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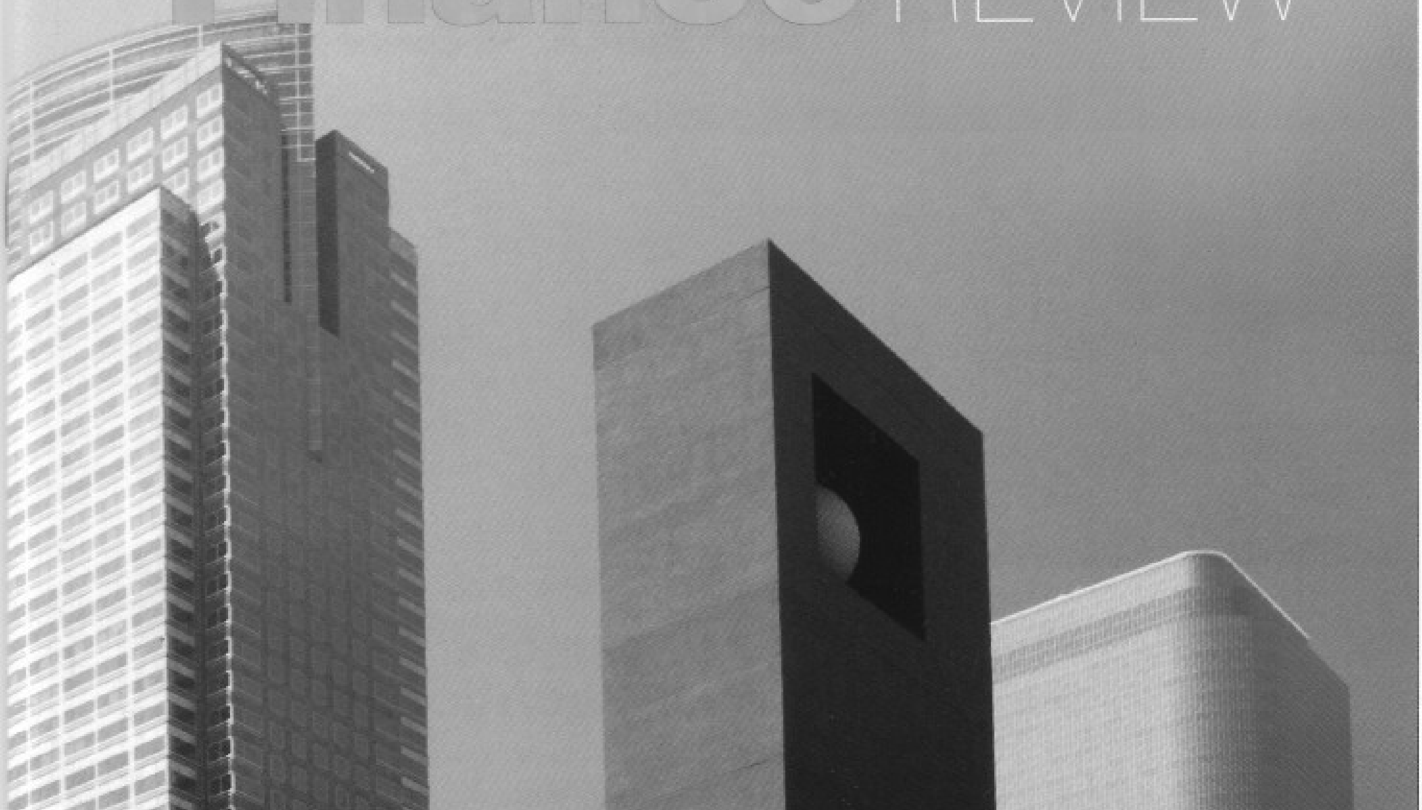

The Seven Deadly Sins
of Corporate Ethics Programs

Excessive Corporate Cash Holdings

The Power of Financial Analysts' Research Reports

CORPORATE
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ESTIMATING THE PROBABILITIES OF DEFAULT FOR CALLABLE BONDS: A DUFFIE-SINGLETON APPROACH

A model for estimating the default risks for callable corporate bonds, based on Duffie and Singleton's reduced-form approach, considers three essential elements: default-adjusted interest rate, default risk, and call provision.

DAVID WANG

Default risk has always been a major topic of concern for financial intermediaries and any agents committed to a financial contract. The standard theoretical paradigm for modeling default risks is the contingent claims approach pioneered by Black and Scholes.[1] Much of the literature follows Merton[2] by explicitly linking the risk of a firm's default to the variability in the firm's asset value. Although this line of research has proven very useful in addressing the qualitatively important aspects of estimating default risks, it has been less successful in practical applications. The lack of success owes to the difficulty of modeling realistic boundary conditions. These boundaries include both the conditions under which default occurs and, in the event of default, the division of the value of the firm among claimants. Firms' capital structures are typically quite complex and priority rules are often violated. In response to these difficulties, an alternative modeling approach has been pursued in a number of articles, including Madan and Unal,[3] Jarrow and Turnbull,[4] and Duffie and Singleton.[5] At each instant, there is some probability that a firm defaults on its obligation. This is called the instantaneous probability of default. The processes of both this probability and the recovery rate determine the value of default risk. Although these processes are not formally linked to the firm's asset value, there is presumably some underlying relation, thus Duffie and Single-

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ton describe this alternative approach as a reduced-form model. Although it is hoped that this class of models will be useful in practical applications, empirical implementation of this type of model is in its infancy. The few analyses that have been done to date fit such models to aggregate yield indexes, hence the ability of these models to estimate accurately the default risks associated with particular instruments is unknown.[6] This article is an effort to determine whether one such model can successfully estimate the default risks for callable corporate bonds.

AT EACH INSTANT, THERE IS SOME PROBABILITY THAT A FIRM DEFAULTS ON ITS OBLIGATION.

Methodology

First, I develop the pricing model for defaultable callable bonds by adopting Duffie and Singleton's reduced-form approach.[7] According to Duffie and Singleton, defaultable bonds can be valued by discounting at a default-adjusted interest rate, R:

$$R = r + hL, \qquad (1)$$

where r is the risk-free interest rate, h is the hazard rate (i.e., the instantaneous probability of default), and L is the loss rate (i.e., the expected fractional loss in the market value) if default were to occur. That is, the price at time 0 of a defaultable discount bond is:

$$B_0 - E_0^Q[\exp(-\int_0^T R_t dt)X], \qquad (2)$$

where X is the face value, T is the maturity time, and E_0^Q is the risk-neutral, conditional expectation at date 0.

Applying the classical replicating-portfolio technique, the pricing model for a defaultable bond can be shown as:

$$\frac{\partial B}{\partial R}[a(b-R)-\sigma\sqrt{R}\lambda]+\frac{\partial B}{\partial t}+\frac{1}{2}\frac{\partial^2 B}{\partial R^2}(\sigma\sqrt{R})^2 - RB = 0 \qquad (3)$$

This is the partial differential equation (PDE) for the price of a defaultable discount bond, $B(R, t)$, when the default-adjusted interest rate, R, is assumed to follow a Cox, Ingersoll, and Ross[8] model:

$$dR=a(b-R)dt+\sigma\sqrt{R}dz, \qquad (4)$$

where the drift and the diffusion parameters are constants. This model incorporates mean reversion. The default-adjusted interest rate is pulled to a level b at rate a. The standard deviation is proportional to $\sqrt{R}$. This ensures that the default-adjusted interest rates are always nonnegative.

On a coupon date, the bond value must jump by the amount of the coupon payment. Hence, to incorporate coupon payments into the model, I impose a jump condition:

$$B(R,t_C^-)=B(R,t_C^+)+K_C, \qquad (5)$$

where a coupon of K_C is received at time t_C. A callable bond is one that the issuer can call back on specified dates for a specified amount. For simplicity, I assume that a callable bond is called so as to minimize its market value. During the time window of callability, the issuer minimizes the market value of a callable bond by exercising the option to call back the bond if and only if its market value, if not called, is higher than the strike price on the call.[9] Hence, to incorporate call options into the model, I impose a constraint on the bond's value:

$$B(R, t_D) \le X_D, \qquad (6)$$

where X_D is the call price and t_D is the call date. To find a unique solution of Equation (1), I must impose one final condition and two boundary conditions. The final condition corresponds to the payoff at maturity and so for a coupon-paying bond:

$$B(R, T) = P_T + K_T, \qquad (7)$$

where a principal amount of P_T and a coupon payment of K_T are received at maturity. The first boundary condition, when the default-adjusted interest rate, R, approaches to zero percent, can be stated as:

$$B(R,t) = B(R,T)e^{-R(T-t)} = B(R,T). \qquad (8)$$

The second boundary condition, when the default-adjusted interest rate, R, approaches to infinity, can be stated as:

EXHIBIT 1 Summary Statistics for Corporate Bond Data

Variable	Mean	Standard Deviation	Minimum	Maximum
Rating (numbers)	5.08	1.58	1.00	9.00
Maturity (years)	8.47	5.11	1.00	29.50
Coupon payment (dollars)	92.41	17.43	53.15	142.50
Age (years)	3.03	1.76	0.10	12.10
Issue size (dollars)	1.46×10^8	1.18×10^8	8.40×10^4	1.62×10^9

Note: ***Ratings*** **are translated onto a linear scale in descending order from 1 to 9, where Aaa = 1, Aa = 2, A = 3, Baa = 4, Ba = 5, B = 6, Caa = 7, Ca = 8, and C = 9.** ***Maturity*** **is defined as the number of years until the maturity date.** ***Coupon payment*** **is defined as the annual coupon payment in dollar amount.** ***Age*** **is defined as the number of years since the issue date.** ***Issue size*** **is defined as the total amount issued in dollar amount.**

EXHIBIT 2 Parameter Values for Default-Adjusted Interest Rate Model

Parameter	Value
a (Reverting speed)	40%
b (Reverting level)	8%
σ (Volatility)	5%
λ(Market price of risk)	-10%
r_0 (Initial rate)	5%

$$B(R,t) = B(R,T)e^{-R(T-t)} = 0. \qquad (9)$$

Second, I back out the implied instantaneous probability of default from the pricing model for defaultable callable bonds. I solve the pricing model numerically by using the fully implicit finite difference method.

Assuming a constant risk neutral mean-loss rate hL and a constant loss rate L, the hazard rate h (i.e., the instantaneous probability of default) can be backed out from the bond price observed in the market. I assume that there is only one value for the instantaneous probability of default that equates the theoretical bond price to its market price. This is called the implied instantaneous probability of default. I build the following constrained optimization model to calculate the implied instantaneous probability of default:

$$\begin{aligned} &min \quad f(h) \\ &s.t. \quad 0\% \leq h \leq 100\%, \end{aligned} \qquad (10)$$

where $f(h) = |U - V|$, h is the implied instantaneous probability of default, U is the market price of the bond, and V is the theoretical price of the bond given by the model. The optimization is accomplished using the golden section search algorithm.

Data

I consider callable, senior unsecured bonds with semiannual coupons, no variation in coupon payments over time, no sinking fund provisions, and original maturities of under 30 years. All bonds in the dataset are rated by Moody's and have at least one year remaining to maturity. Bond prices are adjusted for accrued interest. I examine month-end bond-price observations during the period beginning January 1993 and ending December

EXHIBIT 3 Bond Characteristics

Rating	Numerical rating (Aaa = 1, …, C = 9)
Maturity	Maturity date—Quote date (years)
Coupon payment	Annual coupon payment (dollars)
Age	Quote date—Issue date (years)
Issue size	Total amount issued (dollars)

EXHIBIT 4 The Relationship Between Ratings and Estimated Default Probabilities

	Estimated Default Probabilities			
Ratings	**Mean**	**Standard Deviation**	**Minimum**	**Maximum**
Aaa	0.8444%	0.2537%	0.0000%	1.0957%
Aa	0.8715%	0.3735%	0.0000%	1.3980%
A	0.9095%	0.3717%	0.0000%	1.5999%
Baa	0.9432%	0.5289%	0.0000%	1.9193%
Ba	1.4264%	0.2249%	0.5005%	1.9408%
B	1.8296%	6.1839%	0.5736%	100.0000%
Caa	9.5016%	24.3369%	0.7426%	100.0000%
Ca	31.7416%	40.7494%	0.9552%	100.0000%
C	62.2630%	34.0624%	1.2743%	100.0000%

2002. I use a sample of 355 firms. The number of bonds in the sample is 567. Across all firms and all months, the number of bond prices in the sample is 19,391. Summary statistics about the corporate bond data used in this study are presented in Exhibit 1.

As for the parameter values for the default-adjusted interest rate model, I assume the values that are consistent with Chen.[10] These are listed in Exhibit 2. Moody's finds that during the period of 1970 to 1998, senior unsecured bondholders on average receive approximately 50 percent of par in the event of default. That is, the recovery rate, $\hat{R}$, is approximately 50 percent. Therefore, in line with Moody's evidence, I assume that the loss rate, L, or, $1-\hat{R}$, is 50 percent.

Result

To understand the influence of various factors on the estimated default probabilities, I examine the relationship between the estimated default probabilities and a set of bond characteristics. These are listed in Exhibit 3. Ratings are the most widely observed measure of a bond's default risk. Exhibit 4 presents the relationship between ratings and estimated default probabilities. As expected, estimated default probabilities rise with lower ratings and fall with higher ratings. For investment-grade bonds, the average estimated default probabilities range from 0.8444 to 0.9432 percent. For speculative-grade bonds, the average estimated default probabilities range from 1.4264 to 62.2630 percent.

To test for the statistical significance of the relationship between estimated default probabilities and bond characteristics, I estimate the following regression model:

$$EstimatedDefaultProbability = \beta_0 + \beta_1 Rating + \beta_2 Maturity + \beta_3 CouponPayment + \beta_4 Age + \beta_5 IssueSize + \varepsilon \quad (11)$$

EXHIBIT 5 Regression Analysis

Independent Variables	Regression Coefficients and Associated *t*-Statistics: Investment-Grade Bonds	Speculative-Grade Bonds	All Bonds
Rating	4.55×10^{-4}	1.66×10^{-4}	4.37×10^{-4}
	(15.48)*	(9.10)*	(36.87)*
Maturity	-5.18×10^{-4}	-5.82×10^{-4}	-5.46×10^{-4}
	(-196.53)*	(-94.46)*	(-215.51)*
Coupon payment	-1.54×10^{-5}	-4.86×10^{-5}	-3.43×10^{-5}
	(-0.75)	(-43.72)*	(-35.46)*
Age	-5.03×10^{-7}	5.93×10^{-5}	6.58×10^{-5}
	(-0.06)	(6.66)*	(10.96)*
Issue size	2.46×10^{-12}	1.15×10^{-14}	1.70×10^{-12}
	(15.43)*	(0.10)	(18.59)*
Adjusted R^2	87.46%	50.61%	81.80%

* Denotes statistical significance at the 5% level.

Note: This table reports the OLS estimation results. The *t*-statistics are given in parentheses.

The results are reported in Exhibit 5, which confirms the positive relationship between ratings and estimated default probabilities as discussed above. Exhibit 5 shows that the relationship between maturities and estimated default probabilities is downward sloping. The intuition behind this is that the longer the maturity, the higher the probability that the rating of a bond increases.[11] This explains that estimated default probabilities can be lower for longer maturity bonds. Moreover, less risky firms within a rating class tend to issue longer-maturity bonds.[12] Therefore, the estimated default probability is underestimated for the longer maturities, such that the downward sloping relationship in fact only follows from using less risky bonds for the longer maturities. Coupon payment is a borrowing cost. It reflects the bond's default risk. The higher the bond's default risk, the larger the coupon payment. However, Exhibit 5 shows a negative relationship between coupon payments and estimated default probabilities. The reason for this is most likely the result of multicollinearity. Coupon payment is highly correlated with other bond characteristics. For example, for all bonds, the correlation between coupon payment and rating is 77.92 percent, and the correlation between coupon payment and maturity is -51.95 percent. This multicollinearity problem often results in coefficients that have incorrect signs or magnitude.[13] Age is used as a proxy for liquidity.[14] Newly issued bonds are considered to be more liquid than older bonds. Exhibit 5 shows a positive relationship between ages and estimated default probabilities: the older the bond, the lesser the bond's liquidity, and therefore the higher the estimated default probability. Issue size indicates the level of debt obligation. The larger the issue size, the larger the debt obligation. Exhibit 5 shows a positive relationship between issue sizes and estimated default probabilities: the larger the issue size, the larger the debt obligation, and therefore the higher the estimated default probability.

From the perspective of statistical significance, the results are encouraging. For all bonds, all coefficients are significant at the 5-percent level. For investment-grade bonds, all coefficients are significant at the 5-percent level except for coupon payment and age. For speculative-grade bonds, all coefficients are significant at the 5-percent level except for issue size. The adjusted R^2 is 81.80 percent for all bonds, 87.46 percent for investment-grade bonds, and 50.61 percent for speculative-grade bonds. The adjusted R^2 indicates that bond characteristics explain a significant portion of the variation in estimated default probabilities, especially for investment-grade bonds.

The results from regression analysis are a clear indication that there is a strong relationship between estimated default probabilities and bond characteristics.

Conclusion

This article presents a model for estimating the default risks for callable corporate bonds. The model is developed based on Duffie and Singleton's reduced-form approach.[15] The model considers three essential elements: default-adjusted interest rate, default risk, and call provision. The default-adjusted interest rate is assumed to follow a Cox, Ingersoll, and Ross model.[16] The default risk is modeled as a constant spread, with the magnitude of this spread impacting the probability that governs the arrival of the default event. The call provision is modeled as a constraint on the value of the bond in the fully implicit finite difference scheme. The empirical evidence is encouraging. First, the estimated default probabilities are consistent with Moody's ratings. The estimated default probabilities rise with lower ratings and fall with higher ratings. Second, the relationship between the estimated default probabilities and other bond characteristics is consistent with the intuition. The estimated default probabilities are negatively correlated with maturity and positively correlated with coupon payment, age, and issue size.

The model is by no means a complete success. Both the default risk and the recovery rate in the event of default may vary stochastically through time. In addition, the default risk process may be correlated with the default-free term structure.[17] To improve the model, one can assume that the default risk follows a stochastic process, with a modification that allows the default risk process to be correlated with the default-free term structure.

In summary, this article can be used both as a benchmark for default risk models for callable corporate bonds and as a direction for future research. More generally, this article provides support for the idea that the class of reduced-form models can estimate the default risks for callable corporate bonds successfully. ■

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