

ADJUSTMENT OF THE WACC WITH SUBSIDIZED DEBT IN THE PRESENCE OF CORPORATE
TAXES: THE N-PERIOD CASE

(Draft)

Ignacio Vélez-Pareja
Politécnico Grancolombiano
Bogotá, Colombia
ivelez@poligran.edu.co

Joseph Tham
Duke University
ThamJx@duke.edu

Viviana Fernández
Universidad de Chile
vfernand@dii.uchile.cl

First version: March 8, 2005

This version: April 6, 2005

Tham is with the Duke Center for International Development in the Sanford Institute For Public Policy at Duke University. Vélez-Pareja is Dean of the School of Industrial Engineering and Finance professor at Politecnico Grancolombiano, Bogota, Colombia. Fernández is Assistant Professor, Department of Industrial Engineering, Universidad de Chile.

ABSTRACT

In the Weighted Average Cost of Capital (WACC) applied to the free cash flow (FCF), we assume that the cost of debt is the market, unsubsidized rate. With debt at the market rate and perfect capital markets, debt only creates value in the presence of taxes through the tax shield. In some cases, the firm may be able to obtain a loan at a rate that is below the market rate. With subsidized debt and taxes, there would be a benefit to debt financing, and the unleveraged and leveraged values of the cash flows would be unequal. The benefit of lower tax savings are offset by the benefit of the subsidy. These two benefits have to be introduced explicitly.

In this paper we present the adjustments to the WACC with subsidized debt and taxes and the cost of leveraged equity for multiple periods. We demonstrate the analysis for both the WACC applied to the FCF and the WACC applied to the capital cash flow (CCF). We use the calculation of the Adjusted Present Value, APV, to consider both, the tax savings and the subsidy. We show how all the methods match.

KEY WORDS

Adjusted Present Value, APV, weighted average cost of capital, discounted cash flow, DCF equity value, cost of equity, WACC, subsidized debt with taxes, valuation of cash flows, project evaluation, project appraisal, firm valuation, cost of capital, cash flows, free cash flow, capital cash flow.

JEL CLASSIFICATION

D61, G30, G31, G32, H43

INTRODUCTION

In the Weighted Average Cost of Capital (WACC) applied to the free cash flow (FCF), we assume that the cost of debt is the market, unsubsidized rate. With debt at the market rate and perfect capital markets, debt only creates value in the presence of taxes through the tax shield. In some cases, the firm may be able to obtain a loan at a rate that is below the market rate. In a previous work we showed how to adjust the WACC in the presence of a subsidy and no taxes. There we showed that plugging the lower cost of debt into the WACC formula is not the correct approach to measuring the value creation due to the subsidy. With subsidized debt and taxes, there would be a benefit to debt financing, and the unleveraged and leveraged values of the cash flows would be unequal. The benefit of lower tax savings TS, are offset by the benefit of the subsidy. These two benefits have to be introduced explicitly.

In this paper we present the adjustments to the WACC and the cost of leveraged equity for multiple periods with subsidized debt and taxes. We demonstrate the analysis for both the WACC applied to the FCF and the WACC applied to the capital cash flow (CCF). We use the calculation of the Adjusted Present Value, APV, to consider both, the TS and the subsidy. We show how all the methods match.

The issue of the effect of subsidy in interest rate on the WACC is not widely dealt in the literature. Ross et al, 1999 mention the effect on value and propose to use the APV method and Damodaran 1996 suggests including the value of the subsidy in the cash flow.

This paper is organized as follows: In Section One we derive the expressions for the cost of capital in the presence of subsidy and corporate taxes for multiple periods and illustrate it with an example. In Section Two we conclude.

SECTION ONE

First we derive the cost of leveraged equity, K_e . Let V^L be the leveraged value, let V^{Un} be the unleveraged value, let V^{TS} the value of the TS, let T the corporate tax rate and let V^{LSub} be the value of the interest subsidy. Then, with respect to the end of year 0, the leveraged value equals the sum of the unleveraged value, plus the value of the TS and the value of the interest subsidy.

$$V^L_0 = V^{Un}_0 + V^{TS}_0 + V^{LSub}_0 \quad (1)$$

Using the APV approach, it would be very easy to estimate the value of the subsidized debt. Let Kd^{NS} be the cost of the non subsidized debt, and let Kd^{Sub} be the cost of the subsidized debt. The value of the debt at the end of year 0 is D_0 .

Let L^{Sub}_1 be the interest subsidy at the end of year 1 and TS_1 be the TS at the end of year 1. Then the interest subsidy equals the value of the debt times the difference between the two interest rates adjusted for taxes and the TS are the cost of unsubsidized debt times the debt, D_0 and times the tax rate, T .

$$L^{Sub}_1 = D_0(Kd^{NS} - Kd^{Sub}) \quad (2)$$

and

$$TS_1 = Kd^{Sub} \times T \times D_0 \quad (3)$$

The expression for the value of the interest subsidy is as follows, where λ is the appropriate discount rate for the interest subsidy.

$$V^{LSub}_0 = L^{Sub}_1 / (1 + \lambda) = D_0(Kd^{NS} - Kd^{Sub}) / (1 + \lambda) \quad (4)$$

The expression for the value of the TS is as follows, where ψ is the appropriate discount rate for the TS.

$$V_0^{TS} = Kd^{Sub} \times T \times D_0 / (1 + \psi) = D_0 \times T \times Kd^{Sub} / (1 + \psi) \quad (5)$$

DERIVATION OF KE

Let CCF_1 be the capital cash flow at the end of year 1 with financing. At the end of year 1, the capital cash flow equals the sum of the FCF, plus the TS and the interest subsidy.

Then,

$$CCF_1 = FCF_1 + L_1^{Sub} + TS_1 \quad (6)$$

Also, at the end of year 1, the capital cash flow equals the sum of the cash flow to equity (CFE) and the cash flow to debt (with the subsidized interest rate).

$$CCF_1 = CFE_1 + CFD_1 \quad (7)$$

Putting these two equations together, we obtain,

$$CCF_1 = CFE_1 + CFD_1 = FCF_1 + L_1^{Sub} + TS_1 \quad (8)$$

The corresponding value relationship is as follows.

$$V_0^L = E_0 + D_0 = V_0^{Un} + V_0^{LSub} + V_0^{TS} \quad (9)$$

Substituting the appropriate value expressions for each of the cash flow items in equation 8, we obtain,

$$E_0 \times (1 + Ke) + D_0 \times (1 + Kd^{Sub}) = V_0^{Un} \times (1 + Ku) + V_0^{LSub} \times (1 + \lambda) + V_0^{TS} (1 + \psi) \quad (10)$$

where Ke is the cost of leveraged equity and Ku is the cost of unleveraged equity.

Applying equation 9 to equation 10, we obtain,

$$E_0 \times Ke + D_0 \times Kd^{Sub} = V_0^{Un} \times Ku + V_0^{LSub} \times \lambda + V_0^{TS} \times \psi \quad (10.1)$$

$$E_0 \times Ke + D_0 \times Kd^{Sub} = (E_0 + D_0 - V_0^{LSub} - V_0^{TS}) \times Ku + V_0^{LSub} \times \lambda + V_0^{TS} \times \psi \quad (10.2)$$

Rearranging, we obtain,

$$E_0 \times Ke = E_0 \times Ku + D_0 \times (Ku - Kd^{Sub}) + V^{LSub}_0 \times (\lambda - Ku) + V^{TS}_0 \times (\psi - Ku) \quad (11)$$

Substituting equation 4 and 5 into equation 11, we obtain the expression for the Ke.

$$E_0 \times Ke = E_0 \times Ku + D_0 (Ku - Kd^{Sub}) + D_0 (Kd^{NS} - Kd^{Sub}) (\lambda - Ku) / (1 + \lambda) + [D_0 \times T \times Kd^{Sub} / (1 + \psi)] (\psi - Ku) \quad (12.1)$$

$$Ke = Ku + (Ku - Kd^{Sub}) D_0 / E_0 + (Kd^{NS} - Kd^{Sub}) [(\lambda - Ku) / (1 + \lambda)] D_0 / E_0 + [T \times Kd^{NS} / (1 + \psi)] (\psi - Ku) D_0 / E_0 \quad (12.2)$$

But

From (4) $V^{LSub}_0 = (Kd^{NS} - Kd^{Sub}) / (1 + \lambda)$ and

From (5) $V^{TS}_0 = Kd^{Sub} \times T \times D_0 / (1 + \psi) = D_0 \times T \times Kd^{Sub} / (1 + \psi)$

then

$$Ke = Ku + (D/E) (Ku - Kd^{Sub}) + V^{LSub}_0 (\lambda - Ku) / E + V^{TS}_0 (\psi - Ku) / E \quad (12.3)$$

If we assume that the appropriate discount rate for the interest subsidy and for the TS is equal to the cost of unleveraged equity, then the third and fourth terms in equation 12.2 are zero.

DERIVATION OF WACC^{CCF}

We now derive the WACC for the capital cash flow, CCF. From (8) we can write the following

$$V^L_0 \times (1 + WACC^{CCF}) = CCF_1 = FCF_1 + LSub_1 + TS_1 \quad (13)$$

and

$$V^L_0 \times (1 + WACC^{CCF}) = CCF_1 = V^{Un}_0 (1 + Ku) + V^{LSub}_0 \times (1 + \lambda) + V^{TS}_0 \times (1 + \psi) \quad (14a)$$

As per (9) then

$$V^L_0 \times WACC^{CCF} = V^{Un}_0 \times Ku + V^{LSub}_0 \times \lambda + V^{TS}_0 \times \psi \quad (14b)$$

and

$$V^L_0 \times WACC^{CCF} = (V^L_0 - V^{LSub}_0 - V^{TS}_0) \times Ku + V^{LSub}_0 \times \lambda + V^{TS}_0 \times \psi \quad (14c)$$

Rearranging terms

$$V^L_0 \times WACC^{CCF} = V^L_0 \times Ku + V^{LSub}_0 \times (\lambda - Ku) + V^{TS}_0 \times (\psi - Ku) \quad (14d)$$

Dividing by V^L_0

$$WACC^{CCF} = Ku + (V^{TS}_0 / V^L_0) (\lambda - Ku) + V^{LSub}_0 / V^L_0 (\psi - Ku) \quad (14e)$$

DERIVATION OF $WACC^{FCF}$

Now we derive the WACC to be applied to the FCF. As before, from (8) we can write the following

$$V^L_0 \times (1 + WACC^{FCF}) + LSub_1 + TS_1 = FCF_1 = CFE_1 + CFD_1 = CCF \quad (15a)$$

$$V^L_0 \times (1 + WACC^{FCF}) + V^{LSub}_0 \times (1 + \lambda) + V^{TS}_0 \times (1 + \psi) = V^L_0 \times (1 + WACC^{CCF}) \quad (15b)$$

Replacing the expression for $WACC^{CCF}$ we have

$$V^L_0 \times (1 + WACC^{FCF}) + V^{LSub}_0 \times (1 + \lambda) + V^{TS}_0 \times (1 + \psi) = V^L_0 \times (1 + Ku + (V^{TS}_0 / V^L_0) (\lambda - Ku) + V^{LSub}_0 / V^L_0 (\psi - Ku)) \quad (15c)$$

But

$$V^{LSub}_0 \times (1 + \lambda) = Sub \text{ and}$$

$$V^{TS}_0 \times (1 + \psi) = TS$$

Then

$$V^L_0 \times (WACC^{FCF}) + Sub + TS = V^L_0 \times (Ku + (V^{TS}_0 / V^L_0) (\lambda - Ku) + V^{LSub}_0 / V^L_0 (\psi - Ku)) \quad (15d)$$

Dividing by V^L_0

$$WACC^{FCF} + Sub / V^L_0 + TS / V^L_0 = Ku + (V^{TS}_0 / V^L_0) (\lambda - Ku) + V^{LSub}_0 / V^L_0 (\psi - Ku) \quad (15e)$$

Rearranging terms

$$WACC^{FCF} = Ku + (V^{TS}_0 / V^L_0) (\lambda - Ku) + V^{LSub}_0 / V^L_0 (\psi - Ku) - TS / V^L_0 - Sub / V^L_0 \quad (15f)$$

In summary we have

Table 1a. Summary of formulae for different discount rates

Cash Flow	Discount rate
CFE	$K_e = K_u + (D/E)(K_u - K_d^{\text{Sub}}) + V^{\text{LSub}}_0(\lambda - K_u)/E + V^{\text{TS}}_0(\psi - K_u)/E$
CCF	$WACC^{\text{CCF}} = K_u + (V^{\text{TS}}_0/V^{\text{L}}_0)(\lambda - K_u) + V^{\text{LSub}}_0/V^{\text{L}}_0(\psi - K_u)$
FCF	$WACC^{\text{FCF}} = K_u + (V^{\text{TS}}_0/V^{\text{L}}_0)(\psi - K_u) + V^{\text{LSub}}_0/V^{\text{L}}_0(\lambda - K_u) - \text{TS}/V^{\text{L}}_0 - \text{Sub}/V^{\text{L}}_0$

From this summary, we can obtain simpler formulations depending on the assumptions regarding the discount rate for TS and subsidy. For instance, if we assume that ψ and λ are equal to K_u , then the formulae for the different costs are

Table 1b. Formulae assuming $\lambda = \psi = K_u$

Cash Flow	Discount rate
CFE	$K_e = K_u + (K_u - K_d^{\text{Sub}})D/E$
CCF	$WACC^{\text{CCF}} = K_u$
FCF	$WACC^{\text{FCF}} = K_u - \text{TS}/V^{\text{L}}_0 - \text{Sub}/V^{\text{L}}_0$

The formula for K_e resembles the typical formulation of K_e when ψ is K_u , except that K_d is replaced by K_d^{Sub} . For the CCF we have $WACC^{\text{CCF}}$ equal to K_u ; this is what is expected when we use the CCF and assume K_u as the discount rate for TS. Finally, for discounting the FCF we have $WACC^{\text{FCF}}$ equal to $K_u - \text{TS}/V^{\text{L}}_0 - \text{Sub}/V^{\text{L}}_0$ and this resembles the adjusted WACC. (See Tham and Velez-Pareja 2004).

We illustrate these ideas with a three period numerical example. The values of the various parameters are shown below. We present the input variables and the final tables after solving the circularity¹.

The input variables are shown in table 2.

¹ See Velez-Pareja and Tham 2001, Tham and Velez-Pareja, 2004 and Velez-Pareja and Tham, 2005

Table 2. Input variables for single period example

Tax rate	20.0%
Cost of unleveraged equity, K_u	15.0%
Debt, D_0	842.67
Market cost of debt, K_d^{NS}	10.0%
FCF, constant	1,230.2
Subsidy on K_d	2.0%
Discount rate for Subsidy, λ	10.0%
Discount rate for TS, ψ	8.0%

Next we calculate the CFD with K_d^{Sub} , the TS, the subsidy and the CFE. These values will be needed to calculate K_e and WACC for FCF and CFE.

Table 3a. K_d^{Sub} , CFD, TS, Subsidy CFE, V_0^{TS} and V_0^{LSub}

Year	0	1	2	3
K_{dsub}		8.0%	8.0%	8.0%
Value of debt	842.669	842.669	842.669	
CFD		67.4	67.4	910.1
TS		13.5	13.5	13.48271
Subsidy		16.9	16.9	16.9
FCF		1,230.2	1,230.2	1,230.2
CFE = FCF + TS + Sub - CFD		1,193.2	1,193.2	350.5
V_0^{TS}	34.7463	24.0432	12.4840	
V_0^{LSub}	41.9119	29.2497	15.3213	

Now we can calculate the value of K_e for every year and we calculate the market value of equity.

Table 3b Leverage D% at market value, Ke and leveraged value of equity

Year	0	1	2	3
D%	29.2%	41.0%	76.8%	
D%/E%	0.412	0.696	3.306	
Ke		17.6658%	19.6126%	37.4975%
Leveraged equity value	2,042.8866	1,210.6236	254.9033	
Leveraged value = Equity+debt	2,885.5560	2,053.2929	1,097.5727	

For instance, for year 1, in the previous table we apply equation

$$K_e = K_u + (D/E)(K_u - K_d^{\text{Sub}}) + V^{\text{LSub}}_0(\lambda - K_u)/E + V^{\text{TS}}_0(\psi - K_u)/E$$

$$15\% + 0.412 \times (15\% - 8\%) + 41.912 \times (10\% - 15\%) / 2,042.887 + 34.746 \times (8\% - 15\%) / 2,042.887 = 17.666\%$$

(allow for rounding errors if the reader tries to replicate this calculation).

Table 4 FCF, WACC^{FCF} and leveraged value

Year	0	1	2	3
FCF		1,230.2	1,230.2	1,230.2
WACC ^{FCF}		13.8%	13.4%	12.1%
PV of FCF @ WACC	2,885.5560	2,053.2929	1,097.5727	

In the case of WACC^{FCF} we have for year 1,

$$K_u + (V^{\text{TS}}_0 / V^{\text{L}}_0)(\psi - K_u) + V^{\text{LSub}}_0 / V^{\text{L}}_0(\lambda - K_u) - \text{TS} / V^{\text{L}}_0 - \text{Sub} / V^{\text{L}}_0$$

$$15\% + (34.7463 / 2,885.5560) \times (8\% - 15\%) + (41.9119 / 2,885.5560) \times (10\% - 15\%) - 13.5 / 2,885.5560 - 16.$$

$$9 / 2,885.5560 = 13.8\%$$

Table 5 Unleveraged values, values of TS and subsidy and APV

Year	0	1	2
Unleveraged value	2,808.8979	2,000.0000	1,069.7674
V ^{TS} ₀	34.7463	24.0432	12.4840
V ^{LSub} ₀	41.9119	29.2497	15.3213
Leveraged value APV	2,885.5560	2,053.2929	1,097.5727

The figures from this table are taken from previous tables except the unleveraged value that is calculated as the present value of the FCF at K_u.

Table 6 Capital Cash Flow, CCF, WACC^{CCF} and leveraged value

Year	0	1	2	3
WACC for CCF		14.84%	14.85%	14.85%
CCF=FCF+TS+Sub		1,260.6	1,260.6	1,260.6
CCF=CFD+CFE		1,260.6	1,260.6	1,260.6
PV(CCF)	2,885.5560	2,053.2929	1,097.5727	

The CCF is derived from data from table 2. The WACC^{CCF} is derived using the next equation. For year 1 we have.

$$K_u + (V_0^{TS}/V_0^L)(\lambda - K_u) + V_0^{LSub}/V_0^L(\psi - K_u)$$

$$15\% + (34.7463/2,885.5560) \times (10\% - 15\%) + (41.9119/2,885.5560) \times (8\% - 15\%) = 14.84\%$$

Now we calculate the leveraged value assuming what is the current practice: to include the K_d^{Sub} in the traditional formula for WACC for the FCF. First we calculate the leveraged value without subsidy. This is what is shown in the next table.

Table 7. Calculation of value using K_d^{NS} and FCF

Year	0	1	2	3
Market cost of debt, K_d^{NS}		10.0%	10.0%	10.0%
Debt (% of leveraged value)	29.6%	41.6%	77.7%	
Debt-equity ratio	0.420	0.711	3.486	
K_e		17.10%	18.56%	32.43%
WACC		14.4%	14.2%	13.4%
FCF		1,230.2	1,230.2	1,230.2
Leveraged value	2,847.4	2,027.4	1,084.4	

Now we calculate the value using the traditional WACC for the FCF and including K_d^{Sub} as the cost of debt.

Table 8. Calculation of value using K_d^{Sub} and FCF

Year	0	1	2	3
Debt (% of leveraged value)	29.7%	41.7%	77.9%	
Debt-equity ratio	0.422	0.715	3.528	
K_e		17.95%	20.00%	39.70%
WACC		14.5%	14.3%	13.8%
FCF		1,230.2	1,230.2	1,230.2
Leveraged value	2,839.7	2,021.9	1,081.5	

Observe that the leveraged value has been reduced compared when we use the traditional WACC and include the K_d^{Sub} . A lower cost of debt destroys value! This is counter evident. This occurs because we have lost part of the value generated by the TS and because the K_e calculation absorbs the reduction of the cost of debt. This means that the subsidy has to be explicitly included in the analysis.

In the next table we present a summary of the different calculations for values:

Table 9. Different values with different methods

Method	Leveraged value	Equity value
No subsidy	2,847.4	1,997.03
With subsidy using K_d^{Sub} in the WACC	2,839.7	2,004.73
With subsidy using new formulation for WACC	2,885.5560	2,042.8866

In the numerical example, we assume that the appropriate discount rate for the interest subsidy λ is the market rate of interest. However, we could also use the subsidized rate K_d^{Sub} or the K_u . For completeness, in the next table we show the consistent results for the two other values for λ , namely K_d^{Sub} and K_u .

It might be argued that the differences in this example are irrelevant. However, we think that it is not a matter of precision; it is a matter of correctness that can be reached without extra cost. More, it is usual to assume that differences are assigned to rounding errors or that the magnitude is negligible or that practical approaches are more

important than theoretical and precise ones. However, while errors could cancel out, sometimes errors cumulate. See for instance Vélez Pareja 2004 and 2005.

Table 10. Results for different values of λ

	Unsubsidized	Subsidized	Ku
λ	10.00%	8.00%	15.00%
Equity, unsubsidized debt	2,004.71	2,004.71	2,004.71
Equity, subsidized debt	2,042.89	2,044.41	2,039.45
Value, using APV, WACC for FCF and WACC for CCF	2,885.56	2,887.08	2,882.12

CONCLUSION

In this paper, we show the adjustments that have to be made to the WACC in the presence of a subsidized loan and taxes. It is interesting to observe that when obtaining a subsidy in the cost of debt, using that lower cost in the WACC is not the correct approach to measure the increase in value due to the subsidy. The adjustments to the WACC and the explicit introduction of the subsidy in the analysis, give the proper result.

BIBLIOGRAPHY

- Damodaran, Aswath, 1996, *Investment Valuation*, John Wiley.
- Ross, S. A., R. W. Westerfield and J. Jaffe, 1999, *Corporate Finance*, Irwin-McGraw-Hill.
- Tham, Joseph and Ignacio Vélez-Pareja, 2004, *Principles of Cash Flow Valuation*, Academic Press.
- Tham, Joseph and Velez-Pareja, Ignacio, 2005, "With subsidized debt how do we adjust the WACC?" *Social Science Research Network*.
- Velez-Pareja, Ignacio, 2004, "Modeling the Financial Impact of Regulatory Policy: Practical Recommendations and Suggestions. The Case of World Bank" *Social Science Research Network*.
- Velez-Pareja, Ignacio, 2005, "Cash Flow Valuation in an Inflationary World: The Case of World Bank for Regulated Firms" *Social Science Research Network*.
- Vélez-Pareja, Ignacio and Joseph Tham, 2000, "A Note on the Weighted Average Cost of Capital WACC", (last version October, 2002). *Social Science Research Network*.
- Velez-Pareja, Ignacio and Tham, Joseph, 2005, "Proper Solution of Circularity in the Interactions of Corporate Financing and Investment Decisions: A Reply to the Financing Present Value Approach" *Social Science Research Network*.