

Adverse Selection In Credit Markets With Costly Screening

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Abstract

We develop a credit market model with adverse selection where risk-neutral borrowers self select because lenders make use of a costly screening technology. The model has some features which are similar to the Rothschild-Stiglitz adverse selection model. If an equilibrium exists it is a separating equilibrium, and there exist parameter values for which an equilibrium does not exist. Equilibrium contracts are debt contracts, and this is robust to randomization, in contrast to results for the costly state verification model. This framework can be extended to permit optimal financial intermediary structures, and it potentially has many applications.

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1. INTRODUCTION

In this paper, we develop a model where the costly screening of borrowers in a credit market with adverse selection yields debt contracts as an optimal financial arrangement. The model has at least two advantages over a widely used and tractable alternative model of debt contracts, the costly state verification model (though it would be difficult to argue that our model strictly dominates). First, debt contracts survive randomization here and, second, the frictions in our environment which imply the optimality of debt are closer to the frictions that play an important role in real-world credit markets. The model can be extended in straightforward ways to yield optimal intermediary structures, and it potentially has other useful applications.

There is now a large literature on optimal financial arrangements under private information. A primary aim of this literature has been to show how private information frictions give rise to the simple financial contracts and intermediary structures observed in practice. A widely used financial contracting setup is the costly state verification (CSV) model, first developed by Townsend (1979). In the CSV model, if attention is restricted to deterministic verification strategies, debt contracts arise as an optimal means for economizing on verification costs, and these costs can then be interpreted as costs of bankruptcy. The CSV model has been extended to study investment (Gale and Hellwig 1985), credit rationing (Williamson 1987a), financial intermediation (Williamson 1986), and macroeconomic issues (Bernanke and Gertler 1989, Williamson 1987b).

There are at least two problems with the CSV model as an explanation for debt contracts and financial intermediation. First, as Townsend (1979) and Mookherjee and Png (1989) have shown, debt contracts are not optimal if stochastic verification is permitted, and this result holds even if attention is restricted to environments where all agents are risk neutral (Border and Sobel 1987, Boyd and Smith 1993). Thus, the model loses some of its appeal if very restrictive assumptions are required to obtain the simple contracts observed

in practice.¹ Second, as a vehicle for studying financial intermediary structures, the CSV model relies on delegated monitoring results (Diamond 1984), whereby intermediation economizes on ex post verification costs; an intermediary's depositors delegate verification to the intermediary. However, the costs that appear to be most important for real world financial intermediaries are not ex post verification costs (i.e. auditing costs) but ex ante costs of information acquisition. For banks, these costs are primarily associated with the screening of loan applicants.

In the model we construct, there are two periods and three types of agents, lenders, good borrowers, and bad borrowers. Each lender is endowed with an investment good in the first period; borrowers receive no endowment, but each has access to an indivisible investment project which takes the investment good as input in period one, and yields a random return in the second period. The investment projects of good and bad borrowers differ according to a property that is somewhat stronger than first-order stochastic dominance. Type is private information. In contrast to standard adverse selection models, such as Rothschild and Stiglitz (1977), where self selection is achieved with risk averse agents who have different preferences (typically, some version of the single-crossing property is necessary for self selection), all agents are risk neutral here. To obtain self selection, lenders must make use of a costly screening technology, which reveals a borrower's type at a cost.

We use an equilibrium concept similar to that studied by Rothschild and Stiglitz (1977). That is, in equilibrium there exists no contract that, given the equilibrium contracts, makes the agents who choose it better off while earning nonnegative expected profits. A contract is a payment schedule/screening probability pair. That is, if a borrower accepts a particular contract, she agrees to make contingent payments in period two as specified by the contract, and submits to random screening in period one. Potentially, there might be a pooling

¹However, Boyd and Smith (1993) argue that the suboptimality arising from the use of deterministic auditing where random auditing would be optimal actually implies a very small welfare loss in practice. To the extent that we accept this argument, it dampens the force of our objection.

equilibrium, where each borrower is offered the same contract and there is no screening, or a separating equilibrium where payment schedules and screening probabilities are different for different types of borrowers.

We show that, if the equilibrium exists, it must be a separating equilibrium. Pooling equilibria do not exist here for much the same reason they do not exist in the Rothschild-Stiglitz model. That is, any pooling equilibrium involves an implicit subsidy of bad borrowers by good borrowers. A contract can always be offered which makes the good type better off while not attracting any bad types and earning nonnegative profits. The separating equilibrium has the property that good borrowers are screened with positive probability, while bad borrowers are not screened. Incentive constraints are binding for bad borrowers and do not bind for good borrowers. A central result is that the unique equilibrium separating contract for good borrowers is a debt contract. This contract is relatively unattractive for bad borrowers, and therefore permits self selection while minimizing the screening probability for good borrowers. Debt contracts are also optimal for bad borrowers, but there exists a continuum of contracts which do just as well.

The separating equilibrium exists provided that the cost of screening is sufficiently low and the fraction of bad borrowers in the population is sufficiently large. A feature of interest here is that, for *any* positive screening cost, if there are few enough good borrowers relative to bad borrowers in the population, then a separating equilibrium exists. Thus, if the adverse selection problem is sufficiently severe, the screening technology will be used no matter how large the screening cost. Some comparative statics results show that the screening probability decreases and loan interest rates increase as the cost of screening increases. That is, as screening becomes more costly, the screening technology is used less intensively, and the higher costs of lending result in an increase in market interest rates. Also, an increase in the risk-free interest rate faced by lenders leads to an increase in loan interest rates and in the screening probability. The screening probability increases in this case as higher interest rates tend to aggravate the adverse selection problem.

The model can be extended in a straightforward way to obtain financial intermediation with “delegated screening,” analogous to the delegated monitoring results obtained with the CSV model (e.g. Williamson 1986). That is, if the investment projects of borrowers are large in scale relative to the endowment of an individual lender, then in general there will be replication of screening by the individual lenders who fund a particular borrower’s project. This replication can be circumvented by a financial intermediary which exploits the law of large numbers, holding a perfectly diversified portfolio of loans and making noncontingent payments to its depositors.

Our model thus yields many results which are reminiscent of those obtained with costly state verification, in particular the optimality of debt contracts and intermediary structures. However, this environment is more appealing in that debt contracts survive the introduction of random screening strategies, and because ex ante screening costs appear to be a much more important component of intermediation costs than are ex post auditing costs. Also, the model permits applications to problems where adverse selection plays an important role in credit markets and financial intermediation. It would be difficult to argue, though, that our model dominates the CSV model in all respects. For example, we rely extensively on risk neutrality for our results. Also, to obtain debt contracts as an equilibrium, we require a monotonicity restriction on payment schedules (see also Innes 1990). This latter assumption can be justified, but only by appealing to features of the environment which are not completely worked out.

Related work that deals with the optimality of debt contracts in an environment different from that of the CSV model is Lacker (1992). Lacker appeals to differences in valuation of collateral by borrowers and lenders to obtain optimal debt contracts. A model with costly screening, which focuses on multiple equilibrium issues is in Gale (1992), and an adverse selection model with costly screening is studied in De Meza and Webb (1988). De Meza and Webb do not deal with the optimal contracting issues which are central to this paper.

The remainder of the paper is organized as follows. In Section 2 we construct the

model, and then define an equilibrium in Section 3. We then characterize the separating equilibrium and pooling equilibrium in Sections 4 and 5, respectively. In Section 6, we study the existence of equilibrium and describe some comparative statics results. We show how the model can be extended to yield financial intermediary structures in Section 7, and derive conclusions in Section 8.

2. THE MODEL

There are two periods, denoted 1 and 2, where investment takes place in period 1 and agents consume in period 2. There are three types of agents: lenders, type g borrowers, and type b borrowers. There is a continuum of borrowers and lenders, with the measure of borrowers being strictly less than the measure of lenders. Among the group of borrowers, a fraction α is type g , and the remaining fraction, $1 - \alpha$, is type b .

Each lender has one unit of an investment good in period 1, which can either go to a borrower in exchange for some promise to pay consumption in period 2, or it can earn a certain return of r units of consumption in period 2 for each unit invested in period 1, through a risk-free investment technology. Lenders maximize the expected value of $u(c, e) = c - e$, where c is consumption in period 2, and e is effort in screening borrowers in period 1.

Borrowers have no endowment in period 1 and maximize the expected value of period 2 consumption. Each borrower has access to an investment project which requires 1 unit of the investment good in period 1 to operate, and which yields a random quantity of the consumption good as output in period 2, if funded. If a borrower of type i funds her project, the return in period 2 is distributed according to the probability distribution function $F_i(\cdot)$, with the corresponding probability density function $f_i(\cdot)$. We assume that $f_i(x) > 0$ for $x \in [0, 1]$, $f_i(x) = 0$ otherwise, that $f_i(x)$ is continuous on $[0, 1]$, and that $\mu_i > r$ for $i = g, b$, where μ_i is the mean investment return faced by borrower i . We also assume

$$\frac{f_g(x)}{f_b(x)} < \frac{f_g(y)}{f_b(y)}; x, y \in [0, 1]; x < y. \quad (1)$$

Condition (1) is essentially identical to the monotone likelihood ratio property often assumed in principal agent problems with moral hazard. In the appendix, we show that condition (1) implies that $F_g(x) < F_b(x)$ for all $x \in (0, 1)$, that is $F_g(\cdot)$ dominates $F_b(\cdot)$ in terms of first-order stochastic dominance. Therefore, condition (1) is stronger than first-order stochastic dominance of $F_b(\cdot)$ by $F_g(\cdot)$. However, there are some probability distributions for which (1) is equivalent to first-order stochastic dominance. For example, if $F_g(x) = x^\alpha$ and $F_b(x) = x^\beta$, with $\alpha > \beta$, then $F_g(x) < F_b(x)$ for all $x \in (0, 1)$ and condition (1) holds. Another example is the exponential case, where $F_g(x) = \frac{1-e^{-\alpha x}}{1-e^{-\alpha}}$ and $F_b(x) = \frac{1-e^{-\beta x}}{1-e^{-\beta}}$, with $\beta > \alpha > 0$.

If borrowers are not screened, then type is private information. However, each lender has access to a technology which allows the lender to observe a borrower's type by incurring a fixed cost γ , in units of effort, where $\gamma \geq 0$. We assume that a given borrower can contact at most one lender in period 1, but that a lender may contact as many borrowers as she likes. This is somewhat similar to the costly state verification technology studied by Townsend (1979), in that there exists a costly technology for revealing information which would otherwise be private. This environment differs, however, in that there is adverse selection rather than moral hazard (as in the costly state verification approach), and because here private information is revealed by the technology before investment occurs rather than after. Also, the return on a borrower's investment project is publicly observable here.

3. EQUILIBRIUM

Our equilibrium concept is similar to that in Rothschild and Stiglitz (1977). Equilibrium contracts written in period 1 between lenders and borrowers consist of payment schedule/screening probability pairs $[R_i(x), \pi_i], i = g, b, x \in [0, 1]$, where $R_i(x)$ denotes the payment made by the borrower to the lender in the event that the return on the borrower's investment is x , and π_i denotes the probability that a lender uses the screening technology to reveal the type of a borrower who claims to be type i . We consider two types of contracts,

separating contracts and pooling contracts. A separating contract is offered by a lender to a particular agent type, while a pooling contract is offered to all agent types. There are potentially two types of equilibria: separating equilibria where all contracts offered by lenders are separating contracts, and pooling equilibria, where all contracts are pooling contracts.

3.1 Separating Equilibrium

A separating equilibrium is a pair of contracts $[R_i(x), \pi_i], i = g, b$, which must satisfy the following conditions, in addition to some other conditions which will be discussed later.

$$0 \leq R_i(x) \leq x, x \in [0, 1], i = g, b. \quad (2)$$

$$x \leq y \Rightarrow R_i(x) \leq R_i(y); x, y \in [0, 1]; i = g, b. \quad (3)$$

$$\int_0^1 R_i(x) dF_i(x) \leq (1 - \pi_j) \int_0^1 R_j(x) dF_i(x) + \pi_j \mu_i; i, j = g, b. \quad (4)$$

$$\int_0^1 R_i(x) dF_i(x) \geq r + \pi_i \gamma, i = g, b. \quad (5)$$

Condition (2) states that the payment schedule must be feasible for each type. Condition (3) imposes monotonicity on the payment schedules. This monotonicity condition is also used by Innes (1990), and he provides several justifications for it that rely on an appeal to implicit features of the environment. Among these justifications are the following. First, suppose that a loan market opens in period 2 after borrowers observe their output realization, but before lenders do. Loans are repaid at the end of the period at a zero interest rate, and these transactions are unobserved by lenders. Then, if the payment schedule for a given type is decreasing over some range, it is in the interest of that agent type to borrow temporarily in period 2 in some states of the world in order to fake a higher quantity of output. Thus, the payment schedule in this case is effectively monotonic, so we can simply impose condition (3). A problem with this justification for monotonicity is that borrowers' participation on the loan market must be limited to borrowing, otherwise they can fake lower levels of output as

well.² A second implicit assumption that will yield monotonicity is to suppose that a lender can unilaterally destroy any or all of the borrower's output in period 2, if he/she wishes. The lender would destroy output only if the payment schedule were decreasing over some range, but destroying output is clearly suboptimal, so payment schedules must be monotonic. This second justification may appear implausible when taken at face value, but it suggests that there may be well-specified (and plausible) moral hazard problems in lending (effort by the lender affects the borrower's return) which would guarantee monotonicity and not change the flavor of our results.

The conditions (4) are incentive compatibility constraints. Here, a borrower of type i has probability π_i of being screened. The left side of (4) is the loss in expected utility for the borrower if she reports her true type. The right side of (4) is the loss in expected utility if the borrower falsifies her type, reporting type j . In this case, with probability $1 - \pi_j$ the agent is not screened and cheating is not detected. In this case, the loss in expected utility is the expected cost of making payments as for a type j borrower. Alternatively, with probability π_j the borrower is screened, in which case cheating is discovered and the borrower is denied a loan. If a loan is denied, then the borrower cannot fund her project (as she can contact only one borrower) and consumes zero, so that the loss in expected utility is μ_i . Condition (5) states that the expected return to a lender from each separating contract is at least as great as from the alternative risk-free investment opportunity. As in the Rothschild-Stiglitz (1977) insurance model, each contract earns nonnegative expected profits; there is no cross-subsidization.

²If there is unconstrained lending and borrowing, and if the participants in the period 2 loan market include agents outside the credit market, then the credit market would shut down entirely. If borrowers can lend and borrow only among themselves in period 2, this would imply that payment schedules must be linear.

3.2 Pooling Equilibrium

Pooling contracts clearly do not involve the use of the screening technology, so that these contracts can be characterized by the payment schedule, denoted by $R(x)$. A pooling equilibrium is then an equilibrium payment schedule $R(x)$ which satisfies the following three properties, in addition to some other conditions to be discussed later.

$$0 \leq R(x) \leq x, x \in [0, 1]. \quad (6)$$

$$x \leq y \Rightarrow R(x) \leq R(y); x, y \in [0, 1]. \quad (7)$$

$$\alpha \int_0^1 R(x) dF_g(x) + (1 - \alpha) \int_0^1 R(x) dF_b(x) \geq r. \quad (8)$$

Conditions (6) and (7) require, respectively, feasibility and monotonicity of the equilibrium payment schedule. These conditions are the counterparts of (2) and (3), respectively. Condition (8) states that the expected return from the equilibrium contract for a lender must be no less than the return on the alternative risk-free investment. Note here that, when a lender offers the contract $R(x)$, the probability of lending to a good borrower is α . Condition (8) is the counterpart of (5) in the separating equilibrium.

3. Remaining Equilibrium Conditions

In either a separating equilibrium or a pooling equilibrium, it must be the case that, given the equilibrium contracts, no contract could be offered which is (i) weakly preferred by a lender to the safe alternative investment, given the borrowers who would accept it, and (ii) strictly preferred by some borrower to the equilibrium contract she would otherwise accept. Therefore, in a separating equilibrium, it must be the case that, for $i = g, b$, there exists no separating contract $[R_i^*(x), \pi_i^*]$, such that

$$0 \leq R_i^*(x) \leq x, x \in [0, 1]. \quad (9)$$

$$x \leq y \Rightarrow R_i^*(x) \leq R_i^*(y); x, y \in [0, 1]. \quad (10)$$

$$\int_0^1 R_i^*(x) dF_i(x) \leq (1 - \pi_j) \int_0^1 R_j(x) dF_i(x) + \pi_j \mu_i; j \neq i. \quad (11)$$

$$\int_0^1 R_j(x) dF_j(x) \leq (1 - \pi_i^*) \int_0^1 R_i^*(x) dF_j(x) + \pi_i^* \mu_j; j \neq i. \quad (12)$$

$$\int_0^1 R_i^*(x) dF_i(x) \geq r + \pi_i^* \gamma \quad (13)$$

$$\int_0^1 R_i^*(x) dF_i(x) < \int_0^1 R_i(x) dF_i(x) \quad (14)$$

Conditions (9)-(12) state that the proposed separating contract must be feasible, monotonic, and incentive compatible for each type, respectively. Condition (13) requires that the proposed separating contract be weakly preferred by a lender to the risk-free alternative asset, while (14) states that the proposed separating contract be strictly preferred by type i to the equilibrium contract for type i .

Similarly, in a separating equilibrium there can exist no pooling contract $R^*(x)$ with the following properties.

$$0 \leq R^*(x) \leq x, x \in [0, 1]. \quad (15)$$

$$x \leq y \Rightarrow R^*(x) \leq R^*(y); x, y \in [0, 1]. \quad (16)$$

$$\alpha \int_0^1 R^*(x) dF_g(x) + (1 - \alpha) \int_0^1 R^*(x) dF_b(x) \geq r. \quad (17)$$

$$\int_0^1 R^*(x) dF_i(x) < \int_0^1 R_i(x) dF_i(x); i = g, b. \quad (18)$$

Here, note that both types must strictly prefer the proposed pooling contract to the equilibrium contract for their type.

In a pooling equilibrium, there can exist no separating contract $[R_i^*(x), \pi_i^*]$ for which the following hold.

$$0 \leq R_i^*(x) \leq x, x \in [0, 1]. \quad (19)$$

$$x \leq y \Rightarrow R_i^*(x) \leq R_i^*(y); x, y \in [0, 1]. \quad (20)$$

$$\int_0^1 R(x) dF_j(x) \leq (1 - \pi_i^*) \int_0^1 R_i^*(x) dF_j(x) + \pi_i^* \mu_j; j \neq i. \quad (21)$$

$$\int_0^1 R_i^*(x) dF_i(x) \geq r + \pi_i^* \gamma \quad (22)$$

$$\int_0^1 R_i^*(x) dF_i(x) < \int_0^1 R(x) dF_i(x) \quad (23)$$

Also, in a pooling equilibrium there can exist no pooling contract $R^*(x)$ with the following properties.

$$0 \leq R^*(x) \leq x, x \in [0, 1]. \quad (24)$$

$$x \leq y \Rightarrow R^*(x) \leq R^*(y); x, y \in [0, 1]. \quad (25)$$

$$\alpha \int_0^1 R^*(x) dF_g(x) + (1 - \alpha) \int_0^1 R^*(x) dF_b(x) \geq r. \quad (26)$$

$$\int_0^1 R^*(x) dF_i(x) < \int_0^1 R(x) dF_i(x); i = g, b. \quad (27)$$

4. CHARACTERIZATION OF THE SEPARATING EQUILIBRIUM

Now that separating and pooling equilibria have been defined, we can proceed to establish some basic properties of these equilibria, starting with the separating case. We first need to show that conditions (5), which play the role of nonnegative expected profit conditions, hold with equality.

Lemma 1 *In a separating equilibrium, $\pi_b = 0$.*

Proof: Suppose not, and suppose that condition (4) is a strict inequality for $i = g$. Then, there exists an alternative separating contract for type b , $[R_b^*(x), \pi_b^*]$, with $\pi_b^* = 0 < \pi_b$, $R_b^*(x) = \delta R_b(x)$, $0 < \delta < 1$, and conditions (9)-(14) hold. Alternatively, suppose that $\pi_b > 0$ and condition (4) is an equality for $i = g$. Consider the pooling contract $R^*(x) = R_b(x)$. Since this contract is monotonic, and since $F_g(x) < F_b(x)$, we have $\int_0^1 R^*(x) dF_g(x) > \int_0^1 R^*(x) dF_b(x) > r$, where the first inequality is due to the monotonicity of $R^*(x)$ and the first-order stochastic dominance of $F_b(x)$ by $F_g(x)$, and the second inequality follows from (5). Therefore, condition (17) holds with strict inequality. In addition, this contract is

strictly preferred by type g , as $\int_0^1 R_g(x) dF_g(x) > \int_0^1 R^*(x) dF_g(x)$. Finally, $R^*(x)$ is weakly preferred by type b . Thus, (15)-(18) hold for $R^*(x)$. \square

The above lemma states that type b borrowers will never be screened in a separating equilibrium. If type b borrowers were screened, then we have one of two cases. First, it might be the case that a separating contract could be offered to type b borrowers with a lower screening probability and a preferable payment schedule. This contract could earn nonnegative expected profits as the lower expected screening costs make up for the reduction in expected payments to the lender. Second, the type b contract could be offered to both types as a pooling contract. This contract is preferred by both types, and earns positive expected profits since it involves no screening and will yield a higher expected profit from a type g borrower than from a type b .

Lemma 2 *In a separating equilibrium, if condition (5) is a strict inequality for given i , then condition (4) is an equality for $j \neq i$.*

Proof: Suppose not. Then, it is possible to offer an alternative separating contract for type i , $[R_i^*(x), \pi_i^*]$, with $\pi_i^* = \pi_i$, and with $R_i^*(x) \leq R_i(x)$, $x \in [0, 1]$ with strict inequality for some $x \in S$ with positive measure, such that (9)-(14) are satisfied. \square

Thus, if there were a separating equilibrium where the contract for agent type i earned positive expected profits, then it must be the case that a type j agent, $j \neq i$, must be indifferent between the type i contract and the type j contract. Otherwise, some lender could offer an alternative contract which type i strictly prefers and which is incentive compatible for type j .

Proposition 3 *In a separating equilibrium, (5) holds with equality for $i = g, b$.*

Proof: Suppose that (5) is a strict inequality for $i = b$. Then, from the previous lemma, (4) holds with equality for $i = g$. Consider the alternative pooling contract $R^{**}(x) = \delta R_b(x)$, where $0 < \delta < 1$, with δ sufficiently close to 1. This contract is strictly preferred by both

types of borrowers and, we have ,

$$\alpha \int_0^1 R_b(x) dF_g(x) + (1 - \alpha) \int_0^1 R_b(x) dF_b(x) > r.$$

Therefore, for δ sufficiently close to 1, $R^*(x)$ satisfies (15)-(18), a contradiction.

Now, suppose (5) is a strict inequality for $i = g$. Since (5) holds with equality for $i = b$, and since $\pi_b = 0$, we have $\int_0^1 R_b(x) dF_b(x) = r$. Therefore, since $\mu_i > r$, the above lemma and (4) for $i = b$ imply that $\pi_g < 1$. Now, consider an alternative separating contract for type g , $[R_g^*(x), \pi_g^*]$, with $R_g^*(x) = \delta R_g(x)$ for all x , and $\pi_g^* > \pi_g$. For properly chosen π_g^* and δ , this alternative contract can be constructed so that (9)-(14) hold for $i = g$, a contradiction. \square

The above proposition states that expected profits on each separating contract must be zero in equilibrium. Given separating contracts which are incentive compatible but which earn positive expected profits for a lender, an alternative contract exists which, if offered, earns nonnegative expected profits and is incentive compatible.

Lemma 4 *In a separating equilibrium, $\pi_g > 0$.*

Proof: Suppose not. Then, given Lemma 1, Proposition 3, and (4), we have $\int_0^1 R_g(x) dF_g(x) \leq \int_0^1 R_g(x) dF_b(x)$, a contradiction given (1) and the monotonicity of $R_g(x)$. \square

Given Lemma 1 and Lemma 3, any separating equilibrium must involve screening for type g borrowers, with no screening for type b borrowers. Without screening of type g borrowers, it is not possible to achieve self selection while satisfying the zero expected profit conditions. However, screening of type b borrowers simply wastes resources.³

Lemma 5 *Condition (4) holds with equality for $i = b$.*

Proof: Suppose not. Then, given Proposition 3, Lemma 1, and Lemma 4, there exists an alternative separating contract for type g , $[R_g^*(x), \pi_g^*]$, with $R_g^*(x) \leq R_g(x)$, $x \in [0, 1]$, and $\pi_g^* < \pi_g$, such that (9)-(14) hold. \square

³De Meza and Webb (1988) also obtain the result that good types are screened and bad types are not, in a model with costly screening and adverse selection.

The above lemma states that the incentive constraint must be binding for a type b borrower. Type g borrowers are screened with positive probability for the purpose of preventing type b agents from falsely reporting their type to be g . Since the type g borrowers effectively incur the expected screening costs, given that each contract earns zero expected profits in equilibrium, type g agents can always be made better off with a lower screening probability and preferable payment schedule if the incentive constraint for type b borrowers is not binding.

Now, since $\pi_b = 0$ in a separating equilibrium and (5) holds with equality for $i = b$, the expected utility of type b borrowers is effectively fixed in a separating equilibrium at $\mu_b - r$. As type g borrowers bear the expected screening costs for their type, to find separating contracts which are optimal (that is, immune from alternative separating contracts which earn nonnegative expected profits while making some agents better off) we need to minimize the verification probability π_g subject to the binding incentive compatibility constraint for type b borrowers and the zero expected profit constraint for type g borrowers. This will serve to relax the incentive constraint for type g borrowers as much as possible. We will ignore (4) for $i = g$ in solving for the optimal contract, and then show that, given the optimal contract, this constraint is not binding. Thus, the separating contract for type $i = g$ is determined by choosing $R_g(x)$ and π_g to solve:

$$\max(-\pi_g)$$

subject to

$$\int_0^1 R_g(x) dF_g(x) = r + \pi_g \gamma, \quad (28)$$

$$r = (1 - \pi_g) \int_0^1 R_g(x) dF_b(x) + \pi_g \mu_b, \quad (29)$$

(2) and (3). From the previous analysis, the separating contracts which solve this problem then only need to be checked to insure that there is not a feasible, monotonic pooling contract which makes each type of borrower better off while earning nonnegative expected profits. We are now ready to prove the main proposition of this section.

Proposition 6 *If a separating equilibrium exists, the unique equilibrium separating contract for type $i = g$ is a standard debt contract. That is, $R_g(x) = \bar{R}_g, x \in [\bar{R}_g, 1]; R_g(x) = x, x \in [0, \bar{R}_g]$ for some $\bar{R}_g \in (0, 1)$.*

Proof: The Lagrangian for the above optimization problem is

$$\begin{aligned} \mathcal{L} = & -\pi_g + \lambda_1[r + \pi_g\gamma - \int_0^1 R_g(x)f_g(x)dx] \\ & + \lambda_2[-r + \pi_g\mu_b + (1 - \pi_g) \int_0^1 R_g(x)f_b(x)dx], \end{aligned}$$

and we want to choose $\lambda_i \neq 0, i = 1, 2, \pi_g \in [0, 1]$, and $R_g(x)$ to maximize \mathcal{L} subject to (2) and (3). We can rewrite \mathcal{L} as

$$\begin{aligned} \mathcal{L} = & -\pi_g + \int_0^1 \left[-\frac{f_g(x)}{f_b(x)} + \frac{\lambda_2}{\lambda_1}(1 - \pi_g) \right] \lambda_1 f_b(x) R_g(x) dx \\ & + \lambda_1(r + \pi_g\gamma) + \lambda_2(-r + \pi_g\mu_b). \end{aligned} \quad (30)$$

If $\lambda_1 > 0$ and $\lambda_2 < 0$ or if $\lambda_1 < 0$ and $\lambda_2 > 0$, then maximizing (30), we get $R_g(x) = 0$, which violates (28). If $\lambda_1 < 0$ and $\lambda_2 < 0$, then maximizing (30), we get $\pi_g = 0$, which violates Lemma 4. We therefore have $\lambda_i > 0, i = 1, 2$. Now, given optimal $\lambda_i, i = 1, 2$, and π_g , we must have $-\frac{f_g(x)}{f_b(x)} + \frac{\lambda_2}{\lambda_1}(1 - \pi_g) < 0$ for some $x \in (0, 1)$, otherwise $R_g(x) = x$ is optimal for all $x \in [0, 1]$, and (29) is violated. Similarly, we must have $-\frac{f_g(x)}{f_b(x)} + \frac{\lambda_2}{\lambda_1}(1 - \pi_g) > 0$ for some $x \in (0, 1)$, otherwise $R_g(x) = 0$ is optimal for all $x \in (0, 1)$, and (28) is violated. Therefore, since $-\frac{f_g(x)}{f_b(x)} + \frac{\lambda_2}{\lambda_1}(1 - \pi_g)$ is continuous and monotonic in x , there exists some $\bar{x} \in (0, 1)$, such that $-\frac{f_g(x)}{f_b(x)} + \frac{\lambda_2}{\lambda_1}(1 - \pi_g) > 0$ for all $x \in (0, \bar{x})$, $-\frac{f_g(x)}{f_b(x)} + \frac{\lambda_2}{\lambda_1}(1 - \pi_g) = 0$ for $x = \bar{x}$, and $-\frac{f_g(x)}{f_b(x)} + \frac{\lambda_2}{\lambda_1}(1 - \pi_g) < 0$ for $x \in [\bar{x}, 1]$. Now, given the monotonicity of $R_g(x)$, it is clear that for optimality we must have $R_g(x) = R_g(\bar{x}), x \in [\bar{x}, 1]$. For $x \in [0, \bar{x}]$, given constraints (2) and (3), the optimal payment schedule is $R_g(x) = x, x \in [0, R_g(\bar{x})]$ and $R_g(x) = R_g(\bar{x}), x \in [R_g(\bar{x}), \bar{x}]$. The proof is concluded by letting $\bar{R}_g = R_g(\bar{x})$. \square

Given the above proposition, the separating contract for a type g borrower is characterized by (\bar{R}_g, π_g) , where \bar{R}_g is interpreted as the gross loan interest rate. From (28) and (29), \bar{R}_g and π_g are determined by the following two equations.

$$\int_0^{\bar{R}_g} x dF_g(x) + \bar{R}_g[1 - F_g(\bar{R}_g)] = r + \pi_g\gamma$$

$$r = (1 - \pi_g) \left\{ \int_0^{\bar{R}_g} x dF_b(x) + [1 - F_b(\bar{R}_g)] \right\} + \pi_g \mu_b$$

Alternatively, using integration by parts to simplify the above two equations, we get

$$\bar{R}_g - \int_0^{\bar{R}_g} F_g(x) dx = r + \pi_g \gamma \quad (31)$$

$$r = (1 - \pi_g) \left[\bar{R}_g - \int_0^{\bar{R}_g} F_b(x) dx \right] + \pi_g \mu_b \quad (32)$$

In the appendix, we show that there exists a unique $\bar{R}_g \in (0, 1)$ and a unique $\pi_g \in (0, 1)$ that solve (31) and (32).

Proposition 7 *In a separating equilibrium, given an optimal separating contract for type $i = g$ satisfying (31) and (32), an optimal separating contract for type $i = b$ is a debt contract characterized by $\bar{R}_b > \bar{R}_g$.*

Proof: We need only show that a debt contract for type $i = b$ satisfies the zero expected profit condition for $i = b$, and the incentive constraint for $i = g$. First, since $\mu_b > r$, there exists a unique $\bar{R}_b \in (0, 1)$ satisfying $\bar{R}_b - \int_0^{\bar{R}_b} F_b(x) dx = r$, so that (5) for $i = b$ is satisfied with equality. Second, since $\pi_g > 0$ in a separating equilibrium, and $\mu_b > r$, from (32) we must have $\bar{R}_g - \int_0^{\bar{R}_g} F_b(x) dx < r$. Now, given that $R - \int_0^R F_b(x) dx$ is an increasing function of R , we therefore have $\bar{R}_g < \bar{R}_b$, which in turn implies that (4) is satisfied for $i = g$. \square

From Propositions 6 and 7, debt contracts are optimal for both types in a separating equilibrium. A debt contract for type $i = g$ is the least attractive monotonic and feasible contract for type $i = b$, as it provides the largest rewards for a type g borrower in the upper tail of the probability distribution of returns, where type g has relatively more probability mass than type b . Thus, a debt contract for type g induces as much self selection as is possible, which minimizes the screening probability for type g , and maximizes the welfare of type g borrowers. For type g borrowers, a debt contract is the unique equilibrium contract. However, a debt contract is only one in a continuum of equilibrium contracts for type b borrowers. Any contract which, given the optimal type g contract, satisfies

the zero expected profit constraint for $i = b$ and the incentive constraint for $i = g$ is an equilibrium contract for type b . In fact, another equilibrium contract for type b is $R_b(x) = 0, x \in [0, x^*]; R_b(x) = x, x \in [x^*, 1]$, where x^* solves $\int_{x^*}^1 x dF_b(x) = r$. This contract is feasible, monotonic, and incentive compatible and it is therefore an equilibrium contract. However, it is as far removed as possible from a debt contract.

A separating equilibrium, if it exists, has some attractive features. First, as we showed above, debt contracts are equilibrium contracts for both types, which is consistent with the widespread use of debt contracts in practice. Second, the optimality of the debt contract for type $i = g$ arises because of the costs of screening borrowers. In credit markets, screening costs appear to be a large component of the costs incurred by financial intermediaries. This second feature is an advantage of this approach in explaining debt contracts, relative to the costly state verification approach. In costly state verification models, for example Townsend (1979), Gale and Hellwig (1985), or Williamson (1986, 1987), the existence of debt contracts is explained by costs to a lender of verifying the ex post return of a borrower. In equilibrium, these costs can be interpreted as bankruptcy costs, i.e. auditing costs. These costs would seem to represent a small fraction of the costs of advancing credit, relative to the fraction accounted for by screening costs.⁴ Third, since screening costs are incurred for the low interest rate contracts, these are the contracts which would be intermediated (with some further assumptions about the scale of investment projects), as we will show later. Thus, low quality borrowers pay high interest rates outside the intermediation sector, while high quality borrowers are served by financial intermediaries in equilibrium.

⁴Some casual evidence is the following. Our conversations with commercial loan officers suggest that, in a commercial loan operation, the ratio of those employed screening loans to those employed in “working out” loans (essentially getting all the bank can before a borrower fails) is in the range of 5 to 8.

5. CHARACTERIZATION OF THE POOLING EQUILIBRIUM

Analysis of the pooling equilibrium is more straightforward than for the separating case, as there is no screening here; that is $\pi_i = 0$ for all i . The (potential) equilibrium contract is then completely described by the payment schedule, $R(x)$, which both types of borrowers face.

The constraint (8) is satisfied with equality in a pooling equilibrium, since otherwise there exists another pooling contract $R^*(x)$ satisfying (24)-(27). That is, given a pooling contract earning positive expected profits, both agents can be made better off with another contract which is feasible, monotonic, and earns nonnegative expected profits. Thus, we can restrict attention to the set of pooling contracts which earn zero expected profits. If a contract within this set is an equilibrium, there can be no other pooling contract earning zero expected profits which makes both types better off, and no incentive compatible separating contract for type i that makes type i better off.

Lemma 8 *An equilibrium pooling contract $R(x)$ solves*

$$\min \int_0^1 R(x) dF_g(x)$$

subject to

$$\alpha \int_0^1 R(x) dF_g(x) + (1 - \alpha) \int_0^1 R(x) dF_b(x) = r, \quad (33)$$

(6), and (7).

Proof: We have already shown that an equilibrium pooling contract must satisfy (33). Suppose that the equilibrium pooling contract does not solve the above optimization problem. Then, we can have one of two cases. First, there may exist a contract $R^*(x)$ satisfying (6) and (7) which both types strictly prefer to $R(x)$, and which satisfies $\alpha \int_0^1 R^*(x) dF_g(x) + (1 - \alpha) \int_0^1 R^*(x) dF_b(x) = r$. Otherwise, there exists a separating contract $[R_g^*(x), 0]$ satisfying (19) and (20) which type g strictly prefers, while type b weakly prefers contract $R(x)$. This contract is constructed so as to satisfy $\alpha \int_0^1 R_g^*(x) dF_g(x) + (1 - \alpha) \int_0^1 R_g^*(x) dF_b(x) = r$. Now, since

$F_g(x) < F_b(x)$ for all $x \in (0, 1)$, and since $R_g^*(x)$ is monotonic, we have $\int_0^1 R_g^*(x) dF_g(x) > r$. Therefore, the contract $[R_g^*(x), 0]$ satisfies (19)-(23). \square

The above lemma states that an equilibrium pooling contract is the pooling contract which maximizes the welfare of type $i = g$ borrowers subject to the zero expected profit constraint. If this were not the case, then, either a pooling contract earning zero expected profits could be found which both borrower types strictly prefer, or there exists an incentive compatible separating contract for type g borrowers which earns positive expected profits while making type g borrowers better off.

Proposition 9 *If a pooling equilibrium exists, the pooling contract, $R(x)$, is a standard debt contract with $R(x) = \bar{R}, x \in [\bar{R}, 1]; R(x) = x, x \in [0, \bar{R}];$ for some $\bar{R} \in (0, 1)$.*

Proof: From the optimization problem in Lemma 8, form the Lagrangian

$$\mathcal{L} = - \int_0^1 R(x) dF_g(x) + \lambda [\alpha \int_0^1 R(x) dF_g(x) + (1 - \alpha) \int_0^1 R(x) dF_b(x) - r].$$

Then, the optimization problem is one of choosing $R(x)$ and $\lambda > 0$ to maximize \mathcal{L} subject to (6) and (7). Now, if $\lambda\alpha - 1 \geq 0$, then $R(x) = x$ would be optimal, but then (33) is not satisfied, so $\lambda\alpha - 1 < 0$. We can then rewrite \mathcal{L} as

$$\mathcal{L} = \int_0^1 \left[-\frac{f_g(x)}{f_b(x)} + \frac{\lambda(1-\alpha)}{(1-\lambda\alpha)} \right] R(x) (1 - \lambda\alpha) f_b(x) dx - \lambda r.$$

If $-\frac{f_g(x)}{f_b(x)} + \frac{\lambda(1-\alpha)}{(1-\lambda\alpha)} < 0$ for all $x \in [0, 1]$, then $R(x) = 0$ for all $x \in [0, 1]$ and (33) does not hold. Alternatively, if $-\frac{f_g(x)}{f_b(x)} + \frac{\lambda(1-\alpha)}{(1-\lambda\alpha)} > 0$ for all $x \in [0, 1]$, then $R(x) = x$ for all $x \in [0, 1]$, and (33) does not hold. Therefore, since $\frac{f_g(x)}{f_b(x)}$ is continuous and strictly increasing in x , there is some $x^* \in (0, 1)$ such that $-\frac{f_g(x)}{f_b(x)} + \frac{\lambda(1-\alpha)}{(1-\lambda\alpha)} > 0, x \in [0, x^*); -\frac{f_g(x)}{f_b(x)} + \frac{\lambda(1-\alpha)}{(1-\lambda\alpha)} < 0, x \in (x^*, 1]; -\frac{f_g(x)}{f_b(x)} + \frac{\lambda(1-\alpha)}{(1-\lambda\alpha)} = 0, x = x^*$. Therefore, given the feasibility and monotonicity constraints on $R(x)$, the optimal contract is $R(x) = x, x \in [0, \bar{R}]; R(x) = \bar{R}, x \in [\bar{R}, 1]$, for some $\bar{R} \in (0, 1)$. \square

The optimal gross loan interest rate, \bar{R} , is determined by the zero expected profit constraint. Substituting in (33) and using integration by parts to simplify, we get

$$\bar{R} - \int_0^{\bar{R}} [\alpha F_g(x) + (1 - \alpha)F_b(x)]dx = r. \quad (34)$$

The left side of (34) is continuous and strictly increasing in \bar{R} . In addition, the left side is less than r for $\bar{R} = 0$ and greater than r for $\bar{R} = 1$. Therefore, (34) has a unique solution in $(0, 1)$. From (34), a useful result is that

$$\frac{d\bar{R}}{d\alpha} = \frac{\int_0^{\bar{R}} [F_g(x) - F_b(x)]dx}{1 - F_b(\bar{R}) - \alpha[F_g(\bar{R}) - F_b(\bar{R})]} < 0. \quad (35)$$

As for an equilibrium separating contract, the optimal pooling contract is a standard debt contract. In contrast to the separating case, where debt was not the unique optimal contract for type $i = b$, here both borrower types face a debt contract which is a unique equilibrium. A debt contract is optimal in a pooling equilibrium, as it makes type g borrowers as well off as possible subject to the constraint that the pooling contract earns zero expected profits. That is, the debt contract minimizes the implicit subsidy that type g borrowers pay to type b borrowers.

6. EXISTENCE OF EQUILIBRIUM AND COMPARATIVE STATICS

In this section we show that, if an equilibrium exists, it is a separating equilibrium. We then explore how the parameters of the model, α , r , and γ , determine whether or not an equilibrium exists, and study how changes in the parameters affect equilibrium interest rates and screening probabilities.

In deriving the equilibrium separating and pooling contracts in the previous two sections, we omitted some conditions which are required for the existence of each type of equilibrium. For the separating equilibrium, we need to check that there exists no pooling contract satisfying (15)-(18), and for the pooling equilibrium, we need to check that there exists no separating contract satisfying (19)-(23).

Proposition 10 *A pooling equilibrium does not exist.*

Proof: Suppose a pooling equilibrium exists, which is characterized by the gross loan interest rate \bar{R} which solves (34). Now, consider a separating contract for type $i = g$ which is a debt contract $[\bar{R}_g^*, \pi_g^*]$. This contract is constructed to satisfy the zero expected profit constraint

$$\bar{R}_g^* - \int_0^{\bar{R}_g^*} F_g(x)dx = r + \pi_g^* \gamma, \quad (36)$$

and the incentive constraint for type $i = b$,

$$\hat{r} = (1 - \pi_g^*)[\bar{R}_g^* - \int_0^{\bar{R}_g^*} F_b(x)dx] + \pi_g^* \mu_b, \quad (37)$$

where $\hat{r} = \bar{R} - \int_0^{\bar{R}} F_b(x)dx < r$. Now, equations (36) and (37) yield a unique solution for \bar{R}_g^* and π_g^* , where, substituting for π_g^* in (37) using (36), we get an equation which solves for \bar{R}_g^* .

$$\bar{R}_g^* - \int_0^{\bar{R}_g^*} F_g(x)dx = r + \gamma \left[\frac{\hat{r} - \bar{R}_g^* + \int_0^{\bar{R}_g^*} F_b(x)dx}{\mu_b - \bar{R}_g^* + \int_0^{\bar{R}_g^*} F_b(x)dx} \right]. \quad (38)$$

Note that the left side of (38) is increasing and continuous in \bar{R}_g^* on $[0,1]$, while the right side is decreasing and continuous in \bar{R}_g^* on $[0,1]$. Therefore, given the values of the left and right sides of (38) at the endpoints of $[0,1]$, there exists a unique solution for \bar{R}_g^* . Now, note that, for $\bar{R}_g^* = \bar{R}$, the right side of (38) equals r , while the left side equals $\bar{R} - \int_0^{\bar{R}} F_g(x)dx > r$. Therefore, we have $\bar{R}_g^* < \bar{R}$. Thus, the alternative separating contract satisfies (19)-(23), a contradiction. \square

Therefore, given any potential pooling equilibrium, there always exists a separating contract which type $i = g$ borrowers strictly prefer, and which attracts only type $i = g$ borrowers while earning nonnegative expected profits. We can thus confine attention to the separating equilibrium.

For the separating equilibrium, which can be characterized by the loan interest rates faced by each type of borrower, $\bar{R}_i, i = g, b$, and the screening probability π_g , solving (31), (32),

and

$$\bar{R}_b - \int_0^{\bar{R}_b} F_b(x)dx = r, \quad (39)$$

we need only check that there is no pooling contract which is feasible, monotonic, earns nonnegative expected profits, and makes both types better off. Consider the pooling contract which is a debt contract characterized by \bar{R} solving (34). This contract clearly makes type b borrowers better off than with the separating contract, since $\bar{R}_b - \int_0^{\bar{R}_b} F_b(x)dx = r > \bar{R} - \int_0^{\bar{R}} F_b(x)dx$. The pooling contract also maximizes the expected utility of type g borrowers in the set of pooling contracts which earns nonnegative expected profits. Therefore, if type g agents do not strictly prefer the pooling contract \bar{R} to the separating contract \bar{R}_g , then the separating contracts characterized by $\bar{R}_i, i = g, b$, and π_g , constitute an equilibrium. That is, an equilibrium exists if and only if $\bar{R} \geq \bar{R}_g$.

We will now show how the parameters α , γ , and r determine existence of equilibrium, and how changes in these parameters affect equilibrium interest rates and the screening probability. First, to study the conditions under which an equilibrium exists, use (32) to substitute for π_g in (31) to obtain an equation which solves implicitly for \bar{R}_g .

$$\bar{R}_g - \int_0^{\bar{R}_g} F_g(x)dx = r + \gamma \left[1 - \frac{\mu_b - r}{\mu_b - \bar{R}_g + \int_0^{\bar{R}_g} F_b(x)dx} \right] \quad (40)$$

In (40), the left side is increasing and continuous in \bar{R}_g on $[0,1]$, while the right side is decreasing and continuous in \bar{R}_g on $[0,1]$. Now note, from (40), that \bar{R}_g is independent of α , while from (34), \bar{R} is decreasing in α . Substituting $\alpha = 1$ in (34), we obtain $\bar{R} = \bar{R}_1$ as the solution to

$$\bar{R}_1 - \int_0^{\bar{R}_1} F_g(x)dx = r. \quad (41)$$

Substituting $\bar{R}_g = \bar{R}_1$ into (40), the left side of (40) is equal to r , while the right side is greater than r . Therefore, we have $\bar{R}_g > \bar{R}_1$. Similarly, substituting $\alpha = 0$ in (34), we obtain $\bar{R} = \bar{R}_0$ as the solution to

$$\bar{R}_0 - \int_0^{\bar{R}_0} F_b(x)dx = r. \quad (42)$$

Now, substituting $\bar{R}_g = \bar{R}_0$ into (40), the left side of (40) is greater than r , while the right side is equal to r . Therefore, we have $\bar{R}_g < \bar{R}_0$. Thus, by continuity and (35), given any r and γ , there exists $\alpha^* \in (0, 1)$ such that an equilibrium exists for $\alpha \in (0, \alpha^*]$, and an equilibrium does not exist for $\alpha \in (\alpha^*, 1)$. Note here that, for any screening cost, no matter how large, if the fraction of type b borrowers in the population is sufficiently large, then an equilibrium exists where screening is worthwhile.

Next, note in (34) that \bar{R} is independent of γ , while in (40) \bar{R}_g is increasing in γ . If $\gamma = 0$, from (40) and (34) we have $\bar{R}_g < \bar{R}$, so that an equilibrium exists. Alternatively, as $\gamma \rightarrow \infty$, from (40) we get $r - \bar{R}_g + \int_0^{\bar{R}_g} F_b(x)dx \rightarrow 0$, or $\bar{R}_g \rightarrow \bar{R}_b > \bar{R}$. Therefore, an equilibrium does not exist as $\gamma \rightarrow \infty$. Thus, from continuity and monotonicity, for given r and α , there exists $\gamma^* > 0$ such that an equilibrium exists for $\gamma \in [0, \gamma^*]$, and an equilibrium does not exist for $\gamma > \gamma^*$. Thus, for any r and α , an equilibrium exists if the screening cost is sufficiently small.

We do not get clear results for how the interest rate r affects the existence of equilibrium. Here, from (34) and (40), changes in r affect both \bar{R} and \bar{R}_g , and the relative effects depend on the other parameters.

From (31), (32), and (39), it is clear that loan interest rates $\bar{R}_i, i = g, b$, and the screening probability π_g are independent of α in equilibrium. Thus, the relative fractions of borrower types in the population are irrelevant for equilibrium interest rates and the screening intensity. From (40), an increase in γ results in an increase in \bar{R}_g . Then, from (32), π_g must decrease when γ increases. Therefore, an increase in screening costs causes the screening technology to be used less intensively (the screening probability falls) and the loan interest rate faced by type g borrowers rises. Note that there is no change in the interest rate faced by type b borrowers, from (39).

To determine the effects of a change in r on $\bar{R}_i, i = g, b$, and on π_g , comparative statics

using (31), (32), and (39) gives

$$\frac{d\bar{R}_b}{dr} = \frac{1}{1 - F_b(\bar{R}_b)} > 0$$

$$\frac{d\bar{R}_g}{dr} = \frac{\mu_b - \bar{R}_g + \int_0^{\bar{R}_g} F_b(x)dx + \gamma}{[1 - F_g(\bar{R}_g)][\mu_b - \bar{R}_g + \int_0^{\bar{R}_g} F_b(x)dx] + \gamma(1 - \pi_g)[1 - F_b(\bar{R}_g)]} > 0$$

$$\frac{d\pi_g}{dr} = \frac{F_b(\bar{R}_g) - F_g(\bar{R}_g) + \pi_g[1 - F_b(\bar{R}_g)]}{[1 - F_g(\bar{R}_g)][\mu_b - \bar{R}_g + \int_0^{\bar{R}_g} F_b(x)dx] + \gamma(1 - \pi_g)[1 - F_b(\bar{R}_g)]} > 0$$

Thus, when the interest rate faced by lenders increases, interest rates on loans increase for both types of borrowers, and the screening probability increases for type g borrowers. The increase in loan interest rates makes the loan contract for type g borrowers look relatively more attractive to type b borrowers than the type b loan contract. Therefore, the verification probability must rise in order to induce self selection.

7. FINANCIAL INTERMEDIATION

Costly state verification models of debt contracts (Townsend 1979, Gale and Hellwig 1985, Williamson 1987a) can be extended in such a way that financial intermediation arises endogenously as an efficient means to economize on verification costs (Diamond 1984, Williamson 1986). The costly verification of returns on investment projects is delegated to a financial intermediary by its depositors, and the intermediary is able to guarantee certain returns to its depositors by diversifying its loan portfolio.

The model constructed here has some advantages over the costly state verification framework as a model of debt contracts, particularly in that debt contracts survive randomization, and because screening costs seem more important for credit market activity than do ex post verification costs (essentially auditing costs). Our model can also be extended to generate financial intermediation endogenously in a straightforward way. In fact, intermediaries here will play a “delegated screening” role which is analogous to the “delegated monitoring” role of a financial intermediary in a costly state verification environment.

We extend the model by changing the investment technology available to each borrower so that it requires $K > 1$ rather than one unit of the investment good to fund an investment project in period 1. This implies that more than one lender is needed to fund any one investment project. We retain the assumption that one borrower can meet at most one lender in period 1. Also, we assume that a given lender can observe the results of his/her screening only.

Suppose first that all lending is done directly. However, since one borrower can meet one lender at most, some coordination among lenders is required. Thus, let lenders who wish to contract with borrowers form coalitions of K members each. Coalition members agree in advance on nothing, except that they will offer their contracts, jointly meet one borrower, and then the borrower will report his/her type to the coalition. A Nash equilibrium has similar properties to the equilibrium for the model with $K = 1$, in that, if an equilibrium exists it is a separating equilibrium, only type g borrowers are screened, and debt contracts are equilibrium contracts. An equilibrium consists of promised payments R_i^D , $i = g, b$, and a screening probability for a type g agent, π_g^D , satisfying

$$rK = (1 - \pi_g^D)^K \left[R_g^D - \int_0^{R_g^D} F_b(x) dx \right] + \left[1 - (1 - \pi_g^D)^K \right] \mu_b \quad (43)$$

and

$$\frac{1}{K} \left[R_g^D - \int_0^{R_g^D} F_g(x) dx \right] = \pi_g^D \gamma + r. \quad (44)$$

Note, in the incentive constraint (43), that each lender in a coalition screens type g borrowers with probability π_g^D , and that a borrower who is denied a loan from any member of the coalition cannot fund his/her project.

Now, suppose that some lenders act as financial intermediaries. That is, they take deposits of the investment good in period 1, make loans to borrowers, and offer depositors returns contingent on the returns on their loan portfolios. Now, if a financial intermediary diversifies across a large number of borrowers, and borrows from a large number of depositors, by the law of large numbers it can guarantee each depositor a certain return in period 2 of r per

unit deposited in period 1. The optimal contract results derived in the previous sections apply here, so that debt contracts are equilibrium contracts for both types of borrowers (and the unique equilibrium contract for type g borrowers). An equilibrium consists of promised payments R_i^I , $i = g, b$, and a screening probability for a type g agent, π_g^I , satisfying

$$rK = (1 - \pi_g^I) \left[R_g^I - \int_0^{R_g^I} F_b(x) dx \right] + \pi_g^I \mu_b \quad (45)$$

and

$$R_g^I - \int_0^{R_g^I} F_g(x) dx = \pi_g^I \gamma + rK. \quad (46)$$

Now, suppose that financial intermediaries offer a contract to type g agents with $R_g^I = R_g^D$, and $\pi_g^I = \min(K\pi_g^D, 1)$. In the case where $K\pi_g^D \geq 1$, from (44), (45), and (46), this contract earns nonnegative expected profits, and the incentive constraint for a type b agent is satisfied as a strict inequality. Therefore, there is some alternative contract with $R_g^I < R_g^D$ and $\pi_g^I < 1$ that earns nonnegative expected profits and is incentive compatible. Therefore, direct lending is dominated in this case by intermediated lending. Alternatively, suppose that $K\pi_g^D < 1$. Then, the proposed contract earns zero expected profits, from (44) and (46), and from (43) and (45), it satisfies the incentive constraint for a type b borrower as a strict inequality if and only if

$$1 - \pi_g^D K < (1 - \pi_g^D)^K. \quad (47)$$

But (47) holds by induction, for any integer $K \geq 2$. Therefore, financial intermediation dominates in this case as well.

In moving from a direct lending arrangement to intermediated lending there are two effects. First, with intermediated lending, screening is delegated to the intermediary, so that, for a given screening probability, expected screening costs are reduced as screening is not replicated by several lenders. Second, since a loan can be denied if any one lender discovers cheating in the direct lending case, each individual lender can screen with a lower probability than with financial intermediation, and this will tend to make expected screening costs lower in the direct lending case than with financial intermediation. Inequality (47)

implies that the first effect outweighs the second, so that financial intermediation is more efficient than direct lending.

Note here that the only contracts which need be intermediated are those to type g borrowers, as these are the contracts for which costly screening takes place. Therefore, this framework yields a financial structure where some lending is intermediated while some lending is not. Again, this is an advantage over related costly state verification environments (e.g Williamson 1986), in that those models typically have all lending intermediated.

8. CONCLUSION

We have developed an adverse selection model of a credit market, where self selection is achieved with risk neutral borrowers through the use by lenders of a costly screening technology. In some ways, the model has features that are similar to the adverse selection model of Rothschild and Stiglitz (1977), where self selection occurs with risk averse agents who have different preferences. For example, if an equilibrium exists in our model, it is a separating equilibrium where good borrowers submit to screening with positive probability. Also, for some parameter values an equilibrium does not exist.

In this model, the unique equilibrium contract for a good borrower is a debt contract. Given the characteristics of the probability distributions of investment returns faced by borrowers, a debt contract is relatively unattractive for bad borrowers, so that a debt contract for good borrowers achieves self selection while minimizing the screening probability for good borrowers, and therefore minimizes expected screening costs. Debt contracts are also equilibrium contracts for bad borrowers if an equilibrium exists, but then there exists a continuum of equilibrium contracts for bad borrowers which are not debt contracts. We conjecture that, if the model were extended to include many types of borrowers whose investment projects are ordered in the same way as they are here, then debt contracts would be optimal for each type except the lowest ranked.

Comparative statics results show that equilibrium loan interest rates and the screening

probability increase as the risk free interest rate faced by lenders increases. Also, an increase in the screening cost results in a decrease in the screening probability and an increase in loan interest rates. An equilibrium exists for sufficiently low screening costs and given a sufficiently large fraction of bad borrowers in the population. Somewhat surprisingly, for any positive screening cost, no matter how large, an equilibrium exists provided the fraction of bad borrowers in the population is large enough.

This model has at least two advantages over the alternative costly state verification model, which also can produce debt contracting as an optimal financial arrangement. First, debt contracts survive here even when random screening is permitted; costly state verification models do not yield debt contracts with random verification strategies. Second, ex ante screening costs would appear to be much more important for the functioning of credit markets than ex post auditing costs. In addition to having these advantages, our model is capable of generating intermediary structures, just as in costly state verification setups (e.g. Williamson 1986). Here, if borrowers' investment projects are large in scale relative to a lender's endowment, it is optimal for perfectly diversified financial intermediaries to be delegated the role of screening borrowers, and this is also an equilibrium financial arrangement.

Given that this model can generate debt contracting and intermediary structures, it potentially has many applications. For example, the model could be extended to study the effects of deposit insurance on the portfolio behavior and screening behavior of banks. It is often asserted that deposit insurance causes banks to take on too much risk and to expend too little effort in screening borrowers. An extension of the model with aggregate risk could be used to evaluate this assertion. Another possible extension would be to embed the model in a dynamic framework to study cyclical behavior. Here, we could make the relative numbers of good and bad borrowers endogenous by giving these agents some alternative to operating their own projects. The endogenous mix of borrower types in the population could then produce an interesting propagation mechanism, and allow us to study "credit crunches."

APPENDIX

Proposition 11 *Condition (1) implies that $F_g(x) < F_b(x), x \in (0, 1)$.*

Proof: First, note that

$$\int_0^1 \left[\frac{f_g(y)}{f_b(y)} - 1 \right] f_b(y) dy = 0. \quad (48)$$

Now, (1) and (48) imply that $\frac{f_g(0)}{f_b(0)} - 1 < 0$ and $\frac{f_g(1)}{f_b(1)} - 1 > 0$. Therefore, there exists $\bar{x} \in (0, 1)$ such that $\frac{f_g(x)}{f_b(x)} - 1 < 0, x \in [0, \bar{x}]$ and $\frac{f_g(x)}{f_b(x)} - 1 > 0, x \in [\bar{x}, 1]$. Therefore, we have

$$F_g(x) - F_b(x) = \int_0^x \left[\frac{f_g(y)}{f_b(y)} - 1 \right] f_b(y) dy < 0. \quad (49)$$

□

Proposition 12 *Given $\mu_b > r$ and $\gamma > 0$, there exists a unique $\bar{R}_g \in (0, 1)$ and a unique $\pi_g \in (0, 1)$ that solve (31) and (32).*

Proof: First, solving (32) for π_g in terms of \bar{R}_g , we get

$$\pi_g = 1 - \frac{\mu_b - r}{\mu_b - \bar{R}_g + \int_0^{\bar{R}_g} F_b(x) dx}. \quad (50)$$

Thus, from (50), given \bar{R}_g we get a unique solution for π_g . Next, substitute for π_g in (31) using (50) to get

$$\bar{R}_g - \int_0^{\bar{R}_g} F_g(x) dx = r + \gamma \left[1 - \frac{\mu_b - r}{\mu_b - \bar{R}_g + \int_0^{\bar{R}_g} F_b(x) dx} \right]. \quad (51)$$

Now, note that the left side of (51) is increasing and continuous in \bar{R}_g on $[0, 1]$, and that right side of (51) is decreasing and continuous in \bar{R}_g on $[0, 1]$. For $\bar{R}_g = 0$, the left side of (51) equals 0, while the right side equals $r(1 + 1/\mu_b) > 0$. For $\bar{R}_g = 1$, the left side of (51) equals $\mu_g > r$, while the right side goes to $-\infty$ as $\bar{R}_g \rightarrow 1$. Therefore, there exists a unique solution to (51) for \bar{R}_g in the interval $(0, 1)$. □

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