Business cycle effects on Portfolio Credit Risk: scenario generation through Dynamic Factor analysis

Andrea Cipollini[a] and Giuseppe Missaglia[b]

**[a]Corresponding author: Queen Mary, University of London, Mile End Road, London E1 4NS.
Phone: +44 20 78825546. E-mail: a.cipollini@qmul.ac.uk**
[b]ICCREA Holding, Rome

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Abstract

In this paper, we focus on measuring the risk associated to a bank loan portfolio. In particular, we depart from the standard one factor model representation of portfolio credit risk. In particular, we consider an hetrogeneous portfolio, and we account for stochastic dependent recoveries. We also examine the influence of either one systemic shock (interpreted as the state of the business cycle) or two systemic shocks (interpreted as demand and supply innovations) on portfolio credit risk. The identification and estimation of the common shocks is obtained by fitting a Dynamic Factor model to a large number of macro credit drivers. The scenarios are obtained by employing Montecarlo stochastic simulation.

Keywords: Risk management, default correlation, Dynamic Factor
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The views in this paper are those of the authors. The usual disclaimer applies: all remaining errors are the sole responsibility of the authors. All the computations have been performed by using Gauss.

Introduction

The proposed new Bank of International Settlement accord (known as Basel 2) provides for greater sensitivity of capital requirements to the credit risk inherent in bank loan portfolios. In light of the Basel 2 accord to reform the regulation of bank capital, there has been an extensive research on credit risk. The latter can be considered as a dominant component of risk for banks. The risk of an individual bank can be measured as the dispersion of future losses to its own portfolio driven by the obligors default. However, the focus of risk measurement is not on the standard deviation of the portfolio loss, but, given an highly asymmetric portfolio loss distribution, the emphasis is on the measurement of the Value at Risk (VaR). This is the minimum loss that a portfolio of credit exposures could suffer one out of one thousand years (if we choose the 99.9% percent rule and a year as the forecast horizon).

A crucial input of a portfolio credit risk model, PCR, is the appropriate characterisation of default correlations to obtain the bank loan portfolio distribution with the relevant percentile (e.g. the minimum capital requirement). Recent research suggests that the probability over upgrading, downgrading the credit quality of a borrower, vary with the business cycle. These are, for instance, the empirical findings, based upon transition matrices calculated using external ratings from Moody's and Standard and Poor's, of Nickell et al. (2000). Similar findings are in Bangia et al. (2000) who concentrate on the ratings of corporate borrowers and in Haldane et al. (2001) who focus on sovereign borrowers. Furthermore, the study of Jordan et al. (2002) and of Cateraineu-Rabell (2002), use transition matrices computed according to either Moody's data or to KMW style ratings. Their findings suggest swings, across the business cycle, in the minimum capital requirements (for a portfolio of 339 loans in a shared national credit program in the United States, the former study, and for a selection of banks in G10 countries, the latter study). Other studies, based upon time series data on internal ratings suggests similar conclusions. In particular, the study of Carling et al. (2001), find a substantial fall improvement in the internal ratings ver the 1994-2000 period, and consequently, a fall in the capital charge of a large Swedish bank. This was found to be associated with the gradual improvement of the Swedish economy after the financial problems of the early 1990s. Segoviano and Lowe (2002), having access to time series data on the ratings assigned by a number of Mexican banks to business borrowers, find large swings in required capital. Finally, the study of Carpenter et al. (2001) conclude that in the Untied States there is very little cyclical impact on capital charges; on the other hand, Ervin and Wilde (2001) find large swings in the minimum capital requirements.

Also, the role of uncertain recoveries is important for the determination of Credit Risk VaR. The empirical study of Hu and Perraudin (2002) shows a negative correlation between probability of default and recovery rate. This finding can, for instance, be explained by observing that both default and recovery are found to depend on the state of the macro-economy (see the work by Gupton et al., 2000 and by Frye, 2000b).

In line with the aforementioned empirical findings, portfolio credit risk models account for the influence of the state of the business cycle on credit risk. The study of Shonbucher (2000), based upon the assumption of homogeneous portfolio, constant recovery and one common shock influencing the systemic component of firm asset values, has provided an analityc solution for the limiting portfolio distribution. However, given the heterogenous nature of the portfolio under examination in this paper and the empirical evidence of stochastic dependent recoveries (provided by the aforementioned studes), we use stochastic simulation to quantify the risk associate to a bank loan portfolio. For this purpose, we follow the method put forward by Krenin et al. (1998) by generating scenarios through stochastic simulation to determine conditional default probability and conditional portfolio loss distribuion. In order to account for default correlation, we average out across all scenarios and we obtain the unconditional portfolio loss distribution. We concentrate only on a “default mode” model, that is the model measures credit lossess arising exclusively from the event of default. Given the heterogeneous feature of the portfolio under examination, we implement Montecarlo simulation (which is standard in the propietary models of Portfolio Credit Risk analysis). The novel aspect of the paper are described as follows. First, the macro scenarios are associated to common shocks identified (as aggregate demand and supply) and estimated by fitting a Dynamic Factor model to a large number of credit crivers. Secondly, we account for the impact of stochastic recoveries dependent on defaults.

The outline of the paper is as follows. Section 2 describes the basic definitions underlying the credit portfolio loss distribution. Section 3 describes the analytic solution to retrieve the unconditional loss distribution. Section 4 and 5 describe the stochastic simulation exercise and the Dynamic Factor modelling approach, respectively. Section 6 describes the empirical results. Section 7 concludes.

2. Credit Portfolio Loss Distribution

The credit portfolio loss L is given by:

$$L = \sum_{j=1}^{N} (D_j * L_j) \tag{1}$$

where N is the number of counterparts, D_j is a default indicator for obligor j (e.g. it takes value 1 if firm j defaults, 0 otherwise). Furthermore, the loss from counterpart j is given by:

$$L_j = \sum_{h=1}^{H} EAD_{hj} * LGD_{hj} \tag{2}$$

where EAD_{hj} is the exposure at default to the h business unit of obligor j. Finally, LGD_{hj} is the corresponding loss given default (equal to one minus the recovery rate, see below).

Since L is a random variable, it is crucial to retrieve its probability distribution to measure portfolio credit risk. For this purpose, from (1) and (2) we can observe that we need to consider as a random variable, at least one from D_j, EAD_{hj}, and LGD_{hj}. In this paper, we concentrate on the stochastic nature of defaults and loss given defaults, treating the exposures as deterministic If the portfolio loss is uncertain in the future, then we can concentrate on few moments of the portfolio loss distribution. First, it can be relevant the measurement of the expected loss (e.g. the sample mean of the overall distribution). However, as in standard portfolio risk analysis, the standard deviation of the total portfolio loss is used to measure risk associated to the bank loan portfolio. However, given highly asymmetric credit portfolio loss distribution, it is customary to measure risk as the difference between the percentile at the target solvency probability (99.9% as suggested by Basel 2) and the expected loss. This is the unexpected loss (economic capital). If the forecast horizon is a year, then the unexpected loss predicts the minimum loss (above the expected one) that can occur in one out one thousand years. Finally, if such an extreme event occurs, the loss is predicted by the expected shortfall, computed as the mean of the distribution values beyond the 99.9% percentile.

3. Stochastic PD's and Credit Portfolio Risk analysis

In this section we treat only defaults as stochastic random variables and we model them being dependent on the state of the business cycle. For this purpose, it is customary, in Portfolio Credit Risk analysis, to implement a factor model specification for asset returns. In particular, the dynamics of the level of firm *j*'s asset value index is given by:

$$A_j = b_j Y + \sqrt{1 - b_j} n_j \qquad (3)$$

where Y is a systematic risk shock affecting simultaneously every firm (parodying the state of the macro-economy) and $?_j$ is an idiosyncratic (firm specific) risk shock. The two shocks in (3) are assumed to have independent standard normal distributions, implying that A_j has a standard normal distribution. The parameter $ß_j$ measures the effects of the common shock on the obligor j.

According to Merton (1974), a firm defaults when its asset value index falls below a threshold c_j. Specifically, define A_j as the creditworthiness of obligor j. Let D_j symbolise the default event of firm j, then we can observe that:

if $A_j < c_j$, then $D_j = 1$; $D_j = 0$ otherwise.

The default boundaries c_j are pre-specified and obtained from the (unconditional) probabilities of default PD_j, given by:

$$PD_j = P(A_j < c_j) = F(c_j) \qquad (4)$$

where F is the cumulative standard normal probability distribution. From eq. (4) it is possible to retrieve the level of threshold c_j, which is given by $F^{-1}(PD_j)$.

The main ingredients of a factor model for Portfolio Credit Risk analysis are the individual unconditional PD's (here obtained from the internal rating system of the bank) together with a measure of the asset correlation (measured by the cross product of the factor loadings in eq. (3)). These two inputs are all we need to measure default correlation. Intuitively, negative realisation of the common shock can lead the asset firm values of different obligors to fall below their corresponding threshold values and then lead these obligors into default.

3.1 Analytic solution for the Credit Portfolio Loss

The Basel II proposal (as of January 2001) for the determination of economic capital is based upon the Schonbucher (2000) analytic solution for the unconditional portfolio loss distribution. For this purpose, the author (op. cit.) implements the factor model specification described above. The starting point is the estimation of the probability of default conditional on a specific realisation of the common shock. Under the assumption of homogenous portfolio[1] (e.g., same exposure, probability of default, loading factor across obligors) and deterministic loss given default, the probability of default for firm *j* conditional on a realisation *y* of the common shock *Y* is given by:

$$\begin{aligned} p(y) &= p\{[A_{jt} < \Phi^{-1}(PD)] \mid Y = y\} = \\ &= P[\sqrt{b}\,y + \sqrt{1-b}n_j < \Phi^{-1}(PD)] \\ &= P\left(v_j < \frac{\Phi^{-1}(PD) - \sqrt{b}\,y}{\sqrt{1-b}} \right) = \Phi\left(\frac{\Phi^{-1}(PD) - \sqrt{b}\,y}{\sqrt{1-b}} \right) \end{aligned} \qquad (5)$$

Therefore, conditioning on a specific realisation of the common shock, we obtain independent defaults across obligors, and, as a consequence, the conditional probability of having exactly *n* defaults is:

$$p\{X = n \mid Y = y\} = \binom{N}{n} \{p(y)^{n} (1 - p(y))^{N-n}\}$$

Furthermore, averaging out across the different realisations of the common shock, the unconditional cumulative distribution is:

$$p(X \le n) = \sum_{m=0}^{n} \int_{-\infty}^{\infty} p(X = n \mid Y = y)\, f(y)dy \qquad (6)$$

[1] More recently, Wehrspohn (2003) provides analytic closed form solution of the limiting distribution and of the credit portfolio loss, under the classical one factor model representation, relaxing the assumption of homogeneous portfolio.

Combining the assumption of homogeneous portfolio with the assumption of an infinitely granular portfolio (where each exposure is set to be equal to *1/N*, with large N), Schonbucher (2000) derives the analytical solution for the unconditional cumulative loss distribution:

$$p\left(\frac{X}{N} \leq x\right) = \Phi\left[\frac{1}{\sqrt{b}}\left(\sqrt{1-b}\,\Phi^{-1}(x) - \Phi^{-1}(p)\right)\right] \tag{7}$$

In particular, the Basel 2 computation for the unexpected loss is based upon considering the 99.9th percentile of the distribution in (7) and by fixing the loading factor *ß = 0.2*.

3.2 Stochastic recovery

Recently, few studies, have taken into account the stochastic feature of the recovery rate as well as defaults. In particular, the dependence between the default events and losses given default is introduced through a single factor that drives both default events and recovery rates. The recovery rate is then modelled by specifying the collateral value distribution (for instance, Frye, 2000a uses a Gaussian collateral value, whereas Pykhtin, 2003, focuses on a log normal distribution). These studies provide a macro type of explanation of an inverse relationship between PD's and recoveries documented, for instance, in Hu and Perraudin (2002). In particular, given a negative cyclical downturn, collateral values as well as asset firm values would fall, and, as a consequence, there would be an increase in the number of defaults and a decrease in the number of recoveries (given their dependence on the collateral). The empirical studies by Altman and Brady (2002), and also by Altman, et al. (2003) find that not only the state of the business cycle, but also contract–specific factors, such as, seniority and collateral, seem to affect recovery rates.

To our knowledge, at the industry level, the computation of Credit Risk Portfolio VaR does not fully account for the stochastic dependence of recoveries from default. Proprietary models employed in Credit Portfolio Risk analysis treat the recovery rate either as deterministic or as stochastic (modelled through a beta distribution), but independent from the probability of default. In line with the study of Altman et al. (2002), we model stochastic dependent recoveries, by imposing a perfect rank correlation between the LGD and the default rate associated with the common shock scenarios. In particular, we sort (in descending order) the number of defaults for each common shock scenario, we associate the corresponding percentiles of *rr* obtained from inverting the beta distribution corresponding to the recoveries sorted in ascending order[2]. For example, when the common shock scenarios produce the largest number of defaults, the recovery rate takes the smallest value. On the other hand, when the common shock scenarios produce the smallest number of defaults, the recovery rate takes the largest value.

4. Stochastic simulation

In this paper, in line with equation (3), we assume that both the common and the idiosyncratic innovations are standard Gaussian. However, we consider an heterogeneous portfolio, and we also treat recoveries as stochastic and dependent on default events. Finally, we also let two common shocks affect the systemic components of the creditworthiness indices. Therefore, we cannot use the analytic solution for the unconditional portfolio loss distribution given in (6) and we need to implement Montecarlo simulation for the generation of the asset returns according to the factor model specification given in (3). Comparing the simulated asset returns with pre-specified thresholds (given the availability of data regarding the one-year unconditional PD's, as explained above) we are able to detect whether, conditioning on a specific macro scenario and a specific realisation of the idiosyncratic shock, an obligor defaults. The common shocks driving the systemic component in (3) are estimated and identified by fitting a Dynamic Factor model, DF (see Stock and Watson, 2002) to a large dataset of macroeconomic variables: the credit drivers. In particular (see below) the identified common shocks are orthogonal to each other and, contrary to the Credit Portfolio View approach developed by Wilson (1997) they allow for interdependencies among the credit drivers[3]. Recently, Pesaran et al. (2004) have suggested a Vector Autoregressive model specification (VAR) in order to allow interdependencies among a relative large dataset of macroeconomic credit drivers, in their Credit Portfolio Risk modelling approach. Our choice of fitting a Dynamic Factor model rather than a VAR to the credit drivers dataset can be explained as follows. First, the exogeneity assumptions used by Pesaran et al. (2004) to handle a relative large number of macro-variables characterising cannot be applied to our dataset (given that most of the time series we consider are specific to only one country: Italy. Second, as shown in Giannone et al. (2003), the estimated impulse response profile of macro-aggregates obtained from a Dynamic Factor model gives a better approximation (in the short-run) than VAR of the impulse response profile corresponding to the reduced form of an equilibrium business cycle models. This occurs when the empirically observed data are contaminated by measurement error. Giannone et al. (2003)

[2]The shape of the beta distribution depends on the parameters a and b, linked to m and σ, which are the sample mean and std. deviation of the recovery rate, respectively as follows: $b=\{[m*(m-1)^2]/s^2+m-1\};\ a=(b*m)/(m-1)$.

explain these findings, acknowledging that the rank reduction feature of the system of endogenous variables is preserved by a DF model, and observing that the extraction of the factors is obtained by minimising the noise (which captures the measurement error) to signal ratio. Finally, the simulation experiment suggested by Pesaran et al. (2004) is based upon the joint draw of shocks to credit drivers and specific to each firm (e.g. idiosyncratic innovations). This simulation procedure can be implemented only when the number of obligors is relatively small. However, in this paper, we deal with a large portfolio of obligors (see section 6.1), and we follow the suggestion of Krenin et al (1998) regarding the generation of different scenarios (see section 5.2).

5. Dynamic Factor model

As anticipated in section 4, the identification and estimation of common shocks is obtained by fitting a Dynamic Factor model to x_{nt}, which is the n dimensional dataset of credit drivers (see Stock and Watson, 2002):

$$x_{nt} = Cf_t + x_t \qquad (7)$$

the first addend of the r.h.s. of (7) is the common component for each credit driver given by the product of the r dimensional vector of static factors f_t and the $n \times r$ coefficient matrix of factor loadings. The factor dynamics is modelled as follows (see Forni et al, 2003):

$$f_t = Bf_{t-1} + Ru_t \qquad (8)$$

where R measures the impact multiplier effect of the q dimensional vector of common shocks u_t on f_t.

5.1 Estimation and identification

The static factor space can be consistently estimated by either the generalised principal component estimator proposed by Forni et al. (2000) or the principal component estimator proposed by Stock and Watson (2002)[4]. In this paper we use the procedure proposed by Stock and Watson which (in

[3] The credit drivers are modelled independently from each other and they are assumed to follow an AR(2) process.
[4] More recently, Kapetanios and Marcellino (2003) have proposed an alternative method, based on a state space model to estimate a large dimensional Dynamic Factor model.

case of a cross section dimension exceeding the time series dimension) gives a consistent estimator of the static factors f_t is given by :

$$f_t = \sqrt{T} W_n \tag{9}$$

where W_n is the $n \times r$ matrix having on the columns the eigenvectors corresponding to the first r largest eigenvalues of the covariance matrix of x_{nt}. In the second stage of the analysis, according to (7), we estimate, by OLS, a VAR(1) on the static factors f_t:

$$f_t = Bf_{t-1} + e_t \tag{10}$$

The structural form impact multiplier matrix $R = KMH$, where:

1) M is the diagonal matrix having on the diagonal the square roots of the q largest eigenvalues of covariance matrix of the residuals e_t.
2) K is the $r \times q$ matrix whose columns are the eigenvectors corresponding to the q largest eigenvalues of covariance matrix of the residuals e_t.
3) H is a $q \times q$ rotation matrix, which (in case of q = 2) is given by:

$$H = \begin{bmatrix} \cos q & -\sin q \\ \sin q & \cos q \end{bmatrix}$$

The identification of the common shocks u_t in (8) is achieved by finding the rotation of the angle q in H which complies with sign restrictions on the impulse response profile of the credit drivers x_{nt}:

$$C(I - BL)R \tag{11}$$

In eq. (11) a consistent estimate (for $n > T$) of the reduced form factor loading matrix, C is obtained by regressing x_{nt} on f_t (see Forni et al., 2003). The sign restrictions used to identify R (and the common shocks u) are along the lines of Uhlig **(**2004). In particular, we select all the rotations of the angle ? that imply, over the 12 months forecast horizon of the impulse response profile, a negative impact of the first shock on the real industrial production index, IP, and a positive impact on the aggregate consumer price, CPI, index. Among the selected rotations, we pick the one delivering the lowest impact, in the first three months of the impulse response forecast horizon (in order to allow a delayed effect from the shocks), on the aforementioned series. This particular rotation would then identify a supply shock. If, for this particular rotation, the impulse response profile corresponding to the second shock implies a positive co-movement between CPI and IP series, then the second shock is identified as demand-side structural form innovation.

5.2 Simulation of the credit worthiness index

Given that the credit drivers used in this paper are observed at monthly frequency and the forecast horizon is a year, we need to project the static factors *12* step ahead. Since $e_t = KMHu_t$ we can derive the h-step ahead projection of the static factors (with $h = 12$) by rolling forward the VAR(1) in (10):

$$f_{t+h} = \left[D^h f_t + D^{h-1} KMHu_{t+h-1} + ... + KMHu_{t+h} \right] \quad (13)$$

Once we obtain an OLS estimate of the $r \times 1$ vector of sensitivities coefficients $ß_j$, by regressing the stock returns obligor j on the r estimated static factors, we are able to project the systemic component of the creditworthiness indices:

$$A_{j,t+h} = b_j f_{t+h} \quad (14)$$

We can observe from (13) and (14) that in line with multifactor models for asset returns (such as Arbitrage Pricing Theory, *APT*, see Ross, 1976) the systemic component (driven by the common shocks) can be split in two parts. The first, described the first addend in the r.h.s of eq. (13), is the predictable component, which is a function of current and past values of the common shocks. These values describe the information set available at time t when the rolling forecasts are produced. The remaining addends in (13) capture the unanticipated systemic component, given that they are a function only of future common innovations.

The unpredictability of the A_j is further enhanced by allowing an idiosyncratic (firm specific) disturbance to affect the asset returns. Consequently, the h step ahead projection of the firm j asset return is given by:

$$A_{j,t+h} = b_j f_{t+h} + n_j \quad (15)$$

where n_j is the idiosyncratic (firm specific) innovation.

In the empirical analysis described below we considered the following different cases. The first involves the simulation of asset returns through eq. (15), through a systemic component driven only by one common shock (interpreted as the state of the business cycle). This is when the dimension of the vector of common shocks, u_t, is unity. In the second and third case we fix the dimension of u_t to two. More specifically, we analyse, in the second case, the marginal contribution of a supply shock, by considering the first column of the structural form dynamic multipliers in (13). Finally, in the third case, we examine the marginal contribution of a demand innovation, by considering the second column of the structural form dynamic multipliers in (13).

We follow Krenin (1998) suggestions on how to deal with the replications in the simulation experiment. More specifically, we carried out 1000 simulations for each scenario, and conditional on each scenario we carried out 1000 simulations for the idiosyncratic component of each obligor creditworthiness index. This gives one million observations and by sorting them in ascending order we are able to obtain the unconditional portfolio loss distribution. Finally, the credit risk measures of interest are computed as follows: the expected loss is obtained by taking the mean value of the simulated unconditional loss distribution. The unexpected loss is computed by subtracting the expected loss from the 99.9th percentile of the simulated distribution. Finally, the expected shortfall is computed by measuring the mean value of the area falling to the right of the 99.9th percentile of the simulated distribution.

6. Empirical analysis

6.1 Data

We consider a corporate portfolio, describing the exposures of an Italian bank towards corporate small and medium sized enterprises, SME. Specifically, in this portfolio, there are 270.000 claims which according to the different type of instruments (such as receivables, trade credit loans, and financial letters of credit) are associated with 150.000 counterparts, which gives 53 billions Euro regarding the committed amount and 31 billions Euro regarding the drawn amount. The obligors with marginal exposure have been grouped in homogenous clusters in terms of rating and economic sector. This allows to consider a portfolio with 9912 obligors (with cluster and non-clusters) which gives a total exposure of 44 billions of Euro. To summarise, we consider an heterogeneous portfolio consisting of 9912 sub-portfolios, with obligors treated identically within each sub-portfolio, but with probability of default, exposure and sensitivity differing across the sub-portfolios.

The data span (monthly frequency) under investigation corresponds to the period after the introduction of EMU, starting in January 1999 and ending in May 2003.We now describe the data regarding the proxy for the creditworthiness index and for the credit drivers.

Given that most of the obligors are non floated in the stock market, we assemble the counterparts in twenty large clusters corresponding to the following Italian MIB sub-sectors stock price indices: Food/Grocery, Insurance, Banking, Paper Print, Building, Chemicals, Transport/Tourism, Distribution, Electrical, Real Estate, Auto, Metal/Mining, Textiles, Industrial Miscellaneous, Plants/Machinery, Financial Services, Finance/Part, Financial Miscellaneous, Public Utility, Media. The returns on these stock indices are used to proxy the creditworthiness indices of each obligor.

We consider a large number of credit drivers. First, in line with CreditMetrics (1997), we consider financial variables, given by the MSCI stock price indices for a number of sectors (e.g. Energy, Materials, Industrials, Consumer Discretionary, Consumer Staples, Health, Financials, Information Technology, Telecommunications, Utilities) corresponding to different geographical areas (World, US, Europe, Emerging Markets). We also add to the stock prices data (which are the only ones considered by CreditMetrics, 1997), other financial, nominal and real macroeconomic credit drivers.

The other financial variables considered are the short term and long term interest rates in Italy (e.g. one, two, three, six, nine, twelve months Italian interbank rates; the MSCI Italian government bond yields for the following maturities: one to three years; three to five years; five to seven years; seven to ten years; over ten years).

The nominal variables are the consumer prices, CPI, and the producer prices, PPI. In particular the CPI indices considered are for all items (e.g. aggregate), and for different following aggregate goods: clothing and footwear; communications; education; electricity and other fuels; energy; food; furnishing; health; restaurants and hotels; insurance; recreation; transport. The PPI Indices are for all items and for the following sectors: basic metals; chemicals; consumer goods durable; non durable; electricity, gas and water, supply; electricity, gas, steam and hot water; energy; food, beverages and tobacco, intermediate goods; machinery and equipment; mining and quarrying; motor vehicles; publishing, printing and reproduction; textiles and raw materials.

The real credit drivers considered are given by the real seasonally adjusted (real) indices for aggregate industrial production and for the following sectors: investment goods, intermediate goods, energy, manufacturing, food, textiles, leather, wood, paper, coke, chemicals, rubber, non metals, metals, machinery, electricity, other, furniture, energy. Finally, among the set of real economic variables, we also include the real effective exchange rate.

The CPI and PPI series have been de-seasonalised by employing monthly deterministic dummies. Stationarity has been achieved by taking the first order differences. Finally, each series in the dataset have been standardised to have zero mean and unit variance.

6.2 Clustering

It is important to observe that in the portfolio under examination (see below), some obligors sharing common features are aggregated in clusters. Each of these clusters contains a large number of obligors, each with a small contribution. In order to estimate the conditional losses for each cluster, we apply the Law of Large Numbers, hence the whole distribution collapses into a single value: the corresponding expected loss. As for the large number of obligors organised in non-clusters and given their relative large exposure, we use Montecarlo simulation to obtain the corresponding conditional portfolio loss distribution. Furthermore, in each scenario, the sum of losses deriving from default of the non cluster obligors (obtained through simulation) and the expected loss from the clusters gives the (conditional) portfolio loss distribution. Finally, the Montecarlo simulation has been based upon the simplifying assumptions that: a) we do not account for the use of financial collateral and of credit risk mitigation techniques; b) we consider the year as the reference temporal horizon; c) we do not consider claims maturing in a period less than a year.

6.3 Credit risk measurement

Standard AIC and BIC criteria to select the number of static factors cannot be employed since they rely on the minimisation of a penalty function only of the time series dimension. Therefore, we employ the method suggested by Bai-Ng (2002), which involves the minimisation of a penalty function depending on both the cross section and time series dimension, and the number of static factors, r, is found to be equal to four. As for the estimation of the sensitivies β of the multifactor model for the asset returns, we use *OLS*, and the the corresponding R^2 are given in Table 1.[5]

[5]It is important to observe that the systemic component of the return series should be $N(0,1)$. Consequently, define b and R^2 the matrix of coefficients measuring the sensitivities in each APT regression and the coefficient of determination of each APT regression, respectively. Then, for the purpose of standardising, we consider the ratio between the

Employing the scenario generation described in section 5, we obtain the simulated loss distributions. As we can observe (see Fig. 1-6) the shape of the unconditional loss distribution is asymmetric and highly skewed to the right.

From the Figures below and Tables 2 and 3 (numbers are in milions of Euros) we can draw the following conclusions. First, by comparing the second and third column of Table 3, we can observe that the Basel II measure of the unexpected loss (obtained from the analytic solution described in equation (7)) approximates closely the economic capital obtained from the simulated loss distribution relaxing only the assumption of homogeneous portfolio. Secondly, by comparing results in Tables 2 and 3, we can observe that the consideration of stochastic dependent recovery shifts to the right the unconditional loss distribution, implying high values for the expected loss, unexpected loss and expected shortfall. Finally, If we disentangle the common shock in two structural shocks: aggregate demand and aggregate supply, then we can observe that the demand shock has an higher impact on the credit risk measure of interests (see the fourth and fifth column of Table 2 and Table 3). This holds for both the case of constant and of stochastic dependent recovery. This last finding can be explained by taking into account that two are the type of recession scenarios driven by the identified common shocks. The first, driven by a demand shock, is a deflationary type recession scenario, given that both output and prices fall. The second recession scenario is driven by a supply shock, and it is described by a fall in output and increase in the price level. In this case, the increase in the price level, redistributing wealth from lenders to borrowers (and, also decreasing the level of real interest rates), can mitigate the depressive effect on the firms cashflows driven by a fall in output. Consequently, the supply shock can have a less severe impact on the financial health status of the obligors, and on the overall risk associate to the bank loan portfolio.

7. Conclusions

Since default probabilities are driven primarily by both firm specific innovations and by how different obligors are tied to business cycles and to the degree of macro financial imbalances in the economy, in this paper we attempt to integrate market risk with credit risk. The estimation and identification of the common shock underlying the business cycle has been obtained by fitting a dynamic factor model to a large number of macroeconomic credit drivers. In line with Basel 2, we focus on the unconditional portfolio distribution, given that defaults across obligors are independent

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unanticipated component of the systemic component and its corresponding variance, and we multiply this ratio by R^2. Whrereas we attach weight $1- R^2$ to the idiosincratic component of the return series.

once we condition on a specific macroeconomic scenario. However, we depart from the homogeneous portfolio, one common shock and constant recoveries assumption underlying the analytic solution of the unconditional loss distribution. This analytic solution is the one adopted by Basel II for the computation of the unexpected loss, in order to measure the minimum capital requirement to cope with losses likely to arise in presence of (extreme) negative macroeconomic scenarios. The empirical results suggests, in general, that, in line with Altman et al. (2002), that ignoring the main feature of recoveries, as stochastic and dependent on default, can imply serious under provision of minimum capital requirements (especially in presence of macro-economic shocks identified as demand side).

References

E. I. Altman and B. Brady (2002): "Explaining aggregate recovery rates on corporate bond defaults". New York University Salomon Center working Paper.

E.I Altman, B. Brady, A. Resti, and A. Sironi (2003) "The link between default and recovery rates: Theory, empirical evidence and implicatons". New York University Stern School of Business Department of Finance working paper Series.

Altman, Resti and Sironi (2002): "The link between default and recovery rates: effects on the procyclicality of regulatory capital", *BIS* working paper 113

Bai and Ng (2002): "Determining the number of factors in approximate Factor models", *Econometrica*, 70(1), 191-221.

Bangia, A., Diebold, F. and T. Schuermann (2000): "Rating migration and the business cycle, with application to credit portfolio stress testing", Wharton, Financial Institution Center, working paper 0026

Blanchard, O. and Quah, D. (1989): "The dynamic effects of aggregate demand and aggregate supply disturbances", *American Economic Review*, 79, 655-73.

Carling, K., Jacobson,T., Lind,J. and K. Rozback (2001): "The internal ratings based approach for capital adequacy determination: an application to Sweden", paper prepared for the workshop on Applied Banking research, Oslo, 12-13 June.

Carpenter, S., Whitsell, W., nd E. Zakrajisek al. (2001): "Capital requirements, business loans and business cycles: an empirical analysis of the standardised approach in the New Basel Capital accord" Board of Governors of Federal Reserve System

Cateraineu-Rabell, E., P. Jackson and D.P. Tsomocos (2002): "Pro-cyclicality and the New Basel Accord: bank's choice of loan rating system" presented at the conference "The impact of economic slowdowns on financial institutions and their regulators" at the Federal Reserve Bank Boston, April.

Ervin, W. and T. Wilde (2001): "Procyclicality in the New Accord", *Risk* (October)

Frye, J. (2000a): "Depressing recoveries", *Risk* (November)

Frye, J. (2000b): "Collateral damage detected", Federal Reserve Bank of Chicago, working paper, *Emerging Issues Series*, October 1-14.

Forni, M., M. Hallin, M. Lippi, and L. Reichlin, (2000) "The generalised dynamic factor model: identification and estimation". *The Review of Economics and Statistics*, 82, 540-554.

Forni, M., Lippi, M., Reichlin L. (2003): "Opening the Black Box: structural factor model versus structural VAR models", *CEPR* discussion paper 4133.

Giannone, D., Reichlin, L. and Sala, L. (2003): VARs, Common Factors and the Empirical Validation of Equilibrium Business Cycle Models, *Journal of Econometrics*, forthcoming.

Gordy, M. (2000): " A comparatve analysis of credit risk models", *Journal of Banking and Finance*, 24, 119-149.

G. Gupton, D. Gates, and L. Carty (2000): " Bank loan losses given default". *Moody's Global Credit Research, Special Comment*.

Haldane, A., G. Hoggarth, and V. Sapporta (2001): "Assessing financial stability, efficiency and structure at the Bank of England", in Marrying the macro and micro prudential dimension of financial stability, *BIS working paper*, no 1. 138-59.

Hu, Y. T. And Perraudin, W. (2002): "The dependence of recovery rates and defaults", Birbeck College working paper.

Jordan, J, J Peek and E Rosengren (2002): "Credit Risk Modeling and the Cyclicality of Capital", paper presented at the BIS conference on Changes in Risk through Time: Measurement and Policy Options, 6 March.

J.P. Morgan (1997), CreditMetrics, technical document.

Kapetanios, G. and M. Marcellino (2003) "A Comparison of Estimation Methods for Dynamic Factor Models of Large Dimensions", Queen Mary, Department Economics, working paper 489

Krenin, A. Merkoulovitch, L. and Zerbs, M. (1998): "Principal Components Analysis in Quasi Montecarlo simulation", *Algo Research Quarterly*, 1(2), 21-29

Merton, R. C. (1974): "On the pricing of corporate debt: the risk structure of interest rates", *Journal of Finance*, 29, 449-470.

Nickell, P., Perraudin, W. and S. Varotto (2000): "Stability of rating transition", *Journal of Banking and Finance*, 124, 203-228.

Pesaran, H., Schuermann, T., Treutler, B.J. and Weiner, S.W. (20034: "Macroeconomics Dynamics and Credit risk: a global perspective", *Journal of Money, Credit and Banking,* forthcoming.

Pykhtin, M. (2003) "Unexpected recovery risk". *Risk*, 16(8):74–78

Ross, S.A. (1976): "The arbitrage pricing theory of capital asset pricing", *Journal of Economic Theory*, 13, 341-360.

Schonbucher, P. (2000): "Factor models for portfolio credit risk" Bonn University working paper

Segoviano and Lowe (2002): "Internal ratings, the business cycle and capital requirements: some evidence from an emerging market economy", *BIS* working paper no. 117.

Stock and Watson: "Macroeconomic Forecasting using diffusion indices", *Journal of Business and Economic Statistics*, 20(2), 147-162

Uhlig, H. (2004): "What are the effects of monetary policy on output? Results from an agnostic identification procedure", *Journal of Monetary Economics,* forthcoming.

Wehrspohn, U. (2003): "Analytical loss distributions of heterogenous portfolios in the asset value of credit risk model"; University of Heidelberg, *Center for Risk and Evaluation* working paper.

Wilson, T. (1997a): "Portfolio Credit Risk, Part I", *Risk* 10 (9), 111-117

Table 1: R^2 from the APT regressions

Food/Grocery	0.21
Insurance	0.44
Banking	0.57
Paper Print	0.27
Building	0.49
Chemicals	0.36
Transport/Tourism	0.23
Distribution	0.27
Electrical	0.58
Real Estate	0.30
Auto	0.30
Metal/Mining	0.19
Textiles	0.46
Industrial Miscellaneous	0.07
Plants/Machinery	0.36
Financial Services	0.33
Finance/Part	0.37
Financial Miscellaneous	0.37
Public Utility	0.53
Media	0.61

Table 2: credit risk measures with constant recovery[6]

	analytic: impact of common shock	simulation: impact of common shock	simulation: impact of demand shock	simulation: impact of supply shock
Expected Loss	330	348.57	340.67	335.94
Unexpected Loss	2418.53	2711.90	4682.29	3841.66
Expected Shortfall	-	4047.75	5874.96	4349.66

Note: numbers are in milions of Euros

Table 3: credit risk measures with stochastic dependent recovery[7]

	simulation: impact of common shock	simulation: impact of demand shock	simulation: impact of supply shock
Expected Loss	534.45	532.16	509.94
Unexpected Loss	9593.14	8376.73	6886.07
Expected Shortfall	11694.21	10412.04	7703.47

Note: numbers are in milions of Euros

[6] For each obligor, we use apply the same LGD rate, equal to 45%, which is suggested by the Basel accord for senor unsecured loans.

[7] The mean and the std. deviation of recovery rates for unsecured loans used to estimate the parameters *a* and *b* of the beta distribution for the recoveries are 0.505 and 0.284, respectively (source: Altman, et al, 2003) .

Figure 1: Unconditional loss distribution: common shock and constant recovery

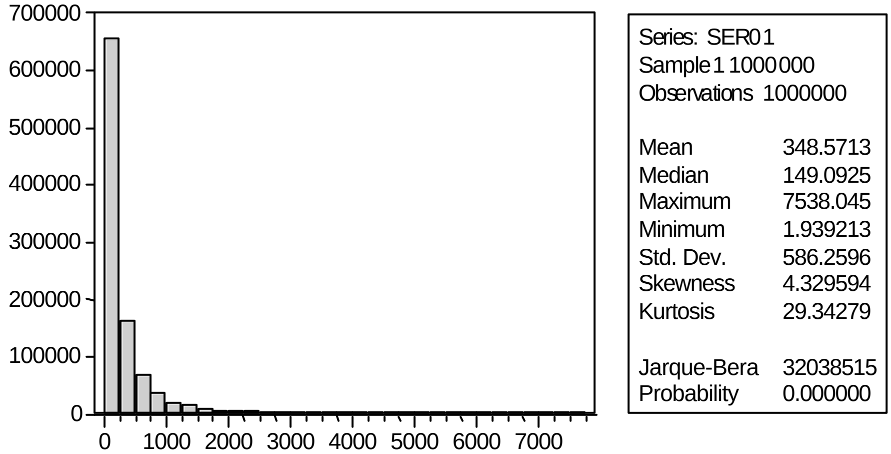


Figure 2: Unconditional loss distribution: demand shock and constant recovery

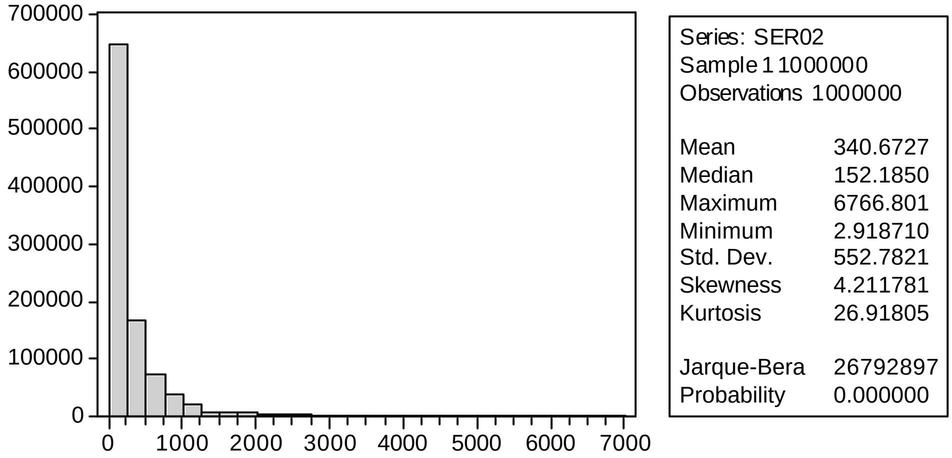


Figure 3: Unconditional loss distribution: supply shock and constant recovery

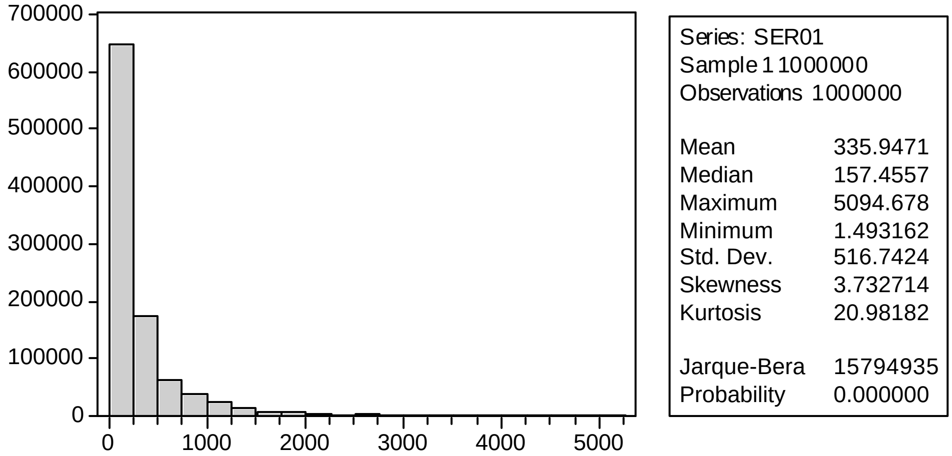
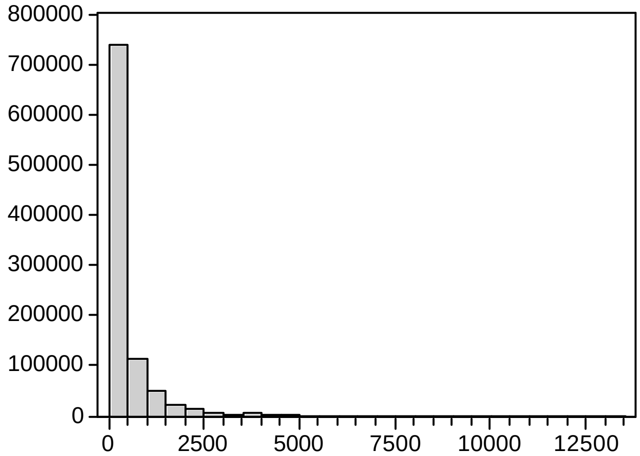
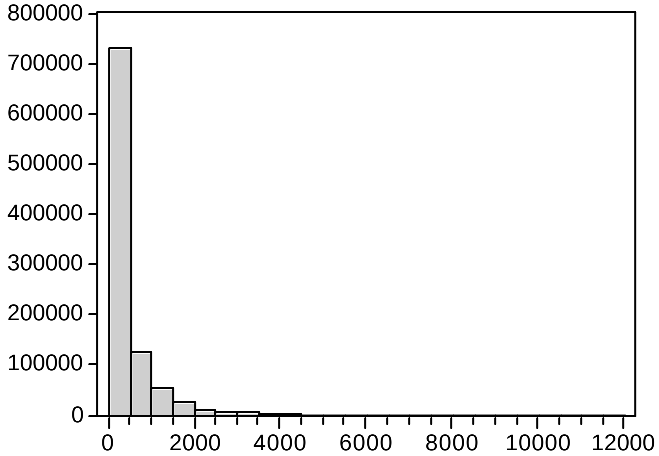


Figure 4: Unconditional loss distribution: common shock and stochastic dependent recovery



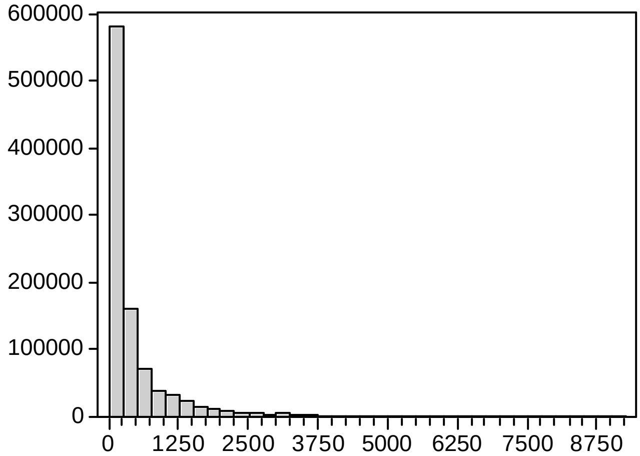
Series: SER01	
Sample 1 1000000	
Observations 1000000	
Mean	534.4526
Median	167.0183
Maximum	13359.48
Minimum	0.913368
Std. Dev.	1037.486
Skewness	4.456326
Kurtosis	30.48648
Jarque-Bera	34789254
Probability	0.000000

Figure 5: Unconditional loss distribution: demand shock and stochastic dependent recovery



Series: SER01	
Sample 1 1000000	
Observations 1000000	
Mean	518.7834
Median	170.4547
Maximum	11992.62
Minimum	1.374711
Std. Dev.	979.6755
Skewness	4.334064
Kurtosis	27.93255
Jarque-Bera	29032015
Probability	0.000000

Figure 6: Unconditional loss distribution: supply shock and stochastic dependent recovery



Series: SER01	
Sample 1 1000000	
Observations 1000000	
Mean	509.9463
Median	177.5033
Maximum	9029.164
Minimum	0.703278
Std. Dev.	915.4072
Skewness	3.862861
Kurtosis	21.95207
Jarque-Bera	17452823
Probability	0.000000