Do Time-Varying Covariances, Volatility Comovement and Spillover Matter?

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Abstract

Financial markets and their respective assets are so intertwined; analyzing any single market in isolation ignores important information. We investigate whether time varying volatility comovement and spillover impact the true variance-covariance matrix under a time-varying correlation set up. Statistically significant volatility spillover and comovement between US, UK and Japan is found. To demonstrate the importance of modelling volatility comovement and spillover, we look at a simple portfolio optimization application. A utility based comparison is used to evaluate the economic performance of the portfolio which considers time varying correlation with volatility comovement and spillover. This paper shows that a portfolio strategy incorporating time-varying correlation with asymmetric volatility comovement and spillover outperforms the constant correlation model without comovement and spillover by yielding the highest level of wealth and utility difference of up to 250 basis points.

JEL Classification: C32, F3, G15

Key words: Volatility comovement, Volatility spillover, Dynamic Conditional Correlation, Multivariate GARCH

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1 Introduction

A burgeoning area of research in financial econometrics is the investigation of time-varying volatility in financial returns series. The motivation underlying studies on volatility comovement and spillover is to understand how joint movements in volatility influence the distribution of portfolio returns as this has implications for daily risk management, portfolio selection and derivative pricing.[1] While comovement in volatility helps us understand transmissions of shocks through the global financial system, we find that there are two effects that influence the volatility of financial markets and their respective assets; volatility comovement and volatility spillover. According to Rigobon and Sack (2003), asset prices are so intertwined, analyzing any single market in isolation ignores important information about its behavior. Changes in the price of an asset in its own market are driven not only by its own volatility shocks, but also by its reaction to shocks in the prices of assets in other markets.

Modelling volatility comovement and spillover between markets gives a better idea of what assets to include and exclude in the portfolio. We examine the importance of modelling volatility comovement and spillover in an asset allocation framework. Standard investment textbooks teach investors to hold assets that are not highly correlated. Correlation numbers themselves do not tell you much about risk or volatility interactions. Moreover, correlation between different assets are time varying. Therefore, explicit modelling of risk interactions is necessary. Modelling of volatility comovement and spillover allows for that and provides more accurate inputs to the classic portfolio formulation problem.

Volatility can be transmitted between assets directly through the lagged conditional variance or indirectly via lagged conditional covariance. In this paper, we define spillover as lagged shocks and comovement as contemporaneous shocks and use the closing times of stock markets as a point of reference in defining the volatility comovement and spillover effect. Distinguishing

[1] Refer to Calvert, Fisher and Thompson (2004)

between volatility spillover and comovement enables us to more accurately characterize how a shock to one market impacts the risk of another market.[2]

Differentiating between volatility comovement and spillover impacts investors holding international portfolios and who wish to hedge against risk. Fleming et al (2001, 2002) find that the utility value of volatility timing can be as much as 50 to 200 basis points per year. They find that volatility timing strategies outperform the unconditionally efficient static portfolios that have the same target expected return and volatility. In terms of volatility comovement and spillover, analyzing time-varying covariances is important for an investor seeking to minimize portfolio risk. Having information about time-varying volatility comovement and spillover differentiates between an informed investor as opposed to the myopic investor who only knows the unconditional volatilities and correlations.

In this paper we aim to answer three key questions which have important implications for an investor holding an international portfolio.

(i) Are there significant volatility interactions in the form of comovement and spillover between US, UK and Japan?

(ii) If significant, how does time varying correlation with volatility comovement and spillover impact an international portfolio of assets?

(iii) Using logarithmic utility as a performance metric, does the time-varying volatility comovement and spillover model perform better?[3]

To answer these questions, we employ the Dynamic Conditional Correlation (DCC) model of Engle (2002) to analyze volatility comovement and spillover. This differs from the benchmark Constant Conditional Correlation (CCC) model of Bollerslev (1990) typically used to estimate covariances. The DCC specification incorporates time varying correlations between the three markets.

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[2] Engle, Ito & Lin. (1990) use the "meteor shower" and "heat wave" concept to test for market dexterity. They reject "heat waves" and conclude that the market suffers only from volatility spillover. Engle et al. (1994) find that except for lagged return spillover from New York to Tokyo for the period after the crash, there is no significant lagged spillover in returns or in volatilities. In our paper, a spillover is similar to a meteor shower and a comovement to heat wave.

[3] Refer to West et al (1993) for utility-based comparisons of foreign exchange rate models

Engle et al. (1994) investigate how returns and volatilities of stock indices are correlated between Tokyo and New York using intradaily data. They find that except for a lagged return spillover from New York to Tokyo for the period after the crash, there was no significant lagged spillover in returns or in volatilities. Koutmos and Booth (1995) study volatility spillover across the London, New York and Tokyo stock markets. They adopt an EGARCH specification for each conditional variance. They find that effects were empirically significant and the volatility spillover was present in all directions. However, the multivariate GARCH aspect was completed using a constant conditional correlation specification. Karolyi (1995) focuses on the volatility of returns on the US and Canadian stock indices. He compares the impulse response functions using four specifications of the volatility process. However, he does not analyze the two conditional variance equations simultaneously in a multivariate set up using time varying conditional covariance. Ng (2000) examines the size and the impact of volatility spillover from Japan and the US to six Pacific-Basin equity markets. By employing four different correlation specifications, she constructs a volatility spillover model which distinguishes between a local idiosyncratic shock, a regional shock from Japan and a global shock from the US and finds significant spillover from the region to the Pacific Basin economies. Balasubramanyan and Premaratne (2003) find that explicit volatility interactions in the form of comovement and spillover are significant from smaller markets to larger markets. This phenomenon is evinced by volatility interactions between Singapore and regional markets such as Hong Kong and Japan. These studies do not analyze volatility comovement and spillover within a same framework using time-varying correlations.

Research involving multivariate GARCH models typically employ a constant covariance specification. The DCC specification on the other hand, allows for the multivariate system to incorporate new information.[4] Numerous studies have shown that correlations between markets are time-varying. Kaplanis (1988) compares the monthly returns of ten markets and find that correlations increase during recessionary periods. King and Wadhwani (1990) find that

[4] Bauwens et al (2003) has a comprehensive table which shows the specification employed in key research papers that use multivariate GARCH approach. All analyses have been done using constant correlation.

international correlations tend to increase during periods of market crises.[5] Longin and Solnik (1995) study the correlations between excess returns on the stock market indices of the G7 countries. Rather than estimating the models simultaneously, they estimate bivariate models for each country paired with the US. They find that correlations have increased over time. Longin and Solnik (2001) show that high volatility per se does not lead to an increase in conditional correlation. Correlation is affected mainly by the market trend. It is only in bear markets that conditional correlation strongly increases; conditional correlation does not increase in bull markets.

By using a DCC specification, our model fills a gap in the literature. The use of DCC specification allows us to incorporate contemporaneous correlations in calendar time rather than merely capture lead or lag relationships.[6] According to Kroner and Ng (1998), unlike several univariate models, none of the popular multivariate models allow for asymmetric effect in the covariance. Such a phenomenon is likely if there is an asymmetric effect in the variance. In accordance to Hentschel (1995), we impose asymmetric volatility comovement and spillover in the conditional variance equations and analyze the effects on the variance covariance matrix.

Our contribution is to capture the volatility interactions and time varying covariance between three markets and their respective assets and to analyze the implications for portfolio and risk management. Issues such as transaction costs and market micro structural aspects have been ignored for the sake of tractability. In our application, only a very basic portfolio with little diversification is considered at this point but we plan to expand to many markets.

The aim of this paper is to analyze volatility spillover and comovement and to translate statistically significant risk interactions between the markets to potential economic benefits, by

[5] One of the earliest contributions was by King and Wadhwani (1990) and Hamao et al. (1990). Subsequent research by Engle and Susmel (1993), King et al. (1994), Ito and Lin (1994) look at the transmission of volatility across international stock markets. Bae and Karolyi (1994) extend the GARCH framework to allow for asymmetric effects of negative ("bad news") and positive ("good news") foreign market returns shocks for volatility. Evidence shows that the magnitude and persistence of shocks originating in New York or Tokyo that transmit to other market are understated if the asymmetric effect is ignored.

[6] Bauwens et al. (2003) mentions in his survey paper, the difficulty that results from interpreting the moving average coefficients because trading hours are not the same between the markets and the conditional correlations do not reflect contemporaneous correlations in the calendar time.

applying a spillover-comovement model to a portfolio optimization problem. Using utility as a performance metric, we evaluate the volatility spillover-comovement model.

We find that a model with time varying correlation with asymmetric volatility comovement and spillover yields the highest level of wealth. This model performs the best. The second best model is the model with time varying correlation with volatility comovement and spillover. Models that incorporate volatility comovement and spillover perform better and yield higher portfolio wealth and utility as opposed to a time varying correlation model that fails to consider volatility interactions.

The remainder of the paper is organized as follows: Section 2 describes the data and some descriptive statistics. Section 3 develops the volatility comovement and spillover model. Section 4 reports the empirical results and Section 5 presents an application of the model to an international portfolio optimization problem. Section 6 concludes.

2 Data

The daily stock returns data come from DataStream.[7] We use daily as opposed to weekly returns in order to capture the informational impact of spillover and comovement. The country indices of US, UK and Japan account for at least 85% of each country's stock market capitalization. The data covers the period 1/02/1984 through 7/22/2004 and are in the respective local currency terms for a total of 5363 usable observations. Table I reports the univariate statistics for the various indices. Japan exhibits the highest standard deviation followed by the US and UK. All three countries have distributions with positive excess kurtosis[8].

Table II shows the opening and closing times for US, UK and Japanese markets. The three markets close in the order of Japan, UK and the US. This ordering has implications on whether the shock is a spillover or comovement. As can be seen, the US and UK markets overlap while the US and Japanese markets are asynchronous. For the purpose of our analysis of volatility comovement and spillover, we use the closing times as a point of reference.

[7] The indices we use are as follows :
US:Dow Jones Industrial Average
UK: FTSE 100
Japan:Nikkei 225

[8] If the kurtosis exceeds 3, the distribution is said to be leptokurtic relative to the normal.

Panel B of Table II shows the unconditional correlation coefficients between daily market returns in local currency terms. Martens and Poon (2001) have shown that using non-synchronous data results in a significant downward bias in correlation when compared to correlations obtained by constructing a sample of all prices at same time GMT. For Japan and US, there is never a time when all three markets are open.

Panel C of Table II shows the unconditional correlation coefficients with US and UK returns series that has been lagged. By doing so, the correlation between US and Japan increases from 0.114 to 0.304. This is consistent with the findings of Martens and Poon (2001). The correlation between UK and Japan falls from 0.256 to 0.202. The correlations between US and UK remain constant in both cases. Figure 1 shows plots of the 3 returns series. The Japanese market is much more volatile than the US and UK markets. This is especially true during the period 1990 through 1998. There are similarities in the returns movement of US and UK, but not between UK and Japan.

3 Empirical Methodology

3.1 Preliminary Analysis Using Variance Decomposition & Impulse Response

In order to study the comovement and spillover of volatility, we first analyze the market dynamics, transmission and propagation mechanism driving these stock markets. We estimate a vector auto regression model (VAR)[9]. The VAR model estimates a dynamic simultaneous equation system, free of *a priori* restrictions on the structure of relationships. Since no restrictions are imposed on the structural relationships between variables, the VAR system can be a flexible approximation to the reduced form of the correctly specified but unknown model of the actual economic structure. As structural models tend to be misspecified, a VAR can be used for the purpose of stylizing empirical regularities among time series data. There are several advantages of using VAR. Any shock to the stock market is typically characterized by an explosive effect to all financial markets. It is difficult to isolate the effect of a shock to any particular market in such a scenario.

[9] The VAR model was developed by Sims (1980) with the purpose of estimating unrestricted reduced-form equations that have uniform sets of lagged dependent variables as regressors.

The advantage of using VAR is that it not only gives us estimates of dependence between the a system of stock markets, but also allows us to shock a particular market and analyze how the shock perpetuates itself throughout the system using the impulse response (IR) analysis. VAR models help to capture the pure effects of artificial shocks introduced by the researcher in a similar manner to dynamic simulations. Another attractive feature of a VAR is that it assumes endogeneity of all variables in the system. If a VAR model is run on the stock return of three countries, then we get a three equation model. The dependent variables are the stock return of the three countries and the independent variables are own past returns and past returns of other countries lagged a certain number of times. The coefficients on these variables give us an idea about whether the stock returns of the different countries are linked in the short run.

After estimating a VAR model, we go on to obtain the variance decompositions. The decomposition of variance of the forecast errors of the returns of a given market indicates the relative importance of the various markets in causing the fluctuations in returns of that market. The decomposition allocates the variance of forecast error into percentages that are accounted for by innovations in all other markets including the market's own innovations. The preliminary analysis is consistent with our comovement and spillover postulate. In the case of Japan; US and UK spillover account for approximately 10 percent of Japan's variance on Day 2. As for UK; Japanese comovement accounts for 3.52 percent of UK's variance and UK's spillover accounts for 9.2 percent the following day. Japanese comovement with US accounts for 1.37 percent of US variance while UK comovement accounts for 15.46 percent of US variance on the same day.

Following the above analysis on variance decomposition, we go on to investigate the pattern of dynamic impulse response of the three markets to shocks from each of the other markets. The results provide insight on the efficiency of each of the markets with respect to the information contained in such shocks. The impulse response coefficients are normalized such that the unit is the standard deviation of the orthogonalized innovation. The initial shock in a variable is set equal to one standard error of innovation at s = 0. The normalization procedure makes it easy to compare the impulse responses across variables in the system. Figure 2 shows the impulse response of Japan, US and UK to each other. When the lower band crosses the horizontal axis, the response becomes statistically insignificant. The markets have been ordered according to closing times with Japan first, followed by UK and the US. The results found are consistent with our definition of lagged shocks being spillover and contemporaneous shocks as being defined as a comovement. Note that the VAR analysis does not involve the modeling of volatility. We are merely using it as a tool to check if our stipulation of contemporaneous and lagged shocks as being comovement and spillover is correct.

As seen in Figure 2, spillover show up as peaks on Day 2 and dies out by Day 3. Comovement, on the other hand, starts to die off on Day 1 and it is gone by Day 2. Having knowledge of the transmission patterns of spillover and comovement can help the investor in devising hedging strategies and managing risk for an international portfolio comprising of stocks from different markets.

3.2 Multivariate GARCH using Dynamic Conditional Correlation Framework

To understand the DCC-GARCH framework, start by writing the conditional variance-covariance matrix of:

$$H_t \equiv D_t R_t D_t \tag{1}$$

where $D_t = diag\left\{\sqrt{h_{it}}\right\}$ is a 3 x 3 diagonal matrix of time-varying standard deviations from univariate GARCH models; and $R_t \equiv \left\{\rho_{ij}\right\}_t$ for i,j = 1,2,3 which is a correlation matrix containing conditional correlation coefficients. The elements in D_t follow the univariate GARCH (p ,q) process in the following manner:

$$h_{it} = \omega_i + \sum_{p=1}^{P_i} \alpha_{ip}\varepsilon^2_{it-p} + \sum_{q=1}^{Q_i} \beta_{iq} h_{it-q} \qquad \forall\, i = 1,2,3 \tag{2}$$

Engle's (2002) DCC(m,n) structure can be written as:

$$R_t = Q_t^{*-1} Q_t Q_t^{*-1} \tag{3}$$

where

$$Q_t = (1 - \sum_{m=1}^{M} A_m - \sum_{n=1}^{N} B_n)\bar{Q} + \sum_{m=1}^{M} a_m(\xi_{t-m}\xi'_{t-m}) + \sum_{n=1}^{N} b_n Q_{t-n} \qquad (4)$$

where $\xi_t = \varepsilon_{it} / \sqrt{h_{it}}$, which is a vector containing standardized errors; $Q_t \equiv \{q_{ij}\}_t$ is the conditional variance-covariance matrix $\bar{Q}$ is obtained from the first stage of estimation; and Q_t^* is a diagonal matrix containing the square root of the diagonal elements of Q_t:

$$Q_t^* = \begin{bmatrix} \sqrt{q_{us-us}} & . & . \\ . & \sqrt{q_{uk-uk}} & . \\ . & . & \sqrt{q_{jp-jp}} \end{bmatrix} \qquad (5)$$

What is of interest to us in R_t is $\rho_{12,t} = q_{12,t} / \sqrt{q_{11,t}q_{22,t}}$, which represents the conditional correlation between each of the returns series for the three countries. The DCC-GARCH framework (2)-(4) is estimated using the maximum likelihood method in which the log-likelihood can be expressed as:

$$L = -\frac{1}{2}\sum_{t=1}^{T}\left(n\log(2\pi) + 2\log|D_t| + \log|R_t| + \varepsilon_t' R_t^{-1}\varepsilon_t\right) \qquad (6)$$

The DCC model is designed to allow for the two-stage estimation of the conditional covariance matrix $\boldsymbol{H_t}$: in the first stage univariate volatility models are fitted for each of the assets and estimates of h_{it} are obtained. In the second stage, the stock market returns are transformed by their estimated standard deviations resulting from the first stage and are used to estimate the parameters of the conditional correlation.[10] The true **H** matrix is generated using univariate GARCH models for the variances, combined with the correlations produced by the **Q**. The correlation estimators are given by (8).

[10] Refer to Capiello, Engle and Sheppard (2003) where asymmetry is introduced into the Q_t equation.

$$\boldsymbol{H_t} = \begin{bmatrix} h_{us,t} & . & . \\ \rho_{us-uk,t}\sqrt{h_{us,t}h_{uk,t}} & h_{uk,t} & . \\ \rho_{us-jp,t}\sqrt{h_{us,t}h_{jp,t}} & \rho_{uk-jp,t}\sqrt{h_{uk,t}h_{jp,t}} & h_{jp,t} \end{bmatrix} \tag{7}$$

$$\rho_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t}q_{jj,t}}} \tag{8}$$

3.3 Volatility Comovement and Spillover

Volatility comovement and spillover are incorporated into the conditional variance equations given by h_{it}. The comovement and spillover affect the D_t matrix. We define lagged shocks as spillover and contemporaneous shocks as comovement[11]. We come up with this definition based on the ordering of market closing times. Using the VAR and Impulse Response techniques as a preliminary analysis, we find that our ordering of markets based on closing times and its implication for whether the shocks are spillover or comovement is justified. The conditional variance equations with volatility spillover and comovement are specified as follows:

$$\begin{aligned} h_{us,t} &= \omega_{us} + \alpha_{us}\varepsilon^2_{us,t-1} + \beta_{us}h_{us,t-1} + \underbrace{\gamma_{us}\varepsilon^2_{uk,t}}_{comovement} + \underbrace{\gamma_{us}\varepsilon^2_{jp,t}}_{comovement} \\ h_{uk,t} &= \omega_{uk} + \alpha_{uk}\varepsilon^2_{uk,t-1} + \beta_{uk}h_{uk,t-1} + \underbrace{\theta_{uk}\varepsilon^2_{us,t-1}}_{spillover} + \underbrace{\gamma_{uk}\varepsilon^2_{jp,t}}_{comovement} \\ h_{jp,t} &= \omega_{jp} + \alpha_{jp}\varepsilon^2_{jp,t-1} + \beta_{jp}h_{jp,t-1} + \underbrace{\theta_{jp}\varepsilon^2_{us,t-1}}_{spillover} + \underbrace{\theta_{jp}\varepsilon^2_{uk,t-1}}_{spillover} \end{aligned} \tag{9}$$

3.4 Asymmetric Volatility Comovement and Spillover

There are several univariate models that capture asymmetric volatility effect. However, few multivariate models capture asymmetric volatility effects. According to Kroner and Ng (1998), if the asymmetric effect in volatility is due to an increase in the information flow following bad news, then the covariance between stock returns should be affected because there will be a change in the relative rate of information flow across firms. In our case, we will look at the

[11] This follows a specification similar to Fratzscher (2001)

information flow across three countries. This would matter for an investor who is holding a portfolio consisting of international assets. A multivariate asymmetric volatility comovement and spillover model using a DCC specification would involve a two step estimation process. We first estimate the $\boldsymbol{Q_t}$ matrix. Then we estimate the asymmetric effects to the multivariate GARCH model with spillover and comovement. In this paper, the asymmetry is applied only to the conditional variance equations. This differs from Capiello, Engle and Sheppard (2003) specification where they apply asymmetry to the correlations. This results in a different covariance matrix compared to the one obtained without asymmetry. The asymmetric terms in a GARCH model change the diagonal terms to

$$\boldsymbol{H_{ii,t}} = c_{ii} + \sum_j a_{ij}\varepsilon_{i,t-1}^2 + \sum_j b_{ij}\boldsymbol{H_{jj,t-1}} + d_{ii}\varepsilon_{i,t-1}^2 \boldsymbol{I}_{\varepsilon>0}\varepsilon_{i,t-1} \quad (10)$$

$$\boldsymbol{H_{ii,t}} = c_{ii} + \sum_j a_{ij}\varepsilon_{i,t-1}^2 + \sum_j b_{ij}\boldsymbol{H_{jj,t-1}} + \underbrace{\theta_{ii}\varepsilon_{j,t-1}^2}_{Volatility\ Spillover} + \underbrace{\psi_{ii}\varepsilon_{j,t-1}^2 \boldsymbol{I}_{\varepsilon>0}\varepsilon_{i,t-1}}_{Asymmetric\ Volatility\ Spillover}$$
$$\underbrace{\gamma_{ii}\varepsilon_{j,t}^2 \boldsymbol{I}_{\varepsilon>0}\varepsilon_{i,t}}_{Volatility\ Comovement} + \underbrace{\eta_{ii}\varepsilon_{j,t}^2 \boldsymbol{I}_{\varepsilon>0}\varepsilon_{i,t}}_{Asymmetric\ Volatility\ Comovement} \quad (11)$$

where $\boldsymbol{I}_\varepsilon$ takes on the value 1 if $\varepsilon < 0$ and the value 0 if $\varepsilon > 0$. A model with asymmetric volatility comovement and spillover will change the diagonal terms to the specification in (11).

4 Results

The primary finding of this paper is that volatility comovement and spillover between the three markets are found to be significant. The parameter estimates of the multivariate DCC-GARCH (1,1) model can be interpreted as weights.

Table IV shows the parameter estimates of the multivariate model with asymmetric spillover and comovement. For US; there are statistically significant volatility comovement effects from UK and Japan. This is shown by $\gamma_{UK} = 0.067$ and $\gamma_{Japan} = 0.004$. Comovement from UK and Japan has an asymmetric effect on US. This means that negative shocks from the US and Japan have a larger impact on US than positive shocks. The asymmetric volatility comovement effects on the US from UK and Japan are given by $\eta_{UK} = 0.034$ and $\eta_{Japan} = 0.003$. The results obtained for US are consistent with the vector autoregression analysis obtained earlier. Volatility movements in the UK have a higher impact on US than volatility movements in Japan.

In the case of UK, US spillover into UK is given by $\theta_{US} = 0.043$ while Japanese comovement effect on UK is given by $\gamma_{Japan} = 0.004$. On examining asymmetric volatility, we find that asymmetric comovement effects from Japan to UK are greater than asymmetric volatility spillover from US into UK as shown by $\eta_{Japan} = 0.212$ while $\psi_{US} = 0.177$.

As for Japan, we find that volatility spillover from UK into Japan is not significant. However, asymmetric volatility spillover from UK to Japan is significant and this is given by $\psi_{UK} = 0.035$. Both volatility spillover and asymmetric spillover from US into Japan is significant and is shown by $\theta_{US} = 0.015$ and $\psi_{US} = 0.003$

As a robustness check, we compare the results obtained for the multivariate DCC-GARCH (1,1) estimation with the results obtained for the variance decomposition analysis performed in the earlier section of the paper. It must be emphasized that the vector autoregression analysis merely identifies the channels of interactions by simulating shocks in one market and analyzing the response of the other markets in the system. The variance decomposition analysis looks at how much of the total variance forecast is attributed to shocks in the other markets. Weights obtained from the multivariate DCC-GARCH (1,1) model are not the same as the percentage values (weights) obtained from the vector autoregression analysis. However, we can view them as ordinal measures and compare the values to see if both the VAR and GARCH analysis arrive at the same outcome.

In Table VI, the value in the first parenthesis shows the weights from either a spillover (seen on second day) or a comovement (seen on the first day), while the value in the second parenthesis shows the weights for the asymmetric spillover or comovement. The table is read as follows: US explains 5.92% of Japan's total variance forecast while UK explains 4.44% of the variance forecast. 5.92% is greater than 4.44%. In the multivariate GARCH analysis, the US's volatility shocks to Japan is given by the parameter estimate $\theta_{US} = 0.015$ while UK's volatility shocks to Japan is given by the estimate $\theta_{UK} = 0.007$. In ordinal terms, 0.015 is greater than 0.007.

Figure 4 shows the plots of volatility for US, UK and Japan for a 20 year period. The Japanese market is much more volatile than the US and UK markets. The US and UK markets exhibit very similar volatility patterns with the UK market being slightly more volatile than the US market.

5 Application of Volatility Comovement and Spillover to Portfolio Optimization

In the previous section, we demonstrated that there are significant conditional second moment interactions between US, UK and Japanese stock markets and their respective assets. In this section, we illustrate the practical importance of modeling volatility comovement and spillover using simple portfolio optimization application. We consider six cases (six different variance covariance matrices in the optimization). The cases are as follows:

Case 1: Constant correlation without comovement - spillover effects (Base Case)

Case 2: Time varying correlation/Dynamic correlation without comovement - spillover effects

Case 3: Constant correlation with comovement - spillover effects

Case 4: Time varying correlation/Dynamic correlation with comovement - spillover effects

Case 5: Constant correlation with asymmetric comovement - spillover effects

Case 6: Time varying correlation/Dynamic correlation with asymmetric comovement - spillover effects

Our objective is to translate the statistical significance of modeling volatility comovement and spillover into economic significance. We consider only a very basic portfolio with little diversification. Significant volatility comovement between US and UK has been identified and US and UK markets are highly correlated over time. However, in line with our objective we do not make any *a priori* decisions on which assets to be held in the portfolio. In order to be consistent with our comovement spillover modelling, I use the same three assets in my portfolio.

Consider maximizing the Sharpe ratio in the absence of a risk free asset. The optimization problem is specified as follows:

$$\max \; w_i R_i \,/\, \sqrt{w_i \Sigma_{ij} w_i'} \qquad (10)$$

$$subject \;\; 1' w_i = 1$$

where $\boldsymbol{w}_{\boldsymbol{i}}$ = daily weights of the portfolio
$\boldsymbol{R}_{\boldsymbol{i}}$ = mean returns (DJIA, Nikkei 225, FTSE 100)
Σ_{ij} = time varying variance covariance matrix

$\boldsymbol{i}$ = 1,2,3...5363(number of daily observations) $\boldsymbol{j}$ = 1,2,3...6 (different cases of the variance covariance matrix). While short sales are permitted, we constraint the extent of short sales permitted with a lower bound of -1 on $\boldsymbol{w}$ (interpreted as shorting of 100%) and an upper bound of 1 (a long position of 100%). Constraints have been imposed on the portfolio optimization problem in order to get reasonable weights. Given that ours is not a well-diversified portfolio, not allowing for short sales result in the weights becoming zero. Hence, short selling is permitted in our model in order to ensure that all assets are used.

The expected return on the daily portfolio is computed by:

$$R_E = w_i{}'.R_i \tag{11}$$

Assume that an investor invests 100 dollars on day one and holds the portfolio for 20 years. End of the period wealth (at the end of 5363 days, compounded on each day) is given by:

$$Wealth_i = 100*(1+R_{E1})(1+R_{E2})+......(1+R_{Ei}) \quad \text{where } i = 1,2,3.....5363. \tag{12}$$

The wealth computation is carried out for the six different cases with the base case being a constant correlation model without any volatility comovement and spillover.

A simple logarithmic utility function is employed to obtain the expected utility level for each of the six cases. In order to capture the economic differences obtained in employing the six different variance covariance matrices, we subtract the base case wealth and utility values from each of the cases. Figure 5 shows the plots of wealth differences and Figure 6 shows the utility differences from the base case.

From Figures 5 and 6, it can be seen that a model with time varying correlation with asymmetric volatility comovement and spillover yields the highest level of wealth. This model performs the best. The second best model is the model with time varying correlation with volatility comovement and spillover. Models that incorporate volatility comovement and spillover perform better and yield higher portfolio wealth and utility as opposed to a time varying correlation model that fails to consider volatility interactions. Models that fail to consider volatility interactions between markets and assets tend to under perform models which do. This has important implications for the kind of risk premium that has to be paid out to an investor to hold such a portfolio. The finding in this paper is consistent with finding found in Mazzotta (2004) though his paper fails to capture time varying volatility interactions between the markets. Our model accounts for not only asymmetry but also asymmetric interactions between the markets and their assets.

6. Conclusion

Overall, we find that financial markets and their assets are both statistically and economically related. Modelling volatility interactions in the form of comovement and spillover within a time varying correlation setup, we obtain the variance covariance matrix and solve a portfolio optimization problem. Despite using a very simple portfolio with only three assets, we find that a portfolio which incorporates time varying correlation with asymmetric volatility comovement and spillover outperforms a portfolio that ignores this information.

Findings from this paper have strong implications for the kind of risk premium that has to be paid to an investor holding and international portfolio. Having information about time-varying volatility comovement and spillover differentiates between an informed investor who knows the volatilities and correlations every day as opposed to the myopic investor who only knows the unconditional volatilities and correlations. This analysis has important implications for portfolio managers.

On a final note, it will be interesting to perform this analysis on a much larger and well diversified portfolio and to also capture asymmetric correlations between the markets. This remains a subject for future research.

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Table I : Univariate statistics on daily market returns 1984:2004

Market	Mean	Std.Dev	Skewness	Excess Kurtosis	Sharpe Ratio
US	0.04*	1.08	-2.67*	63.84*	0.04
UK	0.02*	1.04	-0.74*	10.35*	0.02
Japan	0.03*	1.36	7.93*	7.94*	0.02

* Significant from the null at the 10% level

Table II: Overlaps in timing of stock market operations

Countries	GMT Times	**US**	**Japan**	**UK**
US	14:30-21:00	---		
Japan	00:00-02:00 03:30-06:00	×	---	
UK	08:30-16:30	√	×	---

√ indicates presence of overlapping × indicates no overlapping

Table II,B: Unconditional correlation coefficients between daily returns in local currency terms 1984:2004

Countries	**US**	**UK**	**Japan**
US	---		
UK	0.395 *	---	
JP	0.114 *	0.256 *	---

Table II,C :Unconditional correlation coefficients between daily returns in local currency terms 1984:2004

	US(-1)	**UK(-1)**	**Japan**
US (-1)	---		
UK (-1)	0.395 *	---	
JP	0.304 *	0.202 *	---

* Correlation at 10% level of significance

Table III: Variance decompositions with market ordering based on closing times : Japan, UK, US

		Innovation in market of:			
Variance Decomposition in markets of:	Horizon(days)	Japan	UK	US	Are results consistent?
Japan	1	100.00	0.00	0.00	US spillover into Japan (Yes)
	2	89.64	4.44	5.92	
	3	89.61	4.45	5.94	UK spillover into Japan (Yes)
	10	89.33	4.62	6.04	
UK	1	3.52	96.48	0.00	Japan comoves with UK (Yes)
	2	3.51	87.24	9.24	
	3	3.58	87.10	9.31	US spillover into UK (Yes)
	10	4.00	86.60	9.43	
US	1	1.37	15.46	83.16	Japan comoves with US (Yes)
	2	1.46	15.46	83.09	
	3	1.46	15.43	83.11	UK comoves with US (Yes)
	10	1.57	15.70	82.73	

Table IV: Parameter estimates of multivariate DCC-GARCH(1,1) model **with asymmetric volatility and comovement**

				Asymmetric spillover and comovement effects from:			
Country	ω	α	β	θ volatility spillover	ψ asymmetric volatility spillover	γ volatility comovement	η asymmetric volatility comovement
US	0.032*	0.052*	0.856*	--	--	UK: 0.067*	UK: 0.034*
				--	--	Japan: 0.004*	Japan: 0.003*
UK	0.025*	0.081*	0.847*	US: 0.043*	US: 0.177*	Japan: 0.004*	Japan: 0.212*
Japan	0.011*	0.109*	0.880*	US: 0.015*	US: 0.003*	--	--
				UK: 0.007	UK: 0.035*	--	--
DCC- Alpha-A	0.001*						
DCC-Beta-B	0.988*						
Log-Likelihood: - 22120.44	*Indicates significance at 5% level						

Table V: Parameter estimates of multivariate DCC-GARCH(1,1) model **without asymmetric volatility and comovement**

				Asymmetric spillover and comovement effects from:	
Country	ω	α	β	θ volatility spillover	γ volatility comovement
US	0.026*	0.050*	0.870*	--	UK: 0.064 *
				--	Japan: 0.004 *
UK	0.019*	0.074*	0.859*	US: 0.043*	Japan: 0.004*
Japan	0.010*	0.100*	0.889*	US: 0.017*	--
				UK: 0.004	--
DCC- Alpha-A	0.004*				
DCC-Beta-B	0.991*				
Log-Likelihood: - 22109.59	*Indicates significance at 5% level				

Table VI: Comparison of variance decomposition with DCC-GARCH (1,1) parameter estimates

		Innovation in market of:		
Variance Decomposition in markets of:	Horizon(days)	Japan	UK	US
Japan	1	100.00	0.00	0.00
	2	89.64	4.44 (0.07)(0.035)	5.92 (0.015)(0.003)
	3	89.61	4.45	5.94
	10	89.33	4.62	6.04
UK	1	3.52 (0.004)(0.212)	96.48	0.00
	2	3.51	87.24	9.24 (0.043)(0.177)
	3	3.58	87.10	9.31
	10	4.00	86.60	9.43
US	1	1.37 (0.004)(0.003)	15.46 (0.067)(0.034)	83.16
	2	1.46	15.46	83.09
	3	1.46	15.43	83.11
	10	1.57	15.70	82.73

Figure I: Plots of daily returns series 1994:2004

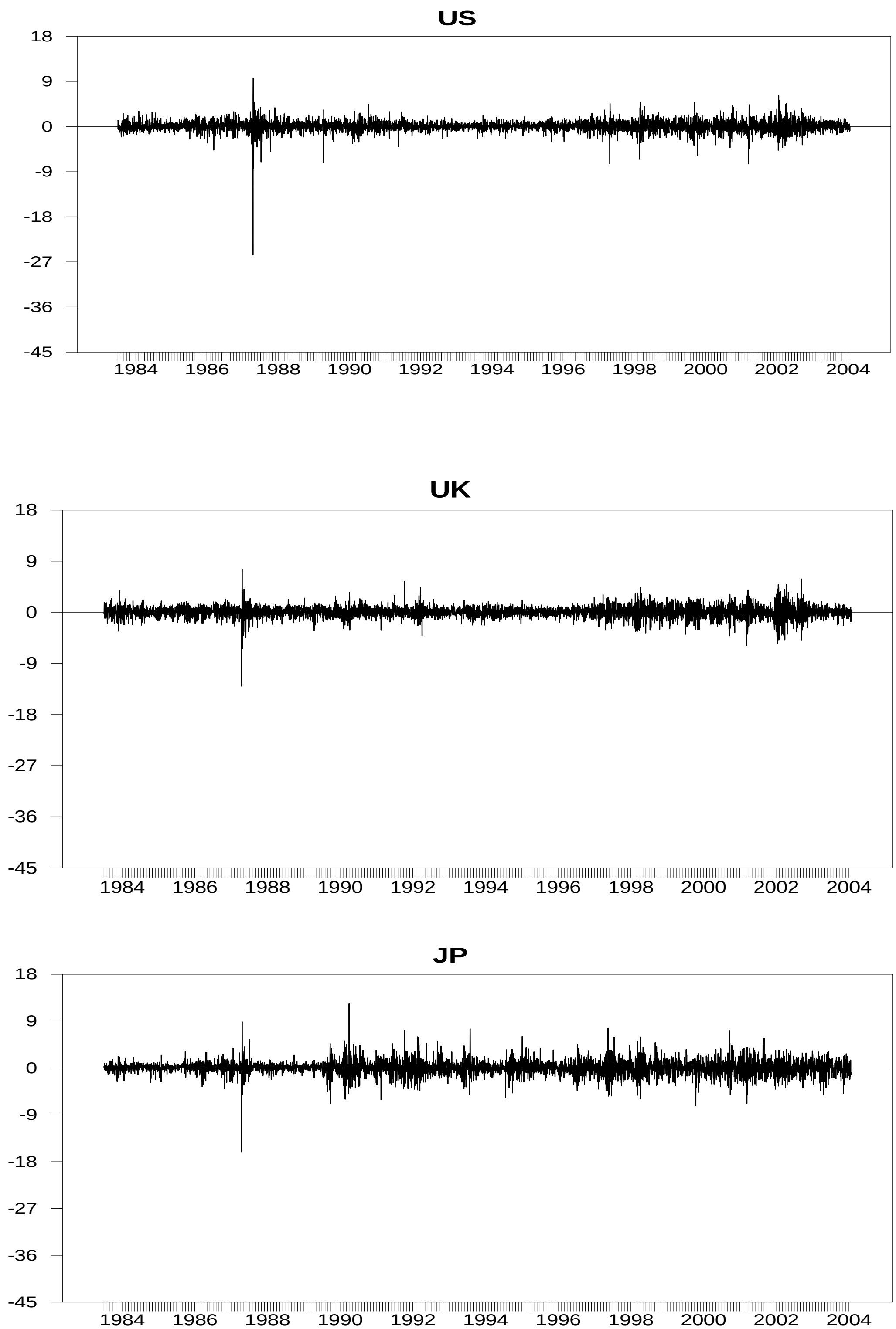


Figure II: Impulse response analysis

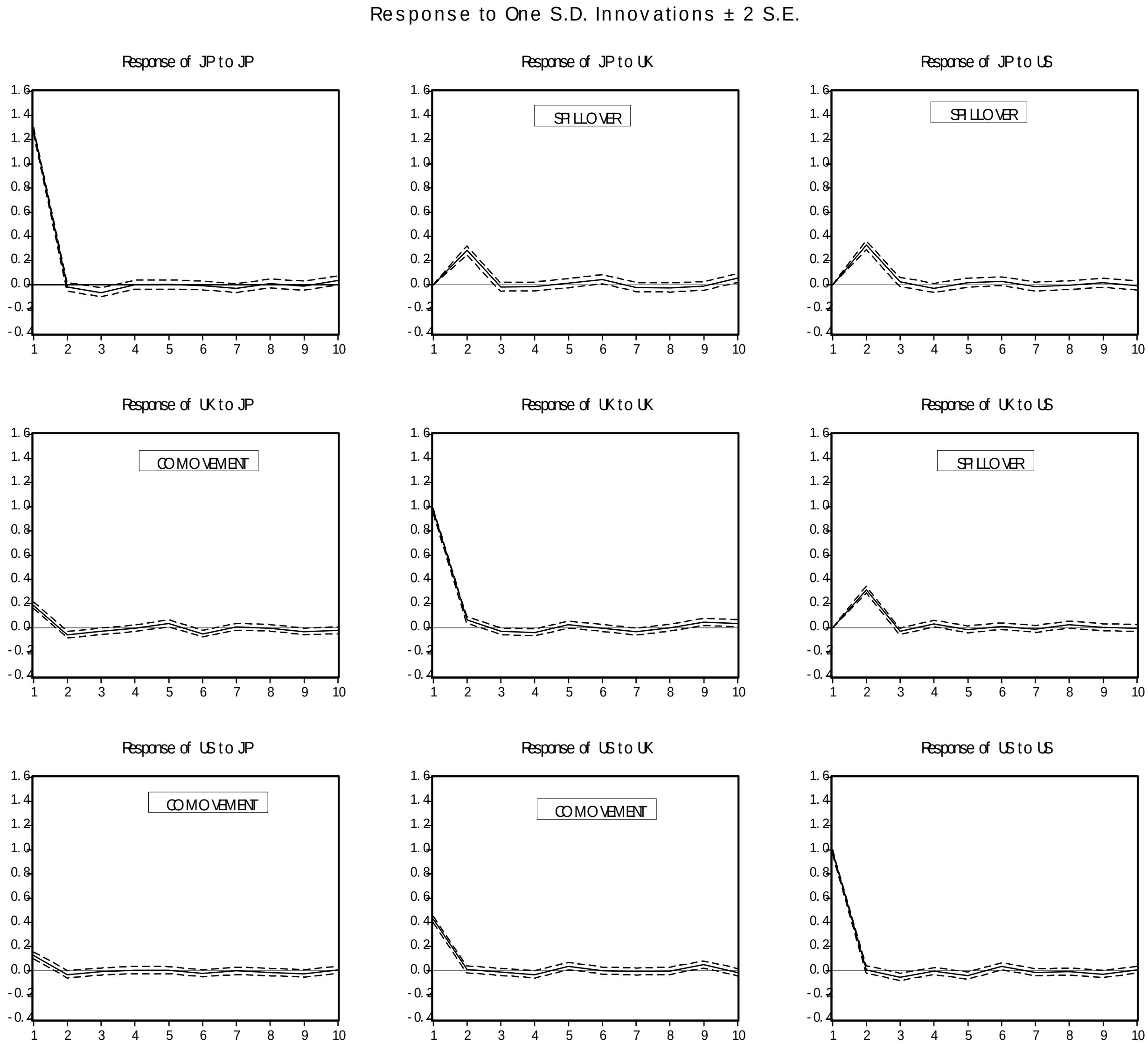


Figure III: The use of time-varying correlations versus constant correlations

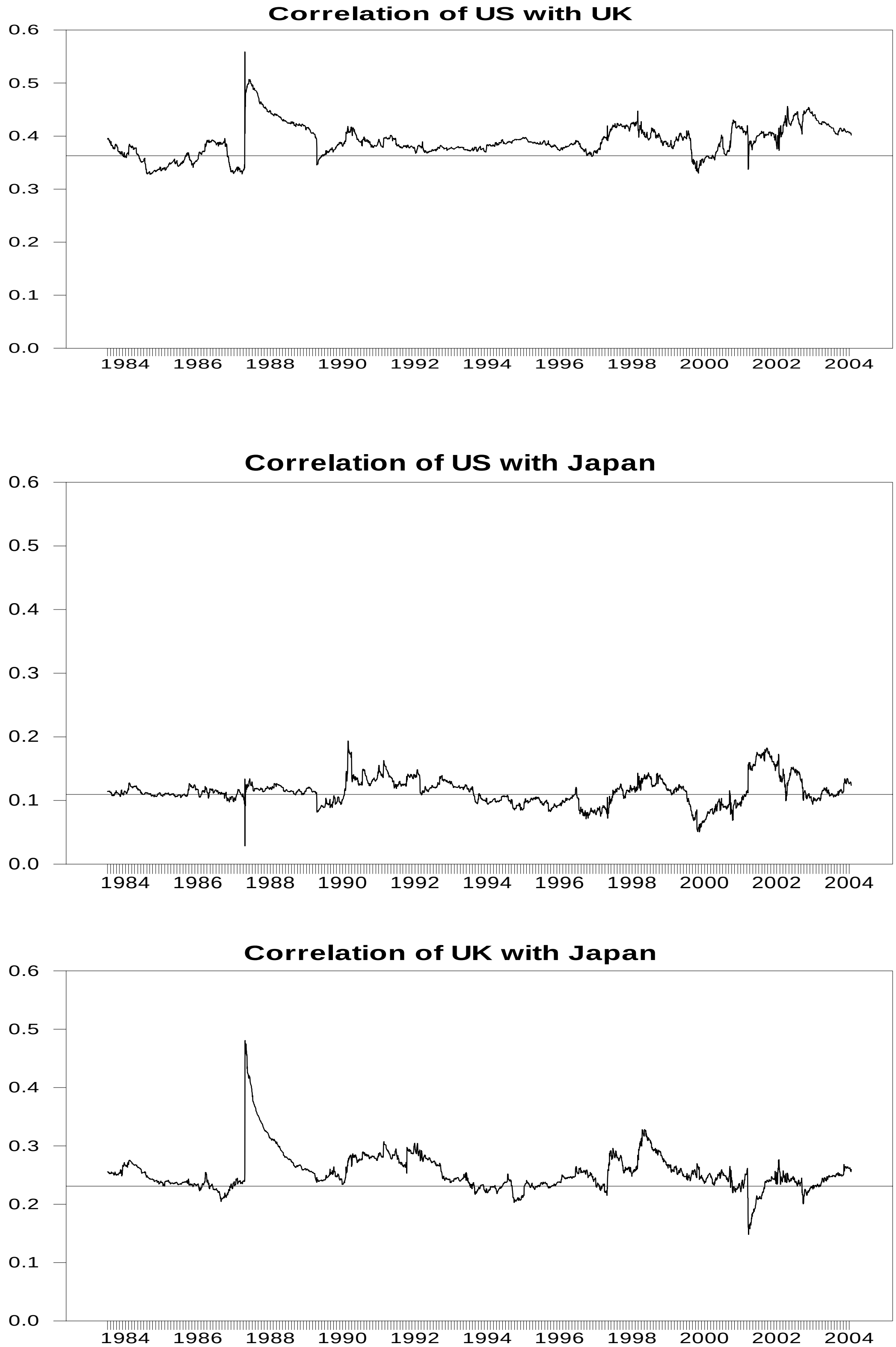
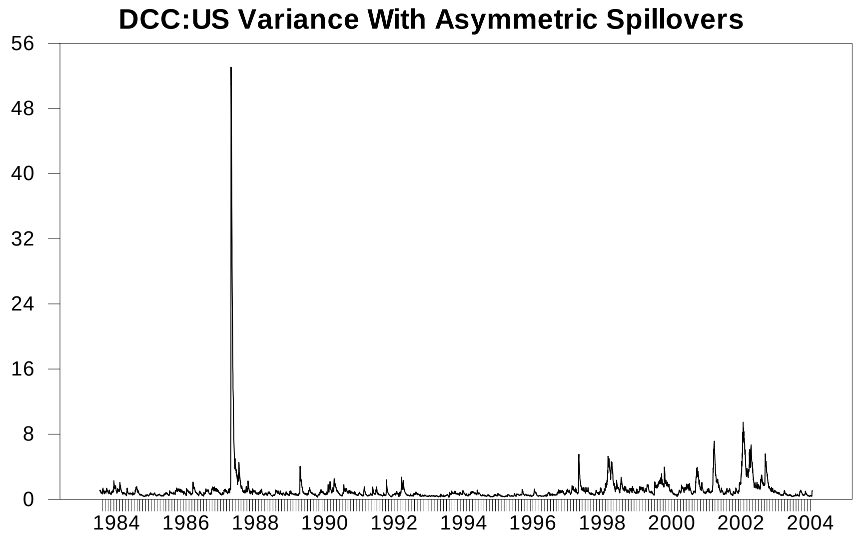
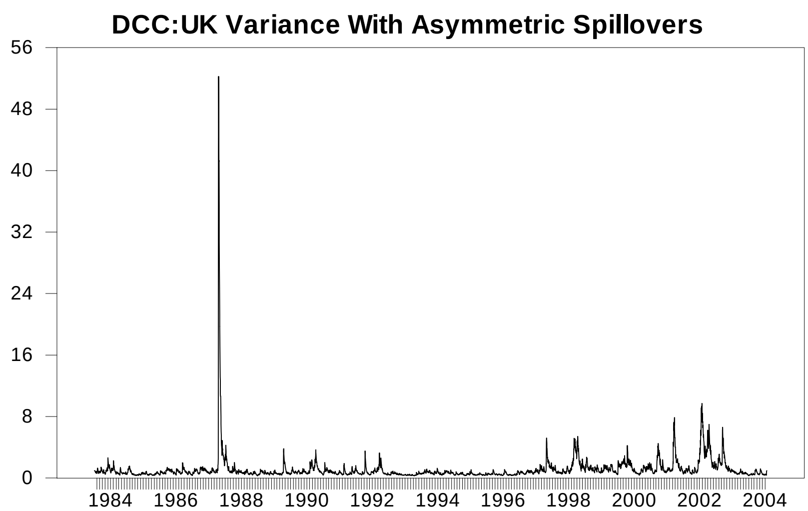


Figure IV: Plots of country volatility with asymmetric spillover using DCC-GARCH





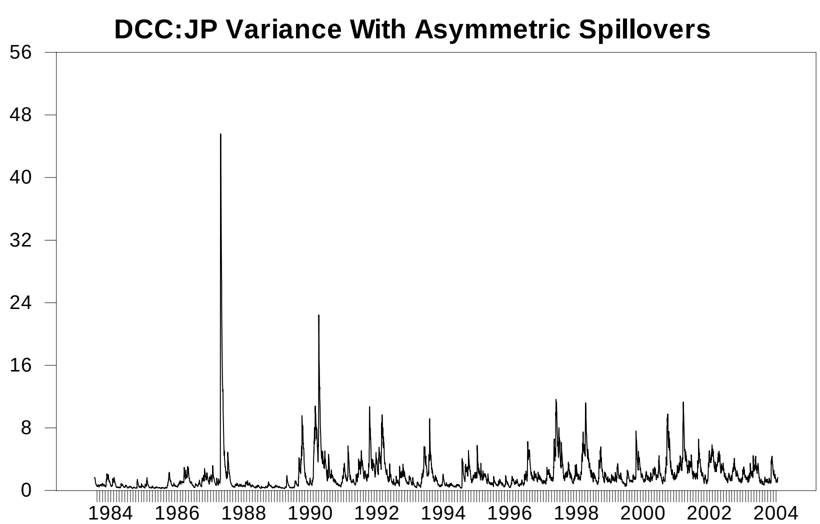


Figure V: Difference in Wealth in Comparison to Case I

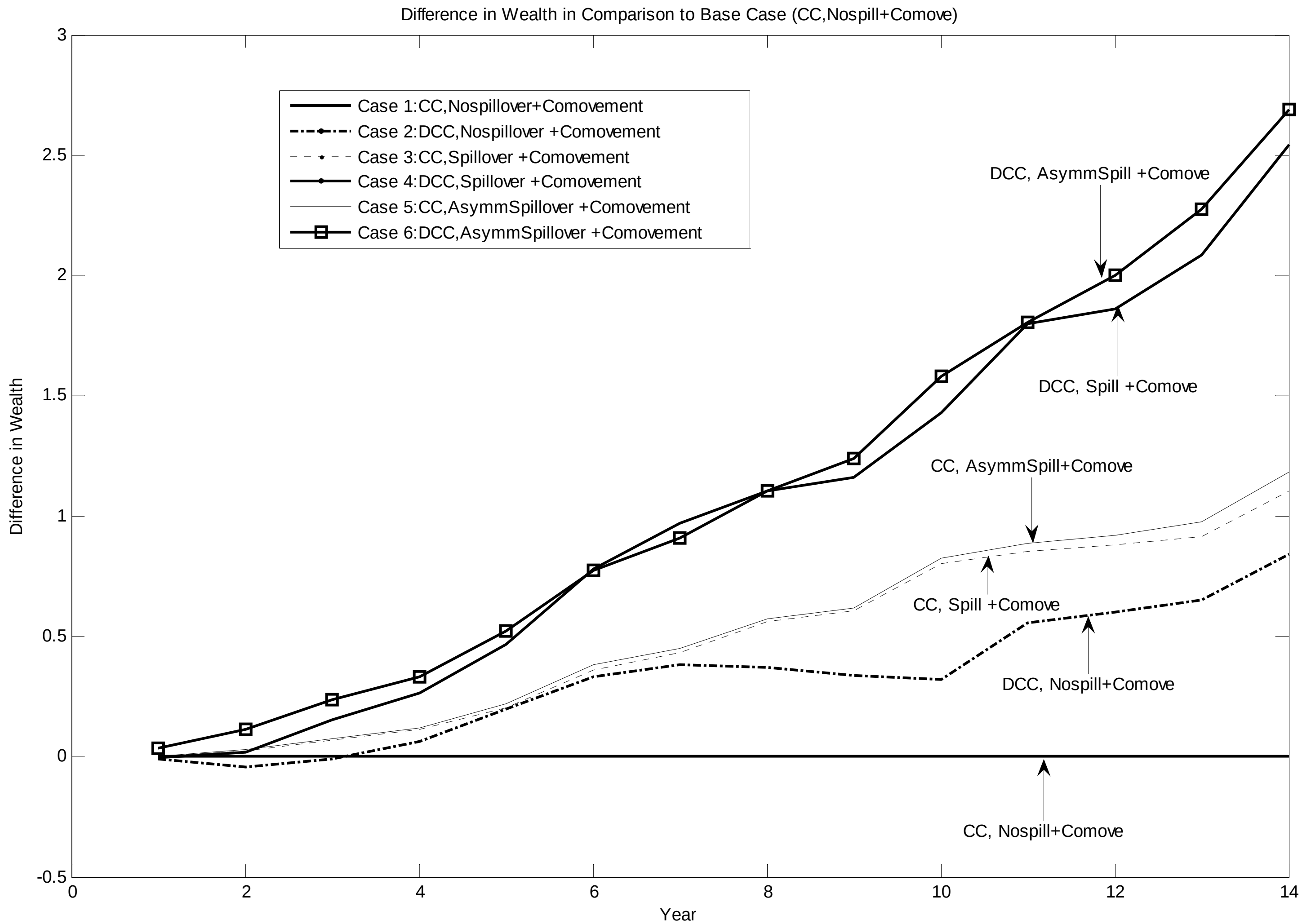


Figure VI: Difference in Utility in Comparison to Case I