# Volume, Volatility, Price, and Profit When All Traders Are Above Average

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## Abstract

Psychological studies establish that people are usually overconfident and that they systematically overweight some types of information while underweighting others. How overconfidence affects a financial market depends on who in the market is overconfident and on how information is distributed. This paper examines markets in which price-taking traders, a strategic-trading insider, and risk-averse market-makers are overconfident. It also analyzes the effects of overconfidence when information is costly. In all scenarios, overconfidence increases expected trading volume and market depth while lowering the expected utility of those who are overconfident. However, its effect on volatility and price quality depend on who is overconfident. Overconfident traders can cause markets to underreact to the information of rational traders. Markets also underreact to abstract, statistical, or highly relevant information, while they overreact to salient, anecdotal, or less relevant information.

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Models of financial markets are often extended by incorporating the imperfections that we observe in real markets. For example, models may not consider transactions costs, an important feature of real markets; so Constantinides (1979), Leland (1985), and others incorporate transactions costs into these models.

Just as the observed features of actual markets are incorporated into models, so too are the observed traits of economic agents. In 1738 Daniel Bernoulli noted that people behave as if they are risk-averse. Prior to Bernoulli most scholars considered it normative behavior to value a gamble at its expected value. Today economic models usually assume agents are risk-averse, though, for tractability, they are also modeled as risk-neutral. In reality people are not always risk-averse, or even risk-neutral; millions of people engage in regular risk-seeking activity, such as buying lottery tickets. Kahneman and Tversky (1979) identify circumstances in which people behave in a risk-seeking fashion. Most of the time, though, most people act risk-averse, and most economists model them so.[1]

This paper analyzes market models in which investors are rational in all respects except how they value information.[2] A substantial literature in cognitive psychology establishes that people are usually overconfident and, specifically, that they are overconfident about the precision of their knowledge. As is the case with risk-aversion, there are well-known exceptions to the rule, but most of the time people are overconfident. Psychologists also find that people systematically underweight some types of information and overweight others.

This paper looks at the effects of overconfidence when price-taking traders, strategic-trading insiders, and market-makers are overconfident and when information is costly. To explore the effects of the overconfidence of these different market participants, three different models are used. These are modifications of well-known models: Diamond and Verrecchia (1981)

[1]Another place where observed behavior has found wide acceptance in economic models is in the discounted utility of future consumption. Nineteenth century economists such as Senior, Jevons, and Böhm-Bawerk believed that, ideally, the present and the future should be treated equally; yet they observed that generally people value present consumption more highly than future [Loewenstein (1992)]. Today, when it may affect the predictions of models, economists usually assume that people discount the utility of future consumption. And people usually do discount the future, but not always. They will, for example, "bite the bullet" and get an unpleasant experience over with, which they could otherwise delay. Loewenstein and Prelec (1991) identify circumstances in which people demonstrate negative, rather than the usual positive, time preference.

[2]In the first model presented here, investors also behave with less than full rationality by trading myopically.

and Hellwig (1980)–price-takers; Kyle (1985)–strategic traders; and Grossman and Stiglitz (1980)–market-makers and costly information.

The main results presented here are:

• Trading volume increases when price-takers, insiders, or market-makers are overconfident. This is the most robust effect of overconfidence. Anecdotal evidence suggests that in many markets trading volume is excessive [Dow and Gorton (1994)]. Recent empirical studies [Odean (1997); Statman and Thorley (1997)] indicate that overconfidence generates trading. From a modeling perspective, overconfidence can facilitate orderly trade even in the absence of noise traders.

• Overconfident traders can cause markets to underreact to the information of rational traders. Markets also underreact when traders underweight new information and overreact when they overweight it. The degree of under- or overreaction depends on the fraction of all traders who under- or overweight the information. A review of the psychology literature on inference finds that people systematically underweight abstract, statistical, or highly relevant information, while they overweight salient, anecdotal or extreme information. This may shed some light on why markets overreact in some circumstances, such as initial public offerings (IPO's) [Loughran and Ritter (1995)], and underreact in others, such as earnings announcements [Bernard and Thomas (1989, 1990)], dividend initiations and omissions [Michaely, Thaler, and Womack (1995)], open-market share repurchases [Ikenberry, Lakonishok, and Vermaelen (1995)], self-tender offers [Lakonishok and Vermaelen (1990)], and brokerage recommendations [Womack (1996)].

• Overconfidence reduces traders' expected utility. When information is costly and traders are overconfident, uninformed traders fare better than informed traders. Overconfidence may cause investors to prefer active management despite evidence that it subtracts value [Lakonishok, Shleifer, and Vishny (1992)].

• Overconfidence increases market depth.

• How overconfidence affects a market depends on who is overconfident. For example, overconfidence improves the quality of prices when an insider is overconfident, but worsens price quality when price-takers are overconfident. And while overconfidence generally increases market volatility, an overconfident market-maker may dampen it. Excess volatility in equity markets has been found by some researchers [Shiller (1981); LeRoy and Porter (1981); Shiller (1989)], though others have questioned these finding [Kleidon (1986); Marsh and Miller (1986)].

# 1 Related Work

A number of researchers have modeled economies in which traders hold mistaken distributional beliefs about the payoff of a risky asset. In Varian (1989) traders' priors have different means. Varian notes that the dispersion of posterior beliefs caused by differing distributional assumptions motivates trade. Harris and Raviv (1993) investigate a multi-period economy in which risk-neutral traders disagree about how to interpret a public signal. The model of price-taking traders presented here differs from Harris and Raviv in that my traders are risk-averse and disagree about the interpretation of private signals. Furthermore, the nature of this disagreement is grounded in psychological research. In Kandel and Pearson (1992), risk-averse traders disagree about both the mean and the variance of a public signal. In this case, the public signal may motivate increased trading even when it doesn't change price. De Long, Shleifer, Summers, and Waldmann (1990) show in an overlapping generations model that traders who misperceive the expected price of a risky asset may have higher expected returns, though lower expected utilities, than rational traders in the same economy. Roll (1986) suggests that overconfidence (hubris) may motivate many corporate takeovers. Hirshleifer, Subrahmanyam, and Titman (1994) argue that overconfidence can promote herding in securities markets. In Shefrin and Statman (1994), traders infer, from past observations, the transition matrix governing changes in the dividend growth rate. Some traders are true Bayesians while others make one of two common errors: they weight recent observations too heavily, thus underweighting prior information, or they commit a gambler's fallacy, expecting recent events to reverse so that short runs of realized events more closely resemble long-term probabilities. When all traders are rational, the market behaves as if it had a "single driver" and prices are efficient. Biased traders can introduce a "second driver" and distort prices.

Benos (1996), Kyle and Wang (1995), and Wang (1995) look at overconfidence in models based on Kyle (1985), but with two informed traders. In Benos, traders are overconfident in their knowledge of the signals of others; they also can display extreme overconfidence in their own noisy signal, believing it to be perfect. Kyle and Wang, and Wang model overconfidence similarly to how it is modeled in this paper, that is, as an overestimation of the precision of one's own information.[3] Gervais and Odean (1997) develop a multi-period model in which a trader's endogenously determined level of overconfidence changes dynamically as a result of his tendency to disproportionately attribute his success to his own ability. In Daniel, Hirshleifer, and Subrahmanyam (1997) rational risk-averse traders trade with risk-neutral traders who overreact to private signals, properly weight public signals, and grow

[3] I learned of Kyle and Wang's work after developing the models in this paper.

more overconfident with success. This results in return-event patterns which are consistent with many market anomalies. My paper differs from these others in that it examines how the effects of overconfidence depend on who, in a market, is overconfident and on how information in that market is disseminated; it also relates market under- and overreactions to the psychological literature on inference.

The rest of the paper is organized as follows: The next section describes some of the literature on overconfidence and on inference and discusses why we should expect to find overconfidence in financial markets. Section 3 presents the models. Section 4 discusses the results. And the final section concludes. Formal statements of the propositions, proofs, and the derivations of equilibria are presented in the appendices. Table 1 provides a summary of notation used in the models.

# 2 Overconfidence

## 2.1 The Case for Overconfidence

Studies of the calibration of subjective probabilities find that people tend to overestimate the precision of their knowledge [Alpert and Raiffa (1982); Fischhoff, Slovic and Lichtenstein (1977); see Lichtenstein, Fischhoff, and Phillips (1982) for a review of the calibration literature]. Such overconfidence has been observed in many professional fields. Clinical psychologists [Oskamp (1965)], physicians and nurses, [Christensen-Szalanski and Bushyhead (1981), Baumann, Deber, and Thompson (1991)], investment bankers [Staël von Holstein (1972)], engineers [Kidd (1970)], entrepreneurs [Cooper, Woo, and Dunkelberg (1988)], lawyers [Wagenaar and Keren (1986)], negotiators [Neale and Bazerman (1990)], and managers [Russo and Schoemaker (1992)] have all been observed to exhibit overconfidence in their judgments. (For further discussion, see Lichtenstein, Fischhoff, and Phillips (1982) and Yates (1990).)

The best established finding in the calibration literature is that people tend to be overconfident in answering questions of moderate to extreme difficulty [Fischhoff, Slovic, and Lichtenstein (1977), Lichtenstein, Fischhoff, and Phillips (1982), Yates (1990), Griffin and Tversky (1992)]. Exceptions to overconfidence in calibration are that people tend to be underconfident when answering easy questions, and they tend to be well-calibrated when predictability is high and when performing repetitive tasks with fast, clear feedback. For example, expert bridge players [Keren (1987)], race-track bettors [Dowie (1976), Hausch, Ziemba, and Rubinstein (1981)] and meteorologists [Murphy and Winkler (1984)] tend to be well calibrated.

Miscalibration is only one manifestation of overconfidence. Researchers also find that people overestimate their ability to do well on tasks and these overestimates increase with the personal importance of the task [Frank (1935)]. People are also unrealistically optimistic about future events. They expect good things to happen to them more often than to their peers [Weinstein (1980); Kunda (1987)]. They are even unrealistically optimistic about pure chance events [Marks (1951), Irwin (1953), Langer and Roth (1975)].[4]

People have unrealistically positive self-evaluations [Greenwald (1980)]. Most individuals see themselves as better than the average person and most individuals see themselves better than others see them [Taylor and Brown (1988)]. They rate their abilities and their prospects higher than those of their peers. For example, when a sample of U.S. students—average age 22—assessed their own driving safety, 82% judged themselves to be in the top 30% of the group [Svenson (1981)].[5] And 81% of 2994 new business owners thought their business had a 70% or better chance of succeeding but only 39% thought that any business like theirs would be this likely to succeed [Cooper, Woo, and Dunkelberg (1988)]. People overestimate their own contributions to past positive outcomes, recalling information related to their successes more easily than that related to their failures. Fischhoff (1982) writes that "they even misremember their own predictions so as to exaggerate in hindsight what they knew in foresight." And when people expect a certain outcome and the outcome then occurs, they often overestimate the degree to which they were instrumental in bringing it about [Miller and Ross (1975)]. Taylor and Brown (1988) argue that exaggerated beliefs in one's abilities and unrealistic optimism may lead to "higher motivation, greater persistence, more effective performance, and ultimately, greater success." These beliefs can also lead to biased judgments.

## 2.2 Inference

Psychologists find that, when making judgments and decisions, people overweight salient information (i.e. information that stands out and captures attention) [Kahneman and Tversky (1993); Grether (1980)]. People also give too much consideration to how extreme information is and not enough to its validity [Griffen and Tversky (1992)]; they "often behave as though

[4]Ito (1990) reports evidence that participants in foreign exchange markets are more optimistic about how exchange rate moves will affect them than how they will affect others. Over two years the Japan Center for International Finance conducted a bi-monthly survey of foreign exchange experts in 44 companies. Each was asked for point estimates of future yen/dollar exchange rates. The experts in import-oriented companies expected the yen to appreciate (which would favor their company), while those in export-oriented companies expected the yen to fall (which would favor their company).

[5]A modest 51% of a group of older Swedish students—average age 33—placed themselves in the top 30% of their group.

information is to be trusted regardless of its source, and make equally strong or confident inferences, regardless of the information's predictive value" [Fiske and Taylor (1991)]. They overweight information which is consistent with their existing theories and are more prone to gather information which supports these theories [Nisbett and Ross (1980)]. They are more confident in opinions based on vivid information [Clark and Rutter (1985)] and weigh cases, scenarios, and salient examples more heavily than relevant, abstract, statistical, or base-rate information [Kahneman and Tversky (1973); Bar-Hillel (1980); Hamill, Wilson, and Nisbett (1980); Nisbett and Ross (1980); Bar-Hillel and Fischhoff (1991); Taylor and Thompson (1982)]. In addition to underweighting base-rate information, people underestimate the importance of sample size [Tversky and Kahneman, 1971; Kahneman and Tversky (1972)] and of regression to the mean, i.e., the tendency of extreme outcomes to be followed by outcomes closer to the population mean [Kahneman and Tversky (1973)].

In general then, we might expect people to overreact to less relevant, more attention grabbing information (e.g., an extreme event, a prominent news article with strong human interest, a rumor) while underreacting to important abstract information.[6] In particular, we might expect people to underestimate the importance of single statistics which summarize a large sample of relevant data (e.g. corporate earnings).

## 2.3 Information

In the following models, traders update their beliefs about the terminal value of a risky asset, $\tilde{v}$, on the basis of three sources of information: a private signal, their inferences from market price regarding the signals of others, and common prior beliefs. The overconfidence literature indicates that people believe their knowledge is more precise than it really is, they rate their own abilities too highly when compared to others, and they are excessively optimistic. To be consistent with these patterns, traders in the model must hold posterior beliefs about the distribution of $\tilde{v}$ that are too precise, value their own information more than others', and expect higher utility than is warranted. In the models, traders overweight their private signals and, therefore, their posteriors are too precise; their own information is valued more than others'; and they overestimate their expected utility.

[6] Reacting to how extreme information is rather than how reliable its source is can have dramatic consequences. On April 11, 1997, The Financial Times of London reported fraud in connection with an offshore fund called the Czech Value Fund, referring to the fund by the abbreviation CVF. Four days later Castle Convertible fund, a small closed-end fund with a diversified portfolio of convertible stocks and bonds trading on the AMEX under the ticker symbol CVF plummeted 32% in 22 minutes. Trading was halted. After the Castle managers assured the exchange that they had no news, trading resumed at close to its preplunge price. Apparently some investors reacted to word of extreme problems rather than to the reliability of that word (New York Times, April 20, 1997, C1, byline Floyd Norris).

To prove most of the propositions in this paper it is sufficient that 1) traders overweight new information relative to prior information and 2) overweight their own information relative to that of others. Both conditions are satisfied if each trader overweights his own signal. These conditions may be further amplified when traders underweight their priors or underweight the signals of others. Common priors reflect base-rate information and the prior opinions of a large group of traders. Such data, and by extension common priors, are likely to be undervalued. The signals of others constitute a large sample. Since large sample inferences are usually undervalued, it is likely that if traders error in valuing the signals of others, they will undervalue these. Most of the propositions are proven only for the situation where each trader overweights his own signals. Often corollaries can be proven for when traders underweight their priors or underweight the signals of others. In the interest of parsimony, these corollaries are not stated formally or proven, though the intuition involved is at times discussed.[7]

The calibration literature discussed above tells us that people overestimate the precision of their information. Overconfidence in one's information is not the only type of overconfidence we might expect to find in the market. Traders could, instead, be overconfident about the way they interpret information rather than about the information itself. For example, traders of a stock might look at signals such as trading momentum, price-earnings ratio, or forecasts of industry trends. These are examples of public information which is available to any trader, but which is valued differently by different traders. Thus, a Graham-and-Dodd style fundamental investor might be aware of recent changes in a stock's momentum but consider its price-earnings ratio to be a more important signal, while a technical trader who follows momentum might believe otherwise. Each is overconfident in his style of analysis and the signal it utilizes. At the same time, each is aware of the beliefs, and perhaps even the signals, of the other.[8]

In the models, traders who believe that their information is more precise than it actually is expect to realize greater utility than they have cause to expect. In this way these models capture some of the spirit of excessive optimism which psychologists have documented.

[7] Previous versions of this paper did explicitly state and prove these corollaries.

[8] Even sophisticated investors may agree to disagree. The Washington Post, (January 7, 1992, page C2, byline Allan Sloan) reports that, during the same time period, the nation's most prominent long-term investor, Warren Buffett, and its most prominent short sellers, the Feshbach Brothers, held, respectively, long and short positions worth hundreds of millions of dollars in Wells Fargo Bank. (Buffett controls the investments of Berkshire Hathaway Inc.; the Feshbachs run an investment fund.) Ostensibly, Buffett and the Feshbachs disagreed about how much the bank would be hurt by its weak loan portfolio. They also differed in their investment horizons. Despite being right about the loans, the Feshbachs lost $50 million when they had to close their positions. As of January 1992, Buffett was about even.

However, optimism is not limited to an inflated opinion of the precision of unbiased signals. A trader might also have false confidence in a biased (misinterpreted) signal or theory. Such misplaced confidence may underlie Odean's (1997) finding that the stocks purchased by a large sample of investors at a discount brokerage house subsequently underperformed, on average, the stocks they sold. These investors appear to have information which they misinterpret.

## 2.4 Overconfidence in Financial Markets

Why might we expect those trading in financial markets to be overconfident? The foremost reason is that people usually are overconfident. The exceptions to overconfidence mentioned above generally do not apply to financial markets. Most of those who buy and sell financial assets try to choose assets that will have higher returns than similar assets. This is a difficult task and it is precisely in such difficult tasks that people exhibit the greatest overconfidence. Not only novices exhibit overconfidence. Griffin and Tversky (1992) write that when predictability is very low, as in the stock market, experts may even be more prone to overconfidence than novices, because experts have theories and models (e.g. of market behavior) which they tend to overweight.[9]

Securities markets are difficult and slow places in which to calibrate one's confidence. Learning is fastest when feedback is quick and clear, but in securities markets the feedback is often slow and noisy. There may even be a trade-off between speed and clarity of feedback whereby short-term traders get quicker, but noisier, feedback, and long-term traders receive clearer feedback but must wait for it. The problem of noisy feedback can be exacerbated by the endogeneity of the evaluation period. Shefrin and Statman (1985) propose and Odean (1996) confirms that investors prefer to sell winners and hold onto their losers. If investors judge their original purchase decisions on the basis of the returns realized, rather than those accrued, then, by holding onto losers, they will judge themselves to have made fewer poor decisions. Furthermore, the feedback from losses will be delayed more than that from gains, further facilitating positive self-evaluations.

Selection bias may cause those participating actively in financial markets to be more overconfident than the general population. People vary in ability and those who believe they have more ability to trade may be more likely to seek jobs as traders or to trade actively on their own account. If people are uncertain judges of their own ability, then we might expect financial markets to be populated by those with the most ability and by those who most overestimate their ability.

[9] This observation may not apply to experts who adhere to computer-based quantitative models [see Dawes, Faust, and Meehl, 1989].

Survivorship bias can also lead to overconfidence in market participants. Unsuccessful traders may lose their jobs or choose to drop out of the market; unsuccessful traders who survive will, on average, control less wealth than successful traders. If traders overestimate the degree to which they were responsible for their own successes—as people do in general [Miller and Ross (1975), Langer and Roth (1975); Nisbett and Ross (1980)]—successful traders may grow overconfident and more wealth will be controlled by overconfident traders. In Gervais and Odean (1997) this self-enhancing bias causes wealthy traders, who are in no danger of being driven from the marketplace, to be overconfident. It is not that overconfidence has made them wealthy, but the process of becoming wealthy has contributed to their overconfidence. An old Wall Street adage, "Don't confuse brains with a bull market," warns traders of the danger of becoming overconfident during a market rally; no doubt this warning is given for good reason.

This paper finds overconfident traders have lower expected utility.[10] It does not necessarily follow that the overconfident will lose their wealth and leave the marketplace. An overconfident trader will make biased judgments which may lead to lower returns. However, an overconfident risk-averse trader will also choose a riskier portfolio than he would otherwise hold and may be rewarded for risk-bearing with greater expected returns. It is possible that the profits of greater risk-tolerance will more than compensate for the losses of biased judgments. Thus, as a group, overconfident traders could have higher expected returns, though lower expected utility, than properly calibrated traders, as is the case in De Long, Shleifer, Summers, and Waldmann (1990).

# 3 The Models

## 3.1 Price-takers

Throughout this paper, expectations which are taken using the distributions that traders believe to be correct are indicated by a subscript "b" (e.g., $\mathrm{var}_\mathrm{b}$). Expectations taken using the distributions which are actually correct are indicated by a subscript "a" (e.g., $\mathrm{var}_\mathrm{a}$). In equilibrium, overconfident traders believe that they are acting optimally, and so they do not depart from the equilibrium. The traders could, in actuality, improve their expected utilities by acting differently, so the equilibria achieved here are not rational expectations equilibria.

The model of price-taking traders is based on Diamond and Verrecchia (1981) and Hellwig (1980). A riskless asset and one risky asset are exchanged in three rounds of trading at times

[10]When objectively measured, expected utility is lower for overconfident traders. However overconfident traders believe that they are maximizing expected utility.

$t = 1$, $t = 2$, and $t = 3$. Consumption takes place only at $t = 4$ at which time the riskless asset pays 1 unit per share and each share of the risky asset pays $\tilde{v}$, where $\tilde{v} \sim N(\bar{v}, {h_v}^{-1})$. The riskless interest rate is assumed to be 0. There are $N$ investors ($i = 1, ..., N$). As a modeling convenience we analyze the limit economy where $N \to \infty$. Thus each investor correctly assumes that his own demand does not affect prices. At $t = 0$ each trader has an endowment of $f_{0i}$ of the riskless asset and $x_{0i}$ of the risky asset. In trading round $t$, trader $i$'s demands for the riskless and the risky asset are $f_{ti}$ and $x_{ti}$. $\bar{x}$ is the per capita supply of the risky asset; it is fixed, known to all, and unchanging. This differs from Diamond and Verrecchia and Hellwig where a stochastic supply of the risky asset provides an exogenous source of noise. $P_t$ is the price of the risky asset in trading rounds 1, 2, and 3. Trader $i$'s wealth is $W_{ti} = f_{ti} + P_t x_{ti}$, for $t = 1$, 2, and 3, and $W_{4i} = f_{3i} + \tilde{v} x_{3i}$. There is no signal prior to the first round of trading at $t = 1$. Prior to trading at $t = 2$ and, again, prior to trading at $t = 3$, trader $i$ receives one of $M$ private signals, $\tilde{y}_{ti} = \tilde{v} + \tilde{\epsilon}_{tm}$, where $\tilde{\epsilon}_{tm} \sim N(0, {h_\epsilon}^{-1})$ and $\tilde{\epsilon}_{21}, ..., \tilde{\epsilon}_{2M}, \tilde{\epsilon}_{31}, ..., \tilde{\epsilon}_{3M}$ are mutually independent. Each signal is received by the same number of traders. ($N$ is assumed to be a multiple of $M$). $\bar{Y}_t = \sum_{i=1}^{N} y_{ti}/N = \sum_{m=1}^{M} y_{tm}/M$ is the average signal at time t.

The assumption that there are $M < N$ signals in any time period is motivated by the observation that when the number of traders is large there are likely to be fewer pieces of information about an asset than there are traders.

Each trader knows that $N/M - 1$ other traders are receiving the same two signals as she. She believes the precisions of these two signals to be $\kappa h_\epsilon$, $\kappa \geq 1$. She believes the precisions of the other $2M - 2$ signals to be $\gamma h_\epsilon$, $\gamma \leq 1$. All traders believe that the precision of $\tilde{v}$ is $\eta h_v$, $\eta \leq 1$; that is, traders underestimate, or correctly estimate, the precision of their prior information. Let $\Phi_{1i} = \{\}$, $\Phi_{2i} = [\, y_{2i} \quad P_2 \,]^T$ and $\Phi_{3i} = [\, y_{2i} \quad y_{3i} \quad P_2 \quad P_3 \,]^T$. Thus $\Phi_{ti}$ represents the information available to trader $i$ (in addition to prior beliefs) at time $t$. Note that a trader's posterior is more precise than that of a rational trader if, after receiving both of her signals, $\eta h_v + 2(\kappa + (M-1)\gamma)h_\epsilon \geq h_v + 2M h_\epsilon$.

Trader $i$'s utility function is $-\exp(-aW_{it})$, thus traders have constant absolute risk-aversion (CARA) with a risk-aversion coefficient of $a$. Traders are assumed to be myopic, that is, they look only one period ahead when solving their trading problem. Thus, at times $t = 1, 2, 3$, trader $i$ solves

$$\max_{x_{ti}} \mathrm{E}[-\exp(-a(W_{t+1\,i}) \mid \Phi_{t\,i}] \quad \text{subject to} \quad P_t x_{ti} + f_{ti} \leq P_t x_{t-1\,i} + f_{t-1\,i}. \tag{1}$$

Since the traders in this model correctly conjecture that they do not affect prices, the only effect of assuming myopia is to eliminate hedging demands [see Brown and Jennings (1989)].

As others, including Singleton (1987) and Brown and Jennings (1989) have found, this simplifies the analysis.

When solving their maximization problems, traders conjecture that prices are linear functions of the average signals:

$$P_3 = \alpha_{31} + \alpha_{32}\bar{Y}_2 + \alpha_{33}\bar{Y}_3 \tag{2}$$

$$P_2 = \alpha_{21} + \alpha_{22}\bar{Y}_2 \tag{3}$$

The conjectures are identical for all traders and the coefficients determine an equilibrium in which the conjectures are fulfilled. Equilibrium is obtained because traders believe that they are behaving optimally though, in fact, they are not. This equilibrium and the proofs for this section are presented in Appendix A.

There is no exogenous noise in this model. The purpose of noise is often to keep traders from using price and aggregate demand to make perfect inferences about the information of others. If rational traders with common priors infer the same aggregate signal, they will have identical posterior beliefs, and, if their endowments and preferences are also identical, they will not trade. If preferences and endowments differ, trading may occur; but it might not occur in response to information, and this runs contrary to what we observe in markets.[11] The absence of exogenous noise in this model demonstrates that, with overconfidence, orderly trading can take place in response to information even when no noise is present. [Varian (1989) has a similar result when traders disagree about the mean of the prior.] Each trader can infer the aggregate signal, but each values his portion of the aggregate differently, arrives at a different posterior belief, and is willing to trade.

In this model, traders can perfectly infer the aggregate signal from price. In practice, traders do not usually make this perfect inference. The certainty in the model would be dispelled if random trading noise traders were added to the economy. However, this certainty results not so much from the lack of noise trading as from the conventional assumption that traders are able to know the preferences of all other traders, to know the distributions of all random variables in the economy (though here these are distorted by overconfidence), and to make perfect inferences from their information. As Arrow (1986) points out, the information gathering and computational demands put on traders in models such as this would, in a more realistic setting, "imply an ability at information processing and calculation that is far beyond the feasible and cannot be well justified as the result of learning and adaptation." It

[11] See Varian (1989) for a discussion of No-Trade Theorems.

may be that the principal source of noise in markets is not that a few (noise) traders don't attempt to optimize their utility, but that most traders are not certain how best to do so.

In lacking exogenous noise, this model is similar to that of Grossman (1976). But in Grossman's model, a trader can infer the aggregate signal $\bar{y}$ from price, and having done so, can ignore his private signal $y_i$ when determining his demand. As Beja (1976) observes, this creates a paradox in which fully informative prices arise from an aggregate demand function which is without information, since, if prices are fully informative, traders have no incentive to consider their private signals when formulating their demand. When traders are overconfident, they can still infer the average signal from price, but they do not ignore their own signal when determining their demand. Each trader considers his signal to be superior to those of others, and since the average signal weights all traders' signals equally, it is not a sufficient statistic to determine an individual trader's demand.

This model is also related to Figlewski (1978) where price-taking traders with different posterior beliefs interact. Figlewski's model does not have an exogenous noise source. To avoid the no-trade dilemma, he assumes traders are unable to infer the information of others from price. Were these traders overconfident, this assumption could be eased and results similar to the ones presented here would follow. In Jaffe and Winkler (1976), risk-neutral informed traders decide to trade after observing a risk-neutral market-maker's bid and ask. The market-maker can expect to lose to all rational investors, and so this market is unstable. Jaffe and Winkler suggest that the introduction of liquidity traders or traders who misperceive their ability–such as the overconfident traders modeled here–could stabilize this market.

As discussed above, overconfidence causes traders to have differing posterior beliefs. The more overconfident traders are, the more differing these beliefs. This leads to the first proposition:

**Proposition 1** *When traders are price-takers, expected volume increases as overconfidence increases, (if $M \geq 2$).*

In all of the propositions, expectations are taken over the true probability distributions. Here we see that as overconfidence increases, traders weight their own signals more and more heavily than they weight those of others when calculating their posterior beliefs. The posterior beliefs are therefore more dispersed and more trading takes place.[12] There is one

[12] In a dynamic setting, such as Shefrin and Statman (1994), volume is determined not simply by differences in beliefs but by the rate of change of those differences [see Karpoff (1986)].

exception to this pattern. If $M = 1$ there is only one (effectively public) signal received by all traders. And since all traders overvalue that signal equally, their beliefs remain homogenous and no trade takes place though price may change. (If traders varied in their overconfidence in the public signal, they would trade.) Expected volume also increases when traders underweight common priors or the signals of others.

**Proposition 2** *When traders are price-takers, volatility of prices increases as overconfidence increases.*

When traders are overconfident, each overvalues his own personal signal. This results in the aggregate signal being overvalued relative to the common prior in the pricing functions, equations (2) and (3), where the coefficients $\alpha_{22}$, $\alpha_{32}$, and $\alpha_{33}$ depend on an increase in $\kappa$. Overweighting the error in the aggregate signal increases the volatility of prices. Decreasing $\eta$ has the same effect, while decreasing $\gamma$ lowers the weight on the aggregate signal and lowers volatility.

Another consequence of biased expectations is that they increase the variance of the difference between price and underlying value, $\mathrm{var}(P - \tilde{v})$. Using this variance as a measure of the quality of prices we have:

**Proposition 3** *When traders are price-takers, overconfidence worsens the quality of prices.*

We will see in the next model that when a strategic insider is overconfident, overconfidence can improve the quality of prices. These two models differ in that the next model has noise traders, but more importantly they differ in how information is distributed. Here all traders receive a signal, in the next model information is concentrated in the hands of a single insider. Even if noise is added to the current model, overconfidence will continue to distort prices, not improve them. This is most easily seen when $M = 1$, that is, when there is one public signal. If the signal is public, noisy demand will obviously not affect traders' information, but overconfidence will continue to distort traders' posterior expectations and, thereby, prices. Prices also worsen when traders underweight common priors or the signals of others.

Distorted expectations reduce expected utilities. When traders are overconfident, their expected utility is lower than when their probabilities are properly calibrated. This is hardly surprising since traders choose their actions in order to optimize expected utility and, when they are overconfident, this attempt to optimize is based on incorrect beliefs. (Similarly, expected utility also declines as $\eta$ and $\gamma$ decrease.) And so:

**Proposition 4** *When price-taking traders are overconfident, their expected utility is lower than if their beliefs are properly calibrated.*

Since there are no noise traders to exploit in this model, the aggregate expected returns from trading must be zero. Overconfidence decreases expected utilities because it results in non-optimal risk sharing; traders who receive the highest signals end up holding too much of the risky asset and others hold too little (given their preferences and the true distributions of signals). Of course each trader believes that she is optimally positioned.[13]

To model overconfidence, I assume that traders overestimate the precision of their private signals. Doing so leads traders to hold differing beliefs and to overestimate the precision of their own posterior beliefs. Diverse posterior beliefs that are held too strongly are sufficient to promote excessive trading, increase volatility, distort prices, and reduce expected utility. For time-series results though, how posteriors are constructed matters. Assuming that people always overweight private signals implies that they always overweight new information. As discussed in section 2.2, this is not the case. People usually overrespond to salient information and underrespond to abstract information. They value valuable information too little and irrelevant information too much. To examine time-series implications of the model, we therefore look at both over- and underweighted signals. For price-taking traders, overvaluing new information leads to price reversals while undervaluing it leads to price trends:

**Proposition 5** *When price-taking traders overvalue (undervalue) new information price changes exhibit negative (positive) serial correlation.*

Any prediction based on this proposition requires an analysis of the type of the information traders were receiving. Note that the serial correlation of returns and of price changes will have the same sign.

All of the traders in this model are overconfident. What would happen if some traders were rational? In general, rational traders would mitigate but not eliminate the effects of the overconfident traders. In markets such as the one modeled here, traders vote with their dollars. As Figlewski (1978) points out "a trader with superior information but little wealth may have his information undervalued in the market price." Due to the assumption of constant absolute risk aversion, wealthy traders in the model trade no more than poor ones and so the impact on price of traders with particular viewpoints depends on their numbers.

[13] If traders are overconfident and $M = 1$, expected utilities will not be affected (although prices will change). Beliefs will be homogenous, albeit mistaken, and traders will hold the same optimal portfolios they would hold if they valued their information correctly.

The mere presence of rational traders does not drive price to its rational value. To change price, traders must be willing to trade. Willingness to trade generally depends on strength of beliefs, risk-tolerance, and wealth. Though possibly endowed with superior information, rational traders may trust their beliefs no more (and possibly less) than overconfident traders. Their wealth and risk tolerance may not exceed those of others. Introducing rational traders into the model reduces trading volume, volatility and the inefficiency of prices. The expected utility of rational traders is greater than that of overconfident traders. Introducing additional overconfident traders who are less overconfident than the existing ones has similar, though less extreme, results.

In the previous theorem, whether price changes are negatively or positively correlated depends on whether traders overvalue or undervalue new information. When rational traders trade with overconfident traders who overvalue their own signals ($\kappa \geq 1$) and undervalue those of others ($\gamma \leq 1$), the information of rational traders will be under-represented in price. Thus prices may trend:

**Proposition 6** *When rational traders trade with overconfident traders who overvalue their own private signals and undervalue those of others, price changes may be positively serially correlated.*

Positive serial correlated price changes are most likely when the precision of the rational traders' signal is high and when overconfident traders significantly undervalue the signals of others.

## 3.2 An Insider

This model of insider trading is based on Kyle (1985). Other than notational differences, the only changes made to Kyle's original model are that the insider's private signal of the terminal value is noisy and that the insider is overconfident.

This is a one-period model in which a risk-neutral, privately-informed trader (the insider) and irrational noise traders submit random market orders to a risk-neutral market-maker. There are two assets in the economy, a riskless asset and one risky asset. The riskless interest rate is assumed to be 0. The terminal value of the risky asset is $\tilde{v} \sim N(\bar{v}, {h_v}^{-1})$. $\bar{v}$ is assumed to equal 0; this simplifies notation without affecting the propositions. Prior to trading a risk-neutral insider receives a private signal $\tilde{y} = \tilde{v} + \tilde{\epsilon}$. $\tilde{\epsilon}$ is normally distributed with mean zero and precision $h_\epsilon$. The insider believes the precision of $\tilde{\epsilon}$ to be $\kappa h_\epsilon$, where $\kappa \geq 1$, and the precision of $\tilde{v}$ to be $\eta h_v$, where $\eta \leq 1$. (Note that all of the propositions in this section are true when $\eta$ is decreasing instead of $\kappa$ increasing.) After observing $\tilde{y}$, the insider demands

$x$ units of the risky asset. Without regard for price or value, noise traders demand $\tilde{z}$ units of the risky asset, where $\tilde{z} \sim N(0, {h_z}^{-1})$. The market-maker observes only the total demand $x + \tilde{z}$ and sets the price $(P)$. The market-maker correctly assumes that the precision of $\tilde{\epsilon}$ is $h_\epsilon$ and that the precision of $\tilde{v}$ is $h_v$. (The propositions do not change if the market-maker, like the insider, believes the precision of the prior to be $\eta h_v$.) After trading, the risky asset pays its terminal value $\tilde{v}$. The insider and the market-maker know the true distribution of $\tilde{z}$ and are aware of each other's beliefs about the precisions of $\tilde{v}$ and $\tilde{y}$.

The insider conjectures that the market-maker's price setting function is a linear function of $x + \tilde{z}$,

$$P = H + L(x + \tilde{z}). \tag{4}$$

He chooses $x$ to maximize his expected profit, $x(\tilde{v} - P)$, conditional on his signal, $\tilde{y}$, and given his beliefs about the distributions of $\tilde{v}$, $\tilde{y}$ and $\tilde{z}$ and the conjectured price function. The market-maker conjectures that the insider's demand function is a linear function of $\tilde{y}$,

$$x = A + B\tilde{y}. \tag{5}$$

She sets price to be the expected value of $\tilde{v}$ conditional on total demand $(x + \tilde{z})$, given her beliefs about the distributions of $\tilde{v}$, $\tilde{y}$, and $\tilde{z}$ and the conjectured demand function.

In Kyle's original model, a linear equilibrium always exists in which the conjectured price and demand functions are fulfilled. Given the assumptions of overconfidence made here, a linear equilibrium exists whenever $\kappa h_\epsilon + 2\eta h_v > \kappa h_v$. (The equilibrium and the proofs for this section are presented in appendix B.) The intuition behind the equilibrium condition is the following. The insider is trying to maximize his profit. His profit increases if he trades more with the same profit margin or if he trades the same amount with a larger margin. If the insider increases his demand, the market-maker shifts the price and thus lowers the insider's expected profit margin. Equilibrium exists at the demand-price pair where the insider believes that, if he increases his demand, the negative effect of the lower expected profit margin will more than offset the gains of greater trading and, if he lowers demand, the losses from trading less will be greater than the gains from a higher expected profit margin. What happens though if the insider and the market-maker disagree too much about the relative precisions of the prior and the private signal? When responding to increased demand, the market-maker won't move prices sufficiently to deter the insider from demanding even more, leading the insider to submit an infinite demand. But then the magnitude of aggregate demand is no longer useful information to the market-maker (though its direction is) and ultimately equilibrium breaks down.

As in Kyle's model, the insider can only influence price through his demand. This assumption is particularly critical when overconfidence is introduced to the model. If the insider could credibly reveal his private signal to the market-maker, then, due to the different weights each attaches to the prior and to the signal, the insider and the market-maker would have different posterior beliefs about the expected value of the terminal payoff. And since they are both risk-neutral, they would each be willing to trade an infinite amount. Infinite trading is a possible problem whenever risk-neutral traders value common information differently. In Harris and Raviv (1993), risk-neutral traders attribute different density functions to a public signal; Harris and Raviv avoid infinite trading by assuming a fixed number of shares are available and that short sales are not allowed. Jaffe and Winkler (1976) avoid infinite trading by assuming only one asset share can change be exchanged. The willingness to trade infinitely is inherent in risk-neutrality, not in overconfidence. Risk-neutrality is assumed here for tractability.

All of the propositions in this section are true when $\eta$ is decreasing instead of $\kappa$ increasing.

**Proposition 7** *Expected volume increases as the insider's overconfidence increases.*

Volume is measured as the expected value of the sum of the absolute values of insider demand and noise trader demand. When the insider is overconfident, he believes that he has received a stronger private signal, $\tilde{y}$, than is actually the case. In calculating his posterior expectation of the final value of the risky asset, he overweights his signal and derives a posterior expectation farther from the prior than he should. Based on this posterior belief, he trades more aggressively than is optimal, thus increasing expected volume.

**Proposition 8** *Market depth increases as the insider's overconfidence increases.*

**Proposition 9** *Volatility of prices increases as the insider's overconfidence increases.*

**Proposition 10** *The quality of prices improves as the insider's overconfidence increases.*

Overconfidence causes prices to be more sensitive to changes in value ($\tilde{v}$) and in the insider's signal ($\tilde{y}$), and less sensitive to changes in informed demand ($\tilde{x}$) and noise trader demand ($\tilde{z}$). The market-maker realizes that the insider is overconfident and that he will trade more in response to any given signal than he would if he were rational. She therefore moves price less in response to changes in total orderflow. That is, she flattens her supply curve thereby increasing market depth (which is measured as the inverse of the derivative of price with respect to order-flow). Since the overconfident insider trades more in response to any given signal, his expected trading increases relative to that of noise traders. Therefore the signal to noise ratio in total demand increases and the market-maker is able to make better inferences about the insider's signal. This enables her to form a more accurate posterior expectation of $\tilde{v}$ and to set a price which is, on average, closer to $\tilde{v}$. This improves the quality of prices, which is measured, as in the previous model, as the variance of the difference of price ($P$) and value ($\tilde{v}$). Because the market-maker can better infer the insider's signal, $\tilde{y}$, the price she sets varies more in response to changes in $\tilde{y}$ than if the insider were rational. This increases the variance of price (volatility). Thus both market depth and volatility rise with overconfidence. This is unusual since an increase of market depth is generally associated with reduced volatility.

**Proposition 11** *The expected profits of the insider decrease as his overconfidence increases.*

The insider's expected profits, $\mathrm{E_a}(x(\tilde{v} - P))$, are equivalent to his expected utility since he is risk-neutral. The insider submits a demand (to buy or to sell) which is optimal given his beliefs about the distributions of $\tilde{v}$ and $\tilde{y}$ and which he believes will maximize his expected profits. He is mistaken about the precision of his knowledge, but, conditional on his beliefs, he behaves optimally. The demand he submits is not, though, the same demand he would submit were he not overconfident, and it is not optimal given the true distributions of $\tilde{v}$ and $\tilde{y}$. Therefore the insider's expected profits are lower than they would be if he were not overconfident.

This model includes an overconfident insider, a rational market-maker, and noise traders. It is assumed that, due to competition, the market-maker expects to earn zero profits. Whatever profits the overconfident insider gives up are passed on the the noise traders in the form of lower losses. Were the rational market-maker profit maximizing, she would benefit from the insider's overconfidence.[14] This model (and the next one) require a source of uncertain demand for the risky asset so that the insider's information is not perfectly deductible from total demand. Noise traders who trade randomly and without regard to price (as in Kyle, (1985)), though they may lack perfect real world analogues, provide an analytically tractable source of uncertain demand. Overconfident, risk-averse, price-taking traders with private signals, such as the traders described in the previous section, could also provide uncertain demand in a market. In that case, if the insider were not too overconfident, he would then profit at the expense of the overconfident price-takers. Unfortunately, replacing noise traders with overconfident price-takers, greatly complicates the model.

[14]Kyle and Wang (1996) show that under particular circumstances when both a rational and an overconfident insider trade with a market-maker, the overconfident insider may earn greater profits than the rational insider. The overconfident insider earns greater profits by "pre-committing" to trading more than his share in a Cournet equilibrium. For this result to hold, traders must trade on correlated information, have sufficient resources and risk-tolerance to trade up to the Cournet equilibrium, know each other's overconfidence, and trade with a third party (e.g. the market-maker). Furthermore, if one trader can trade before the other the result may not hold.

### 3.3 Market-makers and Costly Information

This next model examines the behavior of overconfident market-makers. It also offers an explanation for why active money managers underperform passive money managers: active managers may be overconfident in their ability to beat the market and spend too much time and money trying to do so.

The model is based on Grossman and Stiglitz (1980). Risk-averse traders decide whether or not to pay for costly information about the terminal value of the risky asset; those who buy information receive a common signal; and a single round of trading takes place. The participants in this trading are the traders who buy information (informed traders), traders who do not buy the information (uninformed traders), and noise traders who buy or sell without regard to price or value.[15] As in the previous models, a riskless asset and one risky asset are traded; the riskless interest rate is assumed to be 0; each share of the risky asset pays $\tilde{v}$, where $\tilde{v} \sim N(\bar{v}, {h_v}^{-1})$. Traders believe the precision of $\tilde{v}$ to be $\eta h_v$, where $\eta \leq 1$; that is, they undervalue the common prior. There are $N$ investors ($i = 1, ..., N$). As a modeling convenience we analyze the limit economy where $N \to \infty$. Thus, each investor correctly assumes that his own demand does not affect prices. Each trader has an endowment of $f_{0i}$ of the riskless asset and $x_{0i}$ of the risky asset. $\bar{x} = (\sum_N x_{0i})/N$ is average endowment. As a notational convenience it is assumed that $\bar{x} = 0$ and $\bar{v} = 0$. Prior to trading, traders choose whether or not to pay cost $c$ in order to receive a signal $\tilde{y} = \tilde{v} + \tilde{\epsilon}$, where $\tilde{\epsilon} \sim N(0, {h_\epsilon}^{-1})$. Noise trader demand per (non-noise) trader is $\tilde{z}$, where $\tilde{z} \sim N(0, {h_z}^{-1})$. Thus $-\tilde{z}$ is the supply of the risky asset per trader at the time of trading. In equilibrium, $\lambda^*$ is the fraction of traders who choose to become informed.

All traders, even those who remain uninformed, are overconfident about the signal which they believe to have precision $\kappa h_\epsilon$, where $\kappa \geq 1$. In the previous models, traders were overconfident about their own signals but not those of others. Here everyone believes the information is better than it is, but some decide the price is still too high. It is as if all money managers overestimate their ability to manage money actively, but some decide the the costs of doing so are too high and so, despite their overconfidence, choose to manage passively.[16] In real markets one would expect traders to hold a spectrum of beliefs about

[15] In this section "traders" refers to informed traders and to uninformed traders but not to noise traders who are referred to explicitly as "noise traders." As in the insider model, noise traders could be replaced with overconfident price-takers, such as those discussed in section 3.1. Overconfidence would motivate trading and the model's results would not change significantly. However replacing noise traders with overconfident price-takers greatly complicates the equilibrium without adding much intuition.

[16] In practice some practitioners of passive investing often tout their own skills as superior active managers. For example, Barclays Global Investment Advisors, the largest manager of index funds, has a Global Ad-

the value of costly information. Those who were more overconfident about the information would be more likely to buy it. One could alternatively specify in this model that those traders who did not buy the signal valued it rationally.[17]

Trader $i$'s demand for the risky asset is $x_{1i}$ and for the riskfree asset is $f_{1i}$. So his final wealth is $W_{1i} = x_{1i}\tilde{v} + f_{1i}$. Trader $i$'s utility function is $U(W_{1i}) = -\exp(-aW_{1i})$, where $a$ is the common coefficient of absolute risk-aversion. He maximizes his expected utility by choosing whether or not to become informed, and then, conditional on his information, by choosing his optimal demand subject to the budget constraint. That is, if he is informed, he solves

$$\max_{x_{1I}} \mathrm{E_b}\left[-\exp(-aW_{1I}) \mid y\right] \text{ subject to } x_{1I}P + f_{1I} \leq x_{0I}P + f_{0I} \tag{6}$$

and if he is uninformed he solves

$$\max_{x_{1U}} \mathrm{E_b}\left[-\exp(-aW_{1U}) \mid P\right] \text{ subject to } x_{1U}P + f_{1U} \leq x_{0U}P + f_{0U}, \tag{7}$$

where $i = I$ and $i = U$ indicate prototypical informed and uninformed traders and P is the endogenously determined price of the risky asset. In equilibrium all traders believe that the expected utility of the informed traders is equal to that of the uninformed. Since all traders believe the precision of $y$ is $\kappa h_\epsilon$ and the precision of $v$ is $\eta h_v$, and since the equilibrium is determined by the traders' beliefs, the equilibrium obtained is the same as would occur in a model without overconfidence where the precision of $\epsilon$ was actually $\kappa h_\epsilon$ and that of $v$ was $\eta h_v$. Once again equilibrium holds because the traders believe that they are behaving optimally though, in fact, they are not. The equilibrium and the proofs for this section are presented in Appendix C.

In the previous two models, expected utility drops as overconfidence increases. In this model, where traders are overconfident about a costly signal, it is those who buy the signal who are most hurt by their overconfidence.

**Proposition 12** *When traders overvalue costly information, the expected utility of informed traders may be lower than that of uninformed traders.*

When traders overestimate the value of the costly signal, too many of them are willing to buy it. Its benefits are therefore spread too thin, resulting in lower expected utilities for the

vanced Active Group which actively manages over \$70 billion. And George Sauter who oversees \$61 billion in stock-index mutual funds at Vanguard Group also actively manages Vanguard Horizon Fund Aggressive Growth Portfolio [The Wall Street Journal, February 25, 1997, page C1, byline by Robert McGough].

[17] Assuming that traders who did not purchase the signal valued it correctly would result in a range of possible equilibria rather than a single equilibrium point. Under this specification, it is easy to show that the expected utility of the uninformed traders is as great as, or greater than, that of informed traders.

informed traders. Explicit solutions for the expected utilities of the informed and uninformed traders are given in Appendix C. The proposition states only that the expected utility of the informed may be lower than that of the uninformed. However, I have evaluated these for a wide variety of parameter values and have found that in every case the expected utility of the informed is less than that of the uninformed if $\kappa > 1$ or $\eta < 1$ (and $0 < \lambda < 1$).[18]

Because all traders in this model are equally overconfident in the same signal, overconfidence does not create the dispersion of beliefs which it did in the previous two models. It was this dispersion of beliefs which caused expected volume to rise with overconfidence in these models. Here, expected volume can rise or fall with overconfidence. This is easily understood by looking at boundary cases. When cost is so high that all traders remain uninformed (i.e. $\lambda^* = 0$), the traders do not trade with each other and all trading is done between the uninformed traders and the noise traders. Thus, expected volume equals the expected demand of the noise traders, (i.e. $\sqrt{2/\pi h_z}$). When overconfidence increases sufficiently, some traders, but not all, will become informed. Informed and uninformed traders will now trade with each other and they will also continue to fill the demand of the noise traders, so expected volume will rise. As overconfidence continues to rise, all traders may eventually become informed (depending on the other parameter values), in which case expected volume will fall back to the expected demand of noise traders.[19]

When all traders remain uninformed, or when they all become informed, they function solely as market-makers, buying and selling to meet the order-flow of the noise traders. Since the noise traders have no information, these "market-makers" need not fear asymmetric information but, since they are risk-averse, they do worry about inventory risk. They set price in response to order-flow using an upward sloping supply schedule. In the case where they are uninformed, that schedule is: $P = a\,\mathrm{var_b}(\tilde{v})\,Q$, where $Q$ is supply. As $\eta$ decreases, $\mathrm{var_b}(\tilde{v})$ increases and the slope of the supply curve becomes steeper. Thus, market depth decreases and the volatility of prices increases as $\eta$ decreases. Figure 1 graphs supply curves in two economies. The dashed line represents an economy where $\eta = 1$. The solid line represents an economy where $\eta = .5$. All other parameter values are the same in both economies and all traders are uninformed. One can see that, for any non-zero level of noise

[18] If the uninformed traders are rational rather than overconfident, they optimize correctly. Thus it is trivial to show that their expected utility is at least as high as that of informed traders. If it were not, they would become informed.

[19] When some traders buy information and others don't, this model also does not offer much intuition about how overconfidence affects volatility ($\mathrm{var_a}(P) = C_1(S_b + D)$, where $C_1$, $S_b$, and $D$ are given in Appendix C). Depending on the other market parameters, volatility can be rising or falling in overconfidence. For some parameters it even rises, falls, and then rises again as overconfidence increases. Similar patterns are observed when other market parameters, such as $h_\epsilon$ vary.

trader demand, the absolute value of $P$ will be greater when $\eta = .5$. It follows that volatility is greater when $\eta = .5$. Also, for any change in demand, the magnitude of change in price will be greater when $\eta = .5$; hence the market depth will be less.

When all traders are informed, they act as market-makers who have some information about the terminal value of the risky asset. The supply schedule they set is: $P = \mathrm{E_b}(\tilde{v} \mid y) + a\,\mathrm{var_b}(\tilde{v} \mid y)\,Q$. There are two separate components to this price: $a\,\mathrm{var_b}(\tilde{v} \mid y)\,Q$ is a response to noise trader demand (since $Q = \tilde{z}$) and hedges traders in their capacity as market-makers against inventory risk. $\mathrm{E_b}(\tilde{v} \mid y)$ is a response to the signal $\tilde{y}$ and represents traders' speculations about terminal value. If there were no signal, price would be completely determined by inventory risk; if there were a signal, but no noise trader demand, price would completely determined by the signal. As in the case where all traders are uninformed, a decrease in $\eta$ steepens the supply curve thereby decreasing market depth and increasing the inventory-risk component of volatility. An increase in $\kappa$ flattens the supply curve, thereby increasing market depth and decreasing the inventory-risk component of volatility. This is the only example presented in this paper where underweighting priors may have the opposite effect of overconfidence. Decreasing $\eta$ and increasing $\kappa$ both move $\mathrm{E_b}(\tilde{v} \mid y)$ closer to $y$ and further from $\bar{v} = 0$ thereby increasing the speculative component of volatility. So decreasing $\eta$ increases both the inventory risk component and the speculative component of volatility. But increasing $\kappa$ increases the speculative component of volatility while decreasing the inventory risk component. When expected noise trader demand is low, the speculative component of volatility dominates and increasing $\kappa$ increases volatility. When expected noise trader demand is high, inventory-risk dominates and increasing $\kappa$ decreases volatility.

Figure 2 graphs supply curves in two economies. The dashed line represents an economy where $\kappa = 1$ and the solid line an economy where $\kappa = 2$. All other parameter values are the same in both economies and all traders are informed. The supply curves are conditional on traders receiving a signal of $\tilde{y} = 2$ (one s.d. above the mean signal). The solid line is flatter, which means that market depth is greater when $\kappa = 2$. The two supply curves cross at about $Q = 2$. If demand is less than 2 and greater than about $-1.5$, price will be closer to its unconditional expected value, 0, when $\kappa = 1$ than when $\kappa = 2$. But when the magnitude of noise trader demand is high, (i.e. $\tilde{z} > 2$ or $\tilde{z} < -1.5$) price will be closer to its expected value when $\kappa = 2$ than when $\kappa = 1$. When expected noise trader demand is low, demand will more often fall into the area where the magnitude (and volatility) of price is smaller for $\kappa = 1$. When expected noise trader demand is high, the economy with $\kappa = 2$ will have lower volatility. The following propositions summarize the above discussion.

**Proposition 13** *Market depth is increasing in the overconfidence of a risk-averse market-*

*maker. Volatility increases when expected noise trader demand is high and decreases when it is low. (Precise definitions of high and low expected noise trader demand are given in Appendix C.)*

**Proposition 14** *Market depth decreases and volatility increases when a market-maker underweights his prior beliefs.*

# 4 Discussion

This paper examines the effects of overconfidence in market situations where: 1) information is widely dispersed and traders are price-takers, 2) information is concentrated in the hands of a strategic-trading insider who has sufficient wealth and risk-tolerance that his trades move markets, 3) orderflow contains no information but market-makers are concerned about inventory risk, and 4) information is costly. These situations all arise in equity markets: 1) Individual investors trading on their beliefs place small orders which do not appreciably move the market.[20] 2) Informed insiders trade in anticipation of public news. 3) To control inventory risk, market-makers move prices in response to large informationless trades such as those of index fund managers. 4) Investors choose between buying active management or investing passively. As we have seen, some measures of market activity react the same to the overconfidence of different market participants (e.g. price-takers, insiders); other measures react differently to the overconfidence of different market participants:

*Volume:* Overconfidence increases trading volume. In the model of price-takers, overconfident traders form differing posterior beliefs and trade speculatively with each other. Were these traders rational they would hold identical posteriors and trade only to initially balance their portfolios. Overconfident insiders also trade more aggressively than if they were rational. And, as seen in the model of market-makers, overconfident market-makers set a flatter supply curve. A flatter supply curve encourages more trading when traders are price sensitive. Thus, all three models predict that overconfidence will lead to greater trading volume. While there is anecdotal evidence of excessive trading–e.g., roughly one quarter of the annual international trade and investment flow is traded each day in foreign exchange markets (Dow and Gorton, 1994); the average annual turnover rate on the New York Stock Exchange is currently greater than 50% (NYSE Fact Book for the year 1994)–without an adequate model of what trading volume in rational markets should be, it is hard to prove that aggregate market volume is excessive. Odean (1997) looks at the buying and selling activities of individual

[20]Small trades are considered sufficiently innocuous that Nasdaq market-makers will pay for small trade orderflow.

investors at a discount brokerage. Such investors could quite reasonably believe that their trades have little price impact. On average, the stocks these investors buy subsequently underperform those they sell (gross of transactions costs), even when liquidity demands, risk management, and tax consequences are considered. As predicted by the model of price-taker overconfidence, these investors trade too much. However, overconfidence about the precision of private signals is not enough to explain how these investors identify the wrong stocks to buy and to sell. In addition to overvaluing their information, these investors must also misinterpret it. Statman and Thorley (1997) find that trading volume increases subsequent to market gains. If success in the market leads to become overconfidence–as in Gervais and Odean (1997)–these increases in volume may be driven by overconfidence.

*Efficiency:* Overconfidence can improve or worsen market efficiency. Overconfidence causes the aggregate signal of price-takers to be overweighted. This leads to prices further from the asset's true value, $\tilde{v}$, than would otherwise be the case. Though all available information is revealed in this market, it is not optimally incorporated into price. On the other hand, overconfidence prompts an insider to reveal more of his private information than he otherwise would, enabling the market-maker to set prices which more accurately reflect the asset's true value. However, if the insider's information is time sensitive and becomes public soon after his trades are completed, this gain in efficiency is short-lived.

*Volatility:* Overconfidence can increase or decrease volatility. By overweighting the aggregate signal of the price-takers, overconfidence drives price further from its unconditional mean, $\bar{v}$. This results in increased volatility. By prompting the insider to reveal more of his signal, overconfidence enables the market-maker to move price closer to the true underlying value, $\tilde{v}$, and further from its unconditional value, $\bar{v}$. This, too, increases volatility. However, risk-averse market-makers flatten their supply curves when they are overconfident, just as they would if they were less risk-averse. Flattening the supply curve dampens volatility. Some research suggests that market volatility is excessive [Shiller (1981); LeRoy and Porter (1981); Shiller (1989)], but this is a difficult proposition to prove [Marsh and Merton (1986); Kleidon (1986)]. Pontiff (1997) finds excess volatility for closed-end funds.

*Depth:* Overconfidence increases market depth. When an insider is overconfident, and therefore reveals more of his signal through aggressive trading, the market-maker increases market depth. Overconfidence leads risk-averse market-makers to flatten their supply curves which increases market depth.

*Expected Utility:* Overconfidence lowers expected utilities. Overconfident traders do not properly optimize their expected utilities, which are therefore lower than if the traders were rational. When information is costly, those who choose to become informed fare worse, after paying for information, than those who remain uninformed. In practice the cost of active managers' information must be reflected in their fees. Thus, this finding is consistent with many studies of the relative performance of active (informed) and passive (uninformed) money managers.[21]

*Underreaction and Overreaction:* Overconfident traders who discount the opinions of others can cause markets to underreact to the information of rational traders. Markets also underreact when traders underweight their own new information and overreact when they overweight it. The degree of under- or overreaction depends on what fraction of all traders receive the information and on how willing these traders are to trade. Underreactions occur when all traders undervalue a signal or when only a small fraction of traders overvalue it, but others discount their opinion. Overreactions require that a significant fraction of active traders (those who are most willing to trade) overvalue a signal.

Some documented market return anomalies indicate overreactions to public events while most find underreactions.[22] Fama (1997) points out that if markets occasionally overreact and at other times underreact this could be due to simple chance. Like markets, people, too, sometimes overvalue information and at other times undervalue it. While these valuation errors may appear due to chance, psychologists find that they are systematic. People typically overreact to salient, attention-grabbing information, overvalue cases, anecdotes, and extreme realizations, and overweight irrelevant data. They underreact to abstract statistical information, underestimate the importance of sample size, and underweight relevant data. Markets appear to reflect the same systematic biases as their participants.

Reactions to announcements are considered underreactions when returns in periods following the announcement are of the same sign as returns on the day of the announcement. One of the most robust underreaction anomalies is post-earnings-announcement drift [Ball and Brown

[21] In an early study Jensen (1968) finds underperformance by mutual funds. Lakonishok, Shleifer, and Vishny (1992) document that as a group active equity managers consistently underperformed S&P 500 index funds over the period 1983-1989. They conclude that, after factoring in management fees, active management subtracts value. Using a variety of benchmarks and benchmarkless tests, Grinblatt and Titman (1993 & 1994) find that, at least before fees, some fund managers earn abnormal returns. Malkiel (1995) claims that such results are heavily influenced by survivorship bias. Carhart (1997) also finds little evidence of skilled mutual fund management. Lakonishok, Shleifer, and Vishny ask why pension funds continue to give their money to active managers when index funds outperform active management. They suggest a number of reasons based on agency relationships. They also point out that the pension fund employees may be overconfident in their ability to pick superior money managers.

[22] I wrote the following discussion of market underreactions nearly two and a half years after the original draft of this paper (November 1994) and subsequent to reading more recent working papers on this topic [Barberis, Shleifer, and Vishny (1997), Daniel, Hirshleifer, and Subrahmanyam (1997), and Fama (1997)].

(1968), Bernard and Thomas (1989, 1990)]. Corporate earnings summarize the operations of a company into a single statistic. This statistic is based on a large sample of information and is highly relevant to the value of the company. It is prototypical of the information people typically undervalue: abstract, relevant, and based on a large sample. Markets also underreact to dividends omissions and initiations [Michaely, Thaler, and Womack (1995)]. The decision to omit dividends is generally made reluctantly and in response to significant corporate difficulties. While the omission (or initiation) of dividends is appreciated by investors, it may not be fully appreciated because the bad (or good) news contained in the omission (or initiation) has been condensed into a single event. We might expect a greater reaction when an omission (or initiation) is accompanied by a well-publicized, graphic, portrayal of a company's woes (or good fortune). Like dividend initiations, self-tender offers [Lakonishok and Vermaelen (1990)] and open-market repurchases [Ikenberry, Lakonishok, and Vermaelen (1995)] are positive signals which abstract a wealth of more salient information. In addition to possibly signaling management's sanguine outlook, the announcement of open-market repurchases states that the supply of shares in a company will be reduced. Investors who do not realize that firms face upward-sloping supply curves when they repurchase shares [Bagwell (1992)] and that price is therefore likely to rise, may underreact to the announcement. Finally, the market underreacts to brokerage recommendations [Womack (1996)]. These also condense a great deal of research. While salient details of this research may be told to the brokerage's customers, most investors will, at most, hear only that a recommendation was made.

Most of the documented long-run return patterns following information events are underreactions. One example of market overreaction is to IPO's [Ritter (1991); Loughran and Ritter (1995)]. IPO's differ from the previous events in three important ways. First, there is no direct market reaction to the announcement of the IPO, since the announcement precedes public trading of the firm's stock. IPO's are an overreaction in that newly offered firms tend to subsequently underperform the market; therefore investors who buy IPO's may be overreacting to favorable information about the firm. Second, IPO's are promoted. Company officials and their underwriters are motivated to deliver their message as persuasively as possible (given restrictions imposed by the SEC). Company officers give talks to prospective institutional buyers. They, and the underwriter's brokers, impress buyers with optimistic scenarios about the company's future. People overweight such scenarios, which are more attention-grabbing than other relevant data, such as accounting reports, and more mentally available (having been recounted) than alternative scenarios in which the company fares less well. Third, only buyers react in IPO's. So an IPO's opening day price is determined by those who are most likely to be overconfident about the firm. For most public announcements, if an investor doesn't like the news he can sell the stock if he already owns it, or he can sell it short if he doesn't. In this way his negative opinion influences price. However, investors don't own IPO's prior to an IPO and may find them difficult to borrow for short sale on the day of the offering.[23]

Markets often underreact to announcements of abstract, highly statistical, or highly relevant information. Earnings changes, dividend omissions, brokerage recommendations, and equity issuance are all examples of such information. However, behind each of these events lie many concrete, salient stories: new products succeed, others fail, ad campaigns are waged, employees are fired, scandals emerge. While the sum of these stories is underweighted, the individual parts may, in fact, be overweighted.[24] If markets do systematically overreact, they may do so to highly publicized, graphic news and to rumors.[25]

While brokerage recommendations are delivered in their salient form only to customers, some recommendations are both widely disseminated and attention grabbing. The Wall Street Journal's monthly "Dartboard" column pits the recommendations of four analysts against the random selections of a dart. Many readers follow this contest. The analysts, who's portraits are featured, explain the reasons for their picks. Barber and Loeffler (1993) show that the market overreacts to these recommendations. Similarly, the market overreacts to recommendations made on the popular TV show "Wall Street Week" [Pari (1997)].

Another signal to which we might expect overreactions is price-change. Price-change may be the most salient signal received by investors since it directly contributes to changes in their wealth. It is also a highly publicized signal, instantly available on many computer screens, reported daily in newspapers and other media, and mailed monthly to investors in brokerage statements. Furthermore, many investors may overweight the predictive value of price-changes; they may see deterministic patterns where none exist[26] or follow specific technical trading rules.

[23] Of course some (non-shorting) investors do sell on the day of the offering, but since these either subscribed to the IPO or bought it the same day they sold it, they did not react negatively to the IPO announcement.

[24] As Joseph Stalin put it, "The death of a single Russian soldier is a tragedy. A million deaths is a statistic."

[25] There is some evidence of short-term mean-reversion in returns [Lehmann, (1988); Jegadeesh (1987)]. While such reversions could be due to overreaction to salient news stories, this has not been shown. Mean reversion has also been found at longer horizons [De Bondt and Thaler (1985, 1987) and returns tend to be positively serially corollated at intermediate horizons [Jegadeesh and Titman (1993)]. While these phenomena may be due to over- or underreaction, they have not been conclusively tied to specific information events.

[26] Gilovich, Vallone, and Tversky (1985) show that people hold strong beliefs that random sequences are non-random.

As we saw in the model of price-taking traders, the impact of a private signal depends on how many people receive that signal (and, as Figlewski (1978) points out, on the wealth and risk-tolerance of those traders). The impact of traders, even rational traders, depends on their numbers and on their willingness to trade. The mere presence of rational traders in the market does not guarantee that prices are efficient. Rational traders may be no more willing or able to act on their beliefs than biased traders. Therefore markets with higher proportions of rational traders should be more efficient. If information processing biases are more pronounced in individuals than in institutional traders, then it should come as no surprise that return anomalies are greatest for small firms [see Fama (1997) for a review] which are traded more heavily by individuals.

# 5 Conclusion

Overconfidence can be costly to society. In the models presented here, expected utilities fall with overconfidence because risk-sharing is no longer optimal, too much information is purchased, or noise traders are not optimally exploited. The non-optimal exploitation of noise traders is only a shift in utility, since noise traders benefit from being exploited less. However, non-optimal risk sharing and the expenditure of too many resources on information acquisition represent social losses, as do dead-weight losses from excessive trading.

The effects of overconfidence on a market depend on who is overconfident. Overconfidence increases trading volume and market depth, and decreases the expected utility of overconfident traders. When information is costly, overconfident informed traders fare less well than uninformed traders. Overconfident traders increase volatility, while overconfident market-makers dampen it. Overconfident price-taking traders reduce market efficiency, while overconfident insiders increase it. Overconfident traders can cause markets to underreact to the information of rational traders. Markets underreact to abstract, statistical, and highly relevant information and overreact to salient, but less relevant, information. Thus, like people, markets are predictable in their biases.

# Appendix A (Price-takers)

**Lemma 1** *An equilibrium exists in which the linear price conjectures, equations (2) and (3), lead to linear demand functions. The coefficients of the price conjectures are*

$$\alpha_{31} = \frac{\eta h_v \bar{v} - a\bar{x}}{\eta h_v + 2(\kappa + \gamma M - \gamma)\, h_\epsilon})\tag{A. 1}$$

$$\alpha_{32} = \alpha_{33} = \frac{(\kappa + \gamma M - \gamma) h_\epsilon}{\eta h_v + 2(\kappa + \gamma M - \gamma) h_\epsilon})\tag{A. 2}$$

$$\begin{aligned}\alpha_{21} =\, & \alpha_{31} + (\alpha_{32} + \alpha_{33})\bar{v} - a\bar{x}\,\mathrm{var_b}(P_3 \mid \Phi_2) + \\ & \frac{\bar{v}(\alpha_{33}(\gamma + \gamma M - \kappa) h_\epsilon + \alpha_{32}(\eta h_v + (\kappa + \gamma M - \gamma) h_\epsilon))}{\eta h_v + (\kappa + \gamma M - \gamma) h_\epsilon}\end{aligned}\tag{A. 3}$$

$$\alpha_{22} = \frac{(\kappa + \gamma M - \gamma) h_\epsilon}{\eta h_v + (\kappa + \gamma M - \gamma) h_\epsilon}\tag{A. 4}$$

*and*

$$\mathrm{P}_1 = \alpha_{21} + \alpha_{22}\bar{v} - a\bar{x}\, \alpha_{22}^2 \left( \frac{1}{\eta h_v} + \frac{\gamma + \kappa M - k}{\kappa \gamma h_\epsilon M^2} \right).\tag{A. 5}$$

*Proof:* We will solve the equilibrium first for the third round of trading. Trader $i$ believes $\Phi_{3i}$ has a multi-variate normal distribution. We can calculate the mean and the covariance matrix of this distribution which are $E_{\mathrm{b}}(\Phi_{3i}) = [\, \bar{v} \quad \bar{v} \quad \alpha_{21} + \alpha_{22}\bar{v} \quad \alpha_{31} + (\alpha_{32} + \alpha_{33})\bar{v}\, ]^T$ and

$$\Psi = \begin{bmatrix} \frac{1}{\eta h_v} + \frac{1}{\kappa h_\epsilon} & \frac{1}{\eta h_v} & \frac{\alpha_{22}}{\eta h_v} + \frac{\alpha_{22}}{\kappa h_\epsilon M} & \frac{\alpha_{32}+\alpha_{33}}{\eta h_v} + \frac{\alpha_{32}}{\kappa h_\epsilon M} \\ \frac{1}{\eta h_v} & \frac{1}{\eta h_v} + \frac{1}{\kappa h_\epsilon} & \frac{\alpha_{22}}{\eta h_v} & \frac{\alpha_{32}+\alpha_{33}}{\eta h_v} + \frac{\alpha_{33}}{\kappa h_\epsilon M} \\ \frac{\alpha_{22}}{\eta h_v} + \frac{\alpha_{22}}{\kappa h_\epsilon M} & \frac{\alpha_{22}}{\eta h_v} & \frac{\alpha_{22}^2}{\eta h_v} + \frac{\alpha_{22}^2(\gamma+\kappa M-\kappa)}{\kappa\gamma h_\epsilon M^2} & C_1 \\ \frac{\alpha_{32}+\alpha_{33}}{\eta h_v} + \frac{\alpha_{32}}{\kappa h_\epsilon M} & \frac{\alpha_{32}+\alpha_{33}}{\eta h_v} + \frac{\alpha_{33}}{\kappa h_\epsilon M} & C_1 & C_2 \end{bmatrix},\tag{A. 6}$$

where

$$C_1 = \frac{\alpha_{22}\alpha_{33}}{\eta h_v} + \alpha_{22}\alpha_{32}\left(\frac{1}{\eta h_v} + \frac{\gamma + \gamma M - \kappa}{\kappa\gamma h_\epsilon M^2}\right),\;\; C_2 = \frac{(\alpha_{31+}\alpha_{33})^2}{\eta h_v} + \frac{(\alpha_{32}^2 + \alpha_{33}^2)\,(\gamma + \gamma M - \kappa)}{\kappa\gamma h_\epsilon M^2}.\tag{A. 7}$$

Let $\mathrm{A}^T \equiv \mathrm{cov_b}\,(\tilde{\mathrm{v}}, \Phi_{3i}) = [\, (\eta h_v)^{-1} \quad (\eta h_v)^{-1} \quad \alpha_{22}(\eta h_v)^{-1} \quad (\alpha_{32} + \alpha_{33})(\eta h_v)^{-1}\, ]^T$.

Then, by the projection theorem,

$$\begin{aligned}\mathrm{E_b}\,(\tilde{v} \mid \Phi_{3i}) =\; & \bar{v} + A\,\Psi^{-1}(\Phi_{3i} - \mathrm{E_b}(\Phi_{3i})) \\ =\; & \frac{(y_{2i} + y_{3i})(\kappa - \gamma) h_\epsilon + (\bar{Y}_2 + \bar{Y}_3)\,(\gamma h_\epsilon M) + \eta h_v \bar{v}}{\eta h_v + 2(\kappa + \gamma M - \gamma) h_\epsilon}\end{aligned}\tag{A. 8}$$

and

$$
\begin{aligned}
\mathrm{var}_{\mathrm{b}}\left(\tilde{v} \mid \Phi_3\right) &= (\eta h_v)^{-1} - A\,\Psi^{-1}A^T \\
&= \frac{1}{\eta h_v + 2(\kappa+\gamma M-\gamma)h_\epsilon}.
\end{aligned} \tag{A. 9}
$$

The conditional variance of $\tilde{v}$ is the same for all traders and so the subscript $i$ is dropped in $\mathrm{var}_{\mathrm{b}}\left(\tilde{v} \mid \Phi_3\right)$. Following Grossman (1976), we can solve equation (1) and get demand function

$$
x_{3i} = \frac{\mathrm{E}_{\mathrm{b}}\left(\tilde{v} \mid \Phi_{3i}\right) - P_3}{a\mathrm{var}_{\mathrm{b}}\left(\tilde{v} \mid \Phi_3\right)}. \tag{A. 10}
$$

We calculate the average demand per trader and equate this to the per trader supply, $\bar{x}$. Then solving for $P_3$, we can match coefficients to those of the conjectured second period price function to obtain $\alpha_{31}$, $\alpha_{32}$, and $\alpha_{33}$ as given in equations (A. 1) and (A. 2). Thus the linear price conjectures are fulfilled and equilibrium exists at $t = 3$.

To solve the equilibrium at $t = 2$, we again use the projection theorem, calculating

$$
\begin{aligned}
\mathrm{E}_{\mathrm{b}}(P_3 \mid \Phi_{2i}) = {} & \frac{2(\kappa+\gamma M-\gamma)h_\epsilon\bar{v} + \eta h_v\bar{v} - a\bar{x}}{\eta h_v + 2(\kappa+\gamma M-\gamma)h_\epsilon} + \\
& \frac{(\kappa+\gamma M-\gamma)\left(h_\epsilon(\eta h_v + (\kappa+2\gamma M-\gamma)h_\epsilon)\left(\bar{Y}_2-\bar{v}\right) + h_\epsilon^2(\kappa-\gamma)(y_{2i}-\bar{v})\right)}{(\eta h_v + (\kappa+\gamma M-\gamma)h_\epsilon)(\eta h_v + 2(\kappa+\gamma M-\gamma)h_\epsilon)}
\end{aligned} \tag{A. 11}
$$

and

$$
\mathrm{var}_{\mathrm{b}}(P_3 \mid \Phi_2) = \frac{(\kappa+\gamma M-\gamma)^2 h_\epsilon\left(\eta(\gamma+\kappa M-\kappa)h_v + \left((\kappa-\gamma)^2(M-1) + 2\gamma\kappa M^2\right)h_\epsilon\right)}{\gamma\kappa M^2(\eta h_v + (\kappa+\gamma M-\gamma)h_\epsilon)(\eta h_v + 2(\kappa+\gamma M-\gamma)h_\epsilon)^2}. \tag{A. 12}
$$

Since traders are myopic, trader $i$'s second round demand is

$$
x_{2i} = \frac{\mathrm{E}_{\mathrm{b}}(P_3 \mid \Phi_{2i}) - P_2}{a\mathrm{var}_{\mathrm{b}}(P_3 \mid \Phi_2)}. \tag{A. 13}
$$

Equating per trader demand and per trader supply, solving for $P_2$, and matching coefficients gives us the equilibrium values for $\alpha_{21}$ and $\alpha_{22}$ given in equations (A. 3) and (A. 4). Using the unconditional expectation and variance of $P_2$, we can follow the same steps as above to calculate $P_1$ as given in equation (A. 5). ∎

To simplify the exposition, Propositions 1-5 are proven for the case of $\eta = 1$ and $\gamma = 1$. Proposition 1 states that expected volume increases as overconfidence increases. This is re-stated formally and proven in terms of trading round 2 volume. It is also true for round 3 expected volume.

**Proposition 1** *If $\kappa > 1$, and $M \geq 2$ then*

$$
\mathrm{E}_{\mathrm{a}}\left(\sum_{\mathrm{i}=1}^{N} \frac{|x_{2i} - x_{1i}|}{N}\right) \tag{A. 14}
$$

*is an increasing function of $\kappa$.*

*Proof:* The first step of the proof is to calculate (A. 14), the per capita expected trading volume in trading round 2. Traders have negative exponential utility functions which means that their demand for the risky asset does not depend on their wealth. They have the same prior beliefs about the distribution of $\tilde{v}$. Because they have the same beliefs as well as the same risk aversion, all traders have the same first period demand: $x_{1i} = \bar{x}$. Coefficients from equations (A. 3) and (A. 4) are substituted into equation (3); equations (3), (A. 11), and (A. 12) are then substituted into equation (A. 13) which is substituted into (A. 14). The expectation operator is moved inside the summation and the denominator $N$ is moved outside the expectation. We have then the average expectation of $N$ identical half-normal distributions. Taking expectations and simplifying gives us

$$\mathrm{E_a}\left(\sum_{i=1}^{N}\frac{|x_{2i}-x_{1i}|}{N}\right)=\sqrt{\frac{2(M-1)h_\epsilon}{M\pi}}\cdot \frac{(\kappa-1)\kappa M^2(h_v+2(\kappa+M-1)h_\epsilon)}{a(\kappa+M-1)\left((1-\kappa+\kappa M)h_v+\left(\left(\kappa^2-2\kappa+1\right)(M-1)+2\kappa M^2\right)h_\epsilon\right)} \tag{A. 15}$$

Bearing in mind that $M \geq 2$, and $\kappa > 1$, one can show that, in the given parameter range, the derivative of (A. 15) with respect to $\kappa$ is positive and so (A. 15) is increasing in $\kappa$. ∎

Proposition 2 states that volatility increases when overconfidence increases and when traders undervalue their prior beliefs. Three alternative measures of volatility are $\mathrm{var_a}(P_2)$, $\mathrm{var_a}(P_3)$, and $\mathrm{var_a}(P_3 - P_2)$. The proposition is true for all three measures. It is proven for $\mathrm{var_a}(P_3)$.

**Proposition 2** *If* $\kappa \geq 1$ *and* $\mathrm{var_a}(P_3)$ *is an increasing function of* $\kappa$.

*Proof:* Substituting coefficients from equations (A. 1) and (A. 2) into equation (2), we can calculate

$$\mathrm{var_a}(P_3)=\frac{2h_\epsilon(\kappa+M-1)^2(h_v+2h_\epsilon M)}{h_vM(h_v+2h_\epsilon(\kappa+M-1))^2}. \tag{A. 16}$$

In the given parameter range, the derivative of (A. 16) with respect to $\kappa$ is positive. ∎

The quality of prices can be measured as $\mathrm{var_a}\left(\mathrm{P_t}-\tilde{v}\right)$ for $t = 2$ or $3$. As this variance increases, the quality of prices worsens. Proposition 4 and its proof are given here in terms of $t = 3$. The proposition is also true for $t = 2$; the proof is analogous.

**Proposition 3** *If* $\kappa \geq 1$, $\mathrm{var_a}\left(\mathrm{P_3}-\tilde{v}\right)$ *is an increasing function of* $\kappa$.

*Proof:* Substituting coefficients from equations (A. 1) and (A. 2) into equations (2), we can calculate

$$\mathrm{var_a}\left(\mathrm{P_3}-\tilde{v}\right)=\frac{h_vM+2h_\epsilon\left((\kappa-1)^2-2M+2\kappa M+M^2\right)}{M(h_v+2(\kappa-1+M)h_\epsilon)^2}. \tag{A. 17}$$

In the given parameter range, the derivative of (A. 17) with respect to $\kappa$ is positive. ∎

**Proposition 4** *If* $M \geq 2$ *traders' expected utilities will be lower when* $\kappa > 1$ *than when* $\kappa = 1$.

*Proof:* In this model, traders can infer the aggregate signal. So if they have perfectly calibrated probability beliefs (i.e. $\kappa = \eta = \gamma = 1$) their posterior beliefs will be identical. Because they have the same beliefs as well as CARA utility functions with the same risk aversion, perfectly calibrated traders will hold the same amount of the risky asset in equilibrium (i.e. $\bar{x}$); this maximizes their utility. This is the position to which traders, whether overconfident or perfectly calibrated, trade in the first round of trading where there is no signal (i.e $x_{1i} = \bar{x}$). Following steps similar to those used to obtain (A. 15), we can calculate the expected net trading subsequent to the first round of trading. This is

$$\mathrm{E_a}\left(\sum_{\mathrm{i=1}}^{N} \frac{|x_{3i} - x_{1i}|}{N}\right) = \frac{2(\kappa-1)}{a}\sqrt{\frac{(M-1)h_\epsilon}{M\pi}}. \tag{A. 18}$$

We see that when traders are perfectly calibrated, they do not trade in later rounds and continue to hold their optimal portfolio of the risky asset, $\bar{x}$. However, if traders are overconfident (i.e. $\kappa > 1$) and if $M \geq 2$, they are expected to trade in later rounds, thus departing from their optimal portfolio and reducing their expected utility. ∎

**Proposition 5** $\mathrm{Cov_a}((P_3{-}P_2),(P_2 - P_1))$ *is a decreasing function of* $\kappa$ *and* $\mathrm{cov_a}((P_3{-}P_2),(P_2 - P_1)) = 0$ *when* $\kappa = \eta = \gamma = 1$.

*Proof:* Noting that $P_1$ is a constant and substituting coefficients from equations (A. 1), (A. 2), (A. 3), and (A. 4) into equations (2) and (3), we can calculate

$$\mathrm{cov_a}((P_3 - P_2),(P_2 - P_1)) = -\frac{(\kappa - \gamma + \gamma M)^2(\kappa + (M-1)\gamma - (M\eta))h_\epsilon^2}{M(\eta h_v + (\kappa - \gamma + \gamma M)h_\epsilon)^2(\eta h_v + 2(\kappa - \gamma + \gamma M)h_\epsilon)}, \tag{A. 19}$$

which has the opposite sign of $\kappa - \gamma + M(\gamma - \eta)$. So (A. 19) is 0 if $\kappa = \gamma = \eta = 1$, negative if $\kappa - \gamma > M(\eta - \gamma)$, and positive if $\kappa - \gamma < M(\eta - \gamma)$. Note that when $\eta = \gamma = 1$, the sign of (A. 19) is the opposite of $\kappa - 1$. The derivative of (A. 19) with respect to $\kappa$ is negative. ∎

Adding rational traders to the economy greatly complicates the expression for covariance. For simplicity, and without altering the basic finding that returns may be positively serially correlated when overconfident traders trade with rational traders, Proposition 6 is proven for an economy in which $N$ overconfident traders and $N/M$ rational traders receive private signals in period 2 and $\tilde{v}$ is revealed in period 3; therefore $P_3 = \tilde{v}$. As above, each overconfident trader receives one of $M$ possible signals, $\tilde{y}_{2i} = \tilde{v} + \tilde{\epsilon}_{2m}$; rational traders receive signal $y_{2r} = \tilde{v} + \tilde{\rho}_2$. Overconfident traders believe that $\rho \sim N(0, (\gamma h_\rho)^{-1})$, while rational traders hold correct distributional beliefs about their signals and those of others.

**Proposition 6** *Let* $\eta = 0$. *If overconfident traders trade with rational traders, then* $\mathrm{cov}_a((P_3 - P_2, (P_2 - P - 1))$ *is positive if* $(1-\gamma)(1+\gamma M)h_\rho > (\kappa + 1 + \gamma(M-1))(\kappa + \gamma(M-1) - M)h_\epsilon$.

*Proof:* We can determine the equilibrium as was done in Lemma 1. Then we can calculate

$$\mathrm{cov_a}(P_3 - P_2, P_2 - P_1) = \frac{M\left((1-\gamma)(1+\gamma M)h_\rho - (\kappa + \gamma(M-1)+1)(\kappa + \gamma(M-1) - M)h_\epsilon\right)}{(M(\kappa + \gamma(M-1)+1)h_\epsilon) + (1+\gamma M)h_\rho + (1+M)h_v)^2} \tag{A. 20}$$

which is positive when $(1-\gamma)(1+\gamma M)h_\rho > (\kappa + \gamma(M-1)+1)(\kappa + \gamma(M-1) - M)h_\epsilon$. ∎

# Appendix B (An Insider)

**Lemma 2** *If $\kappa h_\epsilon + 2\eta h_v > \kappa h_v$, an equilibrium exists in which the insider's linear price conjecture, equation (4), and the market-maker's linear demand conjecture, equation (5), are fulfilled. In equilibrium the coefficients of equations (4) and (5) are*

$$A = 0 \tag{B. 1}$$

$$B = \sqrt{\frac{\kappa h_v h_\epsilon}{h_z(\kappa h_\epsilon + 2\eta h_v - \kappa h_v)}} \tag{B. 2}$$

$$H = 0 \tag{B. 3}$$

$$L = \frac{1}{2(\eta h_v + \kappa h_\epsilon)}\sqrt{\frac{\kappa h_\epsilon h_z(\kappa h_\epsilon + 2\eta h_v - \kappa h_v)}{h_v}} \tag{B. 4}$$

*Proof:* The insider submits a demand, $x$, which he believes will maximize his expected profit. To do this he solves

$$\max_x \mathrm{E_b}(x(\tilde{v} - P) \mid y) = \max_x \mathrm{E_b}(x(\tilde{v} - (H + L(x+z))) \mid y) \tag{B. 5}$$

where equation (4) has been substituted for $P$. Taking first order conditions and solving for $x$ we have

$$x = \frac{\mathrm{E_b}(\tilde{v} \mid y) - H}{2L}. \tag{B. 6}$$

We can calculate

$$\mathrm{E_b}(\tilde{\mathrm{v}} \mid y) = \frac{\kappa h_\epsilon y}{\eta h_v + \kappa h_\epsilon}. \tag{B. 7}$$

Substituting (B. 7) into (B. 6) we get

$$x = \frac{-H}{2L} + \frac{\kappa h_\epsilon}{2L(\eta h_v + \kappa h_\epsilon)} y \tag{B. 8}$$

And so if the linear conjectures hold,

$$A = \frac{-H}{2L} \qquad \text{and} \qquad B = \frac{\kappa h_\epsilon}{2L(\eta h_v + \kappa h_\epsilon)} \tag{B. 9}$$

The market-maker sets price equal to the expected value of $\tilde{v}$ given the order-flow she observes. We can calculate

$$\mathrm{P} = \mathrm{E_b}(\tilde{\mathrm{v}} \mid x+z) = \frac{-ABh_\epsilon h_z}{B^2 h_z(h_\epsilon + h_v) + h_\epsilon h_v} + \frac{Bh_\epsilon h_z}{B^2 h_z(h_\epsilon + h_v) + h_\epsilon h_v}(x+z) \tag{B. 10}$$

So if the conjectures hold,

$$H = \frac{-ABh_\epsilon h_z}{B^2 h_z(h_\epsilon + h_v) + h_\epsilon h_v} \qquad \text{and} \qquad L = \frac{Bh_\epsilon h_z}{B^2 h_z(h_\epsilon + h_v) + h_\epsilon h_v} \tag{B. 11}$$

The four equations in (B. 9) and (B. 11) have four unknowns. When $\kappa h_\epsilon + 2\eta h_v \geq \kappa h_v$, they have one real non-negative solution: equations (B. 1)-(B. 4). Thus the conjectures are fulfilled and an equilibrium exists. ∎

Expected trading volume is $\mathrm{E_a}(|x| + |z|)$:

**Proposition 7** *If $\kappa \geq 1$, $\eta \leq 1$, and $\kappa h_\epsilon + 2\eta h_v > \kappa h_v$, $\mathrm{E_a}(|x| + |z|)$ is an increasing function of $\kappa$.*

*Proof:* Substituting equations (B. 3) and (B. 4) into equation (B. 8), and substituting equation (B. 8) for $x$, we can calculate

$$\mathrm{E_a}(|x| + |z|) = \sqrt{\frac{2}{\pi h_z}} + \sqrt{\frac{2\kappa(h_\epsilon + h_v)}{\pi h_z(\kappa h_\epsilon + 2\eta h_v - \kappa h_v)}} \tag{B. 12}$$

When $\kappa h_\epsilon + 2\eta h_v > \kappa h_v$, the derivative of (B. 12) with respect to $\kappa$ is positive. ∎

Market depth is measured as the inverse of the derivative of price with respect to orderflow (i.e. $(x + \tilde{z})$):

**Proposition 8** *If $\kappa \geq 1$, $\eta \leq 1$, and $\kappa h_\epsilon + 2\eta h_v > \kappa h_v$, $\left(\frac{d\,P}{d(x+\tilde{z})}\right)^{-1}$ is an increasing function of $\kappa$.*

*Proof:* Substituting equations (B. 3) and (B. 4) into equation (4), and differentiating with respect to $(x + \tilde{z})$ gives us

$$\left(\frac{d\,P}{d\,(x+\tilde{z})}\right)^{-1} = 1/L. \tag{B. 13}$$

Substituting equation (B. 4) for $L$, the derivative of $1/L$ with respect to $\kappa$ is positive when $\kappa h_\epsilon + 2\eta h_v > \kappa h_v$. ∎

Volatility is measured as the variance of price:

**Proposition 9** *If $\kappa \geq 1$, $\eta \leq 1$, and $\kappa h_\epsilon + 2\eta h_v > \kappa h_v$, $\mathrm{var_a}(P)$ is an increasing function of $\kappa$.*

*Proof:* Substituting equations (B. 1) and (B. 2) into equation (5), and equations (5), (B. 3) and (B. 4) into equation (4), we can calculate

$$\mathrm{var_a}(P) = \frac{\kappa h_\epsilon}{2h_v(\kappa h_\epsilon + \eta h_v)}. \tag{B. 14}$$

The derivative of (B. 14) with respect to $\kappa$ is positive. ∎

Quality of prices is measured as the variance of the difference between price and true underlying value:

**Proposition 10** *If $\kappa \geq 1$, $\eta \leq 1$, and $\kappa h_\epsilon + 2\eta h_v > \kappa h_v$, $\mathrm{var_a}(P - \tilde{v})$ is a decreasing function of $\kappa$.*

*Proof:* Substituting equations (B. 1) and (B. 2) into equation (5), and equations (5), (B. 3) and (B. 4) into equation (4), and then into $\mathrm{var_a}(P - \tilde{v})$, we can calculate

$$\mathrm{var_a}(\mathrm{P} - \tilde{v}) = \frac{\kappa h_\epsilon + 2\eta h_v}{2h_v(\kappa h_\epsilon + \eta h_v)}. \tag{B. 15}$$

The derivative of equation (B. 15) with respect to $\kappa$ is negative. ∎

would result in a range of possible equilibria rather than a single equilibrium point. In this case it is

Trader $i$'s demand for the risky asset is $x_{1i}$ and for the riskfree asset is $f_{1i}$. So his final wealth is $W_{1i} = x_{1i}\tilde{v} + f_{1i}$. Trader $i$'s utility function is $U(W_{1i}) = -\exp(-aW_{1i})$, where $a$ is the comm)-(B. 4) into equations (4) and (5), and then into $\mathrm{E_a}(x(\tilde{v} - P))$, we have

$$\mathrm{E_a}(x(\tilde{v} - P)) = \frac{1}{2(\kappa h_\epsilon + \eta h_v)} \sqrt{\frac{\kappa h_\epsilon(\kappa h_\epsilon + 2\eta h_v - \kappa h_v)}{h_v h_z}}. \tag{B. 16}$$

When $\kappa h_\epsilon + 2\eta h_v > \kappa h_v$, the derivative of (B. 16) with respect to $\kappa$ is negative. ∎

# Appendix C (Market-makers and Costly Information)

The following expectation is needed in this section. Let $x$ be a normally distributed random variable with mean $\mu$ and variance $\sigma^2$, then

$$\mathrm{E}\left(e^{Ax^2+Bx+C}\right) = \frac{1}{\sqrt{(1-2A\sigma^2)}} \exp\left\{-\frac{1}{2\sigma^2}\left(\frac{-\left(\mu + B\sigma^2\right)^2}{1-2A\sigma^2} + \mu^2\right) + C\right\}, \tag{C. 1}$$

which can be obtained by completing the square.

These expectations can be easily calculated:

$$\mathrm{E_b}(\tilde{v} \mid \tilde{y}) = \frac{\kappa h_\epsilon}{\eta h_v + \kappa h_\epsilon}\tilde{y} \equiv \mu_\mathrm{b} \qquad \mathrm{E_a}(\tilde{v} \mid \tilde{y}) = \frac{h_\epsilon}{h_v + h_\epsilon}\tilde{y} \equiv \mu_\mathrm{a} \tag{C. 2}$$

$$\mathrm{var_b}(\tilde{v} \mid \tilde{y}) = \frac{1}{\eta h_v + \kappa h_\epsilon} \equiv r_{\mathrm{ib}} \qquad \mathrm{var_a}(\tilde{v} \mid \tilde{y}) = \frac{1}{h_v + h_\epsilon} \equiv r_{i\mathrm{a}} \tag{C. 3}$$

$$\mathrm{var_b}(\mu_\mathrm{b}) = \frac{\kappa h_\epsilon}{(\eta h_v + \kappa h_\epsilon)(\eta h_v)} \equiv S_\mathrm{b} \qquad \mathrm{var_a}(\mu_\mathrm{b}) = \frac{\kappa^2 h_\epsilon(h_v + h_\epsilon)}{h_v(\eta h_v + \kappa h_\epsilon)^2} \equiv S_\mathrm{a} \tag{C. 4}$$

Since $\tilde{y}$ is normally distributed, $\mu_\mathrm{b}$ is also normally distributed. Define

$$\bar{r} \equiv \frac{r_{\mathrm{ib}}\,\mathrm{var_b}(\tilde{v} \mid P)}{\lambda \mathrm{var_b}(\tilde{v} \mid P) + (1-\lambda) r_{\mathrm{ib}}}. \tag{C. 5}$$

**Lemma 3** *There exists an equilibrium in which each informed trader's demand for the risky asset is*

$$x_{1\mathrm{I}} = \frac{\mathrm{E_b}(\tilde{v} \mid \tilde{y}) - P}{a\,\mathrm{var_b}(\tilde{v} \mid \tilde{y})}, \tag{C. 6}$$

*each uninformed trader's demand for the risky asset is*

$$x_{1\mathrm{U}} = \frac{\mathrm{E_b}(\tilde{v} \mid P) - P}{a\,\mathrm{var_b}(\tilde{v} \mid P)}, \tag{C. 7}$$

*price is*

$$P = \bar{r}\left(\lambda \frac{\mu_{\mathrm{b}}}{r_{\mathrm{ib}}} + (1-\lambda)\frac{\mathrm{E}_{\mathrm{b}}(\tilde{v} \mid P)}{\mathrm{var}_{\mathrm{b}}(\tilde{v} \mid P)}\right) + a\bar{r}\tilde{z}, \tag{C. 8}$$

*and the fraction of traders who choose to become informed is*

$$\lambda^* = \begin{cases} 0 & if \quad \left(1 - \frac{r_{\mathrm{ib}}\,(\mathrm{e}^{2\mathrm{ac}}-1)}{S_{\mathrm{b}}}\right) \leq 0, \\ a\sqrt{\left(1 - \frac{r_{\mathrm{ib}}\,(\mathrm{e}^{2\mathrm{ac}}-1)}{S_{\mathrm{b}}}\right)\frac{r_{\mathrm{ib}}}{(\mathrm{e}^{2\mathrm{ac}}-1)\,h_z}} & if \quad \left(1 - \frac{r_{\mathrm{ib}}\,(\mathrm{e}^{2\mathrm{ac}}-1)}{S_{\mathrm{b}}}\right) \in \left[0, \frac{(\mathrm{e}^{2\mathrm{ac}}-1)\,h_z}{a^2\,r_{\mathrm{ib}}}\right], \\ 1 & if \quad \left(1 - \frac{r_{\mathrm{ib}}\,(\mathrm{e}^{2\mathrm{ac}}-1)}{S_{\mathrm{b}}}\right) \geq \frac{(\mathrm{e}^{2\mathrm{ac}}-1)\,h_z}{a^2\,r_{\mathrm{ib}}}. \end{cases} \tag{C. 9}$$

*Proof:* The derivation of this equilibrium roughly follows Grossman and Stiglitz (1980) and Demski and Feltham (1994). Solving equation (6) gives us equation (C. 6) (see Grossman, 1976). Assume for the moment that, given traders' distributional beliefs and conditional on observing $P$, $\tilde{v}$ is normally distributed. (We will see below that the assumption is self-fulfilling.) Then solving equation (7) gives us equation (C. 7).

Informed trader demand per trader times the fraction of traders who are informed plus uninformed trader demand per trader times the fraction of traders who are uninformed must equal noise trader supply per trader. That is

$$\lambda \frac{\mathrm{E}_{\mathrm{b}}(\tilde{v} \mid \tilde{y}) - P}{a\,\mathrm{var}_{\mathrm{b}}(\tilde{v} \mid \tilde{y})} + (1-\lambda)\frac{\mathrm{E}_{\mathrm{b}}(\tilde{v} \mid P) - P}{a\,\mathrm{var}_{\mathrm{b}}(\tilde{v} \mid P)} = -\tilde{z}. \tag{C. 10}$$

Solving equation (C. 10) for $P$ gives us equation (C. 8). Let

$$\tilde{\zeta} \equiv \tilde{\mu} + \frac{a r_{ib}}{\lambda}\tilde{z}. \tag{C. 11}$$

Substituting equation (C. 11) into equation (C. 8) gives us

$$P = C_1\,\tilde{\zeta} \quad \text{where} \quad C_1 \equiv \frac{\bar{r}\lambda}{r_{\mathrm{ib}}} + \frac{\bar{r}(1-\lambda)}{\mathrm{var}_{\mathrm{b}}(\tilde{v} \mid P)}G_1, \quad G_1 \equiv \frac{S_{\mathrm{b}}}{S_{\mathrm{b}} + D}, \quad \text{and} \quad D \equiv \left(\frac{a\,r_{\mathrm{ib}}}{\lambda}\right)^2 h_z^{-1}. \tag{C. 12}$$

If equation (C. 8) holds, then $P$ is a linear function of $\tilde{\zeta}$ and the two are informationally equivalent. $\tilde{\zeta}$ is a linear combination of two normally distributed random variables and is normally distributed itself. Therefore, given traders' distributional beliefs, $(\tilde{v} \mid P) = (\tilde{v} \mid \tilde{\zeta})$ is normally distributed, as was assumed, and has mean and variance:

$$\mathrm{E}_{\mathrm{b}}(\tilde{v} \mid \tilde{\zeta}) = G_1\,\tilde{\zeta}, \quad \text{and} \quad \mathrm{var}_{\mathrm{b}}(\tilde{v} \mid \tilde{\zeta}) = \frac{1}{\eta h_v} - G_1 S_{\mathrm{b}} \equiv r_u. \tag{C. 13}$$

To solve for $\lambda^*$ we observe that in equilibrium traders are indifferent between buying and not buying information; thus,

$$\mathrm{E_b}[U(W_{1\mathrm{I}})] = \mathrm{E_b}[U(W_{1\mathrm{U}})]. \tag{C. 14}$$

Bearing in mind that $\tilde{\zeta}$ has the same information content as $P$, we can calculate

$$\begin{aligned}
\mathrm{E_b}[U(W_{1i})] &= \mathrm{E_b}\left[\mathrm{E_b}\left[\mathrm{E_b}[-\exp\{-a\,W_{1I}\} \mid \tilde{y}] \mid \tilde{\zeta},\lambda\right]\right] && \text{(C. 15)}\\
&= \mathrm{E_b}\left[\mathrm{E_b}\left[-\exp\left\{-a\mathrm{E_b}[W_{1I} \mid \tilde{y}] + \frac{a^2}{2}\mathrm{var_b}[W_{1I} \mid \tilde{y}]\right\} \mid \tilde{\zeta},\lambda\right]\right] && \text{(C. 16)}\\
&= \mathrm{E_b}\left[\mathrm{E_b}\left[-\exp\left\{-a\left(f_{0I} + x_{0I}P - c + \frac{(\mu_\mathrm{b} - P)^2}{2ar_{i\mathrm{b}}}\right)\right\} \mid \tilde{\zeta},\lambda\right]\right] && \text{(C. 17)}\\
&= \mathrm{E_b}\left[-e^{ac}\sqrt{\frac{r_{i\mathrm{b}}}{r_u}}\exp\left\{-a\left(f_{0i} + x_{0i}P + \frac{\left(E(\tilde{v} \mid \tilde{\zeta},\lambda) - P\right)^2}{2ar_u}\right)\right\}\right] && \text{(C. 18)}\\
&= e^{ac}\sqrt{\frac{r_{i\mathrm{b}}}{r_u}}\mathrm{E_b}[U(W_{1U})]. && \text{(C. 19)}
\end{aligned}$$

Equation (C. 17) is obtained from (C. 16) by substituting $f_{0I}+x_{0I}P+x_{iI}(\tilde{v}-P)$ for $W_{1I}$, equation (C. 6) for $x_{1i}$, and taking expectations. To obtain equation (C. 18) we multiply out $(\mu_\mathrm{b}-P)^2$, apply equation (C. 1), substitute from (C. 3),s $\mathrm{E_b}(\tilde{v} \mid \tilde{\zeta}) = \mathrm{E_b}(\mu_\mathrm{b} \mid \tilde{\zeta})$. From equations (C. 14) and (C. 19) we have

$$1 = e^{ac}\sqrt{\frac{r_{i\mathrm{b}}}{r_u}}. \tag{C. 20}$$

Substituting for $r_{i\mathrm{b}}$ and $r_u$ in equation (C. 20), solving for $\lambda$, and noting that $\lambda$ cannot be negative or larger than 1 gives us equations (C. 9). ▮

**Proposition 11** *There exist values of $h_v$, $h_\epsilon$, $h_z$, $a$, and $c$, such that if $\eta = 1$ and $\kappa > 1$, or if $\eta < 1$ and $\kappa = 1$, and $0 < \lambda^* < 1$, then $\mathrm{E_a}[U(W_{1I})] < \mathrm{E_a}[U(W_{1U})]$, where $i = I$, and $i = U$ represent prototypical informed and uninformed traders.*

*Proof:* To calculate the actual expected utility of the informed trader, we take iterative expectations using the actual distributions of $\tilde{v}$ and $\tilde{\epsilon}$. Equation (C. 1) is used to solve each expectation. The result is:

$$\begin{aligned}
\mathrm{E_a}[U(W_{1I})] &= \mathrm{E_a}\left[\mathrm{E_a}\left[\mathrm{E_a}[U(W_{1I}) \mid \tilde{y}] \mid \tilde{\zeta},\lambda\right]\right]\\
&= -(1 - 2C_3\mathrm{var_a}(\mu_\mathrm{b} \mid \tilde{\zeta}))^{-\frac{1}{2}}(1 - 2C_4\mathrm{var_a}(\tilde{\zeta}))^{-\frac{1}{2}} \cdot \qquad \text{(C. 21)}\\
&\quad \exp\left\{a(c - f_{0I}) + (aC_1x_{0I})^2\mathrm{var_a}(\tilde{\zeta})/(2(1 - 2C_4\mathrm{var_a}(\tilde{\zeta})))\right\}
\end{aligned}$$

where

$$C_2 = \left(1 + \frac{(\eta h_v + \kappa h_\epsilon)}{\kappa(h_v + h_\epsilon)}\right)(\eta h_v + \kappa h_\epsilon) - \frac{(\eta h_v + \kappa h_\epsilon)^2}{h_v + h_\epsilon}, \quad C_3 = \frac{(\kappa - 2)(\eta h_v + \kappa h\epsilon)^2}{2\kappa(h_v + h_\epsilon)}, \tag{C. 22}$$

$$C_4 = \frac{r_{ia} - 2r_{ib}}{2r_{ib}^2} C_1^2 + \frac{\left(G_2 + C_2 C_1 \mathrm{var_a}(\mu_b \mid \tilde{\zeta})\right)^2}{2\mathrm{var_a}(\mu_b \mid \tilde{\zeta})(1 - C_3 \mathrm{var_a}(\mu_b \mid \tilde{\zeta}))} - \frac{G_2^2}{2\mathrm{var_a}(\mu_b \mid \tilde{\zeta})}, \tag{C. 23}$$

$$G_2 = \frac{S_a}{S_a + D}, \quad \mathrm{var_a}(\mu_b \mid \tilde{\zeta}) = S_a(1 - G_2), \quad \text{and} \quad \mathrm{var_a}(\tilde{\zeta}) = S_a + D. \tag{C. 24}$$

Similarly we can calculate the actual expected utility of the uninformed trader:

$$\begin{aligned} \mathrm{E_a}[U(W_{1U})] &= \mathrm{E_a}\left[\mathrm{E_a}\left[\mathrm{E_a}[U(W_{1U}) \mid \tilde{y}] \mid \tilde{\zeta}, \lambda\right]\right] \\ &= -(1 - 2C_5 \mathrm{var_a}(\tilde{\zeta}))^{-\frac{1}{2}} \cdot \\ &\quad \exp\left\{-af_{0I} + (aC_1 x_{0U})^2 \mathrm{var_a}(\tilde{\zeta}) / (2(1 - 2C_5 \mathrm{var_a}(\tilde{\zeta})))\right\} \end{aligned} \tag{C. 25}$$

where

$$C_5 = \frac{(G_1 - C_1)^2}{2r_u^2} \mathrm{var_a}(\tilde{v} \mid \tilde{\zeta}) - \frac{(G_1 - C_1)(G_3 - C_1)}{r_u}, \tag{C. 26}$$

$$G_3 = \frac{\eta S_b}{S_a + D}, \quad \text{and} \quad \mathrm{var_a}(\tilde{v} \mid \tilde{\zeta}) = \frac{1}{h_v} - \frac{\eta^2 S_b^2}{S_a + D}. \tag{C. 27}$$

If equation (C. 21) < equation (C. 25) for any parameter set for which $\eta = 1$, $\kappa > 1$ and $0 < \lambda^* < 1$, and for any parameter set for which $\eta < 1$,$\kappa = 1$ and $0 < \lambda^* < 1$, then the proposition is true. I've evaluated (C. 21) and (C. 25) for a wide variety of parameter values and found (C. 21) to be less than (C. 25) whenever these parameter conditions are true, suggesting that the expected utility of informed traders is always less than than of uninformed traders if traders are overconfident or if they undervalue their prior information. ▮

In the final proposition $\lambda^*$ and $P$ are written as functions of the economy's parameter values (i.e. $\lambda^*(\kappa, \eta, h_v, h_\epsilon, h_z, a, c)$ and $P(\kappa, \eta, h_v, h_\epsilon, h_z, a, c)$). As in Proposition 10, market depth is measured as the inverse of the derivative of price with respect to orderflow.

**Proposition 12** *Let $\kappa_2 > \kappa_1 \geq 1$, then, for any choice of parameter values $\eta$, $h_v$, $h_\epsilon$, $h_z$, $a$, and $c$ such that $\lambda^*(\kappa_2, \eta, h_v, h_\epsilon, h_z, a, c) = 1$ and $\lambda^*(\kappa_1, \eta, h_v, h_\epsilon, h_z, a, c) = 1$,*

$$\left(\frac{d\,P(\kappa_2,\eta,h_v,h_\epsilon,h_z,a,c)}{d\,\tilde{z}}\right)^{-1} > \left(\frac{d\,P(\kappa_1,\eta,h_v,h_\epsilon,h_z,a,c)}{d\,\tilde{z}}\right)^{-1} \tag{C. 28}$$

*and*

$$\mathrm{var_a}(P(\kappa_2,\eta,h_v,h_\epsilon,h_z,a,c)) > \mathrm{var_a}(P(\kappa_1,\eta,h_v,h_\epsilon,h_z,a,c)) \quad \text{if} \quad \mathrm{E_a}(|\tilde{z}|) < \sqrt{2\phi/\pi}\,, \tag{C. 29}$$

$$\mathrm{var_a}(P(\kappa_2,\eta,h_v,h_\epsilon,h_z,a,c)) = \mathrm{var_a}(P(\kappa_1,\eta,h_v,h_\epsilon,h_z,a,c)) \quad \text{if} \quad \mathrm{E_a}(|\tilde{z}|) = \sqrt{2\phi/\pi}\,, \tag{C. 30}$$

*and*

$$\mathrm{var_a}(P(\kappa_2,\eta,h_v,h_\epsilon,h_z,a,c)) < \mathrm{var_a}(P(\kappa_1,\eta,h_v,h_\epsilon,h_z,a,c)) \quad \text{if} \quad \mathrm{E_a}(|\tilde{z}|) > \sqrt{2\phi/\pi} \tag{C. 31}$$

*where*

$$\phi = \frac{\eta(h_v + h_\epsilon)((\kappa_1 + \kappa_2)\eta h_v + 2\kappa_1 \kappa_2 h_\epsilon)}{a^2(2\eta h_v + (\kappa_1 + \kappa_2)h_\epsilon)}. \tag{C. 32}$$

*Proof:* Substituting $\lambda^* = 1$ into equation (C. 5), and equation (C. 5) and $\lambda^* = 1$ into equation (C. 12), we find that $P = \tilde{\zeta}$. Differentiating $P$ with respect to $\tilde{z}$, taking the inverse, and substituting from equation (C. 3) gives us market depth:

$$\left(\frac{d\,P}{d\,\tilde{z}}\right)^{-1} = \frac{\eta h_v + \kappa h_\epsilon}{a}, \tag{C. 33}$$

which increasing in $\kappa$. Bearing in mind that $\mathrm{E_a}(|\tilde{z}|) = \sqrt{2/\pi h_z}$ , we find that $P = \tilde{\zeta}$, equations (C. 4) and (C. 24), and algebraic manipulations give us equations (C. 29)-(C. 31). ∎

**Proposition 13** *Let $\eta_2 < \eta_1 \leq 1$, then, for any choice of parameter values $\kappa$, $h_v$, $h_\epsilon$, $h_z$, $a$, and $c$ such that $\lambda^*(\kappa, \eta_2, h_v, h_\epsilon, h_z, a, c) = 1$ and $\lambda^*(\kappa, \eta_1, h_v, h_\epsilon, h_z, a, c) = 1$, or $\lambda^*(\kappa, \eta_2, h_v, h_\epsilon, h_z, a, c) = 0$ and $\lambda^*(\kappa, \eta_1, h_v, h_\epsilon, h_z, a, c) = 0$, then*

$$\left(\frac{d\,P(\kappa,\eta_2,h_v,h_\epsilon,h_z,a,c)}{d\,\tilde{z}}\right)^{-1} < \left(\frac{d\,P(\kappa,\eta_1,h_v,h_\epsilon,h_z,a,c)}{d\,\tilde{z}}\right)^{-1} \tag{C. 34}$$

*and*

$$\mathrm{var_a}(P(\kappa,\eta_2,h_v,h_\epsilon,h_z,a,c)) > \mathrm{var_a}(P(\kappa,\eta_1,h_v,h_\epsilon,h_z,a,c)) \,. \tag{C. 35}$$

*Proof:* To prove the proposition we look separately at the cases of $\lambda^* = 1$ and $\lambda^* = 0$. If $\lambda^* = 1$ then equation (C. 33) is market depth and is increasing in $\eta$. The variance of $P$ is

$$\mathrm{var_a}(P) \;=\; \frac{\kappa^2 h_\epsilon(h_v + h_\epsilon)}{h_v(\eta h_v + \kappa h_\epsilon)^2} + \frac{a^2}{(\eta h_v + \kappa h_\epsilon)^2 h_z}\,, \tag{C. 36}$$

which is decreasing in $\eta$. This proves the proposition for the first case. If $\lambda^* = 0$, then $P = a\mathrm{var_b}(\tilde{v})\tilde{z}$. Here market depth is simply

$$\left(\frac{d\,P}{d\,\tilde{z}}\right)^{-1} = \frac{\eta h_v}{a}, \tag{C. 37}$$

which is increasing in $\eta$. The variance of $P$ is

$$\mathrm{var_a}(P) = \frac{a^2}{\eta^2 h_v^2 h_z}\,, \tag{C. 38}$$

which is decreasing in $\eta$. This proves the proposition for the second case. ∎

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Table 1: Notation

| | Price-takers | Insider | Costly Information |
|---|---|---|---|
| overconfidence parameter | $\kappa \geq 1$ | $\kappa \geq 1$ | $\kappa \geq 1$ |
| parameter underweighting priors | $\eta \leq 1$ | $\eta \leq 1$ | $\eta \leq 1$ |
| parameter underweighting signals of others | $\gamma \leq 1$ | | |
| number of traders | $i = 1, ..., N$ | 1 insider, 1 market-maker | $i = 1, ..., N$ |
| time | $t = 0, ..., 4$ | $t = 0, 1$ | $t = 0, 1$ |
| number of distinct signals | $m = 1, ..., M$ | 1 | 1 |
| terminal value of risky asset | $\tilde{v} \sim N(0, {h_v}^{-1})$ | $\tilde{v} \sim N(0, {h_v}^{-1})$ | $\tilde{v} \sim N(0, {h_v}^{-1})$ |
| signals | $\tilde{y}_{ti} = \tilde{v} + \tilde{\epsilon}_{tm}$ | $\tilde{y} = \tilde{v} + \tilde{\epsilon}$ | $\tilde{y} = \tilde{v} + \tilde{\epsilon}$ |
| error term in signals | $\tilde{\epsilon}_{tm} \sim N(0, {h_\epsilon}^{-1})$ | $\tilde{\epsilon} \sim N(0, {h_\epsilon}^{-1})$ | $\tilde{\epsilon} \sim N(0, {h_\epsilon}^{-1})$ |
| noise trader demand | | $\tilde{z} \sim N(0, {h_z}^{-1})$ | $\tilde{z} \sim N(0, {h_z}^{-1})$ |
| coefficient of absolute risk aversion | $a$ | | $a$ |
| per capita supply of risky asset | $\bar{x}$ | | $\bar{x}$ |
| price of risky asset | $P_t$ | $P$ | $P$ |
| i's endowment of risky asset | $x_{0i}$ | | $x_{0i}$ |
| i's demand for risky asset | $x_{ti}$ | $x$ | $x_{ti}$ |
| i's endowment of riskless asset | $f_{0i}$ | | $f_{0i}$ |
| i's demand for riskless asset | $f_{ti}$ | | $f_{ti}$ |
| i's wealth | $W_{ti}$ | | $W_{ti}$ |
| trader i's information set | $\Phi_{ti}$ | | |
| fraction of traders who buy information | | | $\lambda$ |

Figure 1.

Supply curves when all traders are uninformed. $P = a\,\mathrm{var_b}(\tilde{v} \mid P)\,Q$, where $P$ is price and $Q$ is quantity, for economies in which $\eta = .5$ (solid line) and $\eta = 1$ (dashed line) and where, for both economies, $\kappa = 1$, $h_v = 2$, $h_\epsilon = 1$, $h_z = .25$, $a = 2$, $c = .2$, and $\lambda = 1$.

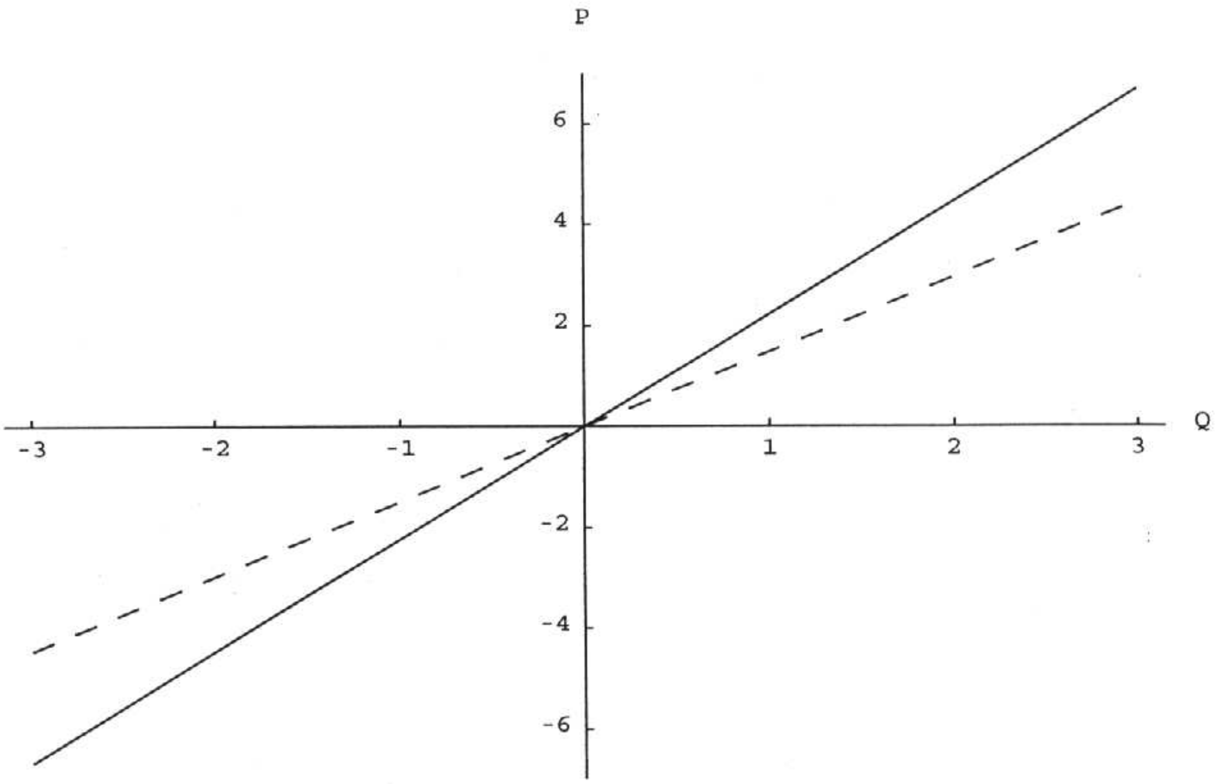

Figure 2.

Supply curves when all traders are informed. $P = \mathrm{E_b}(\tilde{v} \mid y) + a \,\mathrm{var_b}(\tilde{v} \mid y)\, Q$ where $P$ is price and $Q$ is quantity, for economies in which $\kappa = 2$ (solid line) and $\kappa = 1$ (dashed line) and where, for both economies, signal $\tilde{y} = 2$ has been received, $\eta = 1$, $h_v = 2$, $h_\epsilon = 1$, $h_z = .25$, $a = 2$, $c = .09$, and $\lambda = 1$.