Chaos and Nonlinear Forecastability in Economics and Finance *

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Abstract

Both academic and applied researchers studying financial markets and other economic series have become interested in the topic of chaotic dynamics. The possibility of chaos in financial markets opens important questions for both economic theorists as well as financial market participants. This paper will clarify the empirical evidence for chaos in financial markets and macroeconomic series. It will also compare these two concepts from a financial market perspective contrasting the objectives of the practitioner with those of economic researchers. Finally, the paper will speculate on the impact of chaos and nonlinear modeling on future economic research.

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1 Introduction

It has now been almost ten years since economists began searching for chaotic dynamics in economic time series. This search has yielded deeper understandings of the dynamics of many different series, and has led to the development of several useful tests for nonlinear structure. However, the direct evidence for deterministic chaos in many economic series remains weak. This paper will survey the existing results and give some intuition about why they were probably not unexpected. It will also argue that chaotic dynamics still needs to be taken seriously for economic systems, but the tools and methods of study will probably differ from those used in the past.

The possibilities of chaos in economic systems brought an enormous amount of initial interest. The concepts of limited forecastability and complex dynamical properties has very strong intuitive appeal for economics. From forecasting movements in foreign exchange and stock markets, to understanding international business cycles, chaos in economics had a broad range of potential applications. This led to an explosion of empirical work searching for possible chaos in all types of economic and financial time series. These studies have found little or no evidence for chaos in any economic time series, but they have turned up a surprising amount of unexplained nonlinear structure in many series. In hindsight most of these results should have been expected to some extent. Researchers looking at macroeconomic and financial series face certain constraints which make the likelihood of directly seeing chaos small. In macroeconomics the problem of short and noisy time series coming from a system whose dynamics and measurement probes may be changing over time impedes the ability to precisely estimate nonlinear processes. In financial markets traders' ability to perceive complex patterns and trade against them reduces the strength of these patterns yielding series which, although not completely random, are probably some of the most difficult to forecast of all real world time series.

This paper will argue that even though the case for economic chaos appears weak the issue is still a very open question for economic research. Many economic questions are concerned with what fraction of observed fluctuations are coming from exogenous shocks versus underlying structures in the system. In macroeconomics this would be related to the amount of business cycle fluctuations attributable to structures that are part of the dynamics of the system versus outside shocks. Similarly, in financial markets we are concerned with the amount of price movements and trading activity coming from the flow of new information into the system, versus the system generating this through a dynamic of trading and learning. Analysis along these lines will require a greater attention to economic theory along with new empirical tests which put a greater emphasis on how noise is processed in an economic environment.

These issues will be covered in detail in the next three sections. First, the results from macroeconomics will be discussed. Second, financial forecasting will be reviewed. The next to last section presents a brief example of some new directions for empirical work. The final section will discuss some of the most interesting directions for future work in economics.

2 Macroeconomics

The most likely candidates for nonlinear dynamic structures in economics were macroeconomic time series. These series which exhibit a large amount of structure through business cycle fluctuations seemed like a natural place to look for unseen determinism.

The first tests for chaotic dynamics in macro economic series used several different diagnostic tests. Many began the search with an application of the Grassberger-Procacia(1982)[30] dimension estimation algorithm.[1] Most came to the same conclusion. The series were probably not deterministic chaos, but many showed evidence for interesting nonlinear structure. [2]

Initially, the diagnostics estimated were traditional invariants such as information dimension, or lyapunov exponents. Eventually, some of the studies included a test statistic for independence, the BDS test (Brock et al.(1988) [9]). This statistic tests the independence of a time series using the fact that for any independent series, x_t,

$$P(d^m(x_t, x_s) < \epsilon) = P(d^1(x_t, x_s) < \epsilon)^m,$$

$$d^m(x_t, x_s) = \sup_{j=0,m-1} |x_{t+j} - x_{s+j}|,$$

where $P()$ is the probability of each event, and sup is the supremum norm or maximum. This tests the much more restrictive null hypothesis that the series is independent and identically distributed. It is not a test for chaos. It is useful because it is a well defined, and easy to apply test which has power against any type of structure in a series. This feature can be viewed as both a cost and a benefit. On the one hand it can detect many types of nonlinear dependence that might be missed by other tests. On the other hand, a rejection using this test is not very informative. One extension of this test is to use it as a residual diagnostic. The idea is that a certain linear or nonlinear model specification can be tested by looking at

$$\hat{e}_t = x_t - f(x_{t-1}, \ldots, x_{t-l}),$$

where $f()$ is some model specification. The diagnostic test for independence is then applied to $\hat{e}_t$, the estimated model residuals. There is a well defined asymptotic distribution theory for the test which is worked out in [9], and it can also be applied to the residual test in some specifications.[3]

There are many other diagnostic tests for nonlinearity available. Several authors mentioned used the test developed by Tsay(1986) [65]. Also, the bispectral test developed by Hinich(1982) [33] is applied to several macro economic series in Ashley and Patterson(1989) [1] and Barnett and Hinich(1993) [4]. One advantage of this diagnostic is that it has no power against linear stochastic processes. This means that there is no need for estimating residuals before testing. The careful researcher should probably be ready with a toolbox of different tests, knowing a little bit about what the properties of each one is before applying them.

Most of these series suffer from a major limitation of economic data, short, noisy, and possibly nonstationary time series. The number of points available are probably small by any of the data

[1] See Grassberger et al.(1991)[31] for a useful survey of good and bad uses of this algorithm, and Jaditz and Sayers(1993) [36] for a good survey of some of these results from macroeconomics. Papers such as Barnett and Chen(1986) [3], Brock and Sayers(1988) [13], Frank and Stengos(1989) [25], Frank, Gencay, and Stengos(1988) [26], Sayers(1989) [54], and Scheinkman and LeBaron(1989)[55], all are applications to various macroeconomic time series.

[2] There were a few disputes in this literature in that one set of authors did make strong claims about finding chaos in a monetary index(Barnett and Chen(1986)[3]). However, their findings were disputed in Ramsey et al.(1990)[51] which concluded that their claims about chaos were premature. The recent results in Barnett and Hinich(1993)[4] correct some of the criticisms. The claims for chaos are weakened, but it looks like there is some kind of unexplained nonlinearity in these monetary series.

[3] The approach of using residual diagnostics in nonlinearity testing which was first proposed in Brock(1986)[8] has come under some question recently. Theiler and Eubank(1992)[62] find that in some cases analyzing a time series after linear residuals are taken may reduce power against some chaotic null hypothesis.

point requirements that have been previously suggested.[4] Most of these studies check the reliability of their results given the small amount of data available by comparing estimates with distributions from simulated stochastic processes. This methodology draws much of its inspiration from bootstrap techniques of Efron(1979)[22].[5] In general this approach has helped keep researchers honest about what they can confidently claim to have found, and it has probably kept down the number of false positive tests of chaos.

A second related branch of research has attempted to directly fit nonlinear specifications to macroeconomic data. This approach has generally been more successful at finding strong evidence for nonlinearities in these series. Papers such as Beaudry and Koop(1993)[5], Hamilton(1989)[32], McQueen and Thorley(1993) [45], Neftci(1984) [47], Potter(1990) [50], and Terasvirta and Anderson(1992) [61] find evidence for nonlinear behavior in aggregate macro time series. They are generally supportive of the conjecture that business cycles behave differently during expansions and contractions, and that there are differences in the impact on future growth from positive and negative shocks today.

Two recent papers directly use a nonlinear forecasting framework to evaluate nonlinearities in macro economic time series. Granger et al.(1993)[29] and Jaditz and Sayers(1993)[37]. Both papers find that a nonlinear forecasting framework does not add much in terms of out of sample forecast performance. This may suggest some problems in terms of stability/stationarity of the previously documented results.

In macroeconomic series the greatest gains have come from fitting nonlinear models which address certain well known features which are not part of a linear framework. This approach often has detected interesting nonlinearities which were missed using diagnostic testing alone.

3 Finance

Chaos and its implications for forecasting has been even more hotly debated in financial markets. Financial series provide potentially longer and cleaner series on which to do estimation and out of sample testing. Also, the obvious potential gains to forecasting stock price and foreign exchange rate series has drawn a lot of interest.[6] Tests similar to those used for macroeconomic time series have been applied here as well.[7] On the issue of chaos the results often find strong evidence for nonlinear dependence, but no convincing evidence for chaotic dynamics.[8]

The issue of whether a financial series is indeed chaotic may not be of great importance to a financial forecaster who is only interested in adjusting dynamic trading strategies according to apparent predictability in time series. The fact that the previously mentioned diagnostics all found evidence for some kind of nonlinear structure should be a tantalizing indicator for financial

[4] See Smith(1992)[58] and Ruelle(1990)[53]. Most macro series range from about 100-800 data points depending on length and frequency.

[5] Similar techniques have been used in Theiler et al.(1992)[63]. See also Efron and Tibshirani(1993)[23] for a more recent treatment of bootstrapping.

[6] One example of this interest is the recent survey in the Economist(1993)[52].

[7] Blank(1992) [6], Brock et al.(1991) [10], De Grauwe et al.(1993) [16], Decoster et al.(1992) [19], Frank and Stengos(1988) [27], Hinich and Patterson(1985) [34], Hsieh(1991) [35], Mayfield and Mizrach(1992) [44], Peters(1991) [49], and Scheinkman and LeBaron(1989) [56].

[8] One exception here is Peters(1991)[49] who finds strong evidence for low dimensional ($d < 3$) chaos. His results are are from low frequency monthly data sets, and are not backed up by any convincing simulations. Without further evidence from simulations these results should be viewed with skepticism.

forecasters. Just what kinds of structure are in financial time series that these tests are picking up?

One of the largest deviations from pure randomness in financial series is volatility persistence. Return movements are very hard to forecast, but magnitudes of the movements are predictable. This has led to a large literature on trying to model this phenomenon.[9] The fact that stock returns exhibit this sort of structure alone is somewhat of a mystery, but the puzzle was further strengthened by LeBaron(1992,1992b)[40, 39] which showed that autocorrelations in stock and foreign exchange market returns were changing depending on an estimate of recent volatility. LeBaron showed that many financial series followed a process that looked like,

$$r_t = \log(p_t) - \log(p_{t-1})$$

$$r_t = f(\sigma_t^2) r_{t-1} + \epsilon_t$$

$$\sigma_t^2 = \sum_{i=1}^{N} r_{t-i}^2.$$

It turns out that $f()$ is a decreasing function of conditional variance indicating that local predictability in the series is higher during periods of lower volatility. This phenomenon can be used to achieve some small out of sample improvements in forecasts especially in weekly foreign exchange series.[10]

One final curious feature of financial forecasting which may be related to nonlinearities concerns the analysis of technical trading rules. These rules are heuristic forecasting methods which traders claim give them an edge in forecasting the movements of financial markets. Once generally accepted as being worthless by much of the academic community several papers have reopened this debate. [11] The predictability appears to be greatest for foreign exchange markets where the magnitude of trading profitability makes up for reasonable estimates of the costs of trading in these markets. Many of the most successful rules used are related to moving average rules which attempt to follow long range trends. They recommend that a trader buy when the price is above a long range moving average,

$$p_t > \frac{1}{N} \sum_{i=0}^{N-1} p_{t-i},$$

and sell when the price is below.[12]

To summarize these results, there is interesting evidence of potential predictability in many of these series. Before one concludes that there is lots of money to be made forecasting financial series several cautions should be made. First, the actual implementation of a forecasting rule for trading may involve unforeseen costs, and prices taken from recorded data sets may not actually be tradeable. Second, taking on some of these dynamic strategies may involve exposure to extensive

[9] See Bollerslev et al.(1990)[7] for an excellent survey of this large area which includes the ARCH family of models originated by Engle(1982) [24].

[10] Other papers have documented similar out of sample forecast performance for foreign exchange series. One example is Weigend et al.(1991)[66]. Other papers find little or no evidence for forecast improvement (Diebold and Nason(1990)[20]).

[11] Papers such as Brock et al.(1992) [11], Curcio and Goodhart(1992)[15], LeBaron(1991)[38], Levich and Thomas(1993)[43], Schulmeister(1987)[57], Sweeney(1986)[59], and Taylor(1993)[60] all present some evidence that there is predictive power contained in some of the rules used by technical traders.

[12] This is the simplest implementation of these types of rules. Real traders use many more complicated variants of this type of strategy, but this is close to the basic idea.

risks. The large expected returns might be included with a high probability of the strategy losing a considerable amount of money.

This nonlinear forecastability has still ignored the original question about chaos in these series. Given the support for some kind of nonlinear structure the question of chaos still appears very interesting. It is possible that identifying chaos for actual returns series may run up against another barrier. The problem this time is not the lack of data.[13] The problem this time may be related to how much forecastability can be left around in a financial time series. To estimate a lyapunov exponent a researcher needs some idea of forecast degradation over a short horizon. This obviously implies that over the shortest horizon good forecasts were available to analyze how their performance drops after several periods have gone by. Forecasts of this high quality may be unreasonable for financial time series.

As an example of what a small amount of predictability means in a financial market the weekly British Pound/U.S. Dollar foreign exchange series was used. It was sampled every Wednesday from January 1974 through July 1992 at noon New York time. A trader with access to just the correct sign forecast to this series would still have to work hard to claim that the series was chaotic, but I doubt that the trader would care much since the daily return to a strategy of longing or shorting the pound according to the directional forecast would yield a return of about 1% per week, or about 68% compounded over a year. Even if the trader were charged a large transaction cost of 0.5% per trade[14], the strategy would at worst earn about 30% per year.[15] If a positive lyapunov exponent was reliably estimated it would probably imply even greater predictability over the short horizon. It is unlikely that such predictable structure exists for any financial time series.

These are only approximations and conjectures, but they emphasize an important point. For financial series there may be an extremely wide gap between successful nonlinear forecasting, and actual identification of chaotic dynamics in a financial market. The researcher looking for chaos in some of these series probably should cast the search a little wider than just looking a raw price movements. Several possibilities are mentioned in the next section.

4 On the Joint Stability of Volume and Volatility

This section gives a quick example reflecting some of the issues mentioned in the previous section. First, it will directly analyze the dynamics of a dual two component time series. Second, it will analyze the dynamics in a framework that considers the stochastic and deterministic aspects of a nonlinear time series. This approach is inspired by Tong(1990) [64], and Yao and Tong(1992)[67], and looks at the local stability properties of the dual series. The idea here is to consider the properties of a nonlinear stochastic process for stock market trading volume and price volatility. Specifically, to look at some of the local stability properties to see if the dynamical system exhibits any regions where trajectories are spreading apart. Even if a global system is stable it may still be of great interest to understand the stability properties of certain subregions. This is especially true when the system is being continually impacted by outside noise.

The series are constructed from daily NYSE trading volume and volatility constructed using the square of a returns series using the CRSP value weighted index from July 1962 through June

[13] For financial series high quality series are available with millions of data points for high frequency data sets.
[14] Actual costs are probably less than 1/5 this level.
[15] This is a lower bound since it is assuming that trader is trading every day.

1988. Both series are log transformed,

$$s_t = \log(r_t^2),$$

and,

$$v_t = \log(V_t),$$

where r_t is the returns series, and V_t is trading volume.[16] Table 1 shows parameter estimates for a simple 1 lag VAR for the volatility and volume processes. The results show the strong persistence in both series, and also some amount of dynamic cross correlations. The strongest of these being a negative relation from lagged volume to future volatility.

A quick glance at the parameters suggests that the response of this system to new shocks is probably stable. Shocks to the system would be quickly dissipated. One measure of the overall speed at which new shocks to the system disappear is the largest eigenvalue of $A^T A$ where A is the estimated linear system from table 1. Let λ be the largest eigenvalue of $A^T A$. It follows that the deterministic distance one period in the future will be bounded by,

$$||F(x_t + \delta) - F(x_t)|| \leq |\lambda| ||\delta|| + o(||\delta||),$$

where $||.||$ is the Euclidean norm L^2, and $F()$ is the deterministic portion of the dynamics. For this estimated system this number is 0.87. This again shows a strongly stable, and somewhat persistent series, quickly dampening new shocks.

It is possible that the joint dynamics of the two series may depend on the current state of the system. Also, the stability properties of the system may be different in different regions. To explore this, a local linear approximation is used. To get a general interpretation a very simple form of local linear fit will be used. The state space (s_{t-1}, v_{t-1}) is split into four subregions using the median value for each series as the splitting level. These four regions will be referred to as high and low volume and volatility respectively. The two series are contemporaneously correlated so the four regions will not contain the same number of points. There will be more points in the low volume - low volatility region, and in the high volume - high volatility region.

The one lag VAR from table 2 is estimated for each of the four regions in table 2. The results from the first table change dramatically in certain regions. For example, the coefficient of volatility on lagged volume in the low-low region is -1.880, indicating a larger than 1-1 response of volatility on a lagged volume shock. Also, there is some indication of a larger than 1-1 response in the high volatility - low volume region where the coefficient is -1.362. Another interesting pattern that appears is in the high volume region where the lagged volume to volatility channel changes sign from the low volume region, with parameter estimates of 0.547 and 0.174 in the high and low volume periods respectively.

The parameter estimates suggest some instability in the low volume/low volatility region, and this is reflected in a estimate for λ of 3.85 in this region. The estimate is also large for the low volume/ high volatility region with an estimate of 2.37. Both high volume regions give estimates less than 1 reflecting the stability shown in the local linear models fit in those regions. While the local instabilities in parts of the state space do not make this a chaotic system they do suggest that interesting dynamics may occur at certain times. Since the system is driven by noise it will be continuously swept through the unstable regions at different times in its evolution.

It cannot be said that this is indicative of chaos either, but it does display some of the richness of the joint dynamics present in these series. Also, it shows some evidence that there are portions

[16] The trading volume series is detrended using a 100 day moving average.

of phase space over which trajectories are locally spreading. Further tests and details are given in LeBaron(1993)[41].

5 Toward the Future of Chaos in Economics

Many of the results up till now have been very negative about the presence of chaos in economic time series. This final section turns more upbeat in suggesting several untried and promising paths for the future.

The most obvious extension is to move toward new data sets. The most interesting will probably involve more detailed examination of multivariate series such as the study of volume and volatility shown here. Other work will move to direct analysis of components of economic systems from GNP product accounts to the spatial and geographic information on growing economies. These remain relatively uncharted for nonlinear empirical studies in economics.

Many of the early tests used have been generally atheoretic. There have been few connections from empirical results to underlying economic theories. This can be a plus or a minus at times, but in terms of chaotic dynamics the connections drawn from theories to data have been far too rare. The theorist operating in this area will need to use new techniques, since old standards of estimation and diagnostic testing may fail.[17] A few interesting papers have begun to appear which try to make some connections.[18] It is probably likely that extensive simulations from economic models will be compared with empirical data. However, the features matched may not be those from traditional chaos analysis.

Some recent work has extended the set of available nonlinear tools, and several of these tests have not been applied to economic data sets.[19] Although these tests may prove useful still further research is necessary in attempting to understand these series in relation to some of the important economic questions involved. Are business cycle fluctuations driven more by randomness or inherent structures in the economy? Is trade in financial markets self-generating? These questions still remain unanswered.

Another new branch is to look more closely at large scale interconnected systems as metaphors for economic behavior. This "complex systems" approach is used in Bak et al.(1992) [2] and Brock and LeBaron(1993) [12] in modeling some macroeconomic phenomenon. Neither of these papers says much about chaos per se, but there is a connection. One example of this connection is in the paper by Doyon et al.(1993)[21]. In this paper the authors examine the dynamics of a neural network as the overall connectivity is changed. They find a link between nodewise connection strengths and an onset of overall chaotic dynamics in the system. Whether results such as this can impact studies of macroeconomic phenomenon remains to be seen, but they look promising.

In some ways we are only beginning to cut the surface of how to analyze real world data in light of the knowledge about what kinds of dynamics can be generated by nonlinear processes. We have tried to directly move tests from one field to another in an attempt to gain new insights into

[17] Geweke(1993)[28] provides an example of why there may be problems with traditional econometric methods when examining chaotic models.

[18] See for example Chavas and Holt(1991)[14] for their model of hog cycles, and De Grauwe et al.(1993) [16] for their work on foreign exchange markets. Mosekilde and Larson(1988)[46] provide another related attempt to connect a simple chaotic model to some experimental results.

[19] These include Lee, White and Granger(1993)[42], Dechert and Gencay(1992a,b)[18, 17], Nychka et al.(1992)[48], and Sugihara and May(1990).

our home fields. At times this has brought in fresh ideas, and added to the development of some new and interesting diagnostics. However, this transfer should not be the end of the story. We probably should rethink what the important problems were that we were attacking and readjust our searches in these directions. We also need to move closer to understanding and estimating theoretical models for explaining the empirical puzzles that we are interested in.

Table 1
Global Linear Volume Volatility Estimates

$$s_t = a_s + b_s s_{t-1} + c_s v_{t-1}$$

$$v_t = a_v + b_v s_{t-1} + c_v v_{t-1}$$

Dependent	a_i	b_i	c_i
s_t	0.000	0.128	-0.651
	(0.030)	(0.012)	(0.128)
v_t	0.000	0.004	0.665
	(0.002)	(0.001)	(0.009)

Global simple vector autoregression estimated over entire time series. s_t is volatility $\log(r_t^2)$, and v_t is log trading volume (turnover detrended using 100 day moving average).

Table 2

Local Linear Volume Volatility Estimates

$$s_t = a_s + b_s s_{t-1} + c_s v_{t-1}$$

$$v_t = a_v + b_v s_{t-1} + c_v v_{t-1}$$

		Low Volume			High Volume		
	Dependent	a_i	b_i	c_i	a_i	b_i	c_i
Low Volatility	s_t	-0.258	0.071	-1.880	-0.571	0.007	0.547
		(0.104)	(0.026)	(0.391)	(0.119)	(0.032)	(0.493)
	v_t	-0.026	0.002	0.557	0.020	-0.001	0.507
		(0.007)	(0.002)	(0.028)	(0.007)	(0.002)	(0.032)
High Volatility	s_t	-0.538	0.376	-1.362	-0.602	0.389	0.174
		(0.168)	(0.075)	(0.469)	(0.124)	(0.054)	(0.320)
	v_t	-0.038	0.018	0.630	-0.025	0.021	0.674
		(0.012)	(0.006)	(0.035)	(0.010)	(0.004)	(0.026)

Local simple vector autoregression estimated on portions of the time series. s_t is volatility $\log(r_t^2)$, and v_t is log trading volume (turnover detrended using 100 day moving average).

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