Building a Better Fund of Hedge Funds:
A Fractal and α - Stable Distribution Approach

By

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Abstract:
Markowitz's (1952) portfolio theory has permeated financial institutions over the past 50 years. Assuming that returns are normally distributed, Markowitz suggests that portfolio optimization should be performed in a mean-variance framework. With the emergence of hedge funds and their non-normally distributed returns, mean-variance portfolio optimization is no longer adequate. Here, hedge fund returns are modeled with the α -stable distribution and a mean-CVaR portfolio optimization is performed. Results indicate that by using the α - stable distribution, a more efficient fund of hedge funds portfolio can be created than would be by assuming a normal distribution. To further increase efficiency, the Hurst exponent is considered as a filtering tool and it is found that combining hedge fund strategies within a range of Hurst exponents leads to the creation of more efficient portfolios as characterized by higher risk-adjusted ratios. These findings open the door for the further study of econophysics tools in the analysis of hedge fund returns.

Keywords: hedge funds, fund of funds, portfolio optimization, conditional value at risk (CvaR), α - stable distribution, Hurst exponent, fractals

JEL Classification: C61, G11, G23

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Markowitz's (1952) portfolio theory has permeated financial institutions over the past 50 years. Assuming that returns are normally distributed, Markowitz suggests that portfolio optimization should be performed in a mean-variance framework. Many studies, including Madelbrot (1963), Fama (1965), Malevergne and Sornette (2005), Bouchaud and Potter (2000) and Carr et al. (2002) have shown that financial asset returns do not follow a Guassian distribution. This non-normality is clearly seen in the returns of hedge funds. Some studies that describe the "fat-tailed" behavior of hedge funds include Kat and Miffre (2003), Blum et al. (2003), Delornzi and Sui (2004) and Perez (2004). In order to construct an efficient hedge fund portfolio, many have decided to turn away from the mean-variance framework and pursue other avenues. Kat (2002) searches for the optimal fund of hedge funds that offers investors skewness protection, Giamouridis and Vrontos (2005) use a full factor model to construct optimal hedge fund portfolios, Davies et al. (2004) create hedge fund portfolios by incorporating higher moments into a Polynomial Goal Programming optimization model and Morton et al. (2005) use a normal-to-anything (NORTA) method for modeling non-normal return distributions and then perform portfolio optimization with these returns. In this paper, it will be suggested that a conditional value at risk (CVaR) optimizer would result in the creation of a more efficient portfolio. Similar work has been completed by Johri (2004) who compared mean variance optimization to CVaR optimization, Krokhmal et al. (2002) who compared various rebalancing strategies for hedge funds and Agarwal and Naik (2002) who demonstrate, using the mean-CVaR framework, the extent to which a mean-variance framework underestimates tail risk. To improve the results of the CVaR optimizer, hedge fund returns are fitted with an α - stable distribution. Use of the α - stable distribution

and CVaR optimization has been previously utilized in analyzing equity investments (see Tokat et al. (2003)) and its applicability to fund of hedge fund creation has been suggested by Finanalytica Inc[1]. The results from this optimization indicate that the use of a α - stable distribution optimizer creates a far more efficient portfolio than would be achieved with a normal distribution. Having the tools for the creation of an efficient hedge fund portfolio, the Hurst exponent is considered as a means of filtering results. Previously, two studies have considered the Hurst exponent in a hedge fund context, including Amenc et al. (2002) who use the Hurst exponent to check the persistence of returns of the CSFB hedge fund indexes and De Souza and Suleyman (2004) who propose the use of the Hurst exponent in conjunction with the D-statistic to identify managers with persistent good performance. Here we considered different ranges of the Hurst exponent and the effects these ranges have on returns and volatility, finding that selecting returns with certain Hurst exponents does add value to the portfolio creation process. These findings open the door for the incorporation of other econophysics tools in the analysis of hedge funds.

This study is presented in five sections. The second section provides an introduction to the Hurst exponent, α - stable distribution and conditional value at risk. Section three describes the data used and the methodology that was implemented. Section four outlines the empirical results and section five provides the conclusion.

[1] http://www.bravo-group.com/company/clients.jsp

II. Basics

The assortment of quantitative tools for portfolio and risk managers is growing on a daily basis. Most of these tools originate in the academic world and either take time to be incorporated into commercial software packages or are too difficult to implement. For a quantitative tool to enter mainstream asset management, it has to be easy to grasp, implement and sell. The tools used in this study have these characteristics and therefore should be considered viable resources for portfolio and risk managers.

The Hurst Exponent

The Hurst exponent occurs in many areas such as fractals and chaos and has recently made its way into the world of finance. It is a summary statistic of persistence that has been used to characterize long memory processes in time series[1]. The idea of this statistic originates with Hurst (1900 – 1978) who was an English hydrologist who worked on the Nile River Dam project[2]. While designing a dam, he was concerned about adapting the dam's storage capacity to accommodate changes that may occur in the rivers water level. He studied an 847-record of the Nile River's overflows and found that flood occurrences could be characterized as persistent, i.e. heavier floods were followed by above average flood occurrences, while below average occurrences were followed by minor floods[3]. Through this process he developed what has become know as Rescaled Range (R/S) Analysis, one of the ways of finding a time series Hurst exponent. Other methods include Detrended Fluctuation Analysis and spectral analysis. Here, the calculation of

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[2] http://www.bearcave.com/misl/misl_tech/wavelets/hurst/
[3] http://www.fxstreet.com/chat/transcripts/20050510chatsession.asp
[4] http://www.cs.sunysb.edu/~skiena/691/lectures/lecture12/lecture12.html

the Hurst exponent is based on the R/S analysis as found in Amenc et. al (2002)[4]. The steps are as follows:

1) Normalize the return series being considered by subtracting the series mean from each observation, forming R_t

2) Calculate the series $Y_t = \sum_{s=1}^{t} R_s$

3) Calculate $Y_1 = \max(Y_t, 0 \leq t \leq T)$ and $Y_2 = \min(Y_t, 0 \leq t \leq T)$

4) Form the Hurst exponent

$$H = \frac{Log\left(\frac{(Y_1 - Y_2)}{\sigma}\right)}{Log(T)}$$

The value of the Hurst exponent will be between 0 and 1. A process with a Hurst exponent of 0.5 is considered a true random walk while values between 0.5 and 1 are said to follow "persistent" behavior. Values between 0 and 0.5 are said to follow "anti-persistent behavior" also referred to as "mean reverting." The Hurst exponent is related to the concept of fractal dimension (FDI), which measures the roughness (volatility) of a surface. The relationship is:

$$FD = 2 - H$$

A fractal dimension value of 1.5 suggests that the market is acting in a random manner, values between 2 and 1.5 suggested a more jagged (volatile) pattern while values between 1 and 1.5 are much more linear.

[4] Also see Peters (1991).

The α - Stable Distribution

In Mandelbrot (1963), Benoit Mandelbrot found that cotton prices did not exhibit normally distributed behavior as postulated since Louis Bachelier work on security and commodity markets[5]. Mandelbrot suggested that a distribution that better described this price behavior would be the α - stable distribution. Among the fat-tailed distributions, the α - stable distribution is supported by the Central Limit Theorem, which states that α - stable laws are the only possible limit distributions of properly normalized and centered sums of independent, identically distributed random variables[6]. This distribution can account for fat tails and asymmetry, two characteristics that are evident in hedge fund returns. To contrast this distribution with the normal distribution, one only has to look at the moments that are needed to describe a normal distributed variable and a α - stable distributed variable. In the normal case, one only requires the first (μ) and second (σ) moments while in order to describe an α - stable variable one has to have a tail index $\alpha \in (0,2]$, a skewness parameter $\beta \in [-1,1]$, a scale parameter $\sigma > 0$ and a location parameter $\mu \in \Re$. The addition of the tail index and skewness parameters allow the capturing of the fat tails and asymmetry. One of the problems with the α - stable distribution is that it has no closed form for its density and distribution functions. The α - stable distribution can be most conveniently described by its characteristic function

[6] BACHELIER, L. (1900): "Théorie de la Spéculation", Annales Scientifiques de l'Ecole Normale Supérieure, 3rd. Ser. 17: 21-88. Translated in "The Random Character of Stock Market Prices"; edited by Paul Cootner (1964), Cambridge, Mass., MIT-Press.

[7] Borak et al. (2005)

$\phi(t)$ – the inverse Fourier transformation of the probability density function[7]. The most common parameterization of the characteristic function is as follows:

$$\ln\phi(t) = \begin{cases} -\sigma\,|\,t\,|\left\{1 - i\beta\,\mathrm{sign}(t)\tan\dfrac{\alpha\pi}{2}\right\} + i\mu t, & \alpha \neq 1 \\ -\sigma\,|\,\mathrm{t}\,|\left\{1 + \mathrm{i}\beta\,\mathrm{sign(t)}\dfrac{2}{\pi}\ln|\,t\,|\right\} + i\mu t, & \alpha = 1 \end{cases}$$

This creates a problem in simulating a variable from this distribution. To get around this problem, Weron (1996) suggested the following algorithm for constructing a standard α -stable random variable X ~ $S_\alpha(1,\beta,0)$:

- Generate a random variable U uniformly distributed on $(-2/\pi, 2/\pi)$ and an independent exponential random variable E with mean 1;
- For $\alpha \neq 1$ compute:

$$\mathrm{X} = S_{\alpha,\beta} \cdot \frac{\sin\{\alpha(\mathrm{U} + \mathrm{B}_{\alpha,\beta})\}}{\{\cos(\mathrm{U})\}^{1/\alpha}} \cdot \left[\frac{\cos\{\mathrm{U} - \alpha(\mathrm{U} + \mathrm{B}_{\alpha,\beta})\}}{\mathrm{E}}\right]^{(1-\alpha)/\alpha}$$

Where

$$\mathrm{B}_{\alpha,\beta} = \frac{\arctan(\beta\tan\frac{\alpha\pi}{2})}{\alpha}$$

$$S_{\alpha,\beta} = \left(1 + \beta^2\tan^2\left(\frac{\alpha\pi}{2}\right)\right)^{1/2\alpha}$$

[8] Borak et al. (2005)

- For α = 1 compute:

$$X = \frac{2}{\pi}\left[\left(\frac{\pi}{2} + \beta U\right)\tan U - \beta \ln\left(\frac{\frac{\pi}{2} E \cos U}{\frac{\pi}{2} + \beta U}\right)\right]$$

To simulate an α - stable random variable for all admissible values of the parameters α, σ, β and μ use the following property: if $X \sim S_\alpha(1,\beta,0)$ then

$$Y = \begin{cases} \sigma X + \mu, & \alpha \neq 1 \\ \sigma X + \frac{2}{\pi}\beta\sigma \ln\sigma + \mu, & \alpha = 1 \end{cases}$$

Further problems are encountered when fitting a time series to an α - stable distribution. Various methods have been suggested which differ by speed of implementation and accuracy. The most accurate method for parameter estimation is the maximum likelihood (ML) method. For a vector of observations $x = (x_1, \ldots x_n)$, the ML estimate of the parameter $\theta = (\alpha, \sigma, \beta, \mu)$ is obtained by maximizing the log-likelihood function:

$$L_\theta(x) = \sum_{i=1}^{n} \ln \tilde{f}(x_i;\theta),$$

Where $\tilde{f}(x_i;\theta)$ is the α - stable probability density function. The worst estimates are yielded by an estimation method proposed by Press (1972) called the method of moments, which is based on transformations of the characteristic function. A third type of estimation procedure was proposed by Koutrouvelis (1980) who presented a regression-type method which starts with an initial estimate of the parameters and proceeds iteratively until some perspective convergence criterion is satisfied. The method used in this paper is the method proposed by McCulloch (1986) who improved upon the method suggested by Fama and Roll (1971). This method produces good

parameter estimates and is fairly easy to implement. McCulloch (1986) defines two functions, $\phi_1(\alpha,\beta)$ and $\phi_2(\alpha,\beta)$, which are used to back out both α and β. The functions are as follows:

$$\phi_1(\alpha,\beta) = \frac{x_{0.95} - x_{0.05}}{x_{0.75} - x_{0.25}}$$

$$\phi_2(\alpha,\beta) = \frac{(x_{0.95} - x_{0.5}) - (x_{0.5} - x_{0.05})}{x_{0.95} - x_{0.05}}$$

In the above formulas, x_f denotes the f-*th* population quantile.

ϕ_1 measures the relative sizes of the tails and middle of the distribution while ϕ_2 is a measure of the spread between the right part and the left part of the distribution. Estimation is conducted by replacing the population quantiles by their sample counterparts. To solve for α and β, MuCulloch (1986) provides tables of values for the functions $\phi_1(\alpha,\beta)$ and $\phi_2(\alpha,\beta)$ given a grid of values of (α,β). The tables contain values of $\hat{\alpha}$ and $\hat{\beta}$ of structural parameters for given estimated values $\hat{\phi}_1$, $\hat{\phi}_2$ of auxiliary parameters[8]. An estimator of the scale parameter sigma is defined as:

$$\hat{\sigma} = \frac{\hat{x}_{0.75} - \hat{x}_{0.25}}{\hat{\phi}_3}$$

Where the auxiliary parameter $\phi_3 = \phi_3(\alpha,\beta)$ is defined by:

$$\phi_3(\alpha,\beta) = \frac{x_{0.75} - x_{0.25}}{\sigma}$$

McCulloch (1986) also provides a table of values of $\phi_3(\alpha,\beta)$ that allow $\hat{\sigma}$ to be found. Estimation of the location parameter μ, is facilitated through the use of the following equation:

[9] Garcia et al. (2004)

$$\phi_4(\alpha,\beta) = \frac{\mu - x_{0.5}}{\sigma} + \beta \tan\left(\frac{\pi\alpha}{2}\right)$$

For the estimation of $\hat{\phi}_4 = \phi_4(\hat{\alpha},\hat{\beta})$ the estimator μ is found from the previously defined estimators of (α,β,σ):

$$\hat{\mu} = \hat{x}_{0.5} + \hat{\sigma}\left[\hat{\phi}_4 - \hat{\beta}\tan\left(\frac{\alpha\pi}{2}\right)\right]$$

Conditional Value at Risk

The final tool that was used in this study was the conditional value at risk (CVaR) optimization algorithm proposed by Rockafeller and Uryasev (2000). Simply defined, CVaR is the mean loss exceeding value at risk (VaR). CVaR has many interesting features that make it a better measure of risk than VaR. It is applicable to non-symmetric loss distributions, translation invariant, sub-additive, positively homogeneous, monotonic[9]. Conditional value at risk accounts for losses beyond value at risk and is convex with respect to portfolio positions. This measure is also consistent with the mean variance approach in that for normal loss distributions optimal variance and CVaR portfolios coincide[10]. Most importantly, it is easy to control/optimize for non-normal distributions and is computationally simple. Conditional value at risk is computed in the following way:

$$\widetilde{\mathrm{F}}_{\beta}(\mathrm{x},\alpha) = \alpha + \frac{1}{\mathrm{q}(1-\beta)}\sum_{\mathrm{k}=1}^{\mathrm{q}}[-x^{\mathrm{T}}y_k - \alpha]^{+}$$

[10] Rockafellar and Uryasev (2000)
[11] Rockafellar and Uryasev (2000)

α = VaR at α - condience level
β = CVaR confidence level
$-x^{\mathrm{T}} y_k$ = Loss function at scenario k
q = number of scenarios considered

With these three easy to implement yet very powerful tools, great improvements can be gained in the construction of a fund of hedge funds portfolio.

III. Data/Methodology

Currently there are numerous websites that provide information on hedge fund returns. With over 7,000 active hedge funds, it becomes a formidable task to perform analysis on individual managers[11]. To proxy the different hedge fund strategies available, the Hedge Fund Research (HFR) strategy indexes were used. The HFRI Monthly Indexes (HFRI) are equally weighted performance indexes that are utilized by many hedge fund managers as benchmarks[12]. An advantage of the HFRI is that they are broken down to 37 different categories by strategy accounting for over 1,600 hedge funds. In this current study, 30 different indexes were considered. Definitions of the different strategies are available in Appendix A.

Inclusion in the HFRI requires no minimum asset size for a fund and there is no required length of time a fund must be actively trading. To account for survivorship bias, when a fund liquidates or closes, its performance remains in the indexes as of its last performance date[13]. There is much research available that criticizes hedge fund indexes -- see Fung and Hsieh (2002, 2003, 2004), Kat and Brooks (2001) and Amenc and Martellini (2003). The purpose of this paper is not dependent on whether these indexes

[11] http://www.hennesseegroup.com/Press_Client_Releases/release20050112.html
[12] http://www.hedgefundresearch.com
[13] http://www.hedgefundresearch.com

are 100% accurate; they are used here to proxy the statistical characteristics of managers in each of the strategies represented by the indexes. The HFRI data is available in monthly intervals starting in January 1990 for most of the strategies available. In this paper, the period from January 1990 to December 1999 is used as the model calibration period and out-of-sample tests are carried out using the data between January 2000 and December 2004. Table 1 provides a summary of the statistical characteristics of the indexes used.

[Insert Table 1]

Most of the indexes have Hurst exponents above 0.5 and 0.65 indicating that there is some persistence in hedge fund returns. This was also shown by Amenc et al. (2002) who found similar Hurst exponents when they examined the CSFB hedge fund indexes. When looking at the data prior to December 1999, it is interesting to note that the two strategies that have Hurst exponents below 0.5 are both directional, that is, the Non-Hedge Index and the Short Sell Index. When one considers Hurst values below 0.5 as being mean reverting, this is expected. Since these strategies do not hedge, their time series would follow a more volatile pattern as is indicated by their respective fractal dimensions; 1.52 for the Non-Hedge Index and 1.55 for the Short Sell Index. It should also be noted that only one strategy, Fixed Income Index, has a Hurst exponent over 0.65. In both the in sample and out of sample periods, most of the indexes fail various normality tests (exceptions being the Global Marco Index, Market Timing Index, the Short Sell Index, the Statistical Arbitrage Index, and the Energy Sector Index). These

strategies have little in common as their Hurst exponents are not similar and their first and second moments vary considerably. For the rest of the strategy indexes, fat tails and asymmetry are present, making them excellent candidates for the α - stable distribution.

Before the hedge fund indexes are filtered by their Hurst exponents, they are all examined together. Next, one category is created that contains all hedge fund indexes with a Hurst exponent between 0.4 and 0.6, another has hedge fund indexes that have a Hurst exponent between 0.5 and 0.65, a third category has hedge fund indexes with Hurst exponents greater than 0.6 and the final category has hedge fund indexes with Hurst exponents between 0.4 and 0.65. The goal of these categories is to have sets of hedge fund indexes that have varying degrees of persistence and volatility and to see how they combine in a portfolio. With these different categories, portfolios were constructed and tested in the out-of-sample period.

Starting with the original set, a portfolio having a weight of 1/30 to each strategy index was constructed. This portfolio was not rebalanced and the same allocation was used throughout the whole period considered. To keep the allocation of 1/30 constant from month to month, it is assumed that this portfolio is rebalanced continuously, an assumption that is unrealistic but used mainly to form a comparison portfolio. Next, only the "top 20" hedge fund strategies by Sharpe Ratio were considered and assigned an allocation of 5% to each. The Sharpe Ratio considered in this paper is defined as:

$$\text{Sharpe Ratio} = \frac{\mathrm{E}(r) - r_f}{\sigma}$$

$\mathrm{E}(r)$ is the annualized return of the asset
r_f = 3 month US Treasury Bill Rate
σ = annualized standard deviation

Again, this portfolio is assumed to be continuously rebalanced throughout the whole testing period to keep the 5% allocation constant. Having these two base cases, optimization using simulated α- stable distributed returns and simulated normal distributed returns were conducted. As mentioned above, a conditional value at risk optimization was performed which involved maximizing the given expected return for a month subject to a 99% CVaR limit. The reason that a maximize return algorithm was used is because many fund of funds have risk budgets, i.e., VaR or CVaR limits, that they set and would like to know what type of return they can get for these limits. Also, it is easier to explain an optimization where one maximizes return given a risk limit than one that minimizes risk with a preset expected return. The CVaR constraint chosen for this study was 3%, compared to the historical 99% CVaR of 2.12% for the Merrill Lynch Mid-Bond Index. The optimization also involved market beta constraints on the S&P500, the Merrill Lynch Mid Bond Index and the Bear Stearns High Yield Index. The market beta constraint for each one were $-0.2 < \beta_i < 0.2$, effectively creating a market neutral portfolio. Also, a final constraint that was imposed was a maximum allocation to a strategy, which begins at 25% but can increase. This constraint allows for a creation of a portfolio that is not too concentrated in any one hedge fund strategy.

Using the hedge fund strategy index returns, an α- stable distribution was fit to each series and 20,000 simulations were run to create possible portfolio returns[14]. As a comparison, each index was also fit to a normal distribution and 20,000 simulations were conducted to create possible portfolio returns.

[14] For accuracy of α- stable distribution parameters computed, a comparison was made with the parameters obtained using the quantile method provided in the STABLE program created by John Nolan. The comparison indicated that the parameters used in this paper were very similar or identical to those obtained using the STABLE program. For further information on the STABLE program visit: http://academic2.american.edu/~jpnolan/stable/stable.html

Considering the hedge fund indexes with the Hurst exponent between 0.4 and 0.65, a procedure similar to the above was conducted in which each index was fitted with a α - stable distribution, simulations were conducted and optimization performed. Then a normal distribution was fitted and simulations were performed with optimization occurring last. The same occurred for the hedge fund indexes with Hurst exponents between 0.4 and 0.60, between 0.5 and 0.65 and for funds with Hurst over 0.6. Since most hedge funds usually have a waiting period before an investor can redeem, rebalancing was performed quarterly, using the optimization results that were obtained in the previous quarter. Rebalancing involved checking to see whether an index's Hurst exponent had gone out of the boundaries and if so removing it and performing the simulations and optimization. If too few indexes remain, the original indexes are checked to see if any have crossed into the Hurst exponent range being considered. Also, if required, the maximum allocation in the optimization may be changed in order to facilitate the ability of the optimizer to find a solution. Comparison among the different Hurst exponent regimes was conducted using the Sharpe and Sortino ratios. The Sharpe ratio has been described above while the Sortino ratio is as follows:

$$\text{Sortino Ratio} = \frac{\text{E(r)} - \text{MAR}}{\text{DD}}$$

$\text{E}(r)$ is the annualized return of the asset
MAR = minimum acceptable return (3 month US Treasury Bill Rate in this case)
DD = downside deviation of returns

It is acknowledged that the Sharpe ratio and Sortino ratio may not adequately describe the risk-return relationship in non-normally distributed asset returns. The fact remains that these two ratios are widely used and understood by investors and fund managers, 82% for

the Sharpe ratio and 58% for the Sortino ratio, making them a universal gauge of how well a fund is operating[15].

IV. Empirical Results

[Insert Table 2]

Portfolios were constructed using two naïve allocation strategies. Portfolio A had an equal allocation of 1/30 to each hedge fund strategy index while Portfolio B had 5% allocated to the "Top 20" hedge fund strategy indexes ranked by Sharpe ratio. Table 2 illustrates that Portfolio A is a much better risk-adjusted portfolio than Portfolio B. Portfolio A has higher returns and slightly higher volatility resulting in a Sharpe ratio of 1.02 and a Sortino ratio of 1.47 while Portfolio B has a Sharpe ratio of 0.63 and a Sortino ratio of 1.27. Both of these portfolios have loss frequencies of over 25% and their maximum drawdowns are above 5%. Jarque-Bera tests for normality suggest that the return series from both Portfolio A and Portfolio B can be considered as originating from a normal distribution. Next, portfolios were constructed without any filtering by Hurst exponent but that were differentiated by the fact that one considered the hedge fund indexes as being normally distributed (Portfolio D) and the other considered them to be from an α - stable distribution (Portfolio C).

[Insert Table 3]

[15] Edhec European Alternative Multimanagement Practices Survey, 2003

Table 3 shows that the Portfolio C is a much safer portfolio and has both higher Sharpe and Sortino ratios, 1.30 and 2.28, compared to 0.97 and 1.71 for Portfolio D. It should be noted that Portfolio D has a higher annualized return than Portfolio C, 8.22% vs. 7.31%. When a comparison is made between the allocations chosen in the normal distribution optimization against those using the α - stable distribution optimization, it is evident that the former does not take into account skewness and kurtosis and allocates heavily into funds that have high returns. This underestimates risk that some of these hedge fund indexes truly have and this underestimation is seen in the fact that Portfolio D has a loss frequency close to 40% and that its maximum drawdown is more that twice of that experienced by Portfolio C. It is clear that Portfolio D is a very volatile portfolio and that the extra return over Portfolio C is not worth the extra risk. Comparing these two portfolios to portfolios A and B, it is clear that by running a simple optimization, more efficient portfolios can be constructed. To arrive at a safer and better risk adjusted portfolio, the HFR hedge fund strategy indexes were sorted by different Hurst exponents and optimizations were performed.

The first range that was considered involved hedge fund indexes that have Hurst exponents over 0.6. When looking at the market betas of these indexes to high yield and mid bonds, it was evident that a portfolio optimization with the predefined beta constraints would be difficult. Optimization was tried by was unsuccessful. The minimum allocation constraint was raised along with the CVaR and the market beta constraints. Portfolio optimization was eventually successful but it no longer resulted in a market neutral portfolio and had a CVaR of over 5%. This finding suggests that even though these hedge fund indexes displayed persistence, a portfolio of these indexes was

possible only by taking on excessive risk and directional exposure. This characteristic would not sit will with investors who are risk averse. It should be noted that De Souza and Suleyman (2004) performed a similar study and found that combining funds with high Hurst exponents resulted in a portfolio with higher returns and lower volatility. Examination of the strategies with Hurst exponents over 0.6, see Table 1, reveals that these funds do have higher returns but their risks cannot be captured assuming a normal distribution. Also, these funds are driven more by market factors, as evident through their high market betas, than other hedge fund strategies considered. In a period of rising markets these funds would perform well but they would be exposed in downward markets as well, thus not being market neutral as was intended for the portfolios constructed in this study.

[Insert Table 4]

The next range of Hurst exponents considered was between 0.5 and 0.65. These hedge fund indexes still exhibit persistence but to a lesser degree than the above portfolio. Looking at the market betas indicates that there may be some problems with the optimization also. The first portfolio considered involved fitting the hedge fund indexes with a normal distribution. Portfolio optimization does not result in any feasible allocations. Constraints are increased but it is not possible to create a market-neutral portfolio. When the hedge fund indexes are fitted with an α - stable distribution and portfolio optimization performed, feasible allocations were found. This result was a bit surprising. No changes were made to the various constraints yet a solution was found.

Considering this an anomaly, the optimization was performed numerous times always leading to the same results. A reason for this occurring can be attributed to the fact that when a time series is fitted with a normal distribution, only the first and second moment are required while with the α - stable distribution, four parameters are required. The additional information that is obtained with four parameters is evident in simulations, which describe a wider array of possible return outcomes that can occur. This wider array of outcomes has more possibilities than the normal and when the optimization occurs, there are many more scenarios that can be considered, resulting in a feasible allocation. Table 4 has the results for this portfolio, Portfolio E. The Sharpe ratio is 1.26 and the Sortino ratio is 2.37. Comparison to portfolios A, B and C suggests that is this a better risk-adjusted portfolio. When Portfolio E is stacked up against Portfolio C, the winner is not clear. The Sharpe and Sortino ratios for both portfolios are very close as are their annualized returns and volatility. Portfolio E has a lower loss frequency but a higher maximum drawdown. These two portfolios are basically identical. The third range that was used for filtering involved allocating to hedge fund strategies that had Hurst exponents between 0.4 and 0.6.

[Insert Table 5]

Once again two portfolios were constructed, a normally distributed (Portfolio H) and an α - stable distributed (Portfolio G). Portfolio optimization was performed and feasible allocations were found for both Portfolio G and Portfolio H. Table 5 lists the relevant statistics and shows that the Portfolio G performed much better than Portfolio H.

The annualized returns are good for Portfolio G and its annualized volatility is much lower resulting in a Sharpe ratio of 1.46 and a Sortino ratio of 2.46 compared with Portfolio H who has a Sharpe of 0.81 and a Sortino of 1.44. This finding suggests that the benefits of choosing funds within this range are only evident when the strategies are fitted with an α - stable distribution. It seems that out of the chaos that exists in financial returns, some order can be found with the proper tools. When Portfolio G is compared to the other portfolios, it is a better risk-adjusted portfolio. Its returns may not be as high as the other portfolios but it has the lowest volatility and the lowest downside deviation. As a final filtering range, hedge fund indexes are chosen that have Hurst exponents between 0.4 and 0.65. This range is chosen since it is hypothesized that an optimal portfolio will be one that has slightly mean reverting strategies and slightly persistence strategies.

[Insert Table 6]

Again, two portfolio were constructed, a normal one (Portfolio J) and an α - stable one (Portfolio I). Table 6 displays the results and at first glance it is evident that Portfolio I is the most efficient portfolio out of all the ones that were created. The annualized returns on Portfolio I are one of the highest and its annualized volatility is slightly more than Portfolio G's. The loss frequency on Portfolio I is only 16.7%, which indicates a very safe portfolio that can provide steady returns. The Sharpe ratio is 1.50 and the Sortino ratio is 2.79 compared to Portfolio J, which has a Sharpe of 1.12 and a Sortino of 2.01. Comparing Portfolio I to Portfolio C, I is the more efficient portfolio. Considering that the difference between these two portfolios is five funds, which had

Hurst exponents over 0.65, it is evident that including funds with too high persistence adds negative value to the portfolio construction process. Comparing Portfolio I to Portfolio E, it is seen that including fund with Hurst exponents under 0.5 does add value to the portfolio also. It appears as the strategies with Hurst exponents under 0.5 act as hedges and result in a lower volatility while the strategies with Hurst exponents between 0.6 and 0.65 add to the returns of the portfolio. With these results, a comparison was made against the various HFR fund of funds indexes that are available.

[Insert Table 7]

Referring to Table 7, the various characteristics of the fund of fund indexes are available[16]. The diversified index and the general index are not too spectacular, both with unimpressive returns and high volatility. Of relevance are the conservative index and market defensive index considering that the portfolios constructed were market neutral. The conservative index has the lowest annualized volatility and downside deviation but its returns pale in comparison even to the worst performing portfolio that was constructed. Compared to the α - stable distribution portfolios, the conservative index is less efficient and thus inferior. Comparing the market defensive portfolio against the α - stable distribution portfolios results in a similar conclusion. Its Sharpe and Sortino ratios are lower than the α - stable distribution portfolios, it has a high loss frequency and it has a very high maximum drawdown. With these various portfolios, the most efficient, risk-adjusted portfolio is Portfolio I, which had strategies with Hurst exponents between 0.4 and 0.65 with the α - stable distribution.

[16] See Appendix A for a description of these indexes.

The findings presented here show that using the α - stable distribution instead of the normal distribution in creating a hedge fund portfolio does add value to the portfolio creation process. This value is characterized by much lower volatility and less downside risk. These features result in higher Sharpe and Sortino ratios than would occur by building a portfolio assuming a normal distribution. These results can be improved by considering funds with Hurst exponents in the range of 0.4 to 0.65. The combination of the filter and the α - stable distribution results in a very low volatile portfolio that is resistant to downside risk. This finding indicates that combining funds with mean reversion characteristics and funds with persistence characteristics is beneficial, but that the benefits of the characteristics are only available up to a certain limit, mainly 0.65.

[Insert Graph 1]

Besides using the Hurst exponent as a filtering tool, during this study it was found that by setting the upper limit at 0.65, a number of strategies were excluded before they started to suffer losses. Most notably was the statistical arbitrage index that was excluded in September 2002. Graph 1 shows that before the exclusion this strategy was performing well but afterwards it entered a slump that it is still currently in. This finding suggests that the Hurst exponent can also be used as a risk management tool, giving the risk manager some warning of when a strategy may be about to hit a rough patch. It is hypothesized that once a strategy starts to exhibit higher persistence, it is time to examine why this is occurring and whether this is a sign of deeper problems. Financial returns should not follow a smooth line, as there are numerous forces that make this very

unlikely to occur. Funds with high persistence should be viewed with caution and examined closely to see how and why they return are showing no volatility. Ideally, a risk manager would have her own Hurst exponent limit to gauge whether a fund is too persistent.

Conclusion

Mandelbrot (1963) found that cotton prices do not follow a Guassian distribution and are better described by an α - stable distribution. In the forty years since this publication, others have confirmed that many financial time series do not follow a Guassian distribution yet few have started using the α - stable distribution as a substitute. With the emergence of hedge funds and the non-normal distribution associated with their returns, the time is right for the α - stable distribution to be implemented in full force. With its ease of implementation, a result of increased computing power, there is no reason why the α - stable distribution should not be in every portfolio and risk manager's toolbox. It was shown that using an α - stable distribution results in a more efficient portfolio than those using the normal distribution. Going one-step further, the use of fractals, particularly the Hurst exponent, was considered as a filtering tool. By considering different ranges, in particular from 0.4 to 0.6, from 0.4 to 0.65, from 0.5 to 0.65 and the range greater than 0.6, it was found that when a portfolio is composed of strategies from the 0.4 to 0.65 range, a more efficient portfolio can be created.

An implication for risk management was suggested where limits of the Hurst exponent can be used as early warning signals that a hedge fund strategy may be about to experience some turbulence. Overall, the combination of the α - stable distribution and

Hurst exponent does result in a better-constructed portfolio. Such a result would be easy to sell to an institutional investor who would be considering investing into hedge funds but is afraid of extreme losses. With these tools, those extreme losses are mitigated and the worries of investors comforted.

The directions for further research are many. The α - stable distribution and Hurst exponent are part of a larger set of tools available in econophysics. Considering that hedge funds are known to exhibit non-normal distributions, examining the applicability of other econophysics tools seems like a reasonable next step. If something is not normal why force it to be normal? Now hedge funds can be analyzed properly without inappropriate assumptions regarding their distributions, resulting in the construction of superior fund of hedge fund portfolios.

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Appendix A

The following definitions are available from Hedge Fund Research, www.hedgefundresearch.com.

Convertible Arbitrage

Convertible Arbitrage involves purchasing a portfolio of convertible securities, generally convertible bonds, and hedging a portion of the equity risk by selling short the underlying

common stock. Certain managers may also seek to hedge interest rate exposure under some circumstances. Most managers employ some degree of leverage, ranging from zero to 6:1. The equity hedge ratio may range from 30 to 100 percent. The average grade of bond in a typical portfolio is BB-, with individual ratings ranging from AA to CCC. However, as the default risk of the company is hedged by shorting the underlying common stock, the risk is considerably better than the rating of the unhedged bond indicates.

Distressed Securities

Distressed Securities strategies invest in, and may sell short, the securities of companies where the security's price has been, or is expected to be, affected by a distressed situation. This may involve reorganizations, bankruptcies, distressed sales and other corporate restructurings. Depending on the manager's style, investments may be made in bank debt, corporate debt, trade claims, common stock, preferred stock and warrants. Strategies may be sub-categorized as "high-yield" or "orphan equities." Leverage may be used by some managers. Fund managers may run a market hedge using S&P put options or put options spreads.

Emerging Markets

Emerging Markets funds invest in securities of companies or the sovereign debt of developing or "emerging" countries. Investments are primarily long. "Emerging Markets" include countries in Latin America, Eastern Europe, the former Soviet Union, Africa and parts of Asia. Emerging Markets - Global funds will shift their weightings among these regions according to market conditions and manager perspectives. In addition, some managers invest solely in individual regions. Emerging Markets - Asia involves investing in the emerging markets of Asia. Emerging Markets - Eastern Europe/CIS funds concentrate their investment activities in the nations of Eastern Europe and the CIS (the former Soviet Union). Emerging Markets - Latin America is a strategy that entails investing throughout Central and South America.

Equity Hedge

Equity Hedge investing consists of a core holding of long equities hedged at all times with short sales of stocks and/or stock index options. Some managers maintain a substantial portion of assets within a hedged structure and commonly employ leverage. Where short sales are used, hedged assets may be comprised of an equal dollar value of long and short stock positions. Other variations use short sales unrelated to long holdings and/or puts on the S&P 500 index and put spreads. Conservative funds mitigate market risk by maintaining market exposure from zero to 100 percent. Aggressive funds may magnify market risk by exceeding 100 percent exposure and, in some instances, maintain a short exposure. In addition to equities, some funds may have limited assets invested in other types of securities.

Equity Market Neutral

Equity Market Neutral investing seeks to profit by exploiting pricing inefficiencies between related equity securities, neutralizing exposure to market risk by combining long and short positions. One example of this strategy is to build portfolios made up of long

positions in the strongest companies in several industries and taking corresponding short positions in those showing signs of weakness.

Equity Market Neutral: Statistical Arbitrage

Equity Market Neutral: Statistical Arbitrage utilizes quantitative analysis of technical factors to exploit pricing inefficiencies between related equity securities, neutralizing exposure to market risk by combining long and short positions. The strategy is based on quantitative models for selecting specific stocks with equal dollar amounts comprising the long and short sides of the portfolio. Portfolios are typically structured to be market, industry, sector, and dollar neutral.

Equity Non-Hedge

Equity Non-Hedge funds are predominately long equities although they have the ability to hedge with short sales of stocks and/or stock index options. These funds are commonly known as "stock-pickers." Some funds employ leverage to enhance returns. When market conditions warrant, managers may implement a hedge in the portfolio. Funds may also opportunistically short individual stocks. The important distinction between equity non-hedge funds and equity hedge funds is equity non-hedge funds do not always have a hedge in place. In addition to equities, some funds may have limited assets invested in other types of securities.

Event-Driven

Event-Driven is also known as "corporate life cycle" investing. This involves investing in opportunities created by significant transactional events, such as spin-offs, mergers and acquisitions, bankruptcy reorganizations, recapitalizations and share buybacks. The portfolio of some Event-Driven managers may shift in majority weighting between Risk Arbitrage and Distressed Securities, while others may take a broader scope. Instruments include long and short common and preferred stocks, as well as debt securities and options. Leverage may be used by some managers. Fund managers may hedge against market risk by purchasing S&P put options or put option spreads.

Fixed Income: Arbitrage

Fixed Income: Arbitrage is a market neutral hedging strategy that seeks to profit by exploiting pricing inefficiencies between related fixed income securities while neutralizing exposure to interest rate risk. Fixed Income Arbitrage is a generic description of a variety of strategies involving investment in fixed income instruments, and weighted in an attempt to eliminate or reduce exposure to changes in the yield curve. Managers attempt to exploit relative mispricing between related sets of fixed income securities. The generic types of fixed income hedging trades include: yield-curve arbitrage, corporate versus Treasury yield spreads, municipal bond versus Treasury yield spreads and cash versus futures.

Fixed Income: Convertible Bonds

Fixed Income: Convertible Bonds funds are primarily long only convertible bonds. Convertible bonds have both fixed income and equity characteristics. If the underlying common stock appreciates, the convertible bond's value should rise to reflect this

increased value. Downside protection is offered because if the underlying common stock declines, the convertible bond's value can decline only to the point where it behaves like a straight bond.

Fixed Income: Diversified

Fixed Income: Diversified funds may invest in a variety of fixed income strategies. While many invest in multiple strategies, others may focus on a single strategy less followed by most fixed income hedge funds. Areas of focus include municipal bonds, corporate bonds, and global fixed income securities.

Fixed Income: High-Yield

Fixed Income: High-Yield managers invest in non-investment grade debt. Objectives may range from high current income to acquisition of undervalued instruments. Emphasis is placed on assessing credit risk of the issuer. Some of the available high-yield instruments include extendible/reset securities, increasing-rate notes, pay-in-kind securities, step-up coupon securities, split-coupon securities and usable bonds.

Fixed Income: Mortgage Backed Securities

Fixed Income: Mortgage-Backed funds invest in mortgage-backed securities. Many funds focus solely on AAA-rated bonds. Instruments include: government agency, government-sponsored enterprise, private-label fixed- or adjualphastabe-rate mortgage pass-through securities, fixed- or adjualphastabe-rate collateralized mortgage obligations (CMOs), real estate mortgage investment conduits (REMICs) and stripped mortgage-backed securities (SMBSs). Funds may look to capitalize on security-specific mispricings. Hedging of prepayment risk and interest rate risk is common. Leverage may be used, as well as futures, short sales and options.

Macro

Macro involves investing by making leveraged bets on anticipated price movements of stock markets, interest rates, foreign exchange and physical commodities. Macro managers employ a "top-down" global approach, and may invest in any markets using any instruments to participate in expected market movements. These movements may result from forecasted shifts in world economies, political fortunes or global supply and demand for resources, both physical and financial. Exchange-traded and over-the-counter derivatives are often used to magnify these price movements.

Market Timing

Market Timing involves allocating assets among investments by switching into investments that appear to be beginning an uptrend, and switching out of investments that appear to be starting a downtrend. This primarily consists of switching between mutual funds and money markets. Typically, technical trend-following indicators are used to determine the direction of a fund and identify buy and sell signals. In an up move "buy signal," money is transferred from a money market fund into a mutual fund in an attempt to capture a capital gain. In a down move "sell signal," the assets in the mutual fund are sold and moved back into the money market for safe keeping until the next up move. The goal is to avoid being invested in mutual funds during a market decline.

Merger Arbitrage/Risk Arbitrage
Merger Arbitrage, sometimes called Risk Arbitrage, involves investment in event-driven situations such as leveraged buy-outs, mergers and hostile takeovers. Normally, the stock of an acquisition target appreciates while the acquiring company's stock decreases in value. These strategies generate returns by purchasing stock of the company being acquired, and in some instances, selling short the stock of the acquiring company. Managers may employ the use of equity options as a low-risk alternative to the outright purchase or sale of common stock. Most Merger Arbitrage funds hedge against market risk by purchasing S&P put options or put option spreads.

Regulation D
Regulation D Managers invest in Regulation D securities, sometimes referred to as structured discount convertibles. The securities are privately offered to the investment manager by companies in need of timely financing and the terms are negotiated. The terms of any particular deal are reflective of the negotiating strength of the issuing company. Once a deal is closed, there is a waiting period for the private share offering to be registered with the SEC. The manager can only convert into private shares and cannot trade them publicly during this period; therefore their investment is illiquid until it becomes registered. Managers will hedge with common stock until the registration becomes effective and then liquidate the position gradually.

Relative Value Arbitrage
Relative Value Arbitrage attempts to take advantage of relative pricing discrepancies between instruments including equities, debt, options and futures. Managers may use mathematical, fundamental, or technical analysis to determine misvaluations. Securities may be mispriced relative to the underlying security, related securities, groups of securities, or the overall market. Many funds use leverage and seek opportunities globally. Arbitrage strategies include dividend arbitrage, pairs trading, options arbitrage and yield curve trading.

Sector: Energy
Sector: Energy is a strategy that focuses on investment within the energy sector. Investments can be long and short in various instruments with funds either diversified across the entire sector or specializing within a sub-sector, i.e., oil field service.

Sector: Financial
Sector: Financial is a strategy that invests in securities of bank holding companies, banks, thrifts, insurance companies, mortgage banks and various other financial services companies.

Sector: Healthcare/Biotechnology

Sector: Healthcare/Biotechnology funds invest in companies involved in the healthcare, pharmaceutical, biotechnology, and medical device areas.

Sector: Metals/Mining
Sector: Metals/Mining funds invest in securities of companies primarily focused on mining, processing and dealing in precious metals and other commodities. Some funds may employ arbitrage strategies on a worldwide basis.

Sector: Real Estate
Sector: Real Estate involves investing in securities of real estate investment trusts (REITs) and other real estate companies. Some funds may also invest directly in real estate property.

Sector: Technology
Sector: Technology funds emphasize investment in securities of the technology arena. Some of the sub-sectors include multimedia, networking, PC producers, retailers, semiconductors, software, and telecommunications.

Short Selling
Short Selling involves the sale of a security not owned by the seller; a technique used to take advantage of an anticipated price decline. To effect a short sale, the seller borrows securities from a third party in order to make delivery to the purchaser. The seller returns the borrowed securities to the lender by purchasing the securities in the open market. If the seller can buy that stock back at a lower price, a profit results. If the price rises, however, a loss results. A short seller must generally pledge other securities or cash with the lender in an amount equal to the market price of the borrowed securities. This deposit may be increased or decreased in response to changes in the market price of the borrowed securities.

Fund of Funds Composite Index
Fund of Funds invest with multiple managers through funds or managed accounts. The strategy designs a diversified portfolio of managers with the objective of significantly lowering the risk (volatility) of investing with an individual manager. The Fund of Funds manager has discretion in choosing which strategies to invest in for the portfolio. A manager may allocate funds to numerous managers within a single strategy, or with numerous managers in multiple strategies. The minimum investment in a Fund of Funds may be lower than an investment in an individual hedge fund or managed account. The investor has the advantage of diversification among managers and styles with significantly less capital than investing with separate managers.

HFRI FOF: Conservative
FOFs classified as "Conservative" exhibit one or more of the following characteristics: seeks consistent returns by primarily investing in funds that generally engage in more "conservative" strategies such as Equity Market Neutral, Fixed Income Arbitrage, and Convertible Arbitrage; exhibits a lower historical annual standard deviation than the

HFRI Fund of Funds Composite Index. A fund in the HFRI FOF Conservative Index shows generally consistent performance regardless of market conditions.

HFRI FOF: Diversified
FOFs classified as "Diversified" exhibit one or more of the following characteristics: invests in a variety of strategies among multiple managers; historical annual return and/or a standard deviation generally similar to the HFRI Fund of Fund Composite index; demonstrates generally close performance and returns distribution correlation to the HFRI Fund of Fund Composite Index. A fund in the HFRI FOF Diversified Index tends to show minimal loss in down markets while achieving superior returns in up markets.

HFRI FOF: Market Defensive
FOFs classified as "Market Defensive" exhibit one or more of the following characteristics: invests in funds that generally engage in short-biased strategies such as short selling and managed futures; shows a negative correlation to the general market benchmarks (S&P). A fund in the FOF Market Defensive Index exhibits higher returns during down markets than during up markets.

Table 1: Summary Statistics of Hedge Fund Research Strategy Indexes Till December 1999

	Annualized Return	Annualized Volatility	Kurtosis	Skew	Sharpe Ratio	Hurst	FDI
Convertible Arbitrage Index	10.96%	3.53%	3.37	-1.46	1.70	0.5687	1.43
CIS Index	23.54%	39.48%	1.94	-0.11	0.47	0.5886	1.41
Distressed Security Index	15.69%	6.57%	6.56	-0.88	1.63	0.6265	1.37
Event-Driven Index	16.29%	6.71%	7.26	-1.75	1.69	0.5521	1.45
Equity Hedge Index	21.87%	8.68%	1.83	-0.03	1.95	0.5284	1.47
Emerging Markets Index	17.67%	16.44%	4.00	-0.89	0.77	0.6045	1.40
Emerging Markets Asia Index	13.51%	14.36%	1.26	0.27	0.59	0.6495	1.35
Emerging Markets Global Index	19.50%	17.02%	11.84	-2.12	0.85	0.6105	1.39
Emerging Markets Latin America Index	26.23%	21.90%	2.88	0.60	0.97	0.6041	1.40
Equity Market Neutral Index	10.74%	3.11%	0.53	-0.10	1.85	0.5454	1.45
Non-Hedge Index	21.14%	13.54%	1.66	-0.64	1.19	0.4770	1.52
Fixed Income Index	11.66%	3.69%	5.63	-0.52	1.81	0.6516	1.35
Fixed Income Arbitrage Index	9.08%	4.95%	8.94	-1.85	0.83	0.6028	1.40
Fixed Income Convertible Bonds Index	16.28%	9.89%	4.42	-0.70	1.14	0.5443	1.46
Fixed Income Diversified Index	8.89%	3.94%	1.86	0.77	0.99	0.5647	1.44
Fixed High Index	10.67%	7.28%	5.53	-0.83	0.78	0.6472	1.35
Fixed Income Mortgage-Backed Index	10.54%	4.64%	39.75	-5.53	1.20	0.6173	1.38
Merger Arbitrage Index	12.06%	4.64%	14.28	-3.23	1.53	0.5351	1.46
Macro Index	19.24%	9.25%	0.13	0.14	1.54	0.6225	1.38
Market Timing Index	15.19%	6.66%	-0.58	0.10	1.53	0.5283	1.47
Regulation D Index	28.84%	5.20%	7.65	2.20	4.59	0.6405	1.36
RV Arbitrage Index	13.48%	4.12%	10.07	-1.21	2.06	0.5797	1.42
Sector Index	25.73%	11.84%	4.58	-0.22	1.75	0.5891	1.41
Sector Energy Index	28.50%	23.28%	-0.38	0.05	1.01	0.6058	1.39
Sector Finance Index	19.89%	11.95%	11.61	-2.26	1.25	0.6065	1.39
Sector Health Index	20.90%	18.20%	4.49	0.89	0.88	0.5565	1.44
Short Sell Index	0.28%	19.82%	0.74	0.15	-0.24	0.4497	1.55
Sector Real Estate Index	8.87%	7.16%	1.97	-0.46	0.54	0.6071	1.39
Equity Market Neutral Index Statistical Arb	11.34%	3.73%	0.40	0.18	1.71	0.5785	1.42
Sector Technology Index	30.96%	18.28%	1.90	0.31	1.42	0.5900	1.41

Table 2: Equally Weighted Portfolio vs. "Top 20" Portfolio		
	Portfolio A	**Portfolio B**
Annualized Return Since 2000	8.94%	7.72%
Annualized Vol Since 2000	5.77%	5.61%
Loss Frequency	26.66%	33.33%
Drawdown	-5.83%	-5.56%
Hurst	0.57	0.57
Skew	-0.23	0.04
Kurt	-0.11	0.05
Jarque Bera Statistic	0.60	0.02
Sharpe	1.02	0.63
Dstat	0.25	0.27
Downside Deviation	4.23%	3.94%
Sortino Ratio	1.47	1.27

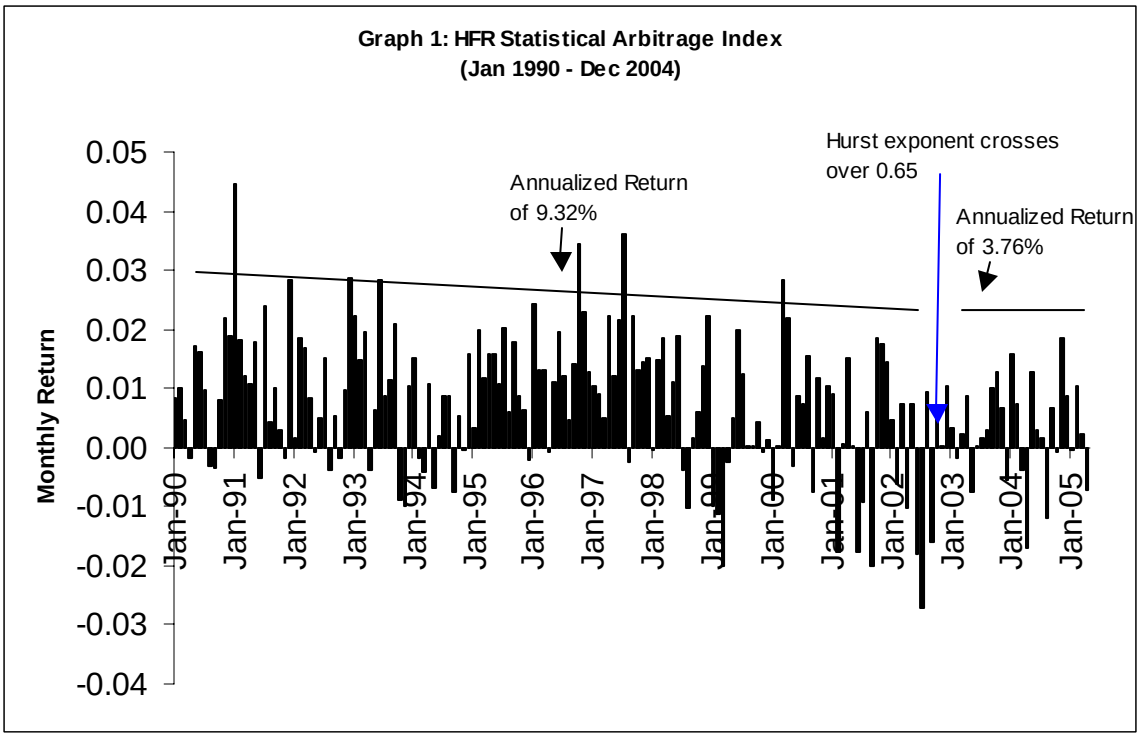
Table 3: Optimization with no Hurst exponent filtering		
	Portfolio C	**Portfolio D**
Annualized Return Since 2000	7.31%	8.22%
Annualized Vol Since 2000	3.55%	5.68%
Loss Frequency	23.30%	38.30%
Drawdown	-1.92%	-4.13%
Hurst	0.57	0.57
Skew	1.73	1.59
Kurt	5.91	4.79
Jarque Bera Statistic	91.06	64.09
Sharpe	1.30	0.97
Dstat	0.14	0.22
Downside Deviation	2.02%	3.24%
Sortino Ratio	2.28	1.71

Table 4: Optimization with Hurst exponent between 0.5 and 0.65	
	Portfolio E
Annualized Return Since 2000	7.60%
Annualized Vol Since 2000	3.90%
Loss Frequency	21.67%
Drawdown	-2.13%
Hurst	0.50
Skew	2.39
Kurt	10.27
Jarque Bera Statistic	250.99
Sharpe	1.26
Dstat	0.13
Downside Deviation	2.07%
Sortino Ratio	2.37

Table 5: Optimization with Hurst exponent between 0.4 and 0.6		
	Portfolio G	**Portfolio H**
Annualized Return Since 2000	7.17%	6.35%
Annualized Vol Since 2000	3.06%	4.49%
Loss Frequency	21.67%	26.67%
Drawdown	-2.13%	-2.98%
Hurst	0.60	0.52
Skew	1.31	1.97
Kurt	4.16	8.10
Jarque Bera Statistic	46.06	157.54
Sharpe	1.46	0.81
Dstat	0.12	0.22
Downside Deviation	1.82%	2.54%
Sortino Ratio	2.46	1.44

Table 6: Optimization with Hurst exponent between 0.4 and 0.65		
	Portfolio I	**Portfolio J**
Annualized Return Since 2000	7.78%	8.69%
Annualized Std Dev Since 2000	3.38%	5.37%
Loss Frequency	16.70%	26.67%
Drawdown	-2.00%	-4.10%
Hurst	0.60	0.56
Skew	2.03	1.80
Kurt	7.46	6.23
Jarque Bera Statistic	140.91	100.79
Sharpe	1.50	1.12
Dstat	0.10	0.18
Downside Deviation	1.82%	2.98%
Sortino Ratio	2.79	2.01

Table 7: Statistics for Various HFR Fund of Hedge Fund Indexes				
	Conservative Index	**Market Defensive Index**	**Diversified Index**	**General**
Annualized Return Since 2000	5.34%	8.27%	4.93%	5.19%
Annualized Vol Since 2000	2.43%	4.85%	4.65%	4.45%
Loss Frequency	26.67%	28.33%	31.67%	33.33%
Drawdown	-1.85%	-6.32%	-6.02%	-4.95%
Hurst	0.60	0.53	0.59	0.58
Skew	-0.06	0.14	0.14	0.41
Kurt	-0.59	0.65	4.20	3.00
Jarque Bera Statistic	0.52	0.53	31.46	16.82
Sharpe	1.08	1.15	0.48	0.56
Dstat	0.18	0.22	0.30	0.29
Downside Deviation	1.73%	3.38%	3.26%	3.04%
Sortino Ratio	1.52	1.65	0.69	0.82


Graph 1: HFR Statistical Arbitrage Index
(Jan 1990 - Dec 2004)
Monthly Return
0.05
0.04
0.03
0.02
0.01
0.00
-0.01
-0.02
-0.03
-0.04
Jan-90
Jan-91
Jan-92
Jan-93
Jan-94
Jan-95
Jan-96
Jan-97
Jan-98
Jan-99
Jan-00
Jan-01
Jan-02
Jan-03
Jan-04
Jan-05
Annualized Return
of 9.32%
Hurst exponent crosses
over 0.65
Annualized Return
of 3.76%