Bailout Policy against Financial Intermediation Failures

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Abstract

Asymmetry in views of depositors and bankers can generate failures of financial intermediation in linking creditors and borrowers, and/or result in excessively high interest rates. Instead of considering asymmetry in assessment of bank's solvency, this paper focuses on asymmetry in views as to whether an insolvent bank will be liquidated or let to continue. Bailout policy has two effects in this respect: first, insurance effect, which lowers market interest rates, and secondly announcement effect, which rules the asymmetry in beliefs out.

The literature on banking crises and panics stresses, that asymmetry in views of depositors and bankers makes crises more severe and limits abilities of regulatory agencies to rescue banks through liquidity provision by monetary policy means (see e.g. Calomiris and Mason, 1997, or Corbet and Mitchell, 2000). However, the asymmetry is treated in a way that depositors (and the regulator) overestimate banks' portfolio's risk relative to the estimation of banks themselves. In this paper, there is no information asymmetry in this sense: all agents have true information about the riskiness of banks' assets. On the contrary, asymmetry is presented in views of depositors and banks regarding outcomes in case of banks' insolvency. When the banks believe they would be allowed to continue even in case of insolvency, depositors assume insolvent banks to be closed in absence of bailout. Symmetric cases of unlimited liability (banks believe to continue and depositors believe to get their deposits repaid in full) and limited liability (banks believe to be closed in case of insolvency, and depositors believe to be repaid only in the amount of bank's assets value) are brie‡y discussed in Sinn (2003). Contrary to Sinn (2003), we understand here unlimited liability not as the case "where banks will always keep their promises" but rather as the case, where the banks expect to be able to do so.

The theory of banking widely uses the principle of limited liability in modelling banking behaviour. Although the literature on limited liability in economics is relatively large (see e.g. the review by Noe and Smith, 1997), its applications to banking are scarce. The research focuses mostly on the ideas that limited liability can give the bankers incentives to take too much risk (e.g. Gollier, Koehl and Rochet, 1995), and/or leads to the excessive interest rates, if intermediation is competitive (e.g. Matutes and Vives, 2000). Other e¤ects of limited liability, as well as the question as to how far the principle of limited liability can hold in practice, su¤er a certain lack of attention. This paper aims, therefore, to contribute in …lling this gap, in which it compares two worlds: one with and one without limited liability, and studies the differences arising in the deposit market. Particular attention will be paid to the question ad to whether limited or unlimited liability allows banks to perform their role as a link between creditors (depositors) and borrowers (…rms) better of worse, and as to whether this performance can be improved.

Before we start theoretical analysis, consider actual practice concerning liability of banks. Dewatripont and Tirole (1993) distinguish between four ways of handling bank failures: (1) liquidation (payo¤ resolution), when the bank is closed and put under receivership, and uninsured claimants are paid o¤ after the assets of the bank are liquidated; (2) merger (purchase and assumption), when a healthy bank purchases all or some of the assets of the failed bank and assumes all or some of the liabilities; (3) government loans or transfers (open bank assistance), when the supervisory agency provides financial assistance to a failed bank in form of loans, asset purchases or cash to restore capital to a positive level; and (4) government ownership (bridge bank), with a supervisory agency acting temporarily as the acquire, taking over the operations of a failing bank and maintaining banking services for the customers.[1] From the point of view of the customers, the bank is closed only in one of the four cases, namely liquidation, in all other cases customers can continue working with the bank, be it rescued, assumed or nationalised. From the point of view of the bank, however, it continues its operations only if it is bailed out through loans or transfers[2], since otherwise it is either liquidated or assumed by other institution, in which cases the failed bank itself is not liable anymore to its customers. This point deserves special attention and we discuss this asymmetry later on.

Most research in the field of bank failure resolutions concentrate on resolving illiquid but solvent banks. Following standard definitions, insolvency occurs when the value of the assets held by a bank is less than the value of the liabilities held; illiquidity occurs however when a bank is not able to meet its current obligations as they come due, so that illiquidity can arise even if a bank is solvent, due e.g. to banking panics. In addition to this, it is possible that insolvency arises when a bank is still liquid, and this case seems to be much less studied in the literature. Gersbach and Wenzelburger (2002) and Vinogradov (2003) have shown, that in a dynamic setting, insolvent banks (whose liabilities are above their assets) can still be liquid due to newly acquired deposits, which provide them with sufficient liquidity to pay out debts to the previous generation of depositors. In fact, as Bennett (2001) writes, "an insolvent bank is more likely to continue to operate in developing economies or economies in transition: one half of the respondents [local banking regulators - D.V.] in this group have allowed insolvent banks to operate, whereas 3 of the 14 [not as little, more than 20% - D.V.] deposit insurers in advanced economies have done so". In general 35% of respondents (10 out of 28) did not deny practice where equity-insolvent depository institutions have been allowed to operate for extended periods. Just one ex-

[1] Bennett (2001) provides a detailed discussion on international evidence on the types of resolution, based upon a survey on deposit insurance practice in 34 countries, including advanced, developing and transitional economies. See also Santomero and Hoffman (1998) for the discussion of problem banks resolution methods and Bhattacharya, Boot and Thakor (1998) for a discussion of theoretical views on bank closure policy.

[2] There is also a possibilty that the regulator follows the policy of forbearance, where the insolvent bank is let to operate further, and interventions are ambiguosly postponed. For the purpose of this paper it is important that all possible schemes are realised in just two outcomes for the failing bank: either continuation or closure.

ample in this connection: during 4 years (1988-92) FDIC allowed insolvent First City Bancorporation (with 59 branches in USA) to operate through open bank assistance, and only in 1992 recurring losses of the bank led to its closure.

These considerations put under the question the arguments in favour of limited liability principle. Excluding limited liability implies, that if banks su¤er losses, and their capital (di¤erence between assets and liabilities) falls below zero, they still continue operating and start the next period with negative capital.[3] On the contrary, under limited liability, banks should have been closed in case of negative capital, which implies that pro…t of the bank is always nonnegative: if it is negative, the bank is closed, and net pro…t of the banker is zero, whereas its losses are borne by bank's creditors, who get only the rest value of the bank.[4]

In terms of failure resolutions above, we focus on the open bank assistance, which allows the bank to continue its operations. Freixas (2000) considers open bank assistance through subsidy, which covers bank's debts in case of bailout, rather than a loan-based rescue. A loan-based rescue instead replaces one liabilities of the bank with other, whereas subsidy (or at least a combination of subsidies and loans) a¤ects incentives of the banks and hence the market situation in general. For the purpose of this paper it is important that the bank in case of bailout obtains the possibility to continue, and therefore bears losses (if any). Such continuation can be possible through credit obtained from a third party (e.g. loans from LOLR, or newly accumulated deposits) to repay current debts. We do not consider the next period after insolvency to exclude possible subsidisation e¤ect, and concentrate only on the incentives generated purely by the possibility of continuation. In this setting the way how the bank obtains means to repay it debts play no role, important is that the bank would start the next period with negative capital in case of losses (as in Gersbach and Wenzelburger, 2002, and Vinogradov, 2003)

As it has been already mentioned, the resolution methods are seen by depositors from another perspective as by banks. Whether depositors receive their funds back, depends on the resolution method chosen. In case of no liquidation (the bank obtains continuation loan, new deposits or even a subsidy from the regulator) deposit repayments are sure. If the failing institution is assumed, through merger, acquisition or nationalisation, its liabilities are not necessarily assumed in full, so that depositors can generally account for

[3] This is consistent with the actual practice. Santomero and Ho¤man (1998) note, that regulators often delay resolution actions in the hope of a turnaround.

[4] Reserve considerations omitted.

partial repayments. And finally, if the failed financial institution is liquidated, its uninsured creditors receive only the rest value of bank's assets. This asymmetry in views of bankers and depositors on possible outcomes, plays a crucial role in the model. To sharpen the effect, we will focus on two possible outcomes: bailout (which leads to continuation) or liquidation performed by the regulating authority. Hence in the world without bailouts depositors have to expect liquidation of an insolvent bank, and consequently deposit repayments in amount of rest value of the bank.

The Paper proceeds as follows. Section 1 introduces the model. Sections 2 and 3 study consequently monopolistic and competitive equilibria in both worlds (with and without limited liability). Section 4 presents a rationale for regulation, and Section 5 studies the effects of bailouts in the world with unlimited liability. Concluding remarks close the paper.

1 The Model

Consider an economy with a continuum of agents [indexed by] $i \in [0;1]$, called households, a banking sector (which can be monopolistic or competitive) and two types of financial assets (one risky an one risk-free). In this world, households do not have access to the market of risky assets, but the banks do.

The model describes two periods: in the first the decision-making and investing is performed, and in the second period one of the two ("H" for "high and "L for "low") states of nature are realised, and payments made. Risky asset brings return of r^H in the second period, if the state of nature "H" is realised, with probability p, and of r^L; if the state of nature "L" is realised, with probability $(1-p)$.

We will assume that

$$r^H > r^F > r^L \tag{A.1}$$

The case of equality is trivial and does not allow for any uncertainty. The case $r^H > r^L \geq r^F$ (and consequently $r^F \geq r^H > r^L$) is not of particular interest since it eliminates uncertainty in agents' decisions: risky asset is preferred to the safe one since it provides higher rate of return in any state of nature (or, respectively, safe asset is always preferred to the risky one).

Moreover we assume that the expected return on risky asset is higher than return on the risk-free asset:

$$pr^H + (1-p)\, r^L > r^F \tag{A.2}$$

Note that this assumption implies that in a world of risk-neutral agents, risky asset is always preferred to the risk-free one; and if households had access to the market of risky assets, they would always invest in this market.

1.1 Banks

We consider intermediated economy, where the existence of banks is justified by the inability of households to access the market for risky assets. In such a world, financial intermediation should provide a link between creditors (households) and borrowers (firms, who are not directly modelled here, but rather presented as some risky project implemented at the market of risky assets).

The banks collect deposits in amount D^d and invest them in portfolio of safe and risky assets. Banks maximise their expected profit.

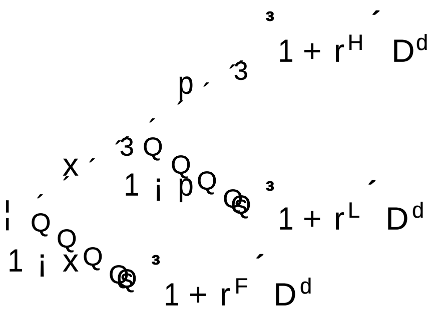

Figure 1: Choice of Banks

The bank is solvent if and only if its rest value is not less than its obligations:

$$V_L \geq \left(1 + r^D\right) D^d$$

Rest value of the bank is determined by the realisation of return on risky asset:

$$V_L = \begin{cases} \left[x\left(1 + r^H\right) + (1 - x)\left(1 + r^F\right)\right] D^d & \text{if } r^H \text{ is realised} \\ \left[x\left(1 + r^L\right) + (1 - x)\left(1 + r^F\right)\right] D^d & \text{if } r^L \text{ is realised} \end{cases}$$

If the bank is insolvent, each depositor receives $\frac{V_L}{D^d}$ so that the "insolvency" return on deposits in this case will be

$$\tilde{e} = \begin{cases} xr^H + (1 - x) r^F & \text{if } r^H \text{ is realised} \\ xr^L + (1 - x) r^F & \text{if } r^L \text{ is realised} \end{cases} \qquad (1)$$

Lemma 1 Given the parameters r^H, r^F, r^L and p, the probability of deposit repayment q is a function of r^D and x:

$$q\left(x; r^D\right) = \begin{cases} 0 & \text{if} \quad r^D > xr^H + (1-x)\, r^F \\ p & \text{if} \quad xr^H + (1-x)\, r^F \geq r^D > xr^L + (1-x)\, r^F \\ 1 & \text{if} \quad r^D \leq xr^L + (1-x)\, r^F \end{cases} \tag{2}$$

Proof. Consider three following possibilities:

1. $r^D > xr^H + (1-x)\, r^F$: In this case
$$V_L \leq \left[x\left(1+r^H\right) + (1-x)\left(1+r^F\right)\right] D^d < \left(1+r^D\right) D^d$$
with probability 1, and the bank is solvent with probability $q = 0$

2. $xr^H + (1-x)\, r^F \geq r^D > xr^L + (1-x)\, r^F$: If r^H is realised (with probability p), then
$$V_L = \left[x\left(1+r^H\right) + (1-x)\left(1+r^F\right)\right] D^d \geq \left(1+r^D\right) D^d$$
and the bank is solvent. If however r^L is realised (with probability $1-p$),
$$V_L = \left[x\left(1+r^L\right) + (1-x)\left(1+r^F\right)\right] D^d < \left(1+r^D\right) D^d$$
and the bank is insolvent, so that the repayments probability is in this case $q = p$

3. $r^D \leq xr^L + (1-x)\, r^F$: The bank is always able to repay its obligations,
$$V_L \geq \left[x\left(1+r^L\right) + (1-x)\left(1+r^F\right)\right] D^d \geq \left(1+r^D\right) D^d$$
with probability $q = 1$.

■

Note on notation Since distinguishing between three cases above is also conducted further in the analysis, we will denote them as follows:

$$\begin{array}{lll} \text{Case} \;\; C_1 & \text{if} & r^D > xr^H + (1-x)\, r^F \\ \text{Case} \;\; C_2 & \text{if} & xr^H + (1-x)\, r^F \geq r^D > xr^L + (1-x)\, r^F \\ \text{Case} \;\; C_3 & \text{if} & r^D \leq xr^L + (1-x)\, r^F \end{array}$$

It is easy to see that C_1 stays for the case, when the banks are never solvent, no matter which state of nature is realised in the second period. Similarly, C_3 denotes the case, when the banks are always solvent, no matter which state of nature is realised in the second period. And the intermediate case C_2 describes such constellations of interest rates, under which the banks are solvent if the state of nature "H" is realised, and insolvent if "L" is realised in the second period.

1.2 Households

All households in the model are equal in their preferences and abilities, so that there is no heterogeneity. The households have to decide whether they invest their endowment Q[5] into the safe asset or into deposit, or combine them in a portfolio with share a of deposits and $(1 - a)$ of safe assets.

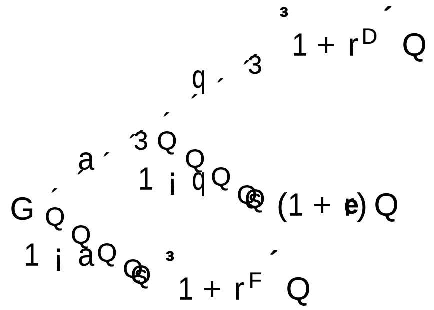

Figure 2: Choice of Households

Households know, that they get their deposits back in full in period 2 with probability q (which at this stage means, that the bank is solvent in the second period), and the interest paid on deposits will be exactly the announced deposit interest rate r^D. With probability $(1 - q)$ depositors cannot get their deposits back in full, instead the rest value (Freixas, 2000, calls this liquidation value) of the bank V_L is equally distributed among all depositors and the interest rate paid on deposits in this case we denote with $\tilde{e}$ (we define it further)

Households maximise their expected profit:

$$G^e = \left\{ a \left(q \left(1 + r^D\right) + (1 - q)(1 + \tilde{e}) \right) + (1 - a) \left(1 + r^F\right) \right\} Q \rightarrow \max_a \quad (3)$$

Total (aggregate) supply of deposits in the economy is hence

$$D^s = \int_0^1 aQdi = aQ \quad (4)$$

[5] Assumption of the fixed amount to be invested by households can be relaxed. The key feature of the model arises due to the corner solution in the problem of the choice of optimal amount of deposits. If we assume the agents decide upon their savings (making Q endogenous), which can be either invested in risk-free asset or deposited, the corner solution with zero deposits still persists, since the choice of households between two types of assets is unchanged. Assuming $Q = const$ however simplifies the presentation.

1.3 Supply of deposits

Total supply of deposits, as defined by (4), is determined by the choice of a by households. We can reformulate the expected profit of depositors (3):

$$G^e = (1 + R_D^e)\,Q$$

with

$$R_D^e = a\left(q r^D + (1-q)\,\tilde{r}\right) + (1-a)\,r^F$$

Given the probability of deposit repayments from (2) and definition of $\tilde{r}$ (1), we can write

$$R_D^e = \begin{cases} a\left[x\left(p r^H + (1-p)\,r^L\right) + (1-x)\,r^F\right] + (1-a)\,r^F & \text{if } C_1 \\ a\left[p r^D + (1-p)\left(x r^L + (1-x)\,r^F\right)\right] + (1-a)\,r^F & \text{if } C_2 \\ a\left(1 + r^D\right) + (1-a)\left(1 + r^F\right) & \text{if } C_3 \end{cases} \quad (5)$$

And the optimisation problem of depositors is equivalent to:

$$R_D^e\left(a; x; r^D\right) \to \max_a \quad (6)$$

The solution of this problem $a = \arg\max R_D^e$ can be found as follows:

1. if $r^D > x r^H + (1-x)\,r^F$ then

$$a = \begin{cases} 1 & \text{if } x\left(p r^H + (1-p)\,r^L\right) + (1-x)\,r^F > r^F \\ a \in [0;1] & \text{if } x\left(p r^H + (1-p)\,r^L\right) + (1-x)\,r^F = r^F \\ 0 & \text{if } x\left(p r^H + (1-p)\,r^L\right) + (1-x)\,r^F < r^F \end{cases} \quad (7)$$

2. if $x r^H + (1-x)\,r^F \geq r^D \geq x r^L + (1-x)\,r^F$ then

$$a = \begin{cases} 1 & \text{if } p r^D + (1-p)\left(x r^L + (1-x)\,r^F\right) > r^F \\ a \in [0;1] & \text{if } p r^D + (1-p)\left(x r^L + (1-x)\,r^F\right) = r^F \\ 0 & \text{if } p r^D + (1-p)\left(x r^L + (1-x)\,r^F\right) < r^F \end{cases} \quad (8)$$

3. if $r^D < x r^L + (1-x)\,r^F$ then

$$a = 0 \text{ since } r^D < r^F \quad (9)$$

To avoid indeterminacy in the choice of depositors, and in order to make presentation easier we will assume further that depositors prefer deposits to risk-free asset as soon as the expected return on deposits is equal or greater than the risk-free rate r^F: [6]

$$a^* = \sup\{\arg\max R^e_D\} \tag{A.3}$$

The discussion above leads to the following proposition:

Proposition 1 Aggregate deposit supply is given by the function

$$D^s = \begin{cases} Q & \text{if} \quad r^D \geq r^F + \frac{(1-p)x}{p}\left(r^F - r^L\right) \\ 0 & \text{if} \quad r^D < r^F + \frac{(1-p)x}{p}\left(r^F - r^L\right) \end{cases}; \tag{10}$$

Proof.

First, recall that aggregate deposit supply is given by (4):

$$D^s = aQ$$

To derive the equation (10), substitute here for the optimal choice of a by households (7 - 9). We use here the equivalence of the two problems:

$$a = \arg\max R^e_D \iff a = \arg\max G^e$$

Note that due to (1) condition $x\left(pr^H + (1-p)\,r^L\right) + (1-x)\,r^F \leq r^F$ in (7) is never met. Hence

$$a = \arg\max G^e = 1 \text{ if } r^d > xr^H + (1-x)\,r^F$$

Moreover, condition $pr^D + (1-p)\left(xr^L + (1-x)\,r^F\right) > r^F$ is automatically satisfied if $r^d > xr^H + (1-x)\,r^F$:

$$\begin{aligned} pr^D + (1-p)\left(xr^L + (1-x)\,r^F\right) &> \\ p\left(xr^H + (1-x)\,r^F\right) + (1-p)\left(xr^L + (1-x)\,r^F\right) &= \\ x\left(pr^H + (1-p)\,r^L - r^F\right) + r^F &> r^F \end{aligned}$$

Condition $pr^D + (1-p)\left(xr^L + (1-x)\,r^F\right) \leq r^F$ is automatically satisfied if $r^d < xr^L + (1-x)\,r^F$:

$$\begin{aligned} pr^D + (1-p)\left(xr^L + (1-x)\,r^F\right) &< \\ p\left(xr^L + (1-x)\,r^F\right) + (1-p)\left(xr^L + (1-x)\,r^F\right) &= \\ xr^L + (1-x)\,r^F &\leq r^F \end{aligned}$$

[6] This assumption only simplifies presentation without change in results.

Finally, condition $pr^D + (1 - p)\left(xr^L + (1 - x)r^F\right) > r^F$ is equivalent to:

$$r^D > r^F + \frac{(1 - p)x}{p}\left(r^F - r^L\right)$$

■

As soon as supply of deposits is determined, we may proceed to the study of monopolistic and competitive demands for deposits from the side of banking sector, and to consequent equilibria.

2 Monopolistic equilibrium

Definition 1: Monopolistic equilibrium under parameters $\left(p; r^F; r^H; r^L\right)$ is a tuple $\left(r_m^{D*}; q^*; x^*; a^*\right)$, which provides

1. $\left(x^*; r_m^{D*}\right) = \arg\max \Pi^e$
2. $a^* = \arg\max G^e$
3. $q^* = q\left(x^*; r_m^{D*}\right)$
4. $D^s = D^d$

Consider now the monopolist bank, which maximises its expected profit

2.1 Limited Liability

If the principle of limited liability holds, expected profit of the bank is given by

$$\begin{aligned} \Pi^e &= p\max\left[\left(xr^H + (1 - x)r^F - r^D\right)D^d; 0\right] + \\ &+ (1 - p)\max\left[\left(xr^L + (1 - x)r^F - r^D\right)D^d; 0\right] \end{aligned}$$

The bank can choose the amount of deposits through setting deposit interest rate according to (10), which reduces the maximisation problem of the bank to:

$$\Pi^e = pQ\max\left[xr^H + (1 - x)r^F - r^D; 0\right] \to \max_{x;r^D} \quad (11)$$

$$\text{s.t. } r^D > r^F + \frac{(1 - p)x}{p}\left(r^F - r^L\right) \quad (12)$$

$$0 \le x \le 1 \quad (13)$$

since if (12) then

$$x r^L + (1 - x) r^F - r^D < \frac{x}{p}\left[r^L - r^F\right] < 0$$

Demand for deposits in this case is given implicitly through the function $r^D = r^D\left(D^d\right)$:

Proposition 2 Optimal choice of the monopolist bank under limited liability is $x = 1$ and $r_m^D = r^F + \frac{(1-p)}{p}\left(r^F - r^L\right)$

Proof.

This result is easily found with Kuhn-Tucker maximisation rule.

■

Condition $r_m^D = r^F + \frac{(1-p)}{p}\left(r^F - r^L\right)$ means that the demand for deposits is arbitrary (infinitely high interest rate elasticity of demand)

Proposition 3 If limited liability principle holds, monopolistic equilibrium in the model is given by:

$$\left\{\begin{array}{c} a^* = 1 \\ x^* = 1 \\ q^* = p \\ r_m^D = r^F + \frac{(1-p)}{p}\left(r^F - r^L\right) \end{array}\right\} \quad (14)$$

Proof. First, we need to check whether these parameters satisfy the definition of equilibrium. Obviously condition $\left(x^*; r_m^{D*}\right) = \arg\max \Pi^e$ is met by construction (monopolistic bank maximises its expected profit). Expected profit of households is in this case

$$G^e = \left(p\left(1 + r^D\right) + (1 - p)\left(1 + r^L\right)\right) Q > 0 \quad (15)$$

And the market for deposits clears by construction (see derivation of the optimal choice of the bank).

Assume now there exists another combination of variables, which still meets the definition of equilibrium. Then by construction this combination of parameters will coincide with the one given in the text of this proposition. Hence the equilibrium is unique.

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2.2 Unlimited Liability

Consider now monopoly bank, which does not rely on the limited liability principle. The bank also maximises its expected profit, but its optimisation problem differs now from the case of limited liability (11):

$$\begin{aligned} \Pi^e &= \left[x \left(p r^H + (1-p)\, r^L \right) + (1-x)\, r^F - r^D \right] D^d \to \max_{x;r^D} \\ \text{s.t. } D^d &= D^s \left(x; r^D \right) \end{aligned}$$

Again deposit supply function (10) allows to reformulate the maximisation problem of the bank:

$$\Pi^e = \left[x \left(p r^H + (1-p)\, r^L \right) + (1-x)\, r^F - r^D \right] Q \to \max_{x;r^D} \tag{16}$$

$$\text{s.t. } r^D \geq r^F + \frac{(1-p)\, x}{p} \left(r^F - r^L \right) \tag{17}$$

$$0 \leq x \leq 1$$

Proposition 4 Optimal choice of the monopolist bank with unlimited liability is

$$\left\{ r_m^D = x \left(p r^H + (1-p)\, r^L \right) + (1-x)\, r^F \right\} \begin{cases} \left\{ \begin{array}{c} x^* = 1 \\ r_m^D = r^F + \frac{(1-p)}{p} \left(r^F - r^L \right) \end{array} \right\} & \text{if } p^2 > \frac{r^F - r^L}{r^H - r^L} \\ \left\{ x^* \in [0;1] \right\} & \text{if } p^2 = \frac{r^F - r^L}{r^H - r^L} \\ \left\{ \begin{array}{c} x^* = 0 \\ r_m^D = r^F \end{array} \right\} & \text{if } p^2 < \frac{r^F - r^L}{r^H - r^L} \end{cases}$$

Proof.

This result again is easy to be found with the help of Kuhn-Tucker optimisation rule.

■

Proposition 5 If limited liability principle does not hold, monopolistic equilibrium in the model is given by:

$$\begin{cases} \begin{Bmatrix} a^* = 1 \\ x^* = 1 \\ q^* = p \\ r_m^D = r^F + \frac{(1-p)x}{p}\left(r^F - r^L\right) \end{Bmatrix} & \text{if} \quad p^2 > \frac{r^F - r^L}{r^H - r^L} \\ \begin{Bmatrix} a^* = 1 \\ x^* \in [0;1] \\ q^* = p \\ r_m^D = x\left(pr^H + (1-p)\,r^L\right) + (1-x)\,r^F \end{Bmatrix} & \text{if} \quad p^2 = \frac{r^F - r^L}{r^H - r^L} \\ \begin{Bmatrix} a^* = 1 \\ x^* = 0 \\ q^* = 1 \\ r_m^D = r^F \end{Bmatrix} & \text{if} \quad p^2 < \frac{r^F - r^L}{r^H - r^L} \end{cases} \qquad (18)$$

Proof.

1. First, we need to check whether these parameters satisfy the definition of equilibrium. Obviously condition $\left(x^*; r_m^{D*}\right) = \arg\max \Pi^e$ is met by construction (monopolistic bank maximises its expected profit). Expected profit of households is in this case

   $$G^e = \left(p\left(1 + r_m^D\right) + (1-p)\left(1 + r^L\right)\right) Q > 0$$

   And the market for deposits clears by construction (see derivation of the optimal choice of the bank).

   Assume now there exists another combination of variables, which still meets the definition of equilibrium. Then by construction this combination of parameters will coincide with the one given in the text of this proposition. Hence the equilibrium is unique.

2. If $p^2 = \frac{r^F - r^L}{r^H - r^L}$; the choice of the bank is arbitrary within the interval $x^* \in [0;1]$. Moreover since $p^2 = \frac{r^F - r^L}{r^H - r^L} > 0$, condition $r_m^{D*} = x\left(pr^H + (1-p)\,r^L\right) + (1-x)\,r^F$ implies $xr^H + (1-x)\,r^F > r^D > xr^L + (1-x)\,r^F$, and therefore $q^* = p$. Expected profit of households is again positive:

   $$G^e = \left(1 + r^F\right) Q > 0$$

3. If $p^2 < \frac{r^F - r^L}{r^H - r^L}$; the choice of the bank is trivial $x^* = 0$ and $r_m^D = r^F$, which implies $q^* = 1$ and $G^e = \left(1 + r^F\right) Q > 0$. Again if any other equilibrium of this type would exist, by construction it would coincide with the tuple (18).

■

Propositions 3 and 5 highlight the difference between the two equilibria. In the case with limited liability assumption (A.2) is sufficient to guarantee that the intermediation (bank) links creditors and borrowers. Unlimited liability however requires from the bank to be more demanding in its estimations of expected return, which implies additional requirement needed to ensure positive expected profit. Hence monopolistic bank can fail under unlimited liability to link creditors and borrowers. We return to this failure after analysis of competitive banking sector.

3 Competitive equilibrium

Definition 2: Competitive equilibrium under parameters $\left(p; r^F; r^H; r^L\right)$ is a tuple $\left(r_c^D; q^*; x^*; a^*\right)$, which provides

1. $a^* = \arg\max G^e$
2. $x^* = \arg\max \Pi^e$
3. $q^* = q\left(x^*; r_c^D\right)$
4. $D^S\left(r_c^D\right) = D^D\left(r_c^D\right)$

Now competitive banks cannot influence deposit interest rate. We distinguish again between limited liability and unlimited liability cases:

3.1 Limited Liability

Optimisation problem of the banks under limited liability is:

$$\begin{aligned}\Pi^e &= p\max\left[\left(xr^H + (1-x)\,r^F - r^D\right)D^d; 0\right] + \\ &\quad + (1-p)\max\left[\left(xr^L + (1-x)\,r^F - r^D\right)D^d; 0\right] \\ \Pi^e &\to \max_{x;D^d}\end{aligned}$$

To find the solution, which determines demand for deposits $D^d = D^d\left(r^D; x\right)$, rearrange the expected profit function of the banks:

$$\Pi^e = \begin{cases} 0 & \text{if } C_1 \\ p\left(xr^H + (1-x)\,r^F - r^D\right)D^d & \text{if } C_2 \\ \left[x\left(pr^H + (1-p)\,r^L\right) + (1-x)\,r^F - r^D\right]D^d & \text{if } C_3 \end{cases}$$

Solution of maximisation problem

$$\pi^e\left(x; D^d; r^D\right) \to \max_{x;D^d} \tag{19}$$

$$\text{s.t. } 0 < x < 1 \tag{20}$$

$$D^d \geq 0 \tag{21}$$

is given by the following proposition:

Proposition 6 Competitive demand for deposits under limited liability is given by a correspondence

$$D^d = \begin{cases} A \in [0; \infty) & \text{if } r^D \geq r^H \\ A \to \infty & \text{if } r^D < r^H \end{cases} \tag{22}$$

Proof.

To prove the proposition we just need to consider again cases C_1, C_2 and C_3 and to solve three correspondent maximisation problems.

■

Condition $a^* = \arg\max G^e$ in the definition of equilibrium above is fulfilled through optimisation problem of households. Condition $x^* = \arg\max \pi^e$ is met by the choice of the banks. We need now to meet the market clearing condition $D^s\left(r_c^{D*}\right) = D^d\left(r_c^{D*}\right)$.

Comparing (10) and (22) we obtain

1. if $r^D < r^H$ equilibrium is impossible since deposit demand is infinitely high and deposit supply bounded from above: $D^s \leq Q$. Banks continue to compete for deposits, increasing r^D.
2. if $r^D \geq r^H$ then automatically[7] $r^D \geq r^F + \frac{(1-p)x}{p}\left(r^F - r^L\right)$, and deposit supply is $D^s = Q$ while deposit demand is arbitrary, which means that the banks collect exactly the amount of deposits supplied by depositors, and equilibrium amount of deposits is $D^* = Q$. Along with that, equilibrium interest rate is indeterminate: $r_c^D \geq r^H$

This proves following

[7] Assume $r^F + \frac{(1-p)x}{p}\left[r^F - r^L\right] \geq r^H$ then $(1-p)x\left[r^F - r^L\right] \geq p\left[r^H - r^F\right]$. But $(1-p)x\left[r^F - r^L\right] \leq (1-p)\left[r^F - r^L\right]$, hence should hold $(1-p)\left[r^F - r^L\right] \geq p\left[r^H - r^F\right]$, which is a contradiction to Assumption (A.2) $r^F < pr^H + (1-p)r^L$.

Proposition 7 In case of limited liability competitive equilibrium is multiple and given by:

$$\left\{\begin{array}{c} a^* = 1 \\ x^* \in [0;1] \\ q^* = 0 \\ r_c^D \geq r^H \end{array}\right\} \quad (23)$$

Proof.

To complete the proof we just need to determine q, which is straightforward (see Lemma1).

■

Note that although probability of deposit repayments is zero, supply of deposits is still positive, since depositors expect to obtain the rest value (liquidation value) of the bank, which is higher than return on safe asset (due to assumption A.2). This result is known also from Matutes and Vives (2000), who write: "when competition is intense banks tend to set deposit rates too high... With perfect competition rates are excessive". Matutes and Vives (2000) attribute this effect to the fact that the bank do not internalise the cost of failure. Indeed, we will see below that relaxing the assumption of limited liability eliminates this problem.[8]

3.2 Unlimited Liability

Competitive banks solve optimisation problem

$$\Pi^e = \left[x\left(pr^H + (1-p)\,r^L\right) + (1-x)\,r^F - r^D\right] D^d \to \max_{x;D^d}$$

Solution of this problem determines again demand for deposits $D^D = D^D\left(r^D; x\right)$. Note that since by assumption $pr^H + (1-p)\,r^L > r^F$ solution of this maximisation problem is $x^* = 1$ and the following proposition is straightforward:

Proposition 8 Competitive demand for deposits under unlimited liability is given by:

$$D^d = \left\{\begin{array}{lll} 0 & \text{if} & r^D > pr^H + (1-p)\,r^L \\ A \in [0;\infty) & \text{if} & r^D = pr^H + (1-p)\,r^L \\ A \to \infty & \text{if} & r^D < pr^H + (1-p)\,r^L \end{array}\right\} \quad (24)$$

[8] Introduction of equity capital in the decision-making by bank would also eliminate the possibility for deposit rates to be higher than r^H, but they will still be higher than the expected rate of return on risky asset.

■

Similarly to the case with limited liability, we need now just to meet the market clearing condition $D^s\left(r_c^{D*}\right) = D^d\left(r_c^{D*}\right)$. Note that due to $x^* = 1$ deposit supply function (10) turns to

$$D^s = \begin{cases} Q & \text{if } r^D \geq r^F + \frac{(1-p)}{p}\left(r^F - r^L\right) \\ 0 & \text{if } r^D < r^F + \frac{(1-p)}{p}\left(r^F - r^L\right) \end{cases}$$

To find equilibrium we have to distinguish between following cases:

1. If $pr^H + (1-p)r^L \geq r^F + \frac{(1-p)}{p}\left(r^F - r^L\right)$ then $\forall Q < \infty$ equality $D^d = D^s$ is fulfilled under competitive interest rate $r_c^D = pr^H + (1-p)r^L$, and the equilibrium amount of deposits in banks is $D^* = Q$. Setting interest rate below or above this level eliminates equilibrium due to $D^d = 0$ or $D^d = A \to \infty$.
2. If $pr^H + (1-p)r^L < r^F + \frac{(1-p)}{p}\left(r^F - r^L\right)$ then (1) $r^D < pr^H + (1-p)r^L$ leads to disequilibrium $D^d \neq D^s$; (2) $r^D \geq r^F + \frac{(1-p)}{p}\left(r^F - r^L\right)$ leads as well to disequilibrium $D^d \neq D^s$. Hence equilibrium can arise only with $pr^H + (1-p)r^L \leq r_c^D < r^F + \frac{(1-p)}{p}\left(r^F - r^L\right)$, which provides $D^s = Q = D^d = D^*$.

This leads to the following proposition:

Proposition 9 In case of unlimited liability competitive equilibrium is given by:

$$\begin{cases} \left\{ \begin{array}{c} a^* = 1 \\ x^* = 1 \\ q^* = p \\ r_c^D = pr^H + (1-p)r^L \end{array} \right\} & \text{if } p^2 \geq \frac{r^F - r^L}{r^H - r^L} \\ \left\{ \begin{array}{c} a^* = 0 \\ x^* = 1 \\ q^* = p \\ r_c^D \in \left[pr^H + (1-p)r^L; r^F + \frac{(1-p)}{p}\left(r^F - r^L\right)\right) \end{array} \right\} & \text{if } p^2 < \frac{r^F - r^L}{r^H - r^L} \end{cases} \tag{25}$$

Proof.

To complete the proof note that

$$pr^H + (1-p)r^L \geq r^F + \frac{(1-p)}{p}\left(r^F - r^L\right) \iff p^2 \geq \frac{r^F - r^L}{r^H - r^L}$$

Furthermore, if $r_c^{D*} = pr^H + (1-p)\, r^L$, it provides $r^L \le r_c^D \le r^H$, which under $x^* = 1$ guarantees $q^* = p$. This proves first part of the proposition.

To complete the proof of the second part note that due to Assumption A.2 $r^F + \frac{(1-p)}{p}\left(r^F - r^L\right) < r^H$. Hence from

$$r_c^D \in \left[pr^H + (1-p)\, r^L ; r^F + \frac{(1-p)}{p}\left(r^F - r^L\right) \right)$$

follows that $r^L \le r_c^D < r^H$ and again under $x^* = 1$ Lemma 1 guarantees $q^* = p$.

■

4 Rationale for regulation

The two economies described here differ only in the validity of limited liability principle. In the world with limited liability, banking system provides a missing link between creditors and borrowers, if banking sector has monopolistic structure. In case of competitive banking system, creditors (households) still deposit with the banks, but whether the banks invest in risky projects, and how much, is indeterminate: $x^* \in [0; 1]$. Moreover competitive equilibrium in case of limited liability produces arbitrarily high deposit interest rate, which plays no role, since depositors know, that the probability of deposit repayments is zero, and count only for the expected liquidation value of the banks, which is still higher than the return from safe asset.

Economy with unlimited liability exhibits two different properties depending on the ratio between p^2 and $\frac{r^F - r^L}{r^H - r^L}$. If $p^2 > \frac{r^F - r^L}{r^H - r^L}$; equilibrium with positive amount of deposits in banks exists and is unique under both monopolistic and competitive banking systems. If $p^2 = \frac{r^F - r^L}{r^H - r^L}$; unique positive equilibrium still exists under competitive banking system, but is multiple under monopolistic banking sector. Already in this case it is possible that the monopolist bank invests only in risk-free asset ($x = 0$) and if the risky asset represents financial obligations of productive firms, this means interruption of the channelling of funds from savers (households) to borrowers (firms). Finally, if $p^2 < \frac{r^F - r^L}{r^H - r^L}$, monopolistic structure of banking sector leaves no other possibility than such equilibrium with broken funds channelling, and competitive banking sector leads to degenerate equilibrium, in which there are no deposits in banking system, which is called disintermediation.

To make clear whether equilibria without investment in risky asset are inefficient, compare intermediated case considered above with the case, where depositors would have direct access to financial market with risky asset. Depositors choose then between two assets: risky and risk-free, and their decision about the share a of their endowment Q to be invested into risky asset looks as follows:

$$a = \begin{cases} 1 & \text{if } pr^H + (1-p)r^L \geq r^F \\ 0 & \text{if } pr^H + (1-p)r^L < r^F \end{cases}$$

And since by assumption (A.2) $pr^H + (1-p)r^L > r^F$, optimal choice of households in the case of direct investment would always be in favour of risky asset. In other words the amount I_{dir} invested in risky asset without intermediation is

$$I_{dir} = Q$$

If, however, the economy is intermediated, and the principle of limited liability does not hold[9], investments I_{int} in risky asset are (generally written to encompass both monopolistic and competitive banking sectors):

$$I_{int} = \begin{cases} Q & \text{if } p^2 > \frac{r^F - r^L}{r^H - r^L} \\ I_{int} \in [0; Q] & \text{if } p^2 = \frac{r^F - r^L}{r^H - r^L} \\ 0 & \text{if } p^2 < \frac{r^F - r^L}{r^H - r^L} \end{cases}$$

Note that there exists a nonempty set of possible values of p

$$0 < \frac{r^F - r^L}{r^H - r^L} < p < \sqrt{\frac{r^F - r^L}{r^H - r^L}} < 1$$

which[10] still meets the assumption (A.2) and hence would lead to positive investment in risky asset in case without intermediation, but leads to zero risky investments in intermediated case. We call such equilibria inefficient.

The intuition behind this inefficiency is that the depositors require from banks higher interest rate than if they would require from issuers of risky assets directly. To avoid inefficient equilibria a regulatory intervention is needed.

The purpose of the regulator could be therefore to provide conditions for positive equilibria, i.e. to guarantee $p^2 > \frac{r^F - r^L}{r^H - r^L}$. Since r^L and r^H are macroeconomic parameters, describing [in particular] productive opportunities of firms, they cannot be regulated endogenously.

[9] The outcomes in the economy with limited liability are described above: $I_{int} = Q$ in case of monopoly or $I_{int} \in [0; Q]$ in case of competitive banking sector independently on macroeconomic parameters p; r^H; r^L; and r^F.

[10] Note that the expression under the square root is always positive due to assumption $r^H > r^F > r^L$.

The regulation of risk-free rate of return r^F could also help against rupture of funds channelling: setting it as close as possible to r^L ensures that $p^2 > \frac{r^F - r^L}{r^H - r^L}$ and hence a unique nondegenerate equilibrium will necessarily exist under both monopolistic and competitive structures of the banking sector. Modelling regulation of r^F requires modelling a financial market of this asset. Along with that this measure would be a short-term market intervention in order to increase or to decrease the rate of return. At the same time the probability of success p describes rather a long-term macroeconomic trend, and tuning financial system to long-term tendencies through short-term interventions seems not to be an appropriate solution.

The only parameter "responsible" for existence of positive equilibrium is p^2, which also seems to be an exogenously given macroeconomic parameter. But recall that p^2 is obtained through two types of probabilities described in the model, i.e. the exogenous probability p of getting r^H on risky asset, and endogenous probability of deposit repayments q. Up to now, the equilibrium value of repayments probability in case of unlimited liability was $q = p$. Instead we suggest to decompose p^2, i.e. to introduce asymmetry between probability q of deposit repayments, and the probability p of investment success. More precisely we suggest to introduce the possibility of bailouts in banking system, which would guarantee that the depositors of bailed-out banks get their deposits in full and with interest accrued on them, independently of the fact whether their bank is solvent or not in the second period. Intuitively, this measure should lower the critical deposit rate, which is a threshold between zero and nonzero deposit supply, and hence this should increase the space for nondegenerate equilibria to appear.

5 Regulated economy

Regulator is responsible for bailouts or liquidation of banks. Bailout presumes paying the debts of the bank out to depositors in order to save the bank. Liquidation means selling bank assets in case of its insolvency and transferring the proceeds to the depositors indemnifying for banks' debts. In general regulator does not promise to save the banks unambiguously, but rather announces some probability of bailouts z.

If the banks in period 2 are not able to repay all their debts, the regulator can intervene and bail the banks out. If the bailout is performed (which probability is z), depositors receive their deposits in full with interest accrued on them. If, however, the bailout is not performed, the banks are liquidated and the repayments to depositors are determined by the value of banks' assets in period 2. Depositors are informed about the values of p, z and the share of risky asset in banks' portfolios x, so that they can form expectations about future repayments on deposits, given the announced deposit rate r^D.

Two bailout scenarios are possible. In case of unconditional bailouts, the regulator saves failing …nancial institutions in period 2 with probability z, whatever the state of the nature in period 2 is. In case of conditional bailouts, the regulator saves failing …nancial institutions with probability z only if "L"-state of nature is realised in period 2. Since in equilibrium banks can fail only if the state of nature "L" is realised, both scenarios lead to the same equilibrium outcome (detailed analysis can be found in Vinogradov, 2004). Therefore all considerations hereinafter are based at the example of unconditional bailouts.

Lemma 2 Given the parameters r^H, r^F, r^L and p, and the probability of bailout z, the probability of deposit repayments q is a function of r^D and x:

$$q\left(x; r^D\right) = \left\{ \begin{array}{lll} z & \text{if} & C_1 \\ p + (1-p)\,z & \text{if} & C_2 \\ 1 & \text{if} & C_3 \end{array} \right\} \qquad (26)$$

Proof.

The proof is similar to Lemma 1.

■

Consider now decisions made by the agents in case LOLR announces in period 1 unconditional bailouts policy: no matter what is the state of the nature in period 2, banks are bailed out with probability z.

5.1 Supply of deposits

Proposition 10 Aggregate deposit supply in economy with bailouts is given by the function

$$D^s = \left\{ \begin{array}{lll} Q & \text{if} & r^D \geq r^F + \frac{(1-p)(1-z)}{p+z(1-p)}x\left(r^F - r^L\right) \\ 0 & \text{if} & r^D < r^F + \frac{(1-p)(1-z)}{p+z(1-p)}x\left(r^F - r^L\right) \end{array} \right\} \qquad (27)$$

Proof.

The proof repeats the proof of Proposition 1.

■

5.2 Expected pro…t of the banks

If unconditional bailouts are performed with the probability z, the expected pro…t of the bank di¤ers now from that in no-bailouts case.

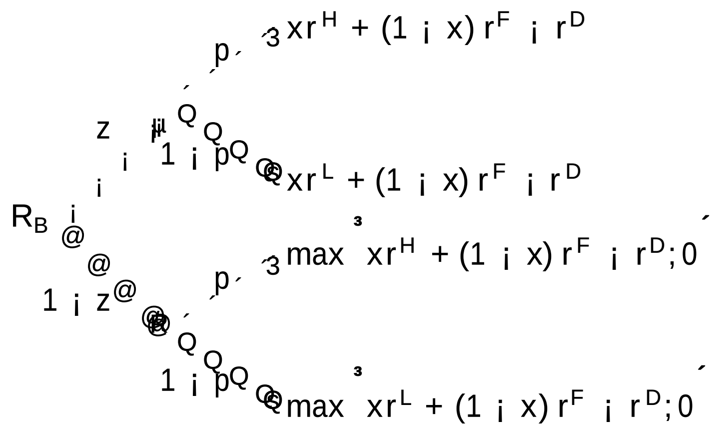

Figure 3: Pro…t/Deposits ratio of banks in case of unconditional bailouts

Indeed, if the bank is bailed out, probability of which is z, the bank obtains additional funds (in a form of continuation loan from LOLR or newly accumulated deposits) and continues its operations, so that its expected pro…t is the same as above. If however the bank is not bailed out, with probability $(1 - z)$, the principle of limited liability is valid, since the bank is liquidated and its debts are covered in amount not exceeding the value of bank's assets. Hence the expected pro…t of the bank accounts now for the possibility of bailouts[11]:

$$\begin{aligned} \Pi^e = \; & z\left[x\left(pr^H + (1-p) r^L\right) + (1-x) r^F - r^D\right] D^d + \\ & (1-z)\, p \max\left(xr^H + (1-x) r^F - r^D; 0\right) D^d + \\ & (1-z)(1-p) \max\left(xr^L + (1-x) r^F - r^D; 0\right) D^d \end{aligned} \tag{28}$$

5.3 Monopoly: demand for deposits and monopolistic equilibrium

Proposition 11 Optimal choice of the monopolistic bank under unconditional bailouts is

$$\begin{aligned} x^* &= 1 \\ r_m^D &= r^F + \frac{(1-p)(1-z)}{p+z(1-p)}\left(r^F - r^L\right) \end{aligned} \tag{29}$$

Proof.

[11] Remember that we concentrate on only one e¤ect of bailout, namely that the bank is let to coninue its operations.

To prove the proposition it suffices to consider again 3 cases C_1, C_2 and C_3 and find a solution of profit maximisation problem in each case. Obviously case C_1 leads to negative profit of the bank and consequent choice of $x^* = 0$ and $r_m^D = r^F$ to eliminate losses. Both cases C_2 and C_3 lead to choice of $x^* = 1$, but C_3 leads to zero profit due to zero deposit supply. Being a price-maker, monopolist bank chooses interest rate to maximise profit, and this choice is (29). It is easy to check that under this choice expected profit of the bank is always positive, and no additional condition is needed.

■

Proposition 12 Monopolistic equilibrium under unconditional bailouts is given by

$$\left\{ \begin{array}{c} x^* = 1 \\ a^* = 1 \\ q^* = p + (1 - p)\, z \\ r_m^D = r^F + \frac{(1-p)(1-z)}{p+z(1-p)} \left(r^F - r^L \right) \end{array} \right\} \qquad (30)$$

Proof. Condition $D^d = D^s$ is met through profit maximisation problem of the bank. It is easy to check, that other conditions from the definition of equilibrium are met as well.

■

With $z = 0$ this result turns into monopolistic equilibrium under limited liability (14). Intuition behind this fact is that announcement of zero probability of bailouts means that there is no other alternative for failing banks as liquidation. This is exactly what limited liability principle is: if the bank fails, its creditors receive only its rest value. Sinn (2003) calls this BLOOS-rule: "you can't get blood out of a stone".

With $z = 1$ this result would represent another symmetric case: when both banks and depositors believe that the banks will continue in the second period, and will be able (for whatever reason) to repay on deposits). Monopolist bank then sets interest rate at $r_m^D = r^F$, and the probability of deposit repayments is $q^* = 1$. We did not analyse this case at the beginning of the paper, since its' analysis just resembles the case of limited liability.

The case, on which we focused, was rather asymmetry in beliefs of depositors and banks, where depositors believe that failing banks will be closed, but the banks believe to continue, and so act within unlimited liability principle. This possibility for asymmetry is now excluded, since the regulator (LOLR or Central Bank) announces its policy with respect to bailing institutions. We can thus expect that this announcement effect would produce certainty about competitive equilibrium, which was previously not the case.

5.4 Competitive banking sector: demand for deposits and competitive equilibrium

Similarly to the competitive case without bailouts, we can prove following:

Proposition 13 Competitive demand for deposits under bailouts is given by

$$D^d = \begin{cases} 0 & \text{if } r^D > \frac{1}{z+(1-z)p}\left(pr^H + z(1-p)r^L\right) \\ D \in [0; \infty) & \text{if } r^D = \frac{1}{z+(1-z)p}\left(pr^H + z(1-p)r^L\right) \\ D \to \infty & \text{if } r^D < \frac{1}{z+(1-z)p}\left(pr^H + z(1-p)r^L\right) \end{cases} \tag{31}$$

Proof.

Each competitive bank maximises its profit but cannot influence r^D. Case C_1 is trivial, since leads to $D^d = 0$. Case C_3 is also trivial and leads to $D^d = D \to \infty$. Case C_2 produces optimal $x^* = 1$, and consequent maximisation of profit with respect to D^d leads to distinguishing between cases with different signs of first derivative of profit (and Lagrangian), as in Proposition 6, and leads to the solution (31).

■

To determine competitive equilibrium we again just need to satisfy the condition $D^d = D^s$.

Proposition 14 Competitive equilibrium under unconditional bailouts is given by the following:

$$\begin{Bmatrix} x^* = 1 \\ a^* = 1 \\ q^* = p + (1-p)z \\ r_c^D = \frac{1}{z+(1-z)p}\left(pr^H + z(1-p)r^L\right) \end{Bmatrix} \tag{32}$$

Proof.

Comparing equations for deposit demand (31) and deposit supply (27) we obtain that the only possibility for equilibrium is $r_c^D = \frac{1}{z+(1-z)p}\left(pr^H + z(1-p)r^L\right)$. Higher interest rate generates excessive supply (under zero deposit demand according to equation 31). Lower interest rate generates excessive (infinitely high) deposit demand. We need only to check, that the deposit supply with r_c^D from (32) is positive. This is guaranteed by the assumption A.2.

■

Note again that under $z \to 0$, this equilibrium is very similar to competitive equilibrium under limited liability. However now we have no failure of financial intermediation, and all funds collected from households are invested in risky asset, as it would be if the households had direct access to the risky asset market. Moreover, with $z \to 0$ competitive equilibrium is unique and characterised by interest rate $r_c^D = r^H$. If $z \to 1$, competitive banking sector offers deposit interest rate exactly at the level of expected rate of return of the risky asset, which is higher than monopolistic equilibrium at the risk-free rate.

6 Conclusions

We started this paper with comparison of financial intermediation systems in two different worlds: one with and one without limited liability. The literature on financial intermediation stresses the role of financial intermediaries in channelling funds from creditors to borrowers if they can not have a direct deal at the financial market. This was also the starting point in this paper. We show however, that intermediation can fail to do this.

In the world with limited liability, monopoly bank successfully links creditors and borrowers and invests all the funds collected from households in risky asset, as would be in case of direct investment. This result changes dramatically as soon as the banking sector is competitive. Competition under limited liability forces banks to choose excessively high (above the highest possible at the risky asset market) deposit interest rate (which can not even be bounded from above), and knowing that this strategy leads to zero profit, banks loose incentives to invest all deposit collected in risky asset. Hence, they freely choose the share of risky asset in their portfolio $0 \leq x \leq 1$, and in general case aggregate investment in risky asset is below its level attained if households have direct access to financial markets.

In the world with unlimited liability (in which, however, depositors do not believe) both monopolistic and competitive equilibria can ensure efficient intermediation (in sense that all funds are channelled from creditors to borrowers). Asymmetry in views of depositors and banks (the first believe that insolvent banks are liquidated, and the latter believe that they will be allowed to operate further) generates, however, another possibility for financial intermediation to fail in channelling funds from depositors to the risky asset market. This failure arises if the probability of success in risky investment is not sufficiently high, but can be corrected through bailout policy of the regulator.

Bailout policy of the regulator has two effects. First it is an announcement effect, which eliminates asymmetry in the beliefs of banks and households. Secondly, it is insurance effect, which lowers both monopolistic and competitive equilibrium interest rates. An announcement effect does not require from the regulator to follow the policy of unambiguous or ambiguous bailouts, instead the probability of bailouts can be chosen at the level of $z = 0$, but the market should still expect that the continuation is possible. A similar effect of the bailout policy can also be found in Hakenes and Schnabel (2004) who write that the result is "independent on whether the government does in fact bail out banks, or whether markets only expect the government to do so. Since the negative impact arises from the expectations of market participants, the government should try to build up a reputation of being committed to a zero-bail-out policy". The question of optimal policy design was not in the focus of this current paper, but the principle of regulator's credibility plays an important role in announcement effect.

Turning to the case of unlimited liability is another way to weaken the principle of limited liability, which causes the problem in competitive case. Matutes and Vives (2000), for example, suggest that a "fair and risk-based deposit insurance makes banks fully liable", which is a kind of endogenisation of the banks' responsibility under limited liability. Allen and Gale (2000) relax limited liability by assuming that the banks can always choose zero amount of deposits to be collected, which turns their profit to be non-negative. However this destructs the link between creditors and borrowers. Competition in financial intermediation generates complex and non-trivial problems, which are reviewed by Allen and Gale (2004), and this current paper contributes to the search for particular solutions.

7 References

Allen F., D. Gale (2000) "Competition in Banking" in Comparing Financial Systems, pp. 231–270, Cambridge and London: MIT Press

Allen F., D. Gale (2004) "Competition and Financial Stability", Journal of Money, Credit and Banking, forthcoming

Bennett R. (2001) "Failure Resolution and Asset Liquidation: Results of an International Survey of Deposit Insurers", FDIC Banking Review Vol. 14, No1, pp. 1–28

Bhattacharya S., A.W.A. Boot, A.V. Thakor (1998) "The Economics of Bank Regulation", Journal of Money, Credit and Banking, Vol. 30 No. 4, pp.745-770

Calomiris C.W., J.R. Mason (1997) "Contagion and Bank Failures During the Great Depression: The June 1932 Chicago Banking Panic", The American Economic Review, Vol. 87 No.5, pp.863-883.

Corbett J., J. Mitchell (2000) "Banking Crises and Bank Rescues: The

Effect of Reputation", Journal of Money, Credit and Banking, Vol. 32 No. 3, pp.474-512

Dewatripont M., J. Tirole (1993) "The Prudential Regulation of Banks", Cambridge, Ma.: MIT Press

Freixas X. (2000) "Optimal Bailout Policy, Conditionality and Constructive Ambiguity", the Bank of Netherlands: Staff Reports No. 49

Gersbach, H., J. Wenzelburger (2002), "Banking and the Macroeconomy", University of Heidelberg, Discussion paper No. 343.

Gollier C., P.-F. Koehl, J.-C. Rochet (1995) "Risk-Taking Behavior with Limited Liability and Risk-Aversion", Wharton Financial Institutions Center, Working Paper 96-13

Hakenes, H., I. Schnabel (2004), "Too Small to Be Saved - Competitive Effects of an Asymmetric Bail-out Policy", University of Mannheim, mimeo.

Matutes, C., X. Vives (2000) "Imperfect Competition, Risk Taking and regulation in Banking", European Economic Review 44, pp. 1–34

Noe T.H., S.D. Smith (1997) "The Buck Stops Where? The Role of Limited Liability in Economics", Federal Reserve Bank of Atlanta Economic Review First Quarter 1997, pp. 46–56

Rochet, J.-C. (1992) "Capital Requirements and the Behaviour of Commercial banks", European Economic Review 36, pp. 1137–1178

Santomero and Hoffman (1998) "Problem Bank Resolution: Evaluating the Options", W.P. 98-05, Financial Institutions Center, The Wharton School, University of Pennsylvania

Sinn, H.-W. (2003) "Risk Taking, Limited Liability and the Competition of Bank Regulators", Finanzarchiv, Vol.59, No3, pp.305-329, reproduced as Ch.7 in Sinn H.-W., "The New Systems Competition", Blackwell, 2003.

Vinogradov D. (2003) "Macroeconomic Evolution after a Shock: The Role for Financial Intermediation", presented at 35th Annual Conference of the Money, Macro and Finance research group, 10 - 12 September 2003, Cambridge, UK.

Vinogradov D. (2004) "Bailout Policy against Financial Intermediation Failures", University of Heidelberg, mimeo

8 Appendix

Proposition 2 Optimal choice of the monopolist bank under limited liability is $x = 1$ and $r^D_m = r^F + \frac{(1-p)}{p}\left(r^F - r^L\right)$

Proof. Lagrangian for the optimisation problem (11) is

$$L = \left(x r^H + (1-x) r^F - r^D \right) + \mu_0 \left(r^D - r^F + \frac{(1-p)\,x}{p} \left(r^F - r^L \right) \right) + \mu_1 x + \mu_2 (1-x)$$

and the Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial x} = r^H - r^F + \mu_0 \frac{(1-p)}{p} \left(r^F - r^L \right) + \mu_1 - \mu_2 = 0 \qquad (33)$$

$$\frac{\partial L}{\partial r^D} = -1 + \mu_0 = 0 \qquad (34)$$

Condition (33) straightforwardly implies $\mu_0 = 1$, which requires $r^D = r^F + \frac{(1-p)x}{p} \left(r^F - r^L \right)$.

Given $\mu_0 = 1$, second KT-condition (34) transforms into

$$p r^H + (1-p) r^L - r^F + p\mu_1 - p\mu_2 = 0$$

Under assumption (1) this implies $\mu_2 > 0$, and consequently $x = 1$. ∎

Proposition 4 Optimal choice of the monopolist bank with unlimited liability is

$$\begin{cases} \left\{ \begin{array}{l} x^* = 1 \\ r_m^D = r^F + \frac{(1-p)}{p} \left(r^F - r^L \right) \end{array} \right\} & \text{if } p^2 > \frac{r^F - r^L}{r^H - r^L} \\ \left\{ \begin{array}{l} x^* \in [0;1] \\ r_m^D = x \left(p r^H + (1-p) r^L \right) + (1-x) r^F \end{array} \right\} & \text{if } p^2 = \frac{r^F - r^L}{r^H - r^L} \\ \left\{ \begin{array}{l} x^* = 0 \\ r_m^D = r^F \end{array} \right\} & \text{if } p^2 < \frac{r^F - r^L}{r^H - r^L} \end{cases}$$

Proof. Lagrangian for the optimisation problem (16) is

$$\begin{aligned} L = {} & x \left(p r^H + (1-p) r^L \right) + (1-x) r^F - r^D + \\ & \mu_0 \left(r^D - r^F - \frac{(1-p)\,x}{p} \left(r^F - r^L \right) \right) + \mu_1 x + \mu_2 (1-x) \end{aligned}$$

and the Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial x} = p r^H + (1-p) r^L - r^F - \mu_0 \frac{(1-p)}{p} \left(r^F - r^L \right) + \mu_1 - \mu_2 = 0 \qquad (35)$$

$$\frac{\partial L}{\partial r^D} = -1 + \mu_0 = 0 \qquad (36)$$

Condition (35) straightforwardly implies $\mu_0 = 1$, which requires $r^D = r^F + \frac{(1-p)x}{p}\left(r^F - r^L\right)$.

Given $\mu_0 = 1$, second KT-condition (36) transforms into

$$p\left(pr^H + (1-p)\,r^L\right) - pr^F - (1-p)\left(r^F - r^L\right) + p\mu_1 - p\mu_2 = 0$$

Hence $\mu_2 > 0$, and consequently $x = 1$ and $r^D_m = r^F + \frac{(1-p)}{p}\left(r^F - r^L\right)$ if

$$p^2 > \frac{r^F - r^L}{r^H - r^L}$$

Expected profit of the bank is in this case positive:

$$\Pi^e = \left[\left(pr^H + (1-p)\,r^L\right) - r^D\right] Q > 0$$

On the contrary, $\mu_1 > 0$, and consequently $x = 0$ and $r^D_m = r^F$ if

$$p^2 < \frac{r^F - r^L}{r^H - r^L}$$

Finally, if $p^2 = \frac{r^F - r^L}{r^H - r^L}$, the bank is indifferent between investing in safe or risky asset, so that x is arbitrary $x \in [0;1]$; and the profit of the bank is zero. In this case we just need to note that

$$\begin{aligned} r^D_m &= r^F + \frac{(1-p)}{p}x\left(r^F - r^L\right) = \left\{p^2 = \frac{r^F - r^L}{r^H - r^L}\right\} = \\ &= x\left(pr^H + (1-p)\,r^L\right) + (1-x)\,r^F \end{aligned}$$

■

Proposition 6 Competitive demand for deposits under limited liability is given by a correspondence

$$D^d = \left\{ \begin{array}{ll} A \in [0;\infty) & \text{if } r^D \geq r^H \\ A \to \infty & \text{if } r^D < r^H \end{array} \right\} \qquad (37)$$

Proof.

Consider first condition C_1. If $r^D > r^H \geq xr^H + (1-x)\,r^F$ than decision of the banks is arbitrary. Assume now that $r^H \geq r^D > xr^H + (1-x)\,r^F$. Profit-maximising banks set $x^* = 1$, and condition C_1 turns to $r^D = r^H$. Optimal choice of the amount of deposits in banks' portfolio is again arbitrary and denoted with $A \in [0;\infty)$.

In case C_2 the banks maximise expected profit given by

$$\Pi^e = p\left(xr^H + (1-x)\,r^F - r^D\right)D^d$$

Lagrangian for the problem (19) is

$$\begin{aligned} L &= p\left(xr^H + (1-x)\,r^F - r^D\right)D^d \\ &\quad + \mu_{01}\left(xr^H + (1-x)\,r^F - r^D\right) + \mu_{02}\left(r^D - xr^L - (1-x)\,r^F\right) \\ &\quad + \mu_1 x + \mu_2(1-x) + \mu_3 D^d \end{aligned}$$

Kuhn-Tucker conditions imply

$$\frac{\partial L}{\partial x} = p\left(r^H - r^F\right)D^d + \mu_{01}\left(r^H - r^F\right) + \mu_{02}\left(-r^L + r^F\right) + \mu_1 - \mu_2 = 0 \quad (38)$$

$$\frac{\partial L}{\partial D^d} = p\left(xr^H + (1-x)\,r^F - r^D\right) + \mu_3 = 0 \quad (39)$$

Assumption $r^H > r^F > r^L$ implies $\mu_2 > 0$ and consequently $x = 1$. Substituting this into the second KT-condition obtain $\mu_3 < 0$, which contradicts the properties of Lagrange multipliers, and hence demand for deposits is infinitely large. Condition C_2 turns in this case into $r^H > r^D \geq r^L$

Finally in case C_3 banks maximise their profit given by $\left[x\left(pr^H + (1-p)\,r^L\right) + (1-x)\,r^F - r^D\right]D^d$.

Lagrangian for this case is

$$\begin{aligned} L &= \left[x\left(pr^H + (1-p)\,r^L\right) + (1-x)\,r^F - r^D\right]D^d \\ &\quad + \mu_0\left(xr^L + (1-x)\,r^F - r^D\right) \\ &\quad + \mu_1 x + \mu_2(1-x) + \mu_3 D^d \end{aligned}$$

Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial x} = \left[\left(pr^H + (1-p)\,r^L\right) - r^F\right]D^d + \mu_0\left(r^L - r^F\right) + \mu_1 - \mu_2 = 0 \quad (40)$$

$$\frac{\partial L}{\partial D^d} = x\left(pr^H + (1-p)\,r^L\right) + (1-x)\,r^F - r^D + \mu_3 = 0 \quad (41)$$

Due to assumption $\left(pr^H + (1-p)\,r^L\right) > r^F$ condition (41) implies $\mu_3 < 0$ and demand for deposits is again infinitely large $D^d \to \infty$. Therefore in condition (40) $\forall r^L, r^F > 0$ and $\forall \mu_0 \geq 0$ $\exists D^d : \mu_2 > 0$ and consequently $x = 1$. Condition C_3 turns into $r^D < r^L$

■