BELIEFS, COMPETITION, AND BANK RUNS[1]

by

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Abstract

Within the framework of Diamond-Dybvig (1983), the optimal (run free) outcome is shown to be the unique forward induction equilibrium. In a version of the model that posits Bertrand

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competition among banks, there are sequential equilibria that imply positive profits. However, the zero-profit contract is supported as the unique equilibrium outcome if the agents' beliefs are restricted to the space of beliefs consistent with the forward induction refinement.

I. Introduction

The paper by Diamond and Dybvig has been very influential both as a theoretical model of a banking system and as a framework for studying banking instability and the design of mechanisms to eliminate bank runs. On the other hand, there has been some discussion as to which of the issues in its title the paper addresses in a satisfactory way. Wallace (1988), for example, questions the deposit insurance policy suggested by Diamond and Dybvig (D-D) as a mechanism that eliminates bank runs and argues that this particular policy is not feasible. The ability of the model to deliver a bank run as an equilibrium outcome of a coordination game played by depositors has also been questioned. One criticism of the bank run outcome is that depositing in the bank while anticipating a run does not seem to be consistent with the behavior of rational individuals. In addition, it is widely recognized that the bank in this model is a central planning solution to the problem of optimal risk sharing. The questions of whether and how this solution can be decentralized have not been dealt with adequately in the literature. In this paper, we discuss this issue and explore the model assuming that agents can choose autarky in the planning period.

In the first section, we consider the model with a social planner bank. The bank offers the optimal contract without aggregate risk. The model has two symmetric sequential equilibria in pure strategies. The first one involves all agents depositing in the bank initially and withdrawing early only if they are impatient. The second one has all agents choosing autarky. There are parameterizations for which there exists a symmetric sequential equilibrium in mixed strategies that involves a bank run with positive probability. We show that only the equilibrium with all agents depositing in the bank initially and withdrawing only if they are impatient survives the forward induction refinement (Cho, 1987).

In the second section of the paper, we discuss some of the difficulties that may arise when we attempt to decentralize the "central planner" bank. We assume that there are two banks competing for deposits in the Bertrand fashion.[2] The game has two stages. In the first stage, before information about the agents' types is revealed, the two banks simultaneously propose contracts, after which each agent chooses a bank. In the second stage, after types are privately observed by depositors, those depositors that are impatient will always withdraw, but the actions of the patient ones will depend on their beliefs about the other agents' actions. An agent might believe, for example, that one of the two competing banks is more "stable" than the other, or that they are both "unstable." We restrict the agents so that each can have a contract with at most one bank. This game has three types of symmetric sequential equilibria in pure strategies. One involves both banks making zero profits and no bank run ever occurring in either bank. A second one involves one of the two banks making positive profits and all agents depositing in that bank.[3] The third equilibrium has all agents choosing autarky. There are parameterizations for which there exist two symmetric equilibria in mixed strategies that involve a bank run with positive probability in either bank. We show that the only sequential equilibrium, symmetric or non-symmetric, that survives the forward

[2] Recall that two competing firms are a sufficient number to imply that both make zero profits. An alternative would be Cournot competition, but it is not clear what competition in quantities would mean in this framework.

[3] The existence of this sequential equilibrium was suggested to us by Ed Green.

induction refinement (Cho, 1987) is the first sequential equilibrium in pure strategies. In this sense, the zero-profit contract is decentralized as the unique equilibrium outcome if the agents' beliefs are restricted to the space of forward induction beliefs. Finally, like in Postlewaite and Vives (1987), we demonstrate that the bank run equilibrium is a correlated equilibrium if agents play correlated strategies in the second stage of the game, after they have deposited in either bank.

II. The Economic Environment

The environment is the D-D environment without aggregate risk.

Time

There are three periods: beginning (0), middle (1), and end (2).

Population and Endowments

There are n agents in the economy, where n is large. Each agent is endowed with one unit of the period 0 good. In period 0 each agent faces a preference shock that determines whether she is impatient or patient. Impatient agents care only about period 1 consumption. Patient agents are indifferent between period 1 and period 2 consumption. We assume that the fraction of patient agents in the economy is α, a constant, and that of impatient agents is $1 - \alpha$. Later we will provide all the details for an expository example where $n = 3$, and $\alpha = \frac{2}{3}$.

Technology

A technology is available in period 0 that can transform the period 0 good to period 1 and 2 goods. The technology set is characterized by a triple (y_0, y_1, y_2) such that: $y_1 \leq -y_0$, and $y_2 \leq (-y_0 - y_1)R$, where $y_0 \leq 0$, $y_1 \geq 0$, $y_2 \geq 0$, and $R > 1$. The technology is, therefore, riskless but illiquid. It provides low levels of output per unit of input if operated for a single period but high levels of output if operated for two periods. No investment in the illiquid technology can start after period 0.

Preferences

Every agent has the same preferences in period 0. In particular, each agent has a state-dependent utility function (with the state private information), which we assume has the form: $U(C) = \begin{cases} u(c_1), & if \quad impatient; \\ u(c_1 + c_2), & if \quad patient; \end{cases}$ where: $u : \Re \rightarrow \Re$ is C^2 in $\Re_{++}$, and $-\frac{cu''(c)}{u'(c)} > 1$.

The Optimal Contract

The optimal insurance contract allows agents to insure against the outcome of being impatient. It is the solution to the following problem:

$$\max(1 - \alpha)u^1(c_1) + \alpha u^2(c_2)$$

$$\text{s.t. } (1-\alpha)c_1 + \alpha\frac{c_2}{R} = 1. \tag{1}$$

As in the D-D model, our assumptions on the preferences guarantee that the optimal contract satisfies the condition that $1 < c_1^* < c_2^* < R$.

III. The Game with a Social Planner Bank

Here we assume that there is a bank in the economy offering the optimal deposit contract in period 0. We assume that the fractions of patient and impatient agents in the bank are the same as the ones in the population.[4] The optimal deposit contract consists of a "short-term" and a "long-term" interest rate: $r_1^* = c_1^*$ and $r_2^* = c_2^*$. For simplicity we assume that if the demand for withdrawals exceeds the bank's resources during period 1, the bank divides its resources among the agents waiting in line[5] and we associate any such outcome with a bank run.

The strategies of the agents in the game are as follows. In the first stage (Figure 1), agents simultaneously choose whether to deposit in the bank (b) or to remain in autarky (A). In the second stage (Figure 2), each agent simultaneously chooses whether to withdraw

[4] The autarky allocation prevails otherwise. This assumption allows for tractability in characterizing the optimal contract and our results do not depend on it. Notice that in the example where $n = 3$ this implies that all agents will choose the same bank or all will choose autarky.

[5] This violates the *sequential service constraint* (Wallace, 1988), which requires that payments to depositors depend only on information about previous withdrawals. However, this is only a simplifying assumption, and our results do not depend on it.

($\bar{w}$) or wait (w), conditional on the event that she is patient. We assume that the agents are expected utility maximizers in all stages of the game and that agents do not randomize during the first stage of the game. The payoffs to agent i given that $n^{'}$ agents deposit in the bank and that k out of $\alpha n^{'}$ patient agents in the bank will choose action $\bar{w}$ are given by[6]:

$$P^i(A^i) = (1-\alpha)u(1) + \alpha u(R) \tag{2}$$

$$P^i(\bar{w}^i, \bar{w}^{-i}) = P^i(\bar{w}^i; k = n^{'}) = u(1) \tag{3}$$

$$P^i(w^i, w^{-i}) = P^i(w^i; k = 0) = (1-\alpha)u(r_1^*) + \alpha u(r_2^*) \tag{4}$$

$$P^i(w^i; 0 < k < n^{'}) = u\left(\max\left\{\frac{[n^{'} - (1-\alpha)n^{'}r_1^* - kr_1^*]R}{\alpha n^{'} - k}, 0\right\}\right) \tag{5}$$

$$P^i(\bar{w}^i; 0 < k < n^{'}) = u\left(\min\left\{r_1^*, \frac{[n^{'} - (1-\alpha)n^{'}r_1^* - kr_1^*]R}{(1-\alpha)n^{'} + k}\right\}\right) \tag{6}$$

[6] Here we use k, the number of $\bar{w}-$players, as the state variable and, for convenience, we assume that agents do not exclude themselves from this calculation. This is consistent with a large population of depositors.

As an example, below we provide a complete description of the expected payoffs of agent 3 for the case where $n = 3$ and $\alpha = \frac{2}{3}$. Since the game is symmetric, the expected payoffs of agents 1 and 2 are similar. Each agent in this example is patient with probability $\frac{2}{3}$. We then have (see Figure 2):

$$P_A^3 = \frac{2}{3}u(R) + \frac{1}{3}u(1) \tag{7}$$

$$P_1^3 = \frac{1}{3}u(r_1^*) + \frac{2}{3}u(r_2^*) \tag{8}$$

$$P_2^3 = \frac{1}{3}u(r_1^*) + \frac{2}{3}u\left(\min\left\{1.5, r_1^*\right\}\right) \tag{9}$$

$$P_3^3 = \frac{1}{3}u\left(\min\left\{1.5, r_1^*\right\}\right) + \frac{1}{3}u(r_2^*) + \frac{1}{3}u\left(\max\left\{0, (3-2r_1^*)R\right\}\right) \tag{10}$$

$$P_4^3 = \frac{1}{3}u\left(\min\left\{1.5, r_1^*\right\}\right) + \frac{1}{3}u\left(\min\left\{1.5, r_1^*\right\}\right) + \frac{1}{3}u(1) \tag{11}$$

$$P_5^3 = \frac{1}{3}u\left(\min\left\{1.5, r_1^*\right\}\right) + \frac{1}{3}u(r_2^*) + \frac{1}{3}u\left(\max\left\{0, (3-2r_1^*)R\right\}\right) \tag{12}$$

$$P_6^3 = \frac{2}{3}u\left(\min\left\{1.5, r_1^*\right\}\right) + \frac{1}{3}u(1) \tag{13}$$

$$P_7^3 = \frac{1}{3}u(1) + \frac{2}{3}u\left(\max\left\{0, (3-2r_1^*)R\right\}\right) \tag{14}$$

$$P_8^3 = u(1). \tag{15}$$

The first claim describes the pure strategy symmetric Nash equilibria of this game.

Claim 1: *The reduced normal form of the game has two symmetric Nash equilibria in pure strategies. The first one has all agents choosing depositing in the bank and waiting. The second one has all agents choosing autarky. More precisely, the two Nash equilibria are:*

$$(b^i, w^i), (A^i),\ i = 1, ..., n.$$

In the case where $n = 3$, these are the only sequential equilibria.

Observe that there are no pure strategy equilibria involving a bank run since in that case autarky offers a higher expected payoff. Next, Claim 2 describes the pure strategy sequential equilibria of the game.

Claim 2: *The game has two types of symmetric sequential equilibria in pure strategies.*

(a) The first one has each agent i, i=1,...,n, taking action $(b,w)^i$*, and having beliefs* μ^i *such that* $E^{\mu,(b,w)}(u^{i(h)}/h) \geq E^{\mu,\pi'}(u^{i(h)}/h)$*, for any strategy* π'^i *for agent i, where* $\pi'^{-i} = (b,w)^{-i}$ *and* μ^i *is consistent with* $(b,w)^{-i}$.

(b) The second one has each agent i, i=1,...,n, taking action A^i *and having beliefs* μ^i *such that* $E^{\mu,(A,\bar{w})}(u^{i(h)}/h) \geq E^{\mu,\pi'}(u^{i(h)}/h)$*, for any strategy* π'^i *for agent i, i=1,...,n, where* $\pi'^{-i} = (A,\bar{w})^{-i}$ *and* μ^i *is consistent with* $(A,\bar{w})^{-i}$.

In the case where n=3, these are the only sequential equilibria.

We now look at the sequential equilibria when agents play mixed strategies in the second stage. The following claim asserts that there may exist a mixed strategy symmetric sequential equilibrium that involves a bank run with positive probability.

Claim 3: *There may exist a symmetric sequential equilibrium in mixed strategies. It involves every agent having* (π,μ)*, with* $\pi = (b,\gamma)$*, where* $\gamma \in \Delta\{w,\bar{w}\}$ *such that:* $E^{\mu,(b,\gamma)}(u^{i(h)}/h) \geq E^{\mu,\pi'}(u^{i(h)}/h)$*, for any strategy* π'^i *for agent i, i=1,...,n, where* $\pi'^{-i} = (b,\gamma)^{-i}$ *and* μ^i *is consistent with* π.

The next claim shows that the game has a unique equilibrium in pure strategies if we restrict the space of beliefs to those consistent with forward induction. This equilibrium is the one that has all agents depositing in the bank and withdrawing early only if impatient.

Claim 4: *The only pure strategy forward induction sequential equilibrium is the first equilibrium in pure strategies.*

Finally, we relate the outcomes of this game to correlated equilibrium, a generalization of the Nash equilibrium solution concept due to Aumann (1974). As an extreme case, this allows for all agents to base their choices on the observation of the realization of the same random variable. The following claim asserts that a bank run is a correlated equilibrium. The proof is given in the appendix.

Claim 5: *For the above game,* $(b^i \bar{w}^i)$, *i=1,...,n, is a correlated equilibrium.*

IV. The Game with Decentralized Banks

Here we assume that there are two banks in the economy offering deposit contracts in period 0. The two banks are identical, but agents may have asymmetric beliefs about other agents' behavior in the two banks. We again require that the fractions of patient and impatient agents in each bank are the same as the ones in the population. The deposit contract of bank j consists of the "short-term" and the "long-term" interest rates: $r^j = (r_1^j, r_2^j) \in F$, where $F \subset \Re_+^2$ is the set of feasible contracts, i.e.,

$$F = \left\{ r^j \in \Re_+^2 : (1-\alpha) r_1^j + \frac{\alpha r_2^j}{R} \leq 1 \right\}, \; j = 1, 2. \tag{16}$$

The payoffs of the two banks are the expected profits. We want to demonstrate that the typical argument leading to zero profits might break down in this example. To this end, we will assume that bank 1 offers the optimal contract (r_1^*, r_2^*) and, therefore, makes zero profits while bank 2 offers an inferior contract $(r_1^{'}, r_2^{'})$ and makes positive profits. Here we restrict our attention to the strategic behavior of the consumers P^i, i=1,...,n. Therefore, ρ, the initial assessment, coincides with information set h_1 in our game (see Figure 1).

In the first stage after the banks propose a contract, agents choose simultaneously which bank to deposit in: bank 1 (b_1), bank 2 (b_2), or autarky (A). In the second stage they simultaneously choose whether to withdraw ($\bar{w}$) or wait (w), conditional on being patient. We assume that the agents are expected utility maximizers in all stages of the game and that they do not randomize during the first stage, when they choose their bank. The resulting game tree is presented in Figure 1 for the case where $n = 3$ and $\alpha = \frac{2}{3}$.

In the previous paragraph we gave a complete description of the expected payoffs of agent 3 in bank 1 for the example case with $n = 3$. Recall that each of the three agents in this example is patient with probability $\frac{2}{3}$. Expected payoffs $P_9^3, ..., P_{16}^3$ are defined the same way with $(r_1^{'}, r_2^{'})$ instead of (r_1^*, r_2^*). The payoffs for the $n-$agent case are defined similarly. Given this payoff structure, we have the following:

Claim 6: *The reduced normal form of the game has three symmetric Nash equilibria in pure strategies. The first one has all agents choosing bank 1 and waiting. The second has all agents choosing bank 2 and waiting. The third one has all agents choosing autarky. More precisely, the three Nash equilibria are:*

$$(b_1^i, w_1^i),\ (b_2^i, w_2^i),\ (A^i),\ i = 1, ...n.$$

In the case where $n = 3$, *these are the only Nash equilibria.*

Recall that only bank 1 offers the optimal contract while bank 2 offers an inferior one. Therefore, Claim 6 states that there exists a pure strategy Nash equilibrium where one of the two banks makes positive profits.[7] Next, Claim 7 describes the pure strategy sequential equilibria of the game.

Claim 7: *The game has three types of sequential equilibria in pure strategies.*

(a) The first one has each agent i, i=1,...,n, taking action $(b_1, w_1, \cdot)^i$, *and having beliefs* μ^i *such that* $E^{\mu,(b_1,w_1,\cdot)}(u^{i(h)}/h) \geq E^{\mu,\pi'}(u^{i(h)}/h)$, *for any strategy* π'^i *for agent i, where* $\pi'^{-i} = (b_1, w_1, \cdot)^{-i}$, $\cdot$ *can be any action, and* μ^i *is consistent with* $(b_1, w_1, \cdot)^{-i}$.

(b) The second one has each agent i, i=1,...,n, taking action $(b_2, \bar{w}_1, w_2)$ *and having beliefs* μ^i*such that* $E^{\mu,(b_2,\bar{w}_1,w_2)}(u^{i(h)}/h) \geq E^{\mu,\pi'}(u^{i(h)}/h)$, *for any strategy* π'^i *for agent i, where* $\pi'^{-i} = (b_2, \bar{w}_1, w_2)^{-i}$, $\cdot$ *can be any action, and* μ^i *is consistent with* $(b_2, \bar{w}_1, w_2)^{-i}$.

[7] These profits cannot be arbitrarily high since in that case this equilibrium will be dominated by autarky.

(c) The third one has each agent i, i=1,...,n, taking action A^i *and having beliefs* μ^i *such that* $E^{\mu,(A,\bar{w}_1,\bar{w}_2)}(u^{i(h)}/h) \geq E^{\mu,\pi'}(u^{i(h)}/h)$*, for any strategy* π'^i *for agent i, where* $\pi'^{-i} = (A,\bar{w}_1,\bar{w}_2)^{-i}$*,* $\cdot$ *can be any action, and* μ^i *is consistent with* $(A,\bar{w}_1,\bar{w}_2)^{-i}$*.*

In the case where n=3, these are the only sequential equilibria.

The next claim asserts that there may exist at most two mixed strategy symmetric sequential equilibria that involve a bank run with positive probability in either bank.

Claim 8: *There can be two types of symmetric sequential equilibria in mixed strategies.*

(a) The first one has each agent i, i=1,...,n, having, (π,μ), with $\pi = (b_1,\gamma)$*, where* $\gamma \in \Delta\{w,\bar{w}\}$ *such that:* $E^{\mu,(b_1,\gamma)}(u^{i(h)}/h) \geq E^{\mu,\pi'}(u^{i(h)}/h)$*, for any strategy* π'^i *for agent i, where* $\pi'^{-i} = (b_1,\gamma)^{-i}$*,* $\cdot$ *can be any action, and* μ^i is consistent with π.

(b) The second one has each agent i, i=1,...,n, having, (π,μ), with $\pi = (b_2,\gamma)$*, where* $\gamma \in \Delta\{w,\bar{w}\}$ *such that:* $E^{\mu,(b_2,\gamma)}(u^{i(h)}/h) \geq E^{\mu,\pi'}(u^{i(h)}/h)$*, for any strategy* π'^i *for agent i, where* $\pi'^{-i} = (b_2,\gamma)^{-i}$*,* $\cdot$ *can be any action, and* μ^i is consistent with π.

The following claim shows that the game has a unique equilibrium if we restrict the space of beliefs to those that are consistent with forward induction. The unique equilibrium implies zero profits for both banks and it does not involve a bank run in either bank.

Claim 9: *The only forward induction sequential equilibrium of the game is the first equilibrium in pure strategies.*

Finally, the next claim asserts that a bank run in either of the competing banks is a correlated equilibrium. The proof is given in the appendix.

Claim 12: *Both* $(b_1, \bar{w}_1)^i$ *and* $(b_2, \bar{w}_2)^i$, *i=1,...,n are correlated equilibria of this game.*

V. Conclusions and Comments

We presented some difficulties that may arise when we attempt to decentralize the central-planner bank in the D-D model, i.e., when we try to support the optimal contract as the outcome of Bertrand competition between profit maximizing banks. We also studied how the model is affected if it is assumed that agents can choose autarky rather than being assumed to deposit in a bank. We showed that the model so amended has no forward induction sequential equilibria involving either bank runs or an autarky choice by the depositors. Finally, in a model that posits both competing banks and consumers who have the option to remain in autarky, we demonstrated how the zero-profit optimal outcome can be supported as the unique outcome of the Bertrand game between competing banks if we restrict the agents' beliefs to the space of beliefs consistent with the forward induction refinement. The model, therefore, provides no grounds for imputing either natural monopoly or instability to banking markets. For this discussion we restricted our attention to symmetric equilibria. Claims 4 and 9, however, state that all other equilibria, symmetric or not, are not forward induction equilibria.

VI. References

Aumann R. (1974): Subjectivity and the Correlation in Randomized Strategies. *Journal of Mathematical Economics* **1**, 67-96.

Aumann R. (1987): Correlated Equilibrium as an Expression of Bayesian Rationality. *Econometrica* **55**, 1-18.

Cho In-Koo (1987): A Refinement of Sequential Equilibrium. *Econometrica* **55**, 1367-1389.

Cho In-Koo, and Kreps D. (1987): Signaling Games and Stable Equilibria. *Quarterly Journal of Economics* **CII**, 179-221.

Diamond D., and Dybvig P. (1983): Bank Runs, Deposit Insurance, and Liquidity. *Journal of Political Economy* **91**, 401-419.

Green E.J. (1995): Implementing Efficient Allocations in a Model of Financial Intermediation. Federal Reserve Bank of Minneapolis.

Kreps D., and Wilson R. (1992): Sequential Equilibrium. *Econometrica* **50**, 863-894.

Morris S., and Shin H.S. (1995): Informational Events That Trigger Currency Attacks. Federal Reserve Bank of Philadelphia Working Paper **95-24**.

Postlewaite A., and Vives X. (1987): Bank Runs as an Equilibrium Phenomenon. *Journal of Political Economy* **95**, 485-491.

Wallace N. (1988): Another Attempt to Explain an Illiquid Banking System : The Diamond and Dybvig Model With Sequential Service Taken Seriously. *Federal Reserve Bank of Minneapolis Quarterly Review* **12**, 3-16.

VII. Appendix. Proofs

Proof of Claim 1: It is easy to see that these are the only symmetric equilibria, since we have that for all i, $P^i(\bar{w}^i, \bar{w}^{-i}) < P^i(A^i, A^{-i}) < P^i(w^i, w^{-i})$. For the case where $n = 3$, to see that there are no other equilibria, symmetric or asymmetric, suppose that there exists an equilibrium of the form $(b, w^1, \bar{w}^2)$. The best response for agent 3 is either (b, w) or $(b, \bar{w})$. In the first case the best response of agent 2 is (b, w). In the second case the best response for agent 1 is autarky. So when $n = 3$ no asymmetric equilibrium exists. ■

Proof of Claim 2: We give a proof of part *(a)* only. The proof of part *(b)* is similar. Clearly, the candidate assessment gives higher expected utility for all $\pi^{'}$ described in part *(a)*. It only remains to show that this is the only pure strategy sequential equilibrium in which agents deposit in the bank. This follows automatically for symmetric equilibria. Next we consider the case where $n = 3$ and look for asymmetric equilibria. Suppose that there exists a sequential equilibrium involving actions of one of the following three types: $(b, \bar{w}^1, \bar{w}^2, \bar{w}^3), (b, w^1, w^2, \bar{w}^3), (b, w^1, \bar{w}^2, \bar{w}^3)$. Any outcome of the first type is dominated by autarky so it cannot be an equilibrium. Similarly, any outcome of the second type is ruled out since it does not involve a best response by agent 3 and, finally, the outcome of the third type involves a non-best response by player 1. Therefore, the equilibrium in *(a)* is the only sequential equilibrium where agents deposit in the bank. This is also the Pareto optimal equilibrium. ■

Proof of Claims 3 and 9: Since here we only need to show existence, we consider only the example case where $n = 3$. The proof is the same for both parts of Claim 9. Here we present a proof of the first part only, concerning bank 1, which can be thought of as the social planner bank of Claim 3. For convenience we drop the subscript 1 in what follows (see Figure 2). We have:

$$\mu(a) = \frac{\gamma^1(w)\gamma^2(w)}{\gamma^1(w)\gamma^2(w) + \gamma^1(w)\gamma^2(\bar{w}) + \gamma^1(\bar{w})\gamma^2(\bar{w})} \tag{17}$$

$$\mu(\alpha) = \gamma^1(w) \tag{18}$$

$$\gamma^3 = \begin{cases} \delta_w, & if \quad P_1^3\mu(a) + P_3^3\mu(b) + P_5^3\mu(c) + P_7^3\mu(d) \\ & \quad > P_2^3\mu(a) + P_4^3\mu(b) + P_6^3\mu(c) + P_8^3\mu(d); \\ \in \Delta(w, \bar{w}), & if \quad P_1^3\mu(a) + P_3^3\mu(b) + P_5^3\mu(c) + P_7^3\mu(d) \\ & \quad = P_2^3\mu(a) + P_4^3\mu(b) + P_6^3\mu(c) + P_8^3\mu(d); \\ \delta_{\bar{w}}, & if \quad P_1^3\mu(a) + P_3^3\mu(b) + P_5^3\mu(c) + P_7^3\mu(d) \\ & \quad < P_2^3\mu(a) + P_4^3\mu(b) + P_6^3\mu(c) + P_8^3\mu(d). \end{cases} \tag{19}$$

For player 3 to mix, we need:

$$P_1^3\mu(a) + P_3^3\mu(b) + P_5^3\mu(c) + P_7^3\mu(d) = P_2^3\mu(a) + P_4^3\mu(b) + P_6^3\mu(c) + P_8^3\mu(d) \tag{20}$$

Assume $\gamma^*(w) = \gamma^1(w) = \gamma^2(w) = \gamma^3(w)$ is the solution to the above equation and the beliefs are given by:

$$\mu(\alpha) = \gamma^*(w) \tag{21}$$

$$\mu(a) = (\gamma^*)^2(w) \tag{22}$$

$$\mu(b) = \mu(c) = \gamma^*(w)\gamma^*(\bar{w}) \tag{23}$$

$$\mu(d) = (\gamma^*)^2(\bar{w}). \tag{24}$$

Then the above equation can be written as:

$$(P_1^3 - P_2^3)(\gamma^*)^2(w) + 2(P_3^3 - P_4^3)\gamma^*(w)\gamma^*(\bar{w}) + (P_7^3 - P_8^3)(\gamma^*)^2(\bar{w}) = 0. \qquad (25)$$

The mixed strategy sequential equilibrium in bank 1 comes from the solution of the following system:

$$\left\{(26); 0 \leq \gamma^*(w) \leq 1; \gamma^*(w) + \gamma^*(\bar{w}) = 1; E^{\mu,(b_1,\gamma^*)}u^i \geq E^{\mu,A}u^i\right\} \qquad (26)$$

.

This type of equilibrium implies a bank run with positive probability. For an example of this equilibrium in bank 1, consider $u(c) = -e^{-\alpha c}$, with $\alpha = 2.25$, and $R = e$. Then $(r_1^*, r_2^*) = (1.5, 9.44)$ and equation (26) reduces to:

$$.014419984\gamma^2 - .581980532\gamma(1-\gamma) - .596400516(1-\gamma)^2 = 0 \qquad (27)$$

which has as a solution $\gamma(w) = .976392433$. The expected utility associated with this equilibrium is $-.035351616$ so it dominates autarky. ■

Proof of Claim 4: We will demonstrate that the symmetric autarky equilibrium is not a forward induction equilibrium. The proof for the other sequential equilibria is similar. The proof relies on the fact that action $\bar{w}^i$ is a *bad deviation* from the equilibrium strategy. This is true since if any agent i chooses action b^i followed by $\bar{w}^i$ he will be, given that the bank operates, strictly worse off for all best responses by other agents at the information sets that can be reached through action $\bar{w}^i$. On the other hand, at the autarky equilibrium agents assign positive probability to an action $\bar{w}^i$ following action b^i by agent i. For the mixed equilibrium, the same argument goes through if we relax the definition of a *bad deviation* by allowing for such an action to be played with positive probability in the equilibrium strategy and by requiring that the agent who deviates is weakly worse off. ■

Proof of Claims 5 and 10: We will prove only that in the case of two competing banks $(b_1, \bar{w}_1)^i$, i=*1,...,n,* is a correlated equilibrium, which is also a proof for the social planner bank case. The proof for $(b_2, \bar{w}_2)^i$ is similar. Suppose that in period 1, after the agents learn their types, the realization of a single toss of a two-sided coin is observed simultaneously by all agents. Suppose that the outcome of this toss is H with probability p and T with probability $p^{'}$. Then the symmetric bank run outcome is achieved if all agents play w_1 or $\bar{w}_1$ respectively according to whether heads or tails came up. On the other hand depositing in bank 1 in period 0 will dominate autarky or depositing in bank 2 if the probability of T is low enough, i.e., if for all agents i we have:

$$p^{'}P^i(b_1^i, b_1^{-i}; T) + pP^i(b_1^i, b_1^{-i}; H) \geq P^i(A^i), \tag{28}$$

$$p^{'}P^i(b_1^i, b_1^{-i}; T) + pP^i(b_1^i, b_1^{-i}; H) \geq P^i(b_2^i, b_2^{-i}; H). \tag{29}$$

By continuity, both of the above inequalities will hold for small enough $p^{'}$. ■

Proof of Claim 6: It is easy to see that these are the only symmetric equilibria, since we have that for all i and j, $P^i(\bar{w}_j^i, \bar{w}_j^{-i}) < P^i(A^i, A^{-i}) < P^i(w_j^i, w_j^{-i})$. For the case where $n = 3$, to see that there are no other equilibria, symmetric or asymmetric, suppose that in either bank j there exists an equilibrium of the form $(b_j, w_j^1, \bar{w}_j^2, \cdot)$. The best response for agent 3 is either (b_j, w_j) or $(b_j, \bar{w}_j)$. In the first case the best response of agent 2 is (b_j, w_j). In the second case the best response for agent 1 is autarky. So when $n = 3$ no asymmetric equilibrium exists. ■

Proof of Claim 7: We give a proof of part *(a)* only. The proof of the other parts is similar. Clearly, the candidate assessment gives higher expected utility for all $\pi^{'}$ described in part *(a)*. It only remains to show that this is the only pure sequential equilibrium in which agents deposit in bank 1. This follows automatically for symmetric equilibria. For the case where $n = 3$

and for asymmetric equilibria, suppose that there exists a sequential equilibrium involving actions of one of the following three types: $(b_1, \bar{w}_1^1, \bar{w}_1^2, \bar{w}_1^3), (b_1, w_1^1, w_1^2, \bar{w}_1^3), (b_1, w_1^1, \bar{w}_1^2, \bar{w}_1^3)$. Any outcome of the first type is dominated by autarky so it cannot be an equilibrium. Similarly, any outcome of the second type is ruled out since it does not involve a best response by player 3 and, finally, the outcome of the third type involves a non-best response by player 1. Therefore, the equilibrium in *(a)* is the only sequential equilibrium where agents deposit in bank 1. ■

Proof of Claim 9: We will demonstrate that the symmetric positive profit equilibrium is not a forward induction equilibrium. The proof for the other sequential equilibria is similar. Once again action $\bar{w}_1^i$ is a *bad deviation* from the equilibrium strategy. This is true since if any agent i chooses action b_1^i followed by $\bar{w}_1^i$ he will be, given that bank 1 operates, strictly worse off for all best responses by other agents at the information sets that can be reached through action $\bar{w}_1^i$. The symmetric equilibrium where bank 2 makes positive profits has all agents assigning positive probability to an action $\bar{w}_1^i$ following action b_1^i by agent i. For the mixed equilibria we again need to relax the definition of a *bad deviation* by allowing for such an action to be played with positive probability and by requiring that the agent who deviates is weakly worse off. ■