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THE SALOMON BROTHERS APPROACH
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EXTRACT

This paper explain, analyze and apply in an example the original paper developed by Kopprasch, Boyce, Koenigsberg, Tatevossian, and Yampol (1987) from The Salomon Brothers Inc. Bond Portfolio Analysis Group. Please, be aware. This paper is for educational issues only. There is a Spanish version in EconWPA.

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1 DURATION AND CONVEXITY FOR NORMAL (NO CALLABLE) BONDS

Bonds are fixed income investments that have a fixed interest rate or coupon, payable on the principal amount. All fixed income investments are evidence of indebtedness which represent a loan or debt between the issuer and the owner or holder of the security.

The value of any bond is the present value of its expected cash flows. In order to find the price you must determine the cash flows and then discount those cash flows at an appropriate rate. But, in practice, this is not so simple for two reasons.

First, holding aside the possibility of default, it is not easy to determine the cash flows for bonds with embedded options. Because the exercise of options embedded in a bond depends on the future course of interest rates, the cash flow is a priori uncertain. The issuer of a callable bond can alter the cash flows to the investor by calling the bond, while the investor in a putable bond can alter the cash flows by putting the bond. The future course of interest rates determines when and if the party granted the option is likely to alter the cash flows.

A second complication is determining the rate at which to discount the expected cash flows. The usual starting point is the yield available on Treasury securities. Appropriate spreads must be added to those Treasury yields to reflect additional risks to which the investor is exposed. Determining the appropriate spread is not simple. The ad-hoc process for valuing an option-free bond (i.e., a bond with no options) once was to discount all cash flows at a rate equal to the yield offered on a new full-coupon bond of the same maturity. Suppose, for example, that one needs to value a 10-year option-free bond. If the yield to maturity of an on-the-run 10-year bond of given credit quality is 8%, then the value of the bond under consideration would be taken to be the present value of its cash flows, all discounted at 8%.

According to this approach, the rate used to discount the cash flows of a 10-year current-coupon bond would be the same rate as that used to discount the cash flow of a 10-year zero-coupon bond. Conversely, discounting the cash flows of bonds with different maturities would require different discount rates. This approach makes little sense because it does not consider the cash flow characteristics of the bonds. Consider, for example, a portfolio of bonds of similar quality but different maturities. Imagine two equal cash flows occurring, say, five years hence, one coming from a 30-year bond and the other coming from a 10-year bond. Why should these two cash flows have different discount rates and hence different present values?

One difficulty with implementing this approach is that might not be zero-coupon securities from which to derive every discount rate of interest. Even in the absence of zero-coupon securities, however, arbitrage arguments can be used to generate the theoretical zero-coupon rate that an issuer would have to pay were it to issue zeros of every maturity. Using these theoretical zero-coupon rates, more popularly referred to as theoretical spot rates, the theoretical value of a bond can be determined. When dealer firms began stripping of full-coupon Treasury securities in August 1982, the actual prices of Treasury securities began moving toward their theoretical values.

In sum, the value of a bond depends on the size of its coupon payments, the length of time remaining until the bond matures and the current level of interest rates. The value of a bond is the present value of its cash flows (coupons and principal) discounted at a suitable interest rate. One convention used to simplify the calculation procedure is to assume a single rate for all cash flows. This is the known as the yield-to-maturity.

The yield-to-maturity (YTM) is that yield which equates the present value of all the cash flows from a bond to the price of a bond. It is an iterative (trial and error) calculation that accounts for the reinvestment of the coupons as well as any capital gain or loss on the price of the bond (which will be redeemed by the issuer at par). Conversely, given the YTM, a price can be calculated. A rise in the YTM will cause the price calculated to decrease, while a fall in the YTM will cause the price to rise. Although it does simplify the calculations, this convention assumes that all the coupons from a bond can be reinvested at the same rate (which is unlikely). The actual return generated by a bond held until maturity depends on the future reinvestment rates at which the coupon payments received are invested.

Bonds changes in price which determine that the investing in bonds is not a secure investment, that is, investing in bonds does not return a fixed rate. The only exception is when the bond is kept to his maturity and pays no coupon. The coupon on fixed income investment reflects prevailing interest rates at time of issuance. When bonds are traded on the secondary market, their price reflects the level of their coupon compared to the current interest rate for a similar bond. Please, see Figure 1.

FIGURE 1

Years	CF	Rate	Price	Coupon	Total	CI
0.0		0.10	$ 1,000		$ 1,000	$ 1,000
1.0	100	0.10	$ 1,000	$ 100	$ 1,100	$ 1,100
2.0	100	0.10	$ 1,000	$ 210	$ 1,210	$ 1,210
3.0	100	0.10	$ 1,000	$ 331	$ 1,331	$ 1,331
4.0	1100	0.10	$ 1,000	$ 464	$ 1,464	$ 1,464

In brief, the determinants of bond price are volatility, namely a bond's term to maturity, initial required market yield, and coupon rate. Whereas the term to maturity is positively related to a bond's price volatility, the other two variables are inversely related to a bond's price volatility. The value of a bond will vary depending on the amount of the cash flows (coupon size), the timing of the cash flows (term to maturity), and the interest rate used for discounting. Duration helps to summarize these variables in a single number.

Duration is a measure of bond price volatility that combines the attributes of both a bond's term to maturity and coupon. Duration is a measure of the average (cash-weighted) term-to-maturity of a bond. There are two types of duration: Macaulay duration and modified duration. Macaulay duration is useful in immunization, where a portfolio of bonds is constructed to fund a known liability. Modified duration is an extension of Macaulay duration and is a useful measure of the sensitivity of a bond's price (the present value of its cash flows) to interest rate movements.

The concept of duration was first introduced by Macaulay (1938) while trying to define the correct measure of the life of a fixed income investment. Because term to maturity ignores the amount and timing of all but the final cash flow, Macaulay standardized the life of a fixed income investment by viewing each payment as a zero-coupon (pure discount) bond. This way, a bond's term to maturity and coupon payments are combined into a single measure of its life, which is its effective maturity (or weighted average maturity) of its cash flows on a present value (PV) basis. Please, see Figure 2

D = SUM [(FC t * t) / (1 + i)^t] / SUM [FC t / (1 + i)^t]

FIGURE 4.2

Years	CF	PV	t * PV	t*(t+1)*FC / (1+i)^(t+2)
1.0	100	90.91	90.91	150.26
2.0	100	82.64	165.29	409.81
3.0	100	75.13	225.39	745.11
4.0	100	68.30	273.21	1128.95
5.0	100	62.09	310.46	1539.47
6.0	1100	620.92	3725.53	21552.64
		1000.00	4790.79	25526.24
		Duration	4.7908	5329.48/1000.00
		D. Modified	4.3553	5.3295/(1+0.1/1)
		Convexity	5.3282	32464.16/5329.48

Macaulay's duration can be used to determine and compare the YTM-price volatility of normal bonds with different maturities and coupons. Several properties of Macaulay's duration can be distinguished from the preceding discussion. These properties show how duration is related to the three key determinants of bond price volatility, namely term to maturity, coupon rate, and YTM. These properties can be summarized as follows:

- Because coupon payments are given weight in calculating duration, Macaulay's duration of a coupon bond is always less than the bond's term to maturity.
- The duration of a zero-coupon bond is exactly equal to its term to maturity. This is because the single payment of the bond is made only when the bond matures.
- If maturity is held constant, a bond with a lower coupon rate has a higher duration than a bond with a higher coupon rate.
- Given a specific coupon rate, a bond with a longer maturity has a higher duration than a bond with a shorter term to maturity. Additionally, duration increases with maturity for bonds selling at par or at a premium. For bonds selling at high discounts, duration may decrease with increasing maturity. This is because bonds selling at a deep discount tend to increase in price as maturity approaches.
- Other things held constant, the duration of a coupon bond is higher if the bond's YTM is lower.

Despite having important uses in bond investments, duration and convexity measures have some serious limitations. First, the interest rate sensitivity of a bond portfolio can only be estimated if there is a change in interest rates that leads to a parallel shift in the yield curve (bonds of different maturities experiencing the same change in YTM). In practice however, changes in interest rates do not lead to an equal change in YTM across bonds of different maturities because the yield curve rarely experiences a parallel shift. Thus two bond portfolios that may have the same duration at the beginning of the investment horizon may end up being affected differently by interest rate changes one period later, depending on how the yield curve has shifted. This problem constrains immunization strategies because it cannot be reflected by duration and convexity measures. Another limitation of duration and convexity is that these measures cannot be used to estimate interest rate sensitivity of bonds with embedded options, such as callable bonds and convertibles. With such bonds, changes in YTM (arising from changes in interest rates) may affect not only the prices of the bonds; but the realization of the cash flows from the bonds as well. This is because the bonds may be called or converted as a result of changes in interest rates.

Modified duration is a measure of the price sensitivity of a bond to interest rate movements. Please, see Figure 2. It is calculated as shown below:

Modified Duration = Macaulay Duration /(1 + y/n), where y = yield to maturity and n = number of discounting periods in year (2 for semi - annual pay bonds)

Then, % Price Change = -1 * Modified Duration * Yield Change

For example, suppose a bond (6 years) was priced initially at par ($100), when the YTM was 10%, with a Macaulay Duration of 4.7908 years. The bond was repriced for an increase and decrease in rates of 5%. The Modified Duration for this bond will be: Dmod = -1 * 4.7908 / (1 + .05) = 5.03 years. Therefore, a change in the yield of +/- 5% should result in a % change in the price of the bond of: -/+ 5.03 * .05 = +/- 0.2515 (+/- 25.15%). Since the bond was initially priced at par, the estimated prices are: $125.6 at 5% and $80.8 at 15%. The actual prices were: $125.38 at 5% and $81.08 at 15%.

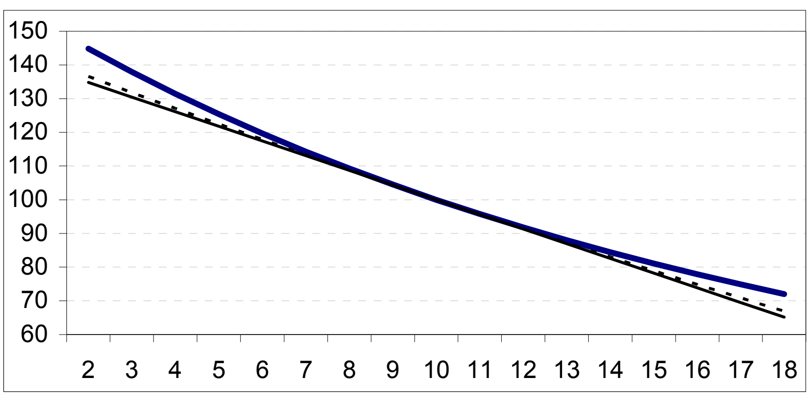
Modified duration indicates the percentage change in the price of a bond for a given change in yield. The percentage change applies to the price of the bond including accrued interest. Please, see Figure 3.

Modified duration became recognized as a better measurement of interest rate risk than Macaulay duration. But it still was not perfect. Most Treasury bonds and corporate bonds pay twice per year. If the semiannual yield was 10%, then the periodic yield is 5%, so the Macaulay duration should be divided by 1.05 to get the modified duration. But not all bonds pay semiannually. For example, Eurobonds pay yearly, and mortgage-backed securities pay monthly. The modified duration of a Eurobond showed its interest rate sensitivity to a change in the annualized rate, but a Treasury bond's modified duration showed its sensitivity to a change in the semiannual rate. This made it difficult to compare the price volatilities of bonds with different payment periods. One way around this problem was to do all of the calculations with semiannual yields.

Duration, in general, is thus a measure of sensitivity of the price of a bond (or other financial asset) to interest rate changes, and can therefore be used to measure interest rate exposure. However, Macaulay's and modified duration can provide fairly accurate estimates of changes in a bond's price only for small changes in YTM. As changes in YTM become larger, the estimation of a bond's price change using the duration measure becomes more inaccurate. This is because duration attempts to estimate a convex relationship with a straight line (the tangent to the convex function of bond price-YTM). To be more accurate, the estimation of a bond's price changes should properly reflect this convexity.

FIGURE 3

Rate	Price	P by D	Error %	P by DC	Error %
2	144.81	134.84	-6.9	136.56	-5.70
3	137.92	130.49	-5.4	131.81	-4.43
4	131.45	126.13	-4.0	127.10	-3.31
5	125.38	121.78	-2.9	122.45	-2.34
6	119.67	117.42	-1.9	117.85	-1.52
7	114.30	113.07	-1.1	113.31	-0.87
8	109.25	108.71	-0.5	108.82	-0.39
9	104.49	104.36	-0.1	104.38	-0.10
10	100.00	100.00	0.0	100.00	0.00
11	95.77	95.64	-0.1	95.67	-0.10
12	91.78	91.29	-0.5	91.40	-0.41
13	88.01	86.93	-1.2	87.18	-0.94
14	84.45	82.58	-2.2	83.01	-1.70
15	81.08	78.22	-3.5	78.90	-2.69
16	77.89	73.87	-5.2	74.84	-3.92
17	74.88	69.51	-7.2	70.83	-5.40
18	72.02	65.16	-9.5	66.88	-7.14



The discrepancy between the estimated change in the bond price and the actual change is due to the convexity of the bond, which must be included in the price change calculation when the yield change is large. However, modified duration is still a good indication of the potential price volatility of a bond.

The previous percentage price change calculation was strictly inaccurate because it failed to account for the convexity of the bond. Convexity is a measure of the amount of whip in the bond's price yield curve and is so named because of the convex shape of the curve. Please, see Figure 2.

C = SUM [(t * (t+1) * FC / (1+i) ^ (t+2)] / P

As with duration, deriving convexity involves the weighting of cash flows (coupons and par value) by a time factor t*(t+1)). After that, the resulting number is divided by the bond's current price and then standardized by multiplying it with a constant of one-half.

Because of the shape of the price yield curve, for a given change in yield down or up, the gain in price for a drop in yield will be greater than the fall in price due to an equal rise in yields. This slight "upside capture, downside protection" is what convexity accounts for. Mathematically DM is the first derivative of price with respect to yield and convexity is the second (or convexity is the first derivative of modified duration) derivative of price with respect to yield. An easier way to think of it is that convexity is the rate of change of duration with yield, and accounts for the fact that as the yield decreases, the slope of the price - yield curve, and duration, will increase. Similarly, as the yield increases, the slope of the curve will decrease, as will the duration. By using convexity in the yield change calculation, a much closer approximation is achieved (an exact calculation would require many more terms and is not useful). Please, see Figure 3.

Using convexity (C) and DM then:

% Price Chg. = -1 * DM * Yield Chg. + C/2 * Yield Chg * Yield Chg.

To improve on accuracy, duration has to be augmented with an additional measure in order to capture the curvature or convexity of a bond's price-YTM relationship. Convexity here denotes the degree to which duration (the slope of the tangent line) changes as YTM changes.

In order to accurately estimate a bond's price change for large changes in YTM, both the first derivative (duration-related) and second derivative (convexity-related) of the price-YTM curve must be considered. Convexity multiplied by the change in YTM squared, is added to the right hand side of the basic duration-YTM sensitivity formula. The added term captures the error in estimating the bond's price change using duration alone, and the approximate percentage price change is estimated.

The first term on the right hand side of equation is linear and represents the slope of the tangent line and thus provides the first-order effects of a change in YTM. The second term is quadratic and convexity-related.

2 CALLABLE BONDS AND ITS VALUATION

Normal bonds are, in general, currently an exception, given that now the majority of these bonds include clauses of diverse type that permit to the issuers to alter the cash flows to pay, under certain circumstances. Among them, there are the bonds that have inserted the option of prepaid (what clearly is related to the level of the rates of interest).

The most common option is an option of early prepaid that permits to the issuer to pay the bonds before the expiration and to replace them with other that imply lower interests.

Thus, a callable bond (or redeemable bond) is a bond in which the issuer has the right to redeem prior to its maturity date, under certain conditions. When issued, the bond will explain when it can be redeemed and what the price will be. In most cases, the price will be slightly above the par value for the bond and will increase the earlier the bond is called. A company will often call a bond if it is paying a higher coupon than the current market interest rates. Basically, the company can reissue the same bonds at a lower interest rate, saving them some amount on all the coupon payments; this process is called refunding. Unfortunately, these are also the same circumstances in which the bonds have the highest price; interest rates have decreased since the bonds were issued, increasing the price. In many cases, the company will have the right to call the bonds at a lower price than the market price. If a bond is called, the bondholder will be notified and have no choice in the matter. The bond will stop paying interest shortly after the bond is called, so there is no reason to hold on to it. Generally, callable bonds will carry something called call protection. This means that there is some period of time during which the bond cannot be called.

When an issuer calls its bonds, it pays investors the call price (usually the face value of the bonds) together with accrued interest to date and, at that point, stops making interest payments. Callable bonds are more risky for investors than non-callable bonds because an investor whose bond has been called is often faced with reinvesting the money at a lower, less attractive rate. As a result, callable bonds often have a higher annual return to compensate for the risk that the bonds might be called early.

There are numerous reasons because an issuer desires to prepay their debt prematurely (some volunteers, other obligatory) but the main one is that that declining interest rates do economic sense for the issuer to substitute new and cheaper debt by old debt with higher rates of interest. Other reasons include the desire of the issuer to eliminate restrictive clauses or to change the structure of financing of the business.

From the point of view of the investor, clearly this prepaid is damaging because the investor loses his investment to high rate of interest and he has to reinvest the money to lower interest rates because his investment horizon is still in place. This is the reason because callable bonds should offer higher interest rate paying a prize with respect to a similar non callable bond.

There are three primary types of call features, including:

1. Optional Redemption. Allows the issuer, at its option, to redeem the bonds. Many municipal bonds, for example, have optional call features that issuers may exercise after a period of years, often ten years.

2. Sinking Fund Redemption. Requires the issuer to regularly set aside money for the redemption of the bonds before maturity.

3. Extraordinary Redemption. Allows the issuer to call its bonds before maturity if certain specified events occur, such as the project for which the bond was issued to finance has been damaged or destroyed.

Therefore, there is a need to value callable bonds modifying duration formulas in order to include such characteristics.

In this paper it will be examined those bonds for which the issuer has the option to manipulate the cash flows of the bond in order to maximize its benefit when the interest rates change, what results in a direct impact in the return that the investor will obtain.

Traditionally, before the 1970s, investors made little attempt to adjust their analysis of fixed income instruments to recognize the effects of embedded options. There were several reasons for this:

1. The structuring of fixed income instruments was more uniform than it is today. For example, most corporate bonds were callable. While specific call provisions might vary, it was rare that an investor would have to compare a callable bond to a non-callable bond.

2. Interest rates were less volatile, so embedded options were worth less.

3. A consistent theory for valuing options had not yet been developed.

However, starting in the 1970's, interest rates became more volatile. This was soon accompanied by a proliferation of new types of fixed income instruments. Many of these entailed complex embedded options. Particularly, since the mid 1980s, a lot of corporate bonds were issued with call features. These features allowed the companies that issued these bonds to call them back at a set price if it was to their advantage. With interest rates began to rising in the late 1970s and early 1980s, very few people paid attention to these calls features. It made little sense for a company to refinance a bond with a 9% coupon rate if the current rate had risen to 14%. But, by the mid 1980s, interest rates started to drop, and companies began refinancing their bonds at lower rates. Investors who thought that they had a long term investment at a high interest rate suddenly found themselves having to reinvest their money at lower rates.

Several investment banks started pricing these bonds differently. Instead of using a yield to maturity, they would calculate the price of the bond assuming that it would be called at the worst possible time. The call date which gave the lowest yield was called the worst call date. They calculated the duration, assuming this worst call date. This was a better estimate of interest rate sensitivity for these bonds. While better, it still turned out to be unsatisfactory. If a callable bond were trading near par, then a small drop in the yield might mean that it would be called in six months, so it would only have a small increase in value. On the other hand, a small increase in interest rates might indicate that it will trade until maturity, so it could have a large drop in value. Some investment banks started looking at one-sided durations, or did what-if analyses, looking at what happens to the bonds under a few interest rate scenarios.

Today, with the wide acceptance of the Black and Scholes Model, it is common for fixed income investors to employ option pricing techniques to value embedded options and adjust their analysis of instruments accordingly. Fixed income instruments are compared, no longer according to their absolute yields, but according to their option-adjusted spreads (OAS).

A callable bond can be seen as a long position in a non callable bond plus a short position in an option call on the underlying bond.

The option call is issued by the investor to the issuer of the bond on the contractual cash flows of the non callable bond that occur after the first date of prepayment. Thus, the option call on the prepayment gives to the holder the right to buy back the bond to a specified price on a determined period.

In the early 1980s, practitioners came to recognize that an option-bearing bond should be viewed as a package of cash flows (i.e., a package of zero-coupon instruments) plus a package of options on those cash flows. For example, a callable bond can be viewed as a package of cash flows plus a package of call options on those cash flows. As such, the position of an investor in a callable bond can be viewed as:

Long a Callable Bond = Long an Option-Free Bond + Short a Call Option on the Bond.

In terms of the value of a callable bond, this means:

Value of Callable Bond = Value of an Option-Free Bond - Value of a Call Option on the Bond.

But this also means that:

Value of an Option-Free Bond = Value of Callable Bond + Value of a Call Option on the Bond.

An early procedure to determine the fairness of a callable bond's market price was to isolate the implied value of its underlying option-free bond by adding an estimate of the embedded call option's value to the bond's market price. The former value could be estimated by applying option pricing theory as applied to interest rate options.

The price of a callable bond is the difference among the price of the non callable bond and the prize of the call option. Thus, the price of a callable bond is calculated as the price of an equivalent not callable bond of similar structure minus the value of the option call semidetached. The value of the option call is reduced because the investor is implicitly selling the option to the issuer of the bond.

A not callable bond is more expensive that a callable bond because the investor values the possibility of to maintain the bond to the expiration without fearing an early prepayment (and a rate of unknown interest for reinvestment) in a scenario of low interest rates. In terms of performance, a not callable bond trades to a lower return in comparison to a callable bond of similar expiration. Investors are expecting to grant some points of return in order to have the protection against a premature repayment of the bonds.

In practice, the option call limits the appreciation of the price of the bond in a bull market. But in a bear market, the option call limits the depreciations of the price of the bond given that the probability of early buyback diminishes.

To the extent that the market yield changes, the value of the not callable component as well as the value of the option call component change:

- When the return of market rises (and the refinancing comes to be less attractive), the value of the not callable component of the bond falls.
- When the return of market diminishes (and the refinancing comes to be more attractive), the value of the not callable component of the bond rises.

In sum, traditionally before the 1980s, investors ignored the characteristic of prepayment (callable feature) of the bonds with this particularity, mainly due the difficulty to value it. Nevertheless, the actual volatility present in the markets does fundamental to have a reference to the issue.

3 THE SALOMON APPROACH

In the mid 1980s, Salomon Brothers developed an option pricing model for callable bonds.

At the heart of the option pricing model is an interest rate process. Instead of looking at a few interest rate scenarios, they looked at thousands of possible interest rate paths through time. These paths are derived from the Treasury curve. For each of these interest rate scenarios, they could predict the probable set of cash flows, and could calculate a present value for those flows. By assigning a probability to each of these paths, an expected present value of the bond can be calculated.

To value such a bond, one must consider the volatility of interest rates, as their volatility will affect the possibility of the call option being exercised. One can do so by constructing a binomial interest rate tree that models the random evolution of future interest rates. The volatility-dependent one-period forward rates produced by this tree can be used to discount the cash flows of any bond in order to arrive at a bond value.

Given the values of bonds with and without an embedded option, one can obtain the value of the embedded option itself. The procedure can be used to value multiple or interrelated embedded options, as well as stand-alone risk control instruments such as swaps, swaptions, caps and floors.

The interest rate paths represent short term Treasury rates through time. If one discounts all of the cash flows by these Treasury rates, and then averages the present values using the probability weights, one will get a present value for the bond. This is the price that someone should pay for this bond if it had the same credit risk as Treasury bonds. But corporate bonds are riskier than Treasury bonds, so investors expect a higher return. If one knew how much higher the return should be, one could just add that value to each of the interest rates in each path. The value that one should add onto the rates is called the option-adjusted spread (OAS). By discounting all of the cash flows by these new rates, one can find the present value of the bond. If the price of the bond is already known, then the process can be reversed to solve for the OAS.

Option-adjusted spread (OAS) is the yield spread of a fixed income instrument that is not attributable to embedded options. For example, a callable bond might be trading at a spread to Treasuries of 200 basis points. Of that, 60 basis points might be attributable to the bond's call feature, with the remaining 140 basis points attributable to such factors as the bond's credit risk, liquidity, etc. For this bond, the option-adjusted spread is 140 basis points.

One of the challenges investors face in comparing the attractiveness of one fixed income investment versus another is the effects of embedded options. For example, if two investments are being compared, one of which is callable and the other of which is not, it is not meaningful to compare their respective yields. The callable instrument's yield will be inflated to compensate investors for the short call option embedded in the instrument. For this reason, yield is a poor indication of the expected return from such an instrument.

Option-adjusted spreads (OAS) were first widely employed in the mortgage-backed securities market. There, investors were offered instruments with extraordinary yields, 500 or 600 basis points over comparable but non-callable Treasuries. To analyze those large spreads, investors needed to somehow subtract out the yield that was attributable to the embedded options. They wanted to know what the yield over Treasuries would be if the exact same instruments did not have embedded options.

The value of option-adjusted spread (OAS) analysis is that it enables investors to judge the degree to which they are being compensated for a particular instrument's credit risk, illiquidity or other such factors. For example, an investor might be comparing two similar bonds. If the bonds had comparable maturity, credit risk and liquidity, the investor might purchase whichever bond had the higher option-adjusted spread, that bond would offer higher compensation for the risks being taken.

Option-adjusted spreads (OAS) are calculated using option pricing models. These models determine the component of an instrument's yield that is attributable to embedded options. That yield is subtracted out to find an option-adjusted yield. When the Treasury yield (or other benchmark yield) for the corresponding maturity is also subtracted out, the result is the option-adjusted spread (OAS).

In the fixed income markets, option pricing is often more an art than a science. This is especially true in the mortgage-backed securities market where broad assumptions must be made about the efficiency with which homeowners will exercise their options to prepay. For this reason, option-adjusted spread (OAS) is not a well defined notion. The option-adjusted spread (OAS) that an investor calculates for an instrument will depend, not only upon the characteristics of that particular instrument, but also upon the assumptions the investor incorporates into his option pricing model. Accordingly, instruments are rarely quoted according to option adjusted spreads (OAS). Sometimes, illustrative option-adjusted spreads (OAS) will be quoted, but it is really up to each investor to analyze option-adjusted spreads (OAS) in a consistent manner for themselves.

Today, option-adjusted spreads (OAS) are calculated for a variety of instruments, including mortgage-backed securities, structured notes and callable corporate bonds. In the United States, spreads are often calculated relative to Treasury yields, but they can be calculated relative to any reasonable yield benchmark.

With these tools, the options pricing model (often called an OAS model) can be used to calculate the true interest rate sensitivity of the bonds. If an investor had a way of estimating the price of the bond for a small change in interest rates, then he could estimate the derivative of the price/yield function by setting up a difference equation. If this value is then divided by (-Price), the true duration can be estimated. For bonds with imbedded options, this value will frequently differ from the modified duration. This new type of duration is sometimes called option-adjusted duration, but is also known as **effective duration**. Note that the shift of the yield curve is a parallel shift of the semiannual yield. If the bond had fixed cash flows, then the effective duration of a bond with fixed cash flows is just the Macaulay duration divided by (1+i/2). For bonds which pay semiannually, this is just the modified duration.

The formula for effective duration is:

Effective duration = - (bond's price when the yield curve is shifted up – bond's price when the yield curve is shifted down) / (2 * amount that the yield curve is shifted * bond's price)

If the bond's price is known, then bond's price when the yield curve is shifted up and bond's price when the yield curve is shifted down can be calculated by doing the following:

Use the Treasury curve to generate the interest rate paths.

1. Find the OAS for the bond.
2. Shift the Treasury curve up by amount that the yield curve is shifted and generate new interest rate paths.
3. Calculate a present value using the calculated OAS. This is bond's price when the yield curve is shifted up.
4. Shift the Treasury curve down by amount that the yield curve is shifted and generate new interest rate paths.
5. Calculate a present value using the calculated OAS. This is bond's price when the yield curve is shifted down"

4 THE SALOMON BROTHERS TERM STRUCTURE - BASED OPTION PRICING MODEL

There are several problems with valuation models. In order to adequately value a callable bond and consequently, the value of the associated option to the bond, the fluctuations of the price of this bond (to the extent that the interest rates change) should be taken into account.

- For short time options, that is, for options in which the time to expiration of the option is a lot shorter than the time to expiration of the bond, the Black-Scholes model can provide a reasonable approximation to the value of those options.
- For long-range options, that is, for options in which the time to expiration of the option itself it is close to the time to expiration of the bond, the Black-Scholes model is not the adequate one to value these options because this model assumes:

    - Constant interest rates, which in reality is clearly inadequate.
    - Lognormal price distributions, which is adequate for stocks because allows to the price to fall close to zero or to rise infinitely in the time. Nevertheless, the behavior of bonds is very different considering that they have a known payment to expiration.
    - Constant prices volatility in the period of analysis, which for bonds is inadequate because their price tends to have high volatility according to the respective volatility of the interest rate, but this volatility, tends to decrease to the extent that the expiration of the bond approaches.

Then, instead of modeling the evolution of the price directly, is preferably to model the evolution of the interest rates structure through the time and then to compute the price of the bonds based on that evolution. Nevertheless, the evolution process of the interest rate that generates the future return rates should be determine in a free and consistent way of arbitration. The model described in this paper, which is utilized to value callable assets, is developed inside a scheme of a free of arbitration binomial tree of interest rates.

Once these problems are resolved, the goals of the modeling process can be confronted.

The first goal is market realism. The movements of the resultant interest rate of the model should resemble those observed in the market. That is, the interest rates should fluctuate through the spectrum of expirations and the yield curve should be able to have flat, positive or negative slope. Consequently, the interest rate process should not rigidly maintain fixed the short-time interest rate while leaves the long-range interest rate varying. Neither, the interest rate evolution should have mere parallels changes respect of the initial yield curve.

The second goal is that conditions of arbitration of the yield curve should not exist. A consistent model of an interest rates structure should not permit some possibility of arbitration among not callable bonds inside a certain bonds risk class. In other words, it should not be permitted to a particular bond always to surpass in return to another bond. If this was possible, an investor would be able to establish a long position in the bond with higher return, financed with a short position in the bond of lower return, with which he would guarantee a risk free profit. Thus, the dynamics of the implied yield curve should be restricted to avoid possibilities of arbitration. If a particular yield curve is imposed in the tree (for example, parallel changes in the yield curve over the initial yield curve), it would arise possibilities of arbitration among bonds of different expirations.

The third goal is the achievement of the Put-Call parity for European interest rates options. An important characteristic of an option appraisal model is that the put and call values for European options should be consistent, that is, the put-call parity should be satisfied. This parity establishes that a position in the underlying assets plus an option put on the underlying assets is equivalent to an option call (same price of exercise and same expiration) on the underlying assets plus a bond term with expiration in the date of exercise of the option (with value of redemption equal to the exercise price and with the same coupon that the underlying assets). This relation should be valid, independent of the model used to reckon the volatility of the yield. The model described in this paper satisfies the put-call parity for options in all dates of expiration.

Now, the model can be presented and explained.

The model generates the temporary structure of interest rates through a binomial process. Using a binomial tree, a distribution of log-normal probabilities of the future paths for the Treasury notes interest rate is created using the volatility of the specified interest rate. Each node of the tree represents a specific point of the time and a specific transformation of the entire temporary structure (this is, the yields to expiration of all the zero-coupon bonds). Some nodes represent the curve of expirations being elevated while others represent it diminishing. Relating the present prices of those Treasury notes, which normally are considered to define the Treasure's yield curve, to its expected forward prices, is possible to spread the short time interest rates in a form that the entire temporary structure is described in a consistent way. This construction of the tree avoids the difficulties of arbitration mentioned further up. Each node in the tree is labeled by the interest rate for the period of a year that begins in that node. In each node, the prices of the long-range instruments that comprise the temporary structure are a function of the one year spot rates of the Treasury note in that node and the future distribution of the one year spot rates of the Treasury notes.

Due to the functional relation between the short and long range interest rates, the model has the property that the volatility of the long-range interest rates is smaller than the volatility of the short range interest rate. To value a bond in any node of the tree, each cash flow in the future is discounted backward the node corresponding to the specific road of the short range interest rates and averaged on all the possible roads. This is equivalent to sequentially average and to discount the prices of the bonds to each adjacent pair of nodes to determine the price of the bonds in the prior node. The tree is built in such a way that the process of averaging and discounting is based on establishing a hedge of zero risk in each node. When this calculation is done for the entire tree, none bond portfolio will dominate to any another bond portfolio inside a portion of the tree. This means that the arbitration of the yield curve is not possible inside the temporary structure. Once the tree has been built in this consistent way, is possible to determine the value of any dependent flow of the interest rate. This method is consistent with recent academic literature on the modeling of the yield curve structure and the option valuation.

In the example that will be presented next, the model is consistent with the yield curve of present Treasury notes in the departure node, due to that the bonds on the yield curve, when they are valued in the way before mentioned, will be valued to their present market prices. The evolution of the interest rates structure is modeled in this way in order to build a consistent option valuation model.

Before of the example, the model should be finished to be explained.

The valuation of any bond, except the Treasury notes that calibrate the tree, requires an adjustment to the short time interest rates to reflect considerations of risk and liquidity. To value a bond correctly, the interest rates in the node for all the tree are moved whether up or down until the price of the bond of the model equalize the market price. The number of basis points that is added or reduced is known as the effective spread. This effective spread (for not callable bonds) is similar to more common yield spreads (prize by risk) that measures the difference between the yield to expiration of the assets and a Treasury note with similar expiration. The conventional yield spread is a constant added to the yield to expiration of the Treasury note, which is a rate used to discount all the future cash flows. In the tree, the effective spread is added to the future interest rates of the Treasury notes. Nevertheless, as with the conventional yield spread to expiration, the effective spread of the short-time bonds tend to be lower than that of those long-range bonds of the same quality of credit.

The model is used as the base on which the callable bonds are valued.

A callable bond is valued in a similar way to the previously described for the not callable bonds. In each node (point of evaluation), the value of the not callable bond is calculated. The value of the option call is calculated using the method of identical discount for the not callable bonds. The method used to discount the option is based on establish a risk free hedge in each node. The risk free hedge implies to replicate the payment of the option with a portfolio containing the underlying not callable bond and a bond of a period. Due to that the tree is built in a way free of arbitration, the put-call parity is maintained in each node for the European options of all dates of expiration. When the call can be exercised, its value is not permitted to fall under the difference among the value of the not callable bond and the price of exercise of the call. This reflects the American nature of the option. The process of option valuation begins from the expiration of the option and works backwards to the departure node. The value of the callable bond in the node of departure (and in all the nodes) is the difference among the price of the not callable bond and the value of the option call.

Dependent instruments of the interest rate that are more complicated than a refunding option can also be evaluated using the general method discussed in this report. The rule of evaluation in each node in the tree depends on the specific characteristics of the implicit option in the bond. The spread used in the tree is changed until the price of the model of the callable bond matches with its market price. The spread simultaneously is used to value the option call and the underlying not callable bond. The spread that correctly values the callable bond in the tree is known as the effective or adjusted option spread (OAS). When the bond is not callable, the term will be utilized only as effective spread.

In an important way, the adjusted option spread (OAS) for callable bonds can be quite different of the yield to expiration spread versus a benchmark Treasure. This is because the option call does the yield to expiration spread larger, while the adjusted option spread specifically removes the component of the yield to expiration spread that is due to the option call. The effective spread permits comparisons among callable bonds with different option calls and among not callable bonds. This provides the investor with a net advantage in basis points of a callable bond or not on a not callable Treasury note.

The valuation of American options differs of the Europeans in a crucial point in the sense that is necessary to incorporate the possibility of early exercise. To understand as this works it is useful to define two values for an option in each node:

- The value of maintenance is the value of the option assuming that the option is not exercised in this node, but exercised optimally from there on.
- The value of exercise is the value of the option in this node assuming that the option is exercised immediately.

Clearly, the value of the option is the higher between the value of maintenance and the value of exercise. The value of maintenance is simply the discounted average of the values in the two nodes in the next level. For option calls, the value of exercise is the excess, if there is it, of the price of the bond on the price of exercise. If the price of exercise is higher, the holder should exercise the option; if the value of maintenance is higher, the option should not be exercised.

The value of the effective spread or adjusted option spread (OAS) is the key for the valuation of the Treasure or corporate bonds. The construction of the tree guarantees that the Treasury note on the yield curve valued by the model to any specified volatility will produce an effective spread of zero basis points. This simply indicates that this bond is correctly valued. If a Treasury note is valued for the model, then the adjusted option spread (OAS) would be zero basis points if this correctly is valued for the market. An adjusted option spread (OAS) of zero basis points means that the not callable bond underlying to the callable bond needs to be discounted to exactly the future short time rates of the Treasure in the tree so that the model values the callable bond (counting for the value of the option call) equal to the market price of callable bond. Nevertheless, the callable bond of the Treasure will have a traditional yield to expiration spread on a similar not callable Treasury notes. This is due to that the option call diminishes the value of the bond.

To illustrate the concept of effective spread, in the following example two Treasury notes with identical expirations and coupon rates are consider, one callable and another not callable:

- If the option call implicit in the callable bond was adequate valued, both bonds would have an effective spread of zero basis points.
- If the adjusted option spread (OAS) of the callable bond is different from this effective spread of zero basis points benchmark, the investor can make value judgments for this callable Treasury note:
    - If the model produces a positive adjusted option spread (OAS), the Treasury callable note is cheap.
    - If this produces a negative adjusted option spread, the callable Treasury note is rich.

The effective spread for not callable corporate bonds is normally positive, reflecting the risk of additional credit and considerations of inherent liquidity in the assets which are not issued by the Treasure. The adjusted option spread (OAS) of a callable corporate bond should be compared with the effective spread of a not callable bond of identical coupon with the same sensibility to the interest rate of the callable bond. The maturity date of this not callable bond would precede the final date of expiration of the callable bond. If the adjusted option spread (OAS) is higher than the effective spread of this not callable benchmark, the callable bond corporate would be cheap. If the adjusted option spread (OAS) is lower that the effective spread of this not callable benchmark, the callable bond corporate would be rich.

The effective spread of a not callable corporate bond will be similar to its spread performance to expiration on the Treasure benchmark.

Given that effective spread is calculated using a binomial tree and that the spread time to expiration not, the two spreads will be generally numerical different, in spite of the fact that nearby. The difference reflects the fact that the spread is added to the future spot rate of the short time Treasure, more than to the yields to expiration itself. Again, as is true for the callable Treasury notes, the adjusted option spread (OAS) for a callable corporate bond will be less than its yield to expiration spread.

The model can also be used to resolve for implied yield volatility, assuming a particular adjusted option spread (OAS). In this application, the underlying not callable bond can be seen as trading to its correct value. The model then can be used to resolve for the yield volatility that would value the callable bond to its present market price. To a particular market price, elevating the selected yield volatility has the effect of diminishing the adjusted option spread (OAS) and vice versa.

5 AN EXAMPLE OF THE MODEL

VALUING A NOT CALLABLE BOND

Risk free rate / Initial period	5.00	
Risk premium / Effective spread	100.00	basis points
Volatility %	10.00	1.11
Interest rates: Up & down probability	0.50	
Price formula in each node	[Prob up * Price up + Prob down * Price down] / (1+interest)	
Bond price	100.00	
Cash flow	10.00	

YEARS RATES

0						6.00					
1					5.43		6.63				
2				4.91		6.00		7.33			
3			4.44		5.43		6.63		8.10		
4		4.02		4.91		6.00		7.33		8.95	

CAPITAL

0						74.56					
1					80.84		77.24				
2				86.55		83.90		80.81			
3			91.65		89.95		87.93		85.55		
4		96.13		95.32		94.34		93.17		91.78	
5	100.00		100.00		100.00		100.00		100.00		100.00

CASH FLOWS

0						42.09					
1					45.09		44.14				
2				37.27		36.72		36.08			
3			28.74		28.48		28.17		27.81		
4		19.61		19.53		19.43		19.32		19.18	
5	10.00		10.00		10.00		10.00		10.00		10.00

NOT CALLABLE BOND PRICE

0						116.65					
1					125.93		121.37				
2				123.82		120.62		116.89			
3			120.39		118.43		116.10		113.36		
4		115.75		114.85		113.77		112.49		110.96	
5	110.00		110.00		110.00		110.00		110.00		110.00

VALUING A CALLABLE BOND

Risk free rate / Initial period	5.00	
Risk premium / Effective spread	100.00	basis points
Volatility %	10.00	1.11
Interest rates: Up & down probability	0.50	
Price formula in each node	[Prob up * Price up + Prob down * Price down] / (1+interest)	
Bond price	90.00	
Cash flow	10.00	

YEARS RATES

0						6.00					
1					5.43		6.63				
2				4.91		6.00		7.33			
3			4.44		5.43		6.63		8.10		
4		4.02		4.91		6.00		7.33		8.95	

CAPITAL

0						74.56					
1					80.84		77.24				
2				86.55		83.90		80.81			
3			91.65		89.95		87.93		85.55		
4		96.13		95.32		94.34		93.17		91.78	
5	100.00		100.00		100.00		100.00		100.00		100.00

CASH FLOWS

0						42.09					
1					45.09		44.14				
2				37.27		36.72		36.08			
3			28.74		28.48		28.17		27.81		
4		19.61		19.53		19.43		19.32		19.18	
5	10.00		10.00		10.00		10.00		10.00		10.00

NOT CALLABLE BOND PRICE

0						116.65					
1					125.93		121.37				
2				123.82		120.62		116.89			
3			120.39		118.43		116.10		113.36		
4		115.75		114.85		113.77		112.49		110.96	
5	110.00		110.00		110.00		110.00		110.00		110.00

CALL PRICE

0						26.65					
1					35.93		31.37				
2				33.82		30.62		26.89			
3			30.39		28.43		26.10		23.36		
4		25.75		24.85		23.77		22.49		20.96	
5	0.00		0.00		0.00		0.00		0.00		0.00

CALLABLE BOND PRICE

0						90.00					
1					90.00		90.00				
2				90.00		90.00		90.00			
3			90.00		90.00		90.00		90.00		
4		90.00		90.00		90.00		90.00		90.00	
5	110.00		110.00		110.00		110.00		110.00		110.00

VALUING A CALLABLE BOND

Risk free rate / Initial period	5.00	
Risk premium / Effective spread	100.00	basis points
Volatility %	10.00	1.11
Interest rates: Up & down probability	0.50	
Price formula in each node	[Prob up * Price up + Prob down * Price down] / (1+interest)	
Bond price	95.00	
Cash flow	10.00	

YEARS RATES

0						6.00					
1					5.43		6.63				
2				4.91		6.00		7.33			
3			4.44		5.43		6.63		8.10		
4		4.02		4.91		6.00		7.33		8.95	

CAPITAL

0						74.56					
1					80.84		77.24				
2				86.55		83.90		80.81			
3			91.65		89.95		87.93		85.55		
4		96.13		95.32		94.34		93.17		91.78	
5	100.00		100.00		100.00		100.00		100.00		100.00

CASH FLOWS

0						42.09					
1					45.09		44.14				
2				37.27		36.72		36.08			
3			28.74		28.48		28.17		27.81		
4		19.61		19.53		19.43		19.32		19.18	
5	10.00		10.00		10.00		10.00		10.00		10.00

NOT CALLABLE BOND PRICE

0						116.65					
1					125.93		121.37				
2				123.82		120.62		116.89			
3			120.39		118.43		116.10		113.36		
4		115.75		114.85		113.77		112.49		110.96	
5	110.00		110.00		110.00		110.00		110.00		110.00

CALL PRICE

0						21.65					
1					30.93		26.37				
2				28.82		25.62		21.89			
3			25.39		23.43		21.10		18.36		
4		20.75		19.85		18.77		17.49		15.96	
5	0.00		0.00		0.00		0.00		0.00		0.00

CALLABLE BOND PRICE

0						95.00					
1					95.00		95.00				
2				95.00		95.00		95.00			
3			95.00		95.00		95.00		95.00		
4		95.00		95.00		95.00		95.00		95.00	
5	110.00		110.00		110.00		110.00		110.00		110.00

VALUING A CALLABLE BOND

Risk free rate / Initial period	5.00	
Risk premium / Effective spread	100.00	basis points
Volatility %	10.00	1.11
Interest rates: Up & down probability	0.50	
Price formula in each node	[Prob up * Price up + Prob down * Price down] / (1+interest)	
Bond price	100.00	
Cash flow	10.00	

YEARS RATES

0						6.00					
1					5.43		6.63				
2				4.91		6.00		7.33			
3			4.44		5.43		6.63		8.10		
4		4.02		4.91		6.00		7.33		8.95	

CAPITAL

0						74.56					
1					80.84		77.24				
2				86.55		83.90		80.81			
3			91.65		89.95		87.93		85.55		
4		96.13		95.32		94.34		93.17		91.78	
5	100.00		100.00		100.00		100.00		100.00		100.00

CASH FLOWS

0						42.09					
1					45.09		44.14				
2				37.27		36.72		36.08			
3			28.74		28.48		28.17		27.81		
4		19.61		19.53		19.43		19.32		19.18	
5	10.00		10.00		10.00		10.00		10.00		10.00

NOT CALLABLE BOND PRICE

0						116.65					
1					125.93		121.37				
2				123.82		120.62		116.89			
3			120.39		118.43		116.10		113.36		
4		115.75		114.85		113.77		112.49		110.96	
5	110.00		110.00		110.00		110.00		110.00		110.00

CALL PRICE

0						16.65					
1					25.93		21.37				
2				23.82		20.62		16.89			
3			20.39		18.43		16.10		13.36		
4		15.75		14.85		13.77		12.49		10.96	
5	0.00		0.00		0.00		0.00		0.00		0.00

CALLABLE BOND PRICE

0						100.00					
1					100.00		100.00				
2				100.00		100.00		100.00			
3			100.00		100.00		100.00		100.00		
4		100.00		100.00		100.00		100.00		100.00	
5	110.00		110.00		110.00		110.00		110.00		110.00

VALUING A CALLABLE BOND

Risk free rate / Initial period	5.00	
Risk premium / Effective spread	100.00	basis points
Volatility %	10.00	1.11
Interest rates: Up & down probability	0.50	
Price formula in each node	[Prob up * Price up + Prob down * Price down] / (1+interest)	
Bond price	105.00	
Cash flow	10.00	

YEARS RATES

0						6.00					
1					5.43		6.63				
2				4.91		6.00		7.33			
3			4.44		5.43		6.63		8.10		
4		4.02		4.91		6.00		7.33		8.95	

CAPITAL

0						74.56					
1					80.84		77.24				
2				86.55		83.90		80.81			
3			91.65		89.95		87.93		85.55		
4		96.13		95.32		94.34		93.17		91.78	
5	100.00		100.00		100.00		100.00		100.00		100.00

CASH FLOWS

0						42.09					
1					45.09		44.14				
2				37.27		36.72		36.08			
3			28.74		28.48		28.17		27.81		
4		19.61		19.53		19.43		19.32		19.18	
5	10.00		10.00		10.00		10.00		10.00		10.00

NOT CALLABLE BOND PRICE

0						116.65					
1					125.93		121.37				
2				123.82		120.62		116.89			
3			120.39		118.43		116.10		113.36		
4		115.75		114.85		113.77		112.49		110.96	
5	110.00		110.00		110.00		110.00		110.00		110.00

CALL PRICE

0						11.65					
1					20.93		16.37				
2				18.82		15.62		11.89			
3			15.39		13.43		11.10		8.36		
4		10.75		9.85		8.77		7.49		5.96	
5	0.00		0.00		0.00		0.00		0.00		0.00

CALLABLE BOND PRICE

0						105.00					
1					105.00		105.00				
2				105.00		105.00		105.00			
3			105.00		105.00		105.00		105.00		
4		105.00		105.00		105.00		105.00		105.00	
5	110.00		110.00		110.00		110.00		110.00		110.00

VALUING A CALLABLE BOND

Risk free rate / Initial period	5.00	
Risk premium / Effective spread	100.00	basis points
Volatility %	10.00	1.11
Interest rates: Up & down probability	0.50	
Price formula in each node	[Prob up * Price up + Prob down * Price down] / (1+interest)	
Bond price	110.00	
Cash flow	10.00	

YEARS RATES

0						6.00					
1					5.43		6.63				
2				4.91		6.00		7.33			
3			4.44		5.43		6.63		8.10		
4		4.02		4.91		6.00		7.33		8.95	

CAPITAL

0						74.56					
1					80.84		77.24				
2				86.55		83.90		80.81			
3			91.65		89.95		87.93		85.55		
4		96.13		95.32		94.34		93.17		91.78	
5	100.00		100.00		100.00		100.00		100.00		100.00

CASH FLOWS

0						42.09					
1					45.09		44.14				
2				37.27		36.72		36.08			
3			28.74		28.48		28.17		27.81		
4		19.61		19.53		19.43		19.32		19.18	
5	10.00		10.00		10.00		10.00		10.00		10.00

NOT CALLABLE BOND PRICE

0						116.65					
1					125.93		121.37				
2				123.82		120.62		116.89			
3			120.39		118.43		116.10		113.36		
4		115.75		114.85		113.77		112.49		110.96	
5	110.00		110.00		110.00		110.00		110.00		110.00

CALL PRICE

0						6.65					
1					15.93		11.37				
2				13.82		10.62		6.89			
3			10.39		8.43		6.10		3.36		
4		5.75		4.85		3.77		2.49		0.96	
5	0.00		0.00		0.00		0.00		0.00		0.00

CALLABLE BOND PRICE

0						110.00					
1					110.00		110.00				
2				110.00		110.00		110.00			
3			110.00		110.00		110.00		110.00		
4		110.00		110.00		110.00		110.00		110.00	
5	110.00		110.00		110.00		110.00		110.00		110.00

VALUING A CALLABLE BOND

Risk free rate / Initial period	5.00	
Risk premium / Effective spread	100.00	basis points
Volatility %	10.00	1.11
Interest rates: Up & down probability	0.50	
Price formula in each node	[Prob up * Price up + Prob down * Price down] / (1+interest)	
Bond price	115.00	
Cash flow	10.00	

YEARS RATES

0						6.00					
1					5.43		6.63				
2				4.91		6.00		7.33			
3			4.44		5.43		6.63		8.10		
4		4.02		4.91		6.00		7.33		8.95	

CAPITAL

0						74.56					
1					80.84		77.24				
2				86.55		83.90		80.81			
3			91.65		89.95		87.93		85.55		
4		96.13		95.32		94.34		93.17		91.78	
5	100.00		100.00		100.00		100.00		100.00		100.00

CASH FLOWS

0						42.09					
1					45.09		44.14				
2				37.27		36.72		36.08			
3			28.74		28.48		28.17		27.81		
4		19.61		19.53		19.43		19.32		19.18	
5	10.00		10.00		10.00		10.00		10.00		10.00

NOT CALLABLE BOND PRICE

0						116.65					
1					125.93		121.37				
2				123.82		120.62		116.89			
3			120.39		118.43		116.10		113.36		
4		115.75		114.85		113.77		112.49		110.96	
5	110.00		110.00		110.00		110.00		110.00		110.00

CALL PRICE

0						1.65					
1					10.93		6.37				
2				8.82		5.62		1.89			
3			5.39		3.43		1.10		0.00		
4		0.75		0.00		0.00		0.00		0.00	
5	0.00		0.00		0.00		0.00		0.00		0.00

CALLABLE BOND PRICE

0						115.00					
1					115.00		115.00				
2				115.00		115.00		115.00			
3			115.00		115.00		115.00		113.36		
4		115.00		114.85		113.77		112.49		110.96	
5	110.00		110.00		110.00		110.00		110.00		110.00

VALUING A CALLABLE BOND

Risk free rate / Initial period	5.00	
Risk premium / Effective spread	100.00	basis points
Volatility %	10.00	1.11
Interest rates: Up & down probability	0.50	
Price formula in each node	[Prob up * Price up + Prob down * Price down] / (1+interest)	
Bond price	120.00	
Cash flow	10.00	

YEARS RATES

0						6.00					
1					5.43		6.63				
2				4.91		6.00		7.33			
3			4.44		5.43		6.63		8.10		
4		4.02		4.91		6.00		7.33		8.95	

CAPITAL

0						74.56					
1					80.84		77.24				
2				86.55		83.90		80.81			
3			91.65		89.95		87.93		85.55		
4		96.13		95.32		94.34		93.17		91.78	
5	100.00		100.00		100.00		100.00		100.00		100.00

CASH FLOWS

0						42.09					
1					45.09		44.14				
2				37.27		36.72		36.08			
3			28.74		28.48		28.17		27.81		
4		19.61		19.53		19.43		19.32		19.18	
5	10.00		10.00		10.00		10.00		10.00		10.00

NOT CALLABLE BOND PRICE

0						116.65					
1					125.93		121.37				
2				123.82		120.62		116.89			
3			120.39		118.43		116.10		113.36		
4		115.75		114.85		113.77		112.49		110.96	
5	110.00		110.00		110.00		110.00		110.00		110.00

CALL PRICE

0						0.52					
1					5.93		1.37				
2				3.82		0.62		0.00			
3			0.39		0.00		0.00		0.00		
4		0.00		0.00		0.00		0.00		0.00	
5	0.00		0.00		0.00		0.00		0.00		0.00

CALLABLE BOND PRICE

0						116.13					
1					120.00		120.00				
2				120.00		120.00		116.89			
3			120.00		118.43		116.10		113.36		
4		115.75		114.85		113.77		112.49		110.96	
5	110.00		110.00		110.00		110.00		110.00		110.00

VALUING A CALLABLE BOND

Risk free rate / Initial period	5.00	
Risk premium / Effective spread	100.00	basis points
Volatility %	10.00	1.11
Interest rates: Up & down probability	0.50	
Price formula in each node	[Prob up * Price up + Prob down * Price down] / (1+interest)	
Bond price	125.00	
Cash flow	10.00	

YEARS RATES

0						6.00					
1					5.43		6.63				
2				4.91		6.00		7.33			
3			4.44		5.43		6.63		8.10		
4		4.02		4.91		6.00		7.33		8.95	

CAPITAL

0						74.56					
1					80.84		77.24				
2				86.55		83.90		80.81			
3			91.65		89.95		87.93		85.55		
4		96.13		95.32		94.34		93.17		91.78	
5	100.00		100.00		100.00		100.00		100.00		100.00

CASH FLOWS

0						42.09					
1					45.09		44.14				
2				37.27		36.72		36.08			
3			28.74		28.48		28.17		27.81		
4		19.61		19.53		19.43		19.32		19.18	
5	10.00		10.00		10.00		10.00		10.00		10.00

NOT CALLABLE BOND PRICE

0						116.65					
1					125.93		121.37				
2				123.82		120.62		116.89			
3			120.39		118.43		116.10		113.36		
4		115.75		114.85		113.77		112.49		110.96	
5	110.00		110.00		110.00		110.00		110.00		110.00

CALL PRICE

0						0.07					
1					0.93		0.00				
2				0.00		0.00		0.00			
3			0.00		0.00		0.00		0.00		
4		0.00		0.00		0.00		0.00		0.00	
5	0.00		0.00		0.00		0.00		0.00		0.00

CALLABLE BOND PRICE

0						116.58					
1					125.00		121.37				
2				123.82		120.62		116.89			
3			120.39		118.43		116.10		113.36		
4		115.75		114.85		113.77		112.49		110.96	
5	110.00		110.00		110.00		110.00		110.00		110.00

6 CONCLUSIONS

The effective duration and the effective yield are referred to callable bonds. Callable bonds do not respond to the changes in the interest rate in the same way or magnitude as the not callable bonds do. In the standard calculation for the modified duration, none provision is done for the possibility that the payment of the final capital can occur before the established expiration. Thus, the modified duration cannot be a good guide to the answer of the price of a callable bond to changes in the general level of interest rates. Nevertheless, the modified duration based on the yield to expiration is so extensively known and useful that is profitable to begin to work on this base. This can be carried out building a not callable bond that is the most closed to the callable bond. The modified duration and the yield to expiration of this close not callable bond are then defined as the effective duration and the effective yield, respectively, of the callable bond. When it is defined thus, the answer to price of the callable bond will be related to the effective duration and to effective yield in the same way as it does the answer in the price of a not callable bond that is related to the modified duration and the yield to expiration.

Once the adjusted option spread (OAS) is determined, a duration spread is calculated so that to express the sensibility of the callable bond price to the changes in the adjusted option spread (OAS). The duration spread is used to identify a particular not callable bond, to which it will refer here as the reference bond. The reference bond has the same coupon that the callable bond and an effective spread equal to the adjusted option spread (OAS) of the callable bond. In short, the recovery value to expiration of the reference bond is determined by the calendar of the call of the callable bond. The reference bond is valued discounting all its cash flows by the future implicit spot short-time interest rate in the binomial tree. The utilized discount rates are the future spot short-time interest rate of the Treasure plus the effective spread. The key point of this reference bond is that the particular maturity date is chosen in such a way that its duration spread fits the duration spread of the callable bond.

The duration spread of the not callable bond express the sensibility of the price of the not callable bond to the changes in the effective spread. The selected reference bond has a sequence of cash flows that at present could be received by an investor that maintains the callable bond. For not callable bonds, the effective duration is identical to the normally calculated modified duration, given that a not callable bond is its same reference bond. For callable bonds, the price sensibility of the reference bond is lower than that of a callable bond underlying the callable bond. In the Figure 5 and 6, the effective duration of a callable bond is compared with the modified duration of a not callable bond of 28 years to expiration and the modified duration of a not callable bond 3 years. To the extent that the interest rates are raised, the duration of both not callable bonds decreases. Nevertheless, the effective duration of a callable bond first is raised and then falls. This difference in the behavior reflects the changing sensibility to price of the option call implicit in the callable bond.

Given that an investment in a callable bond is equivalent to a long position in the not callable underlying bond and a short position in the option call, the net sensibility of the price of the investment is equal to the sensibility of the price of a not callable bond minus the sensibility of the price of an option call. The model is used to determine the sensibility to the option call price respect to the changes in the market interest rates.

To low performances, the effective duration of a callable bond is a lot lower than the duration of the not callable underlying bond. This is because there is a strong probability that the bond be called before its original expiration.

To the extent that the interest rate is increased, the duration of the not callable bond falls, but the effective duration of the callable bond enlarges. This is because the reduced probability of immediately call tends to extend the expiration of the callable bond far behind of the first date of call. This is the region of negative convexity.

For even higher interest rates, the effect of the option call comes to be sufficiently small so that the convexity of the callable bond come to be positive and its effective duration again decreases in subsequent interest rates increases.

For low performances, the callable bond often is seen as trading to call and for high performances; the callable bond is seen like trading to expiration.

For intermediate performance nevertheless, the bond itself will trade neither in the date of call neither to expiration. The effective duration falls in some place among the duration to the first date of call and the duration to expiration. It is for that the model should be used for ascertain the effective duration or the sensibility of the price of the bond. The model owes consistently modeling the evolution of the yield curve, given that the decision of the issuer of the call depends on the present profile of the yield curve of all the future dates of call.

Elevating the level of volatility always increases the value of the implicit option call in the callable bond. Nevertheless, the effect of to elevate the level of volatility of the effective duration depends on the present level of the interest rates and the shape of the yield curve.

Consider a callable bond that is trading in a situation of low interest rates with high probability of call to the more nearby possible date. This bond will have a low effective duration.

To the extent that a specified level of volatility is raised, the probability that the bond be called to the first date of call is reduced, which increases the effective duration of the bond. If the level of yield is such that the callable bond has a high effective duration, there is a relatively small possibility that the bond be called earlier. Elevating the specific volatility increases the possibility that the bond be call sooner. In this case, the increased volatility reduces the effective duration of the bond.

The determination of the effective spread or of the adjusted option spread (OAS) provides the estimation of the wealth or cheapness related to the yield curve of the Treasure benchmark.

But many investors want or need to know the yield about their investments. For callable bonds, is clear that the yield to expiration is not appropriated. Nevertheless, it is not apparent that the yield at the worst (the lower between the yield to expiration and the yield to call) be neither appropriate, and in fact overrates the probable yield.

Consider a bond trading such that, if the yields do not change a lot, a call is probable earlier in the remaining life of the bond. If both, the yield to call and the yield to expiration are calculated, the yield to call will be lower. If the yields do not change and the bond is called, the investor there will be getting the yield to call on the shortest horizon (to call).

But if the yield enlarges and the bond is not called in it expected date of call, will be probably because the bond is itself trading lower that the price of call. If the bond would be sold in the originally expected date of call, the yield on that period it would be lower that the original “worse” yield.

In the other extreme, consider a discount bond that does not seem to be a probable candidate to call, trading to a particular yield to expiration (lower than the yield to call).

If it is maintained to the expiration, the yield of the bond would be equal to the yield to expiration on all the period. This does not imply that the compounded yield earned by the investor be equal to the yield to expiration.

But if the rates decline dramatically and the bond is called, what is obtained in money should be reinvested to the new lowest rate to the original horizon to expiration. The yield of this combined investment on the entire period would be lower than the original yield to expiration, which was thought to be higher or lower to the yield to the worst.

It has been calculated a measure of performance that it can be called effective yield, which is related to the effective duration and to the adjusted option spread (OAS) of the callable bond. We will refer again to the reference bond, which provides a good parameter for the callable bond because has the same coupon, effective spread and effective duration that the callable bond (although its expiration date is earlier). We define the yield to expiration of this reference bond (to the price calculated previously) to be the effective yield of the callable bond.

An investor could be indifferent between the callable bond and the reference bond. Nevertheless, two factors should be considered. They are: the estimation of the investor of the future volatility of the interest rate and the possible changes of the yield curve.

In spite of the fact that the callable bond and the reference bond have the same coupon, effective duration, effective spread and effective yield, they will react differently to the market relative changes because they have different convexity.

If the interest rates do not change substantially since its starting point, the holder of the callable bond will achieve a better return on a specific horizon in comparison of what would be able to provide the reference bond. This is because the holder of the callable bond has a traditional advantage of the yield to expiration on the reference bond.

In essence, the holder earns a yield on the underlying bond plus the prize on the sold option call. If the interest rates levels change substantially, the holder of a callable bond will have a lower return on the horizon period in comparison to the holder of a reference bond.

The effective yield is not an ambiguous guide to value. Of the same form that occurs with the yield to expiration, bonds with different coupon would be able to trade with lightly different yields; this is because of the effect of taxes and of the shape of the yield curve. Thus, the value spread produced by the model is a better guide to value bonds with similar effective durations.

The option adjusted spread (OAS) produced by the model are directly comparable with the mortgage model by Salomon Brothers (both require a volatility of short time Treasury notes). Thus, the callable Treasury notes, the callable corporate bonds and the mortgage bonds now can be valuated in a consistent way.

FIGURE 5: DURATION

RATE	VA	C=125	C=115	C=105
0,01	4,80	0,00	0,00	0,00
0,02	4,57	0,00	0,00	0,00
0,03	4,36	0,00	0,00	0,00
0,04	4,16	0,00	0,00	0,00
0,05	3,97	0,00	0,00	0,00
0,06	3,79	3,02	0,00	0,00
0,07	3,62	3,62	0,00	0,00
0,08	3,46	3,46	0,89	0,00
0,09	3,31	3,31	2,76	0,00
0,1	3,17	3,17	3,17	0,49
0,11	3,03	3,03	3,03	1,36
0,12	2,90	2,90	2,90	1,81
0,13	2,78	2,78	2,78	2,15
0,14	2,67	2,67	2,67	2,43
0,15	2,56	2,56	2,56	2,52
0,16	2,45	2,45	2,45	2,43
0,17	2,35	2,35	2,35	2,34
0,18	2,26	2,26	2,26	2,25
0,19	2,17	2,17	2,17	2,17
0,2	2,09	2,09	2,09	2,09

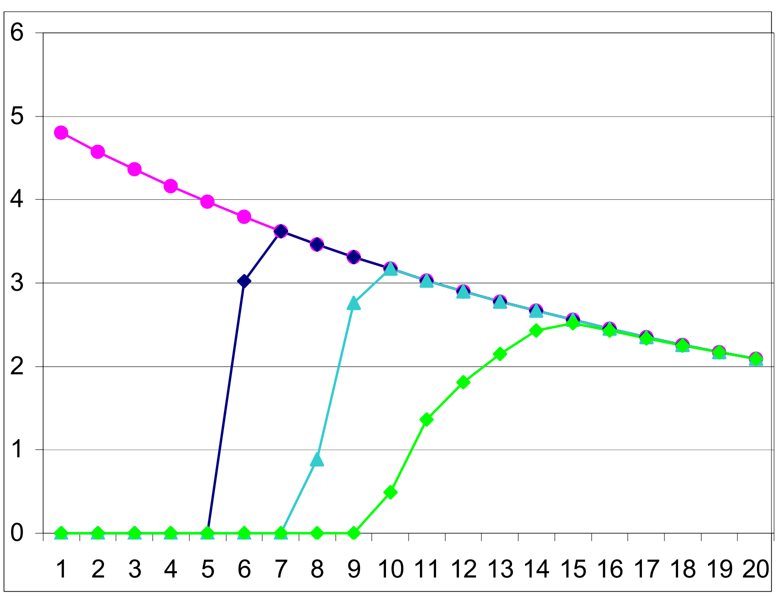
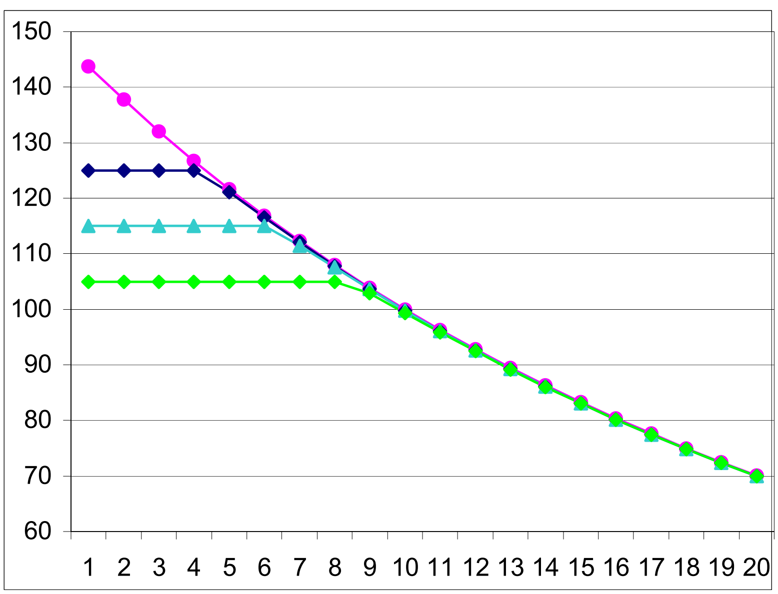


FIGURE 6: CONVEXITY

RATE	VA	C=125	C=115	C=105
0,01	143,68	125,00	115,00	105,00
0,02	137,71	125,00	115,00	105,00
0,03	132,06	125,00	115,00	105,00
0,04	126,71	125,00	115,00	105,00
0,05	121,65	121,04	115,00	105,00
0,06	116,85	116,58	115,00	105,00
0,07	112,30	112,09	111,47	105,00
0,08	107,99	107,77	107,54	105,00
0,09	103,89	103,67	103,62	102,87
0,1	100,00	99,78	99,78	99,30
0,11	96,30	96,09	96,09	95,82
0,12	92,79	92,58	92,58	92,46
0,13	89,45	89,24	89,24	89,17
0,14	86,27	86,07	86,07	86,04
0,15	83,24	83,05	83,05	83,04
0,16	80,35	80,17	80,17	80,17
0,17	77,60	77,43	77,43	77,43
0,18	74,98	74,82	74,82	74,82
0,19	72,48	72,33	72,33	72,33
0,2	70,09	69,95	69,95	69,95



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