Measuring Loss Potential of Hedge Fund Strategies

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Measuring the probabilistic loss that a hedge fund may suffer has been previously analyzed in the literature, mostly through VaR models. In previous research, specific statistical properties that hedge funds' returns usually exhibit, such as negative skewness and positive excess kurtosis have been analyzed extensively. However, other fundamental market risk measures besides VaR, such as time under the water and drawdown have not been explored as extensively. These two measures have important implications for both investors and practitioners. First, assessing the risks of late recovery is as important as quantifying the maximal loss their capital may suffer. Second, estimation of time under the water and maximal loss is key for the estimation of the survival probability, the probability of hitting the stop-loss that may trigger major liquidations and the likelihood of reaching the high-water mark before the end of the year and thus receiving a performance fee. The measurement of market risk in connection with the ARMA time-dependence exhibited by hedge fund returns has also not been explored as extensively. To the best of our knowledge, the literature has modeled hedge funds' market risk neglecting the effect of time-dependence in returns distributions. The purpose of this article is threefold: First, we develop a methodology to statistically infer hedge funds' loss potential according to the aforementioned three market risk measures. Second we derive those estimates under three analytic frameworks: 1) normality and time-independence, 2) non-normality and time-independence, and 3) non-normality and time-dependence. Finally, we assess the accuracy of market risk models based on those three analytical frameworks and the sufficiency of VaR measures.

The data we used are monthly time series from January 1990 until April 2003 published by Hedge Fund Research for 16 indices, MSCI for global equity markets, JPM for global fixed income markets, and Barclays for CTAs, all of them in USD. QisMIXTEST and QisDRAWDOWN are quantitative tools developed by the authors. This article is organized as follows: We will first define drawdown and time under the water. In the following three sections we will estimate drawdown and time under the water under the three analytical frameworks previously mentioned. Finally we will offer some insights regarding our results.[1]

DEFINING DRAWDOWN

In the context of this article we will understand *Drawdown* $(1-\alpha)\%$, $DD_{1-\alpha}$, as the loss a risky investment may suffer with a certain confidence level $1-\alpha$, whatever the investment horizon is. Recall that this definition is very similar to $VaR_{1-\alpha,\Delta T}$, with the difference that VaR does provide the probabilistic loss for a specific time period ΔT.

Let τ be the threshold return which separates profits from losses and

$$\varphi_{dt} := Ln \frac{W_t e^{\tau dt}}{W_{t+dt}}$$

the loss that results of investing W_t during a period dt with respect to the threshold τ. Considering φ_{dt} as a random variable which follows a ψ density function affords the definition of *Drawdown (1 − α)%* as

$$DD_{1-\alpha} := \underset{dt}{Max}\left\{\varphi_{dt}\left|\int_{-\infty}^{\varphi_{dt}} \psi(x) \cdot dx = 1 - \alpha\right.\right\}, \forall dt > 0 \qquad (1)$$

and $VaR_{1-\alpha,\, \Delta T}$ as

$$VaR_{1-\alpha,\Delta T} := \left\{\varphi_{dt}\left|\int_{-\infty}^{\varphi_{dt}} \psi(x) \cdot dx = 1 - \alpha\right.\right\}, dt = \Delta T > 0 \qquad (2)$$

If we define $DD_{1-\alpha}$ in terms of $VaR_{1-\alpha,\, \Delta T}$, the consideration of any τ threshold return is implicitly introduced in the VaR model, so we do not need to consider it in the $DD_{1-\alpha}$ definition.

$$DD_{1-\alpha} := \underset{dt}{Max}\left\{VaR_{1-\alpha,dt}\right\} \geq VaR_{1-\alpha,\Delta T}, \forall dt > 0 \qquad (3)$$

i.e., drawdown is a maximal bound of VaR measures.

Unlike $VaR_{1-\alpha,\, \Delta T}$, $DD_{1-\alpha}$ does not refer to a specific time period $\Delta T > 0$, but to the whole investment horizon, $\forall dt > 0$.

DEFINING TIME UNDER THE WATER

Time Under the Water (1–α)%, $TUW_{1-\alpha}$, is simply the period of time a risky investment may remain with a net asset value (NAV) lower than its target value with a certain confidence level. Given a wealth value W at a time point *t* and a τ threshold return, the time under the water is computed as:

$$TUW_t := Min\{dt \mid W_{t+dt} \geq W_t e^{\tau dt}\}, dt > 0 \qquad (4)$$

which is obviously a random variable provided that W represents the wealth value of a risky investment. Let Θ be the density function for TUW_t. Then,

$$TUW_{1-\alpha} := \left\{dt\left|\int_{-\infty}^{dt} \Theta(x) \cdot dx = 1 - \alpha\right.\right\} \qquad (5)$$

EXHIBIT 1
Comparing Results

		Time Under the Water	
		(1)<(2)	(2)<(1)
Drawdown	(1)<(2)	I	II
	(2)<(1)	III	IV

Source: Authors.

Likewise, $TUW_{1-\alpha}$ can be also defined in terms of VaR with the same implications with respect to τ.

$$TUW_{1-\alpha} := \{dt \mid VaR_{1-\alpha,\, dt} = 0\}, dt > 0 \qquad (6)$$

CHARACTERIZING HEDGE FUND RETURNS

Brooks and Kat [2002] show that the distributions of hedge funds' monthly returns exhibit statistically significant skewness, kurtosis, and autocorrelation of different orders. A direct consequence of such a result is that mean-variance biased performance measures (such as Sharpe ratio or Treynor ratio) may overestimate the efficiency of hedge funds.

Favre and Galeano [2002] derive a modified Sharpe ratio that takes non-normality into account. Similar analyses were performed by Hwang and Satchell [1999], Amin and Kat [2002], Jurczenko and Maillet [2002a, 2002b], Zsolt [2002], and others. Ineichen [2003] provides a historical analysis of drawdown for different hedge fund indices. Measuring the impact of time-dependence on hedge fund's risk and efficiency has remained an unexplored territory so far.

COMPARING RESULTS

This article will deal with three different systems of assumptions or frameworks. For each two frameworks, results can be compared as shown in Exhibit 1.

Let's assume that investors prefer more return over less and shorter time under the water to longer. Then, in Case I, Result 1 is Pareto-dominant with respect to Result 2. The opposite happens in Case IV, where Result 2 is Pareto-dominant with respect to Result 1. In Cases II and III there is no Pareto dominance and we need to introduce a utility function in order to determine the exact trade-off between drawdown and time under the water. For instance, investors more sensitive to time-to-recovery than losses (e.g., investors who require high liquidity) will generally prefer Case II to Case III.

Exhibit 2 displays the diagrams of the Cases described in Exhibit 1.

I. ASSUMING NORMALITY AND TIME-INDEPENDENCE

Computing Drawdown and Time Under the Water When Normality Holds

Using the equations derived in Appendix A, one can immediately construct an algebraic relation between VaR, drawdown, and time under the water by substituting $Z_\alpha \sigma$ in any of the expressions by

$$Z_\alpha \sigma = -\frac{VaR_{1-\alpha,\Delta T} + \left(E\left(r_P\right) - \tau\right)\Delta T}{\sqrt{\Delta T}} \quad (7)$$

Exhibit 3 shows time under the water (in years) and drawdown for $\alpha = 1\%$, $\alpha = 5\%$, and $\tau = 0\%$ per strategy assuming normality. VaR, mean, and standard deviation figures are computed for a one-month time window. Mean and standard deviation values are shown in Exhibit 4.

For example, with a 99% confidence the loss over one month will not be larger than 3.72% when investing in the hedge fund weighted composite. However, with a small likelihood (1%), an investor will experience a loss larger than 5.17% (assuming no specific time horizon) and it might take about 1.51 years until his wealth has recovered. Also, hedge funds provide more favorable numbers than stocks, using the MSCI Global Equity Index as a proxy. In this case, an investor might lose almost 75% of his wealth and it might take more than 72 years until a loss is completely recovered.[2]

In this context, all three risk measures depend on the mean and the standard deviation of the distribution; a low mean and/or a high standard deviation will lead to a high VaR and a high DD, *ceteris paribus*. However, DD increases much faster than VaR when risk increases, for a given mean (*ceteris paribus*). Also, for a given risk level, DD decreases much faster than VaR with an increase in the expected value of the distribution. Thus, DD is more sensitive to changes in the return distribution, and the ratio between DD/VaR is higher the bigger VaR is. Increasing the confidence level, e.g., from 95% to 99%, increases the VaR as well as the DD. However, since DD is again more sensitive, the ratio increases as well.

Although in the case of normal distributions the three risk measures reflect the same information, it might be illustrative for investors and practitioners to consider the three different aspects.

Our main conclusion to these results is that, even though drawdown and time under the water may differ from VaR in the "normal" case, both of them can be determined by applying the mathematical relation that bind them, thus being redundant to VaR.

II. CONSIDERING NON-NORMALITY, ASSUMING TIME-INDEPENDENCE

Modeling Hedge Fund Returns Through Mixture of Normal Distributions

Non-normal returns have been widely modeled through mixture of normal distributions.[3] The assumption underlying this procedure is that the unconditional return distribution is the result of two or more processes that may happen with a certain probability. The influence of higher moments is illustrated in Exhibits 5 and 6, where we observe a trade-off between co-skewness and co-kurtosis that mixture of normals can model (cf. López de Prado and Rodrigo [2004]).

The estimation of the parameters for mixture of normal distributions out of historical data is described in Hamilton [1991]. Applying this quasi-Bayesian approach on a time series of monthly returns, we derived the results displayed in Exhibit 7. Mean and standard deviation have been annualized.

The average squared error is small while there is no significant improvement in estimating a mixture of more than two distributions. The mixtures estimated in Exhibit 7 are characteristic in the sense that each of them provides the same four principal moments about the mean as computed in Exhibit 4. Exhibit 8 illustrates the fitting of the cumulative distribution for the HFR Weighted Composite Index.

Computing Drawdown and Time Under the Water When Normality Does Not Hold

As it is not possible to assume normality in the distribution of hedge funds' returns, drawdown (DD) and time under the water (TUW) cannot be computed analytically:[4] When returns do not follow a normal distribution, the stochastic components of returns do not comply with a Wiener process; thus NAV cannot be modeled through a geometric Brownian motion (GBM)

Exhibit 2
Comparing Results

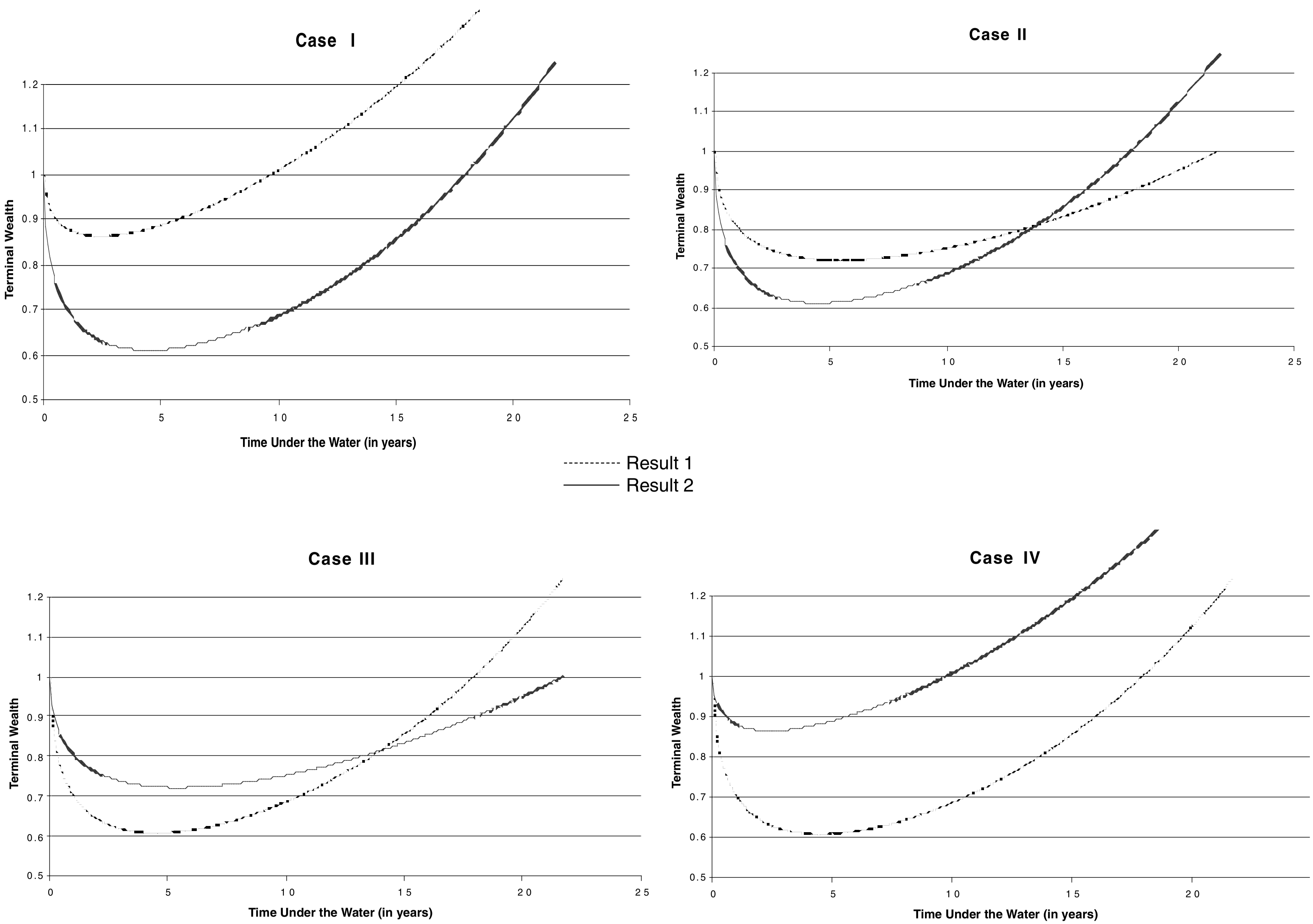


EXHIBIT 3

VaR, Drawdown, and Time Under the Water per Strategy Assuming Normality in Returns' Distributions

	TUW (99%)	DD (99%)	VaR (99%)	TUW (95%)	DD (95%)	VaR (95%)	DD/VaR (99%)	DD/VaR (95%)
HFR Convertible Arbitrage	0.49	1.38%	1.34%	0.24	0.69%	0.67%	3.14%	2.88%
HFR Distressed	1.15	3.94%	3.10%	0.57	1.97%	1.86%	26.94%	5.96%
HFR Emerging Markets	6.49	23.08%	9.27%	3.25	11.54%	6.21%	148.89%	85.81%
HFR Equity Hedge	1.64	6.94%	4.85%	0.82	3.47%	3.02%	43.05%	15.04%
HFR Equity Market Neutral	0.61	1.48%	1.38%	0.30	0.74%	0.74%	7.12%	0.25%
HFR Equity Non-Hedge	4.92	18.94%	8.58%	2.46	9.47%	5.69%	120.90%	66.50%
HFR Event-Driven	1.42	4.72%	3.46%	0.71	2.36%	2.13%	36.18%	10.99%
HFR Fixed Income Arbitrage	1.62	3.40%	2.38%	0.81	1.70%	1.48%	42.60%	14.77%
HFR Fund of Fund Index	1.93	4.77%	3.14%	0.96	2.38%	1.98%	51.81%	20.43%
HFR Fund Weighted Composite	1.51	5.17%	3.72%	0.75	2.58%	2.29%	38.97%	12.61%
HFR Macro	1.39	5.80%	4.29%	0.70	2.90%	2.62%	35.31%	10.49%
HFR Market Timing	1.64	5.14%	3.59%	0.82	2.57%	2.23%	43.27%	15.17%
HFR Merger Arbitrage	0.97	2.55%	2.12%	0.48	1.28%	1.24%	20.48%	2.93%
HFR Relative Value Arbitrage	0.51	1.56%	1.51%	0.25	0.78%	0.76%	3.66%	2.26%
HFR Short Seller	63.61	100.00%	14.62%	31.80	52.39%	10.18%	583.85%	414.73%
HFR Statistical Arbitrage	1.07	2.44%	1.96%	0.53	1.22%	1.17%	24.20%	4.61%
CTA Barclays	8.04	15.26%	5.58%	4.02	7.63%	3.76%	173.45%	102.87%
JPM Global Bond	3.54	6.56%	3.41%	1.77	3.28%	2.23%	92.41%	47.10%

Source: Authors, based on time series of monthly returns.

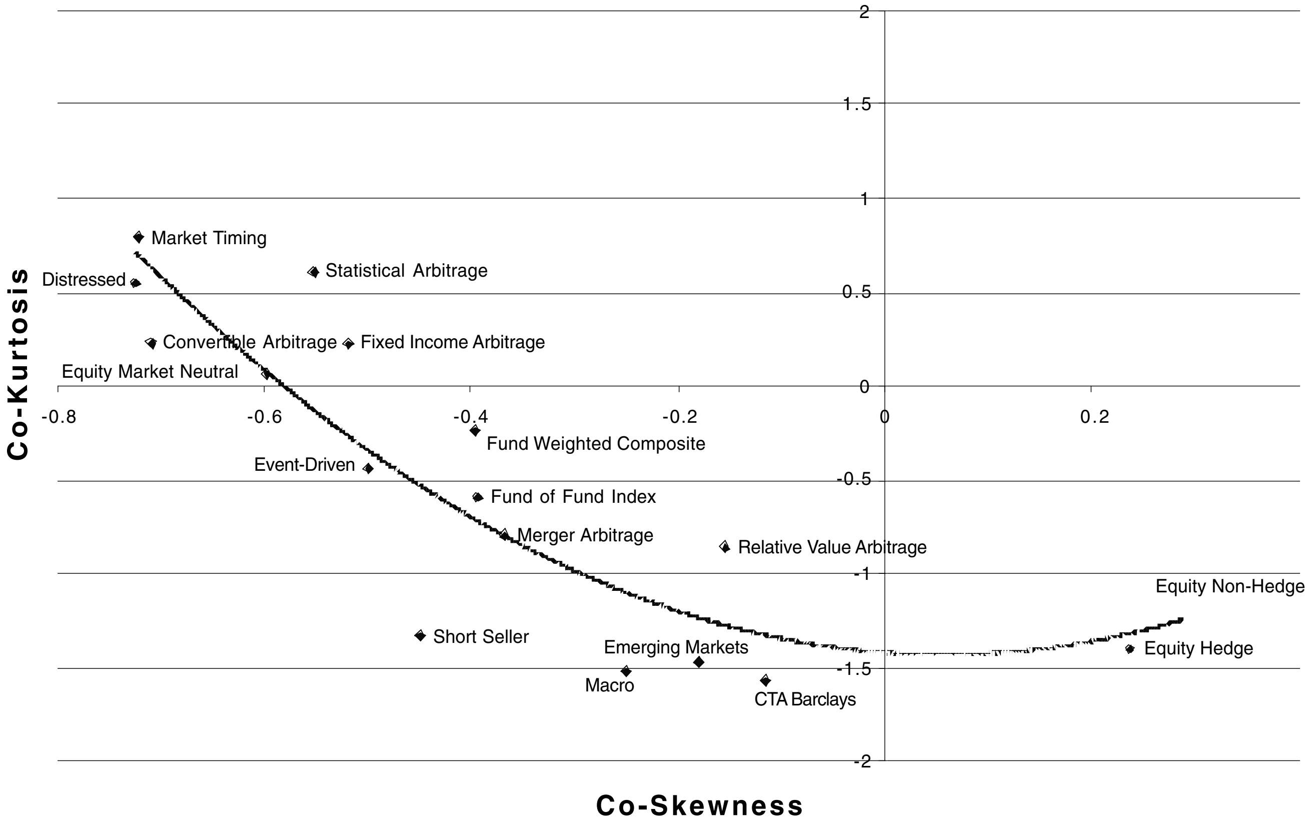
EXHIBIT 4

Sorting HFR Strategies According to the Co-Skewness and Co-Kurtosis Relative to the Market

	Mean	St Dev	Skewness	Ex. Kurtosis	Beta	Correl	Co-Skew	Ex. Co-Kurt
HFR Convertible Arbitrage	0.9%	1.0%	-132.3%	315.4%	6.5%	29.2%	-39.4%	-59.0%
HFR Distressed	1.1%	1.8%	-63.9%	544.9%	14.8%	35.3%	-71.0%	23.1%
HFR Emerging Markets	1.2%	4.5%	-76.0%	360.4%	63.7%	61.9%	-51.9%	22.7%
HFR Equity Hedge	1.4%	2.7%	16.9%	114.9%	38.3%	62.0%	-36.9%	-79.2%
HFR Equity Market Neutral	0.8%	0.9%	5.2%	19.7%	2.3%	10.7%	-18.1%	-146.6%
HFR Equity Non-Hedge	1.3%	4.2%	-47.6%	50.2%	68.6%	70.7%	-50.0%	-43.7%
HFR Event-Driven	1.1%	2.0%	-134.1%	470.4%	26.5%	58.8%	-72.5%	55.0%
HFR Fixed Income Arbitrage	0.7%	1.3%	-167.4%	903.6%	-17.8%	-23.2%	-15.5%	-85.5%
HFR Fund of Fund Index	0.8%	1.7%	-26.5%	388.8%	16.6%	42.4%	-39.7%	-23.4%
HFR Fund Weighted Composite	1.1%	2.1%	-61.6%	261.9%	32.4%	67.7%	-59.8%	6.5%
HFR Macro	1.4%	2.4%	45.6%	8.0%	22.6%	40.4%	-25.1%	-151.7%
HFR Market Timing	1.0%	2.0%	14.9%	-54.9%	30.0%	65.7%	-11.6%	-156.6%
HFR Merger Arbitrage	0.9%	1.3%	-276.2%	1168.6%	12.4%	42.0%	-72.2%	79.6%
HFR Relative Value Arbitrage	1.0%	1.1%	-100.5%	1055.4%	8.9%	35.5%	-55.2%	61.1%
HFR Short Seller	0.5%	6.5%	4.9%	113.3%	-97.1%	-65.0%	33.1%	-101.5%
HFR Statistical Arbitrage	0.8%	1.2%	-13.3%	45.1%	13.5%	50.5%	-45.0%	-132.7%
CTA Barclays	0.6%	2.7%	38.5%	35.9%	-13.4%	-21.9%	23.6%	-139.8%
JPM Global Bond	0.6%	1.7%	19.6%	1.9%	100.0%	100.0%	19.6%	1.9%

Source: Authors, based on time series of monthly returns (1990.01 - 2003.03).

EXHIBIT 5
Co-Skewness Versus Co-Kurtosis Relative to the Market



Source: Authors, based on time series of monthly returns.

EXHIBIT 6
Substitution of Co-Skewness for Co-Kurtosis in HFR Indices and Barclays CTA

Ex. Co-Kurtosis	Coefficients	Standard Error	t Stat	P-value
Intercept	-1.394361126	0.158953037	-8.772157806	2.71952E-07
Co-Skewness^2	4.068362243	0.577469738	7.045152286	3.96991E-06
Adjusted R Square	0.752452924			

Source: Authors, based on Hedge Fund Research and Barclays time series of monthly returns.

diffusion process.[5] A wide-range consequence of not having an analytical solution to compute VaR, drawdown, and time under the water is that the algebraic link between these three disappears, and VaR is no longer able to fully represent all three dimensions of market risk.

A numerical solution to this problem comes from computing via Monte Carlo a significant number of wealth paths, by generating random returns which comply with the assumed distribution function. Wealth paths are simulated as shown below:

$$W_t = W_{t-1} \cdot e^{r_t} \text{ where } r_t = \varepsilon_t \rightarrow \sum_{i=1}^{m} N(\mu_i, \sigma_i) \cdot P_i \Big| \sum_{i=1}^{m} P_i = 1 \quad (8)$$

where ε_t is white noise that distributes as a mixture of m normal distributions.

EXHIBIT 7

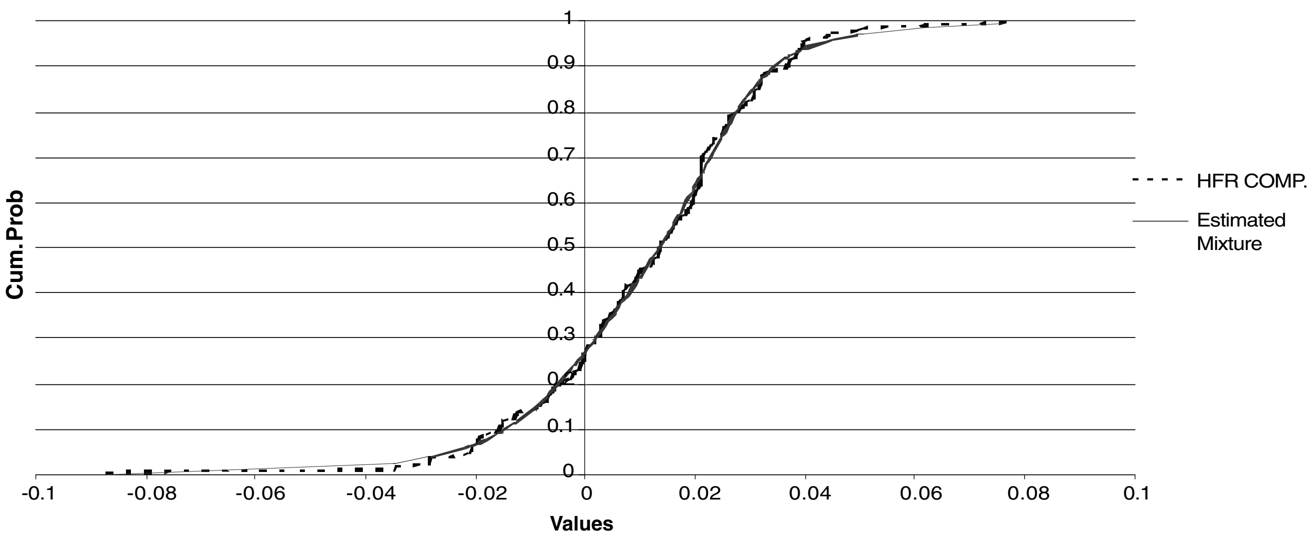
Estimating the Parameters for the Mixture of Normal Distributions Out-of-Time Series

	Mean (1)	StDev (1)	Mean (2)	StDev (2)	Prob	Avg. SE
HFR Convertible Arbitrage	0.14	0.02	-0.018	0.045	0.82	0.01%
HFR Distressed	0.15	0.039	0.057	0.1234	0.83	0.03%
HFR Emerging Markets	0.23	0.082	-0.055	0.251	0.71	0.02%
HFR Equity Hedge	0.15	0.079	0.228	0.1418	0.82	0.01%
HFR Equity Market Neutral	0.0925	0.029	0.1095	0.0375	0.73	0.01%
HFR Equity Non-Hedge	0.27	0.12	-0.54	0.1375	0.86	0.04%
HFR Event-Driven	0.187	0.031	-0.027	0.115	0.74	0.05%
HFR Fixed Income Arbitrage	0.112	0.02417	-0.051	0.0989	0.86	0.03%
HFR Fund of Fund Index	0.105	0.0375	0.066	0.104	0.79	0.02%
HFR Fund Weighted Composite	0.17	0.05	-0.01	0.115	0.77	0.01%
HFR Macro	0.045	0.06	0.45	0.0735	0.7	0.02%
HFR Market Timing	0.235	0.0567	-0.105	0.037	0.67	0.02%
HFR Merger Arbitrage	0.14	0.021	-0.169	0.0937	0.89	0.01%
HFR Relative Value Arbitrage	0.13	0.021	0.055	0.092	0.88	0.03%
HFR Short Seller	0.081	0.275	0.038	0.1	0.63	0.02%
HFR Statistical Arbitrage	0.105	0.037	0.063	0.0541	0.79	0.01%
CTA Barclays	0.141	0.102	-0.077	0.045	0.7	0.01%
JPM Global Bond	0.0113	0.05	0.23	0.0541	0.71	0.02%

Source: Authors, based on time series of monthly returns.

EXHIBIT 8

Estimating the Mixture for HFR Weighted Composite



Source: Authors, based on QisMIXTEST software for estimating mixture of normal distributions.

Then, we can bootstrap via Monte Carlo ψ, Θ density functions for DD and TUW. Applying definitions set for $DD_{1-\alpha}$ and $TUW_{1-\alpha}$, these risk values can be easily computed via Monte Carlo maximization in several alternative ways.[6]

Our calculations are based on 10,000 simulations carried out on a monthly frequency[7] until $VaR_{1-\alpha,\,dt^*} \geq 0$. When this happens, the simulation is finished, providing as results $TUW_{1-\alpha} = dt^*$ and $DD_{1-\alpha} = Max\{VaR_{1-\alpha,\,dt}\} \geq VaR_{1-\alpha,\Delta T}$, $\forall dt \in (0, \infty)$. No yearly withdrawal rate is considered and no simulations are excluded due to the "end of path effect." Readers interested in exploring the accuracy of this approach may contact the writers about receiving an analytic study.

For a standard Pentium IV computer each of these 19 Monte Carlo experiments should be completed in no

EXHIBIT 9

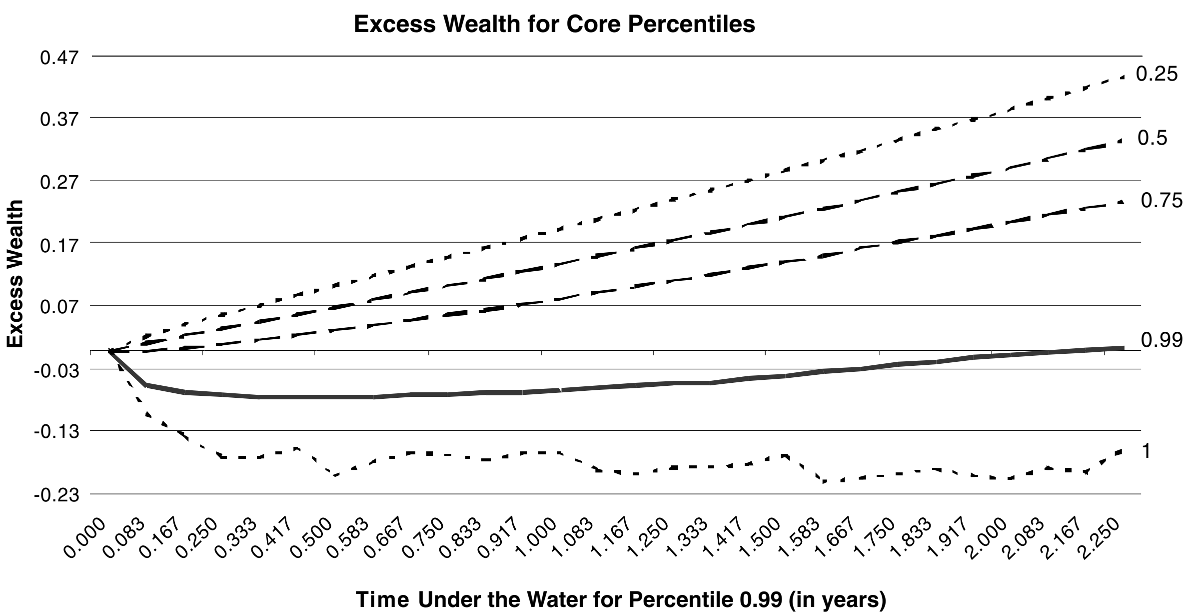
Risk Measures per Strategy Modeling Non-Normality via Mixture of Normal Distributions

	TUW (99%)	DD (99%)	VaR (99%)	TUW (95%)	DD (95%)	VaR (95%)	DD/VaR (99%)	DD/VaR (95%)
HFR Convertible Arbitrage	0.67	2.16%	2.16%	0.33	0.96%	0.92%	0.00%	4.64%
HFR Distressed	1.42	5.54%	5.10%	0.67	2.46%	1.69%	8.61%	45.76%
HFR Emerging Markets	6.25	22.60%	13.63%	3.17	11.77%	7.31%	65.84%	61.01%
HFR Equity Hedge	1.75	7.34%	5.11%	0.92	3.38%	2.90%	43.72%	16.42%
HFR Equity Market Neutral	0.67	1.33%	1.31%	0.33	0.70%	0.67%	1.19%	3.86%
HFR Equity Non-Hedge	4.83	18.28%	10.34%	2.50	9.47%	6.38%	76.80%	48.48%
HFR Event-Driven	1.92	6.88%	6.10%	0.83	3.38%	3.11%	12.82%	8.68%
HFR Fixed Income Arbitrage	2.00	5.52%	4.61%	0.92	2.31%	1.48%	19.77%	55.88%
HFR Fund of Fund Index	2.17	5.59%	4.46%	1.08	2.51%	1.83%	25.29%	37.28%
HFR Fund Weighted Composite	2.25	7.48%	5.77%	1.08	3.47%	3.09%	29.71%	12.28%
HFR Macro	1.25	4.55%	3.42%	0.67	2.29%	2.18%	32.96%	4.89%
HFR Market Timing	1.75	4.67%	2.93%	0.92	2.49%	2.04%	59.34%	22.16%
HFR Merger Arbitrage	1.50	5.30%	5.02%	0.75	2.28%	1.72%	5.67%	32.77%
HFR Relative Value Arbitrage	0.75	3.27%	3.21%	0.33	0.66%	0.43%	1.74%	52.56%
HFR Short Seller	64.75	67.24%	16.37%	32.58	41.87%	10.51%	310.78%	298.37%
HFR Statistical Arbitrage	1.08	2.60%	2.20%	0.58	1.26%	1.18%	18.20%	6.76%
CTA Barclays	8.17	14.48%	5.28%	4.08	7.55%	3.33%	174.30%	126.87%
JPM Global Bond	3.25	5.75%	3.08%	1.67	3.08%	2.06%	86.73%	49.60%

Source: Authors, based on QisDRAWDOWN.

EXHIBIT 10

Drawdown and Time Under the Water for HFR Fund Weighted Composite Index



Source: Authors, QisDRAWDOWN.

more than two hours. Results are displayed in Exhibit 9, where VaR is expressed for a one-month time window. Exhibit 10 illustrates the results for the HFR Fund Weighted Composite Index, where a drawdown of 7.48% is achieved after six months but recovery takes about 2.25 years with a confidence of 99%.

Compared to the normal case, risk values are higher, indicating that modeling hedge funds' returns under the assumption of normality may result in underestimation of their risk. This is especially true for "presumably" protected strategies like HFR Fixed Income Arbitrage or HFR Relative Value Arbitrage, where the level of risk

EXHIBIT 11

Differences Between Mixture of Normal Distribution Model and Normal Model

	TUW (99%)	DD (99%)	VaR (99%)	Case (99%)	TUW (95%)	DD (95%)	VaR (95%)	Case (95%)
HFR Convertible Arbitrage	36.26%	56.80%	61.73%	I	39.64%	37.30%	-100.00%	I
HFR Distressed	23.62%	40.62%	64.36%	I	25.09%	-9.06%	-68.05%	III
HFR Emerging Markets	-3.76%	-2.05%	46.99%	IV	2.01%	17.73%	-55.78%	I
HFR Equity Hedge	6.97%	5.84%	5.34%	I	-2.68%	-3.83%	1.56%	IV
HFR Equity Market Neutral	10.17%	-10.41%	-5.17%	III	-5.93%	-9.21%	-83.24%	IV
HFR Equity Non-Hedge	-1.83%	-3.50%	20.57%	IV	0.02%	12.17%	-36.47%	I
HFR Event-Driven	35.01%	45.86%	76.06%	I	43.29%	46.34%	-64.56%	I
HFR Fixed Income Arbitrage	23.36%	62.43%	93.39%	I	35.76%	-0.05%	-53.60%	III
HFR Fund of Fund Index	12.38%	17.23%	42.04%	I	5.42%	-7.52%	-51.18%	III
HFR Fund Weighted Composite	49.39%	44.87%	55.21%	I	34.32%	34.72%	-23.75%	I
HFR Macro	-10.26%	-21.60%	-20.22%	IV	-21.14%	-16.93%	-6.63%	IV
HFR Market Timing	6.53%	-9.23%	-18.39%	III	-3.09%	-8.63%	37.14%	IV
HFR Merger Arbitrage	55.42%	107.84%	136.97%	I	78.98%	38.76%	-72.31%	I
HFR Relative Value Arbitrage	48.41%	108.92%	112.86%	I	-16.06%	-43.73%	-52.50%	IV
HFR Short Seller	1.78%	-32.76%	11.95%	III	-20.09%	3.26%	-46.77%	II
HFR Statistical Arbitrage	1.40%	6.58%	11.99%	I	3.28%	1.21%	-24.82%	I
CTA Barclays	1.53%	-5.07%	-5.37%	III	-0.95%	-11.43%	0.49%	IV
JPM Global Bond	-8.12%	-12.27%	-9.60%	IV	-5.97%	-7.54%	-6.14%	IV

Source: Authors.

underestimation due to the assumption of normality is between 50%–100% of the total drawdown.

Finally, Exhibit 11 compares risk measures based on non-normal distributions with the similar measures assuming normality in returns' distribution. There is a predominance of Case I for a confidence level of 99%, i.e., both DD and TUW look more unfavorable. For 95% confidence level, there are only five Cases IV, which means that for most of the strategies either drawdown, time under the water, or both risk measures increase when non-normality is considered.

Therefore, we may assert that risk models assuming normality of returns' distribution usually underestimate the risk of hedge fund strategies. However, the size and direction of the error will depend on what dimension of market risk, what strategy, and what confidence level you look at. Since VaR, drawdown, and time under the water are not analytically linked when the assumption of normality does not hold, each of these figures provide complementary information regarding the hedge funds' risk profile.

III. CONSIDERING NON-NORMALITY AND TIME-DEPENDENCE

Time-Dependence of Non-Normally Distributed Returns

Hedge fund returns time series exhibit not only non-normality but also several forms of time-dependence. Brooks and Kat [2002] report the presence of significant serial correlation in hedge fund returns. This is a feature with important consequences when measuring market risk and therefore should not be neglected. We will model time-dependence of logarithmic returns r_t through a Non-Gaussian ARMA(p, q) process, i.e., an ARIMA (p, 1, q) process on the prices W_t:

$$r_t = Ln\frac{W_t}{W_{t-1}} = LnW_t - LnW_{t-1} \tag{9}$$

and following the standard Box-Jenkins notation (Box and Jenkins [1976]),

$$r_t = c + \sum_{i=1}^{p} \phi_i \, r_{t-i} + \varepsilon_t - \sum_{j=1}^{q} \theta_j \, \varepsilon_{t-j}$$

which can be expressed as a finite differential equation

$$\left(1 - \sum_{i=1}^{p} \phi_i \, L^i\right) r_t - c = \left(1 - \sum_{j=1}^{q} \theta_j \, L^j\right) \varepsilon_t \tag{10}$$

where ε_t is white noise that distributes as a mixture of normal distributions:

$$\varepsilon_t \rightarrow \sum_{i=1}^{m} N(\mu_i, \sigma_i) \cdot P_i \Bigg| \sum_{i=1}^{m} P_i = 1$$

$$E(\varepsilon_t) = \sum_{i=1}^{m} \mu_i P_i = 0, \forall t$$

$$E(\varepsilon_t^2) = \sum_{i=1}^{m} \sigma_i^2 P_i = \sigma^2, \forall t$$

$$E(\varepsilon_t \varepsilon_s) = 0, \forall_t \neq s \quad (11)$$

In the next section we explain how to estimate the necessary parameters in order to conduct this non-Gaussian ARMA Monte Carlo process, i.e., $\boldsymbol{\phi}$, $\boldsymbol{\theta}$, $\boldsymbol{\mu}$, $\boldsymbol{\sigma}$, $\boldsymbol{P}$.

Estimating the Non-Gaussian ARMA Process

We have divided the procedure to estimate our non-Gaussian ARMA model into four sequential steps:

1. Initial specification of the model.
2. LS estimation of the ARMA (p, q) coefficients,[8] $\boldsymbol{\phi}$, $\boldsymbol{\theta}$.
3. From Step 1 until model is validated, following a stepwise factor-selection algorithm.[9]
4. Quasi-Bayesian estimation of the parameters of a mixture of normal distributions on the residuals of the ARMA process, i.e., $\boldsymbol{\mu}$, $\boldsymbol{\sigma}$, $\boldsymbol{P}$. This is precisely the same method applied earlier on the returns time series, but now applied on the $\hat{\varepsilon}_t$ ARMA estimated residuals.

Hamilton [1994] demonstrates that, in cases where ε follows a mixture of normal distributions (but also in the more general case of Markov chains), the maximum likelihood ARMA estimator is asymptotically unbiased and optimal.[10] Since our time series can be regarded as large samples, the LS estimators of the ARMA process are assumed to follow a Gaussian distribution although ε follows a mixture of normal distributions.

The four-steps estimation procedure described above generates a stochastic process mimicking the statistical properties of hedge fund returns indices, in particular those related to non-normality and time-dependence. For each hedge fund return index, Exhibits 12 and 13 provide the ARMA and mixture's parameters that generate a stochastic process with the same moments and ARMA coefficients as exhibited by its returns time series.

Non-Gaussian ARMA Monte Carlo

The non-Gaussian ARMA process estimated in the previous section has been fitted after 160 observations, and any additional draw of the stochastic process should be based on an initial simulated sample of that length. Monte Carlo specifications are as for the time-independent case.

Results are shown in Exhibit 14. For HFR Equity Market Neutral, HFR Market Timing, HFR Short Seller, CTA Barclays, and MSCI Global Equity we provide no results because these return indices do not exhibit significant ARMA time-dependence, i.e., the model degenerates into a mixture of normal as discussed in Section II.

Exhibit 15 shows that for all the strategies but one and for both confidence levels the comparison between non-Gaussian ARMA results and mixture of normal distributions results is described by only two cases, I and IV. Recall that in the section "Comparing Results" we justified that Cases I and IV were precisely those of Pareto dominance. In fact, *the appearance of this dichotomy between Cases I and IV when comparing results simply means that assuming time-independence in hedge fund returns is misleading for estimating both risk measures, drawdown and time under the water, and that the error happens for both indicators in the same direction:* underestimating the risks involved in strategies belonging to Case I, overestimating the risks involved in strategies belonging to Case IV.

What are the implications of missing Cases II and III? Lacking cases of non-Pareto dominance means that both risk measures, drawdown and time under the water, move in the same direction once the hypothesis of time-independence is relaxed. There is a clear explanation for that observation: The stationary ARMA processes exhibited by hedge fund strategies affect the modeling of returns in the way of smoothing their time series. This is a well-known property of hedge fund returns indices[11] incorporated in our model, with the consequence that a higher (lower) drawdown implies longer (shorter) time under the water.

For illustration purposes, we compare in Exhibit 16 the Non-Gaussian ARMA model with the normal model. Comments are similar to those regarding Exhibit 15.

CONCLUSIONS

This study has tackled three important dimensions of the market risks involved in hedge funds' investments: drawdown, time under the water, and VaR. These are different but complementary measures of market risk. VaR refers to the probabilistic loss comparing today's wealth with a punctual forward wealth. Drawdown represents the maximum probabilistic loss that an investment may suffer, without restricting its value to a specific investment horizon. Time under the water shows how long it may take to recover

EXHIBIT 12
ARMA Parameters

	Intercept	AR(1)	AR(2)	AR(3)	MA(1)	MA(2)	MA(3)
HFR Convertible Arbitrage	**0.013**	**0.561293**		-0.148588			
HFR Distressed	**0.0112**				**-0.544525**		**0.193687**
HFR Emerging Markets	0.015	**0.308357**					
HFR Equity Hedge	**0.014472**	0.155912					
HFR Equity Market Neutral							
HFR Equity Non-Hedge	**0.012749**				-0.160561		
HFR Event-Driven	**0.010971**				**-0.3158**		
HFR Fixed Income Arbitrage	**0.007006**				**-0.418308**		
HFR Fund of Fund Index	**0.01**	**0.314524**					
HFR Fund Weighted Composite	**0.0132**	**0.24763**					
HFR Macro	**0.017**	0.159457					
HFR Market Timing							
HFR Merger Arbitrage	**0.008682**				**-0.315652**		-0.174732
HFR Relative Value Arbitrage	**0.0101**				**-0.215929**	-0.176709	
HFR Short Seller							
HFR Statistical Arbitrage	**0.009**	**0.216156**					
CTA Barclays							
JPM Global Bond	**0.0055**				**-0.289599**		

Note: Parameters in bold represent Prob<1%. For the rest, Prob<5%.
Source: Authors, E-Views.

EXHIBIT 13
Mixture of Normal Distributions Fitted on ARMA Residuals

	Mean (1)	StDev (1)	Mean (2)	StDev (2)	Prob
HFR Convertible Arbitrage	0.043706	0.01	-0.199106	0.05	0.82
HFR Distressed	0.01581	0.02	-0.07719	0.1234	0.83
HFR Emerging Markets	0.08265	0.073	-0.20235	0.25	0.71
HFR Equity Hedge	-0.01404	0.079	0.06396	0.1418	0.82
HFR Equity Market Neutral					
HFR Equity Non-Hedge	0.1134	0.12	-0.6966	0.15	0.86
HFR Event-Driven	0.05564	0.025	-0.15836	0.115	0.74
HFR Fixed Income Arbitrage	0.02282	0.015	-0.14018	0.0989	0.86
HFR Fund of Fund Index	0.00819	0.0325	-0.03081	0.104	0.79
HFR Fund Weighted Composite	0.039914	0.043	-0.119741	0.115	0.75
HFR Macro	-0.1215	0.059	0.2835	0.0735	0.7
HFR Market Timing					
HFR Merger Arbitrage	0.03399	0.01	-0.27501	0.0937	0.89
HFR Relative Value Arbitrage	0.009	0.018	-0.066	0.093	0.88
HFR Short Seller					
HFR Statistical Arbitrage	0.00882	0.0353	-0.03318	0.0543	0.79
CTA Barclays					
JPM Global Bond	-0.063423	0.048	0.155277	0.054	0.71

Source: Authors, QisMIXTEST.

from a loss with a certain confidence level. In a "normal" world, the last two risk measures can be derived out of VaR, and therefore they may be disregarded as redundant.

We have provided further evidence regarding the existence of conditional volatility regimes which justify the modeling of non-normally distributed hedge funds' returns via mixture of normal distributions. In particular, we cannot underestimate the importance of the four principal moments of hedge fund returns, since there seems to be a strong relation between them for each strategy and a trade-off among them when comparing any two strategies. The estimated mixture of normal's parameters

EXHIBIT 14

Risk Measures per Strategy Modeling Returns Through a Non-Gaussian ARMA Process

	TUW (99%)	DD (99%)	VaR (99%)	TUW (95%)	DD (95%)	VaR (95%)	DD/VaR (99%)	DD/VaR (95%)
HFR Convertible Arbitrage	0.58	3.32%	3.25%	0.33	1.68%	1.68%	2.18%	0.00%
HFR Distressed	2.75	10.80%	5.35%	1.25	4.78%	2.19%	101.84%	118.45%
HFR Emerging Markets	4.25	16.00%	13.95%	2.00	8.32%	7.57%	14.68%	9.86%
HFR Equity Hedge	1.67	6.17%	5.30%	0.83	3.27%	3.04%	16.32%	7.66%
HFR Equity Market Neutral								
HFR Equity Non-Hedge	7.75	26.10%	10.94%	3.75	12.97%	6.64%	138.61%	95.31%
HFR Event-Driven	3.50	12.52%	6.22%	1.75	5.66%	3.14%	101.34%	80.39%
HFR Fixed Income Arbitrage	4.33	10.10%	4.40%	2.08	4.01%	1.83%	129.62%	119.11%
HFR Fund of Fund Index	1.50	4.55%	4.55%	0.75	1.85%	1.85%	0.00%	0.00%
HFR Fund Weighted Composite	1.42	5.83%	5.83%	0.75	2.84%	2.70%	0.00%	5.18%
HFR Macro	0.92	3.65%	3.40%	0.50	2.17%	2.04%	7.28%	6.20%
HFR Market Timing								
HFR Merger Arbitrage	4.50	12.12%	5.10%	2.08	4.12%	1.64%	137.61%	151.34%
HFR Relative Value Arbitrage	1.83	4.99%	3.28%	0.67	1.14%	0.49%	52.06%	132.30%
HFR Short Seller								
HFR Statistical Arbitrage	0.83	2.31%	2.28%	0.50	1.26%	1.20%	1.26%	4.89%
CTA Barclays								
JPM Global Bond	7.83	12.48%	3.22%	4.17	5.96%	2.14%	287.48%	178.38%

Authors, QisDRAWDOWN.

EXHIBIT 15

Differences Between Non-Gaussian ARMA and Mixture of Normal Distribution Models

	TUW (99%)	DD (99%)	VaR (99%)	Case (99%)	TUW (95%)	DD (95%)	VaR (95%)	Case (95%)
HFR Convertible Arbitrage	-12.50%	53.57%	50.30%	II	0.00%	74.51%	82.61%	I
HFR Distressed	94.12%	94.96%	4.90%	I	86.34%	94.22%	29.59%	I
HFR Emerging Markets	-31.99%	-29.23%	2.35%	IV	-36.84%	-29.34%	3.56%	IV
HFR Equity Hedge	-4.76%	-16.06%	3.72%	IV	-9.09%	-3.06%	4.83%	IV
HFR Equity Market Neutral								
HFR Equity Non-Hedge	60.34%	42.79%	5.80%	I	50.00%	36.90%	4.08%	I
HFR Event-Driven	82.61%	81.97%	1.97%	I	110.00%	67.58%	0.96%	I
HFR Fixed Income Arbitrage	116.67%	82.99%	-4.56%	I	127.27%	73.80%	23.65%	I
HFR Fund of Fund Index	-30.77%	-18.58%	2.02%	IV	-30.77%	-26.36%	1.09%	IV
HFR Fund Weighted Composite	-37.04%	-22.11%	1.04%	IV	-30.77%	-18.15%	-12.62%	IV
HFR Macro	-26.67%	-19.79%	-0.58%	IV	-25.00%	-5.25%	-6.42%	IV
HFR Market Timing								
HFR Merger Arbitrage	200.00%	128.44%	1.59%	I	177.78%	80.50%	-4.65%	I
HFR Relative Value Arbitrage	144.44%	52.72%	2.18%	I	100.00%	73.52%	13.95%	I
HFR Short Seller								
HFR Statistical Arbitrage	-23.08%	-11.21%	3.64%	IV	-14.29%	-0.09%	1.69%	IV
CTA Barclays								
JPM Global Bond	141.03%	116.94%	4.55%	I	150.00%	93.31%	3.88%	I

Source: Authors, QisDRAWDOWN.

afford the modeling of non-normally distributed returns consistent with the four principal moments observed in hedge fund time series. The error obtained in this estimation has been negligible even for the simplest case of two distributions conforming the mixture.

Time-dependence of hedge fund returns is also a statistical property reported in the literature. We have estimated for each strategy the parameters corresponding to a non-Gaussian ARMA process with the same moments and ARMA time-dependence as exhibited by hedge fund indices.

Since it is not possible to provide an analytical solution for the computation of drawdown and time under the water whenever normality or time-independence assumptions on returns' distribution do not hold, we have developed a Monte Carlo simulation based on a non-

EXHIBIT 16

Differences Between Non-Gaussian ARMA and Normal Models

	TUW (99%)	DD (99%)	VaR (99%)	Case (99%)	TUW (95%)	DD (95%)	VaR (95%)	Case (95%)
HFR Convertible Arbitrage	19.23%	140.81%	143.09%	I	36.28%	143.69%	150.72%	I
HFR Distressed	139.96%	174.15%	72.41%	I	118.18%	142.95%	17.85%	I
HFR Emerging Markets	-34.54%	-30.68%	50.44%	IV	-38.40%	-27.92%	21.91%	IV
HFR Equity Hedge	1.88%	-11.16%	9.26%	III	1.90%	-5.66%	0.81%	III
HFR Equity Market Neutral								
HFR Equity Non-Hedge	57.42%	37.79%	27.56%	I	52.36%	36.94%	16.74%	I
HFR Event-Driven	146.55%	165.42%	79.52%	I	146.58%	140.14%	47.75%	I
HFR Fixed Income Arbitrage	167.27%	197.22%	84.58%	I	157.03%	135.95%	23.59%	I
HFR Fund of Fund Index	-22.20%	-4.55%	44.90%	IV	-22.19%	-22.37%	-6.51%	IV
HFR Fund Weighted Composite	-5.94%	12.84%	56.82%	II	-0.39%	9.95%	17.72%	II
HFR Macro	-34.19%	-37.11%	-20.68%	IV	-28.20%	-25.28%	-22.27%	IV
HFR Market Timing								
HFR Merger Arbitrage	366.27%	374.79%	140.75%	I	331.80%	223.05%	32.30%	I
HFR Relative Value Arbitrage	262.79%	219.07%	117.51%	I	163.88%	45.66%	-35.88%	I
HFR Short Seller								
HFR Statistical Arbitrage	-22.00%	-5.37%	16.07%	IV	-6.39%	3.19%	2.92%	II
CTA Barclays								
JPM Global Bond	121.45%	90.32%	-5.49%	I	135.62%	81.77%	-3.95%	I

Source: Authors, QisDRAWDOWN.

Gaussian ARMA stochastic process.

Finally, results achieved in this study indicate that:

1. ***Whenever normality or time-independence assumptions do not hold, VaR does not capture all dimensions of market risk:*** There is an algebraic correspondence between VaR, drawdown, and time under the water when normality and time-independence assumptions hold, and the three can be analytically computed by using mean and variance. Whenever either one or both hypotheses are relaxed, there is no analytical solution nor algebraic correspondence between VaR, drawdown, and time under the water. In such a case, VaR is not able to fully capture all market risk dimensions, and its measure must be complemented with others like drawdown and time under the water.
2. ***Drawdown and VaR measures may notably differ for hedge fund investments:*** A relatively low VaR value can coexist with a four times larger drawdown figure. Thus, investors should be aware that VaR models, even when considering non-normality and time-independence of hedge fund returns for a high confidence level, may hide much higher likely losses than expected.
3. ***To apply higher confidence levels does not compensate for the error committed when the non-normality or time-dependence is neglected:*** Some practitioners apply higher confidence levels in an attempt to reduce the errors committed when normality or time-independence is assumed. This practice delivers inaccurate results when computing VaR as well as drawdown and time under the water figures.
4. ***Relaxing the normality hypothesis makes hedge funds riskier. Neglecting time-dependence of returns does not always underestimate hedge funds' risks:*** Assuming normality of hedge funds' returns generally leads to the underestimation of risks. However, assuming time-independence of hedge funds returns does not always underestimate risks, but whatever the direction of the estimation error, drawdown and time under the water are affected in the same way.

In a future study we will examine the loss potential of hedge funds in the context of a diversified portfolio and the precise impact of moments and co-moments on coherent risk figures. Readers interested in this line of research may find a mathematical derivation of a portfolio optimization algorithm dealing with non-normality and time-dependence of hedge funds' returns in López de Prado and Rodrigo [2004].

APPENDIX A

Analytical Derivation of Drawdown and Time Under the Water for Normally Distributed Returns

Time Under the Water for Normally Distributed Returns

Let's consider the following financial variables:

W_t = Investor's wealth at time point t.
r_P = Portfolio P's return.
τ = Investor's threshold return.
σ_P = P portfolio's volatility.
α = Significance level.
δ = Yearly withdrawal rate of funds. If $\delta < 0$, there is an inflow.
Z_α = Critical value for distribution N(0, 1) that verifies that $P(Z \le Z_\alpha) = \alpha$.

Assuming that $r_P \sim N(E(r_P), \sigma_P)$, we can compute for a confidence level $1 - \alpha$ the time necessary to achieve a return τ for portfolio P. Such a time interval is called *Time Under the Water (1 – α)%*, $TUW_{1-\alpha}$.

- Case 1: Let's suppose that $\delta = 0$. Thus, provided that $E(r_P) > \tau$ and $Z_\alpha < 0$:

$$W_t \cdot e^{\tau TUW_{1-\alpha}} = W_t \cdot e^{\left(E(r_P) + Z_\alpha \frac{\sigma_P}{\sqrt{TUW_{1-\alpha}}}\right) TUW_{1-\alpha}} \quad \text{(A-1)}$$

which can be easily solved for $TUW_{1-\alpha}$ as:

$$TUW_{1-\alpha} = \left(\frac{Z_\alpha \sigma_P}{E(r_P) - \tau}\right)^2 \quad \text{(A-2)}$$

- Case 2: When $\delta \neq 0$, there is no analytical solution for $TUW_{1-\alpha}$ However, when a solution exists, it is possible to compute a numerical one by applying Newton-Raphson's algorithm on the expression:

$$f(TUW_{1-\alpha}) = W_t \cdot e^{\tau \cdot TUW_{1-\alpha}} - W_t \cdot e^{\left((E(r_P) - \delta) + \frac{\delta}{TUW_{1-\alpha}} + Z_\alpha \frac{\sigma_P}{\sqrt{TUW_{1-\alpha}}}\right) \text{TY}\Omega_{1-\alpha}} \quad \text{(A-3)}$$

where $f(TUW_{1-\alpha})$ is the loss value with respect to the threshold and the solution comes from $TUW_{1-\alpha}$'s value which makes $f(TUW_{1-\alpha}) = 0$.

Drawdown for Normally Distributed Returns

With the same notation, assuming that $r_P \sim N(E(r_P), \sigma_P)$, we can compute for a confidence level $1 - \alpha$ a the maximum loss that Portfolio P may suffer. Such a loss is called *Drawdown (1 – α)%*, $DD_{1-\alpha}$, and can be expressed as:

$$DD_{1-\alpha} = \underset{dt}{\text{Max}}\left\{\left((\tau - E(r_P) + \delta) - \frac{\delta}{dt} - Z_\alpha \frac{\sigma_P}{\sqrt{dt}}\right)dt\right\}, \forall dt > 0 \quad \text{(A-4)}$$

The solution of this optimization problem for dt is:
1) Necessary condition of first order:

$$\tau - E(r_P) + \delta - \frac{1}{2} Z_\alpha \sigma_P dt^{-\frac{1}{2}} = 0 \Leftrightarrow dt = \left(\frac{Z_\alpha \sigma}{2(\tau - E(r_P) + \delta)}\right)^2 \quad \text{(A-5)}$$

2) Sufficient condition of second order:

$$\frac{1}{4} Z_\alpha \sigma_P dt^{-\frac{3}{2}} < 0 \Leftrightarrow Z_\alpha < 0 \quad \text{(A-6)}$$

and substituting dt in the maximized expression, it is a sufficient condition that $Z_\alpha < 0$ and $E(r_P) > \tau + \delta$ for deriving that

$$DD_{1-\alpha} = \frac{(Z_\alpha \sigma)^2}{4(E(r_P) - \delta - \tau)} \quad \text{(A-7)}$$

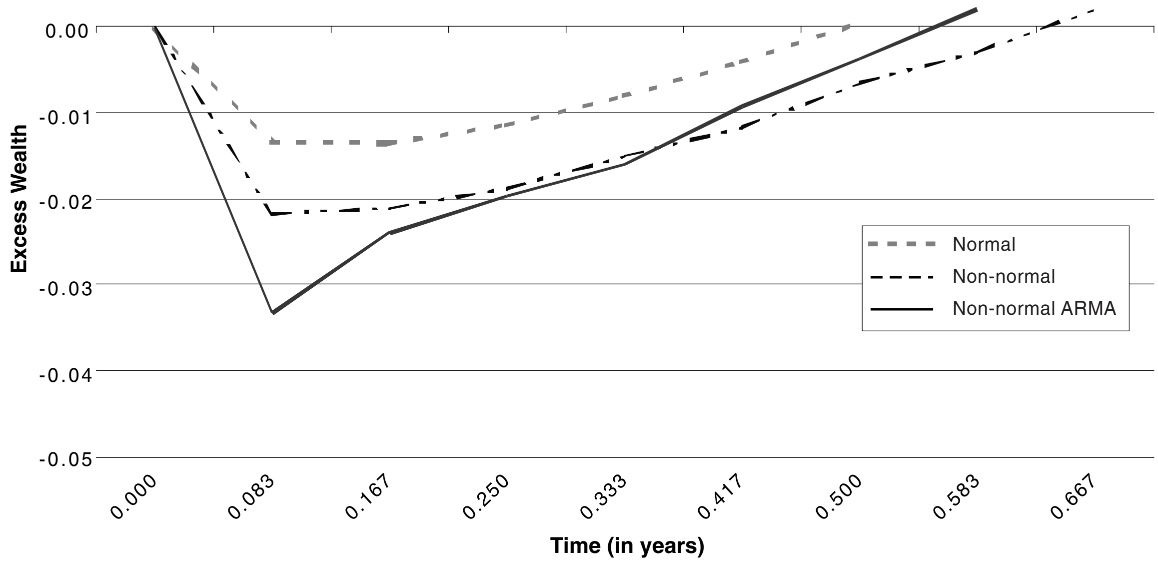
APPENDIX B

Simulation Results per Strategy

For each index, the following exhibits afford the comparison between results from the three alternative analytical frameworks:

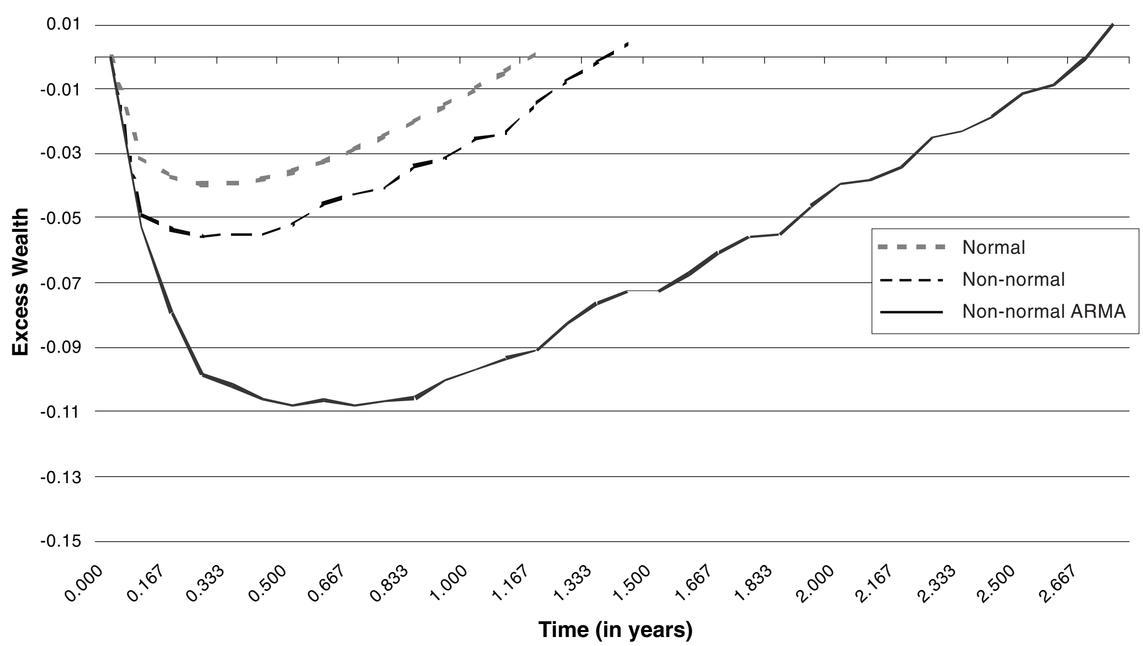
- Normal: As discussed in Section I.
- Non-Normal: As discussed in Section II.
- Non-Normal ARMA: As discussed in Section III.

EXHIBIT B-1
Drawdown and Time Under the Water (in years) for HFR Convertible Arbitrage



Source: Authors, QisDRAWDOWN.

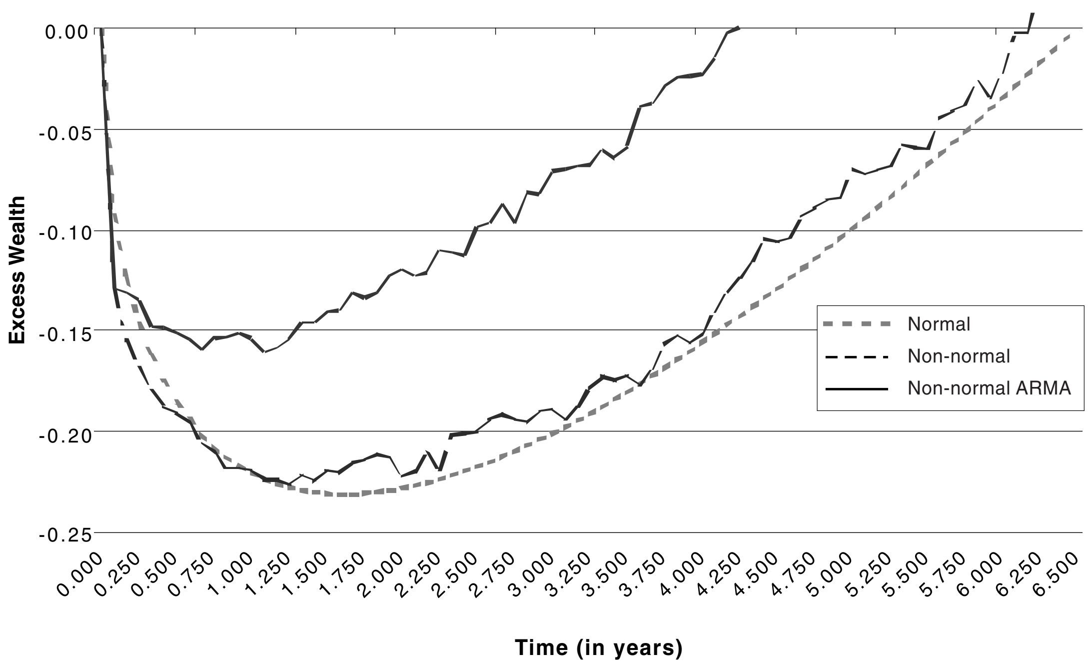
EXHIBIT B-2
Drawdown and Time Under the Water (in years) for HFR Distressed



Source: Authors, QisDRAWDOWN.

EXHIBIT B-3

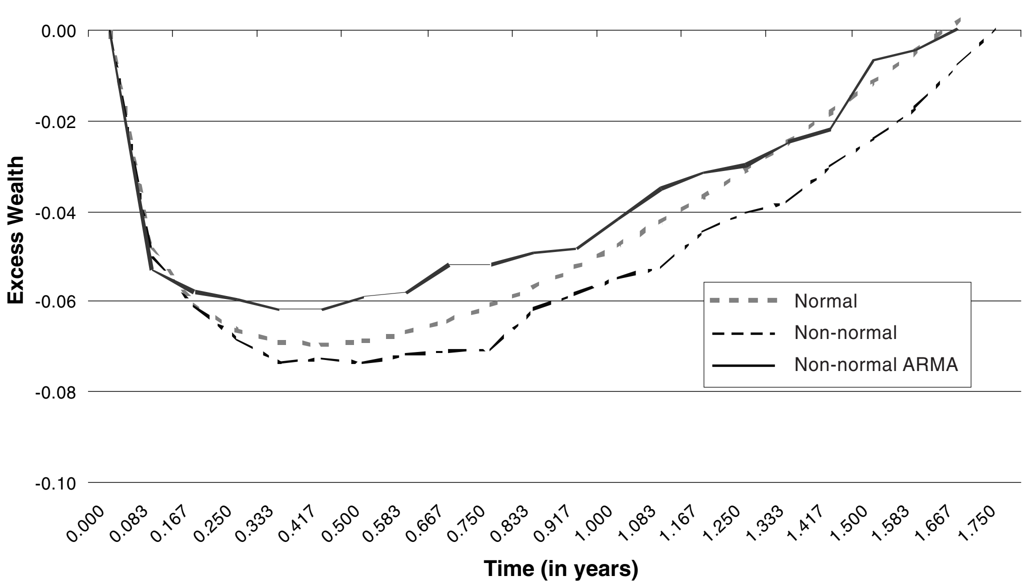
Drawdown and Time Under the Water (in years) for HFR Emerging Markets



Source: Authors, QisDRAWDOWN.

EXHIBIT B-4

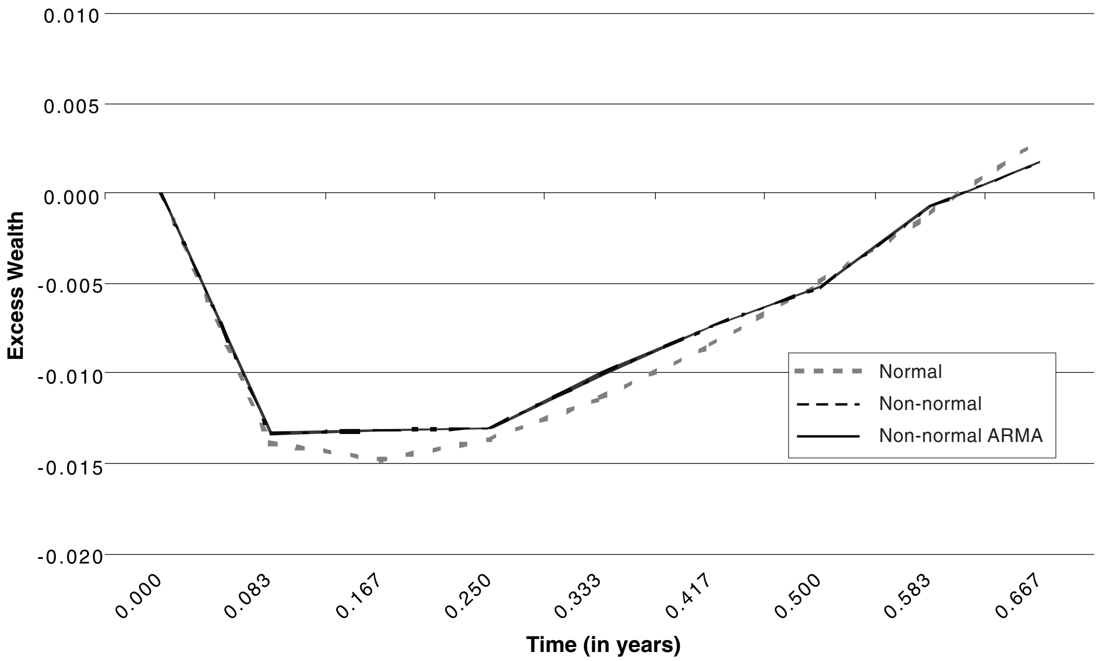
Drawdown and Time Under the Water (in years) for HFR Equity Hedge



Source: Authors, QisDRAWDOWN.

EXHIBIT B-5

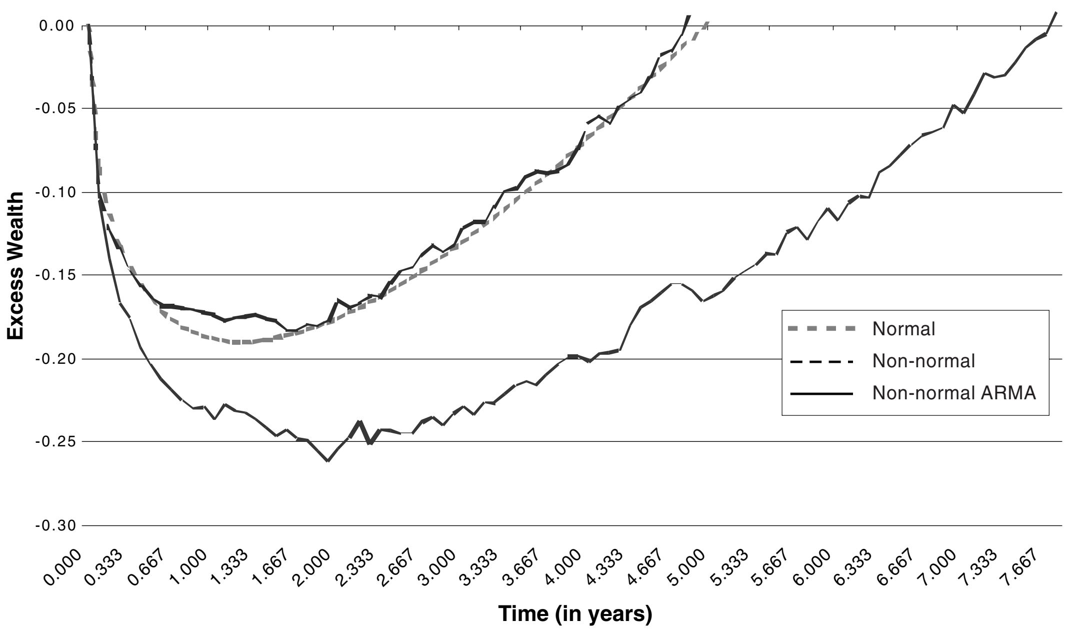
Drawdown and Time Under the Water (in years) for HFR Equity Market Neutral



Source: Authors, QisDRAWDOWN.

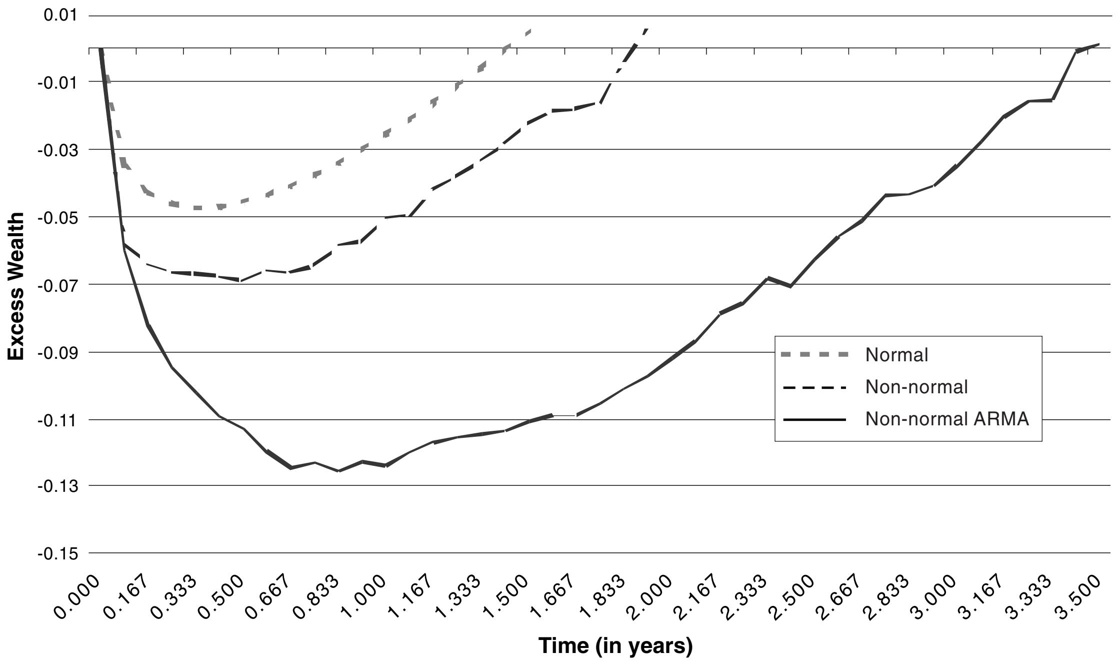
EXHIBIT B-6

Drawdown and Time Under the Water (in years) for HFR Equity Non-Hedge



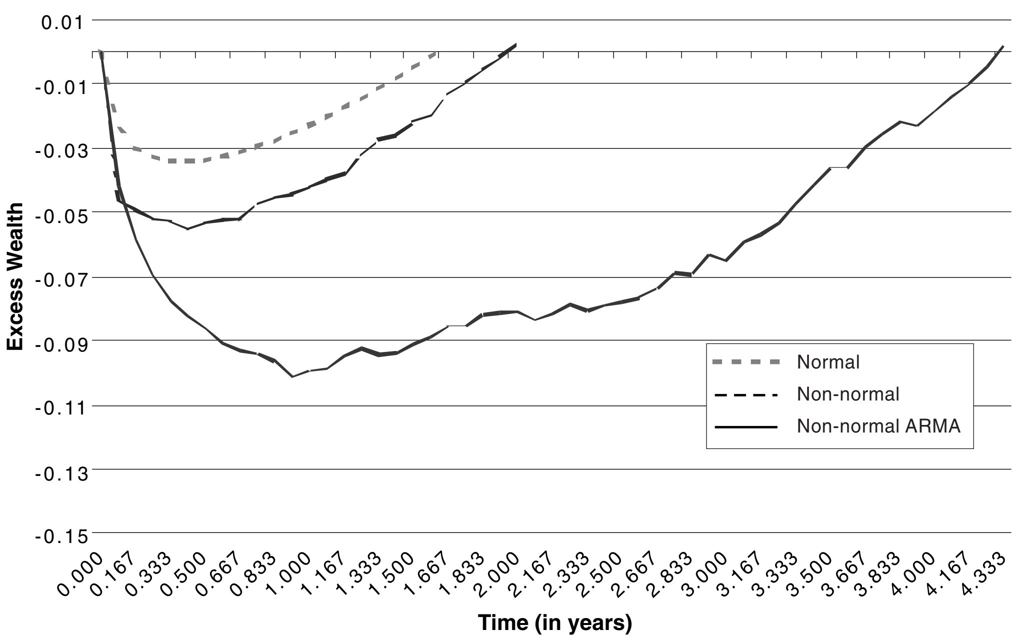
Source: Authors, QisDRAWDOWN.

EXHIBIT B-7
Drawdown and Time Under the Water (in years) for HFR Event-Driven



Source: Authors, QisDRAWDOWN.

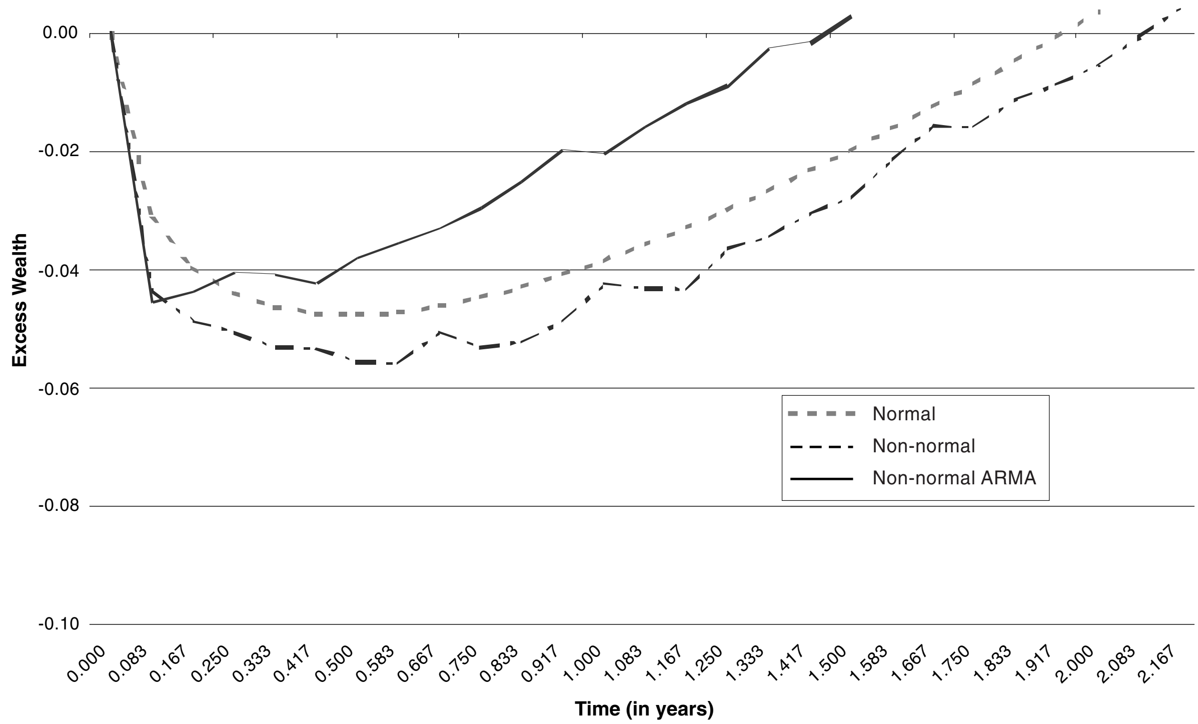
EXHIBIT B-8
Drawdown and Time Under the Water (in years) for HFR Fixed Income Arbitrage



Source: Authors, QisDRAWDOWN.

EXHIBIT B-9

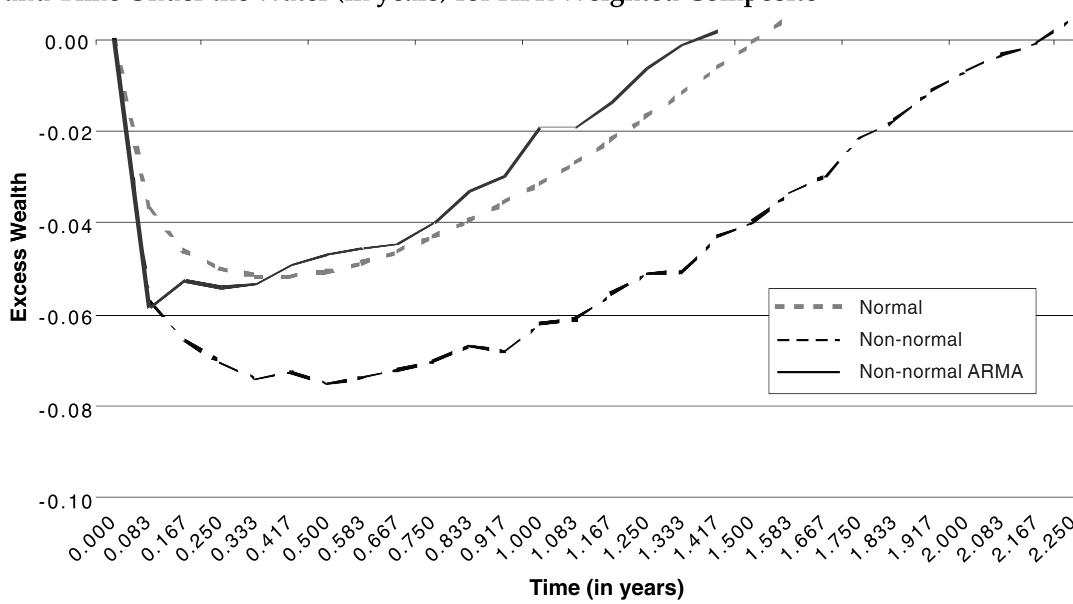
Drawdown and Time Under the Water (in years) for HFR Fund of Fund



Source: Authors, QisDRAWDOWN.

EXHIBIT B-10

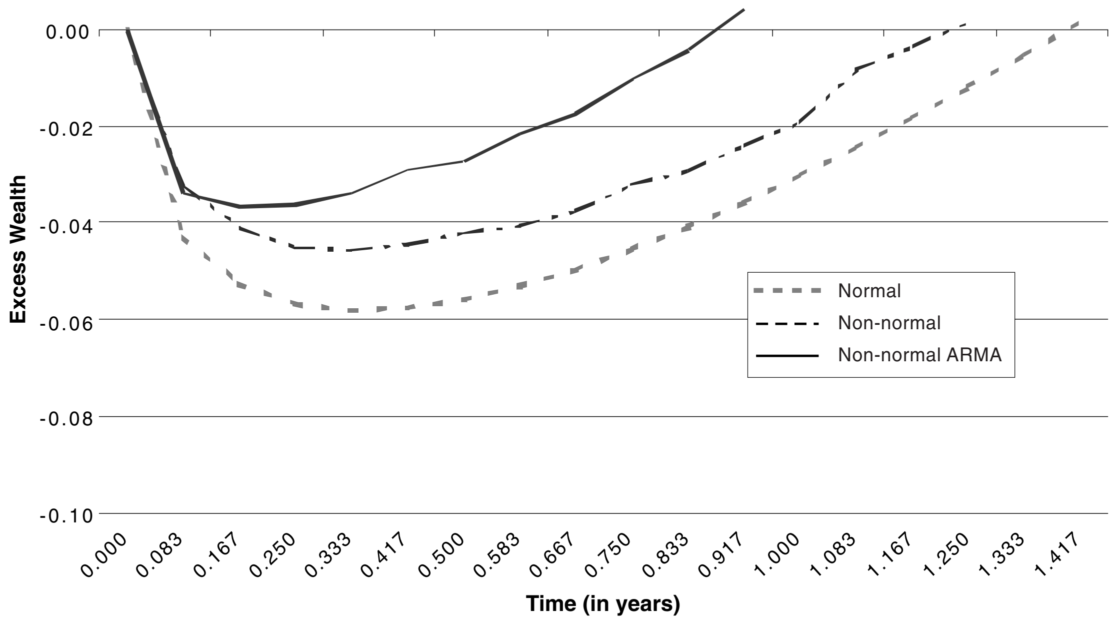
Drawdown and Time Under the Water (in years) for HFR Weighted Composite



Source: Authors, QisDRAWDOWN.

Exhibit B-11

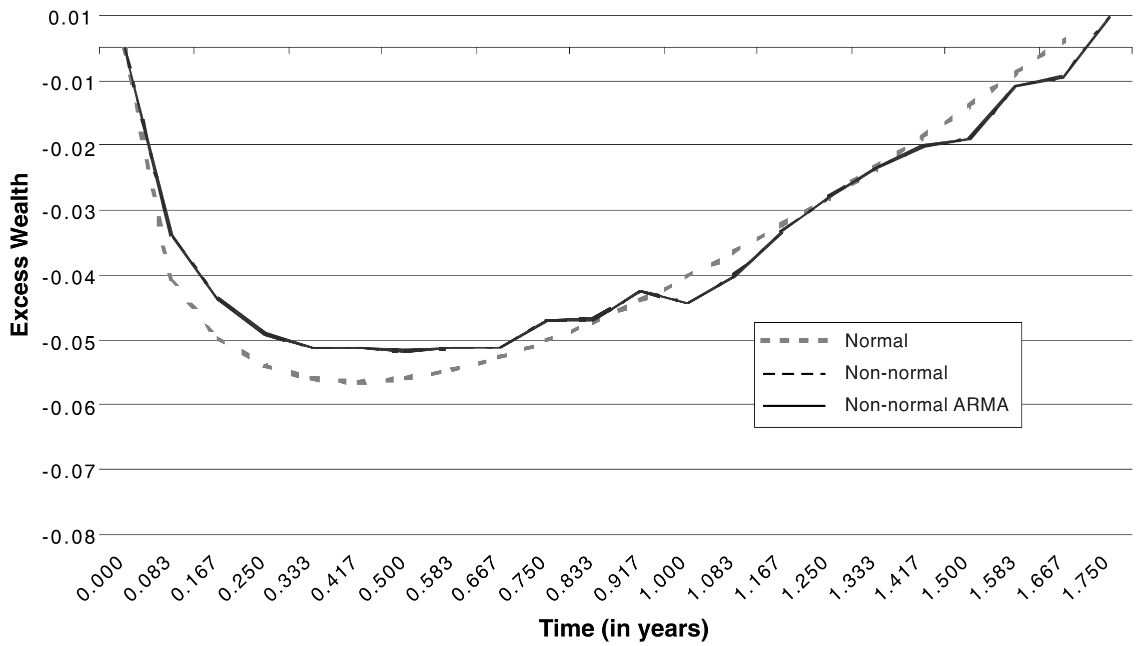
Drawdown and Time Under the Water (in years) for HFR Macro



Source: Authors, QisDRAWDOWN.

Exhibit B-12

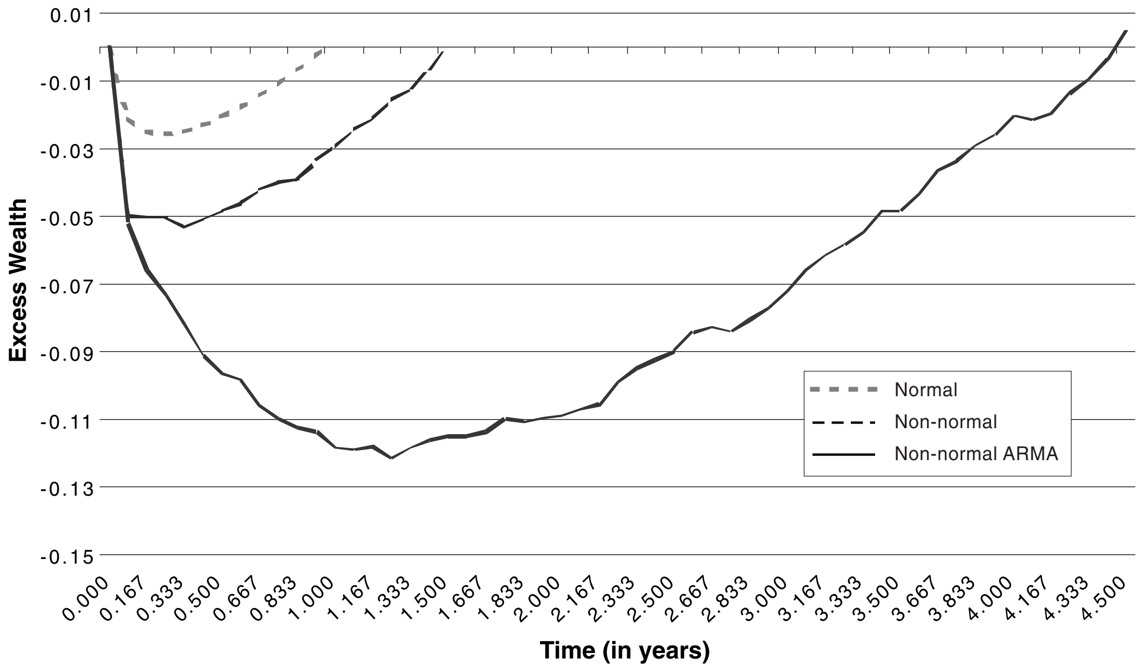
Drawdown and Time Under the Water (in years) for HFR Market Timing



Source: Authors, QisDRAWDOWN.

EXHIBIT B-13

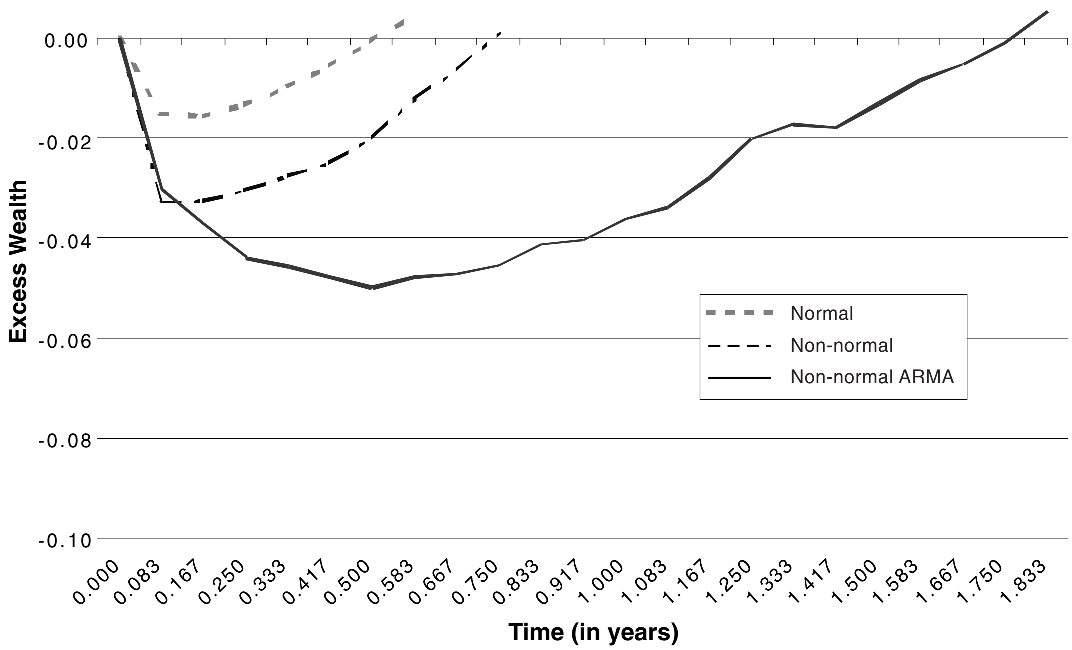
Drawdown and Time Under the Water (in years) for HFR Merger Arbitrage



Source: Authors, QisDRAWDOWN.

EXHIBIT B-14

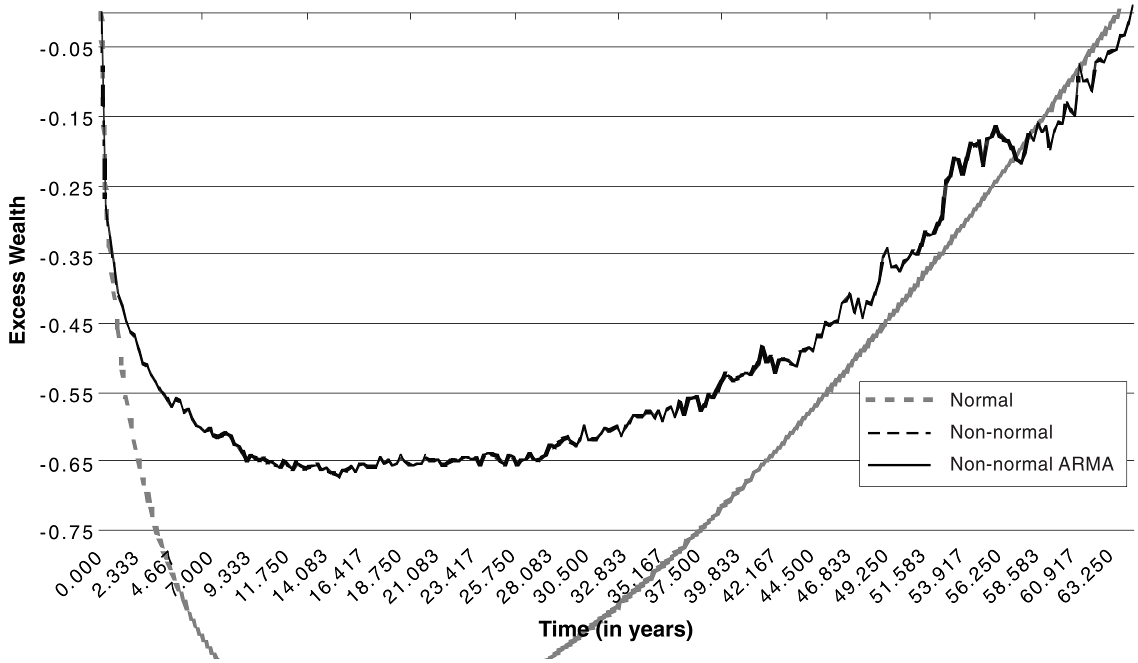
Drawdown and Time Under the Water (in years) for HFR Relative Value Arbitrage



Source: Authors, QisDRAWDOWN.

Exhibit B-15

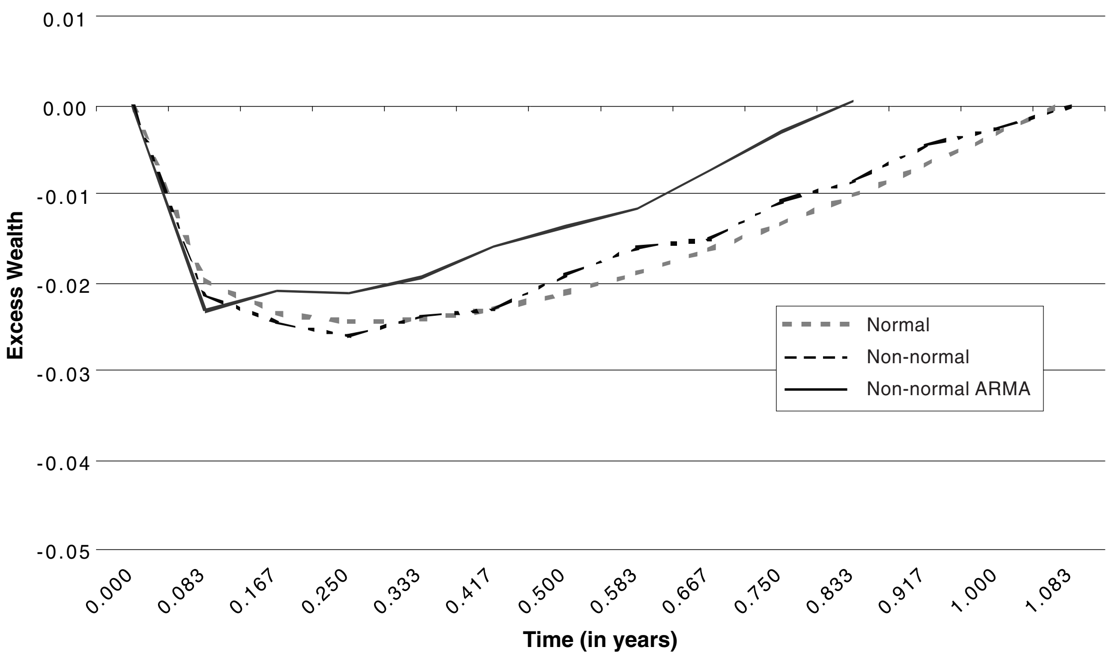
Drawdown and Time Under the Water (in years) for HFR Short Seller



Source: Authors, QisDRAWDOWN.

Exhibit B-16

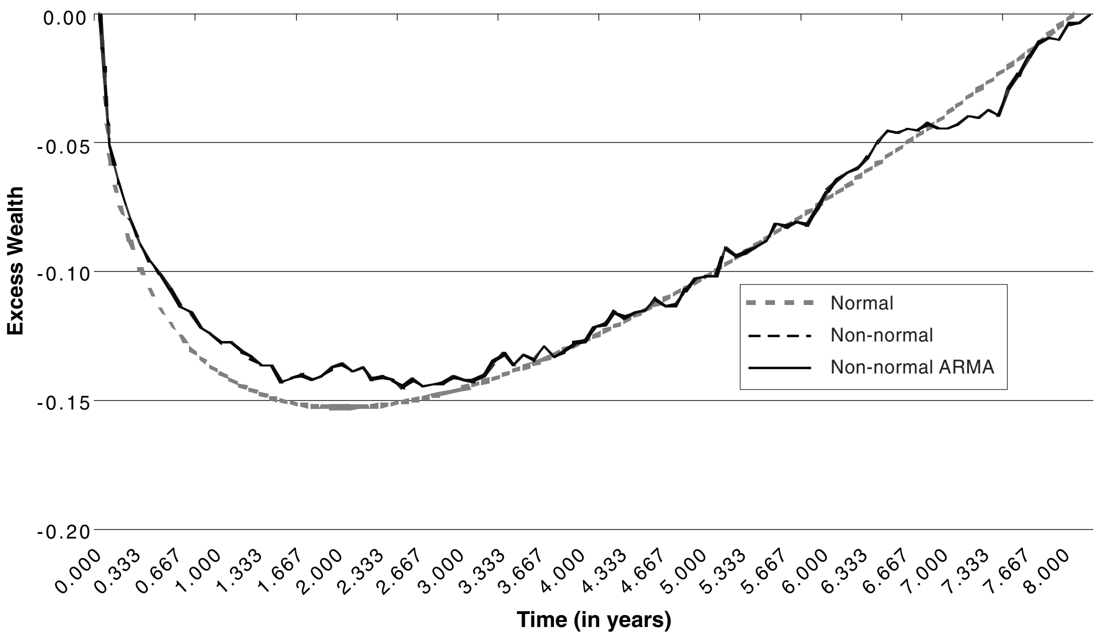
Drawdown and Time Under the Water (in years) for HFR Statistical Arbitrage



Source: Authors, QisDRAWDOWN.

Exhibit B-17

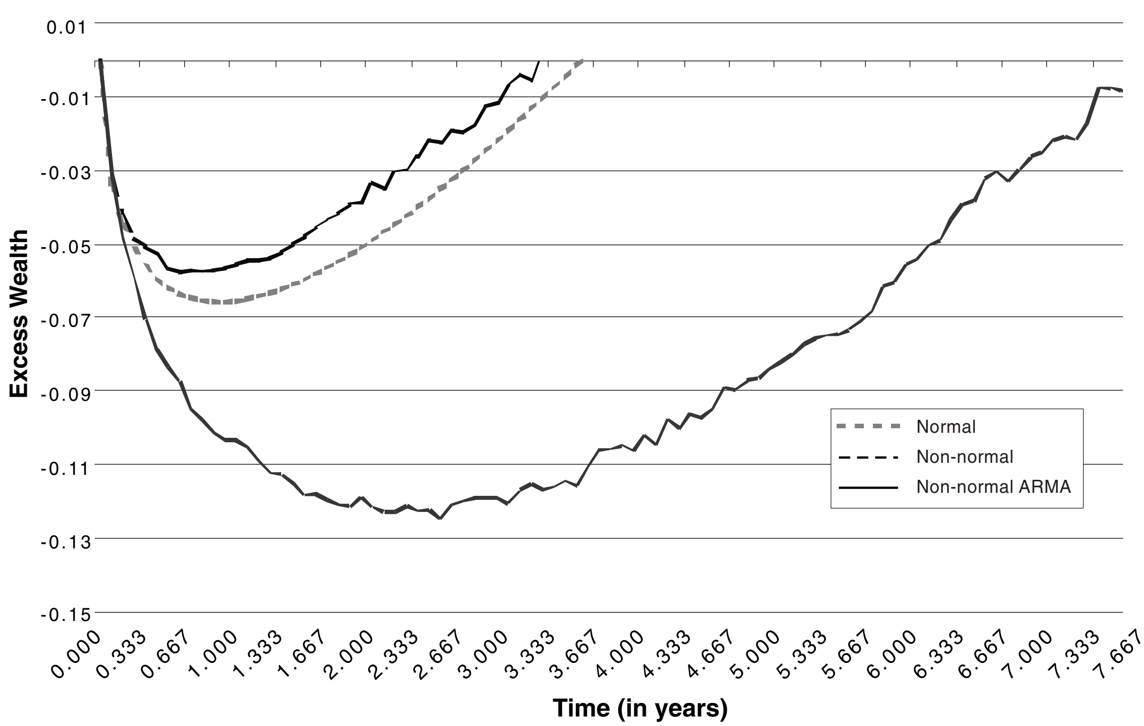
Drawdown and Time Under the Water (in years) for CTA Barclays



Source: Authors, QisDRAWDOWN.

Exhibit B-18

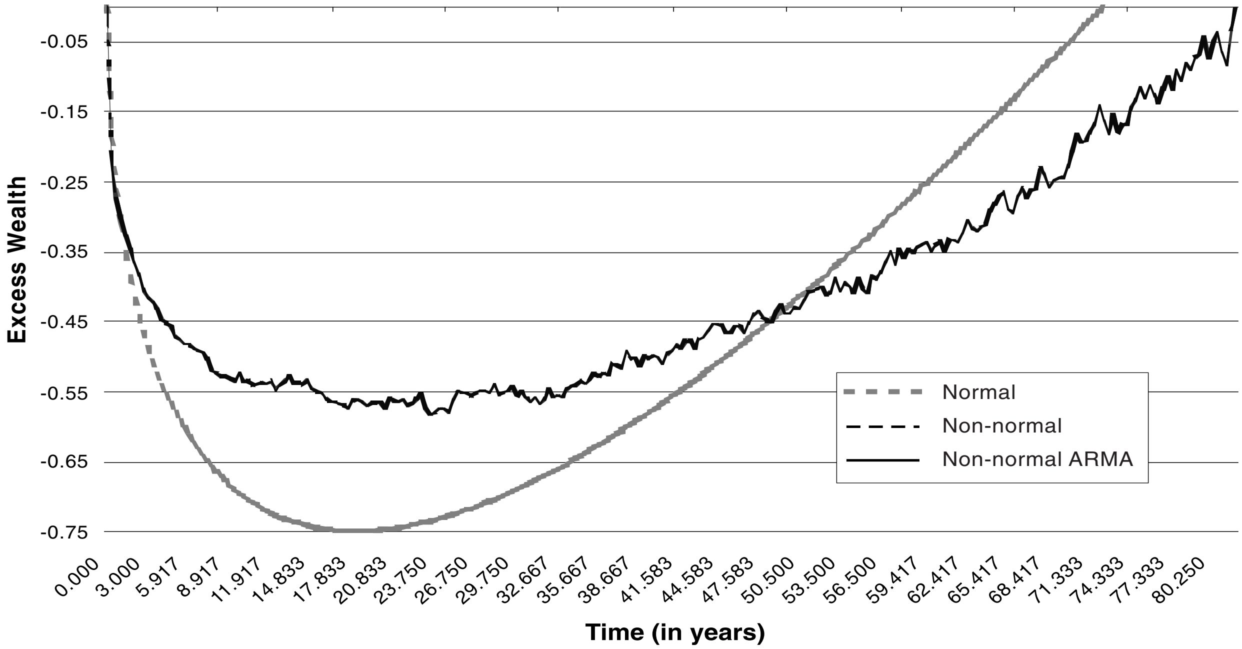
Drawdown and Time Under the Water (in years) for JPM Global Bond



Source: Authors, QisDRAWDOWN.

EXHIBIT B-19

Drawdown and Time Under the Water (in years) for MSCI Global Equity



Source: Authors, QisDRAWDOWN.

ENDNOTES

[1]We gratefully acknowledge the contributions of Heinz Müller (Universität St.-Gallen, Switzerland), Pascal Guillet (head of asset allocation services at UBS Wealth Management), and Laurent Favre (CEO, AlternativeSoft).

[2]However, note that the historical characteristics of the different assets used as input for the calculation might not reflect the future accurately.

[3]Cf. Alexander [2001] for numerous examples.

[4]An exception must be noted for the case that returns are modeled through mixtures of zero-mean normal densities, since $P(X < -c) = \Sigma p_i P(Z < -c/\sigma_i) = \alpha$. However, that's not appropriate for hedge funds.

[5]As can be easily proved by applying Îto's lemma on GBM expression.

[6]Cf., for instance, Jäckel [2002].

[7]It is possible to investigate different frequencies; however, we apply monthly iterations since this is the frequency of our data.

[8]Hamilton [1994], Section 8.2.

[9]Greene [2002], p. 245.

[10]For a strict proof of this result, cf. Hamilton [1994], pp. 685-689.

[11]Brooks and Kat [2002].

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