On Log-Periodic Crashes

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Abstract

A recent theory suggests that crashes are deterministic and governed by log-periodic formulas [2, 3]. One- and two-harmonic equations are in practice employed to fit daily data. Low frequency is theoretically critical for the log-periodicity hypothesis in that the crashes result from the slow build-up of long-range correlations. We extend this theory by putting forward a three-harmonic log-periodic equation that encompasses the one- and two-harmonics and that adjusts better not only to daily data but also to a higher frequency financial series.

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1. Introduction

Recently some geophysicists have suggested that financial crashes, like material rupture [1], are deterministic and governed by log-periodic formulas [2, 3]. This theory has received reasonable media coverage and a best-selling book [4] is out on the pop-science shelves of bookstores.

This is not so surprising because its straightforward implication is for financial crashes to be predictable. Indeed the discoverers of log-periodicity have made the sanguine claim [2] of having picked out the signals prior to the Wall Street crashes of 1929, 1962, and 1987, as well as the 1997 crash on the Hong Kong stock exchange. And they also claim to have forecasted the Nasdaq high-tech bubble burst in April 2000 and correctly predicted the sudden upturn of the Japanese Nikkei index in January 1999.

Searching for log-periodic signals is not widespread in the financial world because their pioneering discoverers do not make public the operational details of crash prediction "but instead offer a few common sense guidelines" [2].

Technically what is available is one-harmonic and two-harmonic log-periodic equations that are usually employed to fit daily data [3]. Low frequency is key for the log-periodicity hypothesis in that crashes would result from slow build-up of long-range correlations. Self-reinforcing imitation between traders in a bull market leads to a bubble. This slow build-up of stress pushes the market to a critical time interval. After a threshold known as the critical point, many traders place the same order (i.e. sell) at the same time, thereby provoking the crash [4]. Here imitation makes the system periodic on the eve of a crash. In that sense crashes are outliers with properties that are statistically distinct from the rest of the population.

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The log-periodicity hypothesis of crashes that are outliers departs from conventional statistics that approaches extreme events. If log-periodicity is present at certain times in financial data then this is suggestive that these periods of time present scale invariance in their time evolution. (And this scaling has nothing to do with that related to the power law tails of returns [5].)

We extend this theory by putting forward a three-harmonic log-periodic formula that encompasses the previous one- and two-harmonics. Our formula outperforms the previous ones in that it adjusts better not only to the usual daily data but also to a higher frequency financial series.

Section 2 of this Letter presents the data and adjusts the log-periodic formulas to them. And Section 3 concludes.

2. Data and analysis

We take two financial series of distinct frequencies. A data set for the daily exchange rate between the Brazilian *real* and the US dollar and another one for the intraday, 15-minute spaced *real*-dollar rate.

The data set for the daily frequency is for the period from 2 January 1995 to 31 December 2003. The set comprises 2259 data points obtained from the Federal Reserve website. Gaps between the trading days are considered. Doing so produces an unequally time-spaced data set that is fitted by a non-linear regression using SAS (www.sas.com). Neglecting the gaps does not change results a great deal, however.

The 15-minute set comprises 9327 data points from 9:30AM of 19 July 2001 to 4:30PM of 14 January 2003. Gaps between office hours are also taken into account as above, though neglecting this does not significantly change results.

Log-periodic cycles with a smooth trend component are generally described by a sum of log-periodic harmonics, i.e.,

$$\ln Z(\tau) = A + B\tau^{\lambda} + \sum_{j=1}^{J} C_j \tau^{\lambda_j} \cos[j\theta_j \ln(\tau) + \phi_j], \qquad (1)$$

where τ is the time starting with the onset of a bubble. Term $A + B\tau^{\lambda}$ is the trend across time, and A, B, and λ give its shape. Parameters θ_j, C_j, and ϕ_j are angular log-frequency, amplitude, and phase of the j^{th} harmonic respectively. We set $\tau = t - t_c > 0$, where t_c is critical time. If $j = 1$ ($j = 2$) in (1), the one- (two-) harmonic log-periodic model [3] obtains. We suggest a three-harmonic log-periodic model considering $\lambda = \lambda_j$ and $\theta = \theta_j$.

The critical time is chosen by optimization from an arbitrarily taken time interval. For the daily series, the optimal t_c is searched from 26 June 2000 to 26 July 2001 in the context of every log-periodic model. The optimal t_c is found to be 31 July 2000 in the one-harmonic model, 11 August 2000 in the two-harmonics model, and 11 August 2000 in the three-harmonics model.

The optimal t_c of the intraday series is searched from 9:30AM of 28 May 2002 to 4:15PM of 17 June 2002 within the one-harmonic log-periodic model. Unlike in the daily series, we could not find an optimal t_c within the two- and three-harmonics models. Thus we decide to consider the one-harmonic model as a benchmark and take the same t_c for the higher-order models. The optimal t_c is found to be 11:00AM of 3 June 2002 in the one-harmonic model, and this is also used in the two- and three-harmonics models.

Figure 1 displays the log of the daily *real*-dollar rate from 31 July 2000 to 31 December 2003 together with its one-, two-, and three-harmonic log-periodic fit respectively. Even with a suboptimal t_c, the three-harmonics formula outperforms the others. (Yet adjustment fails for the series as a whole.)

Figure 2 shows the fit for the intraday data from 11:00AM of 3 June 2002 to 4:30PM of 14 January 2003 using one, two, and our suggested three harmonics. As can be seen, the three-harmonic log-periodic formula adjusts better to the data as well. Parameter values for the fits are presented in Tables 1 and 2.

3. Conclusion

The literature on log-periodicity usually employs daily data and the fits use one- and two-harmonic log-periodic equations. By taking both daily and intraday data from the exchange rate between the Brazilian *real* and US dollar, this Letter shows that a suggested three-harmonic log-periodic formula outperforms the others.

Log-periodic power laws in financial time series are intriguing and important from both a theoretical and practical point of view. At first sight our result in this Letter suggests that the signatures of coming crashes can also be detected at short time scales. But on second thoughts that explanation is at odds with the theory of slow build-up of long-range correlations, tracked precisely by low frequency data. This means that our result points to a different theoretical explanation for the hypothesis that crashes are log-periodic.

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Table 1

Parameter	Estimate ± Standard Error		
	$j = 1$	$j = 2$	$j = 3$
A	0.5813 ± 0.00806	0.5837 ± 0.00822	0.5815 ± 0.00819
B	0.00169 ± 0.000321	0.00220 ± 0.000401	0.00243 ± 0.000431
C_1	–0.00053 ± 0.000092	0.000701 ± 0.000117	0.000780 ± 0.000126
C_2		0.000117 ± 0.000024	–0.00013 ± 0.000027
C_3			0.000077 ± 0.000019
θ	–9.2141 ± 0.0569	–8.7247 ± 0.0511	–8.5762 ± 0.0472
ϕ_1	71.2343 ± 0.3740	45.9033 ± 0.3340	57.4966 ± 0.3089
ϕ_2		–2.2059 ± 0.6898	–1.1958 ± 0.6386
ϕ_3			–120.0 ± 0.9603
λ	0.8551 ± 0.0264	0.8153 ± 0.0253	0.7999 ± 0.0244
t_c	31 July 2000	11 August 2000	11 August 2000

Table 2

Parameter	Estimate ± Standard Error		
	$j = 1$	$j = 2$	$j = 3$
A	0.9406 ± 0.00527	0.9212 ± 0.00440	0.9500 ± 0.00282
B	0.00104 ± 0.000243	0.00135 ± 0.000222	0.000406 ± 0.000065
C_1	0.000263 ± 0.000057	0.000177 ± 0.000029	0.000073 ± 0.000012
C_2		0.000209 ± 0.000030	0.000100 ± 0.000014
C_3			0.000064 ± 9.503E-6
θ	3.6766 ± 0.0535	3.4977 ± 0.0211	3.5673 ± 0.0159
ϕ_1	66.2759 ±0.4828	61.7290 ± 0.2031	61.2662 ± 0.1455
ϕ_2		2.7737 ± 0.3930	1.3608 ± 0.2795
ϕ_3			–4.9071 ± 0.4269
λ	0.5944 ±0.0234	0.5768 ± 0.0162	0.6935 ±0.0162
t_c	11:00AM of 3 June 2002	11:00AM of 3 June 2002	11:00AM of 3 June 2002

Log-periodicity in daily (Table 1) and intraday (Table 2) *real*-dollar rate.
Results for one-, two-, and three- harmonic log-periodic models.

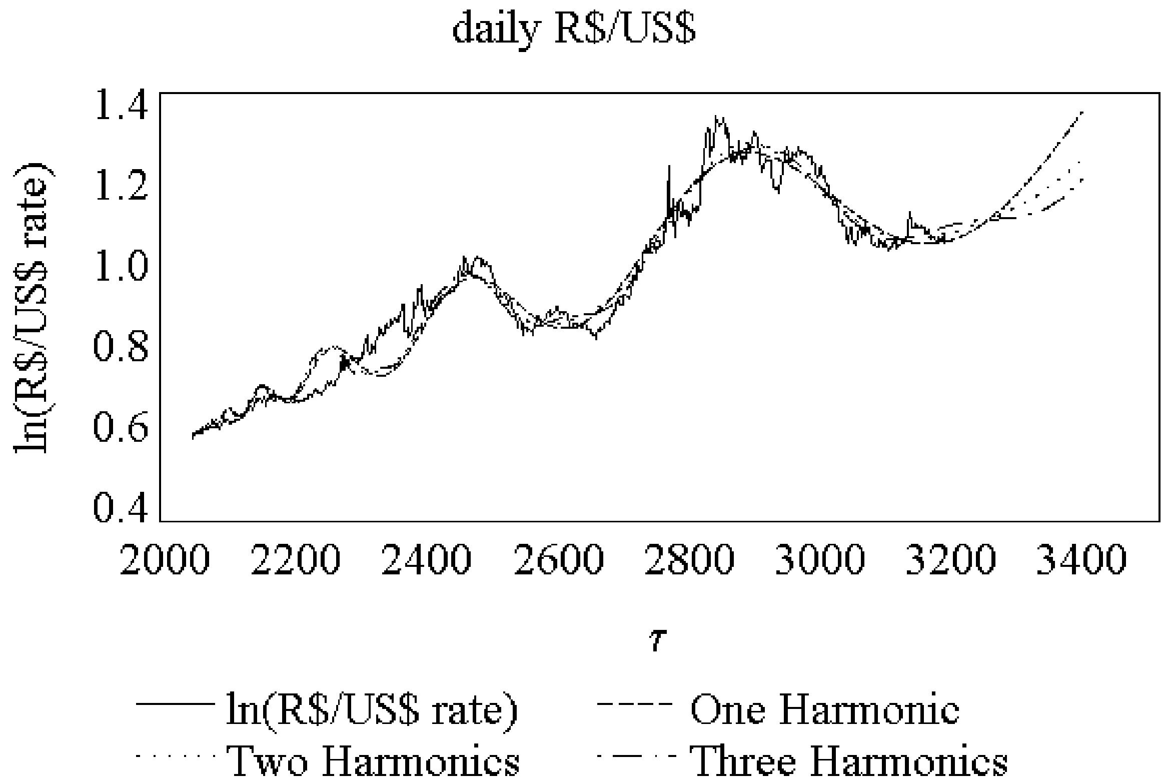

Figure 1. Log of the daily *real*-dollar rate from 28 August 2000 to 26 September 2003 together with its one-, two-, and three-harmonic log-periodic fits. See Table 1.

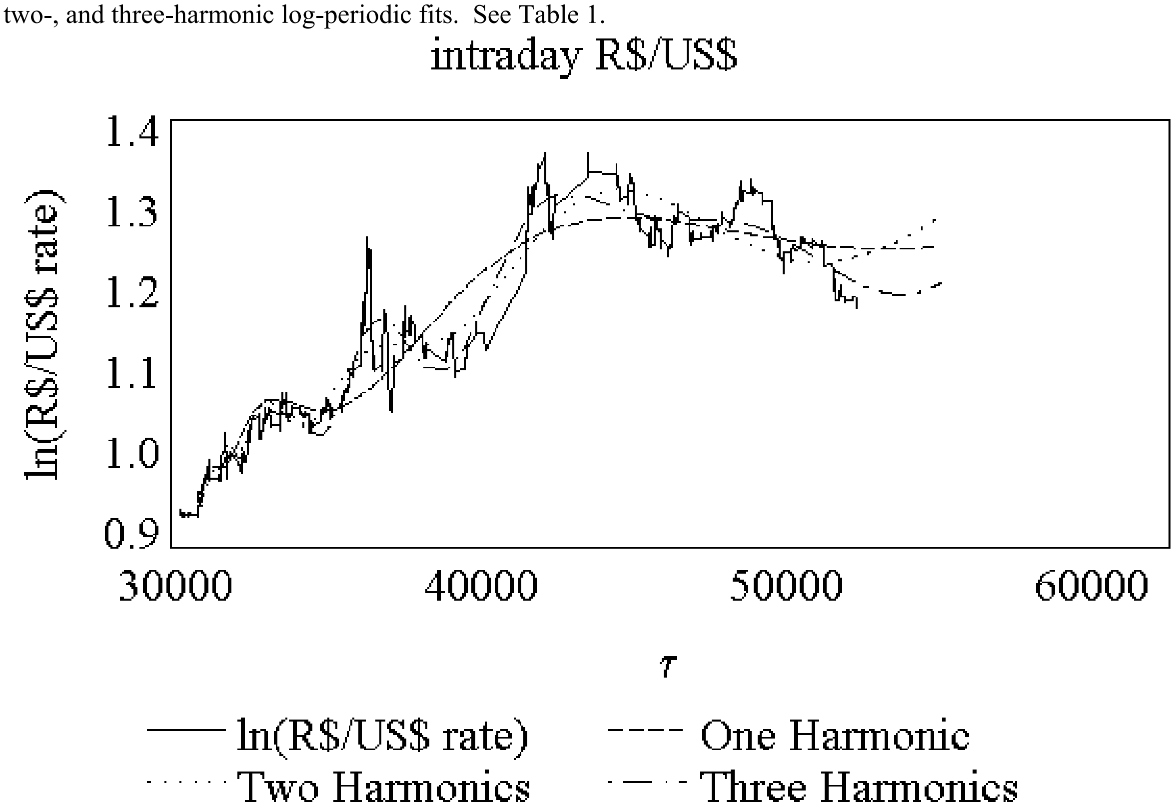

Figure 2. Log-periodic fits of intraday *real*-dollar returns. Our three-harmonic log-periodic formula adjusts better to data. We take 11:00AM of 3 June 2002 as the critical time for all models. See Table 2.