On Risk Premia and Volatility Transmission Across the Stock and Bond Markets

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Abstract

This paper analyzes risk premia and volatility transmission across the stock and bond markets within an expected return beta representation of the conditional capital asset pricing model. Time variation in the market price of risk is characterized by a two state Markov regime switching process, while time variation in conditional betas is characterized by an asymmetric general dynamic covariance process. On the basis of estimated state dependent generalized impulse response functions, we find evidence of a flight to quality phenomenon, whereby investors shift funds from the stock market to the bond market in response to high stock market volatility. Our impulse response analysis also suggests that the degree of risk diversification achieved by cross market hedging is lowest when it is most desirable.

JEL Classification: G11; G12

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1. Introduction

Information arrivals have the potential to generate risk premia and volatility transmission across the stock and bond markets. In response to an information arrival which affects the conditional means and variances of excess returns in at least one market, such as a macroeconomic policy announcement, the principles of optimal portfolio management generally prescribe that investors adjust their holdings in both markets. Such portfolio reallocation may be expected to give rise to dynamic interrelationships across the conditional means, variances and covariances of excess returns on stocks and bonds. Cross market hedging has the potential to amplify and propagate these dynamic interrelationships, which may in turn affect the degree of risk diversification it achieves.

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The nature of risk premia and volatility transmission across the stock and bond markets has profound implications for a variety of investment, risk management, and regulatory policy decisions. Optimal portfolio allocation by investors across the stock and bond markets depends on the conditional means, variances and covariances of excess returns on stocks and bonds, all of which potentially respond to information arrivals. The solvency of financial intermediaries having operations spanning the stock and bond markets can hinge on the degree of risk diversification achieved by cross market hedging, and their assessments of value at risk should reflect dynamic interrelationships across the conditional first and second moments of excess returns on stocks and bonds. Given these implications of risk premia and volatility transmission across the stock and bond markets for investment and risk management decisions, banking regulators should consider them when setting capital adequacy requirements, while market regulators should incorporate them into evaluations of the effects of proposed policy changes.

Despite the potential importance of risk premia and volatility transmission across the stock and bond markets for investment, risk management and regulatory policy decisions, the empirical literature concerning these dynamic interrelationships is rather sparse. Within the framework of an estimated speculative trading model, Fleming, Kirby and Ostdiek (1998) find that shocks to the conditional variances of stock and bond returns are strongly positively correlated. Evidence that the conditional variance of stock returns responds asymmetrically to stock market shocks, while the conditional variance of bond returns responds symmetrically to bond market shocks, is documented by Cappiello, Engle and Sheppard (2003). Within the framework of an estimated multifactor asset pricing model, Scruggs and Glabadanidis (2003) find that the conditional variance of the excess return on stocks responds asymmetrically to both stock and bond market shocks, while the conditional variance of the excess return on bonds is essentially invariant to stock market shocks yet responds symmetrically to bond market shocks. These papers analyze volatility transmission across the stock and bond markets at short horizons.

This paper conducts an empirical analysis of both risk premia and volatility transmission across the stock and bond markets at both short and long horizons. Our analysis is based on the expected return beta representation of the conditional capital asset pricing model or CAPM, which expresses the risk premium on an arbitrary asset as the product of its conditional beta and the market price of risk. Time variation in the market price of risk is characterized by a two state Markov regime switching process, allowing the nature of risk premia and volatility transmission across the stock and bond markets to differ across bull and bear markets. Time variation in conditional betas is characterized by the multivariate generalized autoregressive conditional heteroskedasticity or GARCH class of models. In particular, the dependence of risk premia on conditional betas is characterized by the multivariate GARCH in mean or GARCH-M class of models. Both symmetric and asymmetric multivariate GARCH-M models are considered, allowing the nature of risk premia and volatility transmission across the stock and bond markets to differ in response to positive and negative shocks of equal magnitude. The dynamic effects of these shocks on the conditional means, variances and covariances of excess returns on stocks and bonds are analyzed with state dependent generalized impulse response functions.

Our estimated state dependent generalized impulse response functions exhibit several common features across bull and bear markets. First, the risk premia on stocks and bonds respond asymmetrically to both stock and bond market shocks. Second, in agreement with the results of Scruggs and Glabadanidis (2003), the conditional variance of the excess return on stocks responds asymmetrically to both stock and bond market shocks, while the conditional variance of the excess return on bonds is relatively insensitive to stock market shocks yet responds symmetrically to bond market shocks. Third, the conditional covariance between excess returns on stocks and bonds increases dramatically in response to combined negative shocks to the stock and bond markets, yet declines in response to a negative stock market shock when combined with a positive bond market shock. Our estimated state dependent generalized impulse response functions also exhibit several differences across bull and bear markets. First, reflecting a lower estimated market price of risk, stock and bond risk premia are less sensitive to shocks of given relative magnitudes under a bull market than under a bear market. Second, reflecting lower estimated absolute magnitudes of shocks, the conditional variances and covariances of excess returns on stocks and bonds are also less sensitive to shocks of given relative magnitudes under a bull market than under a bear market.

The organization of this paper is as follows. The next section presents stochastic discount factor and expected return beta representations of the conditional CAPM. An empirical framework based on the expected return beta representation of the conditional CAPM which allows for time variation in conditional first and second moments is developed in section three. The analysis of risk premia and volatility transmission across the stock and bond markets within this empirical framework is the subject of section four. Finally, section five offers conclusions and policy implications.

2. The Theoretical Framework

A central theme of financial economics is the existence of a tradeoff between risk and expected return. The most widely used asset pricing model which quantifies this tradeoff is the capital asset pricing model or CAPM credited to Sharpe (1964) and Lintner (1965a, 1965b), which has both conditional and unconditional versions. The conditional CAPM holds conditional on state variables, and does not imply the unconditional CAPM.

The CAPM predicts a relationship between the intertemporal marginal rate of substitution in consumption and the return on a fully diversified or market portfolio. Consumption based asset pricing models predict that the price of an asset p_t equals the expected present discounted value of its future payoff x_{t+1},

$$p_t = \mathrm{E}_t(m_{t+1}x_{t+1}), \tag{1}$$

where m_{t+1} denotes the intertemporal marginal rate of substitution in consumption or stochastic discount factor. The CAPM expresses the stochastic discount factor as an affine function of the return on the market portfolio R^m_{t+1}:

$$m_{t+1} = a - bR^m_{t+1}. \tag{2}$$

Within the context of the conditional CAPM, parameters a and b are time varying functions of state variables, while these parameters are constant within the context of the unconditional CAPM.

The CAPM has both stochastic discount factor and expected return beta representations. The requirement that the conditional CAPM price both the return on the market portfolio $1 = \mathrm{E}_t(m_{t+1}R^m_{t+1})$ and the risk free rate $1 = \mathrm{E}_t(m_{t+1}R^f_{t+1})$ implies:

$$m_{t+1} = \frac{1}{R^f_{t+1}}\left\{1 - \frac{\mathrm{E}_t(R^m_{t+1} - R^f_{t+1})}{\mathrm{Var}_t(R^m_{t+1})}[R^m_{t+1} - \mathrm{E}_t(R^m_{t+1})]\right\}. \tag{3}$$

Given this stochastic discount factor representation of the conditional CAPM, the requirement that it price the return on an arbitrary asset $1 = \mathrm{E}_t(m_{t+1}R^i_{t+1})$ implies:

$$\mathrm{E}_t(R^i_{t+1} - R^f_{t+1}) = \frac{\mathrm{Cov}_t(R^i_{t+1}, R^m_{t+1})}{\mathrm{Var}_t(R^m_{t+1})}\mathrm{E}_t(R^m_{t+1} - R^f_{t+1}). \tag{4}$$

This expected return beta representation of the conditional CAPM expresses the risk premium on an arbitrary asset $\mathrm{E}_t(R^i_{t+1} - R^f_{t+1})$ as the product of its conditional beta $\beta_{i,t+1} = \mathrm{Cov}_t(R^i_{t+1}, R^m_{t+1}) / \mathrm{Var}_t(R^m_{t+1})$ and the market price of risk $\lambda_{t+1} = \mathrm{E}_t(R^m_{t+1} - R^f_{t+1})$.

3. The Empirical Framework

The expected return beta representation of the conditional CAPM is a cross sectional relationship, and does not impose any restrictions on the time series behaviour of the conditional first and second moments of excess returns. As such, the empirical analysis of risk premia and volatility transmission across the stock and bond markets within the framework of the conditional CAPM requires the imposition of auxiliary assumptions regarding the time series behaviour of the conditional means, variances and covariances of excess returns.

3.1. Specification of the Conditional Mean Function

Let $\boldsymbol{r}_t$ denote an N dimensional vector of excess returns on all assets in the market, measured as returns in excess of the risk free rate, and let $\boldsymbol{H}_t$ denote the conditional covariance matrix of this excess return vector given information available at time $t-1$. Also, let $\boldsymbol{\omega}_t$ denote an N dimensional vector of weights used to construct the market portfolio given information available at time $t-1$. Given normalization $\boldsymbol{\iota}^{\mathsf{T}}\boldsymbol{\omega}_t = 1$, where $\boldsymbol{\iota}$ is an N dimensional vector of ones, it follows that the excess return on the market portfolio is $r_t^m = \boldsymbol{\omega}_t^{\mathsf{T}}\boldsymbol{r}_t$.

Given these definitions, the vector of conditional covariances of returns on all assets with the return on the market portfolio is $\boldsymbol{H}_t\boldsymbol{\omega}_t$, while the conditional variance of the return on the market portfolio is $\boldsymbol{\omega}_t^{\mathsf{T}}\boldsymbol{H}_t\boldsymbol{\omega}_t$. It follows that the vector of conditional betas is $\boldsymbol{\beta}_t = \boldsymbol{H}_t\boldsymbol{\omega}_t / \boldsymbol{\omega}_t^{\mathsf{T}}\boldsymbol{H}_t\boldsymbol{\omega}_t$, and the expected return beta representation of the conditional CAPM may be stated as

$$\boldsymbol{r}_t = \frac{\boldsymbol{H}_t\boldsymbol{\omega}_t}{\boldsymbol{\omega}_t^{\mathsf{T}}\boldsymbol{H}_t\boldsymbol{\omega}_t}\lambda_t + \boldsymbol{\varepsilon}_t, \tag{5}$$

where signal innovation vector $\boldsymbol{\varepsilon}_t$ is an N dimensional process satisfying $\mathrm{E}_{t-1}(\boldsymbol{\varepsilon}_t) = \boldsymbol{0}$ and $\mathrm{E}_{t-1}(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t^{\mathsf{T}}) = \boldsymbol{H}_t$.

Time variation in the market price of risk λ_t is characterized by a two state Markov regime switching process, allowing the market price of risk to exhibit occasional but recurrent shifts across bull and bear markets,

$$\lambda_t = \boldsymbol{\lambda}^{\mathsf{T}}\boldsymbol{\xi}_t, \tag{6}$$

where $\boldsymbol{\xi}_t$ is a selection vector with elements $1-s_t$ and s_t. The regime $s_t \in \{0,1\}$ is the outcome of an unobserved two state, first order Markov chain having vector autoregressive representation:

$$\boldsymbol{\xi}_t = \boldsymbol{P}\boldsymbol{\xi}_{t-1} + \boldsymbol{v}_t. \tag{7}$$

Transition probability matrix $\boldsymbol{P}$ satisfies $\boldsymbol{P}^{\mathsf{T}}\boldsymbol{\iota} = \boldsymbol{\iota}$, and has diagonal elements $p = \mathrm{P}(s_t = 0 \mid s_{t-1} = 0)$ and $q = \mathrm{P}(s_t = 1 \mid s_{t-1} = 1)$. We assume that state innovation vector $\boldsymbol{v}_t$ is intratemporally and intertemporally independent of signal innovation vector $\boldsymbol{\varepsilon}_t$.

3.2. *Specification of the Conditional Variance Function*

Time variation in conditional variances and covariances is a characteristic of the multivariate GARCH class of models. Symmetric multivariate GARCH models restrict conditional variances and covariances to respond equally in magnitude to positive and negative shocks of equal magnitude, while asymmetric multivariate GARCH models relax these restrictions.

There exist many alternative multivariate GARCH models in the literature. A general multivariate GARCH model for N dimensional ordinary innovation vector $\boldsymbol{\varepsilon}_t$ is given by

$$\boldsymbol{\varepsilon}_t = \boldsymbol{H}_t^{1/2}\boldsymbol{z}_t, \tag{8}$$

where conditional covariance matrix $\boldsymbol{H}_t$ depends nontrivially on lagged ordinary innovation vectors and conditional covariance matrices. Standardized innovation vector $\boldsymbol{z}_t$ is an N dimensional independently and identically distributed process satisfying $\mathrm{E}_{t-1}(\boldsymbol{z}_t) = \boldsymbol{0}$ and $\mathrm{E}_{t-1}(\boldsymbol{z}_t\boldsymbol{z}_t^{\mathsf{T}}) = \boldsymbol{I}$, which implies that $\mathrm{E}_{t-1}(\boldsymbol{\varepsilon}_t) = \boldsymbol{0}$ and $\mathrm{E}_{t-1}(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t^{\mathsf{T}}) = \boldsymbol{H}_t$. To complete the model, the functional dependence of conditional covariance matrix $\boldsymbol{H}_t$ on lagged ordinary innovation vectors and conditional covariance matrices must be specified.

A very flexible multivariate GARCH model is the VECH model of Bollerslev, Engle and Wooldridge (1988), in which all nonredundant elements of conditional covariance matrix $\boldsymbol{H}_t$ depend on all nonredundant elements of lagged ordinary innovation cross product matrix $\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}^{\mathsf{T}}$ and lagged conditional covariance matrix $\boldsymbol{H}_{t-1}$,

$$\mathrm{vech}(\boldsymbol{H}_t) = \boldsymbol{W} + \boldsymbol{A}\,\mathrm{vech}(\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}^{\mathsf{T}}) + \boldsymbol{B}\,\mathrm{vech}(\boldsymbol{H}_{t-1}), \tag{9}$$

where the $\mathrm{vech}(\cdot)$ operator stacks the elements of a symmetric matrix contained on and below the principal diagonal into a vector. In the asymmetric VECH model, all nonredundant elements of conditional covariance matrix $\boldsymbol{H}_t$ also depend on all nonredundant elements of lagged asymmetric ordinary innovation cross product matrix $\boldsymbol{\eta}_{t-1}\boldsymbol{\eta}_{t-1}^{\mathsf{T}}$,

$$\mathrm{vech}(\boldsymbol{H}_t) = \boldsymbol{W} + \boldsymbol{A}\,\mathrm{vech}(\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}^{\mathsf{T}}) + \boldsymbol{G}\,\mathrm{vech}(\boldsymbol{\eta}_{t-1}\boldsymbol{\eta}_{t-1}^{\mathsf{T}}) + \boldsymbol{B}\,\mathrm{vech}(\boldsymbol{H}_{t-1}), \tag{10}$$

where $\eta_{i,t} \equiv \max(0, -\varepsilon_{i,t})$. Given that $\boldsymbol{W}$ is a vector of dimension $N(N+1)/2$, while $\boldsymbol{A}$, $\boldsymbol{G}$ and $\boldsymbol{B}$ are square matrices of dimension $N(N+1)/2$, the total parameter count for the symmetric VECH model is $N(N+1)[1+N(N+1)]/2$, while that for the asymmetric VECH model is $N(N+1)[1+3N(N+1)/2]/2$. Restrictions on matrices $\boldsymbol{A}$, $\boldsymbol{G}$ and $\boldsymbol{B}$ which ensure positive definiteness of conditional covariance matrix $\boldsymbol{H}_t$ are difficult to impose.

A reduction in the number of parameters is offered by the diagonal VECH or DVECH model of Bollerslev, Engle and Wooldridge (1988), in which conditional covariance $h_{i,j,t}$ depends only on lagged ordinary innovation cross product $\varepsilon_{i,t-1}\varepsilon_{j,t-1}$ and lagged conditional covariance $h_{i,j,t-1}$:

$$\boldsymbol{H}_t = \boldsymbol{W} + \boldsymbol{A} \odot (\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}^{\mathsf{T}}) + \boldsymbol{B} \odot \boldsymbol{H}_{t-1}. \tag{11}$$

In the asymmetric DVECH model, conditional covariance $h_{i,j,t}$ also depends on lagged asymmetric ordinary innovation cross product $\eta_{i,t-1}\eta_{j,t-1}$:

$$\boldsymbol{H}_t = \boldsymbol{W} + \boldsymbol{A} \odot (\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}^{\mathsf{T}}) + \boldsymbol{G} \odot (\boldsymbol{\eta}_{t-1}\boldsymbol{\eta}_{t-1}^{\mathsf{T}}) + \boldsymbol{B} \odot \boldsymbol{H}_{t-1}. \tag{12}$$

Given that $\boldsymbol{W}$, $\boldsymbol{A}$, $\boldsymbol{G}$ and $\boldsymbol{B}$ are symmetric matrices of dimension N, the total parameter count for the symmetric DVECH model is $2N(N+1)$, while that for the asymmetric DVECH model is $3N(N+1)/2$. Conditional covariance matrix $\boldsymbol{H}_t$ is positive definite if matrices $\boldsymbol{W}$, $\boldsymbol{A}$, $\boldsymbol{G}$ and $\boldsymbol{B}$ are positive definite.

In the BEKK model of Engle and Kroner (1995), all elements of conditional covariance matrix $\boldsymbol{H}_t$ depend on all elements of lagged ordinary innovation cross product matrix $\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}^{\mathsf{T}}$ and lagged conditional covariance matrix $\boldsymbol{H}_{t-1}$:

$$\boldsymbol{H}_t = \boldsymbol{W}^{\mathsf{T}}\boldsymbol{W} + \boldsymbol{A}^{\mathsf{T}}\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}^{\mathsf{T}}\boldsymbol{A} + \boldsymbol{B}^{\mathsf{T}}\boldsymbol{H}_{t-1}\boldsymbol{B}. \tag{13}$$

In the asymmetric BEKK model of Kroner and Ng (1998), all elements of conditional covariance matrix $\boldsymbol{H}_t$ also depend on all elements of lagged asymmetric ordinary innovation cross product matrix $\boldsymbol{\eta}_{t-1}\boldsymbol{\eta}_{t-1}^{\mathsf{T}}$:

$$\boldsymbol{H}_t = \boldsymbol{W}^{\mathsf{T}}\boldsymbol{W} + \boldsymbol{A}^{\mathsf{T}}\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}^{\mathsf{T}}\boldsymbol{A} + \boldsymbol{G}^{\mathsf{T}}\boldsymbol{\eta}_{t-1}\boldsymbol{\eta}_{t-1}^{\mathsf{T}}\boldsymbol{G} + \boldsymbol{B}^{\mathsf{T}}\boldsymbol{H}_{t-1}\boldsymbol{B}. \tag{14}$$

As specified, $\boldsymbol{W}$, $\boldsymbol{A}$, $\boldsymbol{G}$ and $\boldsymbol{B}$ are square matrices of dimension N, with $\boldsymbol{W}$ upper triangular. The total parameter count for the symmetric BEKK model is $N(5N+1)/2$, while that for the asymmetric BEKK model is $N(7N+1)/2$. Conditional covariance matrix $\boldsymbol{H}_t$ is necessarily positive definite.

A reduction in the number of parameters is offered by the factor GARCH or FGARCH model of Engle, Ng and Rothschild (1990), in which conditional covariance $h_{i,j,t}$ depends only on the lagged squared ordinary innovation $\boldsymbol{\omega}^{\mathsf{T}}\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}^{\mathsf{T}}\boldsymbol{\omega}$ and lagged conditional variance $\boldsymbol{\omega}^{\mathsf{T}}\boldsymbol{H}_{t-1}\boldsymbol{\omega}$ of a common factor:

$$\boldsymbol{H}_t = \boldsymbol{W}^{\mathsf{T}}\boldsymbol{W} + \boldsymbol{\delta}\boldsymbol{\delta}^{\mathsf{T}}[\alpha(\boldsymbol{\omega}^{\mathsf{T}}\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}^{\mathsf{T}}\boldsymbol{\omega}) + \beta(\boldsymbol{\omega}^{\mathsf{T}}\boldsymbol{H}_{t-1}\boldsymbol{\omega})]. \tag{15}$$

As specified, $\boldsymbol{W}$ is an upper triangular matrix of dimension N, while $\boldsymbol{\delta}$ and $\boldsymbol{\omega}$ are vectors of dimension N. The BEKK model nests the FGARCH model for $\boldsymbol{A} = \sqrt{\alpha}\boldsymbol{\omega}\boldsymbol{\delta}^{\mathsf{T}}$ and $\boldsymbol{B} = \sqrt{\beta}\boldsymbol{\omega}\boldsymbol{\delta}^{\mathsf{T}}$. In the asymmetric FGARCH model of Kroner and Ng (1998), conditional covariance $h_{i,j,t}$ also depends on the lagged squared asymmetric ordinary innovation $\boldsymbol{\omega}^{\mathsf{T}}\boldsymbol{\eta}_{t-1}\boldsymbol{\eta}_{t-1}^{\mathsf{T}}\boldsymbol{\omega}$ of a common factor:

$$\boldsymbol{H}_t = \boldsymbol{W}^{\mathsf{T}}\boldsymbol{W} + \boldsymbol{\delta}\boldsymbol{\delta}^{\mathsf{T}}[\alpha(\boldsymbol{\omega}^{\mathsf{T}}\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}^{\mathsf{T}}\boldsymbol{\omega}) + \gamma(\boldsymbol{\omega}^{\mathsf{T}}\boldsymbol{\eta}_{t-1}\boldsymbol{\eta}_{t-1}^{\mathsf{T}}\boldsymbol{\omega}) + \beta(\boldsymbol{\omega}^{\mathsf{T}}\boldsymbol{H}_{t-1}\boldsymbol{\omega})]. \tag{16}$$

The asymmetric BEKK model nests the asymmetric FGARCH model for $\boldsymbol{A} = \sqrt{\alpha}\boldsymbol{\omega}\boldsymbol{\delta}^{\mathsf{T}}$, $\boldsymbol{G} = \sqrt{\gamma}\boldsymbol{\omega}\boldsymbol{\delta}^{\mathsf{T}}$ and $\boldsymbol{B} = \sqrt{\beta}\boldsymbol{\omega}\boldsymbol{\delta}^{\mathsf{T}}$. Given normalization $\boldsymbol{\iota}^{\mathsf{T}}\boldsymbol{\omega} = 1$, the total parameter count for the symmetric FGARCH model is $(N^2+5N+2)/2$, while that for the asymmetric FGARCH model is $(N^2+5N+4)/2$. Conditional covariance matrix $\boldsymbol{H}_t$ is positive definite if scalars α, γ and β are positive.

In the constant correlation or CCORR model of Bollerslev (1990), conditional variance $h_{i,i,t}$ depends only on lagged squared ordinary innovation $\varepsilon_{i,t-1}^2$ and lagged conditional variance $h_{i,i,t-1}$, while conditional covariance $h_{i,j,t}$ is proportional to the product of conditional standard deviations $\sqrt{h_{i,i,t}}$ and $\sqrt{h_{j,j,t}}$:

$$\begin{aligned} \boldsymbol{H}_t &= \boldsymbol{D}_t^{1/2}\boldsymbol{R}\boldsymbol{D}_t^{1/2}, \\ \boldsymbol{D}_t &= \boldsymbol{W} + \boldsymbol{A} \odot (\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}^{\mathsf{T}}) + \boldsymbol{B} \odot \boldsymbol{H}_{t-1}. \end{aligned} \tag{17}$$

As specified, $\boldsymbol{D}_t$ is a diagonal matrix of dimension N containing conditional variances $h_{i,i,t}$, while $\boldsymbol{R}$ is a symmetric correlation matrix of dimension N. In the asymmetric CCORR model, conditional variance $h_{i,i,t}$ also depends on lagged squared asymmetric ordinary innovation $\eta_{i,t-1}^2$:

$$\begin{aligned} \boldsymbol{H}_t &= \boldsymbol{D}_t^{1/2} \boldsymbol{R} \boldsymbol{D}_t^{1/2}, \\ \boldsymbol{D}_t &= \boldsymbol{W} + \boldsymbol{A} \odot (\boldsymbol{\varepsilon}_{t-1} \boldsymbol{\varepsilon}_{t-1}^{\mathsf{T}}) + \boldsymbol{G} \odot (\boldsymbol{\eta}_{t-1} \boldsymbol{\eta}_{t-1}^{\mathsf{T}}) + \boldsymbol{B} \odot \boldsymbol{H}_{t-1}. \end{aligned} \tag{18}$$

Given that $\boldsymbol{W}$, $\boldsymbol{A}$, $\boldsymbol{G}$ and $\boldsymbol{B}$ are diagonal matrices of dimension N, the total parameter count for the symmetric CCORR model is $N(N+5)/2$, while that for the asymmetric CCORR model is $N(N+3)$. Conditional covariance matrix $\boldsymbol{H}_t$ is positive definite if correlation matrix $\boldsymbol{R}$ is positive definite, while all elements of matrices $\boldsymbol{W}$, $\boldsymbol{A}$, $\boldsymbol{G}$ and $\boldsymbol{B}$ are positive.

The general dynamic covariance or GDC model of Kroner and Ng (1998) is a hybrid of the BEKK and CCORR models:

$$\begin{aligned} \boldsymbol{H}_t &= \boldsymbol{D}_t^{1/2} \boldsymbol{R} \boldsymbol{D}_t^{1/2} + \boldsymbol{C} \odot \boldsymbol{Q}_t, \\ \boldsymbol{Q}_t &= \boldsymbol{W}^{\mathsf{T}} \boldsymbol{W} + \boldsymbol{A}^{\mathsf{T}} \boldsymbol{\varepsilon}_{t-1} \boldsymbol{\varepsilon}_{t-1}^{\mathsf{T}} \boldsymbol{A} + \boldsymbol{B}^{\mathsf{T}} \boldsymbol{H}_{t-1} \boldsymbol{B}. \end{aligned} \tag{19}$$

As specified, $\boldsymbol{D}_t$ is a diagonal matrix of dimension N containing conditional variances $q_{i,i,t}$, $\boldsymbol{R}$ is a symmetric matrix of dimension N containing ones on the principal diagonal, and $\boldsymbol{C}$ is a symmetric matrix of dimension N containing zeros on the principal diagonal. The GDC model nests a restricted DVECH model for $\boldsymbol{R} = \boldsymbol{I}$, $\boldsymbol{A} = \boldsymbol{I} \odot \boldsymbol{A}$ and $\boldsymbol{B} = \boldsymbol{I} \odot \boldsymbol{B}$, the BEKK model for $\boldsymbol{R} = \boldsymbol{I}$ and $\boldsymbol{C} = \boldsymbol{u}^{\mathsf{T}} - \boldsymbol{I}$, the FGARCH model for $\boldsymbol{R} = \boldsymbol{I}$, $\boldsymbol{C} = \boldsymbol{u}^{\mathsf{T}} - \boldsymbol{I}$, $\boldsymbol{A} = \sqrt{\alpha} \boldsymbol{\omega} \boldsymbol{\delta}^{\mathsf{T}}$ and $\boldsymbol{B} = \sqrt{\beta} \boldsymbol{\omega} \boldsymbol{\delta}^{\mathsf{T}}$, and a restricted CCORR model for $\boldsymbol{C} = \boldsymbol{0}$, $\boldsymbol{W} = \boldsymbol{I} \odot \boldsymbol{W}$, $\boldsymbol{A} = \boldsymbol{I} \odot \boldsymbol{A}$ and $\boldsymbol{B} = \boldsymbol{I} \odot \boldsymbol{B}$. The asymmetric GDC model of Kroner and Ng (1998) is a hybrid of the asymmetric BEKK and asymmetric CCORR models:

$$\begin{aligned} \boldsymbol{H}_t &= \boldsymbol{D}_t^{1/2} \boldsymbol{R} \boldsymbol{D}_t^{1/2} + \boldsymbol{C} \odot \boldsymbol{Q}_t, \\ \boldsymbol{Q}_t &= \boldsymbol{W}^{\mathsf{T}} \boldsymbol{W} + \boldsymbol{A}^{\mathsf{T}} \boldsymbol{\varepsilon}_{t-1} \boldsymbol{\varepsilon}_{t-1}^{\mathsf{T}} \boldsymbol{A} + \boldsymbol{G}^{\mathsf{T}} \boldsymbol{\eta}_{t-1} \boldsymbol{\eta}_{t-1}^{\mathsf{T}} \boldsymbol{G} + \boldsymbol{B}^{\mathsf{T}} \boldsymbol{H}_{t-1} \boldsymbol{B}. \end{aligned} \tag{20}$$

The asymmetric GDC model nests a restricted asymmetric DVECH model for $\boldsymbol{R} = \boldsymbol{I}$, $\boldsymbol{A} = \boldsymbol{I} \odot \boldsymbol{A}$, $\boldsymbol{G} = \boldsymbol{I} \odot \boldsymbol{G}$ and $\boldsymbol{B} = \boldsymbol{I} \odot \boldsymbol{B}$, the asymmetric BEKK model for $\boldsymbol{R} = \boldsymbol{I}$ and $\boldsymbol{C} = \boldsymbol{u}^{\mathsf{T}} - \boldsymbol{I}$, the asymmetric FGARCH model for $\boldsymbol{R} = \boldsymbol{I}$, $\boldsymbol{C} = \boldsymbol{u}^{\mathsf{T}} - \boldsymbol{I}$, $\boldsymbol{A} = \sqrt{\alpha} \boldsymbol{\omega} \boldsymbol{\delta}^{\mathsf{T}}$, $\boldsymbol{G} = \sqrt{\gamma} \boldsymbol{\omega} \boldsymbol{\delta}^{\mathsf{T}}$ and $\boldsymbol{B} = \sqrt{\beta} \boldsymbol{\omega} \boldsymbol{\delta}^{\mathsf{T}}$, and a restricted asymmetric CCORR model for $\boldsymbol{C} = \boldsymbol{0}$, $\boldsymbol{W} = \boldsymbol{I} \odot \boldsymbol{W}$, $\boldsymbol{A} = \boldsymbol{I} \odot \boldsymbol{A}$, $\boldsymbol{G} = \boldsymbol{I} \odot \boldsymbol{G}$ and $\boldsymbol{B} = \boldsymbol{I} \odot \boldsymbol{B}$. Given that $\boldsymbol{W}$, $\boldsymbol{A}$, $\boldsymbol{G}$ and $\boldsymbol{B}$ are square matrices of dimension N, with $\boldsymbol{W}$ upper triangular, the total parameter count for the symmetric GDC model is $N(7N-1)/2$, while that for the asymmetric GDC model is $N(9N-1)/2$. Conditional covariance matrix $\boldsymbol{H}_t$ is positive definite if matrices $\boldsymbol{R}$ and $\boldsymbol{C}$ are positive definite.

3.3. Quasi Maximum Likelihood Estimation

Let $\boldsymbol{\theta} \in \boldsymbol{\Theta} \subset \mathbb{R}^K$ denote a K dimensional vector containing the parameters associated with the conditional mean and variance functions $\boldsymbol{m}_t(\boldsymbol{\theta})$ and $\boldsymbol{H}_t(\boldsymbol{\theta})$, respectively. Also, let $\hat{\boldsymbol{\xi}}_{t|t-1}$, $\hat{\boldsymbol{\xi}}_{t|t}$ and $\hat{\boldsymbol{\xi}}_{t|T}$ denote vectors containing predicted, filtered and smoothed state probabilities $\mathrm{P}(s_t = i \mid \boldsymbol{r}_{t-1},\ldots,\boldsymbol{r}_0;\boldsymbol{\theta})$, $\mathrm{P}(s_t = i \mid \boldsymbol{r}_t,\ldots,\boldsymbol{r}_0;\boldsymbol{\theta})$ and $\mathrm{P}(s_t = i \mid \boldsymbol{r}_T,\ldots,\boldsymbol{r}_0;\boldsymbol{\theta})$, respectively.

Predicted and filtered state probabilities associated with a two state, first order Markov chain may be estimated using the algorithm of Hamilton (1989), while smoothed state probabilities may be estimated using the algorithm of Kim (1993). Given parameter vector $\boldsymbol{\theta}$ and initial conditions $\hat{\boldsymbol{\xi}}_{0|0}$, the predicted and filtered state probabilities may be estimated by iterating forwards on

$$\begin{aligned} \hat{\boldsymbol{\xi}}_{t|t-1} &= \boldsymbol{P}\hat{\boldsymbol{\xi}}_{t-1|t-1}, \\ \hat{\boldsymbol{\xi}}_{t|t} &= \frac{\hat{\boldsymbol{\xi}}_{t|t-1} \odot \boldsymbol{f}_t}{\boldsymbol{\iota}^{\mathsf{T}}(\hat{\boldsymbol{\xi}}_{t|t-1} \odot \boldsymbol{f}_t)}, \end{aligned} \tag{21}$$

where $\boldsymbol{f}_t$ is a vector containing conditional densities $f(\boldsymbol{r}_t \mid s_t = i, \boldsymbol{r}_{t-1},\ldots,\boldsymbol{r}_0;\boldsymbol{\theta})$. Given filtered state probability $\hat{\boldsymbol{\xi}}_{T|T}$, the smoothed state probabilities may be estimated by iterating backwards on

$$\hat{\boldsymbol{\xi}}_{t|T} = \hat{\boldsymbol{\xi}}_{t|t} \odot [\boldsymbol{P}^{\mathsf{T}}(\hat{\boldsymbol{\xi}}_{t+1|T} \div \hat{\boldsymbol{\xi}}_{t+1|t})], \tag{22}$$

where the $\div$ operator performs element by element matrix division. Initial conditions $\hat{\boldsymbol{\xi}}_{0|0}$ are provided by unconditional state probabilities:

$$\begin{aligned} \mathrm{P}(s_0 = 0) &= \frac{1-q}{2-p-q}, \\ \mathrm{P}(s_0 = 1) &= \frac{1-p}{2-p-q}. \end{aligned} \tag{23}$$

These unconditional state probabilities are well defined if the Markov chain is ergodic, which is ensured if the transition probabilities satisfy $p < 1$, $q < 1$ and $p + q > 0$.

The conditional maximum likelihood estimator $\hat{\boldsymbol{\theta}}_T$ of parameter vector $\boldsymbol{\theta}$ maximizes conditional loglikelihood function

$$\mathcal{L}(\boldsymbol{\theta}) = \sum_{t=1}^{T} \ell_t(\boldsymbol{\theta}), \tag{24}$$

where $\ell_t(\boldsymbol{\theta}) = \ln f(\boldsymbol{r}_t \mid \boldsymbol{r}_{t-1},\ldots,\boldsymbol{r}_0;\boldsymbol{\theta})$ and $f(\boldsymbol{r}_t \mid \boldsymbol{r}_{t-1},\ldots,\boldsymbol{r}_0;\boldsymbol{\theta}) = \boldsymbol{\iota}^{\mathsf{T}}(\hat{\boldsymbol{\xi}}_{t|t-1} \odot \boldsymbol{f}_t)$. Under the auxiliary assumption that standardized innovation vector $\boldsymbol{z}_t$ is conditionally multivariate normally distributed, the elements of conditional density vector $\boldsymbol{f}_t$ satisfy:

$$f(\boldsymbol{r}_t \mid s_t = i, \boldsymbol{r}_{t-1},\ldots,\boldsymbol{r}_0;\boldsymbol{\theta}) = (2\pi)^{-N/2} \mid \boldsymbol{H}_t \mid^{-1/2} \exp\left\{-\frac{1}{2}\boldsymbol{\varepsilon}_t^{\mathsf{T}}\boldsymbol{H}_t^{-1}\boldsymbol{\varepsilon}_t\right\}. \tag{25}$$

Maximization of the conditional loglikelihood function is performed numerically subject to the constraints that the transition probabilities satisfy $0 \le p \le 1$ and $0 \le q \le 1$.

Provided that the conditional mean and variance functions are correctly specified and twice continuously differentiable with respect to parameter vector $\boldsymbol{\theta}$, this conditional maximum likelihood estimator is consistent and asymptotically normal, irrespective of the validity of the conditional multivariate normality assumption,

$$\sqrt{T}(\hat{\boldsymbol{\theta}}_T - \boldsymbol{\theta}_0) \overset{d}{\to} \mathcal{N}(\mathbf{0}, \boldsymbol{A}_0^{-1}\boldsymbol{B}_0\boldsymbol{A}_0^{-1}), \tag{26}$$

where $\boldsymbol{m}_t(\boldsymbol{\theta}_0) = \mathrm{E}_{t-1}(\boldsymbol{r}_t)$ and $\boldsymbol{H}_t(\boldsymbol{\theta}_0) = \mathrm{Var}_{t-1}(\boldsymbol{r}_t)$ for some $\boldsymbol{\theta}_0 \in \boldsymbol{\Theta}$. If in addition the conditional mean and variance functions are dynamically complete, then Bollerslev and Wooldridge (1992) show that consistent estimators of $\boldsymbol{A}_0$ and $\boldsymbol{B}_0$ are given by

$$\begin{aligned} \hat{\boldsymbol{A}}_T &= \frac{1}{T}\sum_{t=1}^{T} \boldsymbol{a}_t(\hat{\boldsymbol{\theta}}_T), \\ \hat{\boldsymbol{B}}_T &= \frac{1}{T}\sum_{t=1}^{T} \boldsymbol{b}_t(\hat{\boldsymbol{\theta}}_T)\boldsymbol{b}_t(\hat{\boldsymbol{\theta}}_T)^{\mathsf{T}}, \end{aligned} \tag{27}$$

where $\boldsymbol{a}_t(\hat{\boldsymbol{\theta}}_T) = \nabla_{\boldsymbol{\theta}}\boldsymbol{m}_t(\hat{\boldsymbol{\theta}}_T)^{\mathsf{T}}\boldsymbol{H}_t^{-1}(\hat{\boldsymbol{\theta}}_T)\nabla_{\boldsymbol{\theta}}\boldsymbol{m}_t(\hat{\boldsymbol{\theta}}_T) + \frac{1}{2}\nabla_{\boldsymbol{\theta}}\boldsymbol{H}_t(\hat{\boldsymbol{\theta}}_T)^{\mathsf{T}}[\boldsymbol{H}_t^{-1}(\hat{\boldsymbol{\theta}}_T) \otimes \boldsymbol{H}_t^{-1}(\hat{\boldsymbol{\theta}}_T)]\nabla_{\boldsymbol{\theta}}\boldsymbol{H}_t(\hat{\boldsymbol{\theta}}_T)$ and $\boldsymbol{b}_t(\hat{\boldsymbol{\theta}}_T) = -\nabla_{\boldsymbol{\theta}}\ell_t(\hat{\boldsymbol{\theta}}_T)^{\mathsf{T}}$. If the conditional mean and variance functions are not dynamically complete, then an autocorrelation consistent estimator of $\boldsymbol{B}_0$ is required.

4. Empirical Analysis

Our analysis of risk premia and volatility transmission across the stock and bond markets is based on state dependent generalized impulse response functions derived from an empirical representation of the conditional CAPM. Although this empirical framework invariably does not coincide with the true data generating process, provided that it is dynamically complete in mean and variance, it may be expected to yield empirically valid impulse response dynamics. As such, prior to conducting impulse response analysis of the dynamic effects of shocks on the conditional means, variances and covariances of excess returns on stocks and bonds, we examine the specification of the conditional mean and variance functions associated with an estimated expected return beta representation of the conditional CAPM.

4.1. Model Specification Analysis

The data set consists of monthly excess returns on stock and bond portfolios for the United States over the period January 1942 through December 2004. Details concerning the composition of these stock and bond portfolios are contained in the appendix. Descriptive statistics reported in Table 1 reveal that both the estimated unconditional mean and standard deviation of the excess return on stocks exceed those on bonds, suggesting the existence of a tradeoff between risk and expected return.

Table 1. Descriptive statistics for excess returns

	Mean	Standard Deviation	Skewness	Kurtosis
Stock	0.693	4.229	−0.516	4.720
Bond	0.109	1.517	0.366	6.622

We estimate an expected return beta representation of the conditional CAPM by quasi maximum likelihood over the period February 1942 through December 2004. Our proxy for the market portfolio is an equal weighted portfolio of stock and bond portfolios. Time variation in the market price of risk is characterized by a two state Markov regime switching process, while time variation in conditional betas is characterized by an asymmetric GDC process. Our estimation results are reported in Table 2, where robust t ratios appear in parentheses. With some exceptions, the parameters of the conditional mean and variance functions are precisely estimated, with most of our parameter estimates being statistically significant at conventional levels.

Table 2. Quasi maximum likelihood estimation results

λ_1	λ_2	p	q	$R_{2,1}$	$C_{2,1}$		
0.247	0.613	0.951	0.935	−0.114	1.171		
(0.248)	(0.543)	(2.783)	(2.090)	(−2.828)	(28.221)		
$W_{1,1}$	$W_{1,2}$	$W_{2,2}$	$A_{1,1}$	$A_{2,1}$	$A_{1,2}$	$A_{2,2}$	
1.672	−0.019	−0.001	0.037	−0.116	−0.018	0.393	
(6.237)	(−0.372)	(−0.000)	(0.695)	(−1.041)	(−1.824)	(7.998)	
$G_{1,1}$	$G_{2,1}$	$G_{1,2}$	$G_{2,2}$	$B_{1,1}$	$B_{2,1}$	$B_{1,2}$	$B_{2,2}$
0.370	0.523	−0.006	0.223	0.835	0.183	0.015	0.917
(6.087)	(1.544)	(−0.558)	(2.299)	(17.739)	(2.534)	(1.507)	(62.321)

	$\hat{z}_{1,t\|t-1}$	$\hat{z}_{2,t\|t-1}$	$\hat{z}^2_{1,t\|t-1}$	$\hat{z}^2_{2,t\|t-1}$	$\hat{z}_{1,t\|t-1}\hat{z}_{2,t\|t-1}$
$Q(3)$	2.235	15.124***	0.745	1.128	0.750
$Q(6)$	7.098	17.265***	3.411	8.437	5.112
$Q(9)$	7.805	21.859***	6.374	11.077	5.547
$Q(12)$	8.472	24.594**	8.775	14.517	5.970

$\mathcal{L}(\hat{\boldsymbol{\theta}}_T) = -3291.036$

Note: Rejection of the null hypothesis at the 1%, 5% and 10% levels is indicated by ***, ** and *, respectively.

In order to assess whether our estimated expected return beta representation of the conditional CAPM is dynamically complete in mean and variance, we subject the levels, squares and cross products of the predicted standardized residuals to the autocorrelation test of Ljung and Box (1978). The predicted standardized residual vector $\hat{\boldsymbol{z}}_{t|t-1}$ is related to the predicted ordinary residual vector $\hat{\boldsymbol{\varepsilon}}_{t|t-1}$ by $\hat{\boldsymbol{z}}_{t|t-1} = \hat{\boldsymbol{H}}_{t|t-1}^{-1/2}\hat{\boldsymbol{\varepsilon}}_{t|t-1}$, where $\hat{\boldsymbol{H}}_{t|t-1}$ denotes the predicted conditional covariance matrix. The inverse square root of the predicted conditional covariance matrix is derived using a spectral decomposition as $\hat{\boldsymbol{H}}_{t|t-1}^{-1/2} = \hat{\boldsymbol{P}}_{t|t-1}\hat{\boldsymbol{\Lambda}}_{t|t-1}^{-1/2}\hat{\boldsymbol{P}}_{t|t-1}^{\mathsf{T}}$, where $\hat{\boldsymbol{P}}_{t|t-1}$ is a square matrix containing distinct orthonormal eigenvectors, while $\hat{\boldsymbol{\Lambda}}_{t|t-1}$ is a diagonal matrix containing the corresponding strictly positive eigenvalues. We find little evidence of autocorrelation or autoregressive conditional heteroskedasticity in the predicted standardized residual vector, suggesting that the conditional mean and variance functions are dynamically complete. Thus, the analysis of risk premia and volatility transmission across the stock and bond markets within the framework of our estimated expected return beta representation of the conditional CAPM appears justified.

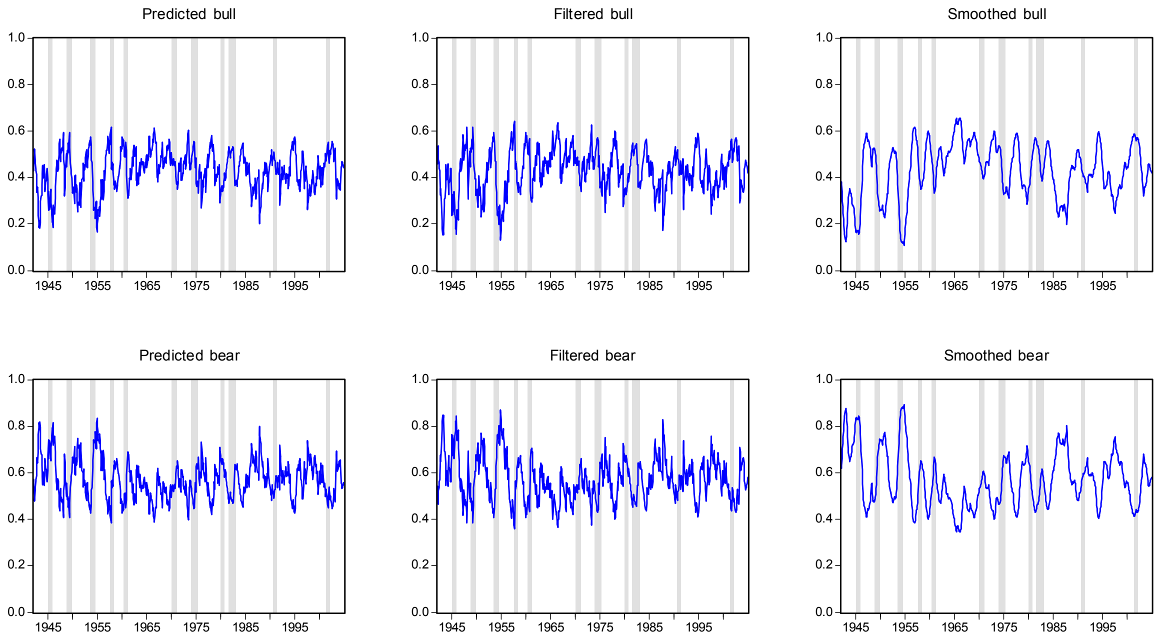
Table 3. Specification test results

Model	Restrictions	Statistic	P Value
Symmetric DVECH	$\boldsymbol{R} = \boldsymbol{I}, \boldsymbol{A} = \boldsymbol{I} \odot \boldsymbol{A}, \boldsymbol{G} = \boldsymbol{0}, \boldsymbol{B} = \boldsymbol{I} \odot \boldsymbol{B}$	108.302	0.000
Symmetric BEKK	$\boldsymbol{R} = \boldsymbol{I}, \boldsymbol{C} = \boldsymbol{u}^{\mathsf{T}} - \boldsymbol{I}, \boldsymbol{G} = \boldsymbol{0}$	1132.234	0.000
Symmetric FGARCH	$\boldsymbol{R} = \boldsymbol{I}, \boldsymbol{C} = \boldsymbol{u}^{\mathsf{T}} - \boldsymbol{I}, \boldsymbol{G} = \boldsymbol{0}$	1132.234	0.000
Symmetric CCORR	$\boldsymbol{C} = \boldsymbol{0}, \boldsymbol{W} = \boldsymbol{I} \odot \boldsymbol{W}, \boldsymbol{A} = \boldsymbol{I} \odot \boldsymbol{A}, \boldsymbol{G} = \boldsymbol{0}, \boldsymbol{B} = \boldsymbol{I} \odot \boldsymbol{B}$	1455.636	0.000
Symmetric GDC	$\boldsymbol{G} = \boldsymbol{0}$	59.341	0.000
Asymmetric DVECH	$\boldsymbol{R} = \boldsymbol{I}, \boldsymbol{A} = \boldsymbol{I} \odot \boldsymbol{A}, \boldsymbol{G} = \boldsymbol{I} \odot \boldsymbol{G}, \boldsymbol{B} = \boldsymbol{I} \odot \boldsymbol{B}$	16.837	0.018
Asymmetric BEKK	$\boldsymbol{R} = \boldsymbol{I}, \boldsymbol{C} = \boldsymbol{u}^{\mathsf{T}} - \boldsymbol{I}$	985.864	0.000
Asymmetric FGARCH	$\boldsymbol{R} = \boldsymbol{I}, \boldsymbol{C} = \boldsymbol{u}^{\mathsf{T}} - \boldsymbol{I}$	985.864	0.000
Asymmetric CCORR	$\boldsymbol{C} = \boldsymbol{0}, \boldsymbol{W} = \boldsymbol{I} \odot \boldsymbol{W}, \boldsymbol{A} = \boldsymbol{I} \odot \boldsymbol{A}, \boldsymbol{G} = \boldsymbol{I} \odot \boldsymbol{G}, \boldsymbol{B} = \boldsymbol{I} \odot \boldsymbol{B}$	1218.222	0.000

Note: All Wald test statistics are asymptotically distributed as χ_r^2, where r denotes the number of exact linear restrictions under test.

Alternative multivariate GARCH models impose different restrictions on the impulse response dynamics of conditional variances and covariances, with symmetric multivariate GARCH models restricting conditional variances and covariances to respond equally in magnitude to positive and negative shocks of equal magnitude. Since the asymmetric GDC model nests a variety of alternative symmetric and asymmetric multivariate GARCH models, it provides a unified framework within which to test the empirical validity of these restrictions. On the basis of robust Wald tests, the results of which are reported in Table 3, we decisively reject both symmetric and asymmetric variants of the DVECH, BEKK, FGARCH and CCORR models, as well as the symmetric GDC model, in favour of the asymmetric GDC model. It follows that there exist statistically significant dynamic interrelationships across the conditional variances and covariances of excess returns on stocks and bonds, and that these dynamic interrelationships are asymmetric.

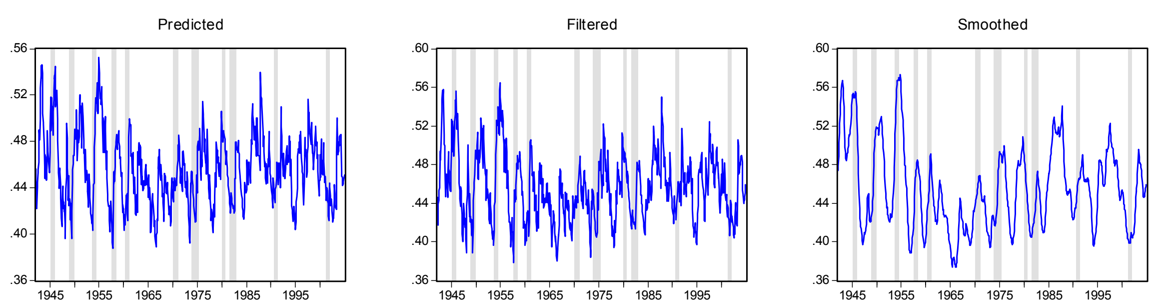
Figure 1. Estimated state probabilities



Note: Shaded regions indicate recessions as defined by the National Bureau of Economic Research reference cycle.

Estimated state probabilities associated with bull and bear markets are plotted in Figure 1. The estimated probability of remaining in a bull market is 0.951, while that of remaining in a bear market is 0.935. Given these estimated transition probabilities, the expected durations of bull and bear markets are 20.481 and 15.314 months, respectively.

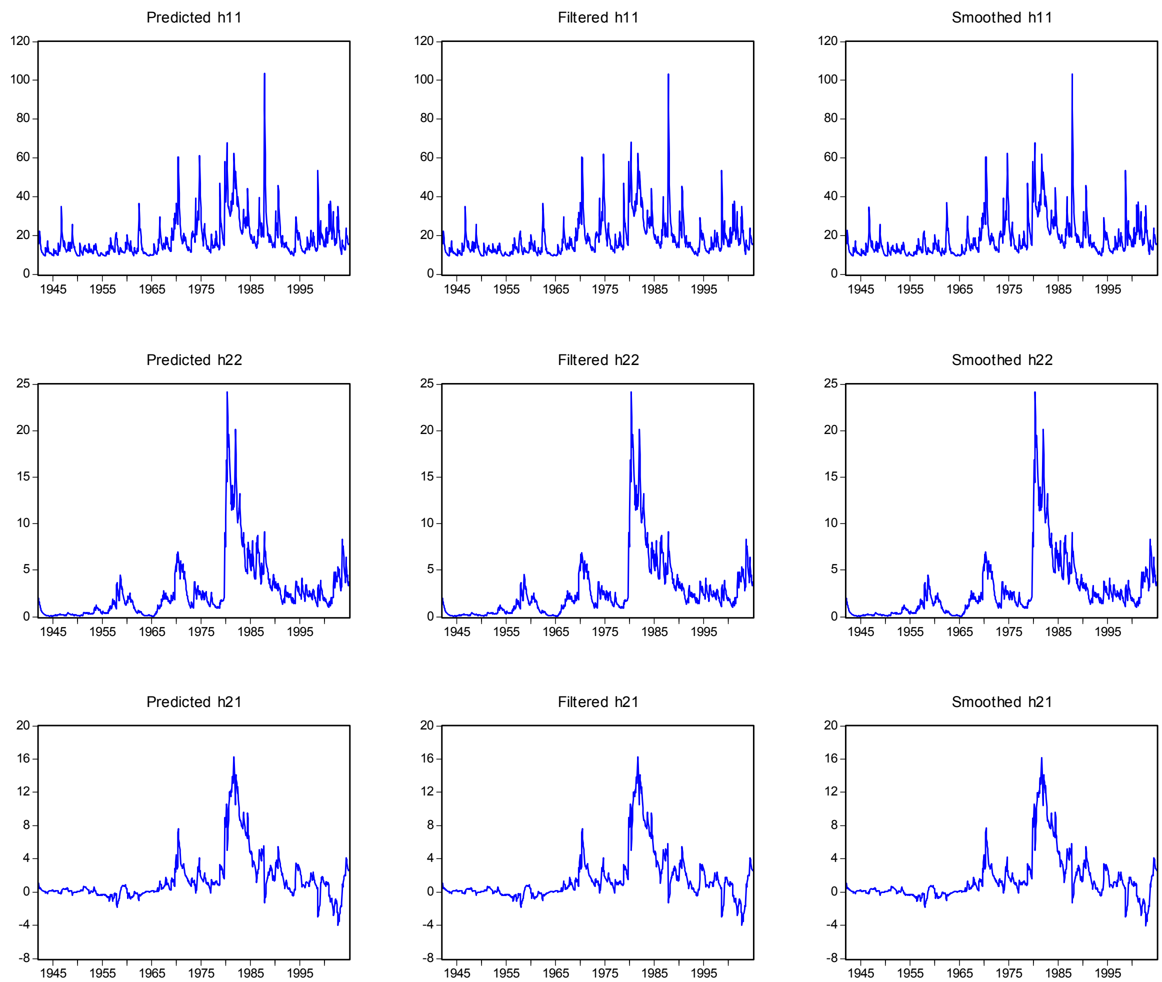
Figure 2. Estimated market price of risk



Note: Shaded regions indicate recessions as defined by the National Bureau of Economic Research reference cycle.

Considerable time variation is exhibited by the estimated market price of risk, which is plotted in Figure 2. The estimated market price of risk in a bull market is 0.247, while that in a bear market is 0.613. Visual inspection of the estimated market price of risk suggests the existence of variation within this range at business cycle frequencies, with bear markets tending to occur during recessions.

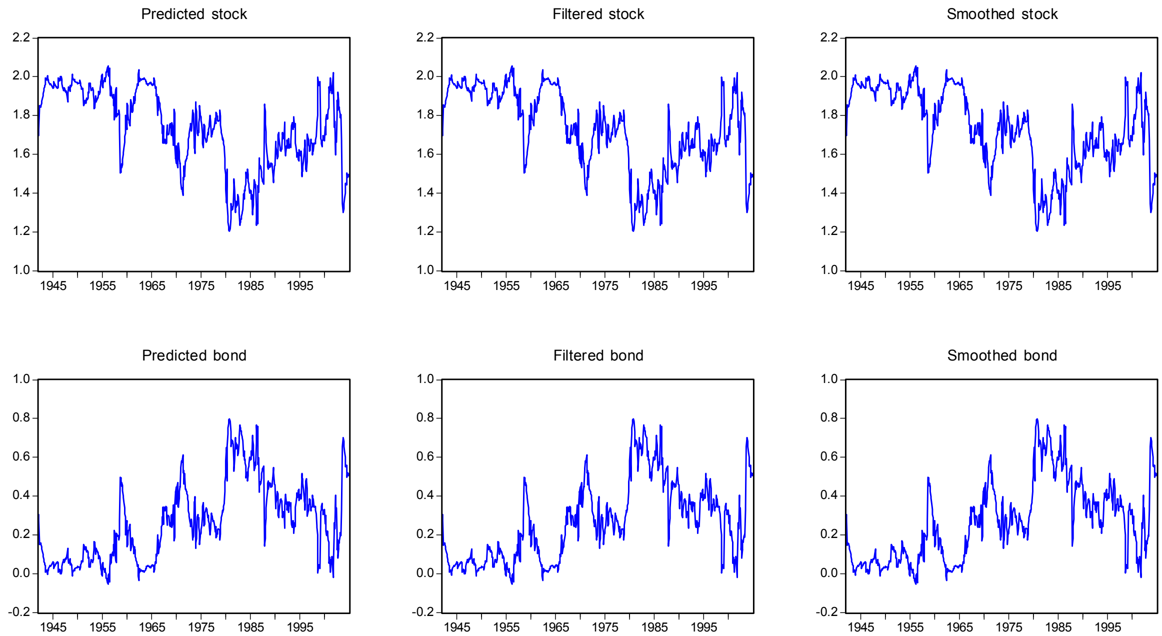
Figure 3. Estimated conditional variances and covariances



Note: The stock conditional variance, bond conditional variance, and conditional covariance are denoted by h_{11}, h_{22}, and h_{21}, respectively.

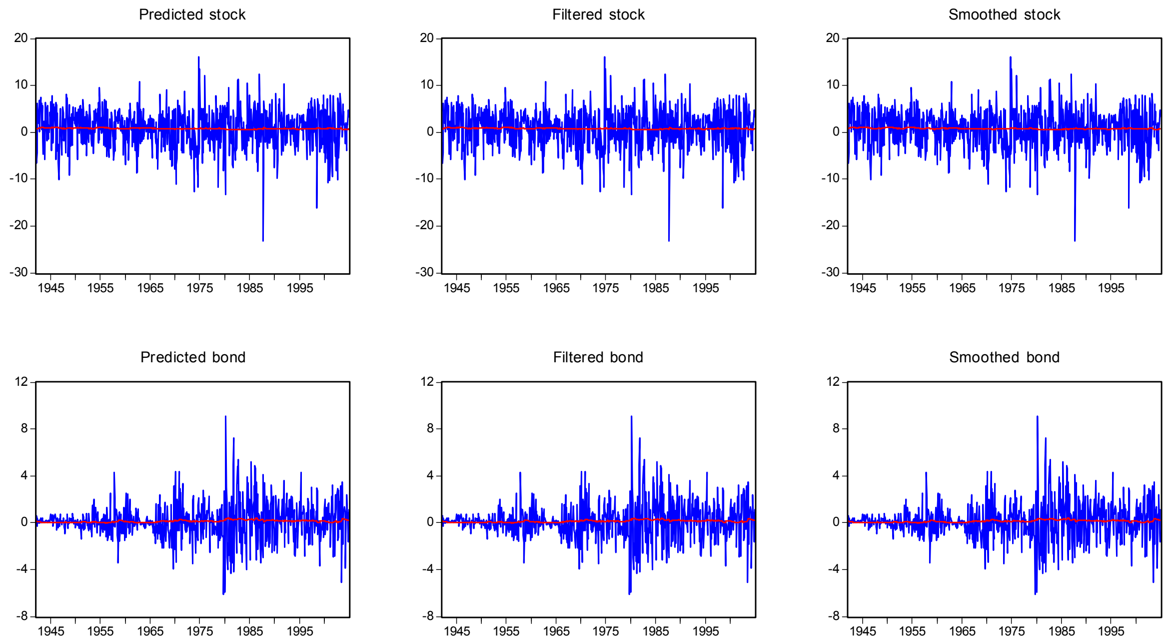
The estimated conditional variances and covariances of excess returns on stocks and bonds are plotted in Figure 3. A prominent feature of the estimated conditional variance of the excess return on stocks is a spike associated with the stock market crash of October 1987. The estimated conditional variance of the excess return on bonds is conspicuously high during the early 1980s, a period of general macroeconomic instability punctuated by a high and volatile risk free rate.

Figure 4. Estimated conditional betas



Considerable time variation is exhibited by the estimated conditional betas of stocks and bonds, which are plotted in Figure 4. As expected, the estimated conditional beta for stocks uniformly exceeds one, while that for bonds is uniformly less than one.

Figure 5. Estimated risk premia



Note: Estimated risk premia are represented by red lines, while blue lines depict excess returns.

Estimated risk premia are plotted versus excess stock and bond returns in Figure 5. Visual inspection reveals that our estimated expected return beta representation of the conditional CAPM accounts for a modest proportion of time variation in excess returns on stocks and bonds.

4.2. Impulse Response Analysis

A flexible tool for analyzing the dynamic effects of shocks is the generalized impulse response function introduced by Koop, Pesaran and Potter (1996). State dependent generalized impulse response functions may be defined as:

$$\begin{aligned} GIRF_{\boldsymbol{r}}(v,\boldsymbol{\delta},i;\boldsymbol{r}_{t-1},\ldots,\boldsymbol{r}_0) &= \mathrm{E}(\boldsymbol{r}_{t+v} \mid \boldsymbol{\varepsilon}_t = \boldsymbol{\delta}, s_t = i; \boldsymbol{r}_{t-1},\ldots,\boldsymbol{r}_0) - \mathrm{E}(\boldsymbol{r}_{t+v} \mid s_t = i; \boldsymbol{r}_{t-1},\ldots,\boldsymbol{r}_0), \\ GIRF_{\boldsymbol{H}}(v,\boldsymbol{\delta},i;\boldsymbol{r}_{t-1},\ldots,\boldsymbol{r}_0) &= \mathrm{E}(\boldsymbol{H}_{t+v} \mid \boldsymbol{\varepsilon}_t = \boldsymbol{\delta}, s_t = i; \boldsymbol{r}_{t-1},\ldots,\boldsymbol{r}_0) - \mathrm{E}(\boldsymbol{H}_{t+v} \mid s_t = i; \boldsymbol{r}_{t-1},\ldots,\boldsymbol{r}_0). \end{aligned} \tag{28}$$

Under the shock profile, the conditional mean and variance functions are perturbed by a fixed shock $\boldsymbol{\delta}$ at time t, while under the benchmark profile, no such shock occurs. If the conditional mean and variance functions are linear conditional on state i, then these generalized impulse response functions are symmetric and history independent.

If the conditional mean and variance functions are nonlinear conditional on state i, then these generalized impulse response functions are generally history dependent. A solution to this conceptual problem is suggested by Koop, Pesaran and Potter (1996), who emphasize that generalized impulse response functions are realizations of random variables for which various conditional versions may be defined. For instance, conditioning on all histories associated with state i renders these generalized impulse response functions history independent:

$$\begin{aligned} GIRF_{\boldsymbol{r}}(v,\boldsymbol{\varepsilon}_t,i) &= \mathrm{E}(\boldsymbol{r}_{t+v} \mid \boldsymbol{\varepsilon}_t, s_t = i) - \mathrm{E}(\boldsymbol{r}_{t+v} \mid s_t = i), \\ GIRF_{\boldsymbol{H}}(v,\boldsymbol{\varepsilon}_t,i) &= \mathrm{E}(\boldsymbol{H}_{t+v} \mid \boldsymbol{\varepsilon}_t, s_t = i) - \mathrm{E}(\boldsymbol{H}_{t+v} \mid s_t = i). \end{aligned} \tag{29}$$

These generalized impulse response functions measure the extent to which mean squared error optimal point forecasts of $\boldsymbol{r}_{t+v}$ and $\boldsymbol{H}_{t+v}$ at time $t-1$ are revised in response to a variable shock $\boldsymbol{\varepsilon}_t$ at time t, conditional on state i.

Since analytic expressions for the conditional expectations associated with these state dependent generalized impulse response functions are unavailable within the nonlinear empirical framework under consideration, we approximate them with nonparametric bootstrap simulations. Following Koop, Pesaran and Potter (1996), history dependent generalized impulse response functions are approximated as

$$\begin{aligned} \widehat{GIRF}_{\boldsymbol{r}}(v,\boldsymbol{\varepsilon}_t,i;\boldsymbol{r}_{t-1},\ldots,\boldsymbol{r}_0) &= \frac{1}{B}\sum_{j=1}^{B}\mathrm{E}(\boldsymbol{r}_{t+v} \mid \boldsymbol{\varepsilon}_t, s_t = i;\boldsymbol{r}_{t-1},\ldots,\boldsymbol{r}_0;\hat{\boldsymbol{\varepsilon}}_{t+1}^{(j)},\ldots,\hat{\boldsymbol{\varepsilon}}_{t+v}^{(j)}) \\ &\quad -\frac{1}{B}\sum_{j=1}^{B}\mathrm{E}(\boldsymbol{r}_{t+v} \mid s_t = i;\boldsymbol{r}_{t-1},\ldots,\boldsymbol{r}_0;\hat{\boldsymbol{\varepsilon}}_{t}^{(j)},\ldots,\hat{\boldsymbol{\varepsilon}}_{t+v}^{(j)}), \\ \widehat{GIRF}_{\boldsymbol{H}}(v,\boldsymbol{\varepsilon}_t,i;\boldsymbol{r}_{t-1},\ldots,\boldsymbol{r}_0) &= \frac{1}{B}\sum_{j=1}^{B}\mathrm{E}(\boldsymbol{H}_{t+v} \mid \boldsymbol{\varepsilon}_t, s_t = i;\boldsymbol{r}_{t-1},\ldots,\boldsymbol{r}_0;\hat{\boldsymbol{\varepsilon}}_{t+1}^{(j)},\ldots,\hat{\boldsymbol{\varepsilon}}_{t+v}^{(j)}) \\ &\quad -\frac{1}{B}\sum_{j=1}^{B}\mathrm{E}(\boldsymbol{H}_{t+v} \mid s_t = i;\boldsymbol{r}_{t-1},\ldots,\boldsymbol{r}_0;\hat{\boldsymbol{\varepsilon}}_{t}^{(j)},\ldots,\hat{\boldsymbol{\varepsilon}}_{t+v}^{(j)}), \end{aligned} \tag{30}$$

where B denotes the number of replications. These nonparametric bootstrap replications draw blocks of consecutive ordinary innovation vectors in intermediate time periods randomly with replacement from blocks of consecutive ordinary residual vectors associated with a given state, and are not dependent on any distributional assumptions. Under this resampling scheme, the length of blocks is set equal to the seasonal frequency to preserve autoregressive conditional heteroskedasticity.

Given these history dependent generalized impulse response functions, history independent generalized impulse response functions are approximated by averaging across all T_i histories associated with state i

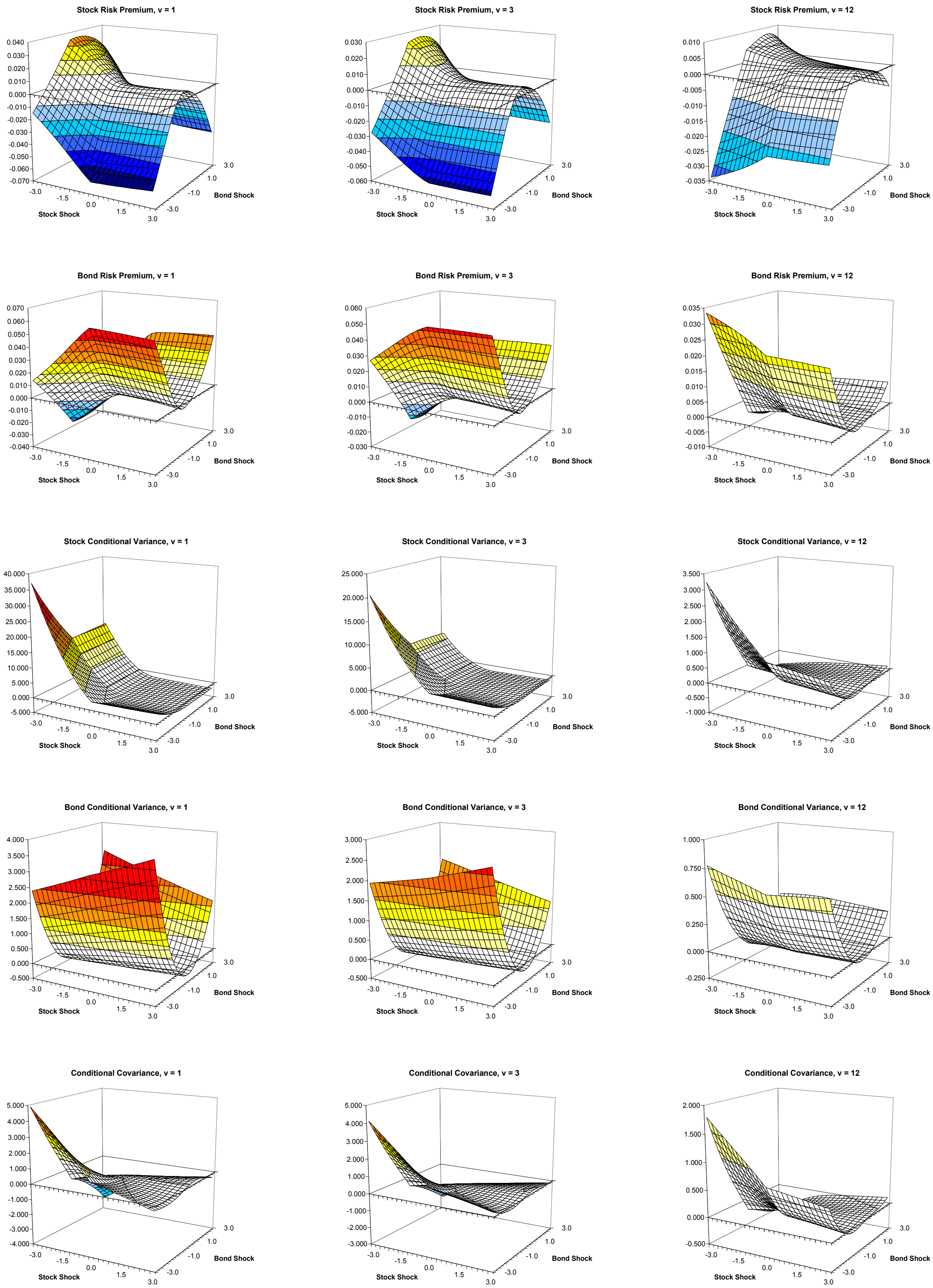
$$\widehat{GIRF}_{\boldsymbol{r}}(v,\boldsymbol{\varepsilon}_t,i)=\frac{1}{T_i}\sum_{t=1}^{T}I(\hat{s}_t=i)\widehat{GIRF}_{\boldsymbol{r}}(v,\boldsymbol{\varepsilon}_t,s_t;\boldsymbol{r}_{t-1},\ldots,\boldsymbol{r}_0),$$
$$\widehat{GIRF}_{\boldsymbol{H}}(v,\boldsymbol{\varepsilon}_t,i)=\frac{1}{T_i}\sum_{t=1}^{T}I(\hat{s}_t=i)\widehat{GIRF}_{\boldsymbol{H}}(v,\boldsymbol{\varepsilon}_t,s_t;\boldsymbol{r}_{t-1},\ldots,\boldsymbol{r}_0), \tag{31}$$

where $I(\hat{s}_t=i)=1$ if $\hat{s}_t=i$ and $I(\hat{s}_t=i)=0$ otherwise. Inferences on the state variable are derived from the estimated smoothed state probabilities, with $\hat{s}_t=i$ if $\mathrm{P}(s_t=i\,|\,\boldsymbol{r}_T,\ldots,\boldsymbol{r}_0;\hat{\boldsymbol{\theta}}_T)>\frac{1}{2}$ and $\hat{s}_t=1-i$ otherwise. Following the recommendation of Koop, Pesaran and Potter (1996), common realizations of the standardized innovation vectors in intermediate time periods are employed across both profiles and histories to reduce the simulation error.

These state dependent generalized impulse response functions may be viewed as generalizations of the news impact surface introduced by Kroner and Ng (1998), which in turn is a generalization of the news impact curve introduced by Engle and Ng (1993). In particular, under the case of a one dimensional state space, our state dependent generalized impulse response functions associated with the conditional variance function at the one step ahead forecast horizon coincide with this news impact surface concept up to a vertical translation. By allowing for state and forecast horizon dependence, in addition to jointly analyzing the conditional mean and variance functions, our state dependent generalized impulse response functions are considerably more informative with regards to the dynamic effects of shocks.

Estimated history independent generalized impulse response functions conditional on bull and bear markets are plotted in Figure 6 and Figure 7, respectively. Note that shocks to the stock and bond markets are measured in terms of the estimated unconditional standard deviations of state dependent ordinary residuals. Consistent with the existence of a tradeoff between risk and expected return, these estimated unconditional standard deviations suggest that bull markets are less volatile than bear markets. In particular, the estimated unconditional standard deviations of shocks to the stock and bond markets are 3.685 and 1.369 under a bull market, as compared with 4.331 and 1.566 under a bear market, respectively.

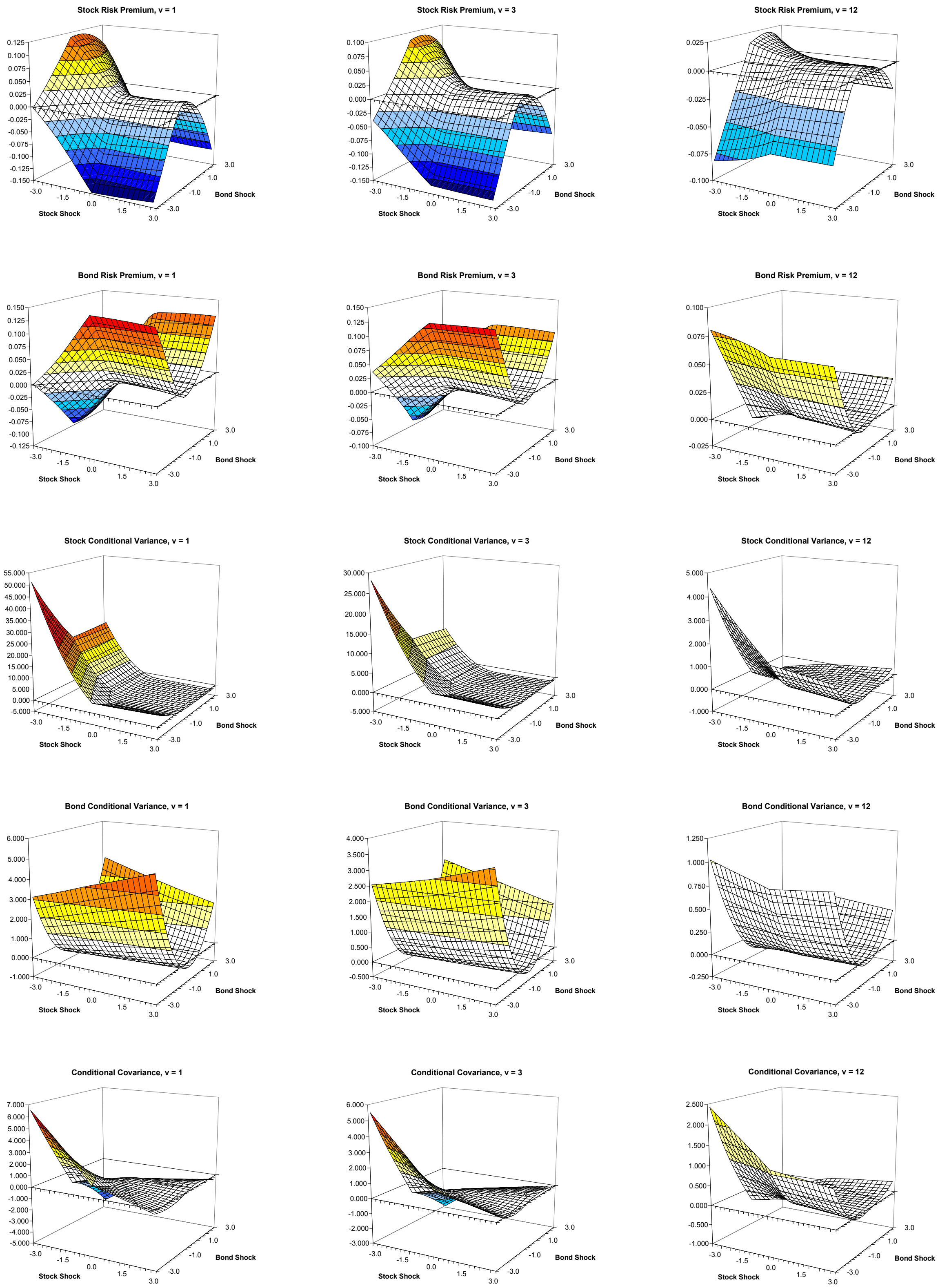
Figure 6. Estimated generalized impulse response functions conditional on bull market



Note: Estimated history independent generalized impulse response functions are based on $B = 99$ nonparametric bootstrap replications.

Our estimated state dependent generalized impulse response functions exhibit several common features across bull and bear markets. First, the risk premium on stocks responds asymmetrically to both stock and bond market shocks. To elaborate, negative stock market shocks generate a positive and economically significant equity risk premium, while positive stock market shocks induce little or no equity risk premium. Furthermore, the equity risk premium is decreasing with bond market shocks of either sign, with negative bond market shocks reducing the equity risk premium to a greater extent than positive bond market shocks of equal magnitude. Second, the risk premium on bonds also responds asymmetrically to both stock and bond market shocks. In particular, negative stock market shocks generate a negative and economically significant term premium, while positive stock market shocks induce little or no term premium. Furthermore, the term premium is increasing with bond market shocks of either sign, with negative bond market shocks raising the term premium to a greater extent than positive bond market shocks of equal magnitude. These are predominantly previously undocumented results. Third, in agreement with the results of Scruggs and Glabadanidis (2003), the conditional variance of the excess return on stocks responds asymmetrically to both stock and bond market shocks, with negative shocks generating considerably greater stock market volatility than positive shocks of equal magnitude. Indeed, stock market volatility is essentially invariant to positive shocks to the stock and bond markets. Fourth, consistent with the results of Scruggs and Glabadanidis (2003), the conditional variance of the excess return on bonds is relatively insensitive to stock market shocks yet responds symmetrically to bond market shocks. However, in conflict with the results of Scruggs and Glabadanidis (2003), the effect of bond market shocks on bond market volatility is asymmetric with respect to stock market shocks, which magnify the effects of bond market shocks of opposite sign. This volatility asymmetry interaction is a previously undocumented result. Fifth, the conditional covariance between excess returns on stocks and bonds increases dramatically in response to combined negative shocks to the stock and bond markets. However, this conditional covariance declines in response to a negative stock market shock when combined with a positive bond market shock. These are also previously undocumented results.

Figure 7. Estimated generalized impulse response functions conditional on bear market



Note: Estimated history independent generalized impulse response functions are based on $B = 99$ nonparametric bootstrap replications.

Our estimated state dependent generalized impulse response functions differ quantitatively but not qualitatively across bull and bear markets. First, reflecting a lower estimated market price of risk, stock and bond risk premia are less sensitive to shocks of given relative magnitudes under a bull market than under a bear market. Second, reflecting lower estimated absolute magnitudes of shocks, the conditional variances and covariances of excess returns on stocks and bonds are also less sensitive to shocks of given relative magnitudes under a bull market than under a bear market. These state dependent asymmetries are predominantly previously undocumented results which would be concealed by state independent generalized impulse response functions.

That our estimated state dependent generalized impulse response functions converge to zero as the forecast horizon v approaches infinity confirms covariance stationarity of the conditional mean and variance functions. In principle, the statistical significance of these generalized impulse response functions may therefore be assessed by estimating confidence surfaces accounting for parameter uncertainty with nonparametric bootstrap simulations. In practice, the high computational cost of such simulations is prohibitive.

5. Conclusion

Although the nature of risk premia and volatility transmission across the stock and bond markets has profound implications for a variety of investment, risk management and regulatory policy decisions, the existing literature lacks a systematic analysis of these issues within a unified empirical framework. This paper analyzes risk premia and volatility transmission across the stock and bond markets within an expected return beta representation of the conditional CAPM. Time variation in the market price of risk is characterized by a two state Markov regime switching process, while time variation in conditional betas is characterized by an asymmetric GDC process. The dynamic effects of these shocks on the conditional means, variances and covariances of excess returns on stocks and bonds are analyzed with state dependent generalized impulse response functions.

The implications of our impulse response analysis of risk premia and volatility transmission across the stock and bond markets for investment, risk management, and regulatory policy decisions may be summarized as follows. First, our finding that negative stock market shocks generate a negative and economically significant term premium while generating considerably greater stock market volatility than positive shocks of equal magnitude is indicative of a flight to quality phenomenon, whereby investors shift funds from the stock market to the bond market in response to high stock market volatility. Second, our result that the conditional covariance between excess returns on stocks and bonds increases dramatically in response to combined negative shocks to the stock and bond markets, an effect which is exacerbated under a bear market, implies that the degree of risk diversification achieved by cross market hedging is lowest when it is most desirable.

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Appendix

The data set consists of monthly excess returns on stock and bond portfolios for the United States over the period January 1942 through December 2004. The stock return corresponds to the one month simple net holding period return including dividends on a value weighted portfolio of all common stocks on the NYSE, AMEX, and NASDAQ listings. The bond return is measured by the one month simple net holding period return on an equal weighted portfolio of Treasury bonds having maturities of 1, 2, 5, 7, 10, 20, and 30 years. The risk free rate corresponds to the one month simple net Treasury bill rate. The stock and bond returns were constructed using data obtained from the Center for Research in Security Prices, while the risk free rate was provided by Ibbotson Associates.

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