REINSURANCE, TAXES AND EFFICIENCY: A CONTINGENT CLAIMS MODEL OF INSURANCE MARKET EQUILIBRIUM

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Abstract. This paper presents an analytical model of underwriting capacity and insurance market equilibrium under an asymmetric corporate tax schedule. It is shown that reinsurance markets enable risk-neutral insurers to allocate tax shields to those firms that have the greatest capacity for utilizing them, in much the same manner as leasing companies share tax shield benefits with lessees in leasing markets. Reinsurance is therefore used as an efficient short-term mechanism to yield the optimal allocation of tax shield benefits. In equilibrium, asymmetric taxes cause the insurance price to be actuarially unfair and the expected return on capital invested in insurance reflects the probability of paying taxes.

Keywords. Reinsurance, option pricing theory, contingent claims,˝asymmetric taxes.

1. Introduction

The economic analysis of insurance and reinsurance markets has been traditionally modeled in an expected utility framework. In the papers by Karl Borch (1960, 1962), risk averse insurers set up a reinsurance pool to share risks according to a rule derived from the first-order conditions for Pareto-optimal allocation. If the insurers involved all have HARA -- class utility functions, then in equilibrium reinsurance will be demanded and supplied on a proportional basis. In the papers by Blazenko (1986) and Eden/Kahane (1990), insurers and reinsurers make optimal reinsurance decisions in a mean-variance framework. In Blazenko (1986), reinsurance provides additional capacity to the market, thus allowing the supply of direct insurance to increase, with a concomitant decline in price. In Eden and Kahane (1990), the joint existence of local insurers and international reinsurers reconciles two conflicting objectives: wide spreading of risks and close monitoring of moral hazard.

Although the expected utility approach to the analysis of reinsurance decision-making has produced a number of important insights into the operation of reinsurance markets, it has also been criticized for 1) failing to provide an adequate basis for modeling the influence of competitive financial markets upon the insurers' behavior, and 2) ignoring the "nexus of contracts" nature of the insurance firm (see Garven (1987)). Main (1982, 1983) and Mayers and Smith (1982, 1990) have proposed a value maximization approach based on tax and agency cost considerations to account for insurance purchases by large, widely held corporations. A similar approach has been used by Garven (1993) to explain the demand for reinsurance by large insurance companies holding diversified portfolios of insurance contracts. Doherty and Tiniç (1981) also criticized the assumption of risk averse behavior by insurers for contradicting the observation that insurance risk is largely diversifiable in the capital market. They argued that reinsurance purchases are more adequately motivated by˝bankruptcy cost considerations in a value maximization framework.

The purpose of this paper is to expand upon this literature by assuming that insurers are risk-neutral and by analyzing the effects of taxes on underwriting capacity and equilibrium in insurance and reinsurance markets. We intend to go beyond intuitive arguments by providing an explicitly theoretical model where reinsurance transactions are economically justified on the basis of the impact that such transactions have upon the tax liabilities of insurers.[1]

The model is based on an option-pricing framework already used in the finance literature in previous work on leasing and on investment decisions. Heaton (1986) shows that the supply of and demand for leasing contracts may be motivated by the existence of incomplete state-contingent tax loss offsets. Green and Talmor (1985) argue that the shareholders of a levered firm will have an incentive to underinvest in risky assets, and to purchase corporate insurance, so as to avoid underutilizing corporate tax shields. Like Main (1983)), we draw upon conceptual analogies between leasing markets and insurance markets and present a model that demonstrates that insurance companies will share risks; i.e., demand or supply reinsurance, depending upon their tax situations. Within this framework, reinsurance is used to allocate tax shields (specifically, claims costs) to those firms that have the greatest capacity for utilizing them, in much the same manner as leasing companies share tax shield benefits with lessees in leasing markets. In our model, we assume that taxes are <u>asymmetric</u> in the sense that tax loss offsets are incomplete. Consequently, it is possible for tax shields to be underutilized, and the government may therefore be viewed as holding a portfolio of contingent claims on the taxable incomes of firms in the economy.

Our analysis implicitly assumes that tax shield under-utilization is an important problem for insurers. Casual empirical evidence suggests that tax shield under-utilization is indeed an important problem for firms generally. Heaton (1987) suggests that the fact that 40 to 60 percent of all U.S. corporation in recent years have paid no taxes lends credibility to the assertion that a full tax loss offset does not always exist for corporations; i.e., taxes are <u>asymmetric</u>. Heaton's

[1]Although we certainly acknowledge the importance of alternative, more traditional explanations for reinsurance (such as its risk-spreading function), our emphasis will be on the role of taxes in motivating reinsurance trade.

observation would appear to be particularly relevant to property-liability insurers in view of the fact that the industry paid approximately zero net federal income taxes during the twenty-five year period preceding the Tax Reform Act of 1986 (see Walker (1991)).

It is important to note that asymmetric taxes represent one of a number of possible factors that may lead to similar results as are presented here. The primary requirement is that there is some cost associated with having low income; consequently, bankruptcy costs as well as agency costs may also play similar roles. Technically, such a cost is a substitute for concave utility. In our model, insurers are assumed to be risk-neutral, but the valuation function is nonlinear due to the convexity of the firm's tax liability. Therefore, by Jensen's inequality, the expected tax payment is greater than the tax on expected income. As Smith, Smithson, and Wilford (1990) have noted, convex tax schedules create a tax benefit from hedging. In our model, reinsurance provides such a hedging mechanism. Consequently, it follows that our paper provides <u>one</u> motivation for reinsurance trade. Concerns with costs of financial distress provide a more traditional explanation. We do not claim that asymmetric taxes are a <u>necessary</u> condition, but we do argue that they may represent a <u>sufficient</u> condition for the existence of a market for reinsurance.

The remainder of the article is organized in the following manner. In the next section, we present a model of the insurance market, where risk-neutral insurers are endowed with a given amount of equity capital, or surplus. They face an asymmetric corporate tax schedule and underwrite optimal shares in a diversified portfolio of accident risks. In equilibrium, the optimal market shares are directly related to the relative financial capacities of the insurers. Section 3 introduces reinsurance. We argue that (given endowed shares in the direct insurance market) reinsurance represents an efficient mechanism to obtain optimal risk-sharing. Reinsurance trade results in a reallocation of tax shields among insurers. The equilibrium set of reinsurance contracts is that set that minimizes the aggregate value of the government's tax claim on the income of the insurance industry. Consequently, the results of our model parallel the results obtained in the capital structure equilibrium theories of Miller (1977) and DeAngelo and Masulis

(1980).[2] Section 4 extends the model to the international reinsurance market by considering "tax clientele" effects that are likely to occur when tax rates are heterogeneous between countries. Section 5 provides a long run extension of the equilibrium argument. Existing firms are allowed to vary their surplus, and new firms may enter the market. This influences the allocative role of reinsurance and the after-tax equilibrium expected return on capital invested in the insurance industry. The sixth section concludes by summarizing our main results and providing suggestions for future research in this area.

2. A model of the insurance market

In this section, we present a model of the insurance market. We begin by presenting the assumptions and notation that will be needed for our analysis.

2.1. Model assumptions and notation

The model has two periods: t=0, the present, and t=1, the future. The economy encompasses a pool of homogeneous accident risks, with realization at t=1. The aggregate accident loss X is normally distributed with expected value E_X and standard deviation σ_X. It is assumed to be stochastically independent of social wealth.

Let p represent the competitive market price of the global accident portfolio. This price is taken as given by any participant in the accident insurance˝market.

There are m insurers supplying accident insurance. At t=0, each insurer j (j = 1, . . ., m) is endowed with an amount of capital (surplus) S_j, and gets a fraction γ_j of the global accident

[2]In the Miller and DeAngelo/Masulis (1980) models, the primary role of corporate debt financing is to minimize the value of the government's tax claim. However, in our model, we do not consider the effects of personal taxation.

portfolio. As this basic model is essentially short term, m and S_j are initially assumed fixed for all j. Moreover, transaction costs are assumed away.

For the sake of simplicity, we will also initially assume that all insurers adopt the same investment policy: at t=0 they invest all their funds (surplus plus premium income) in the riskless asset,[3] with rate of return r. Thus, firms differ only with respect to their endowed surplus S, and γ is the only control variable, where $\gamma \in [0,1]$.

Claims are paid at t=1 and bankruptcy risk is assumed away. Thus, as X is assumed to be uncorrelated with social wealth (and therefore diversifiable in the financial market) each insurer behaves *as if* it is risk neutral.

Finally, each insurer pays taxes on the sum of underwriting profit and investment income at a rate of τ. However, taxes are asymmetric in the sense that government taxes gains and does not rebate losses.[4]

Consequently, as shown in Figure 1, the tax payment has the profile of a European put option, where claims costs represent the underlying asset Z, and the exercise price K is given by the sum of premium and investment income. If $K \leq Z$, no taxes are paid. If $K > Z$, the insurer pays taxes of $\tau[K - Z]$. Therefore, the tax payment equals $\tau MAX[0, K-Z]$.

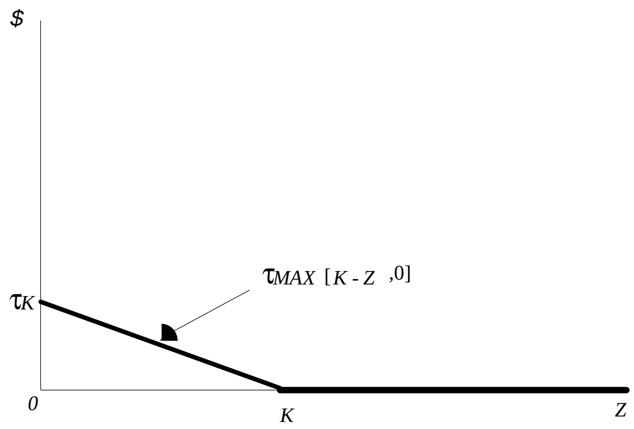

FIGURE 1. Profile of Tax Payment.

[3]This simplifying assumption is not too much at variance with reality. In many countries, insurers are required to invest primarily in safe assets, and more particularly in government bonds.
[4]It may be easily verified that if taxes were symmetric, the optimal value of γ would be either 0 (no insurance supply) or 1 (infinite insurance supply), depending on whether p is less or greater than its actuarially fair value.

2.2. The model

The risk-neutral insurer maximizes the after-tax expected present value of the firm. Ignoring the subscript j, this is given by equation (1):

$$V(\gamma) = S + \gamma p - R^{-1}E_Z - \tau R^{-1}E\{Max[0,K-Z]\}, \tag{1}$$

where

R = 1 + r;
K = rS + γpR; and
Z = γX.

Z, the claims cost, is normally distributed with expected value $E_Z = \gamma E_X$, and standard deviation $\sigma_Z = \gamma\sigma_X$.

Let $P(K,Z) = R^{-1}E\{Max\{0,K-Z]\}$ represent the value of the put option written in favor of the tax authorities. The after-tax present value of the insurer may thus be written:

$$V(\gamma) = S + \gamma(p - R^{-1}E_X) - \tau P(K,Z). \tag{1'}$$

Since Z is normally distributed, we can expand the expression for the put to obtain[5]:

$$P(K,Z) = R^{-1}\gamma\sigma_X[dN(d) + n(d)], \tag{2}$$

where n(•) is the standard normal density function, N(•) is the standard normal distribution function, and

$$d = \frac{K - E_z}{\sigma_z} = \frac{rS + \gamma(pR - E_x)}{\gamma\sigma_x}. \tag{3}$$

From this, we obtain:

$$\frac{\partial P}{\partial \gamma} = R^{-1}\sigma_x[N(d)(d + \gamma\frac{\partial d}{\partial \gamma}) + n(d)]$$

[5]Note that this expression is similar to Brennan's (1979) equation (39) for the discrete-time-neutral valuation of an option when the return distribution of the underlying asset is normal.

$$= R^{-1} [N(d)(pR - E_X) + \sigma_x n(d)], \tag{4}$$

using $\frac{\partial d}{\partial \gamma} = -\frac{rS}{\gamma^2 \sigma_x} < 0$.

Furthermore:

$$\frac{\partial^2 P}{\partial \gamma^2} = -\frac{rS}{\gamma} R^{-1} n(d) \frac{\partial d}{\partial \gamma} > 0. \tag{5}$$

From (1') and (5) we obtain the second order condition: $V''(\gamma) = -\tau(\partial^2 P / \partial \gamma^2) < 0$. Thus, $V(\gamma)$ is concave. We may also remark from (4) that the put value is increasing in γ when $pR \geq E_X$, i.e., whenever p is actuarially fair or unfair.

From (1') and (4) and assuming an interior solution, we obtain the first-order condition for a maximum of V:

$$(pR - E_X)\,[1 - \tau N(d)] = \tau \sigma_X n(d). \tag{6}$$

The left hand side of this equation (6) represents the after-tax expected marginal gain from an increase in γ. The right hand side represents the marginal tax loss from the increased probability that the put ends up in-the-money.

It is instructive to solve equation (6) for p:

$$p = R^{-1} \{E_X + \lambda(\gamma)\, \sigma_X\}. \tag{7}$$

In this expression, $\lambda(\gamma) = \frac{\tau\, n(d)}{1 - \tau\, N(d)}$ is positive and represents the unit risk loading factor. It turns out that the insurer adjusts γ so as to obtain, at the firm level, an actuarially unfair price for insurance, given p, E_X, σ_X and R. Note that if insurance is actuarially fair; i.e., if $pR = E_X$, condition (6) implies that n(d) should be zero. Hence d tends toward infinity. Since $d = rS/\gamma\sigma_X$, this further implies that $\gamma = 0$; i.e., no insurance is supplied. For positive values of γ, condition (6) is only satisfied when $pR - E_X > 0$. This leads to our first proposition:

Proposition 1: In equilibrium, the price of insurance is actuarially unfair.

2.3. Comparative statics

Some comparative statics exercises are useful to check the influence of various parameters on optimal underwriting. This leads to the following proposition:

Proposition 2: The optimal insurance supply is increasing in S, p and r. It decreases in τ and σ_X.

PROOF: We provide the proof for surplus S only. The other results are obtained in the same manner.[6] Differentiating implicitly from the first order condition with respect to γ and S, we obtain:

$$\frac{\partial\gamma}{\partial S} = -\frac{\partial V'/\partial S}{V''(\gamma)}.$$

Since $V''(\gamma) < 0$, this implies that sign $(\partial\gamma / \partial S)$ = sign$(\partial V'/\partial S)$. Now, $\partial V'/\partial S = -\tau(\partial^2 P/\partial\gamma\partial S)$. Differentiating the expression for $\partial P/\partial\gamma$ in equation (4) with respect to S, we find:

$$\partial V'/\partial S = \tau n(d) r^2 S / R\gamma^2\sigma_x > 0. \tag{8}$$

Hence $\partial\gamma / \partial S > 0$. QED.

2.4. Market equilibrium

The market equilibrium condition is written:

$$\textstyle\sum_j \gamma_j = 1. \tag{9}$$

Condition (6) defines the optimal market share γ as a function of six parameters: p, E_X, r, τ, σ_X and S. From the proof of Proposition 2, using (5), (8) and the expression for $V''(\gamma)$, we

[6]The complete proofs are available from either author.

obtain: $\partial\gamma / \partial S = \gamma / S$. Hence γ is linear in S. The optimal value for γ_j, insurer j's market share, may thus be written as a function:

$$\gamma_j = S_j \bullet h(p, \tau, \sigma_X, E_X, r).$$

This follows from the fact that insurers differ only with respect to their endowed capital. Using this expression in (9), we obtain:

$$\Sigma_j \gamma_j = h(\bullet)\, \Sigma_j S_j = 1.$$

Consequently, $h(\bullet) = 1/\Sigma_j S_j$, and

$$\gamma_j = S_j / \Sigma_j S_j = s_j, \qquad (10)$$

where s_j represents the share of insurer j's surplus in the total˝surplus of the insurance industry.

Equation (10) defines an optimal sharing rule, which we express in the following proposition:

Proposition 3: In equilibrium, the share of insurer j in the accident insurance market is equal to its share in the industry's capital: $\gamma_j = s_j$.

Note that the loading λ introduced in equation (7) should be the same for all firms, since we assume price-taking behavior among insurers. However, we noted that λ is firm-specific since it depends on d, which in turn depends on γ. This is not true any more when the market is in equilibrium. For insurer j,

$$d_j = \frac{rS_j + \gamma_j (pR - E_x)}{\gamma_j \sigma_x}$$

In equilibrium, $S_j = \gamma_j \Sigma_j S_j$. Hence we obtain:

$$d_j = \frac{r\Sigma_j S_j + pR - E_x}{\sigma_x},$$

which turns out to be firm-independent.

The dynamics of equilibrium may be described as follows: If $\Sigma_j \gamma_j < 1$, there is an excess demand in the market. The excess demand drives the price (the loading) up. The optimal γ is thus increased for all firms. From (3), the change in d is the joint result of the increases in p and γ:

$$\frac{dd}{dp} = \frac{\partial d}{\partial p} + \frac{\partial d}{\partial \gamma}.\frac{\partial \gamma}{\partial p} = -\frac{R\gamma}{\lambda r S} < 0.$$

Thus d decreases, but since $\partial\lambda / \partial d < 0$, the increased loading is validated in equation (7). The excess demand eventually vanishes.

3. Reinsurance and efficiency

In the previous section's model, market equilibrium infers a precise distribution of demand across insurers. Specifically, each firm takes a share in total demand corresponding exactly to that firm's share in the insurance industry's total capital or surplus.

However, since insurers are assumed to be identical in all respects except for surplus endowments, and since bankruptcy risk has been assumed away, consumers are indifferent with respect to the choice of an insurer. Consumers will be distributed randomly among insurers, and it would occur only by coincidence that the *ex ante* distribution of consumers would replicate the optimal allocation of risks among insurers.[7] Therefore, the equilibrium process not only implies the usual adjustment of supply and demand, but also a reallocation of consumers among insurers. Those insurers facing an excess demand at the equilibrium price deny coverage to some consumers, who must shop around until they meet an insurer with excess capacity.

[7]One could alternatively invoke location theory: consumers might favor local insurers. However, the geographical distribution of insurers may be different from the geographical distribution of consumers, e.g., for tax reasons (see section 4).

In practice, such a reallocation would imply costs ignored in our model: the marketing costs of attracting additional consumers, or the agency costs (e.g., loss of goodwill) from refusing coverage to some consumers.

The insurers in our model could be given the opportunity to adjust their optimal supply to the observed distribution of consumers. Surplus provides the key to this solution. From equation (10) above, it is clear that optimal risk-sharing obtains either by adjusting γ for a given value of S, or proceeding the other way around. However, this would not be easily implemented in practice. Surplus cannot be adjusted in the short term, except for large transaction and agency costs that would probably exceed the above-mentioned costs of consumer reallocations.

The insurance market provides a simpler and more efficient method of consumer reallocation, namely <u>reinsurance</u>. This mechanism may be easily introduced into our model by redefining γ as <u>net</u> underwriting. Let α_j represent the endowed market share of the jth insurer ($\alpha_j > 0,\ j = 1, \ldots, m$) and let β_j represent the fraction of the market which insurer j reinsures ($\beta_j \leq \alpha_j$).[8] If $\beta_j > 0$, the insurer cedes a part of his insurance portfolio; if $\beta_j < 0$, he assumes reinsurance. Net underwriting is thus $\gamma_j = \alpha_j - \beta_j$, and β_j represents the new control variable. Dropping subscript j, the after-tax value of the firm may be restated as:

$$V(\beta) = S + (\alpha - \beta)(p - R^{-1}E_X) - \tau P(K,Z), \tag{11}$$

where

$K = rS + (\alpha - \beta)pR$; and
$Z = (\alpha - \beta)X$.

Note that the law of one price guarantees an identical price for the portfolio of risks on both the direct insurance and the reinsurance markets. If two different prices prevailed, arbitrage opportunities would arise and invalidate the reallocation mechanism. For example, if the reinsurance price exceeded the direct insurance price, insurers would have an incentive to

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[8]This is quota-share reinsurance. The quota-share is $q_j = \beta_j/\alpha_j$.

withdraw from the direct insurance market and specialize in supplying reinsurance. In conformity with observed practice, we assume that quota-share reinsurance occurs on original terms.

Proceeding from (11) as in the previous section, we obtain equations (2), (3), (6) and (7), where γ is simply replaced by $\alpha-\beta$. Thus Proposition 1 still holds.

The market equilibrium conditions, $\Sigma_j\ \beta_j = 0$ and $\Sigma_j\ \alpha_j = 1$, combined with the comparative static results:

$$\frac{\partial\beta}{\partial\alpha} = 1 \quad \text{and} \quad \frac{\partial\beta}{\partial S} = -\frac{\alpha-\beta}{S},$$

yield:

$$\beta_j = \alpha_j - s_j. \tag{12}$$

Equation (12) is essentially a restatement of equation (10) in section 2, defining the optimal sharing rule of Proposition 3.

However, equation (12) adds an additional dimension. It implies that in equilibrium, given α, high surplus firms will tend to reinsure low surplus firms. In particular, if all insurers underwrite the same share of the direct insurance market; i.e., if $\alpha_j = 1/m$ for all j, then we obtain: $\beta_j = (1/m) - s_j$. Defining the average surplus as $\overline{S} = \Sigma_j\ S_j / m$ and the average surplus share as $\bar{s} = \overline{S} / \Sigma_j S_j$, we obtain $\bar{s} = 1/m$. Hence,

$$\beta_j = \bar{s} - s_j.$$

The above equation implies that in equilibrium, with identical *ex ante* direct market shares, firms with higher than average surplus shares provide reinsurance cover, whereas firms with lower than average surplus shares demand reinsurance cover. Hence the proposition:

Proposition 4: In equilibrium, given α, high surplus firms reinsure low surplus firms.

It is instructive to link Proposition 4 to the tax feature of our model. Insurers with high surplus write a more valuable put in favor of the tax authorities (the exercise price K is higher; i.e.

the put is more "in-the-money"). Compared with low surplus firms, they incur less risk of not being able to fully utilize tax shields. Consequently, they are able to assume more risk by providing reinsurance to low surplus firms, which might experience large unused tax shields if actual claims happened to be unexpectedly large.

In our model, reinsurance is thus used as an efficient mechanism to reallocate (tax) risks among insurers. This contrasts with the motivation traditionally proposed in the insurance literature, where reinsurance is used for technical reasons: thanks to reinsurance, insurers are able to construct more balanced portfolios of net underwriting. We set aside this motivation right from the beginning, by assuming that insurers take share in a diversified portfolio of risks. We do not claim that the traditional motivation for reinsurance does not play a role in practice. We simply show that this traditional motivation is not a necessary condition for reinsurance trade.

Our reinsurance model presents interesting analogies with work by Karl Borch:

- In Borch (1962), insurers are endowed with direct insurance portfolios. They are risk averse and use reinsurance to reallocate insurance risks among themselves. A Pareto-optimal sharing of risk occurs when all insurers participate in a reinsurance pool in proportion to their risk tolerance. In contrast, our insurers are risk neutral (i.e., risk aversion is not necessary to motivate reinsurance). However (as in Borch (1962)), market equilibrium results in an optimal sharing rule: each insurer participates in the market in proportion to his share in the industry's total surplus. The driving force of risk allocation is the endowed financial capacity of each insurer, instead of risk aversion.

- In Borch (1985), insurers are also endowed with direct insurance portfolios, but they are assumed to be risk neutral. They demand reinsurance because they must comply with solvency rules, and reinsurance is less costly than having recourse to the capital market. There is a clear analogy with our model: reinsurance and increases in surplus (own capital) are substitutes for reaching a given objective: the solvency constraint in Borch's model, and optimal allocation of tax shields in our model. In both cases reinsurance is used because it is a more efficient mechanism.

Note that the second analogy makes clear that the reinsurance decision is a capital structure decision. By adjusting reinsurance demand or supply, each insurer selects its net amount of insurance leverage for a given level of surplus. An increase in the demand for reinsurance means that net liabilities are lower; i.e., leverage decreases. Thus our model provides a tax motivation for reinsurance which parallels tax-based theories of optimal capital structure in the corporate finance literature (e.g., see Miller (1977) and DeAngelo-Masulis (1980)).

4. International reinsurance and taxes

It is a well-known feature of direct insurance and reinsurance markets that direct insurance transactions occur primarily at the national level, whereas reinsurance is often traded internationally. This is mainly due to non-tariff barriers to trade in services. Most of the time, consumer protection is invoked as a justification for protectionism in the provision of insurance services.[9] However, this argument cannot be used to erect barriers to reinsurance trade. Moreover, freedom in international reinsurance trade is generally considered as a technical requirement for a sufficient spreading of risks.

Given protectionism in the provision of direct insurance services, our model provides an additional explanation for international reinsurance trade. As may be expected from Proposition 2, if other things are equal, insurers in low-tax countries will tend to provide reinsurance cover to insurers in high-tax countries. In this section, we provide a formal proof of this implication by extending our model to two countries: the domestic country and a foreign country.

In the following, variables for the foreign country are denoted by an asterisk, and foreign (re)insurance firms are indexed by k, $k = 1, ..., m^*$. Of course, the model of section 3 is assumed to apply in both countries; but some specific hypotheses are necessary to substantiate the *ceteris paribus* assumption and emphasize the role of taxes. These hypotheses are listed below:

- H1 No foreign exchange risk. The exchange rate is fixed and equals unity.
- H2 Capital flows across countries are free.
- H3 Insurance markets are segmented by non tariff barriers, but international reinsurance trade is free.

[9] See, e.g., Skipper (1987) and Outreville (1989). Note, however, that liberalization of trade in services, including insurance, represented a major issue in the recently concluded "Uruguay Round" negotiations at GATT.

H4 Same insurance risk in both countries: $E_x = E^*_x$ and $\sigma_x = \sigma^*_x$.
H5 All firms have the same surplus: $S_j = S_k^* \equiv S$ for all j and k.
H6 The tax rates are not the same in both countries; viz., $\tau > \tau^*$.
H7 No arbitrage opportunities.

Hypotheses H1, H2 and H7 together imply that r = r*. Hypotheses H3, H4 and H7 together imply that p = p*.

Applying the optimal value of γ from section 2 to firms in each country, we obtain:

$$\gamma_j = \alpha_j - \beta_j = S_j \cdot h\,(p, \tau, \sigma_X, E_X, r), \quad j = 1,...m.$$

$$\gamma^*_k = \alpha^*_k - \beta^*_k = S^*_k \cdot h(p^*, \tau^*, \sigma^*_x, E^*_x, r^*), \quad k = 1,...m^*.$$

Given our hypotheses, this may be written more compactly:

$$\gamma_j = \alpha_j - \beta_j = S \cdot h(\tau), \text{ for all j, and} \tag{14a}$$

$$\gamma_k^* = \alpha^*_k - \beta^*_k = S \cdot h\,(\tau^*), \text{ for all k.} \tag{14b}$$

Proposition 2 and H6 together imply: $h(\tau) < h(\tau^*)$. Therefore,

$$\gamma_j < \gamma^*_k, \text{ all j and k.} \tag{15}$$

Firms in the low-tax country supply more net insurance than firms in the high-tax country.

The market equilibrium conditions are now written:

$$\text{Insurance: } \sum_j \alpha_j = \sum_k \alpha^*_k = 1. \tag{16}$$

$$\text{Reinsurance: } \sum_j \beta_j + \sum_k \beta^*_k = 0. \tag{17}$$

Condition (16) reflect the segmentation of insurance markets, whereas condition (17) reflects the extension of our model to two countries with freedom of reinsurance trade.

We are now ready to prove the next proposition:

Proposition 5: In equilibrium, given α, firms in the low-tax country reinsure firms in the high-tax country.

PROOF: Assume $\alpha_j = \alpha^*_k = \alpha$ for all j and k. (From (16), this implies $m = m^*$). Using (14a) and (14b) we obtain:

$$\beta_j = \alpha - S \cdot h(\tau) = \beta, \text{ for all j, and}$$

$$\beta^*_k = \alpha - S \cdot h(\tau^*) = \beta^*, \text{ for all k.}$$

Since $h(\tau) < h(\tau^*)$, we have: $\beta > \beta^*$. Furthermore, since (17) is now written: $m\beta + m\beta^* = 0$, these two conditions together imply that $\beta > 0$ and $\beta^* < 0$. QED.

Thus, reinsurance provides a contracting mechanism that circumvents constraints imposed by barriers on international insurance trade in a world of differential tax treatment across countries. High-taxed domestic firms provide direct insurance to residents. Then, they cede part of their risks to insurers in low-tax countries, as the latter are willing to assume more risk. This may explain, at least partly, why large reinsurance companies are located in some countries (e.g. Switzerland and Germany), and why captive insurance companies tend to be formed in low-tax domiciles such as Bermuda and the Cayman Islands.

Note however that Proposition 5 is derived under the assumption of identical endowed market shares for all firms in both countries. This is in the spirit of our assumption of identical firms. However, as is clear from relation (15), an insurer in the high-tax country could provide reinsurance cover to an insurer in the low-tax country for α^*_k sufficiently larger than α_j.

5. Surplus and the equilibrium after tax return

Until now, we limited our analysis to the short run. Assuming that each insurer is endowed with a given level of surplus, we argued that it is more efficient for insurers to use reinsurance for the purpose of adjusting to their equilibrium market share, instead of varying their capital to validate an endowed market share α. In the longer run, however, some insurers may realize that they are undercapitalized. This happens if these insurers have to cede regularly a fraction of their insurance portfolio to competitors, thus giving up positive expected profits. They will have an incentive to raise more capital, so as to be able to keep a larger market share for own account. The cost of raising additional capital will be balanced by the additional expected profits over several periods of time.

As a result, more capital will be raised by some firms in the long run; the total surplus of the insurance industry will increase; and reinsurance will be less needed to allocate the tax shield optimally among firms.[10]

In the previous sections, we also considered the number of insurers as given, and equal to m. However, since the equilibrium price of insurance is non actuarial (see Proposition 1), there is obviously an incentive for risk neutral investors to enter the market and add to the number of firms operating in the industry. This will also raise the total surplus of the insurance industry. Concerning reinsurance trade, the entry of new insurers is likely to have however a favorable impact: new insurers with low surplus, but obtaining a market share α in excess of their share in the total surplus of the industry, will purchase reinsurance.

Let $S^* = \Sigma_j S_j$ represent the total surplus of the insurance industry. In section 2.4. above, we noted that this variable enters into the equilibrium expression for d:

$$d = \frac{rS^* + Rp - E_x}{\sigma_x} \tag{18}$$

From equation (7), we obtain:

$$\frac{Rp - E_x}{\sigma_x} = \frac{\tau\, n(d)}{1 - \tau\, n(d)} = \lambda(\tau, d) \tag{19}$$

where λ represents the unit-risk loading factor.[11] Substituting (19) into (18), we obtain a new equilibrium expression for d:

[10]Furthermore, the firms with excess capacity will be able to increase their marketing effort in the long run. This will enable them to take a bigger share of the direct insurance market and to compensate, to a certain extent, the lower reinsurance demand from firms increasing their surplus.

$$d = \frac{rS^*}{\sigma_x} + \lambda(\tau, d). \tag{20}$$

Now, define $E(r_i)$ as the after-tax expected rate of return on capital invested in an insurance firm. By definition,

$$E(r_i) = S^{-1}\{RS + \gamma(Rp - E_X) - \tau E[Max(0, K\text{-}Z)] - S\}.$$

Using (2), (7), (10) and (20), it is easy to derive the equilibrium value for $E(r_i)$:

$$E(r_i) = r[1 - \tau N(d)]. \tag{21}$$

The after-tax equilibrium expected rate of return on insurance is equal to the after-tax rate of return on the riskless asset, adjusted for the probability of paying taxes.[12] Since N(d) is less than one, it follows that the net expected return on insurance is larger than the net return on risk-free investment. This arises in spite of our assumption that insurance underwriting has no systematic risk. Thus asymmetric taxes result in a reward for idiosyncratic risk. The additional return is equal to the tax payment, adjusted for the probability that losses are sustained:

$$E(r_i) - r(1\text{-}\tau) = \tau r(1\text{-}N(d)).$$

Lemma: The equilibrium value of d is increasing in S*.

PROOF: From equation (20), taking r, τ and σ_X as given, the equilibrium value of d is defined by an implicit function:

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[11]Note that this loading factor is Sharpe's reward-to-variability ratio applied to insurance underwriting.

[12]N(d) corresponds to the probability that to the probability that taxes are paid by the insurance industry as a whole. In terms of the notation presented in this paper, N(d) corresponds to the probability that X < K, where K = rS* + Rp (See equation (18)).

$$F(d,\ S^*) = d - \frac{rS^*}{\sigma_x} - \lambda\,(\tau, d) = 0.$$

When S* increases, the total change in d from one equilibrium state to the other is given by:

$$\frac{dd}{dS^*} = -\frac{\partial F / \partial S^*}{\partial F / \partial d} = \frac{r}{\sigma_x}\,(1 - \frac{\partial \lambda}{\partial d})^{-1}.$$

Since $\partial\lambda / \partial d < 0$, $dd / dS^* > 0$. QED.

This lemma allows us to state that when additional capital flows into the insurance industry, N(d) increases, and thus (from (21)) $E(r_i)$ decreases. This occurs either because some firms increase their surplus, or because new firms enter the market, or both. Classically, long run equilibrium is reached when the after-tax expected return on insurance is equal to the after-tax expected return on the other activities with no systematic risk.

The preceding results may be summarized in the following final proposition:

Proposition 6: The equilibrium after-tax expected return on insurance decreases when additional capital flows into this industry. In long run equilibrium, this return is equal to the after-tax expected return on activities with no systematic risk. Idiosyncratic risk is rewarded by an excess return depending on the tax rate and on the probability that the insurance business generates losses.

5. Summary and conclusion

In this paper, we have utilized a contingent claims framework for the purpose of investigating the pricing and incentive effects of asymmetric taxes on net insurance supply. Our analysis provides three contributions to the theory of insurance markets.

The first contribution concerns the issue of underwriting capacity. Although our insurers are assumed to be risk neutral, the equilibrium price of insurance is actuarially unfair (Proposition

1), and market equilibrium is characterized by an optimal sharing rule: the relative financial capacity of each insurer defines his equilibrium market share (Proposition 3).

The second contribution concerns the theory of reinsurance. We have shown that in a short-term two-period model asymmetric taxes are a sufficient (but not necessary) condition to motivate reinsurance trade. Reinsurance is viewed as a mechanism for efficiently allocating tax shields among risk-neutral insurers. We found that well-capitalized firms have a comparative advantage in providing reinsurance services (Proposition 4), as do firms that are domiciled in countries with low tax rates (Proposition 5). These predictions appear to be roughly consistent with stylized facts about the real world market for reinsurance.

Since the reinsurance decision is a capital structure decision, these results strongly parallel those obtained in the capital structure equilibrium theories of Miller (1977) and DeAngelo-Masulis (1980). In equilibrium, a set of reinsurance contracts is chosen which minimizes the value of the government's tax claims.

Our reinsurance model is also reminiscent of work by Green and Talmor (1985) on the incentive effect of asymmetric corporate taxes. In their paper, the tax liability is represented by the value of a call option on the firm's end-of-period assets. The convex shape of this liability provides incentives to underinvest in risky projects, to engage in conglomerate mergers and to purchase corporate insurance. By introducing loss carryback and carryforward provisions, the incentive effects are somewhat mitigated, but the basic argument is not altered since future gains and losses are uncertain and must be discounted. In our paper, the tax liability is represented by the value of a put option on the firm's end-of-period claims payments. The convex shape of this liability provides an incentive to purchase reinsurance. However, in contrast to Green and Talmor (1985), we add a market equilibrium dimension. This results in an optimal allocation of tax shields (expected claims payments) among firms. In equilibrium, the value of the government's tax claims on the insurance industry as a whole is minimized.

The third contribution of our paper concerns the issue of capital flows into the insurance industry. On the assumption that insurance risk is diversifiable in the capital market, the equilibrium after-tax expected return on insurance is equal to the after-tax return on the riskless asset, adjusted for the probability of insurers paying taxes. This expected return is decreasing in the total capital invested in the industry. In the long run, some insurers will use increases in surplus as a substitute for reinsurance purchases. New insurers will also enter the market if the equilibrium after-tax expected return on insurance activity is higher than in comparable sectors. These two effects will increase the total capital invested in the industry, and thus lower its expected rate of return. Long run equilibrium will be reached when the after-tax expected returns are equal across sectors with no systematic risk (Proposition 6).

We believe that the approach followed in this paper constitutes a fruitful application of contingent claims analysis to the study of equilibrium in the insurance and reinsurance markets. However, our model is fairly simple, so as to allow us to concentrate upon the main thrust or our argument. We assumed away systematic risk, risky investments, and transaction costs. We also ignored the traditional motives for reinsurance: diversification of insurance risk and minimization of the probability of insolvency. For this reason, we considered only quota-share reinsurance, which is best suited to risk sharing and tax shield allocation, and ignored other arrangements, such as excess-of-loss or stop-loss reinsurance, which distort the probability distribution of losses. Some of these assumptions could be relaxed to investigate their impact on our results, but at the cost of greater analytical complexity.

In sections 3 and 5, we noted that reinsurance supply, marketing effort, and changes in surplus could be considered as substitutes. It seemed adequate to assume that the reinsurance decision dominates in the short term, but that the two other decisions gain importance in the longer run. A promising avenue for future research would be to model explicitly the joint optimal selection of the three variables. This would however imply the extension of our model in three directions: first, a multiperiod framework; secondly, introduction of a random process for direct insurance demand; and thirdly, explicit consideration of transaction costs.

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