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All submissions for the IABE-2004 Annual Conference will be double blind reviewed on a continual basis. Normally the review process will be completed in about 6-8 weeks after the submission and the author(s) will be informed of the result of the review process. If the paper/case is accepted for the JABE, we will invite the author(s) to submit the manuscript for the JABE and to present the paper at the IABE-2004 Annual Conference. Authors of all other recommended papers/cases and abstracts will be invited to present their submissions at IABE-2004 Annual Conference.

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Journal of Academy of Business and Economics
Volume III, Number 1, 2004

INTRODUCTION

We are excited to present to you a new issue of the *Journal of Academy of Business and Economics,* Volume III, Number 1, 2004. In this issue of the journal (JABE), we have published 25 high quality research articles in Finance, Economics, Accounting, Management and related areas. Each article has successfully undergone a double blind review process. The JABE is a peer-reviewed journal listed in the Cabell's Directory 2004-05 Edition. The journal has ISSN number (ISSN: 1542-8710) issued by the Library of Congress, Washington, DC. The JABE is also listed in the Ulrich's International Periodicals Directory. The JABE will be available online from the Gale Group/ Thomson Publishing. JABE is the flagship publication of the International Academy of Business and Economics (IABE). All rights reserved. ©2004 IABE.

The objective of the journal is to create and provide a worldwide forum for faculty, professionals, and students to publish and share developments in the business, economics, and related fields, particularly relevant at the international level, to help continuously improve teaching, scholarship, and practice.

On behalf of the Executive Committee of the International Academy of Business and Economics, I sincerely thank to all our reviewers for their invaluable timely help in reviewing the papers. The editorial board of the IABE has significantly contributed towards the success of the journal and we commend the editorial board. We express our sincere thanks to all the authors who submitted their papers for review for the journal.

We look forward to your participation and support for continued success of the journal.

Best Regards,

Alan S. Khade, Ph.D.
Managing Editor

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A MODEL TO PRICE PUTTABLE CORPORATE BONDS WITH DEFAULT RISK

David Wang, Hsuan Chuang University, Hsinchu, Taiwan

ABSTRACT

This paper presents a model for pricing puttable corporate bonds that are subject to default risk. The model incorporates three essential ingredients in the pricing of defaultable puttable bonds: stochastic interest rate, default risk, and put provision. The stochastic interest rate is modeled as a square-root diffusion process. The default risk is modeled as a constant spread, with the magnitude of this spread impacting the probability of a Poisson process governing the arrival of the default event. The put provision is modeled as a constraint on the value of the bond in the finite difference scheme. This paper can be used both as a benchmark for models for pricing puttable corporate bonds that are subject to default risk and as a direction for future research.

1. INTRODUCTION

The pricing of defaultable securities has been of interest in the academic and practitioner literature for some time. The standard theoretical paradigm for pricing defaultable securities is the contingent claims approach pioneered by Black and Scholes (1973). Much of the literature follows Merton (1974) by explicitly linking the risk of a firm's default to the variability in the firm's asset value. Although this line of research has proven very useful in addressing the qualitatively important aspects of pricing defaultable securities, it has been less successful in practical applications. The lack of success owes to the fact that firms' capital structures are typically quite complex and priority rules are often violated. In response to these difficulties, an alternative modeling approach has been pursued in a number of papers, including Madan and Unal (1994), Jarrow and Turnbull (1995), Duffie and Singleton (1999). At each instant, there is some probability that a firm defaults on its obligation. This is called the instantaneous probability of default. The processes of both this probability and the recovery rate determine the value of default risk. Although these processes are not formally linked to the firm's asset value, there is presumably some underlying relation, thus Duffie and Singleton describe this alternative approach as a reduced-form model (Duffee, 1999). This paper is an effort to develop one such model for pricing puttable corporate bonds that are subject to default risk.

2. MODEL

I derive the pricing model for defaultable bonds by adopting the reduced-form approach by Duffie and Singleton (1999) and the replicating-portfolio approach by Neftci (2000).

2.1 Reduced-Form Approach

Reduced-form approaches directly assume that defaultable bonds can be valued by discounting at a default-adjusted interest rate. Specifically, I fix some defaultable discount bond that, in the event of no default, pays a face value X at maturity time T. I take as given an arbitrage-free setting in which all securities are priced in terms of some short-term interest rate process r and equivalent martingale measure Q. Under this risk-neutral probability measure, I let h denote the hazard rate for default (i.e., the instantaneous probability of default) at time t and let L denote the loss rate (i.e., the expected fractional loss in the market value) if default were to occur at time t, conditional on the information available up to time t. Under technical conditions, this defaultable discount bond can be priced as if it were default-free by replacing the usual short-term interest rate process r with the default-adjusted short-term interest rate process:

$$R = r + hL. \qquad (1)$$

That is, the price at time 0 of the defaultable discount bond is:

$$B_0 = E_0^Q[\exp(-\int_0^T R_t dt)X], \qquad (2)$$

where E_0^Q denotes risk-neutral, conditional expectation at date 0. This is natural, in that hL is the risk neutral mean-loss rate of the defaultable discount bond due to default. Discounting at the default-adjusted short-term interest rate R therefore accounts for both the probability and timing of default, as well as for the effect of losses on default. A key feature of Equation (2) is that, assuming the risk neutral mean-loss rate process hL being given exogenously, standard term-structure models for default-free debt are directly applicable to defaultable debt by parameterizing R instead of r.

2.2 Replicating-Portfolio Approach
I assume that the default-adjusted term structure R fits a Cox, Ingersoll, and Ross (CIR)-style model (1985). The model is extended to defaultable bonds by assuming a constant risk neutral mean-loss rate hL. Specifically, assume two stochastic differential equations (SDEs) describing the dynamics of two defaultable discount bond prices, $B(t, T_1)$ and $B(t, T_2)$, with maturities T_1 and T_2. The bond prices are driven by the same Wiener process, z. To simplify the notation, I write:

$$B^1 = B(t,\ T_1), \qquad (3)$$
$$B^2 = B(t,\ T_2). \qquad (4)$$

The bond prices are postulated to have the following dynamics:

$$dB^1 = \mu_1\,(B^1,\ t)\,B^1\ dt + \sigma_1\,(B^1,\ t)\,B^1\ dz, \qquad (5)$$
$$dB^2 = \mu_2\,(B^2,\ t)\,B^2\ dt + \sigma_2\,(B^2,\ t)\,B^2\ dz. \qquad (6)$$

Let the dynamics of the default-adjusted interest rate, R, be given by a CIR-style model, a square-root diffusion model:

$$dR = a(b - R)dt + \sigma\sqrt{R}dz, \qquad (7)$$

where $R = r + hL$, and the drift and the diffusion parameters are constants and are assumed to be known. The CIR-style model incorporates mean reversion. The default-adjusted interest rate is pulled to a level b at rate a. The standard deviation is proportional to $\sqrt{R}$. This ensures that the default-adjusted interest rates are always non-negative. Form a risk-free portfolio, P, made of the two bonds, B^1 and B^2, at time t. In particular, I assume that θ_1 units of B^1 are purchased and θ_2 units of B^2 are shorted, for a total portfolio value:

$$P = \theta_1 B^1 - \theta_2 B^2. \qquad (8)$$

Suppose the portfolio weights are chosen as:

$$\theta_1 = \frac{\sigma_2}{B^1(\sigma_2 - \sigma_1)} P \qquad (9)$$

$$\theta_2 = \frac{\sigma_1}{B^2(\sigma_2 - \sigma_1)} P, \qquad (10)$$

where σ_1 and σ_2 are the volatility parameters, $\sigma_1\,(B^1,\ t)$ and $\sigma_2\,(B^2,\ t)$, of the two bonds as described in Equations (5) and (6). As time passes, the portfolio's value will change. Acting as if the portfolio weights are constant, the implied infinitesimal changes will be given by:

$$dP = \theta_1\,dB^1 - \theta_2\,dB^2. \qquad (11)$$

Replacing dB^1 and dB^2 with Equations (5) and (6) gives:

$$\begin{aligned} dP = {} & \theta_1\,[\mu_1\,(B^1,\ t)\,B^1\ dt + \sigma_1\,(B^1,\ t)\,B^1 dz] \\ & - \theta_2\,[\mu_2\,(B^2,\ t)\,B^2\ dt + \sigma_2\,(B^2,\ t)\,B^2\ dz]. \end{aligned} \qquad (12)$$

Substituting for θ_1 and θ_2 from Equations (9) and (10) gives:

$$dP = \frac{\sigma_2}{B^1(\sigma_2 - \sigma_1)} P \; [\mu_1 (B^1, t) B^1 \, dt + \sigma_1 (B^1, t) B^1 \, dz]$$

$$- \frac{\sigma_1}{B^2(\sigma_2 - \sigma_1)} P \; [\mu_2 (B^2, t) B^2 \, dt + \sigma_2 (B^2, t) B^2 \, dz]. \qquad (13)$$

After rearranging and simplifying the notation, *dP* can be written as:

$$dP = \frac{(\sigma_2 \mu_1 - \sigma_1 \mu_2)}{(\sigma_2 - \sigma_1)} Pdt + \frac{(\sigma_2 \sigma_1 - \sigma_1 \sigma_2)}{(\sigma_2 - \sigma_1)} Pdz, \qquad (14)$$

or after dropping the *dz* term:

$$dP = \frac{(\sigma_2 \mu_1 - \sigma_1 \mu_2)}{(\sigma_2 - \sigma_1)} Pdt. \qquad (15)$$

This SDE does not contain a diffusion term and the dynamic behavior of *dP* is riskless. Hence, I can now use the standard argument and claim that this portfolio should not present any arbitrage opportunities and its deterministic return should equal *RPdt*.

$$\frac{(\sigma_2 \mu_1 - \sigma_1 \mu_2)}{(\sigma_2 - \sigma_1)} Pdt = RPdt. \qquad (16)$$

Simplifying *P*, *dt*, and rearranging, the equation becomes:

$$\frac{\mu_1 - R}{\sigma_1} = \frac{\mu_2 - R}{\sigma_2}. \qquad (17)$$

That is, the risk premia of per unit volatility are the same across bonds of different maturities. This result is not unexpected because at the end, all the bonds have the same source of risk given the common *dz* factor. Similar equalities should be true for all bonds as long as their dynamics are driven by the same Wiener process, *z*. This gives a term λ *(R, t)* that is relevant to all bond prices, *B(t, T)*:

$$\frac{\mu_i - R}{\sigma_i} = \lambda(R, t). \qquad (18)$$

This term is called the market price of default-adjusted interest rate risk. Since *B(t, T)* is also a function of *R*, I can apply Ito's lemma:

$$dB(R, t) = \frac{\partial B}{\partial R} dR + \frac{\partial B}{\partial t} dt + \frac{1}{2} \frac{\partial^2 B}{\partial R^2} (\sigma \sqrt{R})^2 dt. \qquad (19)$$

Substituting for *dR* from Equation (7), the equation becomes:

$$dB(R, t) = [\frac{\partial B}{\partial R} a(b - R) + \frac{\partial B}{\partial t} + \frac{1}{2} \frac{\partial^2 B}{\partial R^2} (\sigma \sqrt{R})^2] dt + \frac{\partial B}{\partial R} \sigma \sqrt{R} dz. \qquad (20)$$

This SDE must be identical to Equation (5) or (6), the original equation that drives the bond price dynamics. Simplifying the notation, Equation (5) or (6) can be shown as:

$$dB = \mu \, (B, t) \, B \, dt + \sigma \, (B, t) \, B \, dz. \qquad (21)$$

Equating the drifts in Equations (20) and (21) gives:

$$\mu \, (B, t) \, B = \frac{\partial B}{\partial R} a(b - R) + \frac{\partial B}{\partial t} + \frac{1}{2} \frac{\partial^2 B}{\partial R^2} (\sigma \sqrt{R})^2, \qquad (22)$$

or after rearranging:

$$\mu\,(B,\,t) = \frac{\partial B}{\partial R} a(b-R)\frac{1}{B} + \frac{\partial B}{\partial t}\frac{1}{B} + \frac{1}{2}\frac{\partial^2 B}{\partial R^2}(\sigma\sqrt{R})^2\,\frac{1}{B}. \tag{23}$$

Setting the diffusion coefficients in Equations (20) and (21) equal to each other gives:

$$\sigma\,(B,\,t)\,B = \frac{\partial B}{\partial R}\sigma\sqrt{R}, \tag{24}$$

or after rearranging:

$$\sigma\,(B,\,t) = \frac{\partial B}{\partial R}\sigma\sqrt{R}\,\frac{1}{B}. \tag{25}$$

Equation (18) gives the market price of default-adjusted interest rate risk, $\lambda\,(R,\,t)$, as:

$$\frac{\mu(B,t)-R}{\sigma(B,t)} = \lambda(R,t). \tag{26}$$

Substituting for $\mu\,(B,\,t)$ and $\sigma\,(B,\,t)$ from Equations (23) and (25), the equation becomes:

$$\frac{[\frac{\partial B}{\partial R} a(b-R)\frac{1}{B} + \frac{\partial B}{\partial t}\frac{1}{B} + \frac{1}{2}\frac{\partial^2 B}{\partial R^2}(\sigma\sqrt{R})^2\,\frac{1}{B}] - R}{[\frac{\partial B}{\partial R}\sigma\sqrt{R}\,\frac{1}{B}]} = \lambda(R,t). \tag{27}$$

After rearranging, the equation can be written as:

$$\frac{\partial B}{\partial R}[a(b-R) - \sigma\sqrt{R}\lambda] + \frac{\partial B}{\partial t} + \frac{1}{2}\frac{\partial^2 B}{\partial R^2}(\sigma\sqrt{R})^2 - RB = 0. \qquad \text{Q.E.D.} \tag{28}$$

This is the partial differential equation (PDE) for the price of a defaultable discount bond, *B(R, t)*, when the default-adjusted interest rate, *R*, is assumed to follow a CIR-style model. On a coupon date, the bond value must jump by the amount of the coupon payment. Hence, to incorporate coupon payments into the model, I impose a jump condition:

$$B(R,t_C^-) = B(R,t_C^+) + K_C\,, \tag{29}$$

where a coupon of K_C is received at time t_C. Some bonds have a put feature. This right permits the holder of the bond to return it to the issuing company at any time during specified periods for a specified amount. According to the no-arbitrage argument, to incorporate a put feature into the model, I must impose a constraint on the bond's value:

$$B(R,t_E) \geq X_E\,, \tag{30}$$

where X_E is the put price and t_E is the put date. To find a unique solution of Equation (28), I must impose one final condition and two boundary conditions. The final condition corresponds to the payoff at maturity and so for a coupon-paying bond:

$$B(R,T) = P_T + K_T\,, \tag{31}$$

where a principal amount of P_T and a coupon payment of K_T are received at maturity. The first boundary condition, when the default-adjusted interest rate, *R*, approaches to zero percent, can be stated as:

$$\begin{aligned} B(R,t) &= B(R,T)e^{-R(T-t)} \\ &= B(R,T). \end{aligned} \qquad (32)$$

The second boundary condition, when the default-adjusted interest rate, R, approaches to infinity, can be stated as:

$$\begin{aligned} B(R,t) &= B(R,T)e^{-R(T-t)} \\ &= 0. \end{aligned} \qquad (33)$$

3. METHODOLOGY

I solve the pricing model for defaultable puttable bonds by the fully implicit finite difference method (Hull, 2003; Wilmott, 2000).

Suppose that the number of months to maturity is T. I divide this into N equally spaced intervals of length $\Delta t = T/N$. Δt is fixed at one month. A total of $N+1$ times are, therefore, considered:

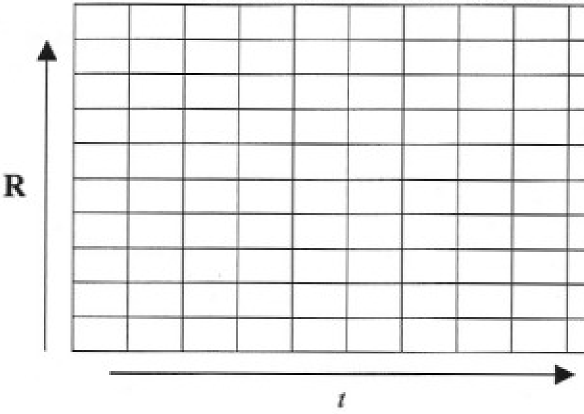
$$0,\ \Delta t,\ 2\Delta t,\ \ldots,\ T.$$

Suppose that R_{max} is a default-adjusted interest rate sufficiently high that, when it is reached, the bond has virtually no value. I define $\Delta R = R_{max}/M$ and consider a total of $M+1$ equally spaced default-adjusted interest rates:

$$0,\ \Delta R,\ 2\Delta R,\ \ldots,\ R_{max}.$$

ΔR is set to be one percent. The time points and default-adjusted interest rate points define a grid consisting of a total of $(M+1)(N+1)$ points as shown in Figure 1. The (i, j) point on the grid is the point that corresponds to time $i\Delta t$ and default-adjusted interest rate $j\Delta R$. I use the variable $B_{i,j}$ to denote the value of the bond at the (i, j) point.

FIGURE 1
THE FINITE DIFFERENCE GRID



Recall that the differential equation for the price of a defaultable puttable bond, $B(R, t)$, is given as:

$$\frac{\partial B}{\partial R}[a(b-R)-\sigma\sqrt{R}\lambda]+\frac{\partial B}{\partial t}+\frac{1}{2}\frac{\partial^2 B}{\partial R^2}(\sigma\sqrt{R})^2-RB=0. \qquad (34)$$

For an interior point (i, j) on the grid, $\frac{\partial B}{\partial R}$ can be approximated by using a symmetric difference approximation:

$$\frac{\partial B}{\partial R} = \frac{B_{i,j+1} - B_{i,j-1}}{2\Delta R}, \tag{35}$$

$\frac{\partial B}{\partial t}$ can be approximated by using a forward difference approximation:

$$\frac{\partial B}{\partial t} = \frac{B_{i+1,j} - B_{i,j}}{\Delta t}, \tag{36}$$

and $\frac{\partial^2 B}{\partial R^2}$ can be approximated by using a backward difference approximation:

$$\begin{aligned}\frac{\partial^2 B}{\partial R^2} &= \left(\frac{B_{i,j+1} - B_{i,j}}{\Delta R} - \frac{B_{i,j} - B_{i,j-1}}{\Delta R}\right) / \Delta R \\ &= \frac{B_{i,j+1} + B_{i,j-1} - 2B_{i,j}}{\Delta R^2}.\end{aligned} \tag{37}$$

Substituting equations (35), (36), and (37) into the differential equation (34) and noting that $R = j\Delta R$ and $B = B_{i,j}$, the corresponding difference equation can be shown as:

$$\begin{aligned}&\frac{B_{i,j+1} - B_{i,j-1}}{2\Delta R}[a(b - j\Delta R) - \sigma\sqrt{j\Delta R\lambda}] \\ &+ \frac{B_{i+1,j} - B_{i,j}}{\Delta t} \\ &+ \frac{1}{2}\frac{B_{i,j+1} + B_{i,j-1} - 2B_{i,j}}{\Delta R^2}(\sigma\sqrt{j\Delta R})^2 \\ &- (j\Delta R)B_{i,j} = 0,\end{aligned} \tag{38}$$

where $i = 0, 1, \ldots, N-1$ and $j = 1, 2, \ldots, M-1$. Rearranging terms, this equation becomes:

$$X_j B_{i,j-1} + Y_j B_{i,j} + Z_j B_{i,j+1} = B_{i+1,j}, \tag{39}$$

where

$$\begin{aligned}X_j &= \frac{1}{2\Delta R}[a(b - j\Delta R) - \sigma\sqrt{j\Delta R\lambda}]\Delta t - \frac{1}{2\Delta R^2}(\sigma\sqrt{j\Delta R})^2\Delta t, \\ Y_j &= 1 + \frac{1}{\Delta R^2}(\sigma\sqrt{j\Delta R})^2\Delta t + (j\Delta R)\Delta t, \\ Z_j &= -\frac{1}{2\Delta R}[a(b - j\Delta R) - \sigma\sqrt{j\Delta R\lambda}]\Delta t - \frac{1}{2\Delta R^2}(\sigma\sqrt{j\Delta R})^2\Delta t,\end{aligned}$$

$i = 0, 1, \ldots, N-1$, and $j = 1, 2, \ldots, M-1$.

The value of the bond at time T is $P_T + K_T$, where P_T is the principal amount and K_T is the coupon payment. Hence,

$$B_{i,j} = P_T + K_T \tag{40}$$

for $i = N$ and $j = 0, 1, \ldots, M$. The value of the bond when the default-adjusted interest rate is zero percent is $B(R,T)$. Hence,

$$B_{i-1,j} = B_{i,j} \tag{41}$$

for $i = 0, 1, \ldots, N$ and $j = 0$. I assume that the bond is worth zero when the default-adjusted interest rate is one hundred percent, so that

$$B_{i,j} = 0 \tag{42}$$

for $i = 0, 1, \ldots, N-1$ and $j = M$. To incorporate coupon payments into the model, I impose a jump condition. Hence,

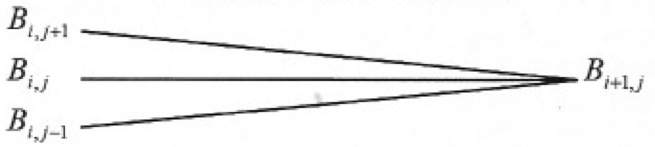
$$B_{i,j} = B_{i,j} + K_C \tag{43}$$

for $i = t_C$ or the coupon date, $j = 0, 1, \ldots, M-1$, and K_C is the coupon payment. To incorporate put features into the model, I impose a constraint on the bond's value. Hence,

$$B_{i,j} \geq X_E \tag{44}$$

for $i = t_E$ or the put date, $j = 0, 1, \ldots, M-1$, and X_E is the put price. Equations (40), (41), and (42) define the value of the bond along the three edges of the grid in Figure 1, where $R = 0\%$, $R = 100\%$, and $t = T$. Equation (39) defines the value of the bond at all other points. Equation (39) shows that there are three unknown bond values linked to one known bond value. See Figure 2. Hence, for each time layer there are $M-1$ simultaneous equations in $M-1$ unknowns; the boundary conditions yield the two missing values for each time layer and the final condition gives the values in the last time layer.

FIGURE 2: THE RELATIONSHIP BETWEEN BOND VALUES IN THE FULLY IMPLICIT FINITE DIFFERENCE METHOD



To find the bond value of interest, go backwards in time, solving for a sequence of systems of linear equations. I compute the solution to the system of linear equations using the Gaussian elimination algorithm. The system of linear equations at time layer i is the following:

$$\left|\begin{array}{ccccccc} Y_1 & Z_1 & & & & & \\ X_2 & Y_2 & Z_2 & & & & \\ & X_3 & Y_3 & Z_3 & & & \\ & & & \cdots & & & \\ & & & & X_{M-2} & Y_{M-2} & Z_{M-2} \\ & & & & & X_{M-1} & Y_{M-1} \end{array}\right| \left|\begin{array}{c} B_{i,1} \\ B_{i,2} \\ B_{i,3} \\ \cdots \\ B_{i,M-2} \\ B_{i,M-1} \end{array}\right|$$

$$= \left|\begin{array}{l} B_{i+1,1} - B_{i,0} X_1 \\ B_{i+1,2} \\ B_{i+1,3} \\ \qquad \cdots \\ B_{i+1,M-2} \\ B_{i+1,M-1} - B_{i,M} Z_{M-1} \end{array}\right| \tag{45}$$

Eventually, $B_{0,1}$, $B_{0,2}$, $B_{0,3}$, …, $B_{0,M-1}$ are obtained. One of these is the bond price of interest. If the initial default-adjusted interest rate does not lie on the grid point, I use a linear interpolation between the two bond prices on the neighboring grid points to find the bond price of interest.

4. CONCLUSION

This paper presents a model for pricing puttable corporate bonds that are subject to default risk. The model incorporates three essential ingredients in the pricing of defaultable puttable bonds: stochastic interest rate, default risk, and put provision. The stochastic interest rate is modeled as a square-root diffusion process. The default risk is modeled as a constant spread, with the magnitude of this spread impacting the probability of a Poisson process governing the arrival of the default event. The put provision is modeled as a constraint on the value of the bond in the finite difference scheme. The model is by no means a complete success. Both the default risk and the recovery rate in the event of default may vary stochastically through time. In addition, the default risk process may be correlated with the default-free term structure (Duffee, 1999). To improve the model, one can assume that the default risk follows a stochastic process, with a modification that allows the default risk process to be correlated with the default-free term structure. In summary, this paper can be used both as a benchmark for models for pricing puttable corporate bonds that are subject to default risk and as a direction for future research.

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