow from bondholders. Companies will also go to the bondmarket when real interest rates or intermediary earnings are low. Our empirical analysis not only confirms these theoretical predictions, but also shows that the distinction between privately held and publicly traded debt is relevant across different maturities.

Observing firms for two decades allows us to explore a number of time related issues. First, we show that the determinants of lender selection are more critical during investment downturns; this suggests that there is a flight to quality during recessions. Another time-related question is whether companies enter and exit the markets for publicly traded debt asymmetrically. Using a simple Markovian econometric model, we find that once a firm has entered the markets for publicly traded debt, it will stay there even after its attributes have fallen well below the original entry threshold. One possible explanation for this is that widespread information about a firm gives it an edge over otherwise identical companies.

Our theory is not new; in fact, Rajan (1992) and Bolton and Sharfstein (1996)'s arguments could justify why intermediaries are good reorganizers. We should admit, however, that our framework inevitably stresses one perspective at the expense of some other. A different, if somewhat complementary view is given by Diamond (1991), Besanko and Kanatas (1993), and Holmstrom and Tirole (1994), who argue that intermediaries are good at preventing managers from investing in suboptimal projects. In Holmstrom and Tirole's model, for example, an entrepreneur can approach a 'generic' or a 'monitoring' investor for funds. The 'monitor' is also called an intermediary because it lends in coalition with generic investors. In Holmstrom and Tirole's setup, a firm with few misaligned incentives avoids intermediaries, since these require a fixed fee for a monitoring effort that will add little value. Holmstrom and Tirole's argument is an appealing description of venture capitalists, who as equity intermediaries are quite involved in the day-to-day affairs of the companies they fund. Their

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argument is more difficult to make for banks or other debt intermediaries, who shun active involvement with non-bankrupt firms since this may subordinate their loans<sup>1</sup>.

Previous empirical studies have used proxies (such as whether a firm has a bond or commercial paper rating) to analyze how companies choose their lenders. Mackie-Mason (1990) showed that large corporations with high R&D and advertising expenditures, a high fraction of tangible assets, high or stable cash flows, high leverage, or paying dividends are more likely to issue bonds than bank loans. Calomiris et al. (1994) showed that large corporations with high and stable cash flows, low inventories, and a low sales-to-capital ratio are more likely to have a commercial paper or a high grade bond rating. The main difference between our study and these others is that our data allows us to explore firms' debt choice using not proxies but actual debt figures that span almost two decades; this enables us to confirm previous results and to show that time and history are important elements that affect how firms choose their lenders. Our theoretical model also allows us to see the empirical results within one unified framework, and to formulate alternative hypotheses more clearly.

The paper's layout is as follows: section 1 presents the theoretical model, its predictions, and an implementable procedure to test them. Section 2 describes our data and its origins. Section 3, the heart of our investigation, begins by looking at how mature and young firms choose their lenders; it then analyzes the economics of creditor selection in more detail. Section 3 also studies whether the determinants of lender choice are stable during business cycles, and the asymmetric nature of entry and exit in the markets for publicly traded debt. The article concludes with a discussion of our results and of future research avenues.

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<sup>1</sup> Weintraub, B.; Resnick, A. (1980), ¶ 5.14, note 247.

## 1 Theoretical Framework

This section develops a model of lender selection and describes a procedure to test it econometrically. In our model, an entrepreneur can approach either bondholders or banks for funds. Although bondholders have a low cost of capital, their reorganization skills are limited. Banks can reorganize distressed firms cheaply but have a high cost of capital. Proposition 1 shows that in this setup companies will sort out according to their attributes, so that “safe” corporations tap the bondmarket directly while “risky” firms borrow from banks. Proposition 2 develops a straightforward mechanism to assess a firm’s choice of lenders based on its observable attributes.

### 1.1 Assumptions

Consider an economy with three types of agents: entrepreneurs, bondholders, and banks; these parties write financial contracts at time  $t = 0$ .

*Entrepreneurs* are risk neutral and have an opportunity cost of capital  $r_f$ . Entrepreneurs have a project with attributes  $x = [v \ e \ z \ \mu \ \sigma \ r_f \ \phi]$ , where  $v$  is the proportion of tangible assets in the project;  $e$  is the fraction of the project that is internally financed;  $z$ ,  $\mu$ , and  $\sigma$  are the project’s size, profitability, and risk<sup>2</sup>;  $r_f$  is the gross riskless rate and  $\phi$  is the bank’s capital.

Since the project requires an investment  $z$ , and since the entrepreneur only has  $ze$  internal funds, he must borrow  $z[1 - e]$  from outside investors. The project’s expected payoff is  $z\mu$ ; its actual payoff,  $z\mu s \in [0, \infty)$ , is realized at  $t = 1$ . The stochastic factor  $s$  has an expected value of one and a density function  $f(s)$  satisfying the increasing hazard rate property<sup>3</sup>. The

<sup>2</sup> In addition to the standard definition of increasing risk by Rothschild and Stiglitz (1970), we assume that the hazard rate increases for higher risk, so  $\rho_\sigma(s) > 0$ .

<sup>3</sup> In fact, the necessary assumption is that  $s\rho(s)$  increases with  $s$ ; we will use the monotone hazard rate property (MHRP) assumption for expositional purposes. The MHRP is widely used in models of incentive contracts. See Fudenberg and Tirole (chapter 7) for more on this.

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hazard rate is defined as  $\rho(s) \equiv f(s)/[1 - F(s)]$ .

*Bondholders*, whose type is indicated by  $\bar{\theta}$ , are competitive and risk neutral. Their opportunity cost of capital is  $r_f + \pi(x; \bar{\theta})$ . We will assume for simplicity's sake that  $\pi(x; \bar{\theta}) = 0$ .

*Banks*, whose type is indicated by  $\underline{\theta}$ , are competitive and risk neutral. We will assume that banks have a higher cost of capital than bondholders, i.e., that banks' capital cost is  $r_f + \pi(x; \underline{\theta})$  with  $\pi(x; \underline{\theta}) > 0$ : this is formally proven by Cantillo (1996), who shows that the agency costs between an intermediary and its depositors are like an added cost of capital. Following Cantillo we also assume that a bank's cost of capital falls as its internal funds increase, so  $\pi_\phi(x; \underline{\theta}) < 0$ , and that other variables have no impact on  $\pi(x; \underline{\theta})$ .

*Verification costs*: Although outside investors can intervene a firm to verify its revenues, this will delay any appropriation of funds to time  $t = 1 + \Delta(\theta, x)$ , where the delay depends on the project's attributes and the lender's type. Gilson et al. (1990) have documented that private and court-administered reorganizations are quite prolonged, taking on average 15 and 28 months to complete. These delays are costly if investors and entrepreneurs get substandard returns in the interim. In particular, one can show that the payoff (in present value terms) of an intervened firm is a fraction of the original income  $z\mu s^4$ . Franks and Torous (1994) and Tashjian et al. (1996) have shown that recovery rates in private reorganizations, in prepackaged bankruptcies, and in formal bankruptcies are 80, 73, and 51 percent; these results imply not only that intervention is quite costly, but that reorganization costs vary considerably for different procedures.

In reduced form, the model we have just described is identical to a costly state verification (CSV) setup with proportional costs. The CSV framework, introduced by Townsend (1979) and used later by Gale and Hellwig (1985), is based on the simple yet powerful argument

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<sup>4</sup> Formally, if interim returns are  $\exp(-k)[r_f + \pi(x; \theta)]$ , with  $k > 0$ , then the payoff at  $t = 1 + \Delta(\theta, x)$  is  $z\mu s [\exp(-k)[r_f + \pi(x; \theta)]]^{\Delta(\theta, x)}$ , and its present value from the perspective of  $t = 1$  is  $\frac{z\mu s [\exp(-k)[r_f + \pi(x; \theta)]]^{\Delta(\theta, x)}}{[r_f + \pi(x; \theta)]^{\Delta(\theta, x)}} = z\mu s \exp[-k\Delta(\theta, x)] \leq z\mu s$

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that when outside investors have less information than insiders, their intervention is costly. In such an environment, the optimal financial instrument is the one that minimizes outside interference, i.e., debt. In a standard debt contract, the entrepreneur makes a fixed payment  $D$  in good states. Bankruptcy is triggered whenever the project's revenues fall below the face value of debt, i.e., when  $s < \frac{D}{z\mu} \equiv b$ . In default, the entrepreneur is intervened by his creditors and forced to pay as much as he can.

Since we have some flexibility in our formulation, we will define  $\theta\delta(x) < 1$  as the fraction of the firm's revenues that are lost in a reorganization. Thus, verification costs are:

$$c(s, x; \theta) = \theta\delta(x)z\mu s \text{ with } \delta_z(x) < 0, \delta_v(x) < 0, \delta_{x_i}(x) = 0 \text{ for other } x_i$$

We will assume that banks can reorganize projects faster than bondholders, so  $\underline{\theta} < \bar{\theta}$ . Indeed, Gilson et al. find that companies who use bank debt are more likely to reorganize privately (i.e., faster and cheaper) than firms who use publicly traded debt. Brown et al. (1992) point out some reasons why bondholders shun private reorganizations:

The literature focuses on two reasons private renegotiation may fail, asymmetric information and a holdout problem. The holdout problem is particularly acute when recontracting with public bondholders because of [...] the Trust Indenture Act of 1939 and the fact that publicly traded debt tends to be diffusely held. The Trust Indenture Act requires that all bondholders approve any alteration of their contract. Thus, while it is in the collective interest of bondholders to restructure, individually they have the incentive to hold out. Bankruptcy law, on the other hand, mitigates the holdout problem by, for example, allowing a reorganization of public debt with only the approval of two thirds of the bondholders.

The second (related) reason why firms may fail to restructure their debt privately is that bondholders are poorly informed about the firm's future prospects. In particular, [...] a so called lemons problem can develop in which bondholders expect management to misrepresent the firm's prospects, so that management can 'cut' a good deal in the private restructuring. In short, it is in shareholders' or management interest to always say that the firm's value is low to reduce the amount bondholders receive in the restructuring. Since information asymmetries are likely to be much less important in a court supervised bankruptcy, bondholders prefer bankruptcy to private renegotiation. In contrast, if private lenders (such as banks and insurance companies) are better informed than public debt holders, they will

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be less likely to use the courts to determine firm value<sup>5</sup>

We assume that intervention is shorter for larger projects ( $\delta_z(x) < 0$ ) as Gilson et al. (1990) have documented. This implies that although reorganization costs may rise with size, *average* intervention expenses fall for larger projects. We also assume that projects with more tangible assets  $v$  need shorter interventions, so  $\delta_v(x) < 0$ ; the intuition for this is that since tangible assets have thicker secondary markets, their disposal takes less time. Finally, and to maintain tractability, we assume that if bondholders and banks team up in a loan, their intervention will last as long as if it had been composed only by bondholders. This assumption rules out simultaneous bank and bond issues; although this is clearly an idealization, our data will confirm that this is true in almost 90 percent of the firm-year observations.

### 1.2 Definition and Characterization of Lending Equilibria

The entrepreneur and lender economic profits  $V(x; b)$  and  $U(x; \theta, b)$  are defined in equations (1) and (2). These functions are all scaled by size.

$$V(x; b) = \mu \int_b^\infty [s - b]f(s)ds - er_f = \mu \int_b^\infty [1 - F(s)]ds - er_f \quad (1)$$

$$U(x; \theta, b) = \mu b[1 - F(b)] + [1 - \theta\delta(x)] \int_0^b \mu s f(s)ds - [1 - e][r_f + \pi(x; \theta)] \quad (2)$$

$\mu b$  stands for the face value of debt and  $b$  for the point at which an entrepreneur defaults.  $b$  will presently become our focus of interest; let us begin by looking at its marginal effect on the lender's economic profit:

$$U_b(x; \theta, b) = \mu[1 - F(b)] - \theta\delta(x)\mu b f(b) \equiv \mu[1 - F(b)][1 - \theta\delta(x)b\rho(b)] \quad (3)$$

An increasing hazard rate  $\rho(s)$  implies that  $U(x; \theta, b)$  is single peaked and quasiconcave with respect to  $b$ . In plain words, lenders cannot raise interest rates indefinitely, since marginal

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<sup>5</sup> Brown, James, Mooradian (1992) pp. 125-126

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lending costs will eventually outweigh marginal benefits. We define a lending equilibrium as:

**Definition 1 (*Lending Equilibrium*)**

An equilibrium  $b(x; \theta)$  is the lowest bankruptcy point where a lender breaks even and an entrepreneur makes positive profits:  $b(x; \theta) = \min\{b' : U(x; \theta, b') = 0 \text{ and } V(x; b') \geq 0\}$ .

When there are many potential lenders, an entrepreneur selects the one who offers the lowest interest rate, or equivalently the lowest  $b$ . Let us define  $X$  as the domain of attributes and  $S(\theta) \subseteq X$  as the subset of attributes where a lending equilibrium exists. Proposition 1 characterizes the bank and bond lending equilibria.

**Proposition 1** Define  $S(\theta) \subseteq X$  as the set of attributes where a lending equilibrium exists:

1. If  $(x_i, x_{-i}) \in S(\theta)$  then all  $(x'_i, x_{-i}) \in S(\theta)$  where  $x'_i$  satisfies  $(x'_i - x_i)U_{x_i} \geq 0$ .
2.  $S(\bar{\theta}) \subset S(\underline{\theta})$  if  $\pi(x; \underline{\theta}) \leq [\bar{\theta} - \underline{\theta}]\delta(v, z) \int_0^{\hat{b}} s f(s; x) ds$
3. Equilibrium rates  $b(x; \theta)$  satisfy the single crossing property,  $\text{sgn}(\frac{d^2 b}{dx_i d\theta}) = \text{sgn}(b_{x_i}) = \text{sgn}(-U_{x_i})$ .

Figure 1: Single Crossing Property for Lender Rates

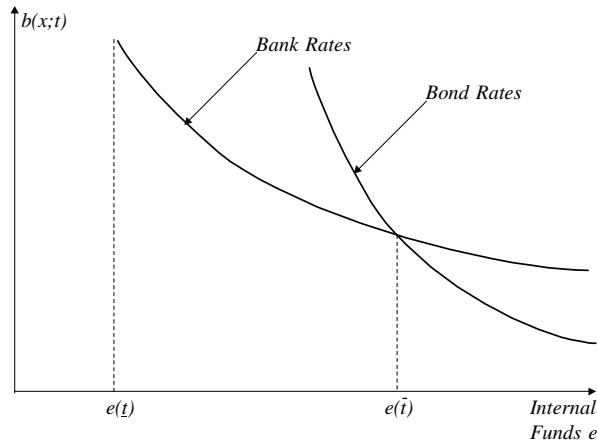


Figure 1 illustrates the proposition’s results; it plots the rates charged by banks and bondholders to entrepreneurs with internal funds  $e$ . Proposition 1.1 demonstrates that if a company with funds  $e'$  is not rationed, then all firms  $e > e'$  are not rationed either.

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Proposition 1.2 ensures that banks can lend wherever bondholders can; Cantillo (1996) arrives at this result with an endogenously generated premium. Proposition 1.3 says that banks and bondholders charge less to cash rich corporations *and* that bondholders cut their rates faster than banks. This is the single crossing property, which is central for our analysis, since it neatly separates bank and bond financed companies in two well defined sets. For instance, cash rich corporations tap the bondmarket directly because they rarely default and need little verification; in these circumstances bondholders can offer a better interest rate than intermediaries, since all that matters is who has the lowest cost of capital. Companies with moderate resources, on the other hand, cannot afford to switch away from banks since eliminating these middlemen induces more damaging verification at the hand of bondholders.

### 1.3 An Implementable Solution:

How can we exploit our theoretical results? One way is to construct a decision function  $d(x)$ , which computes bondholders' payoff if they were to match the competitive rate  $\underline{b}$  charged by banks.  $d(x)$  is defined in equations (4) and (5):

$$d(x) \equiv \frac{U(x; \bar{\theta}, \underline{b})}{1 - e} = \pi(x; \underline{\theta}) - \frac{[\bar{\theta} - \underline{\theta}] \mu \delta(x) \int_0^{\underline{b}} s f(s) ds}{1 - e} \quad \forall x \in S(\underline{\theta}) \quad (4)$$

$$U(x; \underline{\theta}, \underline{b}) = \mu \underline{b} [1 - F(\underline{b})] + [1 - \underline{\theta} \delta(x)] \int_0^{\underline{b}} \mu s f(s) ds - (1 - e) [r_f + \pi(x; \underline{\theta})] \equiv 0 \quad (5)$$

$d(x) > 0$  means that bondholders are making a profit where banks are just breaking even. Since bondholders can underprice banks in this situation, it is clear that a firm with attributes  $x$  taps the bondmarket directly. Proposition 2 shows that, under some regularity conditions, the demand for securities  $h(x)$  is closely related to the decision function  $d(x)$ . We define  $h(x)$  to take a value of 1 if a firm borrows from bondholders and of 0 if it borrows from banks.

**Proposition 2** *Assume  $U_b(x; \bar{\theta}, \underline{b}) > 0$  for  $x \in S(\bar{\theta})$ . In that case  $h(x) = 0$  if  $d(x) < 0$  and  $h(x) = 1$  if  $d(x) > 0$  for all  $x \in S(\underline{\theta})$ .  $d_{x_i}(x) > 0$  for  $x_i = z, v, e, \mu$  and  $d_{x_i}(x) < 0$  for  $x_i = \sigma, r_f, \phi$*

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Figure 2: Decision variable  $d(x)$  as a function of firm attributes

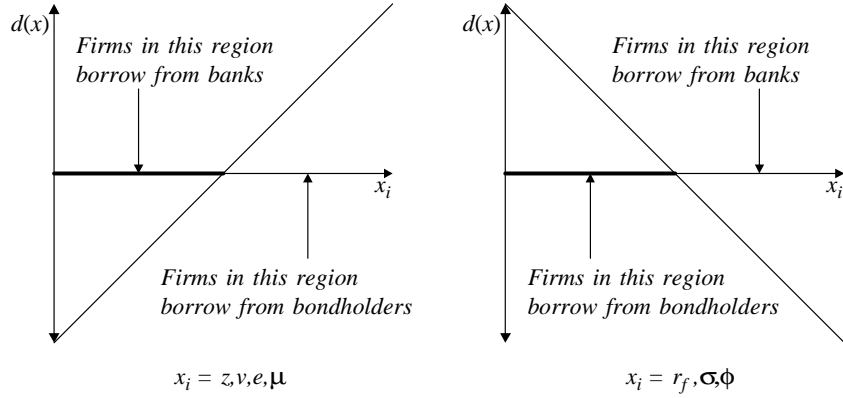


Figure 2 illustrates the proposition's results, i.e., that large corporations with a high proportion of tangible assets, high cash flow or profitability borrow directly from the bondmarket since  $d_{x_i}(x) > 0$  for  $x_i = z, v, e, \mu$ . In contrast, firms with risky projects or operating in an environment with high real interest rates and high intermediary earnings borrow from banks, since  $d_{x_i}(x) < 0$  for  $x_i = \sigma, r_f, \phi$ . Section 3 examines whether our theoretical framework can explain the actual choices made by companies. Before, however, we need to describe the nature and origins of our data.

## 2 Data Description

We will work with two data sets. First, there is a balanced panel of 291 corporations with uninterrupted annual data from 1974 to 1992, which we call the older set. This set contains extensive time and cross-sectional data not available elsewhere, and is composed of mature firms. The second set is an unbalanced panel of 5554 companies with at least one year of data between 1985 and 1992. Although the newer data set is not as rich as the older, it will help as a 'control', i.e., as a way to examine if there are important behavioral differences between young and mature firms.

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**Selection of the Older Set:** This set contains 291 companies with continuous information from 1974 to 1992, or 5529 firm-year observations. To select these corporations we excluded firms in agriculture, public utilities, transportation, financial services, and industries with SIC codes 8000 or higher - which include government, legal, health and educational services; this left 5775 companies. We then required that firms have uninterrupted data from 1974 to 1992 on basic balance sheet items (sales, earnings, liabilities, etc.); this reduced the sample to 576 firms, and filtered out young corporations. The third step was to exclude firms with significant merger or acquisition activities; we defined this as a change of 25 percent or more in the company's gross physical capital stock for reasons other than physical investment or retirements. Thus, we excluded corporations when  $|K_t - K_{t-1} - I_t + R_t| > 0.25K_{t-1}$  where  $K_t$  is the gross book value of physical capital,  $I_t$  is physical investment and  $R_t$  denotes retirements; this left 320 firms, less 29 that were not in Moody's manuals to total 291 companies. It is important to stress that our set is composed of mature corporations, whose data is publicly available to the capital markets. It is also important to mention that though many of these firms are not entrepreneur controlled (as the CSV model would imply), they rarely finance themselves with equity, as our model implies. The following variables were obtained only for the older set :

*Publicly traded and privately held debt outstanding*, was obtained from Moody's manuals. Moody's itemizes, rates, and discusses every public debt issued by a firm. Private placements, capitalized leases, commercial paper, or industrial revenue bonds are sometimes itemized, in which case Moody's makes this clear. Finally, Moody's lists *other debt* which may be subdivided into loans, mortgages, capitalized leases, industrial revenue bonds, and foreign bond issues. We coded as publicly traded debt those issues that Moody's itemized and where it made clear that it was not a private placement, an industrial revenue bond, a capitalized lease, or commercial paper. We added to this any international issue of publicly traded

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bonds which are normally listed in Moody's *other debt* section. We classified as privately held all remaining items, with the exception of commercial paper, industrial revenue bonds, and capitalized leases. These items were excluded from our measures of long term debt because they are either short term (e.g., commercial paper) or their nature - whether they are publicly traded or privately held - is unclear.

The ratio of relative bond usage is  $m_{it} = D_{it}^{\bar{\theta}} / (D_{it}^{\bar{\theta}} + D_{it}^{\theta})$ , where  $D_{it}^{\bar{\theta}}$  and  $D_{it}^{\theta}$  are the publicly traded and privately held long term debt outstanding. A firm is borrowing solely through bonds when the ratio is one.

*Age* is the difference between 1992 and the company's earliest recorded establishment date in Moody's historical summary.

We obtained variables indicative of a firm's corporate governance to appraise the importance of alternative monitoring theories of banking.

*Family control dummy* takes a value of 1 if the company is controlled by a family. We constructed this variable by looking at the 1974 and 1992 board of directors in Moody's manuals. We defined a firm as family controlled if there were two or more members of the 1992 board with the same last name, or if the 1974 and 1992 board each had a member with the same last name, but different first name.

The *concentration of large shareholders* was obtained from Compact Disclosure and proxy statements for 1994. It aggregates the stakes held by large equityholders (i.e., shareholders owning more than 5% of the firm's class A stock)

The *concentration of institutional investors* was obtained from Compact Disclosure and proxy statements for 1994. It computes the concentration of class A shares held by institutional investors, as defined in Compact Disclosure.

**Selection of the Newer Set:** We retrieved all corporations listed in Compustat between 1985 to 1992. We excluded the same industries as in the older set, and we ruled out

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mergers as before. However, we did not require that companies have uninterrupted published information. This left 5554 firms with at least one observation between 1985 and 1992. The following variables were obtained from Compustat for the newer and the older sets.

*The bond rating dummy* equals one if a firm has a bond rating. When a corporation is rated, it almost always has a positive amount of publicly traded debt. In the older set there were only 18 out of 5529 observations where a company had a bond rating and no publicly traded debt. There were also 135 observations where a firm had some public debt and no bond rating. Since these two deviations represented less than 3 percent of the observations, we will assume that a bond rating is equivalent to having some publicly traded debt outstanding.

The *commercial paper rating dummy* equals one if a firm has a commercial paper rating.

*Size* is defined as the logarithm of deflated sales. Our deflator is the average producer price index, which takes a value of 1 in 1990.

*Cash Flow* is defined as “operating income” scaled by sales. Operating income equals sales less general expenses less cost of goods sold less taxes plus extraordinary items. We did not subtract interest payments from sales since this is probably an endogenous variable, inasmuch as firms choose their leverage and lenders simultaneously.

One problem with this untransformed variable are the outliers that affect corporations with low sales; such companies usually report losses and thus generate very large negative values. It may also happen that firms have low sales and a high income (perhaps from extraordinary items); this will produce positive outliers. In either case, something has to be done to deal with this variable in the new set, since its 1992 skewness and kurtosis were -62.4 and 4808. Our solution was to transform cash flow logistically, so that this variable would now lie between 0 and 1, since  $e = 1/[1 + \exp(-CF_{it})]$ . Even though the old set had no serious outliers, we transformed cash flow in both samples to be consistent.

As a robustness check, we ran regressions using the untransformed cash flow, but capping

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or trimming the outliers; our results were then unchanged.

*Profitability and Risk:* We calculated the median cash flow - the cash flow of the typical company - for each two digit SIC industry group, using the firms in our newer set on a year by year basis. Industry profitability was defined as the median industry cash flow, averaged between 1985 and 1992. The standard deviation of this gave us a measure of risk by industry group. We matched a firm's profitability and risk with the profitability and risk of its associated two-digit SIC industry group.

*Tangible assets* were defined as net property plant and equipment as a fraction of total assets. Both variables are measured at book value and taken at the beginning of the company's fiscal year.

Table 1: Summary of Variables Used

Variable	Description
<b>Ratio<sub>t</sub></b> ( $m_{it}$ ) (*)	% of long term debt held in publicly traded instruments
<b>Bond Rated<sub>t</sub></b>	Dummy variable, equals 1 if the firm has a bond rating
<b>Commercial Paper Rated<sub>t</sub></b>	Dummy variable, equals 1 if the firm has a c.p. rating
<b>Size<sub>t</sub></b>	log(Sales)
<b>Cash Flow<sub>t</sub></b>	Operating income divided by sales, transformed
<b>Industry Profitability</b>	Median Cash flow of the two digit SIC industry
<b>Industry Risk</b>	St. Deviation cash flow of the two digit SIC industry
<b>Tangible Assets<sub>t</sub></b>	Property, plant & equipment as a fraction of total assets
<b>Real Interest Rate<sub>t</sub></b>	One year T-Bill yield less expected inflation
<b>Financial Int. Earnings<sub>t</sub></b>	Intermediary undistributed profits over total assets
<b>Family Controlled</b> (*)	Dummy variable, equals 1 if the firm is family controlled
<b>Large Shareholders</b> (*)	% of shares held by 'insiders' as defined by SEC
<b>Institutional Investors</b> (*)	% of shares held by institutional investors
<b>Age</b> (*)	1992 less earliest recorded establishment date

(\*) Available only for the older set. All variables with subscript t vary year by year

The following variables were constructed from the Federal Reserve:

*Risk free real rate* is defined as the one year T-Bill rate less the one year expected inflation.

We took the Livingston index of inflation expectations from the Federal Reserve Bank of Philadelphia, and adjusted it to be a one year forecast. The interest rate series is matched to the end of each firm's fiscal year.

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*Intermediary earnings*, from the Federal Reserve's flow of funds accounts, are defined as undistributed profits for *all* financial intermediaries as fraction of their beginning-of-year assets.

### 3 Empirical Results

This section investigates how firms choose their lenders, and whether this fits the theoretical model we have developed. Section 3.1 compares the behavior of mature and young companies. Section 3.2 investigates the economics of lender selection, the stability of the determinants of lender choice during business cycles, and the entry and exit into the bond market. Section 3.3 repeats this analysis for short term obligations.

#### 3.1 Lender Selection by Mature and Young Firms

This subsection investigates if there are important differences in the way mature and young companies select their lenders. To give an idea of the static differences between the older and newer data sets, table 2 shows summary statistics for 1992. These numbers reveal that even firms which are large enough to be tracked by Compustat rarely use publicly traded debt. In 1992, the median corporation in the newer set had \$ 67 million in sales, and yet less than 15 and 7 percent of these firms had a bond or commercial paper rating. For the older set, 32 and 28 percent of the companies had a bond or commercial paper rating in 1992. The difference is due to the fact that the firms in the older set are more mature, larger, and have a higher proportion of tangible assets than companies in the newer set. For example, the median age of a firm in the older set is 66 years. The fact that a corporation survives twenty years and is thus in the old set tells us much about its default rate; it is low. It is clear that there are significant *static* differences between our data sets; we have yet to test if there are important *behavioral* disparities between mature and young firms.

To test whether old and young firms have different sensitivities to corporate attributes,

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Table 2: Summary Statistics for the Old and New Data Sets, 1992

<b>Older Set, 291 Firms</b>	<b>Mean</b>	<b>Median</b>	<b>Min.</b>	<b>Max.</b>	<b>Std. Dev.</b>
<b>Bond Rated</b>	0.320	0.000	0.000	1.000	0.467
<b>Commercial Paper Rated</b>	0.271	0.000	0.000	1.000	0.446
<b>Ratio <math>m_{it}</math></b>	0.226	0.000	0.000	1.000	0.358
<b>Size</b>	6.246	6.279	1.182	10.94	1.866
<b>Cash Flow</b>	0.525	0.523	0.462	0.655	0.018
<b>Industry Profitability</b>	0.521	0.519	0.506	0.563	0.009
<b>Industry Risk</b>	0.002	0.002	0.001	0.019	0.003
<b>Tangible Assets</b>	0.414	0.376	0.066	0.919	0.166
<b>Leverage</b>	0.214	0.194	0.000	1.243	0.160
<b>Family Controlled</b>	0.461	0.000	0.000	1.000	0.499
<b>Large Shareholders</b>	0.358	0.291	0.000	1.000	0.267
<b>Institutional Investors</b>	0.449	0.489	0.000	0.887	0.227
<b>Age</b>	66.97	66	22	155	30.33
<b>Newer Set, 3194 Firms</b>	<b>Mean</b>	<b>Median</b>	<b>Min.</b>	<b>Max.</b>	<b>Std. Dev.</b>
<b>Bond Rated</b>	0.142	0.000	0.000	1.000	0.349
<b>Commercial Paper Rated</b>	0.067	0.000	0.000	1.000	0.250
<b>Size</b>	4.191	4.206	-4.43	11.78	2.520
<b>Cash Flow</b>	0.496	0.520	0.000	1.000	0.116
<b>Industry Profitability</b>	0.525	0.519	0.506	0.624	0.018
<b>Industry Risk</b>	0.003	0.001	0.001	0.035	0.005
<b>Tangible Assets</b>	0.299	0.227	0.000	0.989	0.240
<b>Leverage</b>	0.287	0.223	0.000	4.489	0.345

The variables are described in table 1

we ran separate probits for the newer and older sets assessing the probability of being rated in 1992. Since these probits are based on cross sectional attributes, the interest rate and intermediary earning variables are not used. We also took the 1985 to 1992 average of any time-varying regressor since a 1992 bond rating may cover old obligations.

Table 3 shows that wherever a variable is significant, its effect is as predicted by the CSV model; size and cash flow emerge as the most significant factors in a firm's selection of lenders. We found that long term tangible assets measured a firm's overall collateralizability better than inventories. Another interesting result is that market-to-book ratios do not have a significant impact on the selection of debt instruments once we control for other effects; the reason for this is that market-to-book ratios capture both a company's expected growth ( $\mu$  in our model), and the fraction of intangible assets ( $v$  in our model). These two variables

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Table 3: Cross Sectional Probit Regressions for Old and New Data Sets, 1992

Estimation Method	Probit	Probit	Probit	Probit
Dependent Variable (1≡Rated, 0≡Not)	Bond Rated <sub>92</sub>	C.Paper Rated <sub>92</sub>	Bond Rated <sub>92</sub>	C.Paper Rated <sub>92</sub>
Data Set	Old	Old	New	New
No. Observations	291	291	4817	4817
R <sup>2</sup>	<b>0.5285</b>	<b>0.6274</b>	<b>0.3515</b>	<b>0.3662</b>
Constant	-15.835 (10.070)	-14.387 (12.661)	-9.9119* (0.9892)	-8.0121* (1.4604)
Size†	0.7427* (0.0794)	1.1248* (0.1376)	0.5132* (0.0209)	0.6548* (0.0348)
Cash Flow†	4.1687 (7.6758)	27.510* (10.657)	2.6603* (1.1310)	6.8747* (1.8607)
Industry Profitability	15.674 (21.070)	-16.178 (27.192)	8.1841* (2.0506)	-3.1955 (3.2544)
Industry Risk	-9.247 (57.516)	-44.314 (82.232)	6.3499 (8.3101)	-17.4448 (15.9976)
Tangible assets†	0.2175 (0.7566)	-0.2635 (0.9334)	0.2469 (0.1801)	0.7407* (0.27207)

Standard errors are in parentheses. \* significantly different from zero at the 5% level. (†) Average value for 1985-1992. We are running a probit regression that assesses the probability of having a rating, depending on the firm's attributes. The regressors are described in table 1.

produce contradictory effects, as shown in proposition 2.

We performed a likelihood ratio test to verify that the explanatory variables affect young and mature firms similarly. The test says that one cannot reject that the slope coefficients for the newer and older sets are identical<sup>6</sup>. In other words, combining the two data sets will give the same slope coefficients, provided that each sample has different intercepts. The inequality in intercepts may clarify the role that age plays in lender selection, particularly if we can assume that the only unmeasured factor distinguishing the two samples is the difference in firms' age. We found that the intercepts differed by 0.286<sup>7</sup>, and that the probability of being rated would on average rise by 3.06 percent if a company switched from the new to the old set. Figure 3 shows how switching from the new to the old data set affects the probability of being rated. The likelihood ratio test simply quantifies our assertion that even though there

<sup>6</sup> The test uses the bond rating data and is done at the 5 percent confidence. The p-value of the test is 0.087

<sup>7</sup> The t-statistic of the difference was 2.604

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are important static differences between young and mature firms, their marginal behavior is essentially the same.

Figure 3: The effect of age on the likelihood of being rated. Firms from the new set are ordered by improving attributes in the x-axis. The dotted line plots the company's likelihood of being bond rated. The solid line plots the firm's likelihood of being rated if it had come from the old data set.

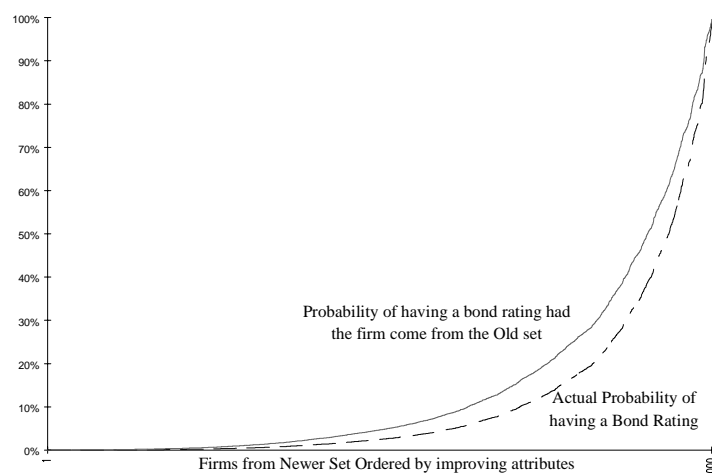


Table 3 suggests that the explanatory variables affect bond and the commercial paper markets similarly. For instance, firms with high cash flows issue bonds and commercial paper rather than bank debt; we know that commercial paper has a shorter maturity than bank loans, and that bank loans have a shorter maturity than bonds. How can maturity drive these results? One explanation is that companies with high cash flows prefer debt with extreme maturities. A more natural interpretation of the data is that firms with high cash flows issue publicly traded obligations, whether they are short or long term.

We run a likelihood ratio test to verify that the commercial paper and bond markets are similar to each other. This test indicates that one cannot reject that the slope coefficients for the bond and commercial paper probits are the same<sup>8</sup>. The intercept is allowed to

<sup>8</sup> The hypothesis test uses the old data set and is done at the 5 percent confidence. The p-value for the test is 0.061

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vary because firms with identical attributes are more likely to have a bond rather than a commercial paper rating. The difference in intercepts would occur if banks' reorganizational advantage is more pronounced for short term ventures than for long term investments, so that companies have to be all the safer to justify going to the commercial paper market.

Although the reaction of bond and commercial paper ratings to different explanatory variables is statistically the same, some intriguing facts emerge from table 3. For instance, commercial paper is more sensitive than bonds to cash flow changes, while the reverse is true for movements in industry profitability. This makes sense, since all that bondholders care about is the company's long term viability. Commercial paper investors, on the other hand, care only about the firm's short term solvency, for which cash flow is a good indicator.

The subsection has shown that even though maturity plays an important role in the process of lender selection, it is still true that the fundamental distinction between publicly traded and privately held debt is relevant across different time horizons. This subsection has also shown that young and old firms look quite different, and yet their *behavioral* patterns are similar. In the following subsections, we will study how companies choose their lenders from a time series perspective.

### 3.2 Lender Selection with Long-Term Debt

#### 3.2.1 A lagged dependent variable regression

We start with some basic accounting identities: debt at time  $t$  equals past debt plus new issues less retirements, as shown in equation (6). The superscripts  $\bar{\theta}$  and  $\underline{\theta}$  indicate whether debt is publicly traded or privately held. Long term debt is the sum of publicly traded and privately held obligations:  $D_{it} \equiv D_{it}^{\bar{\theta}} + D_{it}^{\underline{\theta}}$ . To start, we assume that retirements are a fraction of past debt, a fraction that does not vary across debt types or firms, so that  $R_{it}^j = [1 - \tilde{\alpha}]D_{it-1}^j : j = \underline{\theta}, \bar{\theta}$ . In addition, we assume that debt issuance is proportional to

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outstanding debt:  $I_{it} = \lambda D_{it-1}$ . The issuance of different types of debt is governed by  $h(x)$  defined in proposition 2:  $I_{it}^{\bar{\theta}} = h(x_{it})I_{it}$  and  $I_{it}^{\underline{\theta}} = [1 - h(x_{it})]I_{it}$ . The ratio  $m_{it}$  of bonds to total long term debt behaves as in equation (7).

$$D_{it}^j = D_{it-1}^j + I_{it}^j - R_{it}^j \quad I_{it}^j \geq 0 \quad R_{it}^j \in [0, D_{it-1}^j] \quad j = \underline{\theta}, \bar{\theta} \quad (6)$$

$$m_{it} = \frac{D_{it}^{\bar{\theta}}}{D_{it}} = \frac{\tilde{\alpha} D_{it-1}^{\bar{\theta}} + \lambda h(x_{it}) D_{it-1}}{[\tilde{\alpha} + \lambda] D_{it-1}} = \alpha m_{it-1} + [1 - \alpha][\beta' x_{it}] \quad (7)$$

Equation (7) also assumes that  $h(x_{it})$  is linear. Thus, simple accounting identities and strong assumptions imply that the ratio can be estimated as a lagged dependent variable regression; the estimates are shown in table 4. The results support our framework, since all coefficients are in the direction predicted by the theory and most are significant. The third column of table 4 shows that the inclusion of a firm's age and ownership structure does not affect other coefficients<sup>9</sup>.

We should note that even though the explanatory power in table 4 is quite high, much of this power comes from the lagged dependent variable. Without the lagged variable, the  $R^2$  for the model with and without the governance variables would drop to 0.392 and 0.373 respectively.

Although each one of the corporate governance variables is statistically insignificant, it seems that the more closely held a firm is, the more likely it is to rely on privately held obligations. How can we explain this? One possibility is that closely held corporations require more prolonged interventions; in our model, the delay function would have to satisfy  $\delta_{x_i}(x) > 0$  for  $x_i =$  family firms, insider shareholders and  $\delta_{x_i}(x) < 0$  for institutional investors. The only data on this subject, from Gilson et al. (1990), indicates that companies with a greater number of *shareholders* have a higher chance of restructuring privately (i.e., faster). This supports our model.

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<sup>9</sup> We cannot reject the hypothesis that the coefficients of the financial attributes remain the same after including the governance variables. The hypothesis test is done at the 5 percent confidence. The p-value for the test is 0.6314

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Can these results shed any light on the different theories of banking? This is always a difficult question to ask since banks are probably good monitors and good reorganizers. Furthermore, ‘risky’ companies (i.e., firms where default is likely and costly) probably suffer moral hazard problems as well, i.e., are companies where unmonitored managers will choose projects that are detrimental to creditors or outside shareholders.

Table 4: Lagged Dependent Variable Regression: Old Data Set, 1975-1992

Estimation Method	LDV	LDV
Dependent Variable	Ratio $m_{it}$	Ratio $m_{it}$
No. of Observations	5238	5238
R <sup>2</sup>	0.8840	0.8842
$\alpha$	0.9094* (0.0091)	0.9069* (0.0094)
Constant	-3.0958* (1.5084)	-2.5247 (1.5028)
Size <sub>t</sub>	0.1113* (0.0095)	0.0915* (0.0133)
Cash Flow <sub>t</sub>	3.9348* (1.5150)	3.2691* (1.5024)
Industry Profitability	1.5448 (3.1195)	1.2254 (3.0430)
Industry Risk	-7.3420 (9.1468)	-6.8912 (8.9078)
Tangible Assets <sub>t</sub>	0.1823 (0.1353)	0.2254 (0.1333)
Real Interest Rate <sub>t</sub>	-3.8893* (1.1813)	-3.8452* (1.1464)
Financial Int. Earnings <sub>t</sub>	-0.5447* (0.1467)	-0.5375* (0.1435)
Family Controlled		-0.0377 (0.0371)
Large Shareholders		-0.0808 (0.0636)
Institutional Investors		0.1629 (0.1070)
Age		0.0002 (0.0007)

Heteroskedastic consistent standard errors in parentheses. \*significantly different from zero at the 5% level. We are estimating  $m_{it} = \alpha m_{it-1} + [1 - \alpha][\beta' x_{it}]$ , where  $m_{it}$  is firm  $i$ 's use of publicly traded debt at time  $t$ , and  $x_{it}$  are its attributes. The p-value for the test that the error term from this model is not AR1, is 0.1803, which means we can accept the hypothesis that there is no AR1 component. The variables are explained in Table 1.

One could test the two theories of banking by directly validating their assumptions. This

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has already been done by Gilson et al. (1990) who have shown that banks are indeed good at reorganizing firms. However, we still lack a direct analysis that quantifies how much value banks add as monitors.

Another test of the two theories of banking is to look at companies with few ex-ante incentive misalignments but high ex-post reorganization costs. Family firms could be one such case: their ex-ante incentives seem well aligned with those of outside equityholders and debtholders, since families normally own a large stake in the company they control, and dread the nonpecuniary effects of bankruptcy. Ceteris paribus, a family firm is less likely to need banks if all that mattered was their superior monitoring. Nevertheless, we find that, if any, the effect goes the other way.

Table 4 shows that age does not affect lender choice significantly. This is not surprising, given the age of the firms in the old set: it probably does not matter whether a firm is 91 or a 100 years old; it matters a lot whether a company is 1 or 10 years of age. These results and those of the previous subsection are thus consistent with Petersen and Rajan (1994), who show that the impact of age on lender choice is most important when companies are young, and that by their thirtieth year, marginal increases in age are unimportant.

To assess the economic significance of our results, table 5 displays the change in the ratio  $m_{it}$  (in standard deviations) for a one standard deviation increase in the explanatory variable. For cross-sectional attributes we consider a *permanent* rise in the variable. We also consider a one year shift for time-varying regressors. For example, table 5 predicts that  $m_{it}$  rises by 0.162 standard deviations when cash flow rises permanently by one standard deviation. Hence, if a company with no bonds experiences a permanent 0.06 increase in its cash flow, it would replace 6 percent of its privately held debt with bonds. Size, cash flow, tangible assets, the fraction of institutional investors, interest rates, and financial intermediary earnings have the most powerful economic effects on a firm's choice of lenders.

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Table 5: The Economic Impact of Corporate Attributes on Debt Selection  
Across Firms: Permanent Shock

Size	Cash Flow	Industry Profitability	Industry Risk	Tangible Assets
0.590	0.162	0.040	-0.076	0.110
Family Controlled	Large Shareholders	Institutional Investors	Age	
-0.066	-0.076	0.130	0.024	

Across Time: One-Period Shock

Size	Cash Flow	Tangible Assets	Interest Rate	Intermediary Earnings
0.028	0.023	0.067	-0.081	-0.103

The table displays the change in the ratio  $m_{it}$  (in standard deviations) for a one standard deviation increase in the explanatory variable; we use the estimates in table 4, column 3 to create this table. The first two rows consider a permanent increase in the explanatory variable, while the last row considers a one period increase in the explanatory variable. The variables are explained in Table 1.

### 3.2.2 Stability of the Coefficients during Business Cycles

This subsection investigates if the determinants of debt choice have a differential impact across the business cycle. We will first relax our previous assumptions by allowing debt retirements and issues to vary across time, so  $R_{it}^j = [1 - \tilde{\alpha}_t]D_{it-1}^j$  and  $I_{it} = \lambda_t D_{it-1}$ , where  $j = \underline{\theta}, \bar{\theta}$ . The issuance of debt types is governed by the security demand  $h(x)$  defined in proposition 2:  $I_{it}^{\bar{\theta}} = h(x_{it})I_{it}$  and  $I_{it}^{\underline{\theta}} = [1 - h(x_{it})]I_{it}$ . The ratio now becomes:

$$m_{it} = \frac{D_{it}^{\bar{\theta}}}{D_{it}} = \frac{\tilde{\alpha}_t D_{it-1}^{\bar{\theta}} + \lambda_t h(x_{it}) D_{it-1}}{[\tilde{\alpha}_t + \lambda_t] D_{it-1}} = \alpha_t m_{it-1} + [1 - \alpha_t] h(x_{it}) \quad (8)$$

Equation (8) says that the ratio  $m_{it}$  is more sensitive to changes in corporate attributes when companies issue more debt or when they retire less securities. If we do not control for this, we may wrongly conclude that the determinants of lender choice matter more in recessions, simply because in these periods firms may be issuing more debt. We construct  $\alpha_t = \tilde{\alpha}_t / [\tilde{\alpha}_t + \lambda_t]$  from our older set, where  $1 - \tilde{\alpha}_t$  and  $\lambda_t$  are the median rates of retirement and issuances of long term debt in year  $t$ . Equation (9), which assumes that  $h(x_{it})$  is linear, introduces a time varying parameter  $\varphi_t$  to allow for the possibility that the determinants of

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lender choice vary over time. Estimates of equation (9) with an additive normal error are shown in table 6.

$$m_{it} = [\alpha_t + \varphi_t]m_{it-1} + [1 - \alpha_t - \varphi_t][\beta'x_{it}] \quad (9)$$

Table 6: Stability of coefficients: Old Data Set, 1975-1992

Estimation Method Dependent Variable No. of Observations $R^2$	NLS Ratio <sub>t</sub> 5238 0.8872				
		$\varphi_t$		$\varphi_t$	
<b>Constant</b>	-2.9618* (1.5006)	$\varphi_{75}$	-0.0006 (0.0667)	$\varphi_{87}$	0.0419 (0.0457)
<b>Size<sub>t</sub></b>	0.0972* (0.0138)	$\varphi_{76}$	0.1948* (0.0191)	$\varphi_{88}$	0.1170* (0.0238)
<b>Cash Flow<sub>t</sub></b>	3.3984* (1.5148)	$\varphi_{77}$	0.2213* (0.0103)	$\varphi_{89}$	0.0678 (0.0511)
<b>Industry Profitability</b>	1.9867 (3.0333)	$\varphi_{78}$	0.1929* (0.0167)	$\varphi_{90}$	0.1022* (0.0334)
<b>Industry Risk</b>	-12.2517 (8.6450)	$\varphi_{79}$	0.1637* (0.0201)	$\varphi_{91}$	0.1324* (0.0219)
<b>Tangible Assets<sub>t</sub></b>	0.2788* (0.1415)	$\varphi_{80}$	0.1588* (0.0270)	$\varphi_{92}$	0.1110* (0.0248)
<b>Real Interest Rate<sub>t</sub></b>	-6.4648* (1.9750)	$\varphi_{81}$	0.1749* (0.0152)		
<b>Financial Int. Earnings<sub>t</sub></b>	-0.5666* (0.1631)	$\varphi_{82}$	0.1763* (0.0196)		
<b>Family Controlled</b>	-0.0353 (0.0404)	$\varphi_{83}$	0.0407 (0.0504)		
<b>Large Shareholders</b>	-0.1156 (0.0696)	$\varphi_{84}$	0.1302* (0.0215)		
<b>Institutional Investors</b>	0.1190 (0.1162)	$\varphi_{85}$	0.0748 (0.0491)		
<b>Age</b>	0.0006 (0.0008)	$\varphi_{86}$	0.0997* (0.0384)		

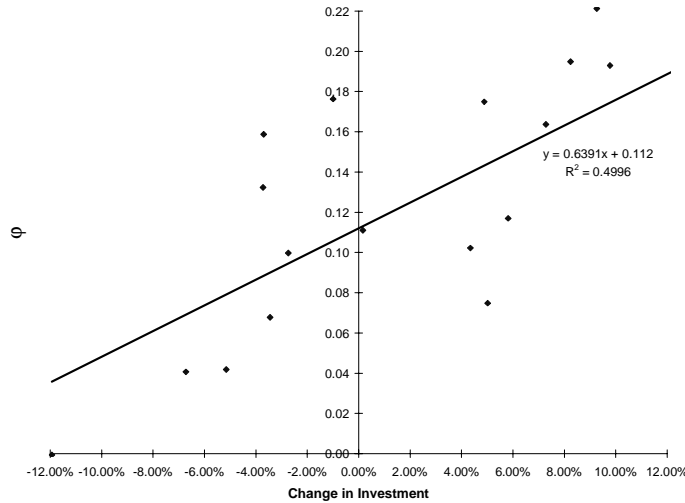
Heteroskedastic consistent standard errors are in parentheses. \*significantly different from zero at the 5% level. We are estimating  $m_{it} = [\alpha_t + \varphi_t]m_{it-1} + [1 - \alpha_t - \varphi_t][\beta'x_{it}]$  where  $m_{it}$  is firm  $i$ 's use of publicly traded debt at time  $t$ ,  $x_{it}$  are its attributes, and  $\varphi_t$  is a time varying parameter to be estimated. We construct  $\alpha_t = \tilde{\alpha}_t / [\tilde{\alpha}_t + \lambda_t]$  from our older set, where  $1 - \tilde{\alpha}_t$  and  $\lambda_t$  are the median rates of retirement and issuances of long term debt in year  $t$ ; other variables are explained in Table 1

What can we make of these results? Figure 4 plots the estimated coefficients  $\varphi_t$  against the percentage change in property plant and equipment investment by non financial corporations, as reported by the flow of funds accounts. There is a strong positive correlation, and the slope

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coefficient has a t-statistic of 4.00. The fact that  $\varphi_t$  is small during investment downturns means that the determinants of lender selection are more important during these periods. This suggests that markets classify firms more sharply during recessions, and that small changes in attributes have a big impact on the type of debt issued. To illustrate this, consider the impact of a temporary one standard deviation increase in size and cash flow in 1975 and 1984, the years with the largest fall and increase in investment. An increase in size raised  $m_{it}$  by 0.12 standard deviations in 1975, while a similar change in 1984 raised the ratio by only 0.03 standard deviations. Similarly, a cash flow increase raised  $m_{it}$  by 0.04 standard deviations in 1975, while a similar increase in 1984 left  $m_{it}$  virtually unchanged.

Figure 4: Variation of  $\varphi_t$  during business cycles;  $\varphi_t$  are estimated from  $m_{it} = [\alpha_t + \varphi_t]m_{it-1} + [1 - \alpha_t - \varphi_t][\beta'x_{it}]$  where  $m_{it}$  is firm  $i$ 's use of publicly traded debt,  $\alpha_t$  is estimated from the retirement and issuance data, and  $x_{it}$  are corporate attributes. The estimated  $\varphi_t$  are shown in Table 6



One explanation for these results is that the probability of default rises unevenly during recessions, i.e., that downturns hurt weak companies more than blue-chip corporations. An uneven increase in the probability of default accentuates each of the lenders' comparative advantage, so that firms that were previously in a 'lender twilight' are now suddenly and

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clearly sorted out. This would also mean that an attribute improvement during a recession raises the firm's survival rate more than a similar improvement in a boom.

### 3.2.3 Entry and exit into the bond market

This subsection explores if companies enter and exit the markets for publicly traded debt asymmetrically. This is the case if a firm becomes widely known after it has used the public debt markets. Easily available information lowers intervention costs, and eclipses banks' comparative advantage. From a theoretical vantage point, this implies that the delay function  $\delta(x)$  is history dependent, with  $\delta(x;\text{previous exposure}) < \delta(x;\text{no exposure})$ . We model this econometrically by allowing different entry and exit thresholds. First, we use accounting identities to link levels and issues of debt:

$$\begin{aligned} D_{it}^{\bar{\theta}} &\equiv \tilde{\alpha}_{it}^{\bar{\theta}} D_{it-1}^{\bar{\theta}} + I_{it}^{\bar{\theta}} \\ D_{it}^{\theta} &\equiv \tilde{\alpha}_{it}^{\theta} D_{it-1}^{\theta} + I_{it}^{\theta} \end{aligned}$$

where  $D_{it}^{\bar{\theta}}$  are firm  $i$ 's bonds outstanding at time  $t$ ,  $1 - \tilde{\alpha}_{it}^{\bar{\theta}}$  is firm  $i$ 's bond retirement rate at time  $t$ , and  $I_{it}^{\bar{\theta}}$  is firm  $i$ 's new issues of bonds at time  $t$ . Bank loans behave analogously. We assume different rates of retirement for bonds and bank loans, but hold these retirement rates fixed across companies and time, so  $\tilde{\alpha}_{it}^{\bar{\theta}} = \tilde{\alpha}^{\bar{\theta}}$  and  $\tilde{\alpha}_{it}^{\theta} = \tilde{\alpha}^{\theta}$ . We constructed the median bond retirement rate from corporations which only had bonds outstanding. This retirement rate was 0.0393, implying that a typical bond has a maturity of 25.5 years. The median retirement rate for intermediated debt, which we constructed analogously, was 0.1318; this implies that privately held obligations have a maturity of 7.6 years. We estimated the issuance of different debt instruments,  $I_{it}^{\bar{\theta}}$  and  $I_{it}^{\theta}$ , by using data on the outstanding debt, the above retirement

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rates and accounting identities; we then estimated the demand for bonds  $\hat{h}(x_{it})$  as follows:

$$\hat{h}(x_{it}) = \begin{cases} 0 & \text{if } I_{it}^{\bar{\theta}} < I_{it}^{\theta} \text{ bank loans are issued} \\ 1 & \text{if } I_{it}^{\bar{\theta}} \geq I_{it}^{\theta} \text{ bonds are issued} \end{cases}$$

In the theoretical section we assumed that firms issue either bank or bond debt; this turns out to be a good approximation of reality, since only 12 percent of the observations had a positive amount of issues of both securities. We will assume that the decision function is observed with noise so that the true variable is  $d(x_{it}) + \epsilon_{it}$  where  $\epsilon_{it}$  is a mean zero error with a normal distribution  $G(\cdot)$ . There are four cases that we need to study to set up the appropriate likelihood function:

- (A)  $\hat{h}(x_{it-1}) = 0$  and  $\hat{h}(x_{it}) = 0$ . In this case, bonds were not issued in the current or previous period. Since  $\hat{h}(x_{it}) = 0$ , proposition 2 says that  $d(x_{it}) + \epsilon_{it} < 0$ . This occurs with a probability  $G(-d(x_{it}))$ .
- (B)  $\hat{h}(x_{it-1}) = 0$  and  $\hat{h}(x_{it}) = 1$ . In this case, bonds had not been issued at time  $t - 1$ , but some were issued at time  $t$ . Proposition 2 says that this happens only if  $d(x_{it}) + \epsilon_{it} > 0$ . This occurs with a probability  $1 - G(-d(x_{it})) = G(d(x_{it}))$
- (C)  $\hat{h}(x_{it-1}) = 1$  and  $\hat{h}(x_{it}) = 0$ . In this case, the company issued bonds last period and none currently. If we believe in an asymmetric entry and exit, the decision function must be not only negative but lower than  $-c$ , so that  $d(x_{it}) + \epsilon_{it} < -c$  to exit. This occurs with a probability  $G(-c - d(x_{it}))$ .
- (D)  $\hat{h}(x_{it-1}) = 1$  and  $\hat{h}(x_{it}) = 1$ . In this case, bonds were issued in both the current and previous periods. Since the firm had previous exposure, they only need  $d(x_{it}) + \epsilon_{it} > -c$  to stay in the market. This occurs with probability  $1 - G(-c - d(x_{it})) = G(c + d(x_{it}))$

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We assume that  $d(x_{it})$  is linear, so  $d(x_{it}) = \beta'x_{it}$ . The log likelihood is given in equation (10), and its estimates are shown in table 7.

$$\begin{aligned} \log(L_{it}) = & [1 - \hat{h}(x_{it-1})][1 - \hat{h}(x_{it})]\log[G(-\beta'x_{it})] + [1 - \hat{h}(x_{it-1})]\hat{h}(x_{it})\log[G(\beta'x_{it})] \\ & + \hat{h}(x_{it-1})[1 - \hat{h}(x_{it})]\log[G(-c - \beta'x_{it})] + \hat{h}(x_{it-1})\hat{h}(x_{it})\log[G(c + \beta'x_{it})] \end{aligned} \tag{10}$$

Table 7: Entry-Exit to Bond Market: Old Data Set, 1975-1992

Estimation Method	Maximum Likelihood	Maximum Likelihood
No. of Observations	5516	5516
<b>Constant</b>	-12.7564*	-13.0853*
	(2.6816)	(-2.7963)
<b>Size</b>	0.3433*	0.2822*
	(0.0174)	(0.0213)
<b>Cash Flow</b>	5.7916*	3.6669*
	(1.6976)	(1.7598)
<b>Industry Profitability</b>	12.3364*	15.1881*
	(5.3777)	(5.5635)
<b>Industry Risk</b>	-2.9762	-10.2025
	(14.5098)	(15.0420)
<b>Tangible Assets</b>	-0.1177	0.0028
	(0.1993)	(0.2026)
<b>Real Interest Rate</b>	-2.8166*	-3.2087*
	(1.6011)	(1.6188)
<b>Financial Int. Earnings</b>	-0.6418*	-0.6907*
	(0.1881)	(0.1900)
<b>Family Controlled</b>		-0.1226*
		(0.0572)
<b>Large Shareholders</b>		-0.1851
		(0.1207)
<b>Institutional Investors</b>		0.8797*
		(0.1631)
<b>Age</b>		0.0002
		(0.0009)
<b>c (asymmetric threshold)</b>	0.9832*	0.9413*
	(0.0581)	(0.0585)

Standard errors in parentheses. \*significantly different from zero at the 5% level. We maximize the likelihood in equation (10). The parameter  $c$  measures the asymmetry of entry and exit into the bond markets; the other variables are explained in Table 1.

Apart from the coefficient on tangible assets, which is not significant, all the parameter estimates conform with the CSV model's predictions. Moreover, corporate governance vari-

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ables have the same sign as in the lagged dependent variable regressions. The family firm dummy and the concentration of institutional investor are statistically significant. The constant  $c$  was positive as predicted by the model, so firms will continue to use publicly traded debt even after their attributes have dropped below the entry threshold.

An alternative account for these results is based on a strategic refinancing argument: this argument says that a firm issues bonds if its attributes cross a threshold; such a company would then lock in a very good yield. Suppose now that the firm's attributes deteriorate: while it is true that bank loans are cheaper than issuing new bonds, the cheapest source of funds are the (mispriced) old bonds. This induces the firm to stay in the bondmarket even after its attributes have deteriorated past the entry threshold. We try to rule this story out by analyzing bond *issues* rather than bond *levels*, and by looking at the commercial paper market. The short term nature of commercial paper would preclude any explanation based on strategic refinancing; we will study this issue in the next subsection.

### 3.3 Lender Selection with Short-Term Debt

It is simple to link debt levels to debt issues for short term instruments, since this type of obligation fully retires within a year (i.e.,  $R_{it}^j = D_{it-1}^j$ ). We define new debt issues as  $I_{it} \equiv \lambda_{it} D_{it-1}$  without loss of generality. The issuance of different types of obligations is governed by the security demand  $h(x)$  defined in proposition 2:  $I_{it}^{\bar{\theta}} = h(x_{it}) I_{it}$  and  $I_{it}^{\theta} = [1 - h(x_{it})] I_{it}$ . Thus, the ratio of commercial paper to total short term debt becomes:

$$m_{it} = \frac{D_{it}^{\bar{\theta}}}{D_{it}} = \frac{\lambda_{it} h(x_{it}) D_{it-1}}{\lambda_{it} D_{it-1}} = h(x_{it}) \quad (11)$$

An excellent indicator of  $h(x_{it})$  is whether a firm has a commercial paper rating or not. A firm will have a rating only if it has some commercial paper outstanding; this is possible only if the firm issued such an obligation during the year. Thus  $h(x_{it}) = 1$  if the firm has a commercial paper rating, and  $h(x_{it}) = 0$  if it has no such rating. We will begin by estimating

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yearly probits of  $h(x_{it})$  on a linearized decision function  $d(x_{it})$  as defined in proposition 2.

Table 8: Cross Sectional Probit Regressions for the New Data Set, 1985-1992

<b>Dependent Variable: C.Paper Rated<sub>t</sub></b>								
<b>Year</b>	<b>1985</b>	<b>1986</b>	<b>1987</b>	<b>1988</b>	<b>1989</b>	<b>1990</b>	<b>1991</b>	<b>1992</b>
<b>No. of Observations</b>	<b>3103</b>	<b>2874</b>	<b>3043</b>	<b>3022</b>	<b>2975</b>	<b>2992</b>	<b>3058</b>	<b>3194</b>
<b>R<sup>2</sup></b>	<b>0.33</b>	<b>0.35</b>	<b>0.37</b>	<b>0.40</b>	<b>0.41</b>	<b>0.40</b>	<b>0.39</b>	<b>0.41</b>
Constant	-10.9* (1.66)	-11.0* (1.65)	-10.6* (1.58)	-10.7* (1.57)	-8.73* (1.50)	-9.08* (1.50)	-7.31* (1.52)	-8.22* (1.53)
Size <sub>t</sub>	0.56* (0.03)	0.57* (0.04)	0.58* (0.03)	0.61* (0.04)	0.63* (0.04)	0.63* (0.04)	0.62* (0.04)	0.65* (0.04)
Cash Flow <sub>t</sub>	3.57 (2.60)	5.71* (2.25)	6.12* (2.19)	4.40* (2.21)	4.47* (1.90)	5.52* (1.76)	6.53* (1.79)	6.10* (1.91)
Industry Profitability	7.72* (3.71)	5.42 (3.54)	3.87 (3.57)	5.31 (3.34)	1.16 (3.37)	0.80 (3.28)	-3.67 (3.35)	-1.93 (3.38)
Industry Risk	-54.2* (25.7)	-39.8 (23.9)	-54.2* (22.0)	-23.9 (17.3)	-19.6 (16.6)	-22.1 (16.0)	-16.0 (16.0)	-13.9 (16.3)
Tangible Assets <sub>t</sub>	0.42 (0.32)	0.50 (0.32)	0.83* (0.30)	0.88* (0.30)	0.81* (0.28)	0.89* (3.19)	0.88* (0.29)	0.85* (0.28)

Standard errors in parentheses. \*significantly different from zero at the 5% level. We are running yearly probits that assess the probability of having a commercial paper rating based on one's attributes. The variables are described in table 1

Table 8 shows the results for the new data set from 1985 to 1992, the years for which we have commercial paper ratings. The results again support the CSV model: coefficient estimates had the correct sign in thirty eight out of forty cases, and the two 'wrong' signs were statistically insignificant. The strongest cash flow coefficients occurred in 1987 and 1991, the years with the two largest investment downturns in the period. This is consistent with our findings in subsection 3.2 - that financial attributes matter most in downturns.

### 3.3.1 Entry and Exit into the Commercial Paper Market

We will now explore if firms enter and exit the commercial paper markets asymmetrically, repeating the analysis in subsection 3.2. Table 9 shows the estimates of a log likelihood function as in equation (10). Companies with large sales or high cash flows are more likely to issue commercial paper. Note that industry profitability and risk have an opposite effect to the one predicted by the theory (although this was not true for the cross-sectional regressions).

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The constant  $c$  was positive, as predicted by the model.

Table 9: Entry-Exit to Commercial Paper Market: New Data Set, 1986-1992

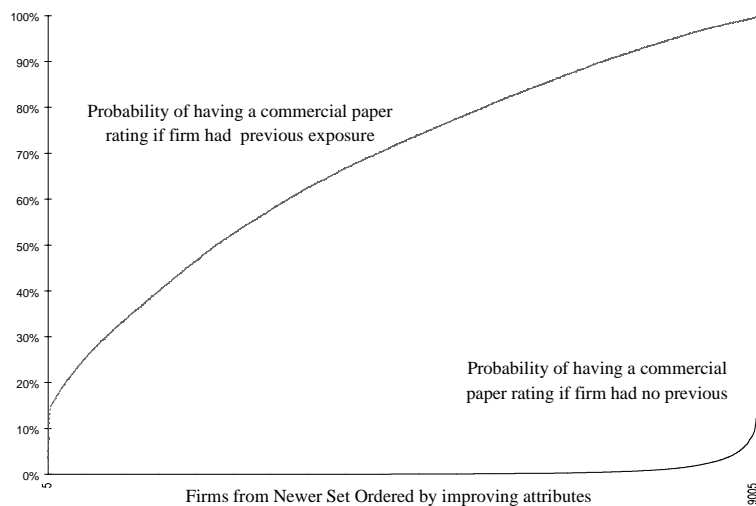
Estimation Method No. of Observations	Maximum Likelihood 20,888
<b>Constant</b>	-3.2907* (1.1884)
<b>Size<sub>t</sub></b>	0.3335* (0.0251)
<b>Cash Flow<sub>t</sub></b>	4.8223* (1.2380)
<b>Industry Profitability</b>	-7.6024* (2.5278)
<b>Industry Risk</b>	7.7158 (10.4276)
<b>Tangible Assets<sub>t</sub></b>	0.4810* (0.2268)
<b>Real Interest Rate<sub>t</sub></b>	0.0389 (0.0323)
<b>Financial Int. Earnings<sub>t</sub></b>	-1.3427* (0.4467)
<b>c (asymmetric threshold)</b>	3.8819* (0.0873)

Standard errors in parentheses. \*significantly different from zero at the 5% level. We maximize the likelihood in equation (10). The parameter  $c$  measures the asymmetry of entry and exit into the bond markets; the other variables are explained in Table 1.

To appraise these results, figure 5 plots the probability of being rated depending on a firm's history. Companies are ordered by improving attributes in the x-axis. The bottom line shows the likelihood of being rated, assuming that a firm had no exposure to the commercial paper market during the last year. The upper line displays the probability of being rated for a firm with identical attributes but with previous exposure to the commercial paper market. Figure 5 shows that once a firm has entered the commercial paper market it is likely to stay there even after its attributes have deteriorated well below the entry threshold. Figure 5 demonstrates that history is enormously important in determining a firm's choice of lenders.

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Figure 5: Probability of having a commercial paper rating depending on whether a firm was /was not in the market the previous year



## 4 Conclusion

We have developed a theoretical model that seems to explain very well how firms select their lenders. The model is based on the simple insight that publicly traded and privately held debt have advantages that dominate in different situations. The advantage of privately held debt is that it allows for less damaging intervention in distress. Publicly traded obligations, on the other hand, bypass the intermediary by offering the security directly to investors; this factor is especially valuable when a firm is less likely to default, i.e., when the services of the middleman are less needed. We expect that those variables that make a firm less likely to default (size, high and stable cash flows, high profitability, ample collateral, low real interest rates) will induce it to tap the bondmarket directly. The evidence presented in this article broadly supports this mode of analysis. Our empirical work uncovered other results worth mentioning.

We found that the same type of factors induce firms to choose commercial paper and

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corporate bonds. This shows that the distinction of publicly traded and privately placed debt is economically relevant at the short and long end of the maturity spectrum.

We discovered that the determinants of lender choice are most crucial during investment downturns. We do not fully understand, however, the cause for this. Finally, we showed that once a firm has entered the markets for publicly traded debt, it will stay there even after its attributes have fallen well below the original entry threshold. It would be interesting to explore more fully how exposure to publicly traded markets affects a firm's intervention prospects.

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## Appendix A

Before we begin the proofs, let us redefine the lenders economic profit as

$$\begin{aligned} U(x; \theta, b) &= \mu b[1 - F(b)] + [1 - \theta\delta(x)] \int_0^b \mu s f(s) ds - [1 - e][r_f + \pi(x; \theta)] \\ &= \mu \int_0^b [1 - F(s)] J(x; s, \theta) ds - [1 - e][r_f + \pi(x; \theta)] \end{aligned}$$

Where  $J(x; s, \theta) \equiv 1 - \theta\delta(x)s\rho(s)$  is the so-called virtual surplus that is widely used in mechanism design. We will identify an increase in risk in the manner of Rothschild and Stiglitz (1970), (so  $\int_{-\infty}^k F_\sigma(s; x) ds > 0$  and  $\int_{-\infty}^{\infty} F_\sigma(s; x) ds = 0$ ), plus  $\rho_\sigma(s) > 0$ .

Table 10: Partial Derivatives of Payoff Functions

Argument $x_i$	$U_{x_i}(x; \theta, b)$	$V_{x_i}(x; \theta, b)$
$b$	$\mu[1 - F(b; x)]J(x; b, \theta)$	$-\mu[1 - F(b)] < 0$
$v$	$-\mu\theta\delta_v(x) \int_0^b s f(s; x) ds > 0$	0
$e$	$[r_f + \pi(x; \theta)] > 0$	$-r_f < 0$
$z$	$-\mu\theta\delta_z(x) \int_0^b s f(s; x) ds > 0$	0
$\mu$	$\int_0^b [1 - F(s; x)] J(x; s, \theta) ds > 0$	$\int_b^\infty [1 - F(s; x)] ds > 0$
$\sigma$	$-\mu \int_0^b [F_\sigma(s; x) J(x; s, \theta) + \theta\delta(x)s\rho_\sigma(s)[1 - F(s; x)]] ds < 0^{10}$	$-\mu \int_b^\infty F_\sigma(s; x) ds > 0$

## A Proof of Proposition 1

We will first state and prove a lemma that relates the existence of a lending equilibrium and the value of the dual of the lenders maximization problem, defined below

$$M(x; \theta) \equiv \max_b \left[ \begin{array}{l} U(x; \theta, b) \\ \text{s.t. } V(x; b) \geq 0 \end{array} \right]$$

Let us also define  $b_u(x; \theta)$  as the point where the lender's utility attains a maximum, i.e., where  $U_b(x; \theta, b_u) \equiv 0$ . The existence and uniqueness of  $b_u(x; \theta)$  will be shown in the main proof.

**Lemma 1**  $x \in S(\theta)$  if and only if  $M(x; \theta) \geq 0$

**Proof.** (i) If  $M(x; \theta) < 0$  investor  $\theta$  can never break even from lending to a firm with attributes  $x$ . This implies that no lending takes place so  $x \notin S(\theta)$ . (ii) If  $M(x; \theta) > 0$ , there exists a profit maximizing rate  $b^*$  satisfying the following properties:  $b^* \leq b_u(x; \theta)$ ,  $U(x; \theta, b^*) > 0$  and  $V(x; b^*) \geq 0$ . We claim that  $U(x; \theta, b) < 0$  for all  $b < [1 - e]r_f/\mu$ ; to see this, rewrite equation (2):

$$U(x; \theta, b) = [1 - F(b)][\mu b - (1 - e)r_f] + \int_0^b [\mu s - (1 - e)r_f] f(s) ds - \theta\delta(x) \int_0^b s f(s) ds - (1 - e)\pi(x; \theta)$$

Since the first and second terms of this equation are negative for  $b < [1 - e]r_f/\mu$ , and the last two terms are always negative, we establish our claim. By the intermediate value theorem there exists a

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unique point  $b \in ([1 - e]r_f/\mu, b^*)$  where  $U(x; \theta, b) = 0$ . Since the entrepreneurs' utility rises as  $b$  falls, their participation constraint is satisfied at  $b$ , for  $V(x; b) > V(x; b^*) \geq 0$ . Thus we have  $x \in S(\theta)$ . (iii) If  $M(x; \theta) = 0$  one can verify that the  $b^*$  which solves the maximization also satisfies the definition of equilibrium and thus  $x \in S(\theta)$ . ■

A) If  $(x_i, x_{-i}) \in S(\theta)$  then all  $(x'_i, x_{-i}) \in S(\theta)$  where  $x'_i$  satisfies  $(x'_i - x_i)U_{x_i} \geq 0$

**Proof.** : We will show that the maximal payoff is monotonic in its arguments with  $\text{sgn}(M_{x_i}) = \text{sgn}(U_{x_i})$ . Define the maximal lender payoff as:

$$\begin{aligned} M(x; \theta) &= \max_{b \in P(x)} U(x; \theta, b) \\ P(x) &= \{b : V(x; b) \equiv \mu \int_b^\infty [s - b]f(s)ds - er_f \geq 0\} = \{b : b \leq b_v(x)\} \end{aligned}$$

Where  $b_v(x)$  is defined by  $V(x; b_v) \equiv 0$ . Since  $U(x; \theta, b)$  is quasiconcave with respect to  $b$ , we solve this as a Lagrange multiplier problem:

$$\begin{aligned} L &= U(x; \theta, b) + \lambda[b_v(x) - b] \\ L_b &= U_b(x; \theta, b) - \lambda = [1 - F(b)]J(x; b, \theta) - \lambda = 0 \\ L_\lambda &= b_v(x) - b \geq 0 \quad \lambda \geq 0 \\ \lambda L_\lambda &= 0 \end{aligned}$$

if  $L_\lambda > 0$  we have an unconstrained problem, and hence  $U_b(x; \theta, b_u) = 0 \Rightarrow J(x; b_u, \theta) \equiv 0$ . The existence of a unique unconstrained maximization rate  $b_u(x; \theta)$  hinges on the increasing hazard rate property. We find that  $\partial b_u/\partial x_i > 0$  for  $x_i = v, z, \sigma$  and  $\partial b_u/\partial \theta < 0$  using the implicit function proposition. If  $L_\lambda = 0$ , entrepreneurs' participation constraint is binding and the solution to the problem is  $b_v$  where  $V(x; b_v) \equiv 0$ . We find that  $\partial b_v/\partial x_i > 0$  for  $x_i = \mu, \sigma$  and  $\partial b_v/\partial x_i < 0$  for  $x_i = e, r_f$  using the implicit function proposition. The constrained maximization rate  $b^*$  is given in equation (A.1); we apply this to look at the derivatives of the dual  $M(x; \theta) \equiv U(x; \theta, b^*)$  :

$$b^*(x; \theta) = \min [b_v(x), b_u(x; \theta)] \tag{A.1}$$

$$M_{x_i}(x; \theta) = U_{x_i}(x; \theta, b^*) + U_b(x; \theta, b^*) \frac{db^*}{dx_i} \tag{A.2}$$

When  $b^* = b_u$  the second argument of equation (A.2) is zero (this is an application of the envelope proposition) and thus  $M_{x_i} = U_{x_i}$ . We need to show that  $\text{sgn}(M_{x_i}) = \text{sgn}(U_{x_i})$  when the participation constraint is binding ( $b^* = b_v$ ). To use this constraint we exploit the fact that  $V(x; b_v) \equiv 0$  to use the implicit function proposition

$$\begin{aligned} U_b(x; \theta, b^*) \frac{db^*}{dx_i} &= \mu[1 - F(b)]J(x; b, \theta) \left[ \frac{-V_{x_i}}{V_b} \right] = [1 - \theta\delta(x)b\rho(b)]V_{x_i} \\ M_{x_i}(x; \theta) &= U_{x_i}(x; \theta, b^*) + J(x; b, \theta)V_{x_i}(x; \theta, b^*) \end{aligned}$$

When  $b^* = b_v < b_u$  we have that  $U_b(x; \theta, b^*) > 0 \Rightarrow J(x; b, \theta) > 0$ . This fact and the derivatives in table 2.1 imply that  $\text{sgn}(M_{x_i}) = \text{sgn}(U_{x_i})$ , and thus  $M(x; \theta)$  is a monotonic function of its arguments  $x$ . Hence if  $M(x_i, x_{-i}; \theta) \geq 0$  then  $M(x'_i, x_{-i}; \theta) \geq 0$  for all  $x'_i$  satisfying  $(x'_i - x_i)U_{x_i} \geq 0$ . This result and lemma 1 prove proposition 1A. ■

B)  $S(\bar{\theta}) \subset S(\underline{\theta})$  if  $\pi(x; \underline{\theta}) \leq [\bar{\theta} - \underline{\theta}]\delta(\underline{v}, \underline{z}) \int_0^{\hat{b}} sf(s; x)ds$ ,  $\hat{b}$  as defined in equation (A.5)

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**Proof.** First we write the difference in payoffs if both lenders charge the same rate,  $b^*(x; \bar{\theta})$  - defined in equation (A.1) - that maximizes bondholders' economic payoff for  $\forall x \in S(\bar{\theta})$ :

$$U(x; \underline{\theta}, b^*) - U(x; \bar{\theta}, b^*) = \mu[\bar{\theta} - \underline{\theta}]\delta(x) \int_0^{b^*(x; \bar{\theta})} sf(s; x)ds - [1 - e]\pi(x; \underline{\theta}) \geq 0 \quad (\text{A.3})$$

The assumption that the intermediaries are better off at the profit maximizing rate is true if the following condition is satisfied:

$$\pi(x; \underline{\theta}) \leq [\bar{\theta} - \underline{\theta}]\delta(\underline{v}, \underline{z}) \int_0^{\hat{b}} sf(s; x)ds \leq \mu[\bar{\theta} - \underline{\theta}]\delta(x) \frac{\int_0^{b^*(x; \bar{\theta})} sf(s; x)ds}{[1 - e]} \quad (\text{A.4})$$

$$\hat{b} \equiv \min [b_v(\underline{\mu}, \underline{\sigma}, \bar{e}, \bar{r}_f), b_u(\underline{v}, \underline{z}, \underline{\sigma}; \bar{\theta})] \leq b^*(x; \bar{\theta}) \quad (\text{A.5})$$

The inequalities in equations (A.4) and (A.5) follow from  $\mu > 1$ ,  $\partial b_v / \partial x_i > 0$  for  $x_i = \mu, \sigma$ ,  $\partial b_v / \partial x_i < 0$  for  $x_i = e, r_f$  and  $\partial b_u / \partial x_i > 0$  for  $x_i = v, z, \sigma$ . Equation (A.3) implies that  $M(x; \underline{\theta}) \geq U(x; \underline{\theta}, b^*(\bar{\theta})) \geq U(x; \bar{\theta}, b^*(\bar{\theta})) \equiv M(x; \bar{\theta}) \forall x \in S(\bar{\theta})$ . This result and lemma 1 prove proposition 1B. ■

C) Equilibrium rates  $b(x; \theta)$  satisfy  $\text{sgn}(\frac{d^2 b}{dx_i d\theta}) = \text{sgn}(b_{x_i}) = \text{sgn}(-U_{x_i})$ .

**Proof.** At the equilibrium bankruptcy  $b(x; \theta)$ ,  $U(x; \theta, b) \equiv 0$ . From our discussion in lemma 1, the equilibrium satisfies  $b(x; \theta) < b_u(x; \theta)$ ; thus  $J(x; b, \theta) > J(x; b_u, \theta) \equiv 0$  and thus  $U_b(x; \theta, b) = \mu[1 - F(b)]J(x; b, \theta) > 0$ . We assume that we are at a crossing point, so  $b_\theta(x; \theta) = 0$ ; this assumption implies:

$$\frac{db(x; \theta)}{d\theta} = -\frac{U_\theta(x; b, \theta)}{U_b(x; \theta, b)} = 0 \Rightarrow U_\theta(x; b, \theta) = 0 \quad (\text{A.6})$$

$$U_\theta(x; b, \theta) = -\mu\delta(x) \int_0^b sf(s)ds - (1 - e)\pi_\theta(x; \theta) = 0$$

Given that there is crossing point, we will prove that it is unique. First, use the implicit function proposition to obtain  $b_{x_i}$ ; then, use  $U_{b\theta} = -\mu[1 - F(b)]\theta\delta(x)b\rho(b) < 0$

$$\frac{db}{dx_i} = -\frac{U_{x_i}(x; \theta, b)}{U_b(x; \theta, b)} \Rightarrow \text{sgn}(b_{x_i}) = \text{sgn}(-U_{x_i}) \quad (\text{A.7})$$

$$\frac{d^2 b}{dx_i d\theta} = \frac{U_{x_i}U_{b\theta} - U_b U_{x_i\theta}}{U_b^2} = -\frac{\delta(x)b\rho(b)U_{x_i} + J(x; b, \theta)U_{x_i\theta}}{\mu[1 - F(b)][J(x; b, \theta)]^2} \quad (\text{A.8})$$

For  $x_i = r_f, \phi$  we have  $U_{x_i\theta} = 0$  so equation (A.8) reduces to:

$$\frac{d^2 b}{dx_i d\theta} = -\frac{\delta(x)b\rho(b)U_{x_i}}{\mu[1 - F(b)][J(x; b, \theta)]^2} \Rightarrow \text{sgn}\left(\frac{d^2 b}{dx_i d\theta}\right) = \text{sgn}(-U_{x_i})$$

For  $x_i = v, z$  we have  $\theta U_{x_i\theta} = U_{x_i}$  so that equation (A.8) simplifies to:

$$\frac{d^2 b}{dx_i d\theta} = -\frac{U_{x_i}}{\theta\mu[1 - F(b)][J(x; b, \theta)]^2} \Rightarrow \text{sgn}\left(\frac{d^2 b}{dx_i d\theta}\right) = \text{sgn}(-U_{x_i})$$

For  $x_i = \mu, \sigma$  we have:

$$\begin{aligned} \frac{d^2 b}{d\mu d\theta} &= -\frac{\delta(x) \int_0^b [1 - F(s; x)][b\rho(b) - s\rho(s)]ds}{\mu[1 - F(b; x)][J(x; b, \theta)]^2} < 0 \\ \frac{d^2 b}{d\sigma d\theta} &= \frac{\delta(x) \int_0^b [[b\rho(b) - s\rho(s)]F_\sigma(s) + s\rho_\sigma(s)[1 - F(s)]] ds}{[1 - F(b)][J(x; b, \theta)]^2} > 0 \end{aligned}$$

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Thus  $b_{\mu\theta} < 0$ ,  $b_{\sigma\theta} > 0$ ,  $\text{sgn}(b_{x_i\theta}) = \text{sgn}(-U_{x_i})$ . For  $x_i = e$  equation (A.8) reduces to

$$U_e(x; b, \theta) = (r_f + \pi) = \frac{\mu \int_0^b [1 - F(s; x)] J(x; s, \theta) ds}{1 - e} \quad (\text{A.9})$$

$$U_{e\theta}(x; b, \theta) = \pi_\theta(x; \theta) = -\frac{\mu \delta(x) \int_0^b s f(s) ds}{1 - e} \quad (\text{A.10})$$

$$\frac{d^2 b}{ded\theta} = \frac{\delta(x) b \rho(b) [r_f + \pi] + J(x; b, \theta) \pi_\theta(x; \theta)}{\mu [1 - F(b)] [J(x; b, \theta)]^2}$$

$$\frac{d^2 b}{ded\theta} = -\frac{\delta(x) [b \rho(b) \int_0^b [1 - F(s)] J(x; s, \theta) ds - J(x; b, \theta) \int_0^b s f(s) ds]}{(1 - e) [1 - F(b)] [J(x; b, \theta)]^2}$$

$$\frac{d^2 b}{ded\theta} = -\frac{\delta(x) [\int_0^b [1 - F(s)] [b \rho(b) J(x; s, \theta) - s \rho(s) J(x; s, \theta)]]}{(1 - e) [1 - F(b)] [J(x; b, \theta)]^2}$$

$$\frac{d^2 b}{ded\theta} = -\frac{\delta(x) [\int_0^b [1 - F(s)] [b \rho(b) - s \rho(s)]]}{(1 - e) [1 - F(b)] [J(x; b, \theta)]^2} < 0$$

Thus  $\text{sgn}(b_{e\theta}) = \text{sgn}(-U_e)$ . These cases establish the single crossing property condition. ■

## B Proof of Proposition 2

Assume  $U_b(x; \bar{\theta}, \underline{b}) > 0$ . In that case  $h(x) = 0$  if  $d(x) < 0$  and  $h(x) = 1$  if  $d(x) > 0$  for all  $x \in S(\underline{\theta})$ .  $d_{x_i}(x; b) > 0$  for  $x_i = z, v, e, \mu$  and  $d_{x_i}(x; b) < 0$  for  $x_i = \sigma, r_f, \phi$

**Proof.** We will go by cases to prove the relationship of  $h(x)$  and  $d(x)$ :

A)  $x \in S(\underline{\theta})/S(\bar{\theta})$  (i) Since  $x \in S(\underline{\theta})$  there exists a  $\underline{b}(x; \underline{\theta})$  where  $U(x; \underline{\theta}, \underline{b}(x; \underline{\theta})) = 0$  and  $V(x; \underline{b}(x; \underline{\theta})) > 0$ . (ii) Using lemma 1 and the fact that  $x \notin S(\bar{\theta})$ , then any  $b$  such that  $V(x; b) > 0$  implies that  $U(x; \bar{\theta}, b) < 0$ . (iii) Since at the equilibrium rate  $\underline{b}(x; \underline{\theta})$  we have  $V(x; \underline{b}(x; \underline{\theta})) > 0$  it then follows from (ii) that  $U(x; \bar{\theta}, \underline{b}(x; \underline{\theta})) < 0$ . Hence  $d(x) < 0$  from equation (4). (iv) Since firms cannot borrow from bondholders then we have  $h(x) = 0$ .

B)  $x \in S(\underline{\theta}) \cap S(\bar{\theta})$  An equilibrium rate exists for both lender types. (i) The bank lending rate  $\underline{b}(x; \underline{\theta})$  satisfies  $\underline{b}(x; \underline{\theta}) < b^*(x; \underline{\theta}) \equiv \min[b_u(x; \underline{\theta}), b_v(x)]$  where  $b_u(x; \underline{\theta})$  is defined in equation (A.1). (ii) Proposition 1A shows that  $U(x; \bar{\theta}, b)$  attains its peak at  $b_u(x; \bar{\theta})$ . (iii) A consequence of the assumption that  $U_b(x; \bar{\theta}, \underline{b}) > 0$  is that  $\underline{b}(x; \underline{\theta}) < b_u(x; \bar{\theta})$ . Facts (i) and (iii) imply that  $\underline{b}(x; \underline{\theta}) < b^*(x; \bar{\theta}) \equiv \min[b_u(x; \bar{\theta}), b_v(x)]$

$d(x) < 0$  implies that  $U(x; \bar{\theta}, \underline{b}) < 0$ . (i) Further,  $U(x; \bar{\theta}, b) < 0$  for all  $b < \underline{b}$  since  $U_b(x; \bar{\theta}, b) > 0$  for all  $b < \underline{b} < b_u(x; \bar{\theta})$ . (ii) Since there exists an equilibrium for bondholders, lemma 1 implies that  $U(x; \bar{\theta}, b^*(x; \bar{\theta})) > 0$ . (iii) The equilibrium rate for bondholders is  $b(x; \bar{\theta}) \in (b(x; \underline{\theta}), b^*(x; \bar{\theta}))$  by the intermediate value theorem. Fact (iii) says that the equilibrium bank rates are lower than the equilibrium bond rates,  $b(x; \underline{\theta}) < b(x; \bar{\theta})$  and companies borrow from banks (i.e.  $h(x) = 0$ ).

$d(x) > 0$  implies that  $U(x; \bar{\theta}, b(x; \underline{\theta})) > 0$ . (i) From the discussion in lemma 1, we know that  $U(x; \bar{\theta}, b) < 0$  for  $b < [1 - e]r_f/\mu$ . (ii) Thus  $b(x; \bar{\theta}) \in ([1 - e]r_f/\mu, b(x; \underline{\theta}))$  by the intermediate value theorem. Fact (ii) says that the equilibrium bond rates are lower than the equilibrium bank rates,  $b(x; \bar{\theta}) < b(x; \underline{\theta})$  so firms borrow from bondholders (i.e.  $h(x) = 1$ ). These cases prove the relationship

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between  $h(x)$  and  $d(x)$ . Regarding the partial derivatives, we find that:

$$d(x) = \frac{U(x; \underline{b}, \bar{\theta})}{1-e} = \frac{\mu \int_0^{\underline{b}} [1 - F(s; x)] J(x; s, \bar{\theta}) ds}{1-e} - r_f = \pi(x; \underline{\theta}) - \frac{[\bar{\theta} - \underline{\theta}] \mu \delta(x) \int_0^{\underline{b}} s f(s; x) ds}{1-e}$$

For  $x_i = v, z, \phi, r_f, \mu, \sigma$  the derivative of equation (4) reduces to:

$$\begin{aligned} d_{x_i}(x) &= \frac{1}{1-e} \left[ U_{x_i}(x; \underline{b}, \bar{\theta}) + U_b(x; \underline{b}, \bar{\theta}) \frac{d\underline{b}}{dx_i} \right] = \frac{1}{1-e} \left[ U_{x_i}(x; \underline{b}, \bar{\theta}) - U_b(x; \underline{b}, \bar{\theta}) \frac{U_{x_i}(x; \underline{b}, \underline{\theta})}{U_b(x; \underline{b}, \underline{\theta})} \right] \\ d_{x_i}(x) &= \frac{1}{[1-e]J(x; \underline{b}, \underline{\theta})} [J(x; \underline{b}, \underline{\theta}) U_{x_i}(x; \underline{b}, \bar{\theta}) - J(x; \underline{b}, \bar{\theta}) U_{x_i}(x; \underline{b}, \underline{\theta})] \\ d_v(x) &= -\frac{[\bar{\theta} - \underline{\theta}] \mu \delta_v(x)}{[1-e]J(x; \underline{b}, \underline{\theta})} \int_0^{\underline{b}} s f(s) ds > 0 \quad d_z(x) = -\frac{[\bar{\theta} - \underline{\theta}] \mu \delta_z(x)}{[1-e]J(x; \underline{b}, \underline{\theta})} \int_0^{\underline{b}} s f(s) ds > 0 \\ d_\phi(x) &= \frac{J(x; \underline{b}, \bar{\theta})}{J(x; \underline{b}, \underline{\theta})} \pi_\phi(x; \underline{\theta}) < 0 \quad d_{r_f}(x) = -\frac{[\bar{\theta} - \underline{\theta}] \delta(x) \underline{b} \rho(\underline{b})}{J(x; \underline{b}, \underline{\theta})} < 0 \\ d_\mu(x) &= \frac{[\bar{\theta} - \underline{\theta}] \delta(x)}{[1-e]J(x; \underline{b}, \underline{\theta})} \left[ \int_0^{\underline{b}} [\underline{b} \rho(\underline{b}) - s \rho(s)] [1 - F(s; x)] ds \right] > 0 \\ d_\sigma(x) &= \frac{-[\bar{\theta} - \underline{\theta}] \mu \delta(x)}{[1-e]J(x; \underline{b}, \underline{\theta})} \int_0^{\underline{b}} [[\underline{b} \rho(\underline{b}; x) - s \rho(s; x)] F_\sigma(s; x) + s \rho_\sigma(s) [1 - F(s)]] ds < 0 \end{aligned}$$

Given our assumptions on stochastic dominance (i.e., that  $\int_0^k F_\sigma(s) ds > 0, \rho_\sigma(s) > 0$ ), it is clear that  $d_\sigma(x) < 0$ . For  $x_i = e$  the derivative of the  $d(x)$  - after using the fact that  $U(x; \underline{\theta}, \underline{b}) = 0$  - reduces to:

$$\begin{aligned} d_e(x) &= \frac{[\bar{\theta} - \underline{\theta}] \delta(x)}{[1-e]^2 J(x; \underline{b}, \underline{\theta})} \left[ \underline{b} \rho(\underline{b}) (1-e) [r_f + \pi(x; \underline{\theta})] - \mu J(x; \underline{b}, \underline{\theta}) \int_0^{\underline{b}} s f(s) ds \right] \\ d_e(x) &= \frac{[\bar{\theta} - \underline{\theta}] \mu \delta(x)}{[1-e]^2 J(x; \underline{b}, \underline{\theta})} \int_0^{\underline{b}} [[1 - F(s)] \underline{b} \rho(\underline{b}) J(x; s, \theta) ds - J(x; \underline{b}, \underline{\theta}) s f(s)] ds \\ d_e(x) &= \frac{[\bar{\theta} - \underline{\theta}] \mu \delta(x)}{[1-e]^2 J(x; \underline{b}, \underline{\theta})} \left[ \int_0^{\underline{b}} [\underline{b} \rho(\underline{b}) - s \rho(s)] [1 - F(s; x)] ds \right] > 0 \end{aligned}$$

This finishes the proof of proposition 2. ■

**Proposition 3** *An approximation to the differential function is given by:*

$$d(x) = \frac{1 - \bar{\theta}\delta(x)k_0}{1 - \underline{\theta}\delta(x)k_0} \pi(x; \underline{\theta}) + \frac{[\bar{\theta} - \underline{\theta}]\delta(x)}{1 - \underline{\theta}\delta(x)k_0} \left[ \frac{\mu k_1}{1 - e} - k_0 r_f \right]$$

## C Proof of Proposition 3

**Proof.** We do an approximation of  $b[1 - F(b)]$ . First, define  $z \equiv \int_0^s \omega f(\omega) d\omega \equiv \Gamma(s)$ ,  $n(z) \equiv s \Rightarrow \Gamma(n(z)) \equiv z$ . The implicit function proposition implies that  $dn/dz = 1/[n(z)f(n(z))]$ . Define  $y \equiv \int_0^b \omega f(\omega) d\omega$  and  $q \equiv \int_0^k \omega f(\omega) d\omega$ . We do a change of variables and an exact Taylor expansion in equation (C.1); here we also do an integration by parts and use  $sf(s) \equiv [1 - F(s)]s\rho(s)$ . In (C.2) we use  $U(x; \underline{b}, \underline{\theta}) = 0$ .

$$\begin{aligned} b[1 - F(b)] &= n(y)[1 - F(n(y))] & (C.1) \\ &= n(q)[1 - F(n(q))] + [y - q][1 - F(n(q^+) - n(q^+)f(n(q^+)))] \frac{dn}{dz} \quad q^+ \in [q, y] \\ &= k[1 - F(k)] + \frac{1 - k^+\rho(k^+)}{k^+\rho(k^+)} \left[ \int_0^b sf(s)ds - \int_0^k sf(s)ds \right] \quad k^+ \in [k, b] \\ &= \frac{k^+\rho(k^+)}{k^+\rho(k^+)} \left[ k[1 - F(k)] + \int_0^k sf(s)ds \right] - \int_0^k sf(s)ds + \frac{1 - k^+\rho(k^+)}{k^+\rho(k^+)} \int_0^b sf(s)ds \\ &= \frac{k^+\rho(k^+)}{k^+\rho(k^+)} \int_0^k [1 - F(s)]ds - \int_0^k [1 - F(s)]s\rho(s)ds + \frac{1 - k^+\rho(k^+)}{k^+\rho(k^+)} \int_0^b sf(s)ds \\ &= \frac{\int_0^k [1 - F(s)][k^+\rho(k^+) - s\rho(s)]ds}{k^+\rho(k^+)} + \frac{1 - k^+\rho(k^+)}{k^+\rho(k^+)} \int_0^b sf(s)ds \\ &= \frac{k_1}{k_0} + \frac{1 - k_0}{k_0} \int_0^b sf(s)ds \quad k_0 \equiv k^+\rho(k^+) \quad k_1 \equiv \frac{\int_0^k [1 - F(s)][k^+\rho(k^+) - s\rho(s)]ds}{k^+\rho(k^+)} \end{aligned}$$

$$\begin{aligned} U(x; \underline{b}, \underline{\theta}) &= \mu \underline{b}[1 - F(\underline{b})] + (1 - \underline{\theta}\delta(x))\mu \int_0^{\underline{b}} sf(s)ds - (1 - e)(r_f + \pi) = 0 & (C.2) \\ &= \frac{\mu k_1}{k_0} + \left[ \frac{1}{k_0} - \underline{\theta}\delta(x) \right] \mu \int_0^{\underline{b}} sf(s)ds - (1 - e)(r_f + \pi) = 0 \end{aligned}$$

$$\mu \int_0^{\underline{b}} sf(s)ds = -\frac{1}{1 - \underline{\theta}\delta(x)k_0} [\mu k_1 - k_0(1 - e)(r_f + \pi)] \quad (C.3)$$

$$d(x) = \pi(x; \underline{\theta}) - \frac{[\bar{\theta} - \underline{\theta}]\mu\delta(x) \int_0^{\underline{b}} sf(s; x)ds}{1 - e} \quad (C.4)$$

$$d(x) = \pi(x; \underline{\theta}) + \frac{[\bar{\theta} - \underline{\theta}]\delta(x)}{1 - \underline{\theta}\delta(x)k_0} \left[ \frac{\mu k_1}{1 - e} - k_0(r_f + \pi) \right]$$

$$d(x) = \frac{1 - \bar{\theta}\delta(x)k_0}{1 - \underline{\theta}\delta(x)k_0} \pi(x; \underline{\theta}) + \frac{[\bar{\theta} - \underline{\theta}]\delta(x)}{1 - \underline{\theta}\delta(x)k_0} \left[ \frac{\mu k_1}{1 - e} - k_0 r_f \right]$$

This concludes the proof of the proposition ■