Methodology and Implementation of Value-at-Risk Measures in Emerging Fixed-Income Markets with Infrequent Trading.

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Abstract

This paper deals with the issue of calculating daily Value-at-Risk (*VaR*) measures within an environment of thin trading. Our approach focuses on fixed income portfolios with low frequency of transactions in which the missing data problem makes *VaR* measures difficult to calculate. We propose and implement a methodology to calculate *VaR* measures with an incomplete panel of prices. The methodology is composed of three phases: Phase I, generates a complete panel of prices, using a term-structure dynamic model of interest rates. Phase II, calculates portfolio *VaR* measures with several alternative methods using the complete panel data generated in phase I. Phase III, shows how to back-test the *VaR* measures obtained in phase II using the original incomplete panel of prices. We provide an empirical implementation of the methodology for the Chilean fixed income market. The proposed methodology seems to provide reliable *VaR* measures for thinly traded markets addressing an important issue for financial risk management in emerging markets.

JEL: C51, C52, G11, G15

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I Introduction.

One important concern of financial institutions is measuring market risks. Moreover, regulatory agencies are requiring them to periodically report risk exposures in order to set up the required capital levels.[1] One of the risk measurement procedures which is becoming a the-facto international standard is the Value-at-Risk (*VaR*), initially proposed to measure only market risks, but later used also for others such as credit and operational risk.

The *VaR* uses econometric techniques to measure the probable loss in value of an investment, within a time interval, under normal market conditions and for a given confidence level[2]. The risk is expressed in money units, which is a simple and easy to understand metric.

In many emerging markets there is an added difficulty for calculating this measure because of the missing data problem associated with thin trading. In these markets assets do not trade every day, thus price panels are incomplete and *VaR* calculations using the traditional methodologies are impossible to perform.

Previous research in this issue is scarce, including Chernobai, Menn, Trück y Rachev (2005) who addressed the problem of incomplete data, but only with an approach towards operational *VaR*. A similar approach is provided by Moscadelli, Chernobai and Rachev (2005). However, to our knowledge, there is no literature related with the evaluation of market risks in thinly traded fixed income markets, which is the focus of this paper.

In this paper we address the issue of how to compute and back-test a *VaR* market risk measurement in a thinly traded fixed income market. We also provide an empirical implementation of the proposed methodology for the Chilean fixed income market. The proposed methodology seems to provide reliable *VaR* measures for thinly traded markets addressing an important issue for financial risk management in emerging markets.

The paper is organized as follows. The next Chapter briefly explains the *VaR* concept. Chapter III outlines the proposed methodology. Chapter IV discusses the econometric approach. Chapter V presents the data. Chapter VI reports empirical results and their interpretation. Finally, chapter VII concludes.

II The Concept of Value-at-Risk.

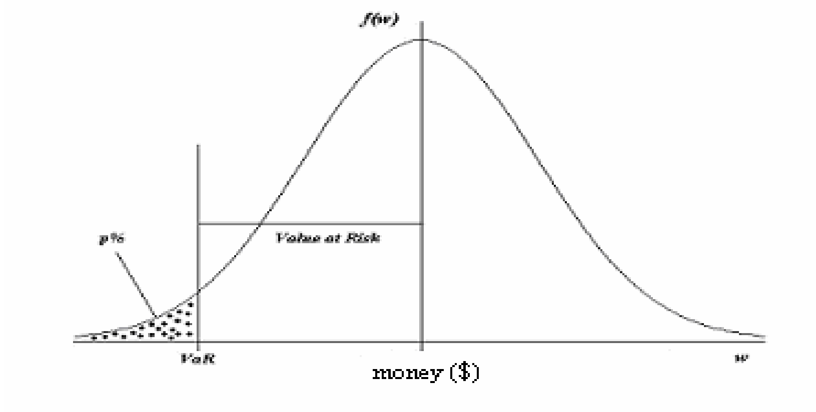
Value-at-Risk is a measure used to estimate how much the value of an asset could decrease over a certain time period for a given confidence level.

Let $w_{t+\Delta t,t}$ be the outcome (in monetary units) of an investment resulting from a price variation in time interval Δt, and $f(w_{t+\Delta t,t})$ the distribution function of the variations of prices for the investment (which is not necessarily known). The *VaR* of an asset (or portfolio of assets), is the quantity of money that could be lost from negative events which occurs with probability 'p' or more (Figure 2.1).

[1] See Jackson, Maude and Perraudin (1997).

[2] A $VaR_{5\%}$= \$-100,000 is equivalent to say that in 5% of the times, there is an expected loss of \$100,000 or more in such investment.

Figure 2.1: Distribution Function of the Variation in Value of an Investment and the *VaR* concept.



The *VaR* can be computed as:

$$P(w_{t+\Delta t,t} \leq VaR_{t+\Delta t,t}) = p \tag{1}$$

then,

$$\int_{-\infty}^{VaR_{t+\Delta t,t}} f(w_{t+\Delta t,t})dw = p \tag{2}$$

To compute the *VaR* we could follow different approaches. The first one, known as *parametric*, adjusts the historical returns of an investment to a known distribution, i.e. a Normal. Once the parameters of the assumed distribution are estimated the *VaR* can be computed using the assumed distribution.

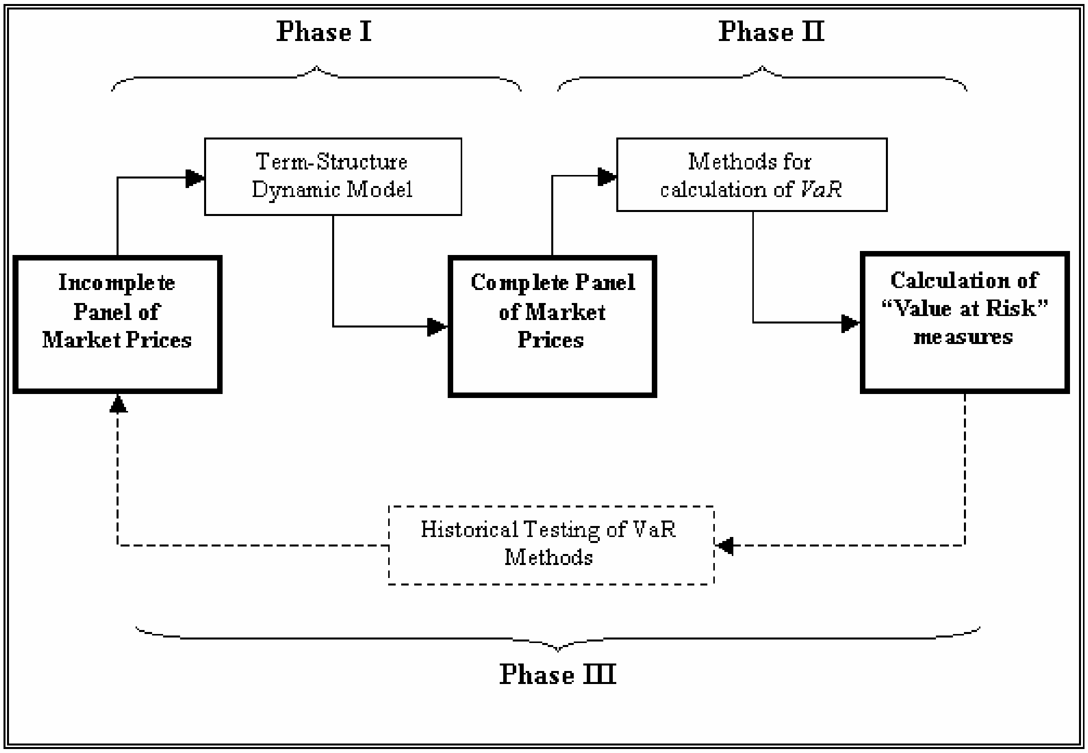
The second one is the historical simulation which is *non-parametric*, and does not assume any distribution of returns, thus no parameter estimations are necessary.

Finally, the Monte Carlo simulation use elements of both of the preceding ones. First, it is necessary to generate a stochastic process to simulate future events. Process parameters are estimated using historical data. After the simulation is performed the *VaR* is calculated.

III The Methodology.

The following proposed methodology is one of the first attempts in the literature to calculate *VaR* measures for a thinly traded market. The methodology is composed of three phases. The first two phases are focused in calculating the *VaR* measures, and the last proposes the back-testing procedure to check for the reliability of the proposed methodology (Figure 3.1).

Figure 3.1: Scheme of the Three-Phase methodology to deal with the problem of calculating and back-testing daily *VaR* measures in thinly traded fixed income markets



3.1 Phase I (Generation of a Complete Panel Data of Prices).

In the case of a thinly traded market, we propose first to generate a complete data panel which later will be used to compute the *VaR* measures. Previous research in emerging fixed income markets (Cortazar, Schwartz and Naranjo, 2003) has shown that dynamic term-structure models are much better than static models (Nelson y Siegel, 1987, Svensson, 1994) for computing missing prices. This is particularly true when we are concerned with obtaining reliable volatility estimates, which is the case when our goal is to compute *VaR* estimates.

We propose choosing a multifactor dynamic term-structure model and calibrate it with the incomplete panel of market prices. It must be noted that Kalman Filter estimation procedures may be used with incomplete panel data and consistent volatility term structure estimates are obtained (Cortazar, Schwartz and Naranjo, 2003). Once the model is estimated we can then compute discount factors for all maturities and construct a complete panel of "model" prices, which we will consider the security "fair" prices.

Then, after we have the complete panel, we have two options in order to calculate the *VaR* measures:

- Take the available prices for the instruments when they were traded and "fill" the holes for the days in which the instruments were not traded with the calculated "fair" prices. This panel would be a mixed one, with actual observed prices for days in which trade was observed, and calculated "fair" prices for days in which trade was absent.

- Use a panel which includes only "fair" calculated prices.

We will later argue why in our in our implementation it is better to use the second panel.

3.2 Phase II (Estimation of "Value at Risk" measures).

Once we have a complete panel of prices, we are able to calculate the *VaR* measures for each asset individually, and for a portfolio of assets as a whole. In this phase, we will calculate daily *VaR* measures using different methods proposed in the literature. We outline the methods used in chapter IV.

3.3 Phase III ("Back Test").

Once we have performed the calculations of *VaR* measures, we proceed to back-test them using historic data. Here, we intend to obtain conclusions regarding two main issues. First, we want to find the coherence of the *VaR* measures obtained with the proposed methodology[3]. Second, we pretend to find which *VaR* calculation method provides a better measurement of market risk for the tested portfolio. The specific tests used are detailed in chapter IV.

IV The Econometric Approach.

Until now, we have conceptualized the sequence of the proposed methodology in order to calculate daily *VaR* measures in a thinly traded fixed-income market. However, to implement our methodology we must choose an econometric approach. First, we choose a dynamic term-structure model and an estimation method to generate a complete panel of "fair" bond prices. Second, we choose the *VaR* estimation methods that will be used in order to calculate the daily *VaR* measures with the "fair" panel of prices. Finally, it is necessary to choose what type of back-testing techniques will be used to asses the reliability of the *VaR* estimations.

We present in this section the econometric techniques that will be used in this study. However, the proposed methodology is open and could be implemented with any of the available econometric techniques.

4.1 The Dynamic Term-Structure Model.

In order to estimate the yield curve needed to generate a complete panel data of "fair" prices, we choose the approach proposed by Cortazar, Schwartz and Naranjo (2003). They provide a method to jointly estimate the current term structure and its dynamics for markets with low-frequency transactions. They use a three-factor generalized-Vasicek model[4] and estimate the model using the Kalman filter with missing data.

With this approach we are able to obtain an estimate of the current term structure even for days with an arbitrary low number of price observations. In what follows we provide a brief description of the approach (see Cortazar, Schwartz and Naranjo, 2003 for a detailed explanation).

[3] A coherent measure expects that the percentage of times in which a losses exceeded the calculated *VaR*, do not exceed the confidence level '*p%*' under which these measures were calculated.
[4] A generalized Vasicek model is a dynamic multifactor mean-reverting Gaussian model of the instantaneous spot interest rate which extends the classic Vasicek (1977). In Vasicek (1977) the interest rate follows an Ornstein-Uhlembeck process and therefore is assumed to revert to a long-run mean.

First, three stochastic unobservable mean-reverting factors called “state variables” are defined. These state variables may be represented with the 3x1 vector $\mathbf{x}_t$. Let δ be a constant. Then, the instantaneous interest rate, $\breve{r}_t$, may be defined as:

$$\breve{r}_t = \mathbf{1}'\mathbf{x}_t + \delta \tag{3}$$

Let the vector of state variables $\mathbf{x}_t$, follow a multifactor Vasicek type process, governed by the following stochastic differential equation:

$$d\mathbf{x}_t = -\mathbf{K}\mathbf{x}_t dt + \mathbf{\Sigma} d\mathbf{w}_t \tag{4}$$

where $\mathbf{K}$=diag(κ_i) and $\mathbf{\Sigma}$=diag(σ_i) are 3x3 diagonal matrices with entries that are strictly positives constants and different. Also, dw_t is a 3x1 vector of correlated Brownian motion increments such that:[5]

$$(d\mathbf{w}_t)'(d\mathbf{w}_t) = \mathbf{\Omega} dt \tag{5}$$

where the (i,j) element of $\mathbf{\Omega}$ is $\rho_{ij} \in [1,-1]$, the instantaneous correlation of the state variables i and j.[6] Under this specification, the state variables have the multivariate normal distribution and each of them reverts to 0, at a mean reversion rate given by k_i.[7] Thus, according to equation (3) the instantaneous interest rate, $\breve{r}_t$, reverts to a long-run mean given by the constant δ.

Cortazar, Schwartz and Naranjo (2003) show that assuming a constant 3x1 vector of market price of risk, $\boldsymbol{\lambda}$, the price of any pure-discount bond is:

$$P(\mathbf{x}_t, \tau) = \exp\left(\mathbf{u}(\tau)'\mathbf{x}_t + v(\tau)\right) \tag{6}$$

where

$$u_i(\tau) = -\frac{1-\exp(-k_i\tau)}{k_i} \tag{7}$$

$$v(\tau) = \sum_{i=1}^{N} \frac{\lambda_i}{k_i}\left(\tau - \frac{1-\exp(-k_i\tau)}{k_i}\right) - \delta\cdot\tau + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{\sigma_i\sigma_j\rho_{ij}}{k_i k_j}\left(\tau - \frac{1-\exp(-k_i\tau)}{k_i} - \frac{1-\exp(-k_j\tau)}{k_j} + \frac{1-\exp(-(k_i+k_j)\tau)}{k_i+k_j}\right) \tag{8}$$

and the equivalent annualized spot rate, is:

$$\breve{R}(x_{t,}\tau) = -\frac{1}{\tau}\left(u(\tau)' x_{t,} + v(\tau)\right) \tag{9}$$

which is a linear function of the state variables. Therefore, under the generalized Vasicek model, spot rates also have the Gaussian distribution.

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[5] A Brownian motion is a continuous-time stochastic process with the properties that between any two dates s and t (s > t), the increment $Z_s - Z_t$ has a normal distribution with mean zero and variance of s – t and the increment is independent of the value of the process at all dates prior to t.

[6] As we are working with three stochastic factors or state variables, we will have the following correlation coefficients: $\rho_{12}, \rho_{13}, \rho_{23}$

[7] In a mean reverting model, every perturbation is on average reduced by half in $\log(2)/k_i$ units of time.

Cortazar, Schwartz and Naranjo (2003) propose to estimate the chosen dynamic model of interest rates using the Kalman filter. A methodology which recursively calculates optimal estimates of the unobservable state variables contained in vector $\mathbf{x}_t$, given all the information available up to some moment in time. In addition, by using Maximum Likelihood methods, consistent estimates of model parameters may be obtained.

To apply the Kalman filter methodology, it is necessary to represent the problem in an state-space approach. The state-space representation includes measurement and transition equations.

The measurement equation relates a vector of observable variables $\mathbf{z}_t$ with a vector of state variables $\mathbf{x}_t$, which in general is not observable, as follows:

$$\mathbf{z}_t = \mathbf{H}_t \mathbf{x}_t + \mathbf{d}_t + \mathbf{v}_t \qquad \mathbf{v}_t \sim N(\mathbf{0}, \mathbf{R}_t) \tag{10}$$

We must recall that the standard Kalman filter assumes a fixed number of observable variables at each time. However, Cortazar, Schwartz and Naranjo (2003) relax this assumption in order to allow for missing observations. So, let m_t be the number of observations available at time t, $\mathbf{z}_t$ is a $m_t x1$ vector, $\mathbf{H}_t$ is a $m_t x3$ matrix, $\mathbf{x}_t$ is a 3x1 vector, $\mathbf{d}_t$ is a $m_t x1$ vector, and $\mathbf{v}_t$ is a $m_t x1$ vector of serially uncorrelated Gaussian disturbances with mean $\mathbf{0}$ and covariance matrix $\mathbf{R}_t$ with dimensions m_t x m_t.

The transition equation describes the dynamics of the state variables:

$$\mathbf{x}_t = \mathbf{A}_t \mathbf{x}_{t-1} + \mathbf{c}_t + \boldsymbol{\varepsilon}_t \qquad \boldsymbol{\varepsilon}_t \sim N(\mathbf{0}, \mathbf{Q}_t) \tag{11}$$

where $\mathbf{A}_t$ is a *3x3* matrix, $\mathbf{c}_t$ is a *3x1* vector, and $\boldsymbol{\varepsilon}_t$ is a *3x1* vector of uncorrelated Gaussian disturbances with mean $\mathbf{0}$ and covariance matrix $\mathbf{Q}_t$. Equations (10) and (11) define the state-space representation.

Once the state-space representation of the model is obtained an extended Kalman filter, which accounts for the missing data and nonlinearities arising from the use of coupon bonds, is applied to calibrate the model.

The calibrated model provides estimates of interest rates for all maturities for each day. Therefore, using this estimated yield curve, "fair" prices for every day *t* and all instruments in the portfolio may be computed.

4.2 The *VaR* Estimation Methods.

In order to calculate the daily *VaR* measures, we will use the following methods:[8]

- Parametric methods in a world of multi-normal distributions (methods of the variance-covariance matrix):
    - Method of the sample variance and covariance, with a window of 250 days for the estimations.
    - Method of exponential decay in the Risk-Metrics version, with a window of 250 days for the estimations.

[8] Here, we only mention the different methods of *VaR* calculation. However, refer to Annex A for a detailed explanation of each method.

- GARCH(1,1) using a variance-covariance matrix decomposition, with a window of 250 days for the estimations.

- Parametric methods accounting for asymmetric and multi-kurtosis effects:
  - Use of t-student distribution, with a window of 250 days for the estimations.
  - "Extreme Value Theory" in its static version, with a window of 400 days for the estimations.[9]
  - "Extreme Value Theory" in its dynamic version, with a window of 400 days for the estimations.
- Non parametric method of historical simulation, with a window of 250 days for the estimations
- Monte Carlo simulation method, using the stochastic structure developed in Cortázar, Schwartz and Naranjo (2003) with a three-factor Vasicek model.

4.3 Back-Testing.

4.3.1 Alternative procedures.

The first relevant measure for Back-testing the calculated *VaR* measures is computing the percentage of times in which a loss has been greater than the calculated *VaR*. By definition, this measure should not be significantly different from the confidence level '*p*' under which the *VaR* measure was calculated. However, to analyze daily *VaR* measures, we would need sets of two consecutive days where trading was observed in order to account for a daily win or loss in the value of the asset. But in a thinly traded market scenario this is not observed frequently. Therefore, it is necessary to develop an ad-hoc testing procedure for the case of thin trading.

One possibility would be to use a reduced sub-sample which includes only consecutive trading days. However, if trading is very thin, this sub-sample would be very small and will require discarding too many data.

A second back-testing procedure that uses all data transactions considers computing the *VaR* measure for periods greater than one day by assuming the following process for asset returns:

$$G = \ln(P) \tag{12}$$

$$dG = (u - \frac{\sigma^2}{2})dt + \sigma dz \tag{13}$$

$$dz = \zeta\sqrt{dt} \qquad \zeta : N(0,1) \tag{14}$$

where u y σ^2 are the mean and variance of returns, P is the price of the asset, t is the time, dz is a Brownian Motion, and ζ is distributed $N(0,1)$. Thus:

$$\Delta G = r_{t+1,t} = \ln(\frac{P_{t+1}}{P_t}) = (u - \frac{\sigma^2}{2})\Delta t + \sigma\zeta\sqrt{\Delta t} \tag{15}$$

[9] As we are modeling extreme observations. We use a window of 400 days instead of 250. We make this because, in this case, with a window of 250 observations estimations are poor because few extreme observations might been realized.

where $r_{t+1,t}$ is the daily return. If we further assume that we invest an amount of money *M* in this asset, we could estimate the variation in value of the asset for tomorrow ($w_{t+1,t}$) as:

$$w_{t+1,t} = \left(e^{(u-\frac{\sigma^2}{2})+\sigma\zeta} - 1 \right) M \tag{16}$$

To estimate the amount of money such that the probability of a loss exceeding that amount would be *p*, we can calculate the inverse of the normal distribution *N(0,1)* for the *p* probability and replace it with *ζ*. We will denote this inverse function with *α*. So this expected loss is the daily *VaR* for a *p* confidence level, as follows:

$$VaR_{t+1,t} = \left(e^{(u-\frac{\sigma^2}{2})+\sigma\alpha} - 1 \right) M \tag{17}$$

Now, if we follow this procedure in which *Δt* could be any period of time (not only one day), then, with *α* obtained from a *N(0,1)*, the *VaR,* for different periods of time is computed as follows:

$$VaR_{t+\Delta t,t} = \left(e^{(u-\frac{\sigma^2}{2})\Delta t+\sigma\alpha\sqrt{\Delta t}} - 1 \right) M \tag{18}$$

This analysis presents, however, some problems. First, empirical evidence suggests (Mandelbrot, 1963, and Fama,1965) that daily logarithmic returns do not follow a normal distribution. Furthermore, recent studies (Dacorogna et.al.,1999) show that returns calculated over longer periods of time, departs more heavily from normality. Therefore, the previous methodology looses reliability as missing data increases.

In addition it seems somehow contradictory to use the normality assumption to handle missing data problems while we use methods like “Extreme Value” because we do not believe that this assumption is consistent with the data.

Therefore, in what follows a simple but efficient method is proposed.

4.3.2 Proposed procedure.

Let a certain amount of money *M* be invested in a single asset which was traded during day *‘t’*, therefore, for that day a price was observed. However, the next day in which the asset was traded was not day *‘t+1’*, but day *‘t+d+1’*.

As mentioned before, this study deals with the calculation of daily *VaR* measures. Therefore it may be interesting to compare the *VaR* for one day ($VaR_{t+d+1,t+d}$), under certain probability level ‘*p*’, with the money won or lost for also one day ($w_{t+d+1,t+d}$):

$$P\left(w_{t+d+1,t+d} \leq VaR_{t+d+1,t+d}\right) = p \tag{19}$$

Therefore, if phase I and II are conducted, it is possible to calculate the desired *VaR*, ($VaR_{t+d+1,t+d}$), with the generated panel of prices obtained in phase I of the methodology.

However, the amount of money won or lost ($w_{t+d+1,t+d}$) cannot be calculated, given that during day *'t+d'* no transaction was observed (the asset was only traded during day *'t+d+1'*). Therefore, it is not possible to compare the result obtained from the *VaR* estimation, with what actually happened in the market (we need two consecutive days in which the asset was traded in order to calculate a daily won or lost).

To face this problem we can start working over equation (19) in the part concerning with the money won or lost as follows:

$$P\left(\left[e^{\ln\left(\frac{P_{t+d+1}}{P_{t+d}}\right)}-1\right]M \leq VaR_{t+d+1,t+d}\,)\right)=p \tag{20}$$

Then,

$$P\left(\frac{P_{t+d+1}-P_{t+d}}{p_{t+d}}M \leq VaR_{t+d+1,t+d}\right)=p \tag{21}$$

Now if we assume that the actual and the "fair" prices follow a similar behavior with respect to their returns, we can write:[10]

$$\ln\left(\frac{P_{t+d}}{P_t}\right)\approx \ln\left(\frac{\hat{P}_{t+d}}{\hat{P}_t}\right)\Rightarrow \frac{P_{t+d}}{P_t}\approx\frac{\hat{P}_{t+d}}{\hat{P}_t}\Rightarrow \tilde{P}_{t+d}\approx P_t\cdot\frac{\hat{P}_{t+d}}{\hat{P}_t} \tag{22}$$

and if we replace (22) in (21), we obtain:

$$P\left(\frac{P_{t+d+1}-P_t\cdot\frac{\hat{P}_{t+d}}{\hat{P}_t}}{P_t\cdot\frac{\hat{P}_{t+d}}{\hat{P}_t}}M \leq VaR_{t+d+1,t+d}\right)=p \tag{23}$$

Therefore, we generate an unobserved price P_{t+d}, from a trading price, in this case P_t . Then we have the following:

$$P\left(\frac{P_{t+d+1}-\tilde{P}_{t+d}}{\tilde{P}_{t+d}}M \leq VaR_{t+d+1,t+d}\right)=p \tag{24}$$

It is important to note that we do not replace directly from the generated "fair" panel of prices, where we also have a price $\hat{P}_{t+d}$, given that doing so could potentially introduce a bias in the testing procedure.

In this way, we calculate the price $\tilde{P}_{t+d}$, under the assumption that the logarithmic returns of the "fair" panel of prices, generated in phase I, exhibit a similar behavior with respect to the real observed

[10] As we are interested in calculating *VaR* measures, is necessary that new panel of returns closely replicate the actual ones. Therefore, is important to test this assumption. We inspect this assumption empirically in section VI.

returns. Consequently, with this information is possible to calculate the daily amount of money won or lost, over the basis of true or market prices which are not necessarily consecutive. The underlying property is represented as follows:

$$P\left(\tilde{w}_{t+d+1,t+d} \leq VaR_{t+d+1,t+d}\right) = p \tag{25}$$

Therefore, what we need to do in order to test the different *VaR* methods is to 'compare' the money won or lost '$\tilde{w}_{t+d+1,t+d}$', with the '$VaR_{t+d+1,t+d}$'. Therefore, if we are calculating the *VaR* for a 5% level, it would be expected that only 5% of the times an actual loss would exceed the value provided by the *VaR*.

We must note that in making a comparison between $\tilde{P}_{t+d}$ (which is a calculated of P_t) and P_{t+d+1} we are taking two market prices in our analysis (a transformation of P_t and P_{t+d+1}). Therefore, our testing procedure relies on actual trading prices directly. A demonstrative example of the procedure is provided in Annex B.

Following the previous scheme, we perform the Kupiec (1995) test. We use this in order to analyze the historic proportion of losses exceeding the *VaR*. We define *Y* as the number of losses which exceeded the *VaR*. Here, *Y* is a random binomial variable with parameters (N_m, p). Being N_m the number of comparisons done between the actual investment outcomes and the calculated *VaR*, and *p* the expected percentage of losses exceeding the *VaR*:

$$\binom{N_m}{Y} \cdot p^Y \cdot (1-p)^{N_m - Y} \tag{26}$$

Based on the above behavior of *Y*. Kupiec (1995), proposed the following test-statistic:

$$K = -2 \cdot \ln\left\{(1-p)^{N_m - Y} \cdot p^Y\right\} + 2 \cdot \ln\left\{\left(1 - \frac{Y}{N_m}\right)^{N_m - Y} \cdot \left(\frac{Y}{N_m}\right)^Y\right\} \tag{27}$$

where *K* is distributed as a χ^2 with one degree of freedom. Under the null (H_0) the proportion of losses exceeding the *VaR* 'Y/N_m' is equal to *p,* meaning that the tested *VaR* method provides a reliable measure. Taking a significance level of 5% for the test, our critical value will be 3.84.

We will perform the Kupiec (1995) test for all of the *VaR* calculation methods outlined in Phase II. For the purpose of performing this test, we will use a sub-sample of daily bond transactions detailed in section V.

As an additional measure, we will calculate "the average *VaR"* for each one of the alternative calculation methods. This measure is relevant, especially for regulated institutions, given that they need to maintain some level of capital conditioned upon their degree of risk exposure. Therefore, these institutions would obviously prefer to measure their risk in some form that would allow them to maintain the minimum possible reserve capital.

A slight variation of the "Conditional Value at Risk" (*CVaR*), is proposed by calculating the average of the amounts in excess of the *VaR*, whenever returns exceeded the calculated *VaR.* The *CVaR* is referred to the average loss in the cases where the loss is greater than the calculated *VaR* (*VaR* plus the quantity of the loss in excess of the *VaR*). However, the measure that we propose is only how much we lost in excess of the forecasted *VaR*, conditioned that the loss was in excess of the *VaR*.

We perform the test for all of the "Value at Risk" methods outlined in Phase II, for confidence levels of 5% and 1%, using daily data from the main secondary market in Santiago, Chile. Results for the tests will be discussed in section VI. This procedure is tested at the asset and not at the portfolio level because thin trading reduces the probability of all assets in the portfolio trading at the same day.

Given that the chosen measures for the historical testing will be calculated for each individual asset, and for each *VaR* calculation method, we need a global measure in order to compare the alternative *VaR* calculation methods. Therefore, we have created "summary indicators" detailed in Annexes C and D.

We must bear in mind that we will perform a Back-Test of the calculated *VaR*s, comparing the wins or losses, but calculated with the "returns" obtained from the "fair" prices obtained in Phase I. Therefore, the reliability of our test will depend on how well do the returns obtained from the "fair" prices, replicate the actual returns observed in the market when trading was present. This is an empirical issue that will be explored in section VI.

V Data Description and Creation of the Testing Portfolio.

The data is divided into three main groups. The first one is composed by the fixed income bonds which will be used to estimate the term structure dynamic model outlined in Phase I. The second one is composed by the fixed income bonds that will be used to construct our testing portfolio in order to implement our methodology. The third one is a sub-sample of the fixed-income portfolio that we will use in order to perform the Back-Test of Phase III.

5.1 Data for the Estimation of the Dynamic Term-Structure Model.

The data consists of all daily transactions at the Santiago Stock Exchange from January 1997 to September 2002 (1430 trading days) of pure-discount bonds and semi-annual amortizing coupon bonds issued by the Chilean government. Pure-discount bonds are usually denominated PRBC ("Pagare Reajustable Banco Central") bonds, and semi annual amortizing coupon bonds are called PRC ("Pagare Reajustable con Cupones") bonds. Both types of bonds are inflation-protected with payments brought to real terms using monthly inflation.[11]

Table 5.1.1 summarizes the data. It can be noted that pure-discount bonds have maturities of less than 1 year while coupon bonds have maturities ranging from 1 to 20 years. Trading frequency is defined as the number of days for which we have at least one transaction of a bond of a specific maturity over all available trading days. A trading frequency of 20% means that at least one bond with that maturity was traded an average of 50 days per year. Standard deviation of observed yields generally decreases as bond maturity increases, which is consistent with mean reversion in interest rates.

[11] In practice this is done by expressing payments in another unit, the UF ("Unidad de Fomento"), which is updated every month using the previous month inflation.

Table 5.1.1: Daily transactions of Chilean government inflation-protected pure discount and coupon bonds from January 1997 to September 2002

Maturity Range (years)	Number of Observations	Average Trading Frequency*	Average Yield**	Yield Standard Deviation**
Pure Discount Bonds (PRBCs)				
0-1	1303	91.18%	5.73%	2.35%
Coupon Bonds (PRCs)				
1-1.5	284	19.86%	6.65%	2.11%
1.5-2.5	457	31.96%	6.25%	1.81%
2.5-3.5	477	33.36%	6.22%	1.39%
3.5-4.5	737	51.54%	5.97%	1.56%
4.5-5.5	561	39.23%	6.36%	1.38%
5.5-6.5	605	42.31%	6.33%	1.19%
6.5-7.5	917	64.13%	6.12%	1.31%
7.5-8.5	1136	79.44%	5.98%	1.23%
8.5-9.5	506	35.38%	6.27%	1.11%
9.5-10.5	603	42.17%	6.45%	0.79%
10.5-11.5	317	22.17%	6.19%	1.04%
11.5-12.5	510	35.66%	6.23%	0.90%
12.5-13.5	311	21.75%	6.12%	0.91%
13.5-14.5	567	39.65%	6.16%	0.80%
14.5-15.5	349	24.41%	5.92%	0.97%
15.5-16.5	373	26.08%	6.04%	0.90%
16.5-17.5	316	22.10%	6.18%	0.78%
17.5-18.5	376	26.29%	6.19%	0.93%
18.5-19.5	609	42.59%	5.95%	0.98%
19.5-20	748	52.31%	6.01%	0.95%

* Trading frequency is defined as the number of days for which there is a transaction of a given bond over all available trading days.
** Continuous Compounding

5.2 Creation of the Testing Portfolio.

A portfolio including 20 PRC bonds with different maturities ranging from 1 to 20 years is created. This will be the testing portfolio in order to apply the proposed methodology and to calculate the *VaR* measures. The portfolio is constructed assuming that UF$10,000 is invested in each of the 20 bonds, thus total portfolio investment is UF$200,000. Also, each day additional investments or divestments in each asset are performed such that the investment of UF$10,000 in each asset remains constant over time.

Daily transactions of PRC bonds from January 1997 to February 2003 (1517 trading days) are used. Table 5.2.1 presents a sub-sample of the traded prices for the 20 assets of our testing portfolio between 03/20/2000 and 05/15/2000 which illustrates the missing data problem common in emerging markets.

Table 5.2.1: Sub-sample of daily prices of PRCs conforming the testing portfolio between 03/20/2000 and 05/15/2000. Bonds have been standardized for every UF$100, given that not all PRCs have been issued with the same face value. PRCs may have been issued with face values of UF$500, UF$1,000, UF$5,000 or UF$10,000. Black spaces represent days in which the instrument was not traded.

Dates	Bonds Differentiated by Maturities in Years																				Trading Prices
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
20-03-2000				96.90			99.65			99.26				98.85							
21-03-2000	100.22		100.13		99.98			99.64		99.48		99.24		99.14						98.79	
22-03-2000	100.18			97.00		95.74						99.45	99.19			98.98	99.00	98.96			
23-03-2000				97.14				99.90		99.70								99.24		99.10	
24-03-2000	100.19							99.97						99.67	99.65	99.63					
27-03-2000		100.26						99.90	99.93	99.97					99.65					99.41	
28-03-2000												99.81								99.56	
29-03-2000		100.35		97.18	100.23			100.05					99.80	99.84	99.77	99.76					
30-03-2000		100.42	100.58	97.29	100.13			100.16		100.06		100.02	100.13	99.96						99.87	
31-03-2000		100.43		97.42				100.23		100.19			99.96					99.81			
03-04-2000				97.40			100.21	100.20	100.22	100.19	100.06		99.96	99.78				99.81	99.80	99.87	
04-04-2000	100.16		100.37	97.42	100.55	96.19		100.20		100.10		100.17	99.96	99.96	99.96						
05-04-2000			100.39				100.18	100.12			100.06			99.84	99.83					99.95	
06-04-2000			100.31	97.38	100.48		100.31				100.06			99.96	99.83	99.76		99.88			
07-04-2000	100.28	100.44	100.36	97.30	100.48			100.05	100.09					99.78	99.71	99.63	99.61	99.59	99.58		
10-04-2000			100.25	97.30	100.23	95.91	100.18	100.09							99.83						
11-04-2000				97.30						100.06		99.96						99.59		99.79	
12-04-2000	100.21			97.26	100.43		100.21	100.09	99.97	100.24	99.97										
13-04-2000	100.33	100.55				95.72		100.20			100.06	100.07					99.68				
14-04-2000								100.27				99.91									
17-04-2000				97.30						99.97							99.54				
18-04-2000		100.49			100.11					99.88		99.76		99.61		99.50	99.34	99.45	99.21	99.64	
19-04-2000				97.30		95.99	100.31	100.09									99.48	99.45			
20-04-2000		100.62		97.36					99.93	99.88											
24-04-2000							100.04			99.79				99.49		99.43				99.41	
25-04-2000									99.69	99.70			99.52	99.37						99.18	
26-04-2000			100.25							99.66		99.55		99.37						99.25	
27-04-2000		100.34						99.83				99.60									
28-04-2000	100.25	100.69	100.52	97.38	100.58	96.30		99.83		99.61		99.55									
02-05-2000										99.53							99.20	99.31			
03-05-2000	100.28	100.24				95.63		99.64												99.18	
04-05-2000		100.14			100.28	95.44		99.72		99.57											
05-05-2000		100.63	100.58	97.27				99.68		99.57			99.52			99.24					
08-05-2000								99.42		99.26										98.64	
09-05-2000	100.19									99.17				98.85		98.66		98.67			
10-05-2000	100.31							99.21		98.95		98.83					98.60		98.47	98.41	
11-05-2000	100.29	100.69				95.17		99.24		99.04							98.60				
12-05-2000				97.20				99.50		99.17							98.73				
15-05-2000														98.79							

Table 5.2.2 describes the complete PRC bond sample for our testing portfolio. We should note that, for example, a PRC issued with maturity of 6 years will pay the first coupon six months after its issuance. Therefore, at this time, the PRC will have a maturity of 5.5 years and not 6 anymore. In the same way, after the second coupon is paid, the PRC will exhibit a maturity of 5 years and so on. Therefore, column 1 of Table 5.2.2 exhibits the observed range of maturities. Then, column 2 classifies the PRC bonds with an approximate maturity taking into account the number of coupons remaining for each PRC until maturity. Column 3 provides the number of days, within our sample of 1517 observations, in which

the bonds were traded. Finally, column 4 provides the percentage of days with respect to the total sample in which the PRCs were traded.

Table 5.2.2: Description of the complete sample for each bond of the testing portfolio. The sample consists of daily PRC bond transactions between January 1997 and February 2003.

Matutity Range (years)	Aproximate Maturity	Number of Days in Which the Asset was Traded	Average Trading Frequency
1-1.5	1	285	18.79%
1.5-2.5	2	480	31.64%
2.5-3.5	3	491	32.37%
3.5-4.5	4	760	50.10%
4.5-5.5	5	585	38.56%
5.5-6.5	6	651	42.91%
6.5-7.5	7	988	65.13%
7.5-8.5	8	1221	80.49%
8.5-9.5	9	538	35.46%
9.5-10.5	10	620	40.87%
10.5-11.5	11	336	22.15%
11.5-12.5	12	523	34.48%
12.5-13.5	13	333	21.95%
13.5-14.5	14	590	38.89%
14.5-15.5	15	387	25.51%
15.5-16.5	16	419	27.62%
16.5-17.5	17	345	22.74%
17.5-18.5	18	422	27.82%
18.5-19.5	19	689	45.42%
19.5-20	20	815	53.72%

5.3 Sub-Sample data used for the Back-test.

Now, we take a sub-sample that will be used to perform the "Back Test" proposed in Phase III.[12] The sub-sample consists of the 1,116 trading days between August 1998 and February 2003. Table 5.3.1 describes the sub-sample. Columns 1 and 2 show the ranges of maturities and the approximate maturities of the instruments. Column 3 shows the number of days each bond traded and Column 4 the average trading frequency. With this information it is possible to calculate the win or loss between day *'t'* and the next day *'t+d+1'* in which the PRC was traded (here *'d'* is variable).

[12] We use a sub-sample and not the whole sample, given that the first *VaR* estimation needs (for the most demanding method) at least 400 observations. See section 4.2 regarding the *VaR* calculation methods used in the study.

Table 5.3.1: Description of the sub-sample used to perform the "Back Test". The sample consists of daily PRC bond transactions between August 1998 and February 2003.

Maturity Range (years)	Aproximate Maturity	Number of Days in Which the Asset was Traded	Average Trading Frequency
1-1.5	1	227	20,34%
1.5-2.5	2	345	30,91%
2.5-3.5	3	369	33,06%
3.5-4.5	4	634	56,81%
4.5-5.5	5	408	36,56%
5.5-6.5	6	432	38,71%
6.5-7.5	7	665	59,59%
7.5-8.5	8	895	80,20%
8.5-9.5	9	350	31,36%
9.5-10.5	10	384	34,41%
10.5-11.5	11	235	21,06%
11.5-12.5	12	375	33,60%
12.5-13.5	13	256	22,94%
13.5-14.5	14	458	41,04%
14.5-15.5	15	351	31,45%
15.5-16.5	16	359	32,17%
16.5-17.5	17	297	26,61%
17.5-18.5	18	347	31,09%
18.5-19.5	19	496	44,44%
19.5-20	20	588	52,69%

VI Empirical Results.

6.1 The complete panel of prices.

Table 6.1 presents the parameter estimates and standard errors of the three-factor generalized Vasicek term-structure dynamic model.

Table 6.1: Parameter estimates and standard errors from daily transactions of Chilean government inflation-protected pure discount and coupon bonds from January 1997 to September 2002.

Parameter	Parameter Estimate	Standard Error
$k1$	0.01820	0.00437
$k2$	0.97969	0.01478
$k3$	2.14709	0.05319
σ_1	0.01930	0.00021
σ_2	0.17974	0.00286
σ_3	0.21104	0.00417
ρ_{12}	-0.79976	0.01105
ρ_{13}	0.38726	0.01093
ρ_{23}	-0.81982	0.00208
δ	0.08044	0.03803
λ_1	0.00004	0.00001
λ_2	-0.01545	0.00404
λ_3	-0.02252	0.00793

The table displays estimates of the mean reversion parameters (k_1, k_2, k_3), the diffusion parameters ($\sigma_1, \sigma_2, \sigma_3$), the correlation coefficients of the state variables ($\rho_{12}, \rho_{13}, \rho_{23}$), the long-run mean of interest rates δ, and the market prices of risk ($\lambda_1, \lambda_2, \lambda_3$). Using these 13 constant parameter estimates, we proceed to estimate the state variables contained in $\mathbf{x}_t$, for each day t, using the recursive estimation technique of the extended Kalman filter described in Cortazar, Schwartz and Naranjo (2003). Using the calibrated model a complete panel of "fair" bond prices is computed.

Table 6.2 presents a sub-sample of the complete panel of "fair" bond prices for the 20 assets of our testing portfolio between 03/20/2000 and 05/15/2000.

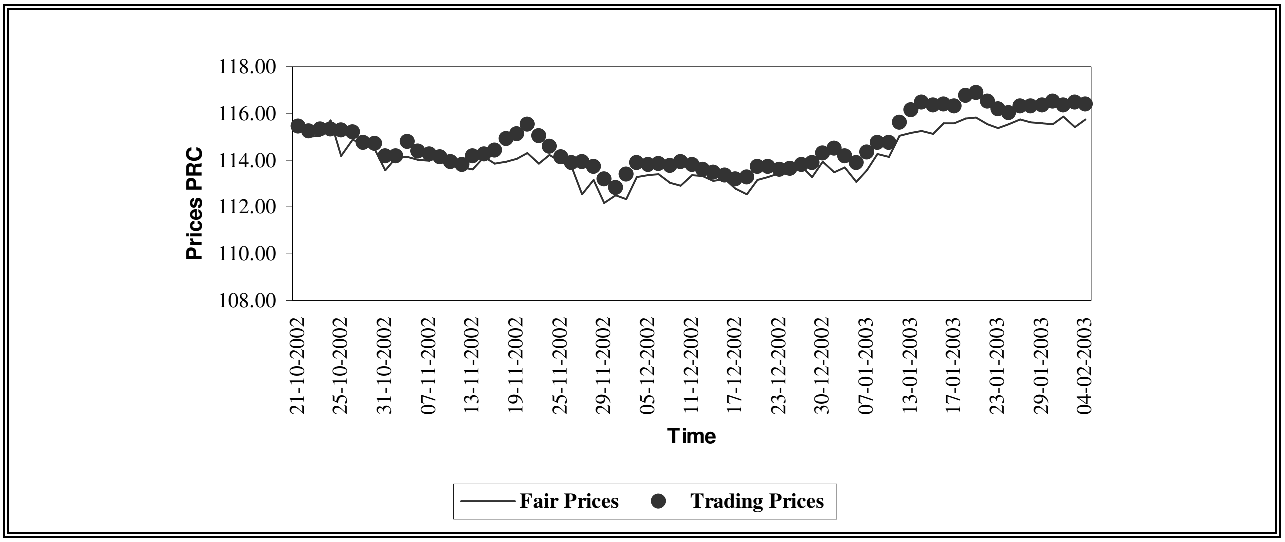
Table 6.2: Sub-sample of daily "fair" prices of PRCs conforming the testing portfolio between 03/20/2000 and 05/15/2000. Bonds have been standardized for every UF$100, given that not all PRCs have been issued with the same face value. PRCs may have been issued with face values of UF$500, UF$1,000, UF$5,000 or UF$10,000.

Dates	Bonds Differentiated by Maturities in Years																				
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Generated "fair" Prices
20-03-2000	100.27	100.16	100.06	99.97	99.88	99.77	99.66	99.53	99.39	99.26	99.12	98.99	98.87	98.77	98.68	98.61	98.56	98.53	98.52	98.55	
21-03-2000	100.25	100.17	100.11	100.04	99.97	99.88	99.77	99.66	99.53	99.41	99.28	99.16	99.05	98.95	98.87	98.80	98.76	98.74	98.74	98.76	
22-03-2000	100.20	100.15	100.11	100.07	100.01	99.94	99.85	99.75	99.63	99.52	99.41	99.30	99.20	99.11	99.04	98.98	98.95	98.94	98.95	98.98	
23-03-2000	100.23	100.20	100.18	100.15	100.11	100.04	99.96	99.86	99.76	99.64	99.54	99.43	99.33	99.25	99.18	99.13	99.10	99.09	99.10	99.14	
24-03-2000	100.21	100.15	100.13	100.11	100.08	100.04	99.99	99.92	99.84	99.76	99.68	99.60	99.54	99.48	99.44	99.41	99.40	99.42	99.45	99.51	
27-03-2000	100.29	100.25	100.23	100.21	100.18	100.14	100.08	100.00	99.92	99.84	99.75	99.67	99.59	99.53	99.48	99.45	99.44	99.45	99.48	99.54	
28-03-2000	100.33	100.29	100.27	100.25	100.22	100.18	100.12	100.05	99.96	99.88	99.79	99.71	99.63	99.57	99.52	99.49	99.47	99.48	99.51	99.57	
29-03-2000	100.36	100.32	100.31	100.29	100.27	100.23	100.18	100.11	100.04	99.96	99.88	99.80	99.73	99.68	99.63	99.61	99.60	99.62	99.65	99.72	
30-03-2000	100.43	100.42	100.42	100.42	100.40	100.36	100.31	100.24	100.16	100.08	100.00	99.92	99.85	99.79	99.74	99.71	99.71	99.72	99.75	99.81	
31-03-2000	100.46	100.46	100.46	100.45	100.43	100.40	100.35	100.28	100.21	100.13	100.05	99.97	99.90	99.84	99.80	99.77	99.77	99.78	99.82	99.88	
03-04-2000	100.44	100.44	100.45	100.45	100.43	100.40	100.34	100.28	100.20	100.12	100.04	99.96	99.89	99.83	99.79	99.76	99.75	99.76	99.80	99.86	
04-04-2000	100.22	100.29	100.35	100.39	100.41	100.40	100.37	100.33	100.26	100.20	100.13	100.06	100.00	99.96	99.92	99.90	99.90	99.92	99.97	100.03	
05-04-2000	100.28	100.32	100.36	100.39	100.39	100.37	100.33	100.27	100.21	100.13	100.06	99.99	99.92	99.87	99.83	99.80	99.80	99.81	99.85	99.92	
06-04-2000	100.26	100.31	100.35	100.38	100.39	100.37	100.34	100.29	100.22	100.15	100.08	100.01	99.95	99.90	99.86	99.84	99.84	99.86	99.90	99.96	
07-04-2000	100.32	100.36	100.40	100.41	100.40	100.37	100.32	100.25	100.17	100.08	99.99	99.91	99.83	99.77	99.72	99.68	99.67	99.67	99.70	99.75	
10-04-2000	100.28	100.29	100.30	100.31	100.30	100.27	100.23	100.17	100.09	100.02	99.94	99.87	99.80	99.75	99.71	99.68	99.68	99.70	99.73	99.80	
11-04-2000	100.35	100.36	100.37	100.38	100.36	100.33	100.28	100.21	100.13	100.05	99.97	99.89	99.82	99.76	99.71	99.68	99.67	99.69	99.72	99.78	
12-04-2000	100.24	100.25	100.28	100.29	100.29	100.27	100.23	100.17	100.11	100.03	99.96	99.89	99.83	99.77	99.73	99.71	99.71	99.73	99.77	99.83	
13-04-2000	100.31	100.37	100.41	100.43	100.42	100.39	100.33	100.26	100.17	100.08	99.99	99.90	99.82	99.75	99.70	99.66	99.64	99.64	99.67	99.72	
14-04-2000	100.41	100.46	100.49	100.50	100.48	100.44	100.37	100.29	100.20	100.11	100.01	99.92	99.83	99.76	99.70	99.66	99.64	99.64	99.66	99.71	
17-04-2000	100.38	100.40	100.42	100.42	100.40	100.36	100.29	100.22	100.13	100.04	99.94	99.85	99.77	99.70	99.65	99.61	99.59	99.59	99.62	99.67	
18-04-2000	100.44	100.43	100.42	100.40	100.36	100.30	100.22	100.13	100.03	99.93	99.82	99.72	99.63	99.56	99.49	99.44	99.42	99.41	99.43	99.47	
19-04-2000	100.44	100.44	100.43	100.41	100.37	100.32	100.24	100.15	100.05	99.95	99.85	99.75	99.66	99.58	99.51	99.47	99.44	99.44	99.45	99.50	
20-04-2000	100.59	100.58	100.56	100.52	100.46	100.39	100.29	100.18	100.07	99.94	99.82	99.71	99.60	99.51	99.43	99.37	99.33	99.31	99.31	99.34	
24-04-2000	100.45	100.42	100.39	100.36	100.31	100.24	100.16	100.06	99.96	99.85	99.74	99.64	99.54	99.46	99.39	99.34	99.31	99.30	99.32	99.36	
25-04-2000	100.38	100.33	100.29	100.25	100.20	100.13	100.05	99.96	99.85	99.75	99.64	99.54	99.45	99.37	99.31	99.26	99.23	99.22	99.24	99.28	
26-04-2000	100.35	100.29	100.25	100.21	100.16	100.09	100.02	99.92	99.82	99.72	99.62	99.52	99.43	99.35	99.29	99.24	99.22	99.21	99.23	99.28	
27-04-2000	100.37	100.32	100.28	100.24	100.19	100.12	100.04	99.94	99.83	99.72	99.61	99.51	99.41	99.33	99.26	99.21	99.18	99.18	99.19	99.23	
28-04-2000	100.34	100.43	100.47	100.46	100.42	100.35	100.25	100.13	100.00	99.86	99.73	99.60	99.48	99.37	99.28	99.20	99.14	99.11	99.10	99.12	
02-05-2000	100.35	100.41	100.43	100.42	100.37	100.30	100.20	100.08	99.95	99.82	99.69	99.56	99.44	99.34	99.25	99.18	99.12	99.09	99.09	99.11	
03-05-2000	100.28	100.25	100.23	100.19	100.14	100.07	99.98	99.87	99.76	99.64	99.52	99.41	99.31	99.22	99.14	99.08	99.05	99.03	99.04	99.07	
04-05-2000	100.23	100.18	100.14	100.10	100.04	99.98	99.89	99.79	99.68	99.57	99.46	99.36	99.26	99.18	99.11	99.06	99.03	99.02	99.03	99.07	
05-05-2000	100.54	100.52	100.48	100.42	100.34	100.24	100.12	99.99	99.85	99.70	99.56	99.42	99.29	99.18	99.08	99.00	98.94	98.90	98.89	98.90	
08-05-2000	100.37	100.31	100.24	100.16	100.08	99.97	99.85	99.72	99.58	99.44	99.30	99.17	99.04	98.93	98.84	98.76	98.71	98.67	98.66	98.68	
09-05-2000	100.25	100.15	100.06	99.99	99.91	99.81	99.70	99.58	99.45	99.32	99.19	99.07	98.95	98.85	98.77	98.70	98.66	98.63	98.63	98.66	
10-05-2000	100.29	100.16	100.05	99.95	99.85	99.73	99.61	99.47	99.33	99.19	99.05	98.91	98.79	98.68	98.59	98.51	98.46	98.42	98.41	98.43	
11-05-2000	100.38	100.37	100.31	100.21	100.09	99.95	99.78	99.60	99.41	99.22	99.03	98.85	98.68	98.53	98.39	98.27	98.17	98.10	98.05	98.03	
12-05-2000	100.39	100.35	100.28	100.19	100.07	99.93	99.78	99.61	99.43	99.25	99.07	98.90	98.74	98.60	98.47	98.36	98.27	98.21	98.17	98.15	
15-05-2000	100.37	100.33	100.27	100.18	100.07	99.94	99.79	99.62	99.45	99.28	99.11	98.94	98.79	98.65	98.53	98.42	98.34	98.28	98.24	98.23	

Table 6.2 complete panel of "fair" prices may be compared with Table 5.2.1 incomplete panel of market prices. We next discuss the issue of whether to use Table 6.2 for *VaR* calculations or rather a mixed panel data which uses market prices whenever available and "fair" prices when not.

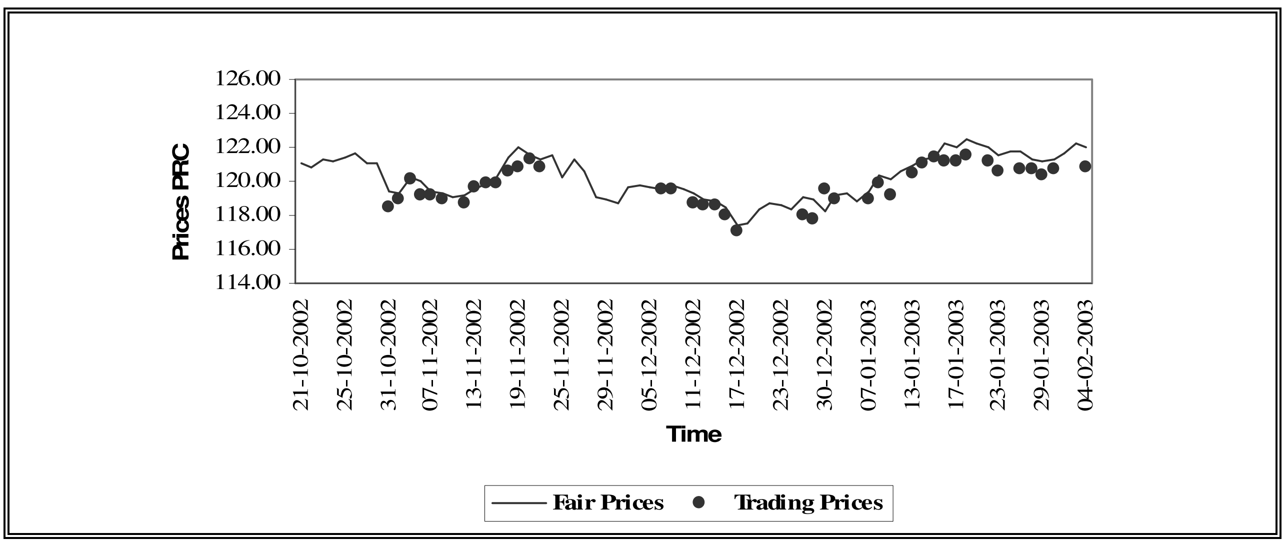
Figure 6.1 plots a sub-sample of the trading prices for the PRC bonds with 8 years maturity, along with the calculated "fair" prices obtained in Phase I of the methodology.

Figure 6.1: Daily trading prices and calculated "fair" prices for PRC bonds with 8 years maturity between October 2002 and February 2003



From the figure, we can appreciate that the generated "fair" prices systematically underestimate the trading prices for the days in which the instruments were traded. This observed bias, is present in all of the bonds of the testing portfolio, but with different directions, however constant over time. For example, Figure 6.2 plots the same information as Figure 6.1 but for bonds with maturity of sixteen years. For these bonds, the "fair" prices overestimate the trading ones.

Figure 6.2: Daily trading prices and calculated "fair" prices for PRCs with 16 years maturity between October 2002 and February 2003



As a more formal comparison between the "fair" prices '$\hat{P}_i$' and the real ones 'P_i', we compute the Mean Error (ME), the Absolute Mean Error (AME), the Square Root of the Mean Squared Error (RMSE), and the U Theil statistic (U) for the whole sample.[13] Table 6.3 displays these measures for the experimental portfolio.

[13] The Mean Error (ME), the Absolute Mean Error (AME), the Square Root of the Mean Squared Error (RMSE), and the U Theil statistic (U) are defined as follows: $ME = \frac{1}{n}\sum_{i=1}^{n}(\theta_i - \hat{\theta}_i)$ $AME = \frac{1}{n}\sum_{i=1}^{n}\left|\theta_i - \hat{\theta}_i\right|$ $RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\theta_i - \hat{\theta}_i)^2}$ $U = \sqrt{\frac{\frac{1}{n}\sum_{i=1}^{n}(\theta_i - \hat{\theta}_i)^2}{\frac{1}{n}\sum_{i=1}^{n}(\theta_i)^2}}$

where θ_i is the real value, in our case the price P_i, and $\hat{\theta}_i$ is the estimated value, here $\hat{P}_i$. The ME, AME and the RMSE are scaled measures with respect to the estimated variable. However, the U Theil does not present this scaling problem and we can appreciate the real magnitude of the error between real and estimated values.

Table 6.3: Mean Error (ME), Absolute Mean Error (AME), Square Root of the Mean Squared Error (RMSE), and U Theil statistic (U) obtained form the differences between "fair" prices $\hat{P}_i$ and trading prices P_i. For every day in which trading was observed between January 1997 and February 2003.

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
ME	-1.26E-02	1.02E-02	1.90E-02	3.02E-02	-3.96E-02	-6.62E-02	-5.27E-02	1.68E-01	2.02E-01	9.26E-03
AME	6.71E-02	1.11E-01	1.52E-01	1.53E-01	2.17E-01	2.06E-01	2.33E-01	2.64E-01	2.85E-01	1.49E-01
RMSE	1.21E-01	1.87E-01	2.60E-01	2.86E-01	3.72E-01	3.36E-01	3.79E-01	4.05E-01	4.85E-01	2.45E-01
U	1.21E-03	1.86E-03	2.58E-03	2.90E-03	3.68E-03	3.45E-03	3.69E-03	3.91E-03	4.73E-03	2.42E-03

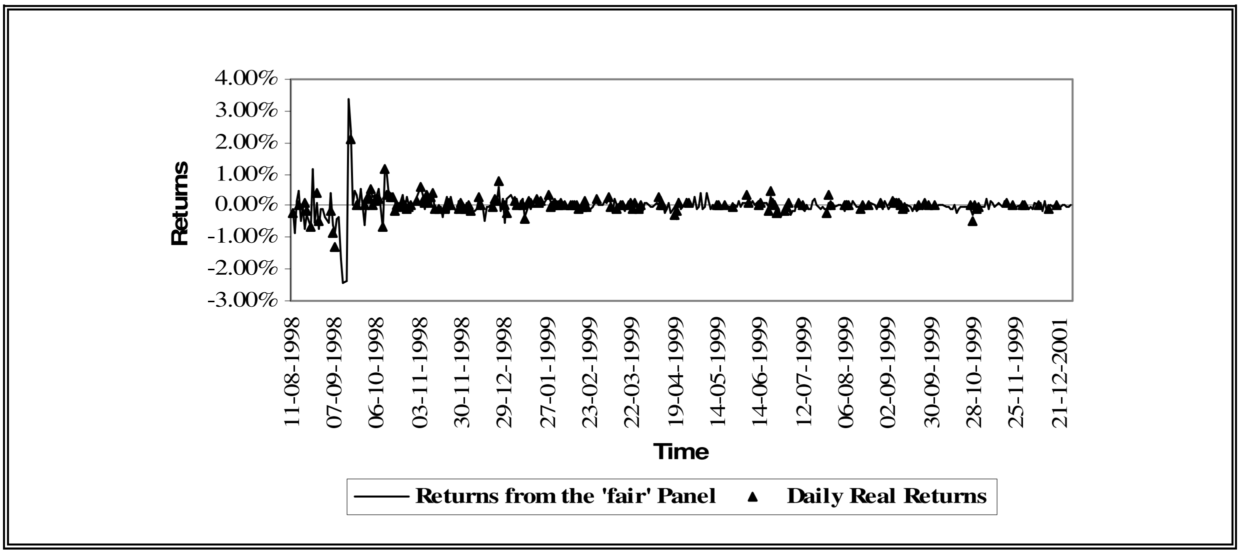
	11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
ME	-6.42E-02	-5.98E-02	-1.81E-01	-1.00E-01	-1.83E-01	-1.17E-01	-9.29E-02	-5.73E-02	1.01E-01	1.44E-01
AME	2.36E-01	2.08E-01	2.84E-01	2.47E-01	3.15E-01	2.81E-01	2.54E-01	2.66E-01	3.56E-01	4.05E-01
RMSE	4.02E-01	3.21E-01	4.63E-01	3.89E-01	4.72E-01	4.20E-01	4.20E-01	4.01E-01	4.91E-01	5.81E-01
U	3.90E-03	3.13E-03	4.45E-03	3.76E-03	4.44E-03	3.96E-03	4.00E-03	3.79E-03	4.55E-03	5.42E-03

If we observe from Table 6.3 the Mean Error (ME),[14] we can see that the values for this measure in comparison with the values for the Absolute Mean Error (AME) are significantly big. For example, if we take a look at columns labeled 8 and 16, corresponding to the PRCs with eight and sixteen years maturity respectively, the absolute value of the ratios ME/AME take values of 0.64 and 0.42 respectively. Therefore, it is apparent that the bias between the trading and the "fair" prices is significant.

Obviously, this systematic bias will affect adversely the *VaR* calculations.[15] Therefore, if we fill the "holes" of the incomplete panel data with "fair" prices, in order to obtain a mixed panel. It would be highly probable to obtain biased estimations. Therefore, this could potentially affect the *VaR* calculation methods, because they will be measuring prices volatility. In that way, we take the decision of using the complete panel of "fair" calculated prices, instead of using a mixed one in order to calculate the *VaR* measures. However, in order to obtain reliable *VaR* measures, is important and essential that the new generated "fair" panel replicates the returns that have been actually observed in the market.

Figures 6.3 and 6.4 plot a sub-sample of daily returns obtained from the "fair" panel, along with daily real returns for days in which the instrument was traded in two consecutive days. Again we plot PRCs with eight and sixteen years maturity. From the Figures, we can appreciate that the "fair" returns replicate very closely the actual returns. In addition, Table 6.4 provides the Mean Error (ME), the Absolute Mean Error (AME), the Square Root of the Mean Squared Error (RMSE), and the U Theil statistic (U) obtained form the differences between "fair" returns and real returns. For every pair wise consecutive days in which trading was observed for the whole sample between January 1997 and February 2003, and for all the instruments in the experimental portfolio.

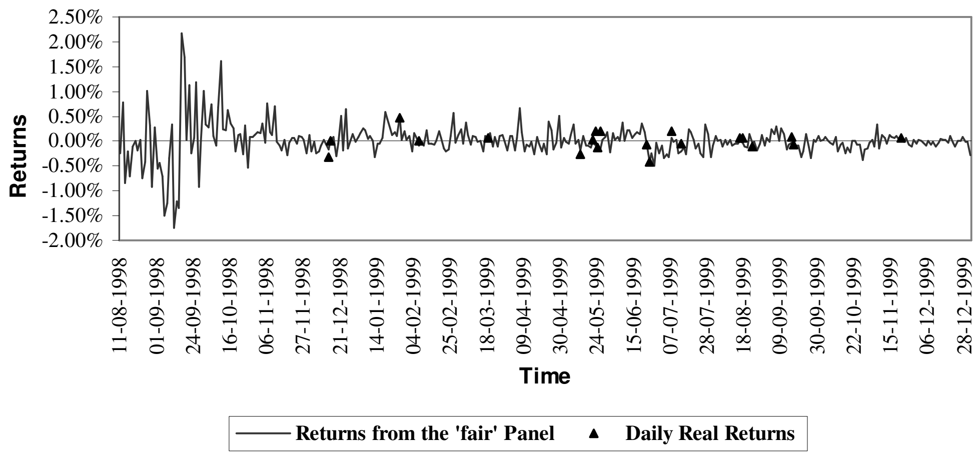
Figure 6.3: Daily real returns and "fair" returns for PRCs with 8 years maturity between August 1998 and December 1999



[14] Note that the ME represents the average of the biases.

[15] It is worth to note that the sign bias for each instrument, is related with the liquidity of the asset. If we recall the "Average Trading Frequency" column of Table 5.2.2 and compare the value with the Mean Error of Table 6.3. We will appreciate that the "fair" prices overestimate the real prices for the less traded instruments, and underestimate the most liquid ones.

Figure 6.4: Daily real returns and “fair” returns for PRCs with 16 years maturity between August 1998 and December 1999



From Table 6.4, we can appreciate the great approximation between “fair” and real returns. As we are working with returns, the U Theil statistic offers the advantage that does not depend on the scales in which variables are expressed. Therefore, if we check the U statistic, the values are very close to zero. This means that the “fair” returns replicate very close the real returns of the market.

In addition, Figures 6.5 and 6.6 plot a sub-sample of returns obtained from the “fair” panel, along with real returns when the instrument was traded between two days, but not necessarily consecutives. We plot PRCs with eight and sixteen years maturity. From the Figures, we can appreciate that the “fair” returns replicate very closely the actual returns. In addition, Table 6.5 provides the Mean Error (ME), the Absolute Mean Error (AME), the Square Root of the Mean Squared Error (RMSE), and the U Theil statistic (U) obtained form the differences between “fair” returns and real returns. For every two days in which trading was observed for the whole sample between January 1997 and February 2003, and for all the instruments in the experimental portfolio.

Table 6.4: Mean Error (ME), Absolute Mean Error (AME), Square Root of the Mean Squared Error (RMSE), and U Theil statistic (U) obtained form the differences between "fair" returns and real returns. For every pair wise consecutive days in which trading was observed between January 1997 and February 2003.

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
ME	1.14E-06	-6.46E-06	-9.38E-06	-3.48E-06	3.45E-06	-5.26E-06	7.53E-07	9.17E-07	-1.02E-07	-3.08E-06	-1.21E-05	6.08E-06	-1.07E-05	-2.82E-06	-1.94E-05	-1.20E-05	-2.05E-06	-1.09E-06	-2.28E-06	3.20E-06
AME	3.15E-05	6.82E-05	8.84E-05	8.23E-05	1.13E-04	9.70E-05	1.02E-04	7.71E-05	9.52E-05	6.24E-05	1.02E-04	7.58E-05	9.39E-05	7.38E-05	9.03E-05	1.13E-04	9.49E-05	1.21E-04	1.25E-04	1.24E-04
RMSE	5.85E-05	1.25E-04	1.52E-04	1.42E-04	2.25E-04	1.67E-04	1.84E-04	1.17E-04	1.58E-04	1.00E-04	2.04E-04	1.10E-04	1.43E-04	1.12E-04	1.33E-04	2.04E-04	1.30E-04	1.74E-04	1.82E-04	1.80E-04
U	2.51E-02	2.87E-02	4.16E-02	4.72E-02	4.54E-02	4.92E-02	3.97E-02	5.85E-02	4.29E-02	4.51E-02	4.39E-02	3.38E-02	3.72E-02	3.41E-02	3.38E-02	4.24E-02	3.45E-02	3.82E-02	4.22E-02	4.03E-02

Figure 6.5: Real returns and “fair” returns for PRCs with 8 years maturity between August 1998 and December 1999

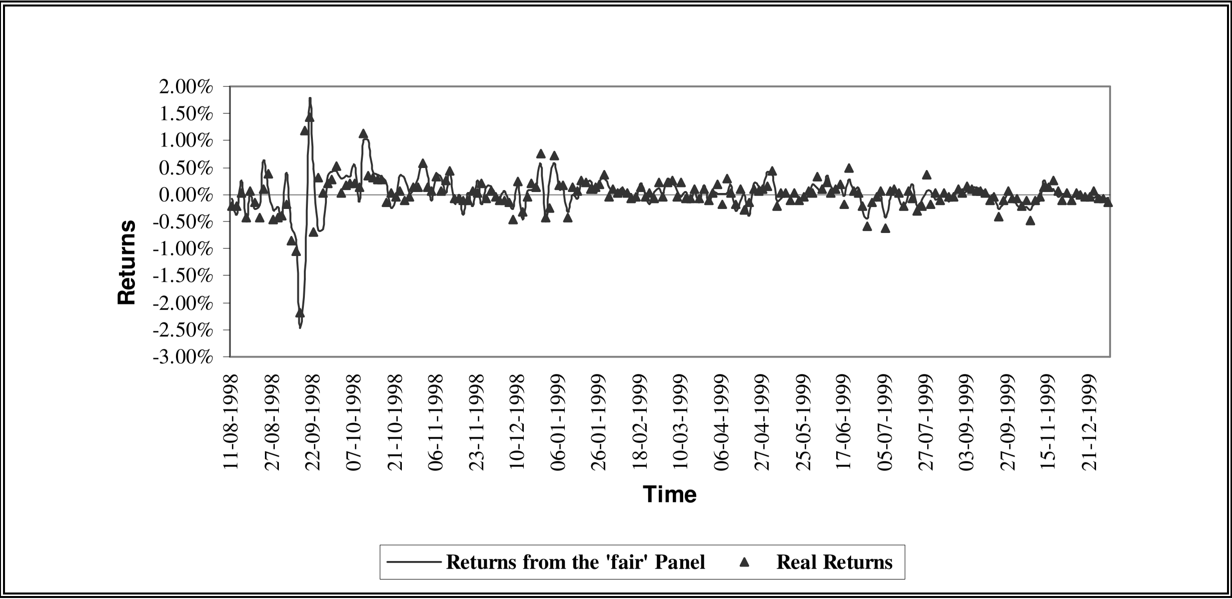


Figure 6.6: Real returns and “fair” returns for PRCs with 16 years maturity between August 1998 and December 1999

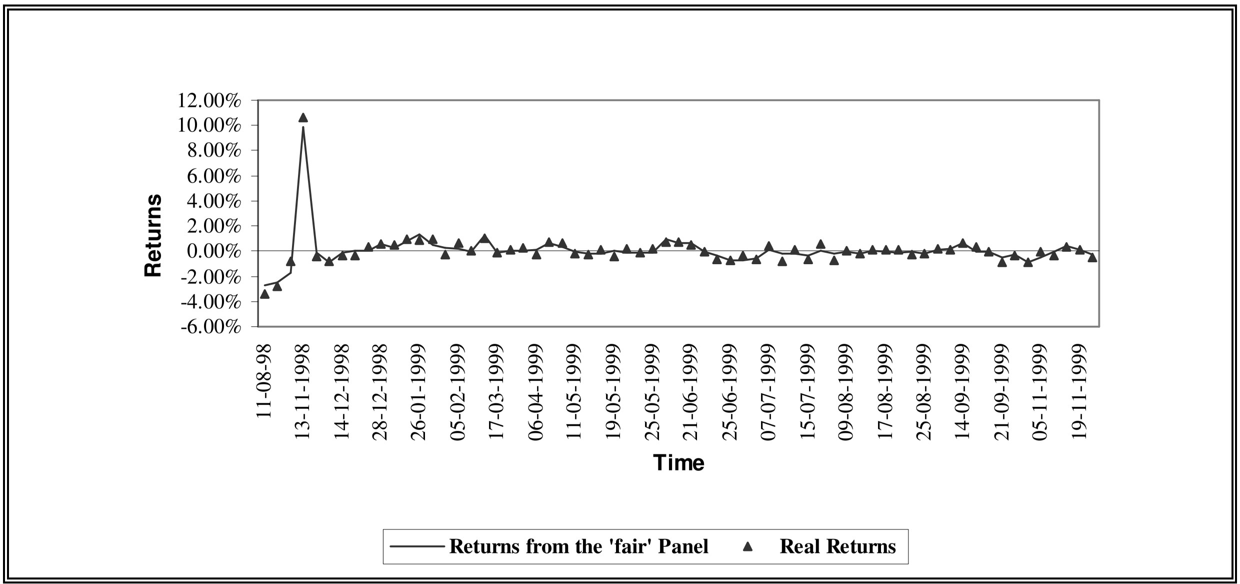


Table 6.5: Mean Error (ME), Absolute Mean Error (AME), Square Root of the Mean Squared Error (RMSE), and U Theil statistic (U) obtained form the differences between "fair" returns and real returns. For every pair wise, not necessarily consecutive days, in which trading was observed between January 1997 and February 2003.

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
ME	1.18E-06	3.47E-07	-3.63E-07	1.32E-07	5.72E-08	-6.20E-07	-1.66E-07	-3.32E-07	-1.96E-06	-6.62E-08	1.61E-06	1.78E-06	2.32E-06	1.57E-06	1.04E-06	1.43E-06	9.65E-07	8.29E-07	4.96E-07	2.51E-07
AME	4.46E-05	8.20E-05	1.16E-04	1.09E-04	1.59E-04	1.40E-04	1.17E-04	8.88E-05	1.17E-04	9.24E-05	1.54E-04	1.05E-04	1.31E-04	1.04E-04	1.39E-04	1.53E-04	1.39E-04	1.58E-04	1.48E-04	1.52E-04
RMSE	8.38E-05	1.50E-04	2.05E-04	2.07E-04	2.86E-04	2.41E-04	2.01E-04	1.42E-04	1.91E-04	1.56E-04	2.80E-04	1.55E-04	1.97E-04	1.61E-04	2.17E-04	2.51E-04	2.05E-04	2.40E-04	2.12E-04	2.20E-04
U	3.60E-02	3.44E-02	5.61E-02	6.86E-02	5.77E-02	7.08E-02	4.36E-02	7.08E-02	5.16E-02	7.00E-02	6.03E-02	4.75E-02	5.13E-02	4.92E-02	5.52E-02	5.23E-02	5.43E-02	5.28E-02	4.92E-02	4.92E-02

From Table 6.5, we can appreciate that the values for the U Theil statistic are again small and close to zero. This clearly indicates the close reflection of the real returns provided by the "fair"returns, even for time periods greater than one day.

Therefore, if we denote by $r_{t+d,t}$ the real return between '*d*' days of a bond traded in the Santiago Stock Exchange. Being $\psi_{t+d,t}$ the "fair" returns between also '*d*' days, the evidence suggests that:

$$r_{t+d,t} \sim \psi_{t+d,t} \tag{34}$$

$$\ln\left(\frac{P_{t+d}}{P_t}\right) \sim \ln\left(\frac{\hat{P}_{t+d}}{\hat{P}_t}\right) \tag{35}$$

This means that the "fair" returns calculated with the "fair" panel, derived from the generalized Vasicek three-factor dynamic term-structure model, replicate very close the actual observed returns of the experimental bond portfolio in the Santiago Stock Exchange. This observation is very important, given that it is a necessary condition to obtain reliable *VaR* measures. In addition, the consistency of the proposed historical testing ("Back-Test") relies on this assumption that appears to be fulfilled empirically.

In short, despite the fact that the calculated "fair" prices slightly departed from the actual trading prices due to the constant over or under estimation observed bias. We have observed that the "fair" returns replicate closely the real returns obtained from the trading prices. Therefore, for the purpose of *VaR* calculations, no bias is foreseen arising from using the "fair" returns.

6.2 Back-testing the *VaR* Measures.

At this point, we proceed to calculate the *VaR* measures using the "fair" panel of bond returns. We calculate the *VaR* measures with the different estimation methods outlined in section 4.2 ("The *VaR* Estimation Methods") and explained in Annex A.

Tables 6.6 and 6.7 display the "Back-Test" summary indicators (see Annex C and D) for the *VaR* measures calculated with several methods for 5% and 1% confidence levels respectively. They display the average percentage loss in excess of the *VaR*, the Kupiec *K* statistic, the Kupiec test result, the average *VaR*, the average loss in excess of the *VaR*, and the maximum excess over the *VaR*.

Table 6.6: "Back-Test" summary indicators of the *Va*R measures calculated with the "fair" panel, against the market data from the Santiago Stock Exchange. Confidence level of 5% for the VaR calculations. Sample between August 1998 and February 2003.

Indicators	Summary Indicators							
	Var-Cov Matrix	RiskMetrics	GARCH(1,1)	t - Student	Hist. Sim.	Static EVT	Dynamic EVT	Monte Carlo
% excess over VaR	6.27%	5.57%	4.84%	6.26%	7.10%	7.48%	5.81%	5.17%
K (Kupiec Test)	1.34	0.28	0.02	1.31	3.49	4.78	0.56	0.02
Reject H_0						X		
Average VaR	-47.99	-42.71	-40.81	-48.12	-40.02	-31.61	-36.61	-37.56
Average Excess over VaR	-25.72	-18.42	-20.62	-25.73	-25.91	-27.80	-20.38	-20.23
Maximum Excess over VaR	-197.01	-141.75	-112.32	-196.97	-201.89	-215.80	-113.27	-115.97

Table 6.7: "Back-Test" summary indicators of the *VaR* measures calculated with the "fair" panel, against the market data from the Santiago Stock Exchange. Confidence level of 1% for the *VaR* calculations. Sample between August 1998 and February 2003.

Indicators	Summary Indicators							
	Var-Cov Matrix	RiskMetrics	GARCH(1,1)	t - Student	Hist. Sim.	Static EVT	Dynamic EVT	Monte Carlo
% excess over VaR	3.21%	2.65%	2.14%	3.21%	2.35%	2.86%	1.71%	2.47%
K (Kupiec Test)	13.23	7.98	4.18	13.23	5.65	9.83	1.79	6.57
Reject H_0	X	X	X	X	X	X		X
Average VaR	-68.87	-47.67	-56.90	-71.52	-78.65	-80.12	-70.93	-70.91
Average Excess over VaR	-28.31	-14.90	-20.06	-28.19	-25.42	-26.86	-16.76	-21.00
Maximum Excess over VaR	-173.97	-96.16	-86.53	-173.89	-120.79	-157.09	-65.38	-89.81

A "perfect" *VaR* measure would exhibit exactly a *p%* average excess over the *VaR* for a *VaR* calculated with a *p%* confidence level. In that way, if we take a look to Table 6.6, we can appreciate that the "best" *VaR* measures are provided by the GARCH(1,1) and the Monte Carlo calculation methods. They offer an average percentage excess over a the *VaR* of 4.84% and 5.17% respectively. These values are very close to the 5% confidence level. On the other hand, methods like the static version of the Extreme Value Theory and the Historical Simulation offer relatively poor *VaR* measures, with average percentage excesses over the *VaR* of 7.48% and 7.10% respectively.

We could also appreciate from Table 6.6 that the Kupiec Test is not rejected at the 5% level for all of the *VaR* calculation methods, but not for the static version of the Extreme Value Theory (EVT). Therefore, with the exception of the static EVT, we can say that the average percentage of losses in excess of the *VaR* are statistically indistinguishable from 5%. This means that the models offer good *VaR* measures when compared with what actually happened in the market. However, if we inspect again the average percentage excess over the *VaR*, we could appreciate that the Risk Metrics method offers a value of 5.57% (very good too). Although the Risk Metrics method is not as good as the GARCH (1,1) or the Monte Carlo, it offers a very good performance and it has the advantage that its calculation is very simple with respect to the other methods.

From Table 6.6, we can also inspect the average excess over the *VaR*. This measure will allow us to observe which method adjusts better to market fluctuations. Therefore, the "best" method is expected to offer the lower level of this measure. We can appreciate that this measure is very similar for all of the methods. However, the Risk Metrics method slightly outperforms the others with an average excess over the *VaR* of $UF 18.42.

If the interested in *VaR* calculations is a bank or financial institution, which needs to report the *VaR* to its supervisory agency, for the purpose of capital requirements. It will be interesting to know which *VaR* calculation method offers an acceptable percentage of excess over the *VaR*, but with a relatively low average value of the *VaR* measure. Of course, a lower average level of reported *VaRs* would mean less capital requirements for the bank or financial institution, and it would have less immobilized resources.

From Table 6.6, we can see that the *VaR* method which provides the lower level of average *VaR* is the static EVT ($UF 31.61). However, we must recall that this method was rejected by the Kupiec Test. Therefore, from the available methods which were not rejected by the test, we can observe that the dynamic EVT offers the lower level of average *VaR* with $UF 36.61.

In short, for *VaR* calculations at the 5% confidence level, it appears that methods as the GARCH (1,1), Monte Carlo, Risk Metrics, and Dynamic EVT perform the best. However, the Risk Metrics method offers better conditions for extreme market fluctuations as the average excess over the *VaR* is the lowest. Also, the dynamic EVT might be preferred in order to report *VaR* measures for the purpose of capital requirements, as the method offers the least average *VaR* measure.

Now we turn our attention to Table 6.7. The *VaR* calculations have been performed at the 1% confidence level. We can appreciate that the only method which is not rejected by the Kupiec test is the dynamic EVT. This means that for all of the other methods the average percentage of excesses over the *VaR* is significantly different from 1%. The poor performance of the alternative *VaR* methods at the 1% level evidences the relevance of the adequate tail modeling of the returns distributions for the 1% level. However, not only tails are important, because if that was the case, the static EVT or the Historical Simulation would also offer good results. Therefore, another important characteristic is the adequate adjustment of the model to the time varying volatility of returns. In that way, we can observe that although the GARCH (1,1) was rejected, it performed better than methods such as the static EVT and Historical

Simulation which exhibited average percentage of excesses over the *VaR* of 2.86% and 2.35% respectively (the GARCH method exhibited an excess of 2.14%).

It is worth to note that the poor performance of the alternative *VaR* methods at the 1% confidence level has been already documented in several studies of both emerging and developed markets. Fernandez (2003) found that, for a Chilean proxy of zero coupon bonds, the dynamic EVT method performed the best. Delfines and Gutierrez (2002) analyzed *VaR* measures for different Argentinean assets such as Brady bonds and Global Government bonds. Their findings suggest similar percentages of excesses over the *VaR* as the ones reported here.

Kiesel et. al. (2000) provide an analysis for emerging markets using Brady bonds (Mexico, Venezuela, Morocco, and Poland, among others). Their findings suggest similar measures as the ones reported here for the 5% and 1% confidence levels. Finally, Bao et. al. (2003) evaluate *VaR* models for Asian emerging markets (Korea, Indonesia, Malaysia, Taiwan, and Thailand). They focus on stock indexes and not on fixed income instruments. However, their calculations and conclusions are again very close from ours for both 5% and 1% confidence levels.

Although the previous studies did not address the problem of incomplete panels of prices, they are useful regarding the conclusions obtained from the alternative *VaR* methods at different confidence levels. Their results are similar to ours. In addition, our historical testing provides coherent values and adjusted to reality. This suggests that our estimations, obtained with the proposed methodology, track closely what is actually happening in the market. However, we must be wise and bear the caveat that further testing of the proposed methodology with different econometric approaches and applied to different markets is needed. We have only provided an analysis for the Chilean particular case, but our results are encouraging and open the horizon for further research. The study offers an alternative approach for financial risk management in low-transaction fixed income markets.

VII Conclusions.

The estimation of daily risk measures has become a crucial issue for financial institutions and regulatory agencies. It is important for implementing and evaluating risk management strategies and regulations in the financial sector. Among the alternative approaches, the Value-at-Risk (*VaR*) has become a the-facto international standard endorsed by many international entities including the Basle Committee.

Much research has been done on how to implement *VaR* measures in developed markets, but very little in emerging markets with thin trading. In this paper we argue that the absence of prices makes computing *VaR* measures very difficult and showing how to deal with this issue is its main focus.

We propose a general methodology in order to calculate and test daily *VaR* measures in thinly traded markets. The methodology is composed of three phases: Phase I, generates a complete panel of prices, using a term-structure dynamic model of interest rates. Phase II, calculates portfolio *VaR* measures with several alternative methods using the complete panel data generated in phase I. Phase III, shows how to back-test the *VaR* measures obtained in phase II using the original incomplete panel of prices. We provide an empirical implementation of the methodology for the Chilean fixed income market.

Our results show that for the calculation of *VaR* measures for a 5% confidence level only one method was rejected by the Kupiec test (the static EVT). It appears that methods such as the GARCH (1,1), Monte Carlo, Risk Metrics, and Dynamic EVT perform the best. However, the Risk Metrics method offers better conditions for extreme market fluctuations as the average excess over the *VaR* is the lowest. Also, the dynamic EVT might be preferred in order to report *VaR* measures for the purpose of capital requirements, as the method offers the least average *VaR* measure.

For the *VaR* calculations with a 1% confidence level, the only method which is not rejected by the Kupiec test is the dynamic EVT. The poor performance of the alternative *VaR* methods at the 1% level evidences the relevance of the adequate left tail modeling of the returns distributions. However, not only tails are important, because if that was the case, the static EVT or the Historical Simulation would also offer good results. Therefore, another important characteristic is the adequate adjustment of the model to the heteroskedasticity of returns. We can observe that although the GARCH (1,1) was rejected, it performed better than methods such as the static EVT or the Historical Simulation.

The methodology is broad and flexible, and could be implemented with different innovations in term-structure dynamic modeling or historical testing analysis. Therefore, it could be applied in any economy with low frequency fixed income markets. It would be interesting to observe future research work using alternative term-structure dynamic models or using data of different fixed income markets.

This study is one of the firsts dealing with the problem of calculating daily *VaR* measures with a panel of incomplete data and may provide a basis for further research in emerging markets where thin trading is a serious issue.

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ANNEX A: The *VaR* Methods.

Parametric methods in a world of multi-normal distributions (methods of the variance-covariance matrix):

The Method of Variance-Covariance is grounded on the assumption of a Multivariate Normal Distribution of returns.

For the case of one asset, from Itô's lemma, we can derive from the logarithmic returns, the *VaR* for one day:

$$VaR_{t+1,t} = \left(e^{(u - \frac{\sigma^2}{2}) + \sigma\alpha} - 1 \right) M \tag{A.1}$$

where u y σ^2 are the mean and variance of returns, α is the inverse of a Normal (0,1) for a p% probability, M is the amount of money invested

To calculate 'u', it could be estimated as the sample measure of the historical returns. Therefore, what could vary is the form of estimating 'σ', existing for those different options shown later.

To calculate the portfolio *VaR*, this is obtained calculating the variance of that portfolio, and replacing it in (A.1). The portfolio variance is obtained using the Variance-Covariance Matrix as follows:

$$\hat{\sigma}^2_{port.} = \boldsymbol{\omega}' \sum \boldsymbol{\omega} \tag{A.2}$$

being $\boldsymbol{\omega}$ y $\boldsymbol{\omega}'$ the weighted vector of the different elements of the portfolio and its transpose respectively, and $\sum$ is the Variance-Covariance Matrix. If we want to estimate the portfolio mean, it could be calculated as the weighted average of the measures of each asset. Using the relative weight of each portfolio element.

Then, to estimate the standard deviations (and also the covariances), we can proceed with the following methods:

Method of the sample variance and covariance

$$\hat{\sigma}^2 = \frac{1}{(T-1)} \cdot \sum_{h=1}^{T} (r_h - \hat{u})^2 \tag{A.3}$$

being T the number of observations, r_h the logarithmic returns, and $\hat{u}$ the sample mean.

$$\hat{\sigma}^2_{i,j} = \frac{1}{(T-1)} \cdot \sum_{h=1}^{T} (r_{h,i} - \hat{u}_i)(r_{h,j} - \hat{u}_j) \tag{A.4}$$

Method of exponential decay in the Risk-Metrics versions

Following J.P. Morgan (1996),

$$\sigma_t^2 = \lambda_{RM} \cdot \sigma_{t-1}^2 + (1-\lambda_{RM}) \cdot r_{t-1}^2$$
$$\lambda_{RM} = [day = 0.94; month = 0.97] \quad \text{(A.5)}$$

$$\sigma_{i,j,t}^2 = \lambda_{RM} \cdot \sigma_{i,j,t-1}^2 + (1-\lambda_{RM}) \cdot r_{i,t-1} r_{j,t-1}$$
$$\lambda_{RM} = [day = 0.94; month = 0.97] \quad \text{(A.6)}$$

then we used the ecuation (A.2) and (A.1).

GARCH(1,1)

Following Bollerslev (1986) for logarithmic returns:

$$r_t = (u - \frac{\sigma^2}{2}) + \sigma_t \cdot Z_t \quad \text{(A.7)}$$
$$\varepsilon_t = \sigma_t \cdot Z_t \quad \text{(A.8)}$$
$$\sigma_t^2 = \eta_0 + \eta_1 \cdot \varepsilon_{t-1}^2 + \eta_2 \cdot \sigma_{t-1}^2 \quad \text{(A.9)}$$

If we see J.P. Morgan (1996), to calculate a portfolio *VaR* we could use the variance-covariance matrix decomposition as follows:

$$\hat{\sigma}^2_{port.} = [\omega_1 \quad \ldots \quad \ldots \quad \omega_n]_{1xn} \cdot \begin{bmatrix} \sigma_1 & 0 & \ldots & 0 & 0 \\ 0 & \sigma_2 & \ldots & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & \ldots & \sigma_n \end{bmatrix}_{nxn} \cdot \begin{bmatrix} 1 & \rho_{12} & \rho_{13} & \ldots & \rho_{1n} \\ \rho_{21} & 1 & \rho_{23} & \ldots & \ldots \\ \rho_{31} & \rho_{32} & 1 & & \ldots \\ \ldots & & & & \ldots \\ \rho_{n1} & \ldots & \ldots & \ldots & 1 \end{bmatrix}_{nxn} \cdot \begin{bmatrix} \sigma_1 & 0 & \ldots & 0 & 0 \\ 0 & \sigma_2 & & & 0 \\ \ldots & & \ldots & & \ldots \\ 0 & & & \ldots & \ldots \\ 0 & & \ldots & \ldots & \sigma_n \end{bmatrix}_{nxn} \cdot \begin{bmatrix} \omega_1 \\ \ldots \\ \ldots \\ \omega_n \end{bmatrix}_{nx1} \quad \text{(A.10)}$$

where the ρ is the Pearson correlation and σ is calculated with (A.9). Then we replace this in equations (A.1) and (A.2).

Parametric methods accounting for asymmetric and multi-kurtosis effects:

These try to model with different distributions, the behavior of logarithmic portfolio returns. Of course, they are not anymore in a world of multinormal distributions. Therefore, we can no longer use the Variance-Covariance Matrix as in the previous examples in order to calculate the portfolio *VaR*. [16]

For that reason, one possibility is to use a simple approximation to model portfolios where the normality assumption no longer holds, this idea was taken from J.P. Morgan (1996). This consists in: If $r_{1,t}$, $r_{2,t}$, …, $r_{c,t}$ are the returns of the *c* asset of a portfolio for time *t*, and that each asset has a weight ω_1, ω_2, …, ω_c, respectively, then the portfolio return is:

[16] This is because the correlation matrix, starts loosing reliability because it is only defined for Multinormal variables.

$$r_{portf,t} = \sum_{i=1}^{c} \omega_i \cdot r_{i,t} \tag{A.11}$$

If the process is repeated for times *t-1, t-2,…, t-k* , where *k* is the size of the time window, then, a series for the portfolio will be maintained, which has implicitly included the correlations for each asset, and to which all the techniques could be applied as it was a single asset.[17]

T - Student

We used the t-Student distribution following the work of Wilson (1993) and Lucas (1997), they propose the possibility of substituting the Normal distribution with a t-Student. The latter has the advantage of adjusting to fat tails better that the former, depending on the degrees of freedom, attaining greater flexibility in the left tail.

"Extreme Value Theory" in its static version

This methodology, pretends to only model the left tail of returns distributions, such modeling is given by the data that is under the threshold 'μ' – look Embrechts, Klüpperberg y Mikosch (1997)- by a transformation of the Generalized Distribution of Pareto (GDP):

$$G_{\xi,\beta(\mu)}(y) = \begin{cases} 1-\left(1+\dfrac{\xi y}{\beta}\right)^{-1/\xi}, \xi \neq 0 \\ 1-\exp\left(\dfrac{-y}{\beta}\right), \xi = 0 \end{cases} \tag{A.12}$$

where ξ and β are the parameters of the GDP.

Following the studies outlined above, and adding the work of Coles (2001) and McNeil and Frey (2000), we can represent the *VaR* as:

$$VaR_p^{(EVT)} = \left[\mu + \frac{\beta}{\xi}\left(\left(\frac{n}{k_\mu}(1-p)\right)^{-\xi} - 1\right)\right]M \tag{A.13}$$

where k_μ is the number of observations in excess of the threshold 'μ'. And *n* is the sample size used for the calculation.

"Extreme Value Theory" in its dynamic version

The static version of the "Extreme Value Theory", like other methods such as sample variance and covariance, and historical simulation; assign the same weight to recent and past data. They do not account for time varying volatility or heteroskedasticity.

In that way, McNeil and Frey (2000) developed a dynamic "Extreme Value Theory" with the purpose that the estimations could be adjusted quickly to market changes. They estimate returns using a GARCH model and maximum likelihood estimation assuming Normal Distribution of error terms.

[17] Recall that "many bonds" are portfolios by their own, composed by different flows or coupons.

Then, they take the residuals $\hat{Z}_t$ of the preceding estimation, and with the residuals taken from the left tail, they adjust them to a Generalized Distribution of Pareto (GDP).

Being '*F(Z)'* the distribution of the obtained residuals, and if we recall equation (A.1), then the inverse of *F(Z)* corresponds to the *α* of that formula, therefore, this values could be used to estimate the *VaR* for a *p%* in that equation.

Then, with the adjusted distribution of residuals, we estimate the inverse function, that will be denoted by $INV(F(Z))_p$, for a *p* confidence level, then this is replaced in equation (A.1) instead of α as follows:

$$VaR_p^{EVT\ Dinamic} = \left(e^{(u-\frac{\sigma^2}{2})+\sigma \cdot INV(F(Z))_p} - 1 \right) M \tag{A.14}$$

Non parametric method of historical simulation:

These methods do not assume a distribution for returns. They take a window of historical data to perform their estimations.

If we assume that an investment is done in one asset, and we take a series of historical returns for the investment, for example in the last 250 days, and it is multiplied times the positions that are being actually taken for the asset. We could then elaborate a histogram with the outcomes of the investment. Then it is enough taking the percentile p%, and that value will deliver the *VaR* .

For the case of one portfolio, it is enough to add in a contemporaneous form, the same series of investment outcomes mentioned for the case of an asset, but using all the assets conforming the portfolio. After this, we can again create the histogram, but now of the portfolio as a whole, and it is enough to take the percentile p% to calculate the *VaR* for the portfolio as a whole.

Monte Carlo simulation method, using the stochastic structure developed in Cortázar, Schwartz and Naranjo (2003) with a dynamic three-factor Vasicek term-structure model:

Following Beder (1995), J.P. Morgan (1996) and Singh (1997), to model the risk using the Monte Carlo technique. The first thing to do is to specify some stochastic process describing the financial variables in study. Here, we take advantage of the assumed process that the term-structure of rates follow, a three factor Vasicek in this case.

Then, having the assumed process, the 'parameters' of that equation must be found. Here we have already estimated them in Phase I of the proposed methodology. Once the task is completed, we simulate the paths of the desired variables. For example, we simulate the prices using "shocks" in the stochastic process assumed (for that we simulate forward a vector of state variables $\mathbf{x}_t$).

With these simulations we can generate a series of returns and therefore a database of investment outcomes. With this information, we can generate a histogram and taking the percentile p%, we obtain the *VaR* measure.

ANNEX B: A demonstrative Example of the "Back Test" Analysis.

To understand the "BackTest" procedure proposed in the document, we provide here a demonstrative example. In this case we use the GARCH (1,1) method. We use a sub-sample ranging from June 8, 2000 through June 20, 2000 for a PRC bond with 8 years maturity.

If we observe Table C.1, the only days in which the daily money won or lost could be calculated, is between t=8 (06/20/2000) and t=7 (06/16/2000)[18], given that only here were two consecutive days in which the asset was traded. Therefore, this is the only time interval in which we are able to perform a comparison with the daily *VaR*.

What we intend to do with the proposed testing procedure, is to capture in some way, relevant information from dates in which not necessarily were two consecutive daily trading prices observed.

For example, day t=5 (06/14/2000) is the first one after t=1 (06/08/2000) in which a transaction happened for a PRC with 8 years maturity. Therefore, what we intend to do from these two market prices is to obtain some information for the testing procedure. We can clearly observe that in t=5, we are unable to calculate the daily money won or lost, given that there was not a market price in t=4.

Table C.1: Sub-Sample of PRC bonds with 8 years maturity between 06/08/2000 and 06/20/2000

Date	Time (t)	Trading prices of PRC bonds with 8 years maturity	Daily money won or lost when investing UF$10.000 in the PRC bond with 8 years maturity.	Daily VaR 5% calculated with the "fair" panel when investing UF$10.000 in the PRC bond with 8 years maturity (GARCH Method)	PRC bonds 8 years maturity prices obtained from term-structure dynamic model.
08-06-2000	**1**	99.68		-14.68	99.86
09-06-2000	**2**			-15.56	99.98
12-06-2000	**3**			-16.48	99.87
13-06-2000	**4**			-15.58	99.88
14-06-2000	**5**	99.86	**?**	-15.58	99.97
15-06-2000	**6**			-15.39	100.02
16-06-2000	**7**	99.86	**?**	-15.43	100.10
20-06-2000	**8**	99.90	4.01	-14.56	100.11

[18] The fact that between these days there are more that one day is explained because by that time there was a weekend and two holidays in Chile and we are only considering working days.

$$\text{Money won or lost}_{5-4} = \left(\frac{P_5 - P_4}{P_4}\right)\cdot 10000$$

$$= \left(\frac{99.86 - P_4}{P_4}\right)\cdot 10000$$

$$= ? \qquad\qquad VaR_{5-4} = -15.58$$

Taking advantage of the empirical observation that the returns for horizons greater than one day, calculated with the market prices and calculated with the "fair" prices are not significantly different:

$$\ln\left(\frac{P_{t+d}}{P_t}\right) \approx \ln\left(\frac{\hat{P}_{t+d}}{\hat{P}_t}\right) \Rightarrow \frac{P_{t+d}}{P_t} \approx \frac{\hat{P}_{t+d}}{\hat{P}_t} \tag{C.1}$$

Then, for the price P_4 we can find the following relation:

$$\frac{P_4}{P_1} \approx \frac{\hat{P}_4}{\hat{P}_1} \tag{C.2}$$

$$\tilde{P}_4 \approx \frac{\hat{P}_4}{\hat{P}_1} \cdot P_1 \tag{C.3}$$

$$\tilde{P}_4 \approx \frac{99.88}{99.86} \cdot 99.68 = 99.70 \tag{C.4}$$

If we observe, this price is obtained from a real trading price, in this case P_1. Therefore, if we replace this value in Table C.1, now we could perform a comparison between "the money won or lost" in t=5, with the daily *VaR* calculated for t=5. (See Table C.2)

We must note that in making a comparison between $\tilde{P}_4$ (which is a transformation of P_1) and P_5 we are taking two market prices in our analysis (P_1 and P_5). Therefore, our testing procedure relies on actual trading prices directly.

Table C.2: Example of the testing procedure proposed.

Date	Time (t)	Trading prices of PRC bonds with 8 years maturity	Daily money won or lost when investing UF$10.000 in the PRC bond with 8 years maturity.	Daily VaR 5% calculated with the "fair" panel when investing UF$10.000 in the PRC bond with 8 years maturity (GARCH Method)	PRC bonds 8 years maturity prices obtained from term-structure dynamic model.
08-06-2000	**1**	99.68		-14.68	99.86
09-06-2000	**2**	[redacted]		-15.56	99.98
12-06-2000	**3**	[redacted]		-16.48	99.87
13-06-2000	**4**	99.70		-15.58	99.88
14-06-2000	**5**	99.86	**16.05**	-15.58	99.97
15-06-2000	**6**	[redacted]		-15.39	100.02
16-06-2000	**7**	99.86	**?**	-15.43	100.10
20-06-2000	**8**	99.90	4.01	-14.56	100.11

$$\text{Money won or lost}_{5-4} = \left(\frac{P_5 - P_4}{P_4}\right)\cdot 10000$$

$$= \left(\frac{99.86 - 99.70}{99.70}\right)\cdot 10000$$

$$= 16.05 \qquad\qquad VaR_{5-4} = -15.58$$

We can observe that in this case, given that in t=5 existed a won and not a loss. The investment outcome was greater than the daily *VaR* calculated for that period.

ANNEX C: Measures for the Historical Testing (confidence level of 5%).

The next tables show the measures for the historical testing calculated for each individual asset, and for each *VaR* calculation method (Section 4.3).

The sub-sample used to perform the "Back Test". The sample consists of daily PRCs transactions between August 1998 and February 2003 (1116 days), with confidence level of 5% for the *VaR* calculations

We must note, however, that the weakness of this procedure is that it can not be tested for the portfolio. This is apparent because is very difficult that all of the assets conforming a portfolio have been traded during the same day. Therefore, we would only be able to perform individual tests for each asset.

Given that the chosen measures for the historical testing will be calculated for each individual asset, and for each *VaR* calculation method. We need a global measure in order to compare the alternative *VaR* calculation methods. Therefore, we have created "summary indicators".

The summary indicators are calculated simultaneously with all of the bonds in line (as it was a single bond). We did not use the average because the percentage of days in which '$w_{t+d+1,t}$' could be calculated (the outcome in monetary units of an investment) is not the same along the sample.

Table C.1: “Back-Test” summary indicators of the *VaR* measures calculated with the “fair” panel, against the market data from the Santiago Stock Exchange. Confidence level of 5% for the *VaR* calculations - Method of the sample variance and covariance. Sample between August 1998 and February 2003.

Indicators	Bonds Differentiated by Maturities in Years																				Summary Indicators
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
% excess over VaR	8.37%	6.69%	5.69%	4.42%	5.88%	5.57%	6.33%	6.48%	5.71%	6.53%	8.51%	5.33%	7.42%	6.77%	5.41%	6.13%	6.40%	5.19%	7.06%	7.48%	6.27%
***K* (Kupiec Test)**	4.55	1.88	0.36	0.47	0.63	0.28	2.28	3.79	0.36	1.73	5.08	0.09	2.77	2.73	0.12	0.90	1.13	0.03	3.94	6.67	1.34
Reject H_0	X										X								X	X	
Average VaR	-41.16	-46.78	-57.05	-53.91	-54.02	-44.36	-43.77	-43.41	-40.74	-35.45	-35.42	-34.85	-40.55	-42.12	-44.35	-43.97	-50.47	-49.82	-55.99	-59.89	-47.99
Average Excess over VaR	-23.79	-28.24	-21.82	-22.68	-38.60	-27.85	-26.59	-18.57	-23.04	-18.12	-20.54	-11.68	-24.89	-16.66	-37.46	-20.54	-32.00	-41.39	-33.69	-32.56	-25.72
Maximum Excess over VaR	-64.42	-77.58	-64.21	-76.88	-177.13	-177.41	-197.01	-115.00	-122.74	-127.38	-66.21	-31.59	-129.27	-82.20	-126.31	-102.42	-119.45	-106.29	-126.41	-141.26	-197.01

Table C.2: “Back-Test” summary indicators of the *VaR* measures calculated with the “fair” panel, against the market data from the Santiago Stock Exchange. Confidence level of 5% for the *VaR* calculations - “RiskMetrics”. Sample between August 1998 and February 2003.

Indicators	Bonds Differentiated by Maturities in Years																				Summary Indicators
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
% excess over VaR	10.13%	6.69%	5.42%	5.52%	6.37%	5.10%	5.27%	5.14%	4.57%	5.74%	7.23%	5.07%	7.81%	5.68%	4.84%	4.74%	5.05%	5.76%	5.04%	4.76%	5.57%
***K* (Kupiec Test)**	9.83	1.88	0.13	0.35	1.49	0.01	0.10	0.04	0.14	0.43	2.18	0.00	3.67	0.42	0.02	0.05	0.00	0.41	0.00	0.07	0.28
Reject H_0	X																				
Average VaR	-40.73	-47.65	-51.44	-44.81	-44.44	-44.54	-37.81	-38.92	-34.38	-37.06	-34.54	-36.34	-35.15	-41.73	-41.42	-39.58	-47.60	-48.38	-50.57	-59.71	-42.71
Average Excess over VaR	-13.15	-16.50	-12.81	-12.14	-22.72	-19.44	-19.78	-16.09	-18.18	-15.81	-15.98	-9.36	-16.80	-15.90	-26.05	-20.97	-22.75	-25.84	-24.37	-27.75	-18.42
Maximum Excess over VaR	-41.93	-52.94	-40.62	-61.76	-114.70	-105.33	-141.75	-79.34	-88.00	-92.48	-44.73	-25.47	-86.43	-64.44	-73.99	-71.53	-56.99	-74.47	-80.15	-91.76	-141.75

Table C.3: “Back-Test” summary indicators of the *VaR* measures calculated with the “fair” panel, against the market data from the Santiago Stock Exchange. Confidence level of 5% for the *VaR* calculations - GARCH(1,1). Sample between August 1998 and February 2003.

Indicators	Bonds Differentiated by Maturities in Years																				Summary Indicators
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
% excess over VaR	7.49%	6.10%	5.42%	4.26%	5.39%	4.41%	4.67%	4.92%	4.57%	4.44%	6.38%	4.00%	5.86%	5.68%	3.70%	3.90%	3.37%	4.32%	5.04%	4.76%	4.84%
***K* (Kupiec Test)**	2.59	0.83	0.13	0.77	0.13	0.33	0.16	0.01	0.14	0.26	0.87	0.85	0.38	0.42	1.36	0.99	1.87	0.35	0.00	0.07	0.02
Reject H_0																					
Average VaR	-40.94	-42.77	-49.33	-43.41	-42.83	-44.62	-38.95	-35.54	-32.42	-33.93	-33.53	-36.52	-33.53	-37.17	-38.23	-39.89	-41.66	-51.04	-50.62	-50.53	-40.81
Average Excess over VaR	-18.29	-24.76	-16.00	-19.46	-26.92	-19.20	-20.18	-18.39	-16.98	-14.66	-17.74	-9.54	-20.86	-12.01	-29.17	-19.44	-29.13	-31.73	-24.37	-28.69	-20.62
Maximum Excess over VaR	-47.33	-56.98	-57.27	-68.75	-112.32	-105.40	-80.62	-86.15	-94.94	-101.06	-48.83	-26.46	-84.64	-79.05	-84.36	-87.72	-65.92	-89.48	-88.80	-95.75	-112.32

Table C.4: "Back-Test" summary indicators of the *VaR* measures calculated with the "fair" panel, against the market data from the Santiago Stock Exchange. Confidence level of 5% for the *VaR* calculations - t de Student. Sample between August 1998 and February 2003.

Indicators	Bonds Differentiated by Maturities in Years																				Summary Indicators
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
% excess over VaR	8.37%	6.69%	5.69%	4.42%	5.88%	5.57%	6.33%	6.48%	5.71%	6.53%	8.51%	5.33%	7.42%	6.55%	5.41%	6.13%	6.40%	5.19%	7.06%	7.48%	6.26%
K (Kupiec Test)	4.55	1.88	0.36	0.47	0.63	0.28	2.28	3.79	0.36	1.73	5.08	0.09	2.77	2.12	0.12	0.90	1.13	0.03	3.94	6.67	1.31
Reject H_0	X										X								X	X	
Average VaR	-57.53	-55.02	-59.39	-51.78	-52.82	-45.72	-48.34	-47.26	-41.52	-36.91	-35.31	-35.47	-40.15	-40.66	-45.81	-47.53	-48.70	-46.38	-49.41	-57.51	-48.12
Average Excess over VaR	-23.77	-28.21	-21.61	-22.66	-38.58	-27.82	-26.46	-18.54	-23.02	-18.10	-20.52	-11.67	-24.87	-17.20	-37.43	-20.52	-31.98	-41.37	-33.66	-32.53	-25.73
Maximum Excess over VaR	-64.40	-77.54	-64.16	-76.83	-177.08	-177.37	-196.97	-114.97	-122.71	-127.36	-66.19	-31.56	-129.25	-82.18	-126.29	-102.39	-119.42	-106.26	-126.37	-141.22	-196.97

Table C.5: "Back-Test" summary indicators of the *VaR* measures calculated with the "fair" panel, against the market data from the Santiago Stock Exchange. Confidence level of 5% for the *VaR* calculations - Method of historical simulation. Sample between August 1998 and February 2003.

Indicators	Bonds Differentiated by Maturities in Years																				Summary Indicators
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
% excess over VaR	8.37%	6.98%	6.78%	5.21%	6.62%	6.50%	6.93%	6.93%	6.29%	6.79%	8.51%	6.13%	8.20%	8.95%	7.12%	7.52%	6.73%	5.48%	7.86%	9.18%	7.10%
K (Kupiec Test)	4.55	2.54	2.21	0.06	2.05	1.87	4.67	6.28	1.13	2.34	5.08	0.95	4.67	12.32	2.96	4.19	1.70	0.16	7.34	17.56	3.49
Reject H_0	X						X	X			X		X	X		X			X	X	
Average VaR	-42.98	-44.58	-44.93	-45.97	-41.09	-42.04	-42.51	-35.92	-34.53	-30.41	-27.59	-28.75	-29.58	-32.27	-38.52	-38.91	-41.35	-42.68	-51.08	-53.94	-40.02
Average Excess over VaR	-25.49	-32.56	-22.88	-22.15	-38.80	-27.72	-27.17	-19.31	-23.01	-19.15	-24.49	-12.51	-25.00	-15.23	-30.15	-19.43	-33.64	-42.88	-34.81	-31.11	-25.91
Maximum Excess over VaR	-68.78	-106.35	-84.39	-93.20	-196.05	-191.90	-201.89	-112.21	-116.50	-124.44	-64.83	-35.80	-128.37	-85.26	-123.62	-99.57	-118.36	-109.24	-129.17	-143.63	-201.89

Table C.6: "Back-Test" summary indicators of the *VaR* measures calculated with the "fair" panel, against the market data from the Santiago Stock Exchange. Confidence level of 5% for the *VaR* calculations - "Extreme Value Theory" in its static version. Sample between August 1998 and February 2003.

Indicators	Bonds Differentiated by Maturities in Years																				Summary Indicators
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
% excess over VaR	7.49%	8.14%	8.13%	5.68%	7.11%	7.19%	8.13%	6.82%	6.57%	5.48%	9.79%	6.40%	9.77%	9.61%	6.27%	7.24%	7.07%	5.48%	8.67%	9.52%	7.48%
K (Kupiec Test)	2.59	6.07	6.45	0.59	3.39	3.87	11.65	5.61	1.66	0.18	8.97	1.43	9.69	16.31	1.10	3.36	2.39	0.16	11.64	20.26	4.78
Reject H_0		X	X			X	X	X			X		X	X					X	X	X
Average VaR	-30.30	-26.08	-29.67	-34.47	-36.10	-32.45	-33.61	-31.25	-32.11	-29.84	-25.74	-23.53	-24.37	-32.38	-30.30	-36.53	-34.85	-38.36	-36.67	-44.90	-31.61
Average Excess over VaR	-31.62	-33.42	-24.09	-24.74	-41.93	-28.40	-28.13	-22.86	-25.17	-24.51	-23.75	-13.48	-23.42	-15.22	-36.40	-21.59	-35.87	-43.09	-34.86	-33.05	-27.80
Maximum Excess over VaR	-70.58	-115.52	-103.17	-109.90	-209.58	-205.43	-215.80	-128.25	-133.65	-136.80	-74.49	-37.17	-138.87	-86.52	-136.52	-109.86	-133.32	-121.99	-134.46	-147.78	-215.80

Table C.7: "Back-Test" summary indicators of the *VaR* measures calculated with the "fair" panel, against the market data from the Santiago Stock Exchange. Confidence level of 5% for the *VaR* calculations - "Extreme Value Theory" in its dynamic version. Sample between August 1998 and February 2003.

Indicators	Bonds Differentiated by Maturities in Years																				Summary Indicators
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
% excess over VaR	8.81%	7.27%	6.78%	4.57%	6.13%	4.41%	5.42%	5.81%	5.43%	5.74%	8.09%	5.07%	7.42%	6.11%	3.99%	5.01%	4.38%	5.48%	6.65%	6.46%	5.81%
K (Kupiec Test)	5.71	3.30	2.21	0.25	1.02	0.33	0.24	1.18	0.13	0.43	4.00	0.00	2.77	1.12	0.81	0.00	0.25	0.16	2.60	2.43	0.56
Reject H_0	X										X										
Average VaR	-33.48	-37.39	-40.93	-42.57	-39.25	-35.37	-39.36	-31.95	-30.40	-29.01	-27.43	-28.83	-33.38	-31.72	-34.58	-40.03	-37.81	-42.77	-42.22	-42.67	-36.61
Average Excess over VaR	-18.35	-28.93	-17.16	-21.98	-28.48	-22.53	-21.11	-17.50	-16.15	-13.01	-16.67	-9.28	-18.52	-13.81	-29.54	-17.69	-25.29	-29.17	-23.20	-25.65	-20.38
Maximum Excess over VaR	-50.46	-86.87	-80.18	-79.99	-113.27	-106.38	-96.07	-96.40	-100.26	-105.26	-49.54	-32.01	-83.50	-84.08	-88.56	-91.91	-66.99	-95.00	-94.98	-97.43	-113.27

Table C.8: "Back-Test" summary indicators of the *VaR* measures calculated with the "fair" panel, against the market data from the Santiago Stock Exchange. Confidence level of 5% for the *VaR* calculations Monte Carlo simulation method, using the stochastic structure developed in Cortázar, Schwartz and Naranjo (2003) with a three-factor Vasicek model. Sample between August 1998 and February 2003.

Indicators	Bonds Differentiated by Maturities in Years																				Summary Indicators
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
% excess over VaR	7.60%	6.54%	5.97%	4.43%	5.94%	4.88%	5.03%	5.09%	4.89%	4.47%	6.96%	4.57%	6.37%	5.73%	3.71%	4.15%	3.61%	4.47%	5.58%	5.30%	5.17%
K (Kupiec Test)	2.81	1.57	0.69	0.45	0.71	0.01	0.00	0.02	0.01	0.23	1.70	0.15	0.93	0.49	1.36	0.58	1.32	0.21	0.34	0.11	0.02
Reject H_0																					
Average VaR	-32.90	-34.36	-38.10	-42.93	-37.13	-35.19	-39.57	-34.64	-30.45	-32.06	-30.58	-30.29	-33.85	-32.77	-32.88	-36.35	-39.26	-41.13	-50.11	-44.84	-37.56
Average Excess over VaR	-21.14	-25.14	-14.33	-17.99	-26.44	-16.69	-18.82	-18.64	-18.48	-13.74	-17.97	-11.44	-20.13	-12.83	-28.17	-21.68	-28.29	-29.26	-21.85	-27.92	-20.23
Maximum Excess over VaR	-46.24	-62.59	-57.60	-72.36	-115.97	-110.94	-81.15	-88.55	-94.88	-105.81	-51.89	-25.50	-85.84	-78.11	-83.86	-82.47	-66.28	-89.31	-88.71	-97.01	-115.97

ANNEX D: Measures for the Historical Testing (confidence level of 1%).

The next tables show the measures for the historical testing calculated for each individual asset, and for each *VaR* calculation method (Section 4.3).

The sub-sample used to perform the "Back Test". The sample consists of daily PRCs transactions between August 1998 and February 2003 (1116 days), with confidence level of 1% for the *VaR* calculations

We must note, however, that the weakness of this procedure is that it can not be tested for the portfolio. This is apparent because is very difficult that all of the assets conforming a portfolio have been traded during the same day. Therefore, we would only be able to perform individual tests for each asset.

Given that the chosen measures for the historical testing will be calculated for each individual asset, and for each *VaR* calculation method. We need a global measure in order to compare the alternative *VaR* calculation methods. Therefore, we have created "summary indicators".

The summary indicators are calculated simultaneously with all of the bonds in line (as it was a single bond). We did not use the average because the percentage of days in which '$w_{t+d+1,t}$' could be calculated (the outcome in monetary units of an investment) is not the same along the sample.

Table D.1: "Back-Test" summary indicators of the *VaR* measures calculated with the "fair" panel, against the market data from the Santiago Stock Exchange. Confidence level of 1% for the *VaR* calculations - Method of the sample variance and covariance. Sample between August 1998 and February 2003.

Indicators	Bonds Differentiated by Maturities in Years																				Summary Indicators
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
% excess over VaR	5.29%	4.36%	2.98%	2.68%	4.17%	2.32%	3.31%	3.13%	3.71%	2.87%	3.83%	2.40%	3.91%	2.62%	3.13%	2.23%	3.37%	3.46%	3.23%	3.23%	3.21%
***K* (Kupiec Test)**	20.93	21.52	9.56	12.40	23.10	5.54	22.39	26.18	15.38	9.03	11.06	5.33	12.59	8.40	10.31	4.06	10.39	12.93	15.65	18.63	13.23
Reject H_0	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
Average VaR	-64.27	-76.76	-74.55	-72.71	-69.08	-66.11	-59.09	-62.49	-54.19	-58.94	-50.28	-53.40	-54.77	-56.50	-62.28	-65.62	-62.45	-67.10	-72.38	-76.71	-68.87
Average Excess over VaR	-22.57	-23.02	-15.92	-16.37	-33.00	-42.05	-30.61	-19.35	-19.16	-24.71	-23.80	-10.01	-27.31	-21.24	-44.91	-32.19	-38.80	-38.29	-41.22	-43.26	-28.31
Maximum Excess over VaR	-52.99	-63.42	-34.78	-48.37	-147.98	-150.37	-173.97	-95.56	-105.26	-111.67	-52.17	-21.59	-114.89	-64.92	-109.22	-85.33	-98.28	-84.65	-99.47	-112.51	-173.97

Table D.2: "Back-Test" summary indicators of the *VaR* measures calculated with the "fair" panel, against the market data from the Santiago Stock Exchange. Confidence level of 1% for the *VaR* calculations - "RiskMetrics". Sample between August 1998 and February 2003.

Indicators	Bonds Differentiated by Maturities in Years																				Summary Indicators
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
% excess over VaR	5.29%	2.91%	2.71%	1.89%	2.94%	2.09%	2.26%	2.57%	2.57%	3.39%	3.40%	2.13%	3.13%	3.06%	2.56%	2.23%	2.69%	2.88%	2.22%	2.55%	2.65%
***K* (Kupiec Test)**	20.93	8.37	7.43	4.04	10.21	3.93	7.85	15.54	6.09	13.69	8.44	3.67	7.47	12.64	6.06	4.06	5.88	8.23	5.52	10.00	7.98
Reject H_0	X	X	X	X	X	X	X	X	X	X	X		X	X	X	X	X	X	X	X	X
Average VaR	-41.52	-51.14	-50.26	-47.89	-46.61	-45.82	-48.27	-38.81	-37.30	-41.43	-36.23	-36.09	-41.69	-37.47	-40.54	-49.03	-46.79	-51.23	-58.33	-53.45	-47.67
Average Excess over VaR	-8.48	-9.66	-6.67	-11.55	-20.68	-19.07	-19.48	-13.95	-13.67	-11.61	-11.58	-7.76	-18.61	-11.84	-23.15	-20.10	-14.95	-21.77	-16.60	-17.96	-14.90
Maximum Excess over VaR	-24.36	-29.47	-19.11	-43.26	-89.23	-80.86	-96.16	-53.44	-56.29	-62.45	-34.15	-13.47	-54.49	-39.87	-42.86	-44.27	-31.79	-43.09	-41.13	-42.84	-96.16

Table D.3: "Back-Test" summary indicators of the *VaR* measures calculated with the "fair" panel, against the market data from the Santiago Stock Exchange. Confidence level of 1% for the *VaR* calculations - GARCH(1,1). Sample between August 1998 and February 2003.

Indicators	Bonds Differentiated by Maturities in Years																				Summary Indicators
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
% excess over VaR	3.52%	3.78%	2.17%	2.21%	2.70%	1.62%	1.96%	2.12%	2.00%	1.57%	2.98%	1.33%	2.34%	1.75%	1.71%	1.39%	2.02%	2.31%	2.42%	2.04%	2.14%
***K* (Kupiec Test)**	8.84	15.76	3.81	6.95	8.10	1.43	4.82	8.62	2.74	1.06	6.07	0.38	3.39	2.11	1.47	0.50	2.41	4.36	7.23	4.94	4.18
Reject H_0	X	X		X	X		X	X			X							X	X	X	X
Average VaR	-52.03	-66.64	-63.31	-70.18	-65.13	-60.69	-58.23	-48.01	-47.84	-47.79	-48.68	-45.82	-56.42	-58.53	-52.55	-55.21	-69.15	-66.67	-74.85	-83.67	-56.90
Average Excess over VaR	-19.14	-16.10	-10.71	-14.98	-25.80	-27.08	-18.89	-20.70	-19.01	-20.29	-16.10	-8.59	-29.74	-19.52	-33.34	-25.56	-16.01	-27.50	-15.01	-24.27	-20.06
Maximum Excess over VaR	-32.53	-35.32	-32.17	-53.05	-86.53	-81.66	-46.09	-60.66	-64.63	-73.23	-39.68	-11.63	-52.96	-61.50	-66.34	-68.28	-37.82	-65.41	-61.94	-72.26	-86.53

<u>Table D.4</u>: "Back-Test" summary indicators of the *VaR* measures calculated with the "fair" panel, against the market data from the Santiago Stock Exchange. Confidence level of 1% for the *VaR* calculations - t de Student. Sample between August 1998 and February 2003.

Indicators	Bonds Differentiated by Maturities in Years																				Summary Indicators
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
% excess over VaR	5.29%	4.36%	2.98%	2.68%	4.17%	2.32%	3.31%	3.13%	3.71%	2.87%	3.83%	2.40%	3.91%	2.62%	3.13%	2.23%	3.37%	3.46%	3.23%	3.23%	3.21%
***K* (Kupiec Test)**	20.93	21.52	9.56	12.40	23.10	5.54	22.39	26.18	15.38	9.03	11.06	5.33	12.59	8.40	10.31	4.06	10.39	12.93	15.65	18.63	13.23
Reject H_0	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
Average VaR	-89.81	-83.15	-79.66	-74.74	-75.93	-67.15	-64.01	-60.79	-61.18	-56.38	-53.41	-60.56	-59.15	-54.47	-57.62	-69.62	-72.76	-77.55	-81.84	-81.10	-71.52
Average Excess over VaR	-22.53	-22.96	-14.93	-16.31	-32.93	-41.94	-30.35	-19.27	-19.12	-24.67	-23.75	-9.97	-27.26	-21.19	-44.85	-32.14	-38.74	-38.22	-41.14	-43.18	-28.19
Maximum Excess over VaR	-52.95	-63.37	-34.68	-48.26	-147.87	-150.27	-173.89	-95.49	-105.20	-111.61	-52.12	-21.56	-114.84	-64.86	-109.16	-85.26	-98.21	-84.58	-99.37	-112.40	-173.89

<u>Table D.5</u>: "Back-Test" summary indicators of the *VaR* measures calculated with the "fair" panel, against the market data from the Santiago Stock Exchange. Confidence level of 1% for the *VaR* calculations - Method of historical simulation. Sample between August 1998 and February 2003.

Indicators	Bonds Differentiated by Maturities in Years																				Summary Indicators
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
% excess over VaR	3.08%	2.03%	1.63%	1.42%	2.70%	1.62%	1.96%	1.90%	2.57%	2.09%	2.98%	1.60%	3.52%	2.84%	2.85%	2.51%	3.03%	3.46%	2.62%	2.89%	2.35%
***K* (Kupiec Test)**	6.41	2.87	1.23	1.00	8.10	1.43	4.82	5.79	6.09	3.50	6.07	1.15	9.91	10.44	8.08	5.81	8.02	12.93	9.10	14.07	5.65
Reject H_0	X				X		X	X	X		X		X	X	X	X	X	X	X	X	X
Average VaR	-89.69	-97.77	-98.53	-96.88	-89.06	-87.83	-77.72	-80.82	-80.64	-83.54	-82.97	-70.02	-73.30	-75.58	-73.82	-71.29	-71.06	-87.82	-93.28	-105.80	-78.65
Average Excess over VaR	-18.74	-18.37	-10.11	-10.87	-25.51	-31.40	-30.75	-19.26	-18.61	-26.36	-24.14	-12.77	-23.56	-19.36	-34.81	-26.70	-25.36	-33.90	-36.96	-36.53	-25.42
Maximum Excess over VaR	-40.16	-48.10	-18.88	-24.86	-90.10	-90.91	-120.79	-82.01	-83.84	-86.00	-39.48	-22.84	-70.52	-65.10	-83.38	-71.11	-47.45	-68.42	-78.48	-94.55	-120.79

<u>Table D.6</u>: "Back-Test" summary indicators of the *VaR* measures calculated with the "fair" panel, against the market data from the Santiago Stock Exchange. Confidence level of 1% for the *VaR* calculations - "Extreme Value Theory" in its static version. Sample between August 1998 and February 2003.

Indicators	Bonds Differentiated by Maturities in Years																				Summary Indicators
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
% excess over VaR	4.85%	3.49%	2.17%	1.74%	3.43%	2.09%	2.86%	2.79%	3.43%	2.35%	2.98%	2.40%	3.52%	2.18%	2.85%	2.23%	3.70%	3.46%	3.43%	3.23%	2.86%
***K* (Kupiec Test)**	17.60	13.12	3.81	2.84	14.93	3.93	15.49	19.55	12.78	5.12	6.07	5.33	9.91	4.84	8.08	4.06	12.97	12.93	18.10	18.63	9.83
Reject H_0	X	X			X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
Average VaR	-97.67	-117.03	-113.46	-121.01	-95.63	-93.32	-88.88	-72.87	-74.46	-64.26	-66.80	-72.61	-67.16	-71.98	-73.50	-85.93	-86.14	-101.16	-95.42	-111.44	-80.12
Average Excess over VaR	-19.23	-22.97	-12.44	-18.58	-33.02	-38.96	-30.44	-18.20	-18.44	-26.74	-27.63	-9.65	-26.83	-22.56	-39.27	-25.61	-30.14	-32.44	-34.30	-39.52	-26.86
Maximum Excess over VaR	-51.38	-61.83	-24.04	-38.23	-125.87	-128.62	-157.09	-84.09	-94.81	-101.86	-44.65	-22.08	-101.23	-62.32	-91.70	-73.46	-80.78	-71.34	-92.28	-105.03	-157.09

Table D.7: "Back-Test" summary indicators of the *VaR* measures calculated with the "fair" panel, against the market data from the Santiago Stock Exchange. Confidence level of 1% for the *VaR* calculations - "Extreme Value Theory" in its dynamic version. Sample between August 1998 and February 2003.

Indicators	Bonds Differentiated by Maturities in Years																				Summary Indicators
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
% excess over VaR	3.08%	2.33%	1.08%	1.26%	1.72%	1.39%	1.36%	1.68%	1.71%	1.31%	2.98%	1.33%	1.95%	1.97%	1.71%	1.11%	1.35%	2.02%	2.22%	2.04%	1.71%
***K* (Kupiec Test)**	6.41	4.46	0.03	0.41	1.74	0.60	0.76	3.43	1.49	0.33	6.07	0.38	1.84	3.36	1.47	0.05	0.33	2.80	5.52	4.94	1.79
Reject H_0	X	X									X								X	X	
Average VaR	-75.38	-91.85	-97.08	-86.78	-90.80	-80.92	-71.66	-64.76	-56.77	-53.80	-51.99	-56.96	-62.29	-67.30	-65.40	-71.10	-77.32	-84.47	-80.26	-96.24	-70.93
Average Excess over VaR	-11.99	-11.24	-9.79	-14.49	-21.89	-17.76	-15.10	-18.35	-15.75	-19.51	-11.48	-3.99	-26.90	-16.36	-21.76	-22.63	-13.83	-26.42	-13.41	-20.59	-16.76
Maximum Excess over VaR	-29.72	-27.66	-21.04	-41.14	-62.84	-60.67	-33.66	-46.64	-39.14	-56.35	-39.91	-9.24	-55.49	-60.92	-64.13	-65.38	-35.74	-62.80	-61.39	-57.33	-65.38

Table D.8: "Back-Test" summary indicators of the VaR measures calculated with the "fair" panel, against the market data from the Santiago Stock Exchange. Confidence level of 1% for the VaR calculations - Monte Carlo simulation method, using the stochastic structure developed in Cortázar, Schwartz and Naranjo (2003) with a three-factor Vasicek model. Sample between August 1998 and February 2003.

Indicators	Bonds Differentiated by Maturities in Years																				Summary Indicators
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
% excess over VaR	4.00%	4.37%	2.53%	2.70%	3.07%	2.12%	1.98%	2.19%	2.34%	1.98%	3.34%	1.58%	2.65%	1.76%	2.28%	1.77%	2.40%	2.63%	2.95%	2.48%	2.47%
***K* (Kupiec Test)**	11.76	21.61	6.16	12.66	11.40	4.15	5.04	9.63	4.61	2.88	8.06	1.08	4.85	2.20	4.24	1.74	4.24	6.45	12.52	9.22	6.57
Reject H_0	X	X	X	X	X	X	X	X	X		X		X		X		X	X	X	X	X
Average VaR	-73.44	-86.98	-93.28	-90.66	-84.09	-76.73	-70.44	-60.88	-59.16	-55.99	-50.11	-52.19	-55.00	-61.45	-65.26	-68.62	-68.06	-75.05	-82.62	-88.12	-70.91
Average Excess over VaR	-20.50	-16.70	-10.78	-17.41	-26.32	-28.95	-19.36	-20.80	-22.15	-18.98	-15.79	-11.26	-29.19	-20.98	-34.25	-25.33	-16.66	-30.04	-15.64	-24.70	-21.00
Maximum Excess over VaR	-38.54	-35.41	-38.11	-55.84	-87.97	-89.81	-51.09	-63.26	-68.22	-79.57	-49.34	-16.74	-59.47	-67.41	-67.04	-73.02	-39.57	-72.79	-68.78	-72.42	-89.81