AN EMPIRICAL ANALYSIS OF EQUITY DEFAULT SWAPS II: MULTIVARIATE INSIGHTS

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Abstract. Equity default swaps (EDS) - contracts that trigger a payment when the underlying equity price falls below a predetermined level - have attracted much attention recently because of their similarities to credit default swaps (CDS) on the one hand, and American digital puts on the other. Particular interest has been received by Collateralized debt obligations (CDOs) referencing a portfolio of EDSs, which not only requires the univariate assessment of the risks inherent in EDSs, but also the analysis of dependencies between EDSs (and other asset classes). In this paper, we specifically address correlation or dependency aspects of EDSs, by applying techniques developed for estimating default correlation. Based on Standard & Poor's CreditPro and Compustat (North America) databases, extensive empirical research is presented. Amongst the main findings are that EDS correlations for standard strikes/barriers of 30% are significantly higher than default correlations, and increase in barrier level, but only for strikes above 50%. This indicates a barrier dependent correlation concept.

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1. Introduction

Recently, *new*[1] equity-credit hybrid derivatives - called Equity Default Swaps (EDS) - have been introduced by a number of banks offering significantly higher spreads than comparative vanilla credit default swaps (CDS). While a payment is triggered on a CDS when a credit defaults, a payment is triggered on an EDS when the underlying equity price falls below a predetermined level. New in the credit markets, EDSs are known in the equity markets as deep "out-of-the-money", long-dated digital American puts with regular instalments. The price decline is often referred to as an *equity event*[2], analogous to a *credit event* within a CDS contract. While any combination of trigger level and recovery rate could be considered, EDS contracts are typically structured in the market with a trigger level set at 30%[3] and a fixed recovery rate of 50% (see Sawyer (2003) and Wolcott (2004) for a general introduction).

While it seems intuitive that EDS contracts should become more "credit-like", with EDS and CDS spreads starting to converge when the strike or barrier approaches zero, research on EDSs pricing and risk management is still in its infancy. The literature on EDS pricing relies so far on methodologies developed either for credit or equity products. Medova and Smith (2004) propose structural credit models, also called firm's value models (see Merton (1974)), that naturally allow the modelling of a firms asset value, credit quality and equity price. Applying the Leland and Toft (1996) model, qualitative and quantitative properties of EDS spreads are discussed. While Albanese and Chen (2004) also discuss the application of models used for CDS (particularly the credit barrier model of Albanese et al. (2003)), they note that from an equity viewpoint, EDSs would be priced naturally using local volatility models. They show that the EDS spreads can differ significantly when these models are used instead of a credit models with jumps.

Complementary to the pricing literature, several techniques have been proposed from a risk perspective. For example, Jobst and Gilkes (2005) discuss historic average EDS behaviour and the application of models based on the stochastic behaviour of the instrument. In particular GARCH models for volatility estimation, and the shortcomings for long-term risk assessment are discussed. As an alternative, de Servigny and Jobst (2005) explore simple pattern recognition/scoring techniques in order to discriminate the risk profiles of EDSs. These techniques are usually used in the credit risk environment, but the authors conclude that the application to EDS is adequate leading to more robust and comprehensive risk assessment than any of the previous approaches. This can be explained by the systematic way in which the full set of available information is incorporated, and by the fact that the period from inception to maturity of the EDS is considered. An extensive empirical study on EDS performance/behaviour from a historic perspective is conducted using Compustat and ratings performance data (the same data sources considered in this paper).

[1]There is an ongoing debate whether or not EDSs are newly structured derivates, or only variations of well established equity barrier options.

[2]Throughout this paper, *equity events* are interchangeably denoted as *equity default*, *equity drop* or *EDS default events*.

[3]In the remainder of this paper, we refer to this trigger level frequently as barrier or strike.

Letting pricing and risk aspects aside, some investors may see added value in the risk characteristics of EDS, especially given the recent tightening of CDS spreads. Assuming higher spreads for EDSs, investors may be interested in executing CDS/EDS carry trading strategies where EDS protection is sold and CDS protection is bought on the same name. If there is no EDS trigger event and subsequent payment prior to the contract maturity, gains will be realized from the difference in EDS/CDS spreads. If both, a credit default and equity drop event occurs, the investor is mostly hedged (apart from a recovery and timing mismatch as the recovery rate on CDSs is usually determined after the event through a market bidding process), while the major loss scenario constitutes of a equity event without a simultaneous (credit) default event (for further details, see Davletova et al. (2004).

While the single-name market for EDSs is still under development, EDSs have also been considered as yield-enhancing instruments in synthetic Collateralized Debt Obligations (CDOs). In a typical CDO referencing a CDS portfolio, a seller of protection is paid a premium in exchange for the commitment to pay a principal amount when losses on a pool of credits exceed a certain threshold. Losses in this context are defined as the notional amount of credits that experience a credit event minus a recovery rate, which is usually determined via a market bidding process. Thus, the portfolio constitutes credit risk to the seller of protection with some additional uncertainty surrounding recovery rates.

In a CDO that references a pool of equities under an EDS contract, the same basic roles exist. The seller is paid a premium in exchange for a principal commitment when losses exceed the threshold amount. In this case, however, losses are defined as the notional amount of equities whose prices fall to the trigger level, minus a predetermined recovery rate. In this way, one of the main criticisms of CDS contracts - namely, uncertain recoveries - is removed, while retaining a view on an extreme price deterioration.

From a risk management and pricing viewpoint, understanding the behaviour of single EDSs is not sufficient and interactions between EDSs (and other asset classes) need to be captured. Furthermore, investors may be interested in hybrid portfolios referencing CDS and EDS contracts simultaneously, which also requires a detailed analysis of the link between CDS and EDS behaviour. In this paper, we conduct a detailed empirical analysis of EDS, and CDS-to-EDS correlation, by applying techniques developed in the credit risk modeling literature to EDS data. We try to gain insight into the similarities or differences of correlations extracted from the two different sources of event data (credit and equity events) and discuss implications for portfolio modelling.

Section 2 discusses alternative correlation estimators that have been proposed for default correlation and are applied in section 3 to historic (credit) default and equity default data. A detailed discussion of the database(es) can be found in section 3.1. Section 4 extends the analysis by investigating the effects of varying barriers or strikes on EDSs, and reports results on a detailed sensitivity analysis. Section 5 discusses implications for portfolio modelling before concluding and outlining future research in Section 6.

2. Default and Event Correlations: Alternative estimators

The accurate estimation of Value-at-risk and other risk measures for loan portfolios, or the risk assessment and valuation of synthetic single-tranche CDOs are both sensitive to the parametrization of portfolio credit risk models. As part of the calibration exercise the accurate estimation of either default correlation directly, or the correlation between latent-variables (or asset value) that induce adequate dependency between default events or default times is desired.[4] For example, a commonly used model for portfolio credit risk assumes correlated latent variables (frequently denoted as asset values) V_i to be multivariate normally distributed, i.e.

$$(V_1, ..., V_N) \sim \Phi^{\Sigma}, \tag{1}$$

where Φ denotes the cumulative distribution function of the multivariate normal distribution with (latent variable) correlation matrix Σ. Furthermore, a default barrier Z_i is defined for each company as $Z_i = \Phi^{-1}(P_i)$, where Φ denotes the standard normal distribution function, and P_i the company's default probability. Within this model setup, the joint probability of default is given by $P(V_1 \leq Z_1, ..., V_N \leq Z_N) = \Phi^{\Sigma}(Z_1, ..., Z_N)$. The correlation between the latent variables induces correlation between the default events (or default times when the standard Gaussian copula default time model of Li (2000) is considered).

Estimating either default or more commonly the latent variable correlation turns out not to be an easy task and several alternative approaches are commonly employed:

- Correlations derived from equity prices or (transformed) asset values,
- Correlations inferred from credit spreads,
- Correlations estimated directly from empirical default observations.

Each of these alternatives has got advantages and disadvantages, leading to possibly quite different estimation results. The advantage of correlations from equity prices - or transformed asset values derived within a structural framework - is clearly data availability and ability to estimate issuer specific co-movements. While this is true for corporate assets, the analysis cannot easily be adopted for other structured finance assets that are frequently contained in CDOs (such as RMBS, ABS, etc.). Most importantly, however, equity prices, which are exposed to trends and market movements independent of the credit quality changes, produce at best very noisy estimates as shown in de Servigny and Renault (2003). Credit spread based estimates have similar properties. While credit spread data is widely available even for non-corporate assets, spread movements are likely to be influenced by market trends or liquidity issues. Unlike equity and spread based correlations, an approach that directly employs actual (observed) default events reduces the possibility of spurious correlation caused by unrelated external factors. Because event based[5] correlations are usually based on large samples spanning at least 20 years of data, they are frequently seen as long-term estimates that should dampen the fluctuations due

[4] We do not challenge the assumption that correlation is a suitable measure of dependency in this paper.

[5] We denote by events, outcomes that either occur or not occur and are binary in nature. Defaults or EDS trigger events are two such examples.

to business cycle and economic effects. They are therefore also denoted as "through-the-cycle" estimates, compared to "point-in-time" estimates that focus more on current market conditions. Although the actual number of observations or companies considered is usually large, the actual number of defaults (events) is often limited. In these situations, a careful assessment of the properties of the estimation techniques under consideration is required.

In this paper, we focus on empirical event based correlations, where we consider typical events as both default and equity drop events (within the same analytic framework). While equity and spread based correlations may reflect current information more efficiently, we favour the event based approach - based directly on historic event observations - within the context of long-term structured products such as CDOs. Although we describe several commonly used estimators in the context of default events, the application to other events (or asset classes) is straightforward. In sections 3 and 4, we employ these estimators to default and EDS data, respectively.

2.1. **de Servigny and Renault (2003): Joint default probability approach.** de Servigny and Renault (2003) estimate joint (pairwise) default probabilities (JPD) from historical default data before calculating empirical default and latent variable[6] correlations. Their approach is based on the work of Lucas (1995) and Bahar and Nagpal (2001) where the joint probability of default (or any discrete event) is given by the ratio of the number of possible pairs of firms in a given group (e.g. risk-class or industry) that actually defaulted to the total number of possible pairs in that group.

Empirical Joint Default Probabilities

For a population of N_t^c obligors at the beginning of year t in group $c \in \mathcal{C}$, $N_t^c(N_t^c-1)/2$ different pairs can be considered (by drawing pairs in an experiment without replacement). Similarly, given D_t^c defaults over a fixed period of time (usually one year), $D_t^c(D_t^c-1)/2$ possible pairs could have defaulted. Then, the joint probability for two firms i and j, belonging to group c, is given by:

$$P_{ij}^c(t) = \frac{D_t^c(D_t^c-1)}{N_t^c(N_t^c-1)}. \tag{2}$$

de Servigny and Renault (2003) outline the possibility of spurious negative correlations of this estimator for rare events (e.g. if $D_t^c = 1$ implies that the joint default probability is estimated equal to zero). They therefore propose to draw pairs of firms according to an experiment with replacement, leading to a new estimator for the joint probability of default:

$$P_{ij}^c(t) = \frac{(D_t^c)^2}{(N_t^c)^2}. \tag{3}$$

[6]We use the terms *latent variable correlation* and *implied asset correlation* interchangably in the remainder of this document.

Similarly, they derive an estimate for the joint default probability of two firms (i and j) in two different groups, $c \in \mathcal{C}$ and $d \in \mathcal{C}$, N_t^c and N_t^d as

$$P_{ij}^{cd}(t) = \frac{D_t^c D_t^d}{N_t^c N_t^d}, \tag{4}$$

where D_t^c and D_t^d denote the number of defaulted firms in groups c and d, respectively.

Consider $t = 1, ..., T$ years of data in total, we obtain a (weighted) average estimator for the joint default probabilities (assuming independence between these cohorts) as

$$\bar{P}^{cd} = \sum_t w_t^{cd} \frac{D_t^c D_t^d}{N_t^c N_t^d}, \tag{5}$$

where w_t^{cd} is the weight representing the relative importance of the population in a given year t. Among possible choices are, $w^{cd} = 1/T$ (equal weighting), or $w_t^{cd} = \frac{\sqrt{N_t^c N_t^d}}{\sum_m \sqrt{N_m^c N_m^d}}$ (weighted according to the size of the group in each year).

Empirical Linear Default Correlation

Use of the standard correlation equation allows us to derive an estimate of the empirical default correlation from joint default probabilities,

$$\rho^{cd} = \frac{\bar{P}^{cd} - \bar{P}^c \bar{P}^d}{\sqrt{\bar{P}^c(1-\bar{P}^c)}\sqrt{\bar{P}^d(1-\bar{P}^d)}}, \tag{6}$$

where $\bar{P}^k$ denotes the average probability of default of companies in group k.

Latent Variable/Implied Asset Correlation

The estimated joint default probabilities can also be used to obtain latent variable correlation, Σ, within the commonly used credit portfolio model presented above. This implied asset correlation is the correlation that needs to be used within the credit portfolio model to recover or match the joint default (or equity) events that have been observed empirically as close as possible.

For two companies, the joint default probability P_{ij} is given within the model by $P_{ij} = \Phi(Z_i, Z_j, \rho_{ij})$, where Z_i and Z_j are the default barriers (as previously defined) that depend on the marginal probability. Hence, the *implied asset correlation*, i.e. the correlation between the latent variables V_i, can be derived by solving $\rho_{ij} = \Phi^{-1}(P_{ij}, Z_i, Z_j)$.

de Servigny and Renault (2003) employ this estimator to Standard & Poor's CreditPro ratings and default database, investigate the stability of correlations, and discuss possible biases (upwards for low correlations and downwards for large correlations) in simulation experiments. Particular focus lies on correlations within (intra) and between (inter) industries, resulting typically in a correlation matrix

$$\Sigma^{Ind} = \begin{pmatrix} \rho_1 & \rho_{1,2} & \cdots & \rho_{1,C} \\ \rho_{2,1} & \rho_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \rho_{C-1,C} \\ \rho_{C,1} & \cdots & \rho_{C,C-1} & \rho_C \end{pmatrix}, \tag{7}$$

where ρ_c denotes the pairwise correlation between two companies in the same industry c, and $\rho_{c,d}$ denotes the correlation between two firms in industry c and d, respectively, and $C = |\mathcal{C}|$.

2.2. **Demey et al. (2004): A maximum likelihood approach(MLE) .** Demey et al. (2004) present a constraint version of the maximum likelihood procedure carried out in Gordy and Heitfield (2002) for estimating default correlation. While Gordy and Heitfield's approach is based on a very flexible multi-factor structure that is unfortunately hardly tractable numerically, Demey et al. (2004) suggest to add a new constraint on the inter-group (or inter-industry) correlations, so that they are able to reduce the number of factors for each group to two. This implies a new MLE that is more adequate for efficient numerical optimisation. We present this constraint *Binomial MLE* approach, and a computationally even less expensive *Asumptiotic MLE* approximation next.

Constraint Factor Structure

Having a very large number of firms to cope with in practice, it is usual to assume that we have identified a (lower) number of factors and rewrite the latent random variables $(V_1, ..., V_N)$ as a linear function of the factors. Under the addtional assumption of a unique correlation between two latent variables among all groups (i.e. $\rho_{c,d} = \rho$ for all $c \neq d$ in equation (7)), and with $\rho \leq \min_{c \in \mathcal{C}} \rho_c$, each latent variable V_i can be written as a function of two (independent) factors F and F_c, repectively:

$$V_i = \sqrt{\rho} F + \sqrt{\rho_c - \rho} F_c + \sqrt{1 - \rho_c} \epsilon_i, \qquad i \in c. \tag{8}$$

This factor setup is also appealing because there is a natural economic interpretation for each term: V_i can be explained by a common factor F which affects each obligor in the same way, and by a specific factor F_c depending on the group (industry or risk-class) of firm i. The unexplained part (noise) is captured by the idiosyncrating term ϵ_i. Unless otherwise stated, F, F_c, and ϵ_i are assumed to be standard normally distributed.

The resulting restricted correlation dynamics ,

$$\Sigma_{MLE}^{Ind} = \begin{pmatrix} \rho_1 & \rho & \cdots & \rho \\ \rho & \rho_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \rho \\ \rho & \cdots & \rho & \rho_C \end{pmatrix}, \tag{9}$$

compared to equation (7), implies, however, efficient numerical optimisation of the MLE compared to the unconstrained model (see Gordy and Heitfield (2002) or Demey et al. (2004) for further details on the unconstrained model).

Conditional Default probabilities

Using the Gaussian assumption for the idiosyncratic term ϵ_i, we can write the probability of default conditional on the factors F and F_c as:

$$P_c(f, f_c) = \Phi\left(\frac{Z_c - \sqrt{\rho} f - \sqrt{\rho_c - \rho} f_c}{\sqrt{1 - \rho_c}} \right), \tag{10}$$

where we also assume that all firms in a given risk class (or group) are equally likely to default and time-invariant, i.e. $Z_i^t = Z_i = Z_c, \forall i \in c$.

Binomial MLE (BinMLE)

Conditional on the factors, the random variable D_t^c representing the default number in risk class c at time t has a binomial distribution with parameters N_t^c - the number of firms in risk class c at time t - and $P_c(f, f_c)$.

We can then derive the conditional likelihood of our observations and derive the unconditional log-likelihood $l_t(\Xi)$ by summing over the distribution each factor F, and F_c, $c = 1, ..., C$:

$$l_t(\Xi) = \log \int_{\mathbb{R}} d\Phi(f) \prod_{c=1}^{C} \int_{\mathbb{R}} Bin^c(f, f_c) d\Phi(f_c), \tag{11}$$

where $Bin^c(f, f_c) = \begin{pmatrix} N_t^c \\ D_t^c \end{pmatrix} P^c(f, f_c)^{D_t^c} (1 - P^c(f, f_c))^{N_t^c - D_t^c}$. Ξ denotes the vector of variables in the optimisation. We can study two estimators. In the firs, the thresholds $Z_c = \Phi^{-1}(\bar{P}^c)$ are known and $\Xi = (\rho, \rho_1, ..., \rho_C)$, only. In the second, we assume the thresholds Z^c are unknown and included in Ξ for joint estimation.[7]

Asymptotic MLE (AsymMLE)

Demey et al. (2004) also develop this likelihood function assuming that the number of firms in each class is large enough to allow us to approximate the random variable representing the default rate by its limit, which leads to a computationally more efficient formulation. Let us note that $\mu^c = \frac{D^c}{N^c}$ is the (average) default rate in class c. When $N^c \to \infty$, and conditional on the factors $F = f$ and $F_c = f_c$, we have according to the law of large numbers $\mu_c \to P^c(f, f_c)$. Under this assymptotic assumption, Demey et al. (2004) derive the following expression for the log-likelihood function:

$$l_t(\Xi) = \log \int_0^1 dy \prod_{c=1}^{C} \phi\left(f(y)\right) \frac{\sqrt{1-\rho_c}}{\sqrt{\rho_c - \rho}} \frac{1}{\phi\left(\Phi^{-1}(\mu^c)\right)}, \tag{12}$$

where

$$f(y) = \frac{Z_c - \sqrt{1-\rho_c}\Phi^{-1}(\mu^c) - \sqrt{\rho}\Phi^{-1}(y)}{\sqrt{\rho_c - \rho}}. \tag{13}$$

In a finite sample, D_c may be equal to zero, which is not possible assymptotically, and Demey et al. (2004) present a way to reconcile the data with the model and also present a generalised formula for the log-likelihood:

$$l_t(\Xi) = \log \int_0^1 \prod_{c \in \mathcal{U}_t} \phi\left(f(y)\right) \frac{\sqrt{1-\rho_c}}{\sqrt{\rho_c - \rho}} \frac{1}{\phi\left(\Phi^{-1}(\mu^c)\right)} \prod_{c \in \bar{\mathcal{U}}_t} \left(1 - \Phi\left(f(y)\right)\right) dy, \tag{14}$$

[7]Throughout the rest of this paper, we report results for the latter approach and refer to Demey et al. (2004) for a comparative analysis. Initial testing shows that the results reported in this paper are not changing significantly under the first approach.

where $\mathcal{U}_t$ denotes the set of groups for which the default rate at time t is strictly positive, and $\bar{\mathcal{U}_t}$ denotes the subset of groups where the default rate is zero and subsequently substituted by a minimum default rate of $\mu_c = \frac{1}{N^c}$, for all $c \in \bar{\mathcal{U}_t}$.

2.3. **Discussion of "estimation bias".**

Because one does't normally know the exact level of correlation a priory, the statistical properties of such estimates are frequently tested within simulation (parametric bootstrap) experiments. de Servigny and Renault (2003) simulate a realistic sample of Automotive companies with varying level of correlation and reveal that the JPD estimator is performing well for default correlations between 2% and 12%.[8] Above 12%, the estimator appears to produce to low correlation, while too high estimates are obtained for very small correlations. The performance improves when T increases, as shown by the authors in repeating the experiment on 50 years of simulated data, instead of the usual 21 years.

We know that the MLE estimators are asymptotically unbiased, however, when applied to small samples, biases may appear. The asymptotic MLE will converge to the true correlation for $T \to \infty$, and $N \to \infty$, while the Binomial estimator converges for $T \to \infty$, only. Demey et al. (2004) investigate stability and bias issues in several bootstrap experiments and reveal that for low default rates (μ^c) and small samples (N^c), the asymptotic estimators may severely underestimate the true default correlation. In a series of experiments, the authors reveal a very encouraging performance of the Binomial MLE approach across a wide range of low frequency, small sample conditions. For example, experiments conducted on average default rates of 200bp and latent variable correlations of 25% reveal that the mean of the bootstrap distribution converges quickly to the true correlation, even for samples as small as 50. In contrast, the asymptotic estimators appear unbiased only for sample sizes of 500 and above. Such sample sizes are quite difficult to obtain when a industry specific split is pursued.

These insights into the properties will prove to be of paramount importance, when all three estimators - the *Joint default probability*, *Binomial MLE*, and *Asymptotic MLE* - are employed to the same data in sections 3 and 4. A good agreement in outcomes should lead to some comfort regarding the stability of the estimates, while we may be able to explain differences by some of the properties discussed above.

3. (Credit) Default and equity event correlation

3.1. **Description of data.**

An extensive analysis of historical data was conducted based on Standard & Poor's CreditPro® ratings and default database linked to Standard & Poor's Compustat® (North America) data. In total, CreditPro contains a ratings history of 9740 companies from the 31 December 1981 to 31 December 2003, and includes 1386 default events. The Compustat database contains approximately 56500 corporations trading in the US or Canada between 1962 and 2003. Of course, not all of these companies are listed and we therefore focus on companies that have listed equity with prices quoted

[8] Note that 2% to 12% default correlation will translate usually into higher implied asset/latent variable correlations when "combined" with the default probabilities of each obligor.

regularly on a major exchange. If multiple equity issues are available, we focus on the primary issue and also exclude "non-vanilla" equities such as American Depository Receipts (ADRs), over-the-counter (OTC) traded equity, mutual or investment trust funds, or exchange traded funds, unless otherwise stated. We also exclude equities where not all price data (monthly close and low) between the first and last observations is available. In the subsequent analysis, up to 12240 equity time series are analysed. When both, credit and price/market information was required, a match of approximately 4500 companies was achieved between both databases, of which approximately 2200 firms also had sufficient and continuous price (monthly closing and running minimum) data. While the coverage is by no means perfect, we are not aware of a larger empirical study on EDSs.

3.2. **Default correlation .** Throughout this section, we employ the three estimators to the CreditPro ratings and default data. We start with one year horizon correlations are estimated at a industry level, based on 66536 annual observations and 1170 default events over the period of 1981 to 2003. At the beginning of each year $t = 1, ..., 23$, we count the number of firms in each industry (N_t^c) and the number of firms that defaulted within the next year (D_t^c) and calculate average marginal and pairwise default probabilities, $\bar{P}^c$ and $\bar{P}^{cd}$. The average number of firms in each rating class, and the average default rate for each CreditPro industry are reported in Table 5.

3.2.1. *JPD approach.* The average joint default probabilities and the corresponding default and latent-variable correlation matrices are shown in Tables 1, 2, and 3, respectively. For example, the historic average probability of a automotive and a energy firm to default together within one year is 0.07%. Combined with the corresponding univariate default probabilities, this translates into a default correlation of 0.49%, and a latent variable correlation of 3.33%.

	Auto	*Cons*	*Ener*	*Fin*	*Chem*	*Health*	*HiTech*	*Ins*	*Leis*	*RealEst*	*Telecom*	*Trans*	*Util*
Auto	0.13	0.10	0.07	0.02	0.09	0.07	0.08	0.01	0.12	0.02	0.22	0.08	0.03
Cons	0.10	0.11	0.05	0.02	0.09	0.06	0.07	0.02	0.12	0.02	0.14	0.08	0.02
Ener	0.07	0.05	0.09	0.01	0.05	0.04	0.05	0.01	0.07	0.01	0.09	0.05	0.02
Fin	0.02	0.02	0.01	0.01	0.02	0.01	0.01	0.00	0.03	0.01	0.03	0.02	0.00
Chem	0.09	0.09	0.05	0.02	0.10	0.05	0.07	0.01	0.11	0.02	0.15	0.08	0.02
Health	0.07	0.06	0.04	0.01	0.05	0.05	0.04	0.01	0.07	0.00	0.11	0.05	0.01
HiTech	0.08	0.07	0.05	0.01	0.07	0.04	0.07	0.01	0.08	0.02	0.11	0.06	0.01
Ins	0.01	0.02	0.01	0.00	0.01	0.01	0.01	0.01	0.02	0.00	0.02	0.02	0.01
Leis	0.12	0.12	0.07	0.03	0.11	0.07	0.08	0.02	0.17	0.04	0.19	0.11	0.03
RealEst	0.02	0.02	0.01	0.01	0.02	0.00	0.02	0.00	0.04	0.02	0.01	0.02	0.00
Telecom	0.22	0.14	0.09	0.03	0.15	0.11	0.11	0.02	0.19	0.01	0.55	0.16	0.09
Trans	0.08	0.08	0.05	0.02	0.08	0.05	0.06	0.02	0.11	0.02	0.16	0.09	0.03
Util	0.03	0.02	0.02	0.00	0.02	0.01	0.01	0.01	0.03	0.00	0.09	0.03	0.02

TABLE 1. Joint default probabilities (in%), 1 year horizon, 1981 - 2003 (JPD).

Throughout the rest of this paper we focus frequently on the average correlation of two firms within the same (intra) and in different (inter) industries, calculated as the simple arithmetic average of all diagonal and off-diagonal elements, respectively. The average intra/inter-industry default correlations are 2.3%/0.6%, which results in 14.6%./4.7% intra/inter-industry latent variable correlation.

	Auto	*Cons*	*Ener*	*Fin*	*Chem*	*Health*	*HiTech*	*Ins*	*Leis*	*RealEst*	*Telecom*	*Trans*	*Util*
Auto	2.18	0.77	0.49	0.35	1.07	1.09	1.07	0.15	0.95	0.22	2.87	0.85	0.92
Cons	0.77	1.17	-0.35	0.42	1.25	0.77	0.74	0.33	0.81	0.94	0.52	0.78	0.13
Ener	0.49	-0.35	2.02	-0.36	-0.05	0.14	0.16	0.29	-0.16	-0.31	-0.31	0.18	0.12
Fin	0.35	0.42	-0.36	0.60	0.50	0.11	0.21	0.00	0.53	1.21	0.22	0.18	0.14
Chem	1.07	1.25	-0.05	0.50	2.26	0.57	1.09	0.43	1.48	1.24	1.63	1.47	0.78
Health	1.09	0.77	0.14	0.11	0.57	1.34	0.48	0.14	0.47	-0.41	1.09	0.60	0.25
HiTech	1.07	0.74	0.16	0.21	1.09	0.48	1.40	0.00	0.59	0.79	0.66	0.64	0.13
Ins	0.15	0.33	0.29	0.00	0.43	0.14	0.00	0.88	0.27	0.42	0.39	0.69	0.43
Leis	0.95	0.81	-0.16	0.53	1.48	0.47	0.59	0.27	1.70	1.70	1.15	1.29	0.65
RealEst	0.22	0.94	-0.31	1.21	1.24	-0.41	0.79	0.42	1.70	4.00	-0.45	0.83	0.28
Telecom	2.87	0.52	-0.31	0.22	1.63	1.09	0.66	0.39	1.15	-0.45	8.37	1.75	3.79
Trans	0.85	0.78	0.18	0.18	1.47	0.60	0.64	0.69	1.29	0.83	1.75	1.54	0.91
Util	0.92	0.13	0.12	0.14	0.78	0.25	0.13	0.43	0.65	0.28	3.79	0.91	2.18

TABLE 2. Empirical default correlation matrix (in%), 1 year horizon, 1981 - 2003 (JPD).

	Auto	*Cons*	*Ener*	*Fin*	*Chem*	*Health*	*HiTech*	*Ins*	*Leis*	*RealEst*	*Telecom*	*Trans*	*Util*
Auto	11.81	4.72	3.33	3.80	6.77	7.39	7.03	1.88	5.38	2.64	13.24	5.46	8.55
Cons	4.72	6.83	-2.67	4.47	7.72	5.43	5.03	4.06	4.63	9.67	2.81	5.06	1.47
Ener	3.33	-2.67	12.55	-5.24	-0.37	1.16	1.28	3.81	-1.14	-4.97	-1.98	1.35	1.48
Fin	3.80	4.47	-5.24	9.36	5.53	1.48	2.60	0.06	5.18	16.50	2.15	2.11	2.58
Chem	6.77	7.72	-0.37	5.53	13.44	4.44	7.57	5.38	8.54	12.63	8.62	9.41	7.82
Health	7.39	5.43	1.16	1.48	4.44	9.99	3.88	2.04	3.22	-7.77	6.51	4.58	3.11
HiTech	7.03	5.03	1.28	2.60	7.57	3.88	9.64	0.04	3.85	9.11	3.92	4.66	1.57
Ins	1.88	4.06	3.81	0.06	5.38	2.04	0.04	14.60	3.13	8.41	4.09	8.03	7.67
Leis	5.38	4.63	-1.14	5.18	8.54	3.22	3.85	3.13	8.60	14.77	5.58	7.51	6.05
RealEst	2.64	9.67	-4.97	16.50	12.63	-7.77	9.11	8.41	14.77	34.15	-5.63	9.10	5.32
Telecom	13.24	2.81	-1.98	2.15	8.62	6.51	3.92	4.09	5.58	-5.63	27.83	9.10	23.18
Trans	5.46	5.06	1.35	2.11	9.41	4.58	4.66	8.03	7.51	9.10	9.10	9.68	8.90
Util	8.55	1.47	1.48	2.58	7.82	3.11	1.57	7.67	6.05	5.32	23.18	8.90	21.89

TABLE 3. Empirical latent variable (asset) correlation matrix (in %), 1 year horizon, 1981 - 2003 (JPD).

de Servigny and Renault (2003), using data from 1981 to 2001, report generally similar results, based on US and NIG data, only. Some subtle differences can be observed that highlight the sensitivity to the data under consideration. They also show that default correlations are sensitive to the horizon under consideration.

Sensitivity to credit quality
We investigate the impact of credit quality by focusing on NIG data, next. The intra- and inter-industry default correlations increase substantially to 6.3% and 1.0%, respectively, while the corresponding latent-variable correlations change more moderately. More specifically, an increase to 19.8% for firms in the same industry, and a slight decrease to 3.9% for firms in different industries is revealed. Combining these results with the bootstrap experiments of de Servigny and Renault (2003), we can support the findings of Gordy and Heitfield (2002) that differences in implied asset correlations are not significant across asset classes.

Sensitivity to weighting scheme
We also check the impact of employing a different weighting scheme employed in equation (5). Using an equal weighting scheme instead of the commonly used *group-size* approach

gives default correlations of 2.57%/0.6% and implied asset correlations of 16.6%/4.3%. We can conclude that applying a different weights has only a minor impact on correlation estimates for the data under consideration.

Sensitivity to risk horizon
So far, the correlations are derived from historic average annual (joint) default rates as discussed in section 2.1. Replacing $\bar{P}^c$ and $\bar{P}^{cd}$ with T-year estimates $\bar{P}^c_T$ and $\bar{P}^{cd}_T$ in equation (6) allows us to investigate the impact of longer horizons on default and implied asset correlation. Table 4 reveals that despite the sharp increase in default correlation when moving from a one year to a three year horizon, changes in the corresponding latent-variable correlations are much more moderate. Although encouraging, in our experience the opposite outcome is obtainable for different sets of data.

	Default Corr.			Implied Asset Corr.		
Horizon	1y	3y	5y	1y	3y	5y
Intra	2.3	4.5	4.7	14.6	15.3	14.3
Inter	0.6	1.6	1.9	4.7	6.4	6.6

Table 4. Correlation for 1, 3, and 5 year horizons (JPD).

3.2.2. *Binomial and Asymptotic MLE .* We apply now the CreditPro default data to the Maximum likelihood estimators discussed in section 2.2. By focusing on industry level correlations again, latent-variable (implied asset) correlations are directly estimated. Table 5 shows industry specific correlation estimates from the CreditPro ratings and default database. Column *AvgN* and *AvgPD* contain the average number of firms in each year in each industry as well as the average default probability. *DefCorr* and *ImpAssCorr* contain the empirical default and implied asset correlation according to section 2.1. *AsyMLE* and *BinMLE* contain the Asymptotic MLE and Binomial MLE results following the approach in section 2.2.[9] The last row contains the correlation of two companies in industries. While this number is a direct output in Demey et al., the average of all industry combinations (pairs) is reported for the JPD estimates. In addition, the average intra-industry correlation - calculated as the simple average of the industry specific estimates - is reported in the row above.

Overall, we can observe a good agreement between all estimators, with average inter-industry correlations of approximately 4% to 7% and average intra-industry estimates of approximately 14% to 17%. Further testing of these estimates to credit quality and horizon changes reveals overall a behavior similar to that observed for the JDP approach. Given the good small-sample properties of the Binomial MLE approach (see Demey et al.

[9]We would like to point out that the results based on Standard & Poor's data reported by Demey et al. (2004) are incorrect. Since then, the authors published a correction note (Demey and Roncalli (2004)), reporting results very similar to ours (e.g. 14.3% and 6.8% intra and inter industry correlation for the Binomial MLE2 estimators). Small differences occur as the period 1981 - 2002 is considered in their study. Furthermore, a different implementation of the numerical integration and optimisation may lead minor differences.

	AvgN	*AvgPD*	*DefCorr*	*ImpAssCorr*	*AsyMLE*	*BinMLE*
Auto	297	2.17	2.18	11.80	12.93	10.84
Cons	354	2.48	1.17	6.80	10.19	7.63
Ener	149	2.20	2.02	12.60	18.76	19.06
Fin	530	0.60	0.60	9.40	16.27	15.93
Chem	113	2.04	2.26	13.40	8.72	6.55
Health	149	1.25	1.34	10.00	11.11	8.44
HiTech	97	1.84	1.40	9.60	11.86	6.55
Ins	260	0.65	0.88	14.60	22.92	13.32
Leis	169	3.07	1.70	8.60	11.32	9.16
RealEst	60	1.11	4.00	34.20	36.71	33.02
Telecom	119	1.97	8.37	27.80	23.49	30.32
Trans	134	2.07	1.54	9.70	9.65	6.55
Util	352	0.40	2.18	21.90	15.94	21.30
$Avg(Ind) = Intra$			2.28	14.65	16.14	14.51
$Inter - Industry$			0.60	4.70	5.00	6.45

TABLE 5. Comparison of latent-variable correlations for default data.

(2004)), it is encouraging to see that the simple JDP approach produces very comparative results (see, for example, Table 6 for horizon dependent Asymptotic MLE estimates).

	Implied Asset Corr.		
Horizon	1y	3y	5y
Intra	16.1	17.3	15.3
Inter	5.0	6.7	6.4

TABLE 6. Correlation for 1, 3, and 5 year horizons (AsyMLE).

3.3. **"Standard EDS" (30% barrier) correlation.** We now apply all three estimators to equity data, first considering the standard EDS with a 30% barrier. The database was filtered to contain 12240 equity time-series from 1962 to 2003. Over that period, we get in total 128.995 annual observations (i.e. starting prices P_t^i for equity i at time t) and 6636 equity events for a 30% barrier within the subsequent year. We frequently focus on a subset of equities for rated corporations only. By joining the CreditPro and Compustat data, we achieve a match on approximately 4500 entities, 2200 of which have also adequate time-series data for monthly closing and low prices. This corresponds to 17812 annual observations and 627 equity events for EDSs with a 30% barrier over the period 1981 to 2003.

Equity events are identified in the following way. For a given date t (e.g. 31. December 2000), the barrier B_t^i for an EDS written on equity i is determined as a fraction $K = 30\%$ of the current price S_t^i: $B_t^i = K \cdot S_t^i$. Depending on the maturity M (typically 1 year), we determine the minimum price $\tilde{S}_{t,M}^i := \min_{(t,M]} S^m$ from monthly low price data contained in the database for every month before maturity. If the minimum price is less or equal the barrier, an equity event is registered.

3.3.1. *JPD approach* . We first consider rated firms only in our analysis, covering the same time period (1981-2003) as in section 3.2.1, but only a subset of approximately 2300 firms.[10] The corresponding joint equity event probabilities, equity event correlations and latent variable correlations are shown in Tables 7, 8, and 9. The average intra- and inter-industry default and latent variable correlations are 8.39%/2.48% and 27.25%/14.22%, respectively. Compared to the results for credit event (default) data, EDS(30%) correlations are significantly higher with implied asset correlations almost twice as high. Within the factor interpretation, EDSs have a significantly higher loading/sensitivity to both, the global and industry specific factor, compared to credit default swaps. Because this finding may have a significant impact on modelling portfolios of CDS and EDS, we investigate the stability of these results for different data and estimators, next.

	Auto	*Cons*	*Ener*	*Fin*	*Chem*	*Health*	*HiTech*	*Ins*	*Leis*	*RealEst*	*Telecom*	*Trans*	*Util*
Auto	0.25	0.16	0.18	0.11	0.20	0.21	0.68	0.12	0.34	0.09	1.04	0.18	0.11
Cons	0.16	0.13	0.16	0.07	0.13	0.16	0.42	0.09	0.22	0.07	0.63	0.12	0.07
Ener	0.18	0.16	0.63	0.14	0.19	0.21	0.48	0.14	0.31	0.11	0.63	0.21	0.07
Fin	0.11	0.07	0.14	0.08	0.12	0.10	0.29	0.07	0.18	0.06	0.40	0.08	0.05
Chem	0.20	0.13	0.19	0.12	0.24	0.18	0.53	0.11	0.34	0.13	0.73	0.14	0.08
Health	0.21	0.16	0.21	0.10	0.18	0.25	0.62	0.12	0.30	0.09	0.87	0.20	0.12
HiTech	0.68	0.42	0.48	0.29	0.53	0.62	2.26	0.28	0.93	0.19	3.28	0.57	0.40
Ins	0.12	0.09	0.14	0.07	0.11	0.12	0.28	0.10	0.17	0.08	0.39	0.10	0.04
Leis	0.34	0.22	0.31	0.18	0.34	0.30	0.93	0.17	0.53	0.17	1.29	0.24	0.14
RealEst	0.09	0.07	0.11	0.06	0.13	0.09	0.19	0.08	0.17	0.15	0.18	0.03	0.02
Telecom	1.04	0.63	0.63	0.40	0.73	0.87	3.28	0.39	1.29	0.18	4.86	0.85	0.58
Trans	0.18	0.12	0.21	0.08	0.14	0.20	0.57	0.10	0.24	0.03	0.85	0.32	0.11
Util	0.11	0.07	0.07	0.05	0.08	0.12	0.40	0.04	0.14	0.02	0.58	0.11	0.08

TABLE 7. Joint equity event - EDS(30%) - probabilities (in%), 1 year horizon, 1981 - 2003 (JPD).

Rated and unrated firms: 1960 - 2003

We consider data on approximately 12240 equities over $t = 1, ..., 40$ years with more than 128.900 annual observations and 6636 EDS(30%) events next. Table 10 reveals only minor differences, some of which may be attributed due to the overall higher (marginal) event rate of 4.99% for all (rated and unrated) firms compared to 3.52% for rated firms, only.

Drivers of EDS comovements

de Servigny and Jobst (2005) develop statistical credit scoring models and show that EDS riskiness can be explained reasonably well by six variables: credit rating, historic equity volatility, market capitalization, the debt-over-equity (leverage) ratio, historic equity return, and the ratio of the current level of the S&P 500 to the highest S&P 500 level of the last 10 years. Depending on the strike price for the EDSs under consideration, the relative importance of these variables may vary, with credit rating and volatility being

[10]Note that the average credit default rate of this subset of approximately 2300 obligors for which equity prices are also available is slightly lower than the default rate for the whole sample. The corresponding default and implied asset correlations, are also slightly different. Intra-industry correlation is estimated at 16% on average, which is close to the estimates for the entire CreditPro sample of almost 10.000 obligors. At an inter-industry level, however, the average correlation is only 1.34%, compared to 4.7% for the large sample. A closer inspection reveals that this low average is closely linked to two industries, Insurance and Real Estate, that are up to -25% correlated to some other industries, as a result of the low number of observations and defaults. By ignoring the extreme negative correlations, an average inter-industry estimate of approximately 3.8% is obtained.

	Auto	*Cons*	*Ener*	*Fin*	*Chem*	*Health*	*HiTech*	*Ins*	*Leis*	*RealEst*	*Telecom*	*Trans*	*Util*
Auto	4.67	3.35	0.87	2.09	3.64	3.22	8.61	1.74	4.52	1.66	10.72	1.96	3.21
Cons	3.35	3.47	1.79	1.67	2.77	3.26	5.77	1.85	3.20	1.60	7.05	1.60	2.42
Ener	0.87	1.79	9.61	2.22	1.88	1.47	1.88	1.48	1.77	1.18	0.87	1.21	0.18
Fin	2.09	1.67	2.22	2.67	3.12	1.80	4.25	1.68	3.11	1.77	4.09	0.83	1.62
Chem	3.64	2.77	1.88	3.12	6.03	2.97	6.69	2.17	5.54	3.94	7.04	1.29	2.02
Health	3.22	3.26	1.47	1.80	2.97	4.22	7.08	1.97	3.36	1.39	7.59	2.48	3.49
HiTech	8.61	5.77	1.88	4.25	6.69	7.08	21.45	2.71	8.60	1.67	25.05	5.76	9.33
Ins	1.74	1.85	1.48	1.68	2.17	1.97	2.71	2.33	2.02	2.50	2.46	0.85	0.91
Leis	4.52	3.20	1.77	3.11	5.54	3.36	8.60	2.02	5.76	3.01	9.12	1.69	2.94
RealEst	1.66	1.60	1.18	1.77	3.94	1.39	1.67	2.50	3.01	8.04	-0.62	-0.98	-0.44
Telecom	10.72	7.05	0.87	4.09	7.04	7.59	25.05	2.46	9.12	-0.62	30.13	6.84	10.74
Trans	1.96	1.60	1.21	0.83	1.29	2.48	5.76	0.85	1.69	-0.98	6.84	5.77	2.99
Util	3.21	2.42	0.18	1.62	2.02	3.49	9.33	0.91	2.94	-0.44	10.74	2.99	4.91

Table 8. Empirical equity event - EDS(30%) - correlation matrix (in%), 1 year horizon, 1981 - 2003 (JPD).

	Auto	*Cons*	*Ener*	*Fin*	*Chem*	*Health*	*HiTech*	*Ins*	*Leis*	*RealEst*	*Telecom*	*Trans*	*Util*
Auto	20.08	16.79	4.30	12.29	17.11	14.89	27.54	9.95	17.92	10.34	31.24	9.72	18.32
Cons	16.79	18.58	9.23	11.15	14.93	16.35	21.82	11.46	14.76	10.96	24.31	8.99	15.95
Ener	4.30	9.23	30.54	11.96	9.14	6.95	6.89	7.93	7.35	7.09	2.96	5.74	1.32
Fin	12.29	11.15	11.96	17.11	17.43	10.76	18.23	11.31	15.41	12.68	16.55	5.43	12.43
Chem	17.11	14.93	9.14	17.43	25.72	14.46	23.41	12.44	21.76	21.04	23.08	7.04	13.24
Health	14.89	16.35	6.95	10.76	14.46	18.36	23.52	10.97	13.97	8.84	23.48	11.81	19.44
HiTech	27.54	21.82	6.89	18.23	23.41	23.52	46.87	11.80	25.18	8.38	50.60	19.72	35.22
Ins	9.95	11.46	7.93	11.31	12.44	10.97	11.80	13.98	10.13	15.78	9.96	5.19	7.29
Leis	17.92	14.76	7.35	15.41	21.76	13.97	25.18	10.13	19.84	15.29	24.84	7.57	15.78
RealEst	10.34	10.96	7.09	12.68	21.04	8.84	8.38	15.78	15.29	36.47	-3.24	-8.63	-5.14
Telecom	31.24	24.31	2.96	16.55	23.08	23.48	50.60	9.96	24.84	-3.24	55.21	21.32	38.47
Trans	9.72	8.99	5.74	5.43	7.04	11.81	19.72	5.19	7.57	-8.63	21.32	22.96	17.16
Util	18.32	15.95	1.32	12.43	13.24	19.44	35.22	7.29	15.78	-5.14	38.47	17.16	28.56

Table 9. Empirical latent variable (asset) correlation matrix for EDS(30%) data (in %), 1 year horizon, 1981 - 2003 (JPD).

	Def Corr		Asset Corr	
	Rated	All	Rated	All
Intra	8.39	7.49	27.25	24.57
Inter	2.48	3.76	14.22	14.39

Table 10. Comparison of correlation estimates (JPD approach) for universe of all companies and a subset or rated firms (JPD).

most explanatory at all levels. Their analysis also suggests that EDSs are more sensitive to the cyclical nature of equities with chances of downturns / shocks in equity markets compared to CDS. Some of this additional systematic risk at the univariate level may also translate into bivariate asset correlation, and explain therefore - at least to some extent - the consistently higher EDS correlations. Of course, this observation may have a significant impact on credit portfolio modelling, some of which will be discussed in section 4.4.

Due to the importance of rating and volatility, EDSs written on firms with a high credit quality, and EDSs written on equities with low volatility can be shown to be less likely to breach the 30% barrier. For example, the average event rate for EDSs written on IG firms is approximately 1% (11324 observations and 123 events), compared to 8% (5521 observations and 432 events) for EDSs written on NIG firms. We estimate correlations on data for NIG and IG as well as low and high volatile equities (based on the larger universe) next. Table 11 reveals that differences in implied asset correlation between IG/NIG and low/high volatility firms is minor, while event correlations behave as expected. Compared to the asset correlations for all rated or non-rated data, however, an increase of intra-industry correlation can be observed.

	NIG		IG		High Vol		Low Vol	
	Event	Asset	Event	Asset	Event	Asset	Event	Asset
Intra	12.84	31.03	6.17	30.60	12.71	33.97	4.27	25.62
Inter	5.01	14.01	1.91	13.0	6.42	19.86	1.44	12.26

TABLE 11. Impact of EDS quality on correlation (JPD).

Sensitivity to weighting scheme

Again, using an equal weighting scheme instead of the commonly used *group-size* approach gives equity event correlations of 9.53%/3.23% and implied asset correlations of 31.40%/14.12%. We observe again a moderate impact with slightly higher intra-industry correlations.

Sensitivity to observation (cohort) frequency

We estimate correlations based on quarterly ($t = 1, ..., 92$) and monthly ($t = 1, ..., 296$) observations in the database of rated firms, next. For a 1 year time-horizon, the estimates are based on non-overlapping observations, while for a higher observation frequency, observations overlap. The impact on both, event and latent-variable correlation is minor. For example, annual asset correlations of 27.25%/14.22% compare to estimates based on quarterly and monthly observations of 25.24%/12.75% and 24.88%/12.44%, respectively.

Sensitivity to risk horizon

We replace $\bar{P}^c$ and $\bar{P}^{cd}$ again with 3-year and 5-year estimates $\bar{P}^c_T$ and $\bar{P}^{cd}_T$ in equation (6). In contrast to the results for default data, event correlations are not increasing significantly when moving from 1 to 3 years and, more importantly, latent-variable correlations are now decreasing in horizon. Although surprising, some of the decrease may be attributed by the dependency of EDSs on the time of writing the contracts. While the year 2000 burst of the equity bubble contributes multiple times for 1 year estimates, multi-year estimates reflect this information only partially, or not at all. For example, 5-year estimates are based on the last sample (cohort) taken on 31. December 1998. Equity markets were at year 2002 levels and 5-year estimates may not fully capture the strongly declining market environment, which is reflected in the average event rates of 3.52%, 9.25%, and 11.29% over 1, 3, and 5 years. Similar results are obtained for the larger dataset of rated and non-rated entities, for example, intra/inter-industry asset correlations decrease from 24.57%/14.39% for 1 year to 17.37%/10.12% for 5 year horizons.

	Event			Asset		
Horizon	1y	3y	5y	1y	3y	5y
Intra	8.39	9.30	6.74	27.25	21.68	16.41
Inter	2.48	3.99	3.55	14.22	10.83	9.26

TABLE 12. EDS(30%) correlation for 1, 3, and 5 year horizons (JPD).

3.3.2. *Binomial and Asymptotic MLE.* We now repeat the analysis of section 3.2.2 with the EDS(30%) data on the subset of rated companies. Table 13 reveals that while the JPD

	AvgN	*AvgEEP*	*DefCorr*	*ImpAssCorr*	*AsyMLE*	*BinMLE*
Auto	113	3.26	5.80	23.00	15.29	20.30
Cons	115	2.28	3.00	17.00	18.01	22.46
Ener	58	4.57	8.10	28.00	27.57	36.13
Fin	85	1.75	2.50	16.00	13.17	17.70
Chem	46	2.79	4.20	21.00	15.79	18.02
Health	72	3.35	3.60	17.00	15.53	20.03
HiTech	55	8.11	22.10	44.00	28.05	36.31
Ins	44	2.17	2.30	14.00	17.68	17.70
Leis	60	5.07	5.30	19.00	16.26	18.16
RealEst	27	1.62	22.50	47.00	52.97	40.13
Telecom	22	12.50	34.40	61.00	39.07	52.87
Trans	24	3.49	1.54	19.00	22.79	24.61
Util	55	1.34	2.18	27.00	19.09	24.85
$Avg(Ind) = Intra$			8.50	27.15	23.17	26.87
$Inter - Industry$			3.70	14.20	9.40	17.60

TABLE 13. Comparison of latent-variable correlations for different estimation techniques and EDS data.

approach and Binomial MLE approach in reasonable good agreement, the Asymptotic MLE approach seems to underestimate the correlations at both, inter and intra industry level. Noticing the lower number of firms under consideration ($AvgN$), this difference may be caused by the small-sample bias of the Asymptotic estimator (and possibly the JPD estimator at the inter-industry level). We will discuss bias issues further in the next section.

4. Barrier/Strike dependent EDS correlations

Whereas credit events are not directly linked to equity performance, equity default swaps depend directly on the observable market prices. Hence after writing the contract initially, subsequent price moves change the distance to the barrier: the option moves either further out-of-the-money or closer to-the-money. Of course, the closer to the barrier an equity is trading, the more likely a subsequent equity event is. While a more detailed analysis of the drivers of equity events for different barriers is presented in de Servingy and Jobst (2005), we investigate the multivariate case next. Unless otherwise stated, we focus again on the universe of rated entities in our databases, and Table 14 shows the number of equity events for different barriers. Throughout this section, we focus on average intra- and inter-industry correlations for ease of presentation.

Barrier(in%)	No Obs.	Nr Events	Event rate (%)
1	17812	8	0.04
5	17812	53	0.30
10	17812	133	0.75
20	17812	335	1.88
30	17812	627	3.52
40	17812	1110	6.23
50	17812	1807	10.14
60	17812	2982	16.74
70	17812	4825	27.09
80	17812	7588	42.60
90	17812	11616	65.21
100	17812	17541	98.48

TABLE 14. Event frequency for different EDS barriers (rated obligors, only).

4.1. **JPD approach.** We start by simply repeating the analysis conducted in section 3.3.1 for different barriers from 10% to 90%. Figures 1a and 1b show the corresponding average intra and inter industry correlation at a risk horizon of 1 year.

[Insert Figures 1a and 1b here]

Equity event correlations, both within and between industries, appear to be increasing the closer to-the-money the strikes are as a result of the significantly higher event rates for higher barriers. A barrier dependence is also revealed for implied asset (latent-variable) correlations, with intra industry correlations that are almost flat for barriers below 50%. At first sight, this behavior may be surprising and contradictory to the widely known and accepted view that correlations may increase significantly when markets decline severely (crash). While it may well be true that price co-movements increase in these scenarios, the correlations we measure here are quite different to price co-movements. Our estimators attempt to explain historically observed joint events rather than regular price co-movements. Within the factor interpretation, high barrier EDSs are much more sensitive to the outcome of the global or industry factors. Adverse factor outcomes of even moderate nature are quite likely to cause EDS events for, say, 70% to 80% barriers, whereas EDSs with low barriers only trigger when these factors perform extremely badly. Overall, the sensitivity to common factors is lower for low barrier EDSs, as these instruments trigger equity events under normal market conditions mainly due to firm-specific reasons.

4.2. **Binomial and Asymptotic MLE.** Table 14 reveals that for low strikes the number of EDS triggers is very low even compared to the average default rate for the CreditPro universe and a first attempt to address possible biases is to apply the Binomial and Asymptotic MLE and focus on the comparison of average latent-variable correlations. Figures 2a and 2b contain the results for intra-industry and inter-industry correlation, respectively, from which several interesting observations follow.

[Insert Figures 2a and 2b here]

Firstly, for barriers below 50%, the Asymptotic MLE approach appears to be significantly biased, leading to a underestimation of correlation. In contrast, the Binomial estimator reveals almost constant inter-industry correlation, in line with the intra-industry results, for barriers below 50%. It is also encouraging that above 50%, all there estimators appear to produce similar outcomes. Overall, it appears the for the rated universe of approximately 2300 equities over the period 1983 to 2003, correlations below 50% are very similar (on average) while a significant level of barrier dependence appears for strikes above 50%.

4.3. **Sensitivity of results.** Throughout this section, we further investigate the stability of estimates by considering larger samples and different time considerations.

Sensitivity to time-horizon
We start by considering the sample of rated entities for extended risk-horizons of 3 and 5 years. Figure 4 confirms that a) event and latent variable correlations are barrier dependent, and b) that the risk horizon has a significant impact, particularly on intra-industry EDS correlation.[11]

[Insert Figure 4 here]

Rated and unrated firms: 1962-2003
We also consider the extended universe of rated and non-rated obligors. Overall, we gain similar insights with implied asset correlations appearing to be barrier dependent. For example, we obtain average intra/inter-industry pairs of 24.57%/14.38% for a 30% barrier compared to 31.31%/21.93% for a barrier of 70%.

Sensitivity to "EDS quality"
We split the sample once again into IG and NIG assets, knowing that credit quality had a significant impact on EDS performance, historically. Correlation is then estimated following the JPD approach, resulting in Figures 3a and 3b.

[Insert Figures 3a and 3b here]

For barriers above 60%, IG correlations increase steeply significantly above NIG estimates, while for lower barriers, NIG correlations are higher (Figure 3a). A possible interpretation may be that moderate price declines (e.g. 10% to 20%) of equities of highly rated firms are mostly caused in bear markets or as a result of general economic swings (systematic risks). If that happens, however, a large proportion of these equities is affected. Correlations for lower rated firms are significantly below IG levels for high barriers as NIG firms are usually more risky and volatility and declines of this magnitude are also frequently caused for firm specific reasons in normal markets. Considering barriers below 60%, we observe steeply declining IG correlations, well below NIG levels. A similar analysis based on the larger universe of rated and non-rated firms where highly volatile and less volatile firms are considered separately reveals qualitatively similar results, as does an analysis for extended horizons.

[11] Due to the larger number of events at for extended horizons, EDS(1%) and EDS(5%) correlations were also estimated.

Even more interesting to note is that IG and NIG implied asset correlation for barriers below 50% are in very good agreement, while significant differences are observed for higher strikes (Figure 3b).[12] Once again, while we would not expect intuitively such severe differences if all of the instruments are driven by the same (systematic) risk factors and subject to the same sources of randomness, the empirical observations reveal the opposite. Interestingly, the change in behaviour at approximately a 50% barrier level is in line with the findings in de Servigny and Jobst (2005) where the performance of scoring models severely deteriorates below 50%, with the impact of equity/market factors (e.g. equity volatility or S&P500 ratio) gaining significance compared to credit factors (e.g. rating or debt-to-equity ratio) for barriers above 50%.

On the relationship between equity and credit events: Sensitivity to equity cycles
Rather than looking at dependency, measured by linear correlation, at an industry level, we are now investigating the firm-specific link between credit and equity events. We present an adjustment to the usual definition of Kendall's Tau[13] that will be applied to binary (CDS/EDS trigger) observations, followed by an analysis of credit and equity events for different barriers. Taking a dynamic view gives some indication of the link between CDS and EDS through time and in different market conditions.

We denote by $d(i,t,T)$ the indicator function (0,1) of default of a (rated) firm i within T years from time T. $e(i,b,t,T)$ denotes the indicator of an equity drop event (drop to a barrier of b%) of firm i within T years from time t. Usually t denotes the end of December in each year from 1983 to 2002, and T is one year.

Given a total of N_t firms at time t, we can consider $\frac{N_t(N_t-1)}{2}$ pairs $[(d_{it}, e_{it}), (d_{jt}, e_{jt})]$ of firms i, j. We denote or label a pair concordant, if $d_{j,t} = d_{i,t}$ and $e_{j,t} = e_{i,t}$, otherwise, we call the pair non-concordant.

By counting the number of concordant and discordant pairs c_t and $\bar{c}_t$, respectively, we calculate Kendall's Tau in the usual way:

$$\tau = \frac{c_t - \bar{c}_t}{c_t + \bar{c}_t}. \tag{15}$$

In the case of the normal distribution, the linear and rank correlation can be linked analytically: $\rho_t = \sin(\pi/2 \cdot \tau_t)$.

In the empirical analysis, we include additional assets, such as OTC traded equities and ADRs, in order to enlarge the sample of (credit) defaulters in the database. Overall, we observe about 311 default events, and 828 EDS events for a 30% barrier. Performing the calculation for each year in our 23 year time-series, we plot the rank correlation τ_t and the corresponding linear correlation coefficient ρ_t in Figures 5 and 6 for two barriers, 10% and 30%.

[Figures 5 and 6 here]

[12]Note that due to the small number of equity events for the IG sample, EDS(10%) is not considered in this experiment.

[13]Notice that Kendall's tau is applied more fequently within the context of continuous random variables, see, for example, Nelsen (1999).

Overall, we observe that up to 1997, the EDS and CDS behaviour is in good agreement. Thereafter, however, there seems to be a de-coupling of the strong correlation between credit and equity events. This could be caused by a greater sensitivity of EDSs to market shocks, and CDSs reacting less to stressful conditions. Of course, for barriers closer to-the-money, the performance deteriorates significantly. Figure 7 shows that this effect can be mitigated when focusing on high quality EDS. For this sample of EDSs written on IG obligors, only, this corresponds to 19 credit and 162 equity events (at a 30% barrier level).

[Figure 7 here]

5. Implications for Portfolio Modelling

We have already discussed frequently used credit portfolio models based on correlated latent-variables. Correlation is either introduced via copula functions (e.g. Li's (2000) Gaussian copula default time model), or through the dependency of each asset on some common factors.

In any case, a one-to-one link between the obligors and the latent-variable process is established, and frequently motivated by the idea of Merton-type (structural) models where equity and debt can be expressed as options on the firms asset value. Intuitively, one would therefore expect a single asset value process that drives both CDS and EDS performance. The findings of the previous sections, however, indicate that such an assumption is not supported by the data. The calibration of factor models, and/or the direct estimation of linear correlations from CDS and EDS data indicates instrument specific correlations/factor loadings, rather than an obligor specific setup. This highlights that the standard modelling framework of one source of uncertainty may be imperfect. Throughout this section, we discuss how multiple instruments (possibly written on the same underlying obligor) could be integrated in portfolio models for CDS and EDS.

5.1. **Copula dependence: Consistency in correlations.** As an alternative to the usual Gaussian dependency (equation 1), alternative joint distributions can be fitted to the data. This is possible while keeping the Gaussian marginals through the use of copula functions that couple the univariate to a multivariate distribution. Dependence of variables is henceforth captured in a wider sense compared to linear correlation (see, e.g. Nelsen (1999)).

One such choice that may prove very useful within the context of EDSs is to fit a bivariate t-distribution with ν degrees of freedom (DoF) to joint EDS events:

$$\bar{P}^{cd} = t_2(Z_i, Z_j, \rho_{cd}, \nu), \tag{16}$$

where t_2 denotes the bivariate t-distribution. In particular, we may keep the correlation coefficients ρ_{cd} consistent with the implied asset correlation estimated from default data and then fit a bivariate t-distribution to the joint EDS event data. For example, given $\bar{P}^{cd}$, for all $c, d \in \mathcal{C}$, we could find the degree of freedom ν that minimizes the distance of the model implied event (using the bivariate-t distribution) to observed joint event probabilities. Figure 10 displays the outcome of such an exercise. As expected, the higher co-movements of EDSs for high barriers translates into lower DoF. Interestingly, a steep decline in DoF can be observed for barriers below 50%, with the exception of 90% barriers, where a return to normality ($\nu \to \infty$) can be revealed.[14]

5.2. **CDS-to-EDS dependence: Empirical analysis and Factor models.** We return now to Gaussian dependency and factors (latent-variables). Having shown that EDSs have a different sensitivity to systematic risks (factors), we continue to model credit and equity default swaps by separate, yet correlated, latent variable processes. Within a factor modelling framework (e.g. equation (8)), the dependence on the same set of factors imposes correlation between CDS and EDS despite varying degrees of factor sensitivity across these instruments.[15]

Two companies, same industry
For example, a CDS and EDS written on two different reference entities i and j belonging to the same industry $d \in \mathcal{C}$ can be modelled via two *pseudo* latent variable processes:

$$V_i^{CDS} = \sqrt{\rho^{CDS}}F + \sqrt{\rho_d^{CDS} - \rho^{CDS}}F_d + \sqrt{1-\rho_d^{CDS}}\epsilon_i, \tag{17}$$

$$V_j^{EDS} = \sqrt{\rho^{EDS}}F + \sqrt{\rho_d^{EDS} - \rho^{EDS}}F_d + \sqrt{1-\rho_d^{EDS}}\epsilon_j, \tag{18}$$

where ρ^{CDS} and ρ_d^{CDS} denote factor sensitivities of a CDS, and ρ^{EDS} and ρ_d^{EDS} denote factor sensitivities for a EDS. Then, the covariance between V_i^{CDS} and V_j^{EDS} is given by[16] $Cov(V_i^{CDS}, V_j^{EDS}) = \frac{Var(V_i^{CDS}+V_j^{EDS}) - Var(V_i^{CDS}) - Var(V_j^{EDS})}{2}$.

Calculating the variance $Var(V_i^{CDS} + V_j^{EDS})$ as

$$Var(V_i^{CDS} + V_j^{EDS}) = 2\sqrt{\rho^{EDS}}\sqrt{\rho^{CDS}} + 2\sqrt{\rho_d^{CDS} - \rho^{CDS}}\sqrt{\rho_d^{EDS} - \rho^{EDS}} + 2$$

leads to the correlation between two firms in the same industry $d \in \mathcal{C}$:

$$\begin{aligned} \rho_{ij}^{dd} &= Corr\left(V_i^{CDS}, V_j^{EDS}\right) \\ &= \sqrt{\rho^{EDS}}\sqrt{\rho^{CDS}} + \sqrt{\rho_d^{CDS} - \rho^{CDS}}\sqrt{\rho_d^{EDS} - \rho^{EDS}}. \end{aligned} \tag{19}$$

Two companies, different industry
By similar argument, the correlation between two companies in different industries c and d can be derived as

$$\rho_{ij}^{cd} = \sqrt{\rho^{CDS}}\sqrt{\rho^{EDS}}. \tag{20}$$

Employing the factor loadings of sections 3.2 and 3.3, we can compute the model implied correlation between the pseudo-latent variable processes (one for CDS, and another one

[14] Initial findings on replacing the standard normal factors with t-distributed factors within the MLE framework indicates possible non-normality in the idiosyncratic term.
[15] Such a setup is similar to the quantile regression framework employed for predicting VaR at difference confidence levels (see, for example, Chernozhukov and Umantsev (2000).
[16] For two random variables X, Y, the variance of the sum of X + Y is given by $Var(X+Y) = Var(X) + Var(Y) + 2Cov(X,Y)$, where $Cov(X,Y)$ denotes the covariance.

for EDS, depending on the level of barrier). For example, using the estimates of the JPD approach, the intra- and inter-industry correlation between the CDS and EDS latent variables are 19.6% and 8.17%, respectively.

Comparison to empirical estimates: JPD approach
To gain some preliminary insight into how these outcomes compare to a direct empirical estimation, we apply the JPD estimator and compute an enlarged matrix capturing CDS an EDS events, simultaneously. For example, the average intra/inter-industry credit, equity, and credit-to-equity correlations for a 30% barrier are shown in Table 15. At first

	CDS	EDS(30%)	CDS-to-EDS Factor Model	CDS-to-EDS JPD estimate
Intra	14.6	27.3	19.6	11.1
Inter	4.7	14.2	8.17	6.1

TABLE 15. Credit-to-equity correlation: 30% barrier (JPD vs factor model).

sight, the empirical CDS-to-EDS correlations are somewhat below the outcomes derived from the factor modelling approach. It is worth noting, however, that the two samples are quite different. The CDS data contains observations from approximately 10000 obligors, while EDS observations are obtained from a subset of approximately 2300 obligors, only.[17] Furthermore, it can be also expected that credit (default) events cause frequently EDS event which potentially biases the empirical (JPD) estimates.

In a similar way, correlation can be obtained for different instruments (e.g. EDSs with different barriers), see Figures 8 and 9 for JPD estimates. For example, the black line denotes the correlation between credit and equity events for barriers of 10% to 90%, whereas blue line indicates the correlation between EDS(10%) and EDS(10%) up to EDS(90%).

[Figures 8 and 9 here]

In summary the factor approach allows to capture dependency between different instruments in a consistent and convenient manner.

5.3. **EDS and CDS: The single obligor case.** In this section, we are interested in modelling a CDS and EDS written on the same underlying obligor i. Due to the empirical results on instrument specific correlations, a single asset value/latent variable process per obligor is not adequate. We therefore propose a factor approach once again as a useful first approximation. Within the usual factor framework, higher correlation is introduced through perfectly correlated idiosyncratic (noise) terms, that is,

$$V_i^{CDS} = \sqrt{\rho^{CDS}}F + \sqrt{\rho_d^{CDS} - \rho^{CDS}}F_d + \sqrt{1 - \rho_d^{CDS}}\epsilon_i, \tag{21}$$

$$V_i^{EDS} = \sqrt{\rho^{EDS}}F + \sqrt{\rho_d^{EDS} - \rho^{EDS}}F_d + \sqrt{1 - \rho_d^{EDS}}\epsilon_i. \tag{22}$$

[17]Re-running the analysis on the subset of 2300 firms for credit events and equity events reveals even lower levels of correlation. This however, may be caused by the fact that the sample of 2300 firms under consideration has on average a lower default rate than the full sample of corporates (see also footnote 10).

Then, the latent variable processes are strongly correlated:

$$\rho_{ii}^{CDS/EDS} = \sqrt{\rho^{CDS}}\sqrt{\rho^{EDS}} + \sqrt{\rho_d^{CDS} - \rho^{CDS}}\sqrt{\rho_d^{EDS} - \rho^{EDS}} + \sqrt{1-\rho_d^{EDS}}\sqrt{1-\rho_d^{CDS}}.$$

For the previous example of a CDS and an EDS(30%), we obtain a correlation of approximately 98% between the two pseudo asset value processes. While this results in the latent variables to move in a very similar way, it does not transform into event correlations of similar magnitude. The calibration of the event barriers to the higher EDS probabilities ensures that the EDS will trigger in most cases much earlier and more often within the proposed simulation scheme.

6. Conclusion

We have discussed and applied several alternative estimation techniques to empirical default (CDS) and equity default (EDS) data. We have shown that despite the different biases of each estimator, the results for both, default and equity event data are reasonably stable.

One of the main findings of this paper is that the latent-variable (implied asset) correlations that impose event correlation within commonly use (credit) portfolio models differ significantly when derived from EDS data as opposed to default data. We find, that for barriers below 50%, correlations are reasonably stable, and increase strongly above 50%. One explanation may be that the higher systematic risks of EDSs found in de Servigny and Jobst (2005) translates into co-movements. This has several important implications for (credit) portfolio modelling and a factor approach is outlined that is capable of capturing many of the requirements discussed.

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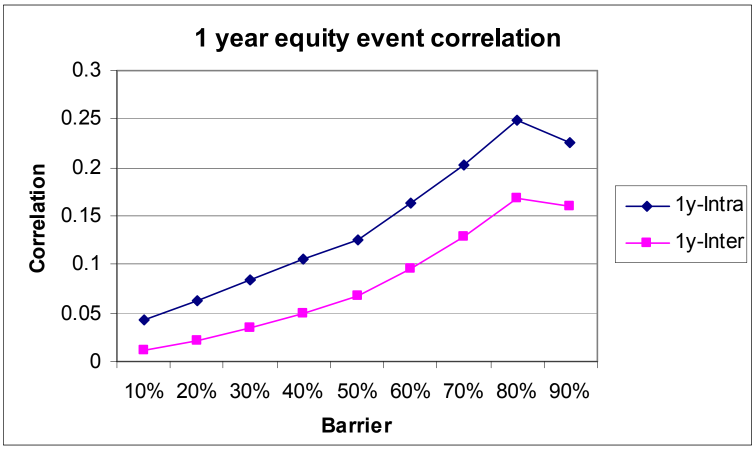

Figure 1a: Equity event correlation for different barriers (1 year horizon, JPD).

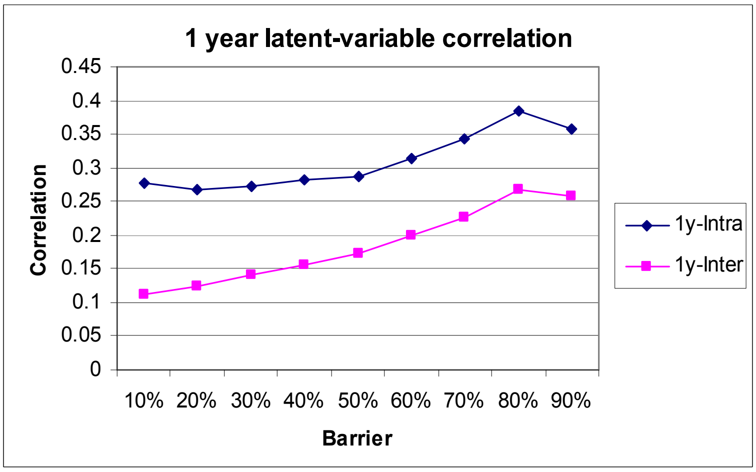

Figure 1b: Latent-variable correlation for different barriers (1 year horizon, JPD).

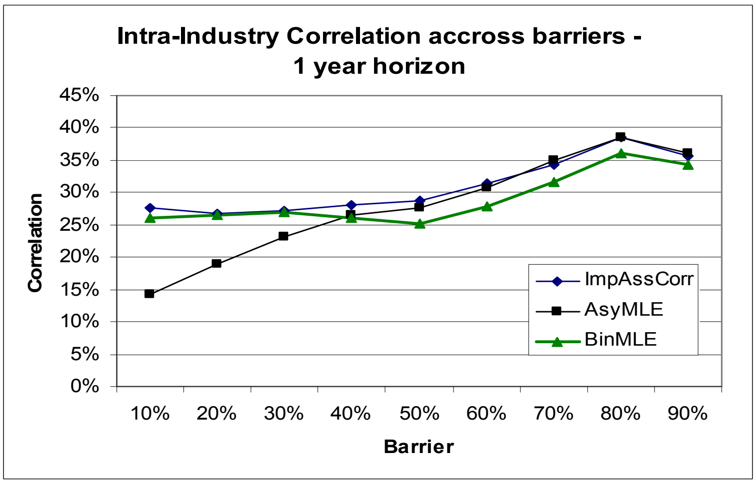

Figure 2a: Intra-Industry Correlation for different barriers and estimators.

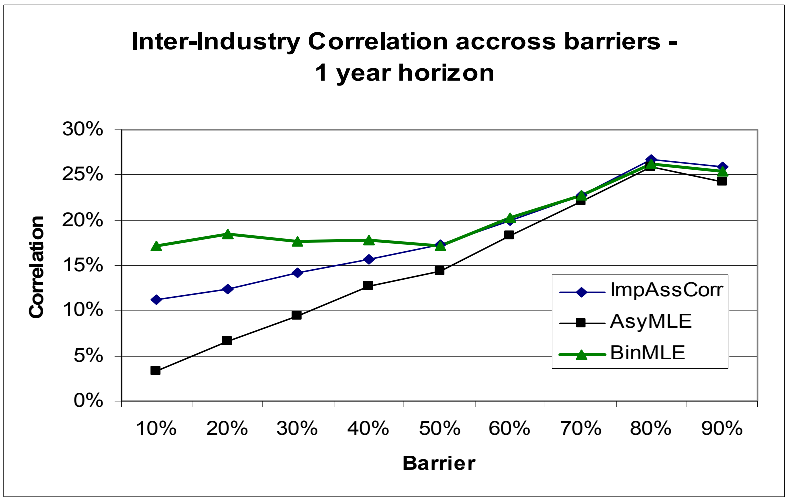

Figure 2b: Inter-Industry Correlation for different barriers and estimators.

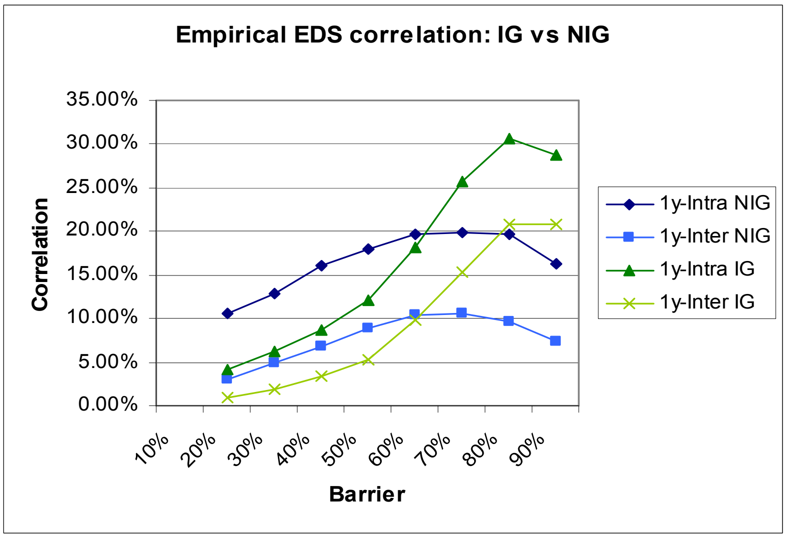

Figure 3a: Empirical EDS correlation for different credit quality (JPD).

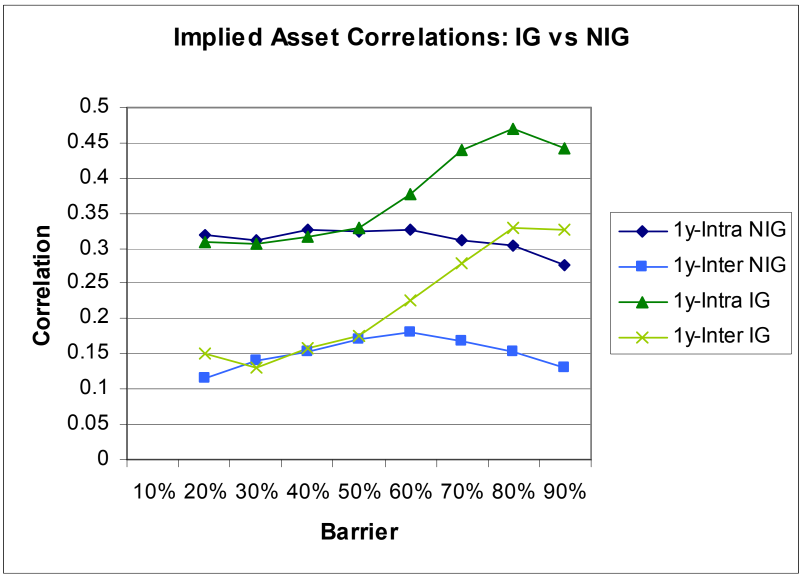

Figure 3b: Latent-variable correlation for different credit quality (JPD).

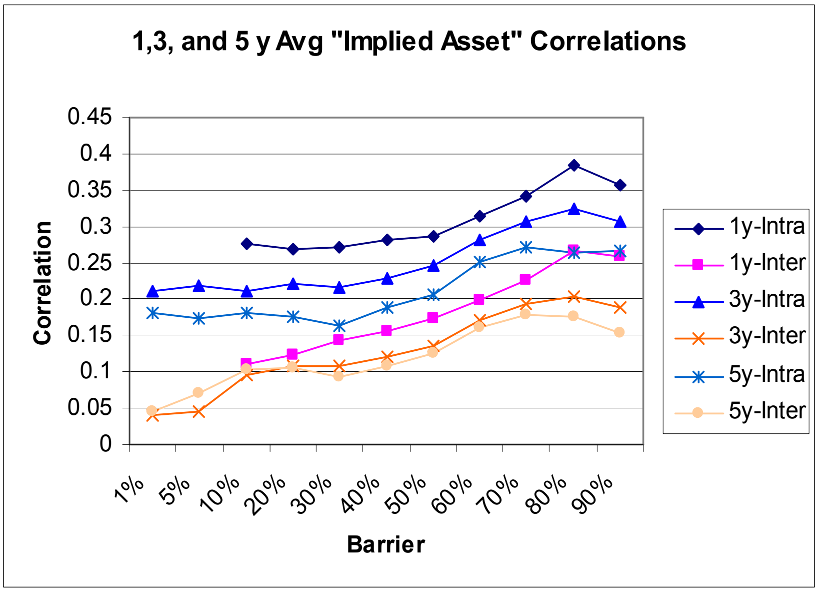

Figure 4. Maturity dependence of correlations (JPD).

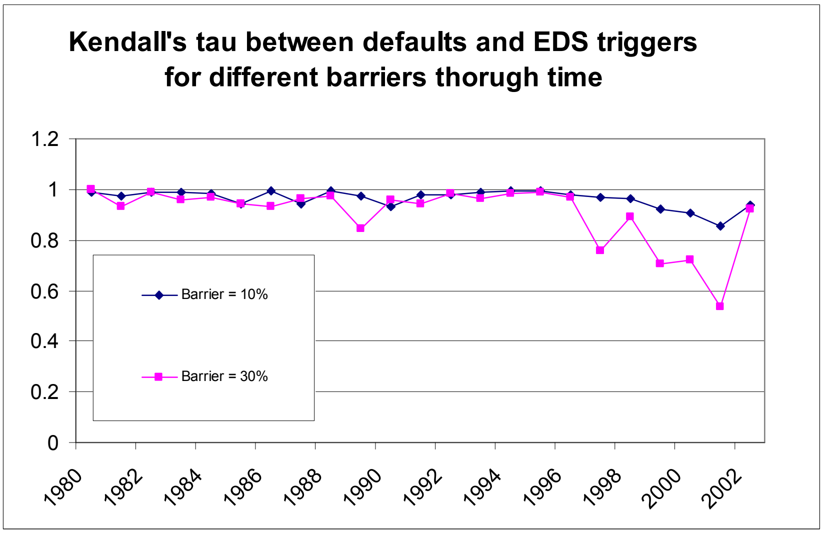

Figure 5. Kendall's tau between CDS and EDS events.

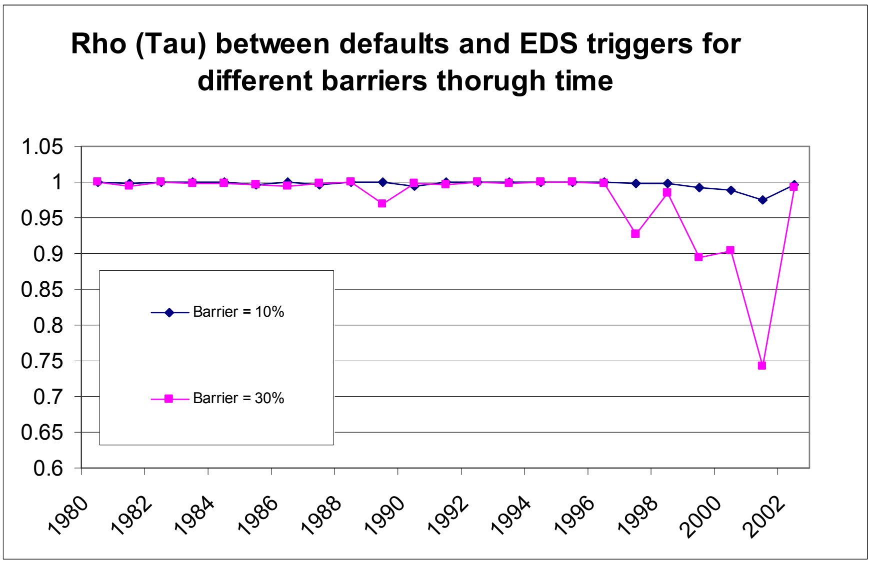

Figure 6. Pearson correlation estimated from Kendall's tau between CDS and EDS events.

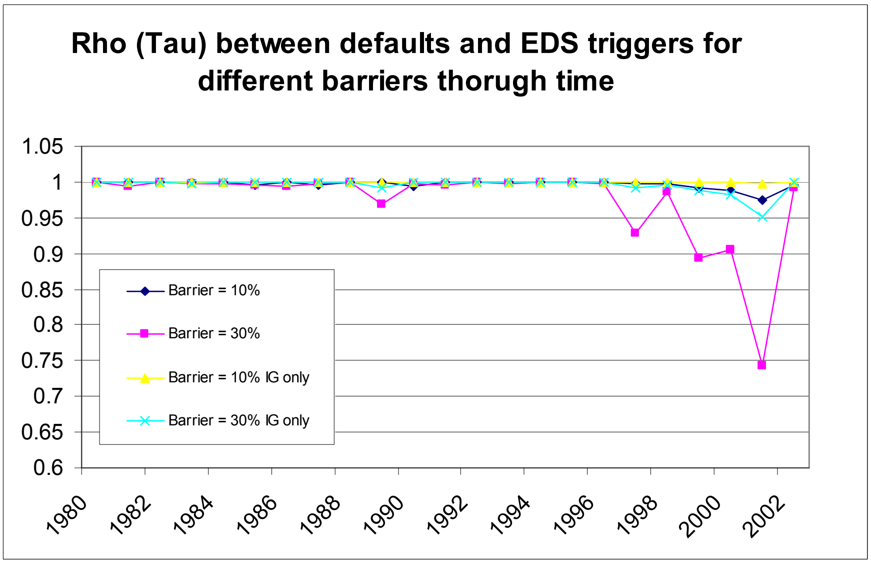

Figure 7. Kendall's tau between CDS and EDS events: All rated obligors vs IG only.

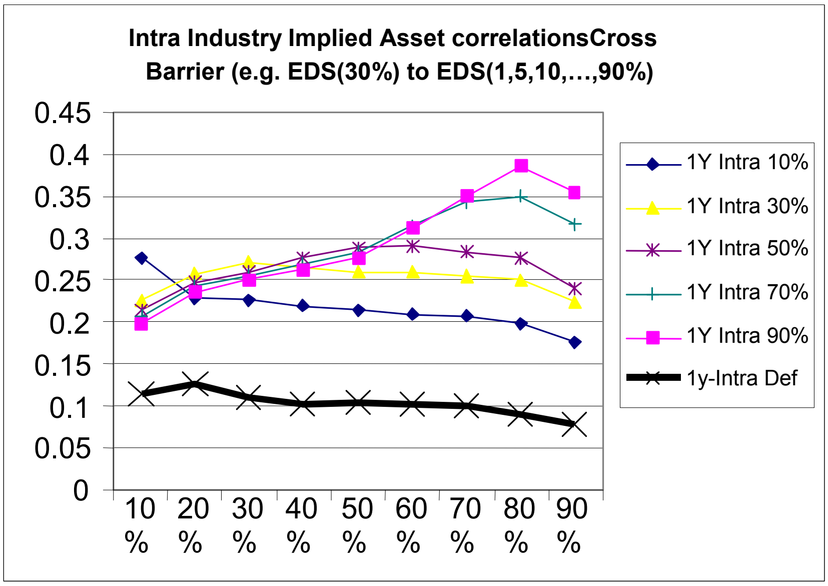

Figure 8.Intra-industry latent-variable correlation between different instruments.

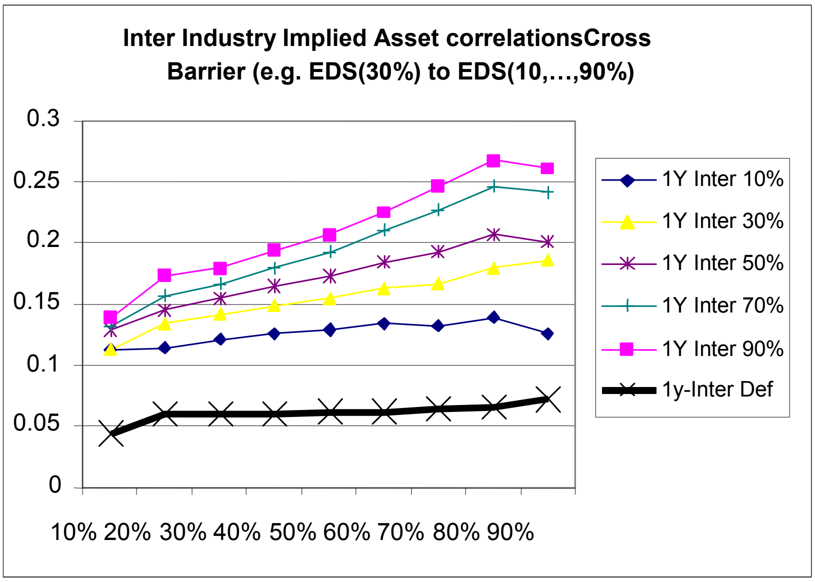

Figure 9.Inter-industry latent-variable correlation between different instruments.

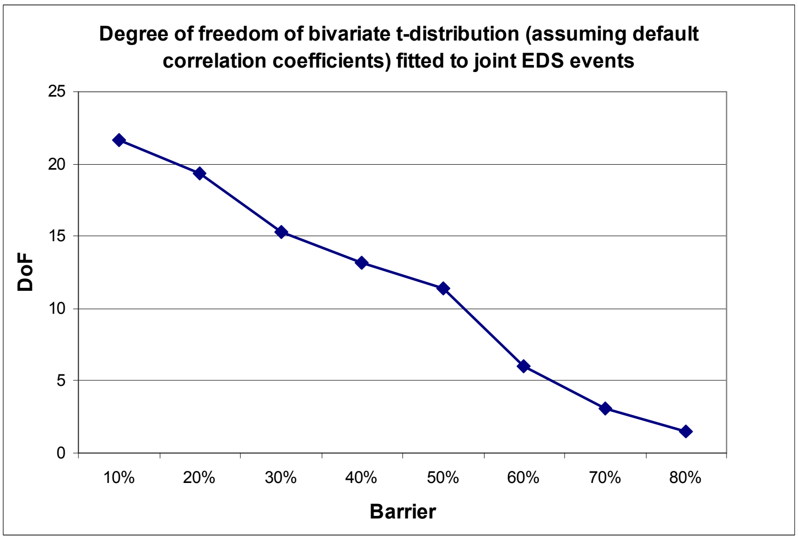

Figure 10: DoF for bivariate t-distribution from joint EDS event calibration.