

**OPTIMAL ASSET REBALANCING IN THE PRESENCE OF
TRANSACTIONS COSTS**

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Abstract

We examine the optimal trading strategy for an investment fund which wishes to maintain assets two assets in fixed proportions, e.g. 60/40 in stocks and bonds. Transactions costs are assumed to be proportional to the amount of each asset traded.

We show that the optimal policy specifies a band about the target stock proportion. As long as the actual stock/bond ratio remains inside this band, no trading should occur. If the ratio goes outside the band, trading should be undertaken to move the ratio to the nearest edge of the band. We compute the optimal band and resulting annual turnover and tracking error of the optimal policy, as a function of transactions costs, asset volatility, the target asset mix, and other parameters. We show how changes in transactions costs and other parameters affect the size of the no-trade band, turnover, and tracking error. Compared to a quarterly rebalancing strategy, an example demonstrates that the optimal strategy can reduce turnover by almost 50%.

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I. Introduction

Many asset funds desire to maintain constant asset proportions, such as a 60/40 ratio of stocks to bonds. As asset values move randomly, asset ratios diverge from the desired target ratio. Funds "rebalance" by trading to restore the asset proportions to their target levels. Rebalancing typically occurs at a regular time interval, e.g. every month or every quarter.

If transactions costs are proportional to the dollar amounts traded, the conventional strategy of periodic rebalancing to the target ratio will not be optimal. Work by Constantinides [1986] and by Dumas and Luciano [1991] (based on a model with power utility functions) shows that the optimal strategy is typically characterized by a "no trade" interval about the target asset ratio. As the ratio of asset values moves randomly within this interval, no trading is needed. When the asset ratio wanders outside the no-trade interval, asset proportions should be adjusted back to the nearest edge of the interval. Subsequently, Dixit [1991] and Dumas [1991] provided further mathematical results for this and related problems.

To illustrate the nature of the optimal policy, we shall later show an example where the target stock/bond ratio is 1.5: a 60/40 stock-to-bond ratio. We show that no readjustments should be made when the ratio remains between 1.44 and 1.56. If the ratio moves (say) to 1.58, then trading should take place to restore the ratio to 1.56. If the ratio moves to 1.42, then the ratio should be restored to 1.44. Thus there may be extended periods with no trading (as the ratio remains inside

the no-trade interval), followed by brief periods in which many trades may be required to keep the ratios at the interval boundary. The ratio should never be moved to 1.50!

Our work differs from previous work in several dimensions. First, we take as given the target asset ratios and focus only on how to trade when divergences from the ratios occur. Rather than using a specific utility function over wealth, we postulate a cost function for deviations from the target ratio. Second, we develop a technique for estimating the expected required turnover of the optimal policy. Finally, we consider two risky assets, rather than a risky and a riskless asset.¹

II. The Optimization Problem

Consider two assets ("stocks" and "bonds"), whose prices S and B follow log random walks

$$(1) \quad dS(t)/S = \mu_S dt + \sigma_S dZ_S(t)$$

$$(2) \quad dB(t)/B = \mu_B dt + \sigma_B dZ_B(t),$$

where dZ_S and dZ_B are the increments of Wiener processes with correlation ρ .

Using Ito's Lemma, we may easily compute the stochastic process for $w(t) = S(t)/B(t)$:

$$(3) \quad dw(t)/w = (\mu_S - \mu_B + \sigma_B^2 - \rho\sigma_B\sigma_S) dt + \sigma_S dZ_S(t) - \sigma_B dZ_B(t)$$

¹ Related problems include the optimal cash management problem examined by Connor and Leland [1995] and Leland and Connor [1995], and the option pricing and replication problem in the presence of transactions costs (see, e.g., Leland [1985] and Hodges and Neuberger [1989]).

which implies

$$(4) \quad E(dw/w) = (\mu_S - \mu_B)dt$$

$$(5) \quad E(dw/w)^2 = (\sigma_S^2 + \sigma_B^2 - 2\rho\sigma_S\sigma_B)dt.$$

Given whatever utility function s/he may have, the investor wishes to hold stocks to bonds in a target ratio w^* . Divergence between the actual ratio, w , and w^* cause the investor to lose utility. The absolute loss will depend on the total dollars invested. But we scale the losses relative to wealth, implying that loss depends only on the divergence between $w(t)$ and w^* . When this divergence is small, the dollar equivalent cost over a time interval dt may be approximated by

$$(6) \quad L = \lambda(w(t) - w^*)^2 dt,$$

where λ is a constant representing the cost per unit of tracking error.²

Over an infinite horizon, the investor wishes to minimize the discounted integral of this cost of divergence, plus any trading costs associated with adjusting $w(t)$. As is well known, trading costs will be infinite if the investor tries to peg $w(t) = w^*$ continuously (see Leland [1985]). Constantinides [1986] and Dixit [1991] show that, in the presence of proportional trading costs, the optimal policy will specify an interval $w_{min} \leq w^* \leq w_{max}$. If the current $w(t)$ lies inside this interval,

² In an expected utility framework, λ scaled to wealth will equal $-U''(W)W/U'(W)$, the Pratt [1964] measure of relative risk aversion. This will be invariant to wealth with the power or log utility functions. However, we assume that loss is a function of $(w - w^*)$.

no trading will occur. If $w(t) > w_{max}$, stocks will be sold and the proceeds used to purchase bonds in order to reduce the ratio to w_{max} . If $w(t) < w_{min}$, stocks will be bought and bonds sold in order to increase the ratio to w_{min} .

Let k_S be the transactions cost per dollar of stocks traded, and k_B be the transactions cost per dollar of bonds traded. Since the portfolio must be self-financing,³ any change in the dollar value of stocks from trading, δS , will be matched by an opposite amount of change in the dollar value of bonds: $\delta B = -\delta S$. Total trading costs δC from a given trade δS in stocks will therefore be

$$(7) \quad \delta C = k_S |\delta S| + k_B |\delta B| = (k_S + k_B) |\delta S|.$$

We may readily ascertain that the change in w due to a trade δS is

$$(8) \quad \delta w = (1 + w) \delta S / B,$$

implying

$$(9) \quad \delta C / |\delta w| = B(k_S + k_B) / (1 + w).$$

Clearly costs are proportional to wealth, since B is an absolute number. Normalizing costs by wealth $W = B + S$ gives (for $\delta c = \delta C / W$)⁴

³ Self-financing means that no additional funds are being contributed or withdrawn. Therefore, any purchases of stocks must be financed by selling bonds, and vice-versa. For simplicity, we assume that transactions costs are paid from outside the fund.

⁴ It is appropriate to normalize by wealth here, since the loss function (6) also was normalized by wealth.

$$(10) \quad \delta c / \delta w = (k_S + k_B) / (1 + w)^2.$$

Let $V(w(t); w_{min}, w_{max})$ be the cost function associated a trading strategy characterized by no trading whenever $w(\tau) + \delta w(\tau) \in [w_{min}, w_{max}]$. That is,

$$V(w(t); w_{max}, w_{min}) = E\left[\int_{\tau=t}^{\text{infinity}} e^{-r(\tau-t)} \lambda (w(\tau) - w^*)^2 d\tau + PV\{transactions\ cost}\right]$$

where $PV\{transactions\ costs\}$ is the present value of the costs associated with implementing the strategy.

When $w(\tau) + dw(\tau) \in [w_{min}, w_{max}]$, the only cost over the time interval $d\tau$ is the cost of not being at the optimal w^* . From the definition of $V(w(t); w_{min}, w_{max})$,

$$(11) \quad V(w(t); w_{min}, w_{max}) = - \int_{\tau=t}^{t+dt} \lambda [w(t) - w^*]^2 d\tau + E[e^{-rdt} V(w(t) + dw(t); w_{min}, w_{max})]$$

Since the optimal strategy moves $w(t)$ to w_{min} if $w(t) + dw(t) < w_{min}$, and to w_{max} if $w(t) + dw(t) > w_{max}$, it follows that for small excursions of $w(t)$ outside the no-trade interval,

$$(12) \quad V(w(t); w_{min}, w_{max}) = V(w_{min}; w_{min}, w_{max}) + \delta c^* / \delta w (w_{min} - w(t))$$

$$(13) \quad V(w(t); w_{min}, w_{max}) = V(w_{max}; w_{min}, w_{max}) + \delta c^{**} / \delta w (w(t) - w_{max})$$

when $w(t) + dw(t) < w_{min}$ and $w(t) + dw(t) > w_{max}$, respectively.

From (10), $\delta c^*/\delta w = (k_S + k_B)/(I + w_{min})^2$, and $\delta c^{**}/\delta w = (k_S + k_B)/(I + w_{max})^2$. From (12) and (13), smoothness of the function V w.r.t. w at $w = w_{min}$ and $w = w_{max}$ implies that

$$(14) \quad V_I(w_{min}; w_{min}, w_{max}) = -\delta c^*/\delta w = -(k_S + k_B)/(I + w_{min})^2$$

$$(15) \quad V_I(w_{max}; w_{min}, w_{max}) = \delta c^{**}/\delta w = (k_S + k_B)/(I + w_{max})^2$$

where $V_k(\bullet; \bullet, \bullet)$ denotes the derivative of V with respect to its k th argument.⁵

Expanding the expectation term of (11) and simplifying gives the differential equation

$$(16) \quad a w V_I[w; w_{min}, w_{max}] + .5 b w^2 V_{II}[w; w_{min}, w_{max}] + \lambda(w - w^*)^2 - r V[w; w_{min}, w_{max}] = 0$$

where

$$a = \mu_S - \mu_B + \sigma_B^2 - \rho \sigma_B \sigma_S; \quad b = \sigma_S^2 + \sigma_B^2 - 2\rho \sigma_S \sigma_B.$$

Equation (16) has the solution

$$(17) \quad V[w; w_{min}, w_{max}] = \lambda \left[\frac{w^2}{r - 2a - b} - \frac{2w w^*}{r - a} + \frac{w^{*2}}{r} \right] + C_1 w^x + C_2 w^y$$

where

$$x = (-2a + b - \text{Sqrt}[(2a - b)^2 + 8br])/2b; \quad y = (-2a + b + \text{Sqrt}[(2a - b)^2 + 8br])/2b,$$

and C_1 and C_2 are determined by the boundary conditions (14) and (15), and depend upon w_{min} and

⁵ Dumas [1991] shows that this condition is not actually a smooth-pasting condition, but the limit of a "value matching" condition. The actual smooth-pasting conditions are given in (18) and (19), or (18') and (19').

w_{max} . The latter are determined by the "smooth pasting" conditions, which require that $V(w; w_{min}, w_{max})$ be minimized w.r.t. w_{min} and w_{max} . This provides the two final conditions needed for optimization, that at the optimal $w_{min} = w_{min}^*$ and $w_{max} = w_{max}^*$:

$$(18) \quad V_2(w_{min}^*; w_{min}^*, w_{max}^*) = 0;$$

$$(19) \quad V_3(w_{max}^*; w_{min}^*, w_{max}^*) = 0.$$

Following Dumas [1991], it can in turn be shown that these conditions imply

$$(18') \quad V_{II}(w_{min}^*; w_{min}^*, w_{max}^*) = 0$$

$$(19') \quad V_{II}(w_{max}^*; w_{min}^*, w_{max}^*) = 0.$$

Solving (17) subject to the conditions (14), (15), (18'), and (19') generates solutions for the optimal strategy parameters w_{min}^* and w_{max}^* , and for the function $V(w; w_{min}^*, w_{max}^*)$. In section IV below, we examine the nature of the optimal solution.

III. Turnover

Following Leland and Connor [1995], we may utilize a further technique to estimate the turnover associated with any particular policy. Consider the special case of $V(w; w_{min}^*, w_{max}^*)$ when $\lambda = 0$.

Call this function $T(w; w_{min}^*, w_{max}^*)$. Now this is the total discounted value of the transactions costs alone, since the penalty for diverging from w^* has been set to zero. The value of T is the net

present value of transactions costs forever, which can be viewed as the capitalized value of annual transactions costs. Therefore annual transactions costs are simply rT , and annual (one-way) turnover is $rT/(k_S + k_B)$. We set $w = w^*$, and define $T^* = T(w^*; w^*_{min}, w^*_{max})$. Our reported turnover results assume $T = T^*$.

IV. Results

We first derive the optimal trading strategy for a base case, with target stock/bond mix equal to 60/40. Where appropriate, all base case rates are in annual units:

$\mu_S - \mu_B =$		3.6%
r	=	7.5%
σ_S	=	20.0%
σ_B	=	10.0%
ρ	=	0.3
w^*	=	1.5
λ	=	0.35⁶
k_S	=	1.0% (one way)
k_B	=	0.5% (one way)

The optimal policy for this set of parameters is

⁶ While we have fixed λ , in a mean-variance world λ varies inversely with the chosen w^* . From the first order conditions for maximizing a function $U(w) = aw - \lambda bw^2$ generates optimal $w^* = a/(2b\lambda)$, or $\lambda = (a/2bw^*)$. For our example, $a = .04$, $w^* = 1.5$, and $b = .038$, implying $\lambda = 0.35$.

$$w_{min} = 1.421$$

$$w_{max} = 1.573$$

Therefore, the optimal policy puts a lower bound on the percentage of stock at $1.421/2.421 = 58.69\%$, and an upper bound on the percentage of stock at $1.573/2.573 = 61.14\%$. No trading should take place as long as the stock percentage remains between these bounds. If the ratio goes beyond the bounds, trading should occur such as to move the ratio to the

nearest boundary. Expected annual one-way turnover for this strategy is 8.95%, costing approximately 13.4 basis points. The average instantaneous standard deviation of tracking errors about $w^* = 0.0440$.⁷ A one standard deviation upward move in w would bring the asset mix to $1.544/2.544 = 60.69\%$. Therefore the annual standard deviation of the tracking error of the stock percentage about 60% is approximately 0.69%.

We can compare this result with the common practice of rebalancing to w^* at a constant time interval δt . Quarterly rebalancing, for example, sets $\delta t = 0.25$. Approximating the lognormal distribution with a normal distribution with zero drift gives an expected percent change in w over the period δt :

$$(20) \quad E[|dw|/w] = (2/\pi)^{0.5} \sigma_w (\delta t)^{0.5}.$$

In the case examined above, $\sigma_w^2 = b = 0.038$ and $w = 1.5$. Over a quarter of a year, the expected change $E[|\delta w|/w]$ from (20) will be 0.1167. From (7), it can be shown that $\delta S/W = \delta S/(B+S) = \delta w/(1+w)^2$. Therefore adjusting $\delta w = 0.1167$ requires adjusting stock (as a percentage of wealth) by an amount equal to $0.1167/(1+1.5)^2 = 1.87\%$. Trading back to w^* each quarter will therefore require an expected annual turnover of four times the quarterly amount, or 7.47%.

The average instantaneous variance of the tracking error $(w(t) - w^*)$ over the quarter will be one-half of the end-quarter variance of $\sigma_w^2 w^2 \delta t$, or 0.0107. Rebalancing to w^* implies that each quarter

⁷ Standard deviation is calculated as follows. We determine total tracking error costs (the integral of the discounted expectation of $\lambda(w-w^*)^2$) as total costs less trading costs: $V(w^*; w^*_{min}, w^*_{max}) - T(w^*; w^*_{min}, w^*_{max})$. This is multiplied by r to generate annual tracking error costs and divided by L to give the annual expectation of $(w-w^*)^2$. The square root of this is the annualized standard deviation.

starts anew with zero instantaneous variance. Therefore the average instantaneous variance over the year equals the average instantaneous variance over each quarter. Taking the square root gives an average instantaneous standard deviation of $(w(t) - w^*)$ of 0.1034. A one standard deviation upward move of w implies an asset mix of $1.6034/2.6034 = 61.59\%$, which in turn implies the standard deviation of the asset mix is about 1.59%. This is to be compared with the average instantaneous standard deviation of about 0.69% for the optimal policy.

Accuracy with quarterly rebalancing is less than the optimal policy, but turnover also is less. By setting λ at a lower level, however, we can find an optimal strategy which has the same turnover as the quarterly rebalancing strategy. Consider decreasing λ until the standard deviation of the tracking error of the optimal strategy is 0.1034, the same as quarterly rebalancing. By setting $\lambda = 0.0276$, the standard deviation of w about w^* is .1034, and the standard deviation of the stock percentage is 1.59%--the same as with the quarterly rebalancing strategy. The strategy has a "no trade" interval about w^* of $[1.307, 1.663]$, implying no trade when the stock proportion remains between 56.7% and 62.5%. But for this strategy, expected turnover is only 3.76%. In sum, the same degree of tracking accuracy can be achieved with an optimal rebalancing strategy as with quarterly rebalancing to w^* , with expected turnover only $3.76/7.47 = 50.3\%$ of the quarterly rebalancing's expected turnover. Cost savings on trading of almost 50% are realized by using the optimal strategy.⁸

V. Comparative Statics

⁸ Given our transactions costs assumptions, total trading costs will fall from approximately 11.2 bps. to 5.6 bps. For funds with taxable investors, the lower turnover can also lower the expected amount of realized capital gains. While we have focused on equal expected accuracy with lower turnover, we would also find equivalent gains if we had examined improved accuracy with equal turnover.

(i) Transactions costs.

Doubling transactions costs increases the size of the no-trade interval to [1.400, 1.592]. Expected annual one-way turnover falls to 7.10%. Halving transactions costs reduces the no-trade interval to [1.438, 1.559], with one-way turnover 11.30%. To a close approximation, *the size of the no-trade interval ($w_{max} - w_{min}$) varies directly as the cube root of transactions costs.*⁹ *Turnover varies inversely with the cube root of transactions costs.* These relationships hold for any given set of parameters, although the constant of proportionality clearly depends upon the chosen parameters. Since cost per unit trade is proportional to transactions costs, *total trading cost will vary with transactions costs raised to the two-thirds (2/3) power.*

(ii) Relative volatility.

Halving the relative variance (from .038 to .019) reduces the no-trade interval to [1.436, 1.557], and one-way turnover reduces to 5.66%. Doubling the relative volatility increases the no-trade interval to [1.402, 1.594] and one-way turnover to 14.2%. To a close approximation, *optimal turnover varies with the variance b of the dw/w process raised to the two-thirds (2/3) power, or equivalently, with the cube root of standard deviation. The size of the no-trade interval is proportional to the cube root of the variance, or equivalently, is proportional to the standard*

⁹ These results can be derived from the total differential of the smoothness conditions (14) and (15) to the second order, noting that several terms equal zero because of conditions (18), (19), (18'), and (19'). The remaining terms can be solved for the changes dw_{min} and dw_{max} which are (approximately) equal to the cube root of dk , the change in transactions costs $k = k_S + k_B$.

deviation raised to the 2/3 power.

(iii) Relative return and the riskfree rate.

Doubling the expected return premium of stocks over bonds marginally reduces the optimal no-trade interval, although not to the second decimal place. Similarly, turnover is marginally affected. Changes in the riskless interest rate also have negligible effects.

(iv) The target stock/bond ratio w^* .

Reducing w^* to 1.0 (a 50/50 stock/bond ratio) gives a no-trade interval of [0.929, 1.064], and lowers annual one-way turnover to 7.00%. The standard deviation of the stock fraction is 0.96%. Compared with our base case, turnover is lower and accuracy is less. Increasing λ will raise turnover and lower accuracy. When $\lambda = .725$, turnover (8.95%) is identical, and both total cost V and turnover T at the new optimum are the same as when $w^* = 1.5$. Thus $V - T$, the costs of the tracking error or "(in)accuracy costs", also remain unchanged. Therefore, with an appropriate adjustment in λ , a no-trade interval exists for each w^* which keeps turnover and (in)accuracy costs invariant to w^* . To a close approximation, the adjustment to λ which leaves turnover and accuracy costs unchanged is proportional to $(w^*/(1+w^*))^4$. Thus if w^* moves from 1.5 to (say) 3, λ must be reduced by a fraction $F = [(1.5/2.5)^4/(3/4)^4] = (6/7.5)^4 = .4096$. Since the original $\lambda = 0.35$, the adjusted λ must equal $0.35 * F = 0.1434$. We conclude that it is always possible to change λ so that *turnover and (in)accuracy costs are invariant to changes in w^* .*¹⁰

¹⁰ Note, however, that the adjustment to λ in footnote 3 will not coincide with the adjustment above. To keep trading

(v) **The cost per unit of tracking error λ .**

When parameters other than λ remain fixed, to a close approximation *turnover is proportional to the cube root of λ , and the no-trade interval is inversely proportional to the cube root of λ .*

(vi) **The tradeoff between turnover, no-trade interval, and tracking error.**

We alter λ and compute turnover and tracking error, keeping other parameters constant. Tracking error is measured by the annualized standard deviation of w . Table I shows this tradeoff, and also shows $(w^*_{max} - w^*_{min})$, the size of the no-trade interval around $w^* = 1.5$.

TABLE I

<i>Cost/unit Tracking Error</i> λ	<i>One-way Turnover</i>	<i>Std.Dev.[w]</i>	<i>No-Trade Interval</i>
5.0	21.81%	1.81%	.0627
2.5	17.30%	2.28%	.0790
1.0	12.73%	3.10%	.1072

and accuracy invariant to changes in w^* , λ must decrease with w^* more rapidly than the inverse proportional rule. If λ follows the inverse proportionality rule, then both turnover and accuracy will increase as w^* rises. Furthermore, constant (in)accuracy costs $(V - T)$ do not mean that the standard deviation either of w or of the fraction invested in stocks remains constant. It is readily shown that, to a close approximation, the standard deviation of w about w^* will be proportional to $[w^*/(1+w^*)]^2$.

0.5	10.09%	3.90%	.1351
0.35	8.95%	4.40%	.1522
0.25	7.99%	4.92%	.1703
0.10	5.86%	6.70%	.2314

It can be verified that, to a close approximation:

(i) *The expected turnover is inversely proportional to the width of the no-trade interval;*

(ii) *The standard deviation of tracking error is inversely proportional to
the expected turnover; ergo*

(iii) *The standard deviation of tracking error is proportional to the width of the
no-trade interval.*

VI. Conclusion

We have shown that the optimal strategy for rebalancing requires the investor to determine a "no trade" interval about the target asset mix, w^* . If the actual asset mix w lies within this interval, no trading is required. If the actual asset mix randomly moves outside the interval, rebalancing should take place--not to w^* , but to the nearest boundary of the no-trade interval $[w_{min}, w_{max}]$. We have derived explicit techniques for determining the optimal no-trade interval. It will depend upon the riskfree interest rate, the expected return differential between stocks and bonds, the relative

volatility of stocks to bonds, the cost per unit of tracking error, and the transactions costs of stocks and bonds. In many cases, the no-trade interval changes with the cube root of the parametric changes.

We have also determined the expected annual turnover associated with the optimal strategy. Turnover also moves with the cube root of various parametric changes. For typical parameters, turnover following an optimal policy with equivalent accuracy is approximately one-half the turnover of a quarterly rebalancing to w^* . Thus the gains to following the optimal strategy are substantial.

As expected, increases in the cost per unit of tracking error will tighten the no-trade interval, implying better accuracy of tracking w^* through time, but higher turnover. We have shown that the accuracy of tracking (as measured by the annual standard deviation of w about w^*) will be inversely proportional to annual turnover.

There are two important ways which this analysis can be generalized. Following the work of Dixit [1991], we can incorporate a fixed charge per transaction as well as a proportional trading cost. The optimal strategy will be defined in terms of two intervals, one inside the other. No trading will take place as long as asset proportions remain in the larger (outer) interval. Once asset proportions move beyond this outer interval, the asset proportion will be adjusted to the nearest boundary of the inside interval. A more difficult generalization will study the optimal trading strategy when there is a *temporary* shift in parameters (e.g. trading costs). Since now we will have a time-dependent optimization problem, numerical techniques will almost surely have to be used.

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