Log-Periodic Crashes Revisited

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Abstract

We revisit the finding that crashes can be deterministic and governed by log-periodic formulas [2, 3]. One- and two-harmonic equations are usually employed to fit daily data during bubble episodes. But a three harmonics has been shown to fit anti-bubbles [5]. Here we show that the three-harmonic formula can work for bubble episodes as well as anti-bubbles. This is illustrated with daily data from the Brazilian *real*-US dollar exchange rate. And we also show that the three-harmonics can fit an intraday data set from that foreign exchange rate.

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1. Introduction

Some geophysicists have suggested that financial crashes, like material rupture [1], can sometimes be deterministic and governed by log-periodic formulas [2, 3]. Their finding has attracted reasonable media coverage and a best-selling book [4] is out on the pop-sci shelves of bookstores.

This is not so surprising because its straightforward implication is for some financial crashes to be predictable. Indeed the discoverers of log-periodicity have made the sanguine claim [2] of having picked out the signals prior to the Wall Street crashes of 1929, 1962, and 1987, as well as the 1997 crash on the Hong Kong stock exchange. And they also claim to have forecasted the Nasdaq high-tech bubble burst on April 2000 and correctly predicted the sudden upturn of the Japanese Nikkei index on January 1999.

One-harmonic and two-harmonic log-periodic equations are usually employed to fit daily data [3]. An exception is a three-harmonic formula used to fit anti-bubbles (bear markets) in the Nikkei (from 1990 to 1998) and future gold prices (from 1980 to 1998) [5].

Cooperative imitation is key for the log-periodicity hypothesis in that crashes would result from the build-up of correlations. Indeed self-reinforcing imitation between traders in a bull market leads to a bubble. The build-up of stress pushes the market to a critical time interval. After a threshold known as the critical point, many traders place the same order (i.e. sell) at the same time, thereby provoking the crash [4]. Imitation makes the system periodic on the eve of a crash. And in that sense crashes are outliers with properties that are statistically distinct from the rest of the population.

The log-periodicity hypothesis of crashes that are outliers departs from the conventional statistics that approaches extreme events. If log-periodicity is present at certain times in financial data then this is suggestive that these periods of time present scale invariance in their time evolution. (And this scaling has nothing to do with that related to the power law tails of returns [6].)

This paper shows that a three-harmonic log-periodic formula (that encompasses the one- and two-harmonics) can fit bubble episodes, too. What is more, it can fit bubbles in frequencies higher than daily ones. To illustrate our case we take one financial series with two distinct frequencies. One data set for the daily exchange rate between the Brazilian *real* and the US dollar and one for the intraday, 15-minute spaced *real*-dollar rate. The three harmonics outperforms the one- and two-harmonics in that it adjusts better not only to the daily data but also to the higher frequency series.

We also provide an example of both bubble and anti-bubble log-periodic behavior relative to a same crash in the daily *real*-dollar rate. This has not been found previously because the crashes considered were disruptive enough to break down symmetry [5].

The rest of the paper is organized as follows. Section 2 presents the data and adjusts the log-periodic formulas to them. And Section 3 concludes.

2. Data and analysis

The data set for the daily frequency of the *real*-dollar series is for the period from 2 January 1995 to 10 June 2005. The set comprises 2,624 data points obtained from the Federal Reserve website. Gaps between trading days are considered. Doing so produces an unequally time-spaced data set that is fitted by a non-linear regression using SAS. Neglecting the gaps does not change results a great deal, however.

The 15-minute set of the *real*-dollar rate comprises 9,327 data points from 9:30AM of 19 July 2001 to 4:30PM of 14 January 2003. Gaps between office hours are also taken into account as above, though neglecting this does not significantly change results.

Log-periodic cycles with a smooth trend component are generally described by a sum of log-periodic harmonics, i.e.,

$$\ln Z(\tau) = A + B\tau^{\lambda} + \sum_{j=1}^{J} C_j \tau^{\lambda_j} \cos[j\theta_j \ln(\tau) + \phi_j], \qquad (1)$$

where τ is the time starting with the onset of a bubble. Term $A + B\tau^{\lambda}$ is the trend across time, and A, B, and λ give its shape. Parameters θ_j, C_j, and ϕ_j are angular log-frequency, amplitude, and phase of the j^{th} harmonic respectively. We set $\tau = t - t_c > 0$, where t_c is critical time. If $j = 1$ ($j = 2$) in (1), the one- (two-) harmonic log-periodic model [3] obtains. The three-harmonic log-periodic model holds if $\lambda = \lambda_j$ and $\theta = \theta_j$. Such log-periodic formulas can be alternatively derived from a Landau expansion up to the third order [5].

For the daily series, the critical time t_c is estimated as 16 December 2002 using the three-harmonic log-periodic model, when the Brazilian currency reached its highest depreciation relative to the dollar. This is seen as a mild crash. Thanks to an exchange-rate bands policy, until December 1998 market participants could then easily use the information of a predictable daily revaluation of the Brazilian currency. Thus the eventual crash of 13 January 1999 cannot be of log-periodic nature; it most probably resulted from the pegging of the exchange rate. After the currency crisis, the *real*-dollar rate was let to float. So the crash that is likely to be log-periodic (and referred to in this paper) is that related to the peak that the *real*-dollar rate reached at the time of president Lula's election. Market participants self-organized imitatively and the result was a depreciating exchange rate that evolved log-periodically. And the bubble eventually burst after the last-minute (credible) conversion of left-wing Lula to economic orthodoxy.

Figure 1 displays the log of the daily *real*-dollar rate from 31 July 2000 to 10 June 2005 together with its one-, two-, and three-harmonic log-periodic fits respectively. The three-harmonics formula outperforms the others for the period ranging from 31 July 2000 to 7 October 2003 (from $t = 2037$ to $t = 3200$). Yet from $t = 3200$ on (7 October 2003−10 June 2005) the fit breaks down.

Given the crash at $t_c = 2905$ (16 December 2002), we have taken $\tau = t - 2905$ and predicted the anti-bubble behavior just by reflecting the fitted curve around $\tau = 0$. Figure 2 shows the bubble, anti-bubble, their log-periodic adjustments, and the crash (vertical line). The log-periodic equations can predict the anti-bubble till $\tau \approx 200$ (3 July 2003). Between this date and 29 April 2004 ($200 < \tau < 500$), the exchange rate stabilizes. Foreign exchange market participants went on standby perhaps to make sure that Lula's conversion to economic orthodoxy was for real. Then the anti-bubble resumes. This is shown in Figure 2 as the shift to the right. Thus although the log-periodic equations fail to predict the anti-bubble pattern, they are still able to track its trend. The anti-bubble is symmetric though exhibiting an intermittent behavior. Thus we provide an example of both bubble and anti-bubble log-periodic behavior relative to a same crash. This has not been found previously because the crashes considered were disruptive enough to break down the symmetry [5]. Here we can find such a case because we are dealing with a mild crash that did not provoke a market rupture.

To illustrate the presence of log-periodicity in high-frequency data (non-existent in literature), we apply our equations to the intraday *real*-dollar rate. Figure 3 shows the fit from 9:30AM of 28 May 2002 to 4:30PM of 14 January 2003 using one, two, and three harmonics. As can be seen, the three-harmonic log-periodic formula adjusts better to the data as well. The critical times were found from 16:30AM of 3 October 2002 to 10:00AM of 28 October 2002 (Table 2).

Parameter values for all the fits are presented in Tables 1 and 2.

3. Conclusion

We revisit the finding that crashes can be deterministic and governed by log-periodic formulas [2, 3]. Literature usually employs one- and two-harmonic equations to fit daily data during bubble episodes. By taking daily as well as intraday data from the exchange rate between the Brazilian *real* and US dollar, this paper shows that a three-harmonic log-periodic formula outperforms the others. If the three-harmonics can fit higher frequency data then the signatures of coming crashes may also be detected at short time scales.

The three harmonics has been already shown to fit anti-bubbles [5]. Yet this paper shows that it can work for bubble episodes as well. It also shows both bubble and anti-bubble log-periodic behavior relative to a same crash, a result that is novel in literature.

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References

[1] D. Sornette, C. Vanneste, Dynamics and memory effects in rupture of thermal fuse networks, Phys. Rev. Lett. 68 (1992) 612-615.

[2] D. Sornette, A. Johansen, Significance of log-periodic precursors to financial crashes, Quant. Finance 1 (2001) 452-471.

[3] D. Sornette, W. X. Zhou, The US 2000-2002 market descent: how much longer and deeper? Quant. Finance 2 (2002) 468-481.

[4] D. Sornette, Why Stock Markets Crash: Critical Events in Complex Financial Systems, Princeton University Press, Princeton, 2003.

[5] A. Johansen, D. Sornette, Financial "anti-bubbles": log-periodicity in gold and Nikkei collapses, Int. J. Mod. Phys. C 10 (1999) 563-575.

[6] R. Mantegna, H. E. Stanley, An Introduction to Econophysics, Correlations and Complexity in Finance, Cambridge University Press, Cambridge, 2000.

Table 1

Parameter	Estimate ± Standard Error		
	$j = 1$	$j = 2$	$j = 3$
A	0.5813 ± 0.00806	0.5837 ± 0.00822	0.5815 ± 0.00819
B	0.00169 ± 0.000321	0.00220 ± 0.000401	0.00243 ± 0.000431
C_1	–0.00053 ± 0.000092	0.000701 ± 0.000117	0.000780 ± 0.000126
C_2		0.000117 ± 0.000024	–0.00013 ± 0.000027
C_3			0.000077 ± 0.000019
θ	–9.2141 ± 0.0569	–8.7247 ± 0.0511	–8.5762 ± 0.0472
ϕ_1	71.2343 ± 0.3740	45.9033 ± 0.3340	57.4966 ± 0.3089
ϕ_2		–2.2059 ± 0.6898	–1.1958 ± 0.6386
ϕ_3			–120.0 ± 0.9603
λ	0.8551 ± 0.0264	0.8153 ± 0.0253	0.7999 ± 0.0244
t_c	3 December 2002	29 November 2002	16 December 2002

Table 2

Parameter	Estimate ± Standard Error		
	$j = 1$	$j = 2$	$j = 3$
A	0.9406 ± 0.00527	0.9212 ± 0.00440	0.9500 ± 0.00282
B	0.00104 ± 0.000243	0.00135 ± 0.000222	0.000406 ± 0.000065
C_1	0.000263 ± 0.000057	0.000177 ± 0.000029	0.000073 ± 0.000012
C_2		0.000209 ± 0.000030	0.000100 ± 0.000014
C_3			0.000064 ± 9.503E-6
θ	3.6766 ± 0.0535	3.4977 ± 0.0211	3.5673 ± 0.0159
ϕ_1	66.2759 ±0.4828	61.7290 ± 0.2031	61.2662 ± 0.1455
ϕ_2		2.7737 ± 0.3930	1.3608 ± 0.2795
ϕ_3			–4.9071 ± 0.4269
λ	0.5944 ±0.0234	0.5768 ± 0.0162	0.6935 ±0.0162
t_c	10:00AM of 28 October 2002	10:15AM of 25 October 2002	16:30AM of 03 October 200202

Log-periodicity in daily (Table 1) and intraday (Table 2) *real*-dollar rate.
Results for one-, two-, and three- harmonic log-periodic models.

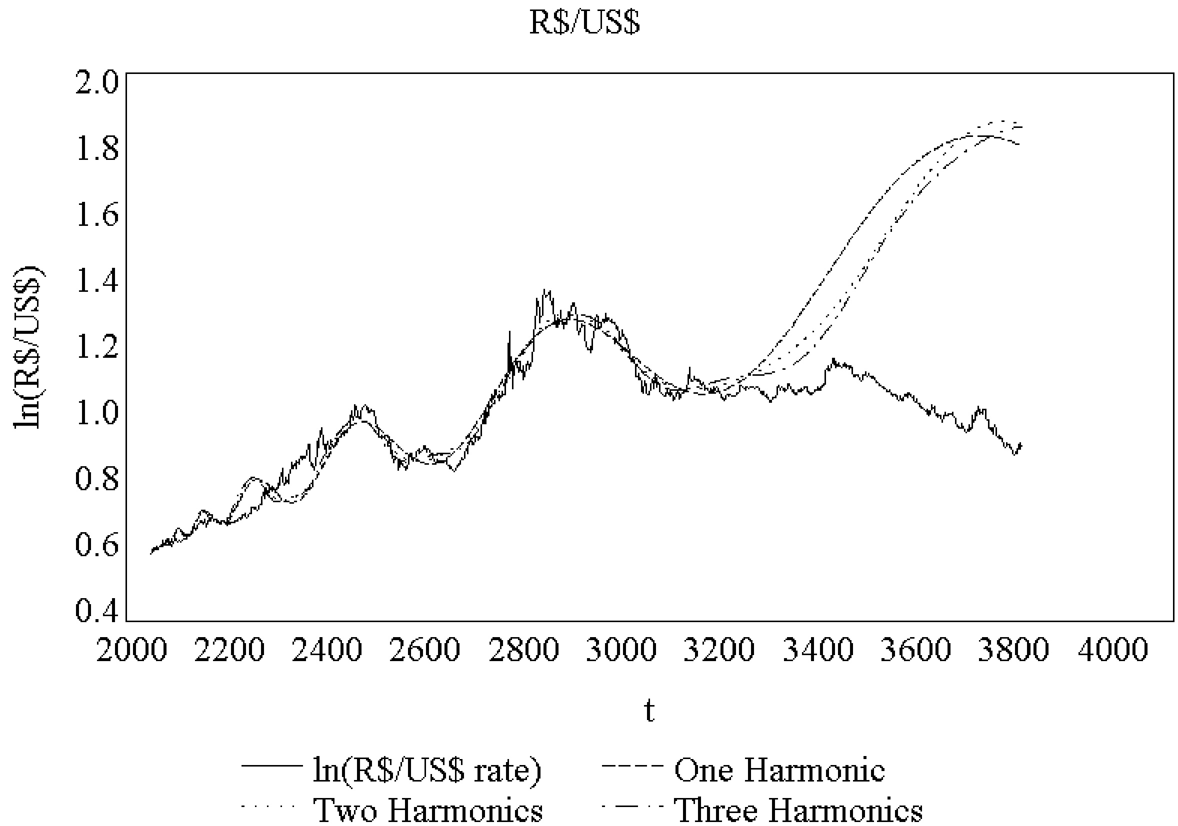

Figure 1. Log of the daily *real*-dollar rate together with its one-, two-, and three-harmonic log-periodic fits, 31 July 2000–10 June 2005. The three harmonics adjusts better to data, but from t = 3200 on (7 October 2003) all the fits break down. See Table 1.

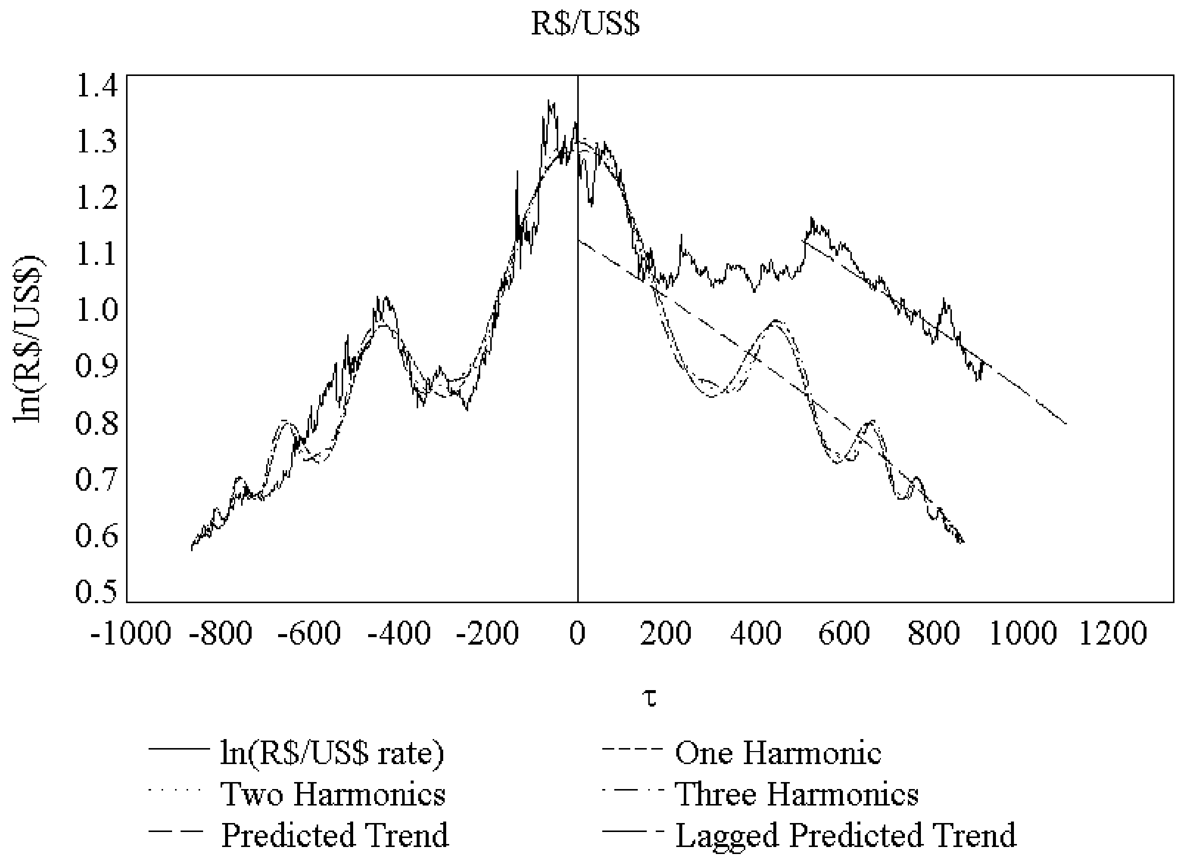

Figure 2. Bubble, anti-bubble, their log-periodic adjustments, and the crash (vertical line). Log-periodicity for the anti-bubble works till $\tau \approx 200$ (3 July 2003). But although the log-periodic equations fail to predict the anti-bubble from this date on, they can still track its trend (negatively sloped line shifted to the right).

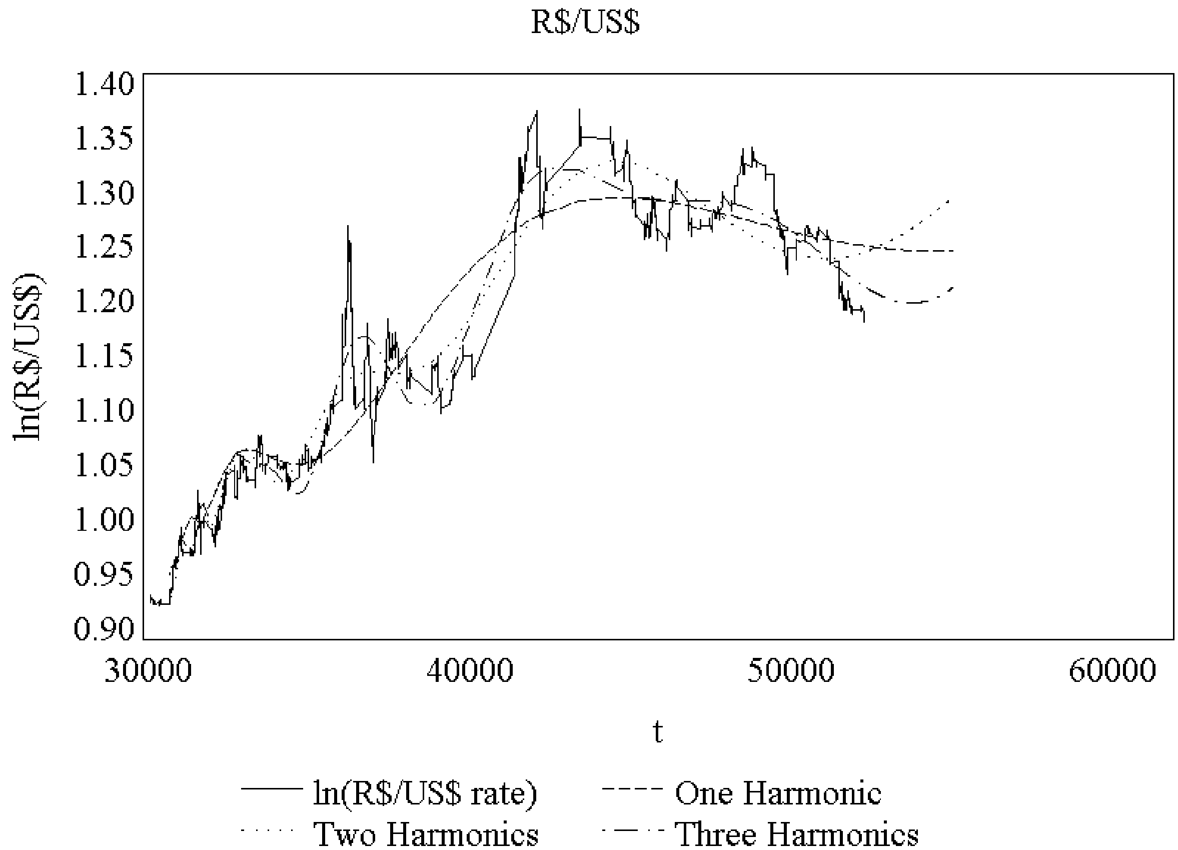

Figure 3. Log-periodic fits of intraday *real*-dollar returns from 9:30AM of 28 May 2002 to 4:30PM of 14 January 2003. The three-harmonic log-periodic formula adjusts better to data. See Table 2.

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