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Comments Welcome

A Theory of Overconfidence, Self-Attribution, and Security Market Under- and Over-reactions

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Abstract

We propose a theory based on investor overconfidence and biased self-attribution to explain several of the securities returns patterns that seem anomalous from the perspective of efficient markets with rational investors. The theory is based on two premises derived from evidence in psychological studies. The first is that individuals are overconfident about their ability to evaluate securities, in the sense that they overestimate the precision of their private information signals. The second is that investors' confidence changes in a biased fashion as a function of their decision outcomes. The first premise implies overreaction to private information arrival and underreaction to public information arrival. This is consistent with (1) post-corporate event and post-earnings announcement stock price 'drift', (2) negative long-lag autocorrelations (long-run 'overreaction'), and (3) excess volatility of asset prices. Adding the second premise leads to (4) positive short-lag autocorrelations ('momentum'), and (5) short-run post-earnings announcement 'drift,' and negative correlation between future stock returns and long-term measures of past accounting performance. The model also offers several untested empirical implications and implications for corporate financial policy.

1 Introduction and Summary

In recent years a large body of evidence on security returns has presented a sharp challenge to the traditional view that securities are rationally priced to reflect all publicly available information. Several studies have phrased conclusions in terms of market underreaction or overreaction to new information. Some of the more notable anomalies include: (1) underreaction to public news events (public-event-date average stock returns of the same size as average subsequent long-run abnormal performance),¹ (2) short-term momentum (positive short term autocorrelation of stock returns, for individual stocks and the market as a whole),² (3) long-term reversal (negative autocorrelation of short-term returns separated by long lags, or “overreaction”),³ (4) unconditional excess volatility of asset prices relative to fundamentals,⁴ (5) short-run post-earnings announcement stock price ‘drift’ in the direction indicated by the earnings surprise,⁵ and (6) abnormal stock price performance in the opposite direction of long-term earnings changes.⁶

There remains strong disagreement over the interpretation of this evidence of predictability. In the absence of arbitrage, a positive pricing kernel exists that prices all assets. The question is how reasonable a rational equilibrium model underlying such a kernel/model

¹Events for which this has been found include stock splits (Grinblatt, Masulis, and Titman (1984), Desai and Jain (1996), and Ikenberry, Rankine, and Stice (1996)), tender offer and open market repurchases (Lakonishok and Vermaelen (1990), Ikenberry, Lakonishok, and Vermaelen (1995)), analyst recommendations (Groth, Lewellen, Scharbaum, and Lease (1979), Bjerring, Lakonishok, and Vermaelen (1983), Elton, Gruber, and Gultekin (1984), Womack (1996), and Michaely and Womack (1996)), dividend initiations and omissions (Michaely, Thaler, and Womack (1995)), seasoned issues of common stock (Loughran and Ritter (1995), Spiess and Affleck-Graves (1995), Teoh, Welch, and Wong (1996)), earnings surprises (at least for a period after the event; (Bernard and Thomas (1989, 1990), Brown and Pope (1996)), public announcement of previous insider trades (Seyhun (1986), Seyhun (1988) and Rozeff and Zaman (1988); however, the abnormal profits to outsiders do not exceed the associated bid-ask and transactions costs), and venture capital share distributions (Gompers and Lerner (1995)). There is also evidence that earnings forecasts underreact to public news, such as quarterly earnings announcements (Abarbanell and Bernard (1991, 1992), Mendenhall (1991)).

²Jegadeesh and Titman (1993), Daniel (1996); ‘Short’ here refers to periods on the order of 6-12 months. At very short horizons there is negative autocorrelation in individual stock returns (Jegadeesh (1990) and Lehmann (1990)), but this may result from bid-ask spreads and other measurement problems (Kaul and Nimalendran (1990)).

³Cross-sectionally, see DeBondt and Thaler (1985, 1987), Chopra, Lakonishok, and Ritter (1992); on robustness issues, see Fama and French (1996). For the aggregate market, see Fama and French (1988) and Poterba and Summers (1988); on the robustness of the finding in the post-WWII period, see Kim, Nelson, and Startz (1988) and Daniel (1996); the latter finds that the long-horizon negative autocorrelation is partly masked by a market momentum effect (positive serial correlation) at approximately a one year horizon.

⁴Shiller (1981), Shiller (1989); on the robustness of this finding, see Kleidon (1986); Marsh and Merton (1986).

⁵Bernard and Thomas (1989, 1990).

⁶Lakonishok, Shleifer, and Vishny (1994), La Porta, Lakonishok, Shleifer, and Vishny (1995); see however DeChow and Sloan (1995).

would be. Based on the high Sharpe-ratios (relative to the market) apparently achievable with simple trading strategies, any asset pricing model consistent with these patterns would have to have extremely variable marginal utility across states.⁷ Moreover, to explain the various cross-sectional predictability results (on size, book-to-market, and momentum, for example) such a model would require that marginal utilities vary strongly with the returns on size, book-to-market and momentum portfolios. No such correlation is obvious in examining the data. Given this evidence, it seems reasonable to consider quasi-rational explanations for the observed return patterns.

Moreover, there are important corporate financing and payout patterns which seem potentially related to market anomalies. Firms tend to issue equity (rather than debt) after rises in market value, and when the firm or industry book/market ratio is low. There are industry-specific financing and repurchase booms, perhaps designed to exploit industry-level mispricings. Transactions such as takeovers that often rely on securities financing are also prone to industry booms and quiet periods.⁸

Although it is not obvious how the empirical securities market phenomena can be captured plausibly in a model based on perfect investor rationality, no psychological (“behavioral”) theory for these phenomena has won general acceptance. Some aspects of the patterns seem contradictory, such as apparent market underreaction in some contexts and overreaction in others. While explanations have been offered for particular anomalies, we have lacked an integrated theory to explain these phenomena.⁹

A criticism often raised by economists against psychological theories is that, in a given

⁷MacKinlay (1995) argues that size/book-to-market strategies produce excessively high Sharpe ratios. Campbell and Cochrane (1994) find that a utility function with extreme habit persistence is required to explain the time series patterns in market returns; presumably any model which explained the cross-sectional patterns as well would require even more extreme variability in marginal utilities.

⁸Section 5 develops further implications for managerial policy. These include the implications that a rational manager acting in the interest of current shareholders should (1) issue after a rise in the firm’s stock price, (2) favor public over rights issues after a price runup, (3) prefer debt over equity over issues after a price rundown, and (4) prefer repurchase over dividends after a price runup. The analysis also implies that if managers act to exploit mispricing, there will be both general and industry-specific financing and repurchase booms.

⁹Daniel, Hirshleifer, and Subrahmanyam (1997) show that the overconfidence approach can also be applied to explain important anomalies relating to risk measures and to fundamental predictors of security returns. These puzzles include: (1) a positive association between price-scaled fundamental measures (dividend yield, earnings/price, and book/market) and future stock market returns, (2) the ability of firm size to predict future returns when size is measured by market value, but not when measured by book value or other non-market measures, (3) domination of β as a cross-sectional predictor of stocks’ future returns by price or by price scaled variables (book/market, earnings/price ratios), and (4) the willingness of investors to hold imperfectly diversified portfolios despite the at-best ambiguous evidence regarding a premium for bearing residual risk.. They also examine the relation between price-scaled fundamental variables such as book/market ratios and corporate financial decisions.

economic setting, the universe of conceivable irrational behavior patterns is much larger than the set of rational patterns. Thus, it is sometimes claimed that allowing for irrationality opens a Pandora’s box of ad hoc stories which will have little out-of-sample predictive power. However, as DeBondt and Thaler (1995) point out, a good psychological finance theory will be grounded on the evidence from psychology about how people actually behave. We think such a theory should be built upon a valid psychological microfoundation that allows for the rational side of investor decisions. To deserve consideration a theory should be parsimonious, explain a *range* of anomalous patterns in different contexts, and generate new empirical implications.

This paper offers a theory of security price anomalies based on *investor overconfidence*, and variations in confidence arising from *biased self-attribution*. The overconfidence theory of security prices is based on a large body of evidence from cognitive psychological experiments and surveys (summarized in Section 2) showing that individuals overestimate their own abilities in various contexts. In our model, an overconfident investor is one who overestimates the precision of his private information signal about security value.¹⁰ If investors are more overconfident about signals with which they have greater personal involvement, they will tend to be overconfident about private information signals (information they receive before some other set of investors) than about public signals (received simultaneously by all). A distinguishing feature of our theory is this differential confidence about different kinds of signals.

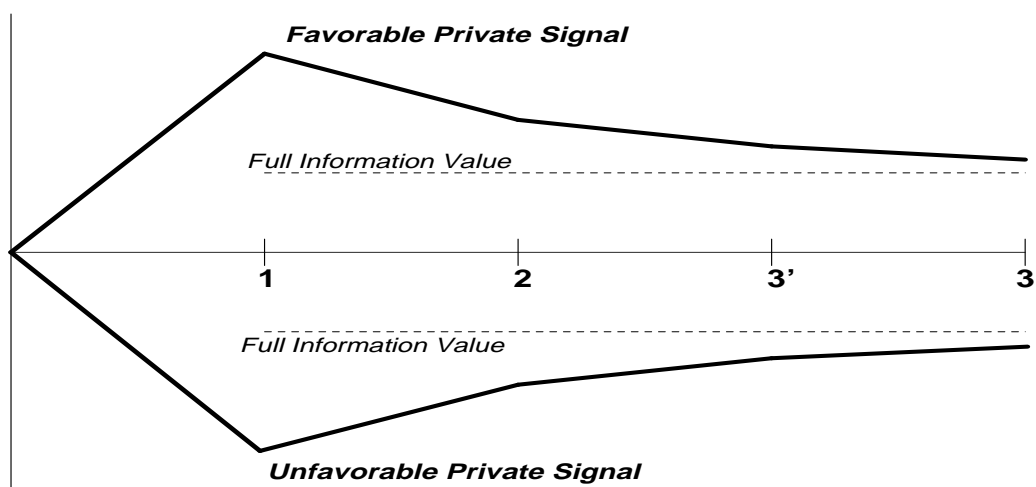
Suppose that traders start at date 0 with a common prior, and then at date 1 informed traders revise their belief about a security or securities based upon a common private signal. Figure 1 illustrates the average price path following a positive (upper curve) or negative (lower curve) private signal. The upper curve is an “impulse-response function;” it shows the expected prices conditional on a private signal of unit magnitude arriving at time 1. The dotted line shows the level the price would move to if investors were fully rational.

Informed traders are assumed to overestimate the precision of their private signal, but not of public signals. As a result, the informed overweight the private signal relative to the prior, causing the stock price to overreact. At date 2, when noisy public information signals arrive, the inefficient deviation of the price will be partially corrected, on average. On subsequent dates, as more public information arrives, the price will, on average, move still closer to the full-information value. We call the part of the impulse response prior to the peak or trough the *overreaction phase*, and the later section the *correction phase*.

The pattern just described around the public information signal implies that the post-

¹⁰For similar specifications of overconfidence, see Kyle and Wang (1996), Odean (1995), and Wang (1995).

Figure 1: Average Price as a Function of Time with Overconfident Investors



public-event abnormal stock price performance will tend to be of the same sign as the abnormal stock price reaction occurring at the event date (consistent with evidence on various public events). Thus, the overall pattern in our basic (static) model is one where stock prices overreact to private information signals and underreact to public signals. In the dynamic version of the model where self-confidence fluctuates in response to confirming or disconfirming public signals, the overreaction to private information can continue over a period of time. Although there remains a tendency to underreact to public information, public information can sometimes trigger further overreaction to a preceding private signal.

The remainder of the introduction describes how this approach can explain other existing anomalies. Several new untested or incompletely-tested implications are provided as well.¹¹

Suppose that rationally managed firms tend to buy securities or signal more strongly

¹¹The model has the following further empirical implications for various corporate events that are either untested or have been confirmed only on a few specific events, based on Propositions 1-2 and Propositions 6-10: (1) post-public-news-event abnormal performance (either including or excluding the event date) that is on average better [worse] when fundamental/price ratios such as book/market are high [low]; (2) A negative correlation between pre-event stock price performance (either including or excluding the event date) and post-event stock price performance; (3) a positive correlation between initial event-date stock price reactions and post-event performance (after normalizing for confidence proxies), (4) less post-event abnormal performance for relatively 'non-selective' corporate events (where occurrence of the event is not based on private information about the firm's prospects); (5) positive abnormal returns starting after the announcement of toehold ownership stakes in a firm; (6) stronger post-earnings drift when the surprise is measured relative to forecasts than when measured relative to past earnings or a time-series prediction based on past earnings; and (7) Greater long-term reversal of price moves occurring on dates when there are no public news events about a firm reported in standard sources than price moves occurring on public event dates.

when managers believe their stock is undervalued by the market, and sell securities or signal less strongly when managers think their stock is overpriced. Then the firm's average post-announcement abnormal stock price performance (following announcement of a share purchase/sale or other corporate action) will be of the same sign as the average initial price reaction. The underreaction of overconfident traders to public signals is consistent with a growing number of studies of various corporate events (see footnote 1).

The combination of overreaction to private signals and underreaction to public signals can explain some empirically-observed time-series patterns in stock prices. Long-run negative stock price autocorrelations arise simply because the overreaction to the private signal at date 1 must eventually reverse as uncertainty resolves. However, in a constant-confidence setting there is an immediate (single-period) overreaction phase and a gradual reversal. In such a setting the unconditional price-change autocorrelations is negative at all lags and horizons. Thus, the basic constant-confidence model is inconsistent with short-term unconditional momentum.¹²

However, short-lag autocorrelations will be positive in a setting with a smooth, multi-period overreaction phase. Outcome-dependent shifts in confidence result in exactly such a delayed overreaction. According to *attribution theory* in psychology (Bem (1965)), when events occur that confirm the validity of an individual's actions, the individual attributes this to his own high ability, while events that disconfirm the action are attributed to external noise or sabotage. This suggests that confidence will vary over time depending on whether an individual's beliefs and actions are confirmed by subsequent events.

If an investor makes a trade based on a private signal, then we say that a later public signal *confirms* a trade if it has the same sign (good news arrives after a buy, or bad news after a sell). Based on attribution theory, we assume that when an investor receives confirming public information, his confidence rises, but disconfirming information causes confidence to fall only modestly, if at all.¹³ When there are two consecutive favorable signals, the second up-move is reinforced by increased confidence. When there are two down-moves, the second down move is reinforced. Thus, outcome-dependent shifts in confidence can bring about positive momentum. Such momentum will be reversed in the long run as public information arrival gradually draws the price back toward fundamentals. This yields a hump-shaped impulse response function for a private signals as illustrated in Figure 2. This

¹²This outcome is similar to that of the 'fads' model of Summers (1986), in which an instantaneous shock drives the stock price away from the fundamental value, and then the price only gradually reverts to the fundamental value. The sudden appearance of the price error leads to negative serial correlation, because the 'hump' or 'trough' is not smoothed on the left.

¹³This also relates to the notion of cognitive dissonance, in which individuals suppress information that conflicts with past choices.

Figure 2: Average Price as a Function of Time with Outcome-Dependent Confidence

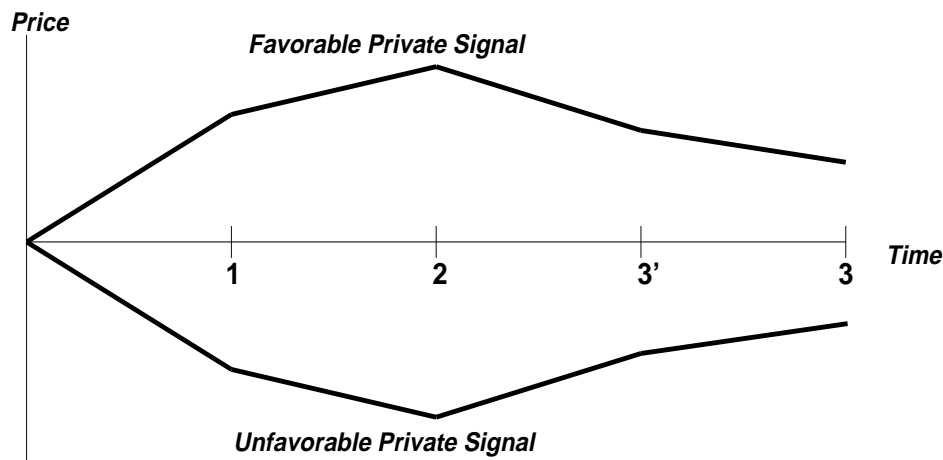


Figure shows two possible date 1 prices, and the paths for expected price conditional on the date 1 move. The figure shows that with outcome-dependent confidence, there is a smooth, multiperiod overreaction phase, and as a result there is positive unconditional price-change autocorrelation at short horizons.

Short-lag autocorrelations are positive for the impulse-response function illustrated in Figure 2, because the econometrician calculates return autocorrelations without conditioning on the dates of private or public information arrival. Pairs of returns drawn from the overreaction phase (dates 0 through 2) are positively autocorrelated, as are pairs drawn from the correction phase (dates 2 through 3). This contributes positively to the econometrician's unconditional autocorrelation. Pairs of price changes that straddle the extremum (dates 1 through 3') are negatively autocorrelated. This contributes negatively to the econometrician's unconditional autocorrelation. Overall, this unconditional autocorrelation will be positive if the extremum-straddling negative autocorrelation is sufficiently small. This will be the case so long as the slopes do not change too dramatically at the extremum.¹⁴ In contrast, short-term returns that are separated by longer lags are likely to straddle the extremum of the impulse-response function, leading to negative autocorrelations. Thus, the pattern of momentum at short lags and reversal at long lags arises naturally from the model.

The static confidence analysis implies possible underreaction to earnings or other financial forecasts or announcements. The dynamic analysis with biased self-attribution implies a lag-dependent pattern. Earnings surprises at first tend to increase confidence, causing

¹⁴In a continuous-time setting, returns at some sufficiently short horizon will be positively autocorrelated so long as the impulse-response function is smooth (has a continuous first derivative)

an same-direction average stock price trend. Eventually, this overreaction reverses. Thus, the analysis is consistent with both short-term trends after earnings announcements and reversals following earnings trends.

A possible objection to models that explain price anomalies as market inefficiencies is that smart investors should be able to profit by trading against the mispricing. If such trading causes wealth to flow from irrational to smart traders, eventually the smart traders may dominate price-setting. For several reasons, we do not find these arguments to be compelling, as discussed at the start of Section 6.

Apart from its specific empirical implications, a special case of the overconfidence approach to securities pricing promises to play a useful theoretical role. The limiting case in which traders observe pure noise but believe they observe a useful signal is a quasi-rational model of noise-trading behavior. By quasi-rational, we mean that investors are rational in all respects but one, that they are overconfident about the quality of their private information. By ‘noise-trading’, we mean trading based on a random variable regarded as a meaningful signal by the individual which actually contains no useful information. Such a microfoundation for noise-trading behavior may be valuable since noise trading underlies both a leading approach to understanding security price anomalies (Shiller (1984), Black (1986), De Long, Shleifer, Summers, and Waldmann (1990a), De Long, Shleifer, Summers, and Waldmann (1991), Campbell and Kyle (1993),¹⁵ and the standard models of security market informational equilibrium (Kyle (1985), Grossman and Stiglitz (1980), and Glosten and Milgrom (1985)).¹⁶ The relation of the confidence approach to the noise approach is discussed further in Section 6.

The remainder of the paper is structured as follows. Section 2 describes psychological evidence of overconfidence and self-attribution bias. Section 3 develops the basic model of overconfidence. Here, we describes the economic setting, define overconfidence. We analyze the equilibrium to derive implications about stock price reactions to public versus private news, short versus long-term autocorrelations, and volatility. Section 4 examines time-variation in overconfidence, to derive implications about the signs of short-term versus long-term return autocorrelations. Section 5 discusses the normative and empirical implications of the model for corporate financial policy. Section 6 discusses further the relation to existing theories. Section 7 concludes.

¹⁵Shiller (1984) proposes that individuals trade based on mistaken but popular models of the economy; Black (1986) that investors trade on noise. De Long, Shleifer, Summers, and Waldmann (1990a) and De Long, Shleifer, Summers, and Waldmann (1991) point out that overconfidence can underlie noise trading, but do not explicitly analyze the effect of overconfidence on prices.

¹⁶Wang (1995) shows (and it is true here as well) that when some traders are overconfident, information-motivated trading can occur without noise traders.

2 Overconfidence and Attribution Bias

The model we present in this paper relies on two behavioral regularities: *overconfidence* and *attribution bias*. In their summary the microfoundations of behavioral finance, DeBondt and Thaler (1995) state that “perhaps the most robust finding in the psychology of judgment is that people are overconfident.” Evidence of overconfidence has been found in several contexts. Examples include psychologists (Oskamp 1965), physicians and nurses (Christensen-Szalanski and Bushyhead 1981, Baumann, Deber, and Thompson 1991), engineers (Kidd 1970), attorneys (Wagenaar and Keren 1986), negotiators (Neale and Bazerman 1990), entrepreneurs (Cooper, Woo, and Dunkelberg 1988), managers (Russo and Schoemaker 1992), investment bankers (Stael von Holstein 1972), and market professionals such as security analysts and economic forecasters (Ahlers and Lakonishok 1983, Elton, Gruber, and Gultekin 1984, Froot and Frankel 1989, DeBondt and Thaler 1990, DeBondt 1991).¹⁷

Our theory assumes that investors view themselves as more able to value securities than they actually are, so that they underestimate their forecast error variance. This is consistent with evidence that people overestimate their own abilities, and perceive themselves more favorably than they are viewed by others.¹⁸ Several experimental studies find that individuals underestimate their error variance in making predictions.¹⁹

The second aspect of our theory is biased self-attribution: the confidence of the investor in our model grows when public information is in agreement with his information, but it does not fall commensurately when public information contradicts his private information. The psychological evidence indicates that people tend to credit themselves for past success, and blame external factors for failure (Fischhoff 1982, Langer and Roth 1975, Miller and Ross 1975, Taylor and Brown 1988). As Langer and Roth (1975) put it, ‘Heads I win, tails it’s chance.’ De Long, Shleifer, Summers, and Waldmann (1991) argue that such biased self-attribution may be important for securities markets:

...overconfidence in the precision of one’s estimate is likely to become more extreme over time as those who succeed attribute their success to their own skill and judgment. ... In asset markets, the richest individuals may well be those who placed large bets on very risky gambles and won. Their success would naturally tend to reinforce their confidence in their own hunches, whether or not such confidence is justified.

However, they do not attempt to model this process of shifting confidence.

¹⁷See Odean (1995) for a good summary of empirical literature pertaining to overconfidence.

¹⁸See Greenwald (1980), Svenson (1981), Cooper, Woo, and Dunkelberg (1988), and Taylor and Brown (1988).

¹⁹See Alpert and Raiffa (1982); Fischhoff, Slovic, and Lichtenstein (1977), and the discussions of Lichtenstein, Fischhoff, and Phillips (1982) and Yates (1990)).

3 The Basic Model: Constant Confidence

In this section we present a simple model in which individuals endowed with private information are *overconfident* about the quality of their signals. Individuals are ‘quasi-rational’ in the sense that they rationally maximize expected utility except for this single mistaken perception. In Section 4, we develop a model with time-varying confidence.

3.1 The Economic Setting

There are two continuous masses of agents: a set of identical risk neutral overconfident traders (the informed, I); and a set of identical risk averse fully rational traders who have exponential utility (the uninformed, U).²⁰ Each individual is endowed with a basket containing security shares, and a riskfree numeraire which is a claim to one unit of terminal-period wealth. With the assumed preferences, the initial distribution of wealth and shares is irrelevant for equilibrium prices. At each date individuals can trade the consumption claims for shares.

There are 4 dates. At date 0, individuals begin with their endowments and identical prior beliefs, and trade solely for optimal risk-transfer purposes. At date 1, I 's receive a common noisy private signal about underlying security value and trade with U 's.²¹ At date 2, a noisy public signal arrives, and further trade occurs.²² At date 3, conclusive public information arrives, the security pays a liquidating dividend, and consumption occurs. All random variables are independent and normally distributed.

The risky security generates a terminal value of θ , which is assumed to be normally distributed with mean $\bar{\theta}$ and variance σ_{θ}^2 . For most of the paper we set $\bar{\theta} = 0$ without loss of generality.

The private information signal received by I 's at date 1 is

$$s_1 = \theta + \epsilon.$$

where ϵ is normally-distributed noise that is independent of θ with variance σ_{ϵ}^2 (or precision

²⁰The assumption that prices are set by a risk neutral set of individuals makes the model tractable. We conjecture that similar effects would operate with risk aversion. It is not possible to have both sets of traders be risk neutral, because they would “agree to disagree” and take infinite trades against each other.

²¹Some previous models with common private signals include Grossman and Stiglitz (1980), Admati and Pfleiderer (1988), and Hirshleifer, Subrahmanyam, and Titman (1994).

²²The model’s assumption that private information precedes public information is not very restrictive.. It is essential for several of the implications that at least some noisy public information arrives after a private signal, but additional public information could arrive earlier as well. Additional public signals preceding or contemporaneous with the private signal could be incorporated within the prior of the current model.

$1/\sigma_\epsilon^2$). The U 's correctly assess the error variance, but I 's underestimate it to be $\sigma_C^2 < \sigma_\epsilon^2$.²³ The differing beliefs about the noise variance are common knowledge to all.

Similarly, let the date 2 public signal be

$$s_2 = \theta + \eta,$$

where η is normally-distributed noise that is independent of θ and ϵ with (correctly estimated) variance σ_p^2 .

Equilibrium is defined as follows:

1. At each date, all individuals maximize expected utility as a function of terminal wealth with respect to their beliefs.
2. Prices are set such that the aggregate demands for the risky and numeraire securities at each date equal aggregate supply.
3. At each date individuals can trade at the market price to modify their bundles of risky and numeraire securities.
4. Individuals make decisions at each date based on their available information, including the market price, and I 's overconfident beliefs about precision.
5. It is common knowledge that I 's believe with certainty that the noise variance of s_1 is σ_C^2 and that U 's believe with certainty it is greater than σ_C^2 .²⁴

²³It is not crucial that the U 's correctly assess the error variance, only that they do not underestimate it as much as the informed do. Also, the uninformed need not be more rational than the informed. The groups may be similar except that the uninformed have no private signal to be overconfident about. Even if it is assumed that all individuals receive private signals, it is not essential to have asymmetry between individuals. Most of the results derived in the paper would also apply in a setting with a continuum of ex ante identical risk averse individuals who receive and are overconfident about information signals of the form

$$s_1 = \theta + \epsilon + \delta_i,$$

where δ_i is independent across individuals. Since a set of discrete individuals will be unable to disentangle $\theta + \epsilon$ from δ_i perfectly, even if they underestimated the variance only of δ_i , not ϵ , this would carry over to overweighting of $\theta + \epsilon$ relative to the prior. Such a model would be broadly similar to, but less tractable than that of the main text. If there is a continuum of individuals, prices would be set solely based on $\theta + \epsilon$, and individuals would be able to disentangle $\theta + \epsilon$ perfectly from δ_i . Such a model is tractable and essentially equivalent to that of the main text. However, to obtain price anomalies it requires underestimation of the variance of ϵ , not just δ_i .

²⁴Since prices are set by overconfident traders, it matters little what the U 's believe, but there must be some disagreement for signal-based trading to occur. The assumption that maintains as much rationality as possible in the model subject to including overconfidence would be that the U 's believe with certainty that the noise variance is the true value σ_ϵ^2 .

3.2 Equilibrium Prices and Trades

Given that the informed traders are risk neutral, prices at each date satisfy

$$\begin{aligned} P_1 &= E_C[\theta|\theta + \epsilon] \\ P_2 &= E_C[\theta|\theta + \epsilon, \theta + \eta], \end{aligned} \tag{1}$$

where the subscript C denotes the fact that the expectation operator is calculated based on the informed traders' confident beliefs. Trivially, $P_3 = \theta$.

The conditional moments in (1) can be calculated by standard properties of normal variables (Anderson 1984, Chapter 2) to be

$$\begin{aligned} P_1 &= \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_C^2}(\theta + \epsilon) \\ P_2 &= \frac{\sigma_\theta^2(\sigma_C^2 + \sigma_p^2)}{D}\theta + \frac{\sigma_\theta^2\sigma_p^2}{D}\epsilon + \frac{\sigma_\theta^2\sigma_C^2}{D}\eta, \end{aligned} \tag{2}$$

where $D \equiv \sigma_\theta^2(\sigma_C^2 + \sigma_p^2) + \sigma_C^2\sigma_p^2$.

3.3 Empirical Implications

In this subsection, we investigate the empirical implications of the constant confidence model. Subsection 3.3.1 examines price reactions to public and private information; Subsection 3.3.2 examines the implications for price-change autocorrelations; and Subsection 3.3.3 examines implications for event-studies.

3.3.1 Overreaction and Underreaction

The basic results of this model are related to the price reaction to public and private information. Overconfidence causes *overreaction* to private information, so that the covariance between the date 1 price change and the date 2 price change, $\text{cov}(P_2 - P_1, P_1 - P_0)$, is negative if investors are overconfident. Intuitively, there is overreaction in response to the signal $\theta + \epsilon$, which is partially corrected by the date 2 public signal, and fully corrected upon release of the date 3 public signal (so that $\text{cov}(P_3 - P_1, P_1 - P_0) < 0$). This price change reversal arises from the continuing correction to the date 1 overreaction.

Proposition 1 *If investors are overconfident, than price moves resulting from private information signals are on average partially reversed in the long run.*

In contrast, stock prices underreact to independent public news arrival. As discussed above, the overreaction at date 1 is partially corrected at date 2, when the public signal is released.

When θ is revealed at date 3, the price will correct fully. Thus, the price changes at the time of and subsequent to the public signal are positively correlated. Formally, $\text{cov}(P_3 - P_2, P_2 - P_1) > 0$ when investors are overconfident ($\sigma_\epsilon^2 > \sigma_C^2$). (Appendix A provides detailed expressions.) This implies the following proposition:

Proposition 2 *In the constant confidence model with overconfident investors, the price reaction to conditionally independent public news arrival is positively correlated with later stock price changes.*

In Subsection 3.3.3, we will see that the assumed independence of the public signal and the error in I 's signal ϵ is important for event study implications. However, in that section we show that Proposition 2's conclusion of positive price change autocorrelation around public signals extends to the case where the public event is related to the mispricing at date 1.²⁵

Based on this analysis, a central theme of this paper is that when overconfidence influences prices, *stock prices overreact to private information arrival and underreact to public information arrival.*²⁶ The overreaction to private information occurs because investors overweight their private signals, so they are drawn too far from their initial prior beliefs. Having done so, they are unduly confident about their revised beliefs. When a noisy public signal arrives, they tend to correct toward true values, but only part way. This underreaction to public news explains the evidence from many event studies of long run stock market under- or over-performance in the same direction as the initial price reaction to public events (see Subsection 3.3.3). The pattern of overreaction-to-private-/underreaction-to-public- signals is potentially testable by examining whether, on dates when there are no public information releases for a firm, its stock price moves tend to be reversed in the long term more than price moves coincident with the release of public information about the firm.

3.3.2 Momentum, Reversals, and Volatility

Many studies calculate autocorrelations without conditioning on the arrival of public information. (The autocorrelations discussed earlier do not condition on the *value* of the public

²⁵Although the price changes are positively autocorrelated, the event that induces them is *uncorrelated* with the subsequent price change. This may seem counterintuitive, but the reason is simple. The mispricing has the same sign as ϵ , the error in the private signal. The public signal $\theta + \eta$ is unrelated to the sign of the mispricing, so a high public signal is just as likely to be associated with a positive as a negative long-run correction. In contrast, the price change when the public signal is announced reflects not just the public signal, but also the mispricing. The more underpriced the security, the more positive on average will be the stock price reaction to further news. Thus, a favorable event-date stock price change is associated with a positive future average trend.

²⁶An interesting analogy is found in the theory of Prendergast and Stole (1996). They find that managers will behave in this fashion in order to build and maintain reputation.

signal, but do rely on the existence of the signal at the specified date.) For example, return autocorrelations in oft-cited studies such as Fama and French (1988) and Jegadeesh and Titman (1993) are unconditional. To calculate an unconditional autocorrelation in the model, consider an equivalent experiment where the econometrician picks consecutive dates for price changes randomly (dates 1 and 2, versus dates 2 and 3). The date 2 and 3 price changes are positively correlated, but the date 1 and 2 price changes are negatively correlated. Suppose that the econometrician is equally likely to pick either pair of consecutive dates. Then the overall autocorrelation is

$$\frac{\sigma_C^4 \sigma_\theta^8 (\sigma_C^2 - \sigma_\epsilon^2)}{2(\sigma_\theta^2 + \sigma_C^2)[\sigma_C^2(\sigma_\theta^2 + \sigma_p^2) + \sigma_\theta^2 \sigma_p^2]^2},$$

which is *negative* under overconfidence.

Proposition 3 *If investors are overconfident, price changes are unconditionally negatively autocorrelated in both the short and long run.*

Thus, the constant-confidence model does not explain unconditional price momentum (as documented, for example, in Jegadeesh and Titman (1993)). However, the unconditional autocorrelation will be positive (consistent with the momentum evidence) if the hump or trough in the average price pattern is smooth. The negative autocovariance of price changes surrounding a smooth extremum will be low in absolute value (as discussed in the introduction; see Figure 2). The model in Section 4 based on biased self-attribution and outcome-dependent confidence leads to a smooth overreaction phase up to the extremum, consistent with short-term momentum.

The basic model also implies that overconfidence can decrease volatility around public announcements. It can be calculated directly that date 2 price change volatility $\text{var}(P_2 - P_1)$ can either increase or decrease in σ_C^2 . (Explicit expressions for the variances of this section are contained in Appendix A.) Intuitively, greater overconfidence causes underweighting of the public signal, which tends to reduce date 2 variance. However, overconfidence causes wider swings at date 1 away from fundamentals, which create a greater need for corrective price moves (and variance) at date 2 (and date 3). In contrast, Odean (1995) finds, in a setting without a noisy public information arrival, that overconfidence increases volatility. This occurs in our model at date 1, where the date 1 price volatility $\text{var}(P_1 - P_0)$ decreases with σ_C^2 , and therefore rises with confidence. The differing results on volatility suggest that, in samples broken down by types of news event, either excess or insufficient volatility may be possible.

Consider again an econometrician who does not condition on the occurrence of private or public news arrival. He will calculate price change variances placing equal weights on

price changes $P_1 - P_0$, $P_2 - P_1$, and $P_3 - P_2$. The unconditional volatility is therefore just the arithmetic mean of $\text{var}(P_3 - P_2)$, $\text{var}(P_2 - P_1)$, and $\text{var}(P_1 - P_0)$. When there is no overconfidence, $\sigma_C^2 = \sigma_\epsilon^2$, this reduces to $\sigma_\theta^2/3$. The excess volatility is the difference between the volatility with overconfidence and $\sigma_\theta^2/3$. This is positive so long as there is overconfidence, $\sigma_C^2 < \sigma_\epsilon^2$.

Proposition 4 *Overconfidence increases volatility around private signals, can increase or decrease volatility around public signals, and increases unconditional volatility.*

Thus, consistent with Odean (1995), overconfidence creates excess volatility. This may help explain the findings of Shiller (1981) and Shiller (1989) that stock prices are too volatile relative to dividends. If investors were *underconfident*, $\sigma_C^2 > \sigma_\epsilon^2$, then (see the expressions in Appendix A) there would be *insufficient* volatility relative to the rational level. Thus, it should not be presumed that excess volatility is a natural consequence of almost any model of imperfectly-rational pricing. Rather, evidence in favor of excess volatility supports models of overreaction to information over models of underreaction.

3.3.3 Event Study Implications

In event studies, public-news-arrival-date security price movements are interpreted as responding to information revealed by the event. Many recent studies (footnote 1) provide evidence of post-event price trends, suggesting that the market price does not fully incorporate information contained in the events.

We now slightly generalize the model for application to event studies. We assume that the date 2 signal is no longer public, but is instead received *privately* by the firm's manager (or other individual such as an analyst), and that this individual takes an action (the 'event') which is publicly observed.

Let $P_2^C(s_2)$ be the valuation that would be placed on the security by an overconfident investor at date 2 were he to observe the signal s_2 in addition to his signal s_1 . (Since we examine events that fully reveal s_2 , this is in equilibrium just the post-event stock price P_2 .) Let $P_2^R(s_2)$ be the comparable valuation that would be set by a fully rational investor. The date 2 *mispricing* then is just the difference $P_2^R(s_2) - P_2^C(s_2)$. The nature of the stock price reaction to an event depends critically on whether the event depends on the date 2 mispricing or not. We thus begin with the following definition:

Definition 1 *An event is a random variable that is independent of all random variables in the model except possibly the information signals s_1 and s_2 . A non-selective event is an event that is independent of the date 2 mispricing $P_2^R(s_2) - P_2^C(s_2)$. A selective event is an event whose occurrence and/or magnitude depends on the date 2 mispricing.*

Examples of non-selective events are public news about demand for a firm’s product or the supply of its inputs, or (perhaps) revelation of a breakthrough in its production processes. A *selective* event, in contrast, contains information about the pricing error and about the post-event price movements. Sophisticated managers or analysts who are not overconfident are likely to initiate selective events, such as repurchasing shares or making buy recommendations, when a firm’s shares are undervalued by the market.

Examples of selective events are a new issue if the firm may have undertaken the new issue because the manager correctly believed the stock to be overvalued by the market, or an analyst’s ‘buy’ recommendation if the analyst correctly believed the stock to be undervalued.

Since the market overweights the private signal, the date 1 and date 2 pricing errors (i.e., the deviations of market price from the price that would obtain if investors were not overconfident) are linear in the private signal error ϵ . Thus, selectivity boils down to whether the manager’s action is independent of ϵ .

Non-Selective Events

Since a non-selective event is an action that is unrelated to the pricing error at date 2, it will not forecast future price movements. This observation can be formalized as follows.

Proposition 5 *If investors are overconfident, then an event is non-selective if and only if it is followed by zero average post-announcement abnormal price changes.*

A simple type of non-selective event is a variable that is linearly related only to s_2 . Such an event is independent of the error term in the private signal ϵ , and will not forecast future price movements, so that $\text{cov}(s_2, P_3 - P_2) = 0$.²⁷

The above discussion suggests that if a manager reports sales, cash flow, or earnings without regard to mispricing, there will be no post-announcement drift following changes in these variables. However, we will show that this conclusion changes if we relax either our assumption that earnings reports are non-selective, or our assumption that earnings changes are measured relative to past earnings, rather than relative to earnings forecasts.

Selective Events

Many public events, such as repurchases, seem to be undertaken by management at least partially in response to market mispricings. These *selective* public events will forecast future price changes. For tractability, we consider selective events that are linear functions of the

²⁷The public signal s_2 is uncorrelated with future price changes, yet as we showed in Proposition 2, the stock price *reaction* to this event is positively correlated with the later price change. Intuitively, the public signal is unrelated to the error in the private signal. So even though it is informative about value, it is unrelated to the market pricing error. Thus, it cannot predict future price changes. The market price change in response to the public event, in contrast, is related to the pricing error; on average it acts to correct it, as does the subsequent price change as well.

date 2 mispricing, so that, for the remainder of this section, the term ‘selective’ will apply to events which satisfy this condition.

Consider a manager who observes P_1 (and therefore s_1) and receives s_2 at date 2. The manager can undertake an exchange offering, and will do so if he can do the exchange at a favorable equity price. In this setting, the size of the offering will fully reveal s_2 to the market. Thus, the manager will condition the size of the offering on the mispricing at date 2, which he knows precisely, since he knows both s_1 and s_2 . It can easily be shown that in this setting the date 2 pricing error is proportional to the expected error in the private signal:

$$\epsilon^* = E[\epsilon | P_1, s_2],$$

where the expectation is again taken with respect to rational beliefs.

When $\epsilon^* < 0$, the manager believes the market has undervalued the firm, so the firm can ‘profit’ by offering debt for equity. The more undervalued the firm, the greater the size of the offering. If $\epsilon^* > 0$, an equity-for-debt swap would be preferred instead. Similarly, we could interpret the event as either a repurchase or new issue of equity, with the size of the equity sale being a monotonically increasing function of ϵ^* .²⁸ The covariance $\text{cov}(P_3 - P_2, \epsilon^*)$ describes the relation between the corporate event and the post-event price change. This covariance is *negative* so long as individuals are overconfident ($\sigma_C^2 < \sigma_\epsilon^2$). Thus, a good news event such as a repurchase (indicating $\epsilon^* < 0$) is associated with a positive event date price change and a positive post-announcement abnormal price change.

Proposition 6 *If investors are overconfident, then selective events that are engaged in when the stock appears underpriced (overpriced) based on the date 1 market price and the date 2 signal will be associated with positive (negative) event-date abnormal price changes and will on average be followed by positive (negative) post-announcement abnormal price changes.*

Proposition 6 sounds similar to Proposition 2, but the content differs. In Proposition 2 price changes were correlated even though the event of public news arrival was uncorrelated with later price changes (see Proposition 5). In Proposition 6, the event itself is correlated with future price changes (see also footnote 27).

This analysis implies long run underperformance following IPOs as well as seasoned issues, under the premise that the announcement of an IPO is a ‘bad news event’ for investors. Since IPO firms are private prior to the event, we cannot measure the event date stock price reaction empirically. However, given the consistent general finding of negative

²⁸We are assuming here that the manager is maximizing the intrinsic value of the firm. The manager’s actions could also be a much more complicated function of both the mispricing and the final stock price, depending on the manager’s incentive function.

stock price reactions to the issuance of equity and equity-like securities, and the evidence of inferior post-IPO accounting performance (Jain and Kini (1994), Mikkelson, Partch, and Shah (1995), Teoh, Wong, and Rao (1996)), it seems likely that that IPO announcement is indeed on average bad news. Under this interpretation, the evidence that IPOs do internationally exhibit long-run average underperformance (even measured from the offering price) for several years after the issue²⁹ is consistent with the model.

We have interpreted the model in terms of firms buying or selling shares to exploit mispricing. An alternative interpretation is that a manager with favorable information ($\epsilon^* < 0$) would like to signal good news to the market, and chooses an action (such as a repurchase, dividend, debt for equity swap, or stock split) to reveal his information. Various signaling models beginning with Ross (1977) have shown that certain financial signals may be more costly for a firm or manager to take if the manager has adverse information than if he has favorable information. With a continuous signal, this typically leads to a fully revealing equilibrium, consistent with our assumption that ϵ^* is revealed to the market at the event date.

Since the ‘event’ in our model is continuous, announcements of firm performance measures such as sales, cash flow or earnings can be interpreted as corporate events. Whether they are selective depends on the assumed behavior of managers and the precise way that changes in these variables are measured.

The earnings event itself, or a managerial earnings forecast, may be selective if managers report earnings (or forecasts) strategically. This will occur if managers report more favorably when they believe that the market is undervaluing their firm.³⁰ There may be settings where managers have such an incentive to manage earnings to correct market mispricing. Suppose managers benefit if the market perceives the firm favorably in the short run, but incur a penalty for a report that proves in the long-run to be over-aggressive. If managers derive rising marginal disutility from a lower stock price (owing to fear of takeover, dismissal, or unwanted attention from activist investors), then the net benefit from issuing a more favorable report may increase when the stock is more undervalued. In such a setting, changes in earnings or managerial forecasts will be selective events, so there will be positive post-announcement abnormal price changes after an earnings increase and negative price changes after an earnings decrease.

A different way in which changes in earnings or other performance variables can be selective events is if the change in the variable is measured relative to an analyst forecast

²⁹See Ritter (1991), Loughran, Ritter, and Rydqvist (1994).

³⁰In general, managers have some discretion over earnings through the use of accounting adjustments (accruals), or by manipulating actual cash flows.

benchmark. If the analyst’s forecast released at date 1 reflects the same private signal used by the informed to set prices, then this benchmark will be related to the market misvaluation. The difference between actual earnings reported at date 2 and the forecast made at date 1 will therefore be negatively correlated with the mispricing, and therefore positively correlated with post-event price changes.³¹

To formalize these ideas, suppose first that instead of reporting or forecasting based on s_2 truthfully, managers report earnings of $s_2^* = s_2 - \kappa\epsilon^*$, where $\kappa > 0$ is a constant. In other words, the manager biases the report or forecast upwards (downwards) for an undervalued (overvalued) stock. It follows that $\text{cov}(P_3 - P_2, s_2^*) = -\kappa\text{cov}(P_3 - P_2, \epsilon^*) > 0$, so higher earnings are associated with greater post-announcement-date price changes.

Next, to focus on the choice of the date 1 benchmark, assume that analysts at date 1 forecast earnings based on the private signal $\theta + \epsilon$.³² The resulting surprise provided by the public signal (earnings) relative to the forecast, $z \equiv \theta + \eta - E[\theta + \eta | \theta + \epsilon]$, is therefore positively correlated with the future price change $P_3 - P_2$ (see Appendix A). Intuitively, the higher the public signal relative to expectations, the higher is the public signal relative to the private signal, so the greater the date 1 undervaluation. This implies a higher expected future price change.³³

Proposition 7 *If either:*

1. *Managers or analysts report or forecast higher (lower) earnings when they perceive the firm to be undervalued (undervalued) [specifically, if they report s_2^* rather than s_2], or*
2. *earnings surprises are measured relative to earnings forecasts that incorporate the private signal, then*

positive (negative) earnings announcements or forecasts will on average be followed by positive (negative) post-announcement abnormal price changes.

Thus, the analysis is consistent with evidence of ‘underreaction’ of stock prices to accounting performance relative to forecasts cited in the introduction (Bernard and Thomas

³¹A similar conclusion applies if we further assume that analysts share the overconfident beliefs of the informed. Elsewhere we are interpreting analyst ‘buy’ recommendations as selective events that act to *correct* mispricing. These two interpretations will be consistent so long as (1) analysts are overconfident, but not as overconfident as the rest of the market; or (2) some but not all analysts are overconfident.

³²We do not require that the analysts be overconfident, only that they possess a signal correlated with the informed signal. However, the evidence reported by Abarbanell and Bernard (1992) of positive serial correlation in forecast errors is consistent with analysts’ sharing the biases of the informed.

³³It is easy to show that the positive correlation between the surprise and the future price change will also obtain if (1) the analyst receives a noisy version of the private signal, or (2) the analyst’s shares the bias of the informed (*i.e.*, overestimates the precision of the informed).

1989). This occurs because measuring performance relative to forecast makes the event selective. Thus, post-earnings drift should be larger when earnings surprises are measured relative to forecast than when they are measured relative to past earnings.

The model's event study predictions also apply to events undertaken by outsiders who have information about the firm. An example is an analyst's recommendation to buy or sell shares of the firm. Thus, the analysis is consistent with evidence on analyst 'buy' and 'sell' recommendations discussed in the introduction. On the other hand, the analysis implies that there should be no 'drift' following non-selective events. For example, regulatory shifts that affect stock prices should not lead to subsequent abnormal performance.

One could add further realistic elements to this model, such as the possibility that no event occurs, or a multiperiod decision of when to time the event. However, the basic intuition would still hold that if managers take advantage of mispricing, the event-date stock price reaction will only partly reverse the market's pricing error (because of overconfidence), so the average post-announcement price change will have the same sign as the initial reaction.

The model implies that mispricing can be measured noisily by fundamental/price ratios such as book/market, dividend yield, or earnings/price ratios. If the fundamental in the numerator is relatively constant, than a market overreaction in the direction of undervaluation will increase these ratios, and an overreaction in the direction of overvaluation will decrease these ratios. The measures are imperfect to the extent that there is noise in the numerator variables (for fuller discussion, see Daniel, Hirshleifer, and Subrahmanyam (1997)). The book/market ratio, for example, can be viewed as an empirical approximation of the quantity $\bar{\theta} - P_1$ of the model. In the model here positive selective events (such as repurchase) are undertaken in response to market undervaluation, and negatively selective events (such as new issues) in response to overvaluation. Thus, the expected size of a selective event should be related to the size of the misvaluation. Furthermore, the probability of a positive event occurring is increasing in the degree of undervaluation. It follows that:

- Proposition 8** 1. *The expected size of a positive (negative) selective event is increasing (decreasing) in the firm's fundamental/price ratios (such as book/market).*
2. *The probability of a positive (negative) selective event occurring is increasing (decreasing) in the firm's fundamental/price ratios.*

Thus, the analysis predicts repurchases and other positive events will tend to occur when market, industry, or firm book/market ratios are high, and equity issuance and other negative selective events will tend to occur when price-scaled ratios are low. This is consistent

with the finding of Pagano, Panetta, and Zingales (1996) that the probability of an IPO is positively related to the market/book ratio in the company's industrial sectors.

The analysis further implies that better pre-event price performance should be associated with worse post-event performance (either including or excluding the event date). This is because mispricing arises from overreaction to private information, and firms are selecting events based on mispricing. Thus, prior performance measures the extent of the mispricing. Formally, this follows because $\text{cov}(P_3 - P_2, P_1 - P_0) < 0$ and $\text{cov}(P_3 - P_1, P_1 - P_0) < 0$.

Proposition 9 *Better pre-event stock price performance is on average associated with worse post-event performance (either including or excluding the event date).*

This is potentially testable on various corporate events. In the context of stock splits, Ikenberry, Rankine, and Stice (1996) report that post-split performance for two-for-one splits (excluding the split month) is negatively related to pre-split performance in their sample.

This analysis also implies that if confidence is approximately constant across time and firms, then the event date price changes should be positively correlated with post-announcement drift. This is just underreaction, and follows under the conditions of Proposition 2. Proposition 2 was based on conditioning on a non-selective news event, namely, the arrival of s_2 . Even though s_2 is private information here, the result is the same because s_2 is fully revealed by the corporate action, so that P_2 is identical in all states to what it would be if s_2 were made public directly. Thus, $\text{cov}(P_3 - P_2, P_2 - P_1)$ is the same in both cases.

Proposition 10 *If confidence is constant across time and firms, then the event date price changes will be positively correlated with post-announcement drift.*

However, cross-sectional differences in confidence across sample firms could reverse this implication, because higher overconfidence will tend to reduce in absolute magnitude the event-date price change, and increase the post-event price change. These effects can in principle be disentangled using a proxy for investors' confidence about the firm such as the book/market ratio.

The model above, which is based on static confidence, does not explain the negative association between past long-term accounting performance (sales and cash-flow growth) and price changes documented by Lakonishok, Shleifer, and Vishny (1994). However, the dynamic model based on time-varying confidence owing to biased self-attribution is consistent with both short-term 'drift' and long-term reversals, as discussed in the next Section.³⁴

³⁴We show in Section 4 that biased self-attribution leads to a more complex relation between public

4 Outcome-Dependent Confidence

The implications described so far were based on a fixed level of confidence by informed traders. However, psychological theory and evidence suggest that confidence varies over time depending on past actions and outcomes (see Section 2). This evidence suggests that events that confirm an individual’s beliefs and actions to boost confidence too much, and events that disconfirm to weaken confidence too little.

In the constant-confidence model of Section 3, the unconditional short-lag autocorrelation calculated by an econometrician who does not know the date of information arrival was negative. As illustrated in Figure 1, this arose because of the strong average reversal for price changes straddling the date 1 extremum of the impulse response function. With outcome-dependent confidence, we will show that the date 1 average overreaction continues at date 2, so that the average reversal at the date-2 extremum of the impulse response function can be smooth, as illustrated in Figure 2. This leads to an average pattern of continuation both for the pair of price moves leading up to the extremum, and those following the extremum. As argued in the introduction, such a pattern is consistent with both short-run momentum and long-term reversal. Similarly, the outcome-dependent analysis can explain both short-run continuation in stock price responses to financial disclosures, and long-run reversal of these responses.

4.1 Biased Self-Attribution and Autocorrelations

Consider an informed individual who initially is not overconfident, and who buys or sells a security based on his private information. A public signal *confirms* his trade if they have the same sign (“buy” and a positive signal, or “sell” and negative signal). Assume that if the later public signal confirms his trade, the individual becomes more confident, and if it disconfirms his confidence decreases by little or remains constant. Then when there are two consecutive price increases, the second up-move is reinforced by increased confidence, which intensifies overreaction. Similarly, there will be a corresponding reinforcement of the second down move when there are two down-moves (depending on the feasibility of short sales). Both the upside effect and the downside contribute to positive autocorrelation (possibly weakened or eliminated by short sales restrictions). There is an opposing effect brought about when good news follows bad or vice versa. By assumption this opposing effect is relatively weak. Thus, overconfidence can, under appropriate parameter restrictions, intensify overreaction and bring about positive autocorrelation in stock prices during the announcements and future price changes that is consistent with evidence both of drift after short-term earnings changes and reversals after long-term earnings changes.

initial overreaction phase.

This point is distinct from the correction-phase positive autocorrelation shown in Subsection 3.3.1. Such correction-phase underreaction obtains here as well. Initial overreaction is reversed in the long run as public information arrival gradually draws the price back toward fundamentals. This yields a hump-shaped pattern for cumulative abnormal price changes after a favorable private signal, and a U-shaped pattern after an unfavorable private signal (see Figure 2).

We present two models with dynamic confidence. The model presented in 4.2 is tractable enough to be analytically solved, but is somewhat stylized. The model presented in Subsection 4.3 allows us to develop more complex implications, but can only be solved using simulation methods.

4.2 The Simple Model with Outcome Dependent Confidence

We retain the assumptions of the basic model, with the following modifications. We still allow for, but no longer require, initial overconfidence, so $\sigma_C^2 \leq \sigma_\epsilon^2$. For tractability, the public signal is now discrete, with $s_2 = \{1, -1\}$ released at date 2. We assume that the precision assessed by the investors at date 2 about their earlier private signal depends on the realization of the public signal in the following way. If

$$\text{sign}(\theta + \epsilon) = \text{sign}(s_2),$$

confidence increases, so the perceived noise variance decreases to $\sigma_C^2 - k$, where $0 < k < \sigma_C^2$. If

$$\text{sign}(\theta + \epsilon) \neq \text{sign}(s_2),$$

confidence remains constant, so the perceived noise variance remains at σ_C^2 .³⁵

The probability of receiving a public signal +1 is denoted by p . For a high value to be a favorable indicator of value, p must tend to increase with θ . However, for simplicity we take the limiting case where the signal is virtually pure noise, so that p is a constant.³⁶

Given normality of all random variables, the date 1 price is

$$P_1 = E_C[\theta|\theta + \epsilon] = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_C^2}(\theta + \epsilon). \quad (3)$$

The date 0 price is the prior mean of zero. If $\text{sign}(\theta + \epsilon) \neq \text{sign}(s_2)$, then confidence is constant. Since the public signal is virtually uninformative, the price (virtually) does not

³⁵More generally, perceived noise variance would rise, but not as much as perceived noise variance decreases when the public signal confirms the private signal. The results would be similar.

³⁶Allowing p to vary with θ creates intractable non-normalities. Appendix B provides a discrete model which derives similar results using an informative public signal.

move at date 2. However, if $\text{sign}(\theta + \epsilon) = \text{sign}(s_2)$, then the new price is calculated using the new level of the assessed variance of ϵ . This price, denoted by P_{2C} , is

$$P_{2C} = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_C^2 - k}(\theta + \epsilon).$$

4.2.1 Empirical Implications of the Simple Model

Explicit calculations and expressions for covariances for this subsection are in Appendix C. It can easily be shown that

$$\text{cov}(P_2 - P_1, P_1 - P_0) > 0.$$

Thus, the model shows that short-term momentum can apply during the overreaction phase, not just the correction phase. As a result,

$$\begin{aligned} \text{cov}(P_3 - P_1, P_1 - P_0) &< 0 \\ \text{cov}(P_3 - P_2, P_2 - P_1) &< 0, \end{aligned}$$

because the dates 1 and 2 overreactions must be reversed in the long-term.

Proposition 11 *If investor confidence varies as a function of outcomes owing to biased self-attribution, then prices can exhibit short-run momentum as a result of continuing overreaction.*

It has been common in casual usage to equate positive autocorrelations with underreaction, and negative autocorrelations with overreaction. This correspondence is valid in some settings. However, fundamentally, these terms describe whether prices move too much or too little in response to information. In the current model prices trend upward (or downward) across two dates as a result of continuing *overreaction* to the date 1 private signal. The date 2 public signal on average leads to further overreaction in the same direction. Thus, while negative autocorrelations are a result of overreaction in our model, positive autocorrelations can result from either underreaction or overreaction.

Intuitively, further dates of noisy public information arrival should eventually cause the mispricing to be corrected (so long as confidence does not explode infinitely). This process leads to momentum during the correction phase, just as in the basic model of Section 3. Thus, there is positive short-term autocorrelation both in the overreaction phase and the correction phase.

Suppose we add an extra public information arrival date $3'$ between date 2 and 3, to show that if the arrival of public information is gradual, then short-term momentum effects

dominate. If date 2 is viewed as the average ‘hump’ or ‘trough’ (see Figure 2), the only negative single-lag autocorrelation is that which straddles the extremum, and all long-lag (single-period-price changes, lag greater than one) autocorrelations are negative. The absolute value of the negative single-lag straddling autocorrelation approaches zero as the public signal becomes arbitrarily noisy. Thus, in an event-unconditional sample, short-term momentum will be observed (positive autocorrelation) and long-term reversal (negative autocorrelation).

To show this, let us add a date 3’ between dates 2 and 3, where a public signal $\theta + \eta$ is released. For simplicity, we assume that overconfidence is not affected by the release of the second public signal. As in Section 3, η is a zero mean, normally distributed variable with variance σ_p^2 , and is independent of all other random variables. The price at date 3’ when overconfidence is not revised at date 2 is given by (2). When overconfidence is revised at date 2, the price at date 3’, denoted by $P_{3’C}$ is given by the same expression as (2), except that σ_C^2 is replaced by $\sigma_C^2 - k$; i.e.,

$$P_{3’C} = \frac{\sigma_\theta^2(\sigma_C^2 - k + \sigma_p^2)}{D}\theta + \frac{\sigma_\theta^2\sigma_p^2}{D}\epsilon + \frac{\sigma_\theta^2(\sigma_C^2 - k)}{D}\eta,$$

where $D \equiv \sigma_\theta^2(\sigma_C^2 - k + \sigma_p^2) + (\sigma_C^2 - k)\sigma_p^2$.

It is easy to show that the remaining single-period-price change autocorrelations are

$$\begin{aligned} \text{cov}(P_3 - P_3', P_2 - P_1) &< 0 \\ \text{cov}(P_3 - P_3', P_1 - P_0) &< 0 \\ \text{cov}(P_3' - P_2, P_1 - P_0) &< 0 \\ \text{cov}(P_3 - P_3', P_3' - P_2) &> 0 \\ \text{cov}(P_3' - P_2, P_2 - P_1) &< 0. \end{aligned}$$

Date 2 is the extremum (the ‘hump’ or ‘trough’ date after which the average correction begins). By (10) and the above, the single-period-price change single-lag autocorrelations that fall entirely within either the overreaction phase or within the correction phase are positive [$\text{cov}(P_2 - P_1, P_1 - P_0)$ and $\text{cov}(P_3 - P_3', P_3' - P_2)$]. The only negative single-period-price change single-lag autocorrelation is the one that straddles the extremum $\text{cov}(P_3 - P_2, P_2 - P_1)$. Increasing the lag forces the autocorrelations to straddle the extremum. We see that all the extremum-straddling single-period-price change autocorrelations are negative. (Moving outside the formal model to a setting where time is subdivided finely, returns at longer lags will still be more likely to straddle the peak or trough, and therefore tend to be negatively autocorrelated.) Thus, the analysis predicts that short-lag autocorrelations will

be positive, and long-lag autocorrelations will be negative. There is momentum during the overreaction, reversal at the hump or trough, and momentum during the correction.³⁷

Under appropriate parameter assumptions, the negative single-lag autocorrelation surrounding the peak or trough will be arbitrarily close to zero. This will occur if either the extra overreaction or the start of the correction is weak. The extra overreaction is small if confidence is boosted only slightly ($k > 0$ small) when an investor's trade is confirmed by public news.³⁸ The initial correction is slight if the further noisy public signal is not very informative (σ_η^2 large). When parameter values are such that this straddling autocorrelation is not too large, it will be outweighed by the positive autocorrelations during the heart of the overreaction phase or the heart of the correction phase. (See the calculations in Appendix C.) In other words, an econometrician calculating autocorrelations unconditionally would find, in a large sample, a positive single-lag autocorrelation. In contrast, longer-lag pairs of price changes that straddle the extremum of the impulse response function will tend to be opposed, because a price change drawn from the overreaction phase tends to be negatively correlated with a price change drawn from the correction phase. Thus, the overconfidence theory provides a joint explanation for both short-term momentum and long-term reversals.

Proposition 12 *If investor confidence changes owing to biased self-attribution, and if overreaction or correction is sufficiently gradual, then stock price changes will exhibit unconditional short-lag positive autocorrelation ('momentum') and long-lag negative autocorrelation ('reversal').*

According to Jegadeesh and Titman (1993), their momentum evidence is "... consistent with delayed price reactions to firm-specific information." The model of Section 3 is consistent with this interpretation if the information arriving is public. However, Proposition 12 suggests an alternative interpretation. When confidence varies, momentum may occur not because the market is slow to react to news, but because the market initially overreacts to the news, and later continues this overreaction even further when further public news arrives.³⁹

³⁷Autocovariances involving long-run price changes that extend to the terminal date, such as $\text{cov}(P_3 - P_1, P_1 - P_0)$, $\text{cov}(P_3 - P'_3, P'_3 - P_0)$, and $\text{cov}(P_3 - P_2, P_2 - P_0)$ will also be negative since these involve overreactions and their corrections.

³⁸If the signal were informative, k would, however, need to be sufficiently large that on average the price does continue to overreact at date 2.

³⁹An alternative possible explanation for momentum is a lead-lag effect coming from delayed reaction to information about common market factors (Lo and MacKinlay (1990)). Jegadeesh and Titman (1993) find that such lead-lag effects do not explain the 3-12 month short-term autocorrelation of stock returns.

4.3 A Dynamic Model of Outcome-Dependent Confidence

We now extend this model to many periods. Because the analytics become intractable, we present numerical simulations. The analysis implies patterns of security price-change autocorrelations consistent with the findings of Section 4.2. It also yields implications for the correlation between public information announcements (such as managers' forecasts or financial reports of sales, cash flows or earnings) and future price changes. Both the short-term and long-term patterns of return autocorrelations described in the introduction, both unconditionally and in relation to public announcements about financial performance measures such as earnings or sales, are consistent with this model of time-varying confidence.

In the model, the investor has a prior on the precision of his private signal, and uses an updating rule that reflects self-attribution bias. Since confirming signals cause the investor to raise his estimate of his signal's precision more than disconfirming signals cause him to revise his estimate downward, on average, the arrival of public information causes his estimate of the precision of his signal to increase.⁴⁰ As a result a price which is already "too high" will, on average, be pushed still higher by public information. It is this feature of updating rule that results in the positive autocorrelation of price changes at short horizons and in positive price changes following earnings increases.

However, after the arrival of *enough* public information, the precision of the cumulative public signal becomes overwhelmingly strong. Thus, so long as investor confidence cannot increase without bounds, even a very overconfident investor will be forced to recognize the truth. As a result the price asymptotes to the true value of the security. This results in long horizon negative autocorrelations in security price changes, consistent with the model in Sections 3 and 4.1. Thus, the model implies that short-term positive changes in financial performance measures such as earnings or sales, will tend to forecast future positive price changes (consistent with Bernard and Thomas (1989, 1990)), yet is also consistent with long-term changes in performance measures forecasting negative future price changes (Lakonishok, Shleifer, and Vishny (1994)).

4.3.1 The Model

We retain the basic structure considered in earlier sections. As before, the (unobservable) value of a share of the firm's stock is $\tilde{\theta} \sim \mathcal{N}(0, \sigma_{\theta}^2)$. The public noise variance σ_{θ}^2 is common knowledge. At time 1, each informed investor receives a private signal $\tilde{s}_1 = \tilde{\theta} + \tilde{\epsilon}$ where

⁴⁰Given biased self-attribution, it may be asked why confidence does not gradually evolve to be extremely strong. The answer lies in the fact that an overconfident individual will disproportionately often receive public information that reflects poorly on his information precision, relative to his expectation. This tendency opposes the effect of self-attribution bias on confidence.

$\tilde{\epsilon} \sim \mathcal{N}(0, \sigma_\epsilon^2)$. The error variance σ_ϵ^2 is incorrectly perceived by the investor. He estimates σ_ϵ^2 using the *ad hoc* rule described below. On dates 2 through T , a public signal $\tilde{\phi}_t$ is released, $\tilde{\phi}_t = \tilde{\theta} + \tilde{\eta}_t$, where $\tilde{\eta}_t$ is *i.i.d* and $\tilde{\eta}_t \sim \mathcal{N}(0, \sigma_\eta^2)$. The variance of the noise, σ_η^2 , is also common knowledge. We also define Φ_t as the average of all public signals through time t , that is:

$$\Phi_t = \frac{1}{(t-1)} \sum_{\tau=2}^t \tilde{\phi}_\tau = \theta + \frac{1}{(t-1)} \sum_{\tau=2}^t \tilde{\eta}_\tau. \quad (4)$$

The average public signal Φ_t is a sufficient statistic for the $t-1$ public signals, and $\tilde{\Phi}_t \sim \mathcal{N}(\theta, \sigma_\eta^2/(t-1))$. As before, an informed investor forms expectations about value rationally (using Bayesian updating) except for his perceptions of his private information precision. In this regard, (1) he may be initially overconfident (*i.e.*, places too high a precision on his own information relative to the public signals) and (2) more importantly, he incorrectly updates his beliefs about the precision of his signal based on the public signals. At time 1, the investor believes that the precision of his signal is $v_{C,1}$, which is greater than the true precision $v_\epsilon = 1/\sigma_\epsilon^2$. Also, we define $\sigma_{C,1}^2 = 1/v_{C,1}$ as the investor's belief about the noise variance of his private signal. The C subscript here denotes that this is not the true variance, but the belief of the *Confident* investor.

At every subsequent release of public information the investor updates his estimate of the variance based on an *ad hoc* updating rule. If the new public signal (ϕ_t) confirms the investor's private signal s_1 , and the private signal is not too far away from the public signal, then the investor becomes more confident in his private signal. If the new public signal disconfirms his private signal, the investor revises the estimated precision downwards, but not by as much. This updating process is consistent with evidence of biased self-attribution. Because of this differential response to public information, the arrival of public information on average causes the investor to revise his estimate of the precision of his private information above his true precision.

The updating rule that we implement is:

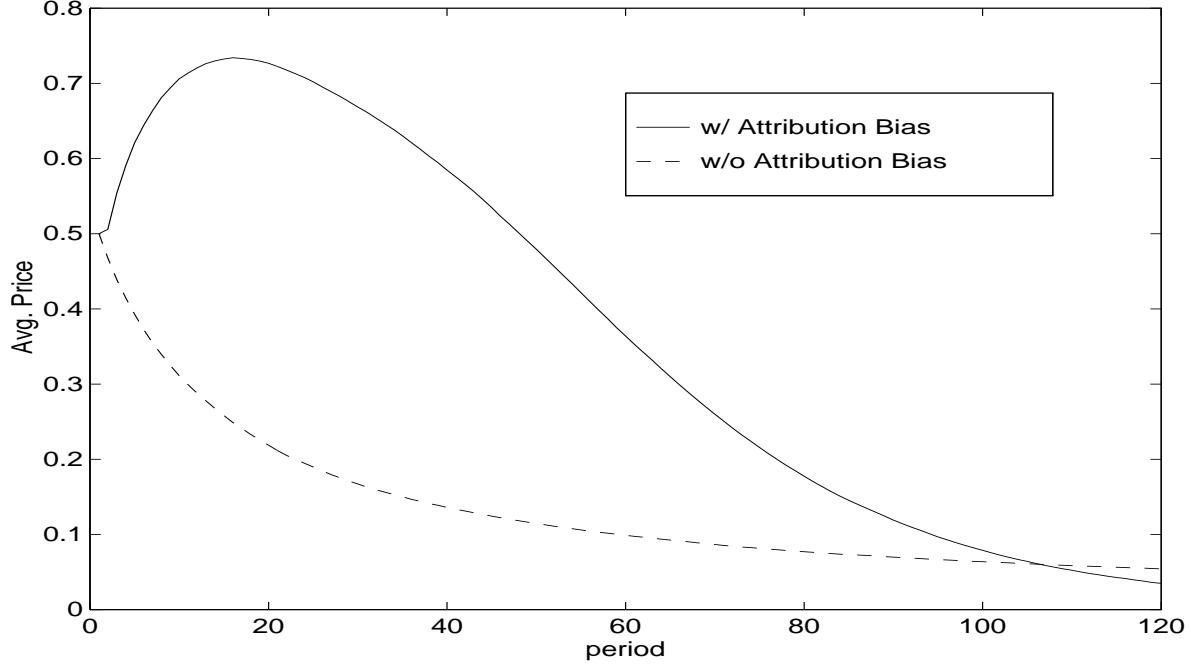
$$\text{if } \begin{cases} \text{sign}(s_1 - \Phi_{t-1}) = \text{sign}(\phi_t - \Phi_{t-1}) \text{ and } |s_1 - \Phi_{t-1}| < 2\sigma_{\Phi,t} & \text{then } v_{C,t} = (1 + \bar{k})v_{C,t-1} \\ \text{otherwise} & v_{C,t} = (1 - \underline{k})v_{C,t-1}, \end{cases}$$

where $\sigma_{\Phi,t}$ is the standard deviation of Φ at time t . Intuitively, if a new public signal ϕ_t deviates from past information in the same direction as the private signal, and the private signal is not too far away from the public signal, then the investor becomes more confident in the private signal. We impose the restriction that $\bar{k} > \underline{k} > 0$. The ratio $(1 + \bar{k})/(1 - \underline{k})$ is an index of the investor's attribution bias.^{41,42}

⁴¹Several alternative *ad-hoc* updating rules consistent with this intuition all led to roughly equivalent simulation results.

⁴²The investor is forming beliefs as if, at each point in time, he knows exactly what the precision of his

Figure 3: Average Price Path Following Private Information Shock



4.3.2 The Equilibrium

Since the investor is risk-neutral and the risk-free rate is zero, at each point in time the stock price is the expectation of its terminal value:

$$P_t = E_C[\tilde{\theta}|s_1, \phi_2, \dots, \phi_t] = E_C[\tilde{\theta}|s_1, \Phi_t].$$

Define $v_\theta = 1/\sigma_\theta^2$, and $v_\eta = 1/\sigma_\eta^2$. The price of the security at time t is given by:

$$\tilde{P}_t = E_C[\tilde{\theta}|s_1, \Phi_t] = \frac{(t-1)v_\eta\Phi_t + v_{C,t}s_1}{v_\theta + v_\eta + v_{C,t}}.$$

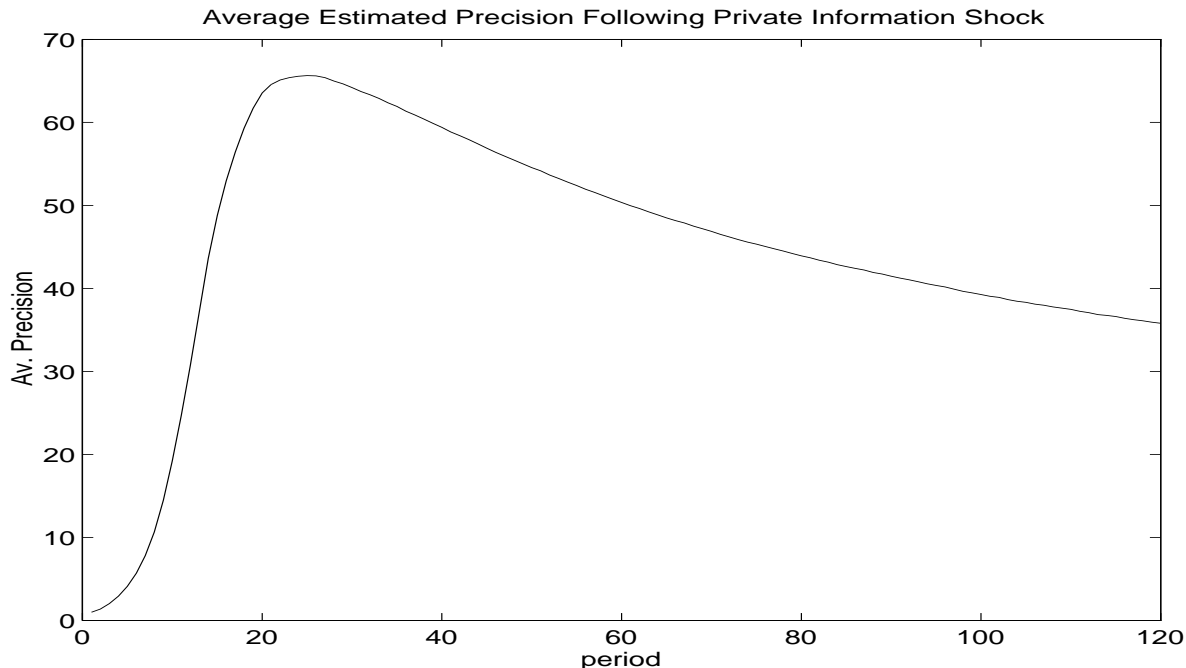
Recall that the precision of Φ is $(t-1)v_\eta$.

4.3.3 Simulation Results and Empirical Implications

For the simulation we used the parameters $\bar{k} = 0.75$, $\underline{k} = 0.1$, $\sigma_\theta^2 = 1$, $\sigma_\epsilon^2 = 1$, $\sigma_\eta^2 = 7.5$. We also make the investor's initial estimate of his precision equal to the true precision of his private signal. We performed this simulation 50,000 times, each time redrawing the

signal is. Rationally he should take into account the fact that $v_{C,t}$ is an estimate. This is done solely for tractability; we expect that the essential results would not change were the investor to incorporate the estimation error in $v_{C,t}$ into his calculations.

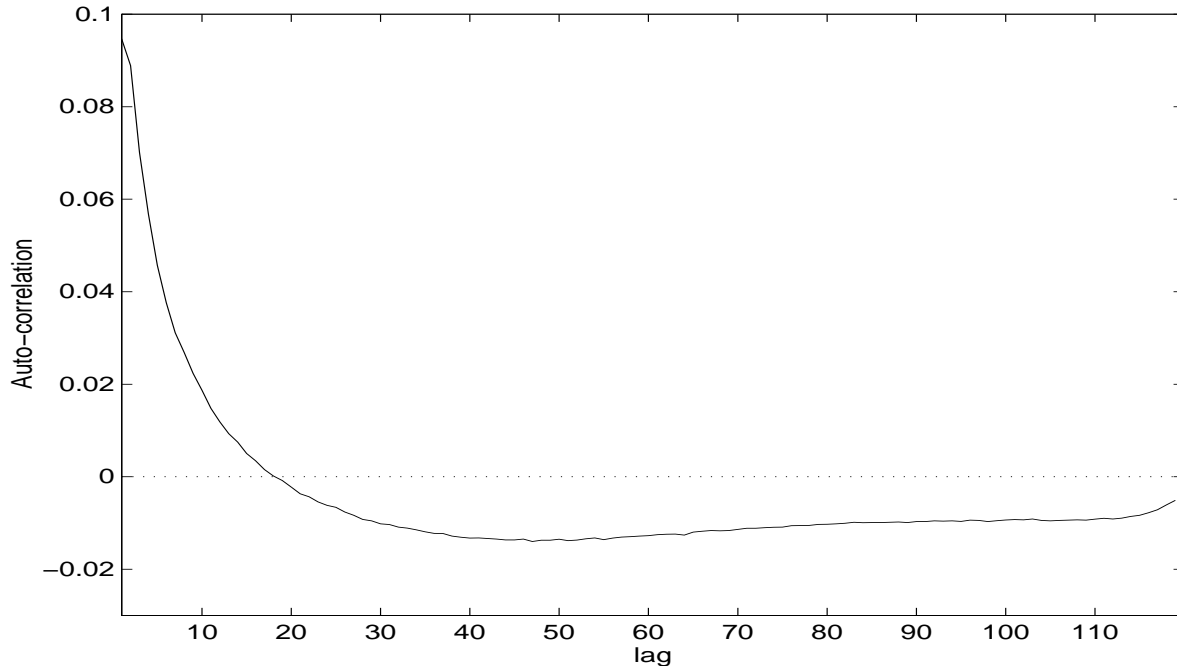
Figure 4:



value $\theta \sim \mathcal{N}(\bar{\theta}, \sigma_{\theta}^{\epsilon})$, the private signal $s_1 = \theta + \epsilon$, and the public information set ϕ_t , for $t = 2, \dots, T$. We set $\bar{\theta}$ to zero without loss of generality. We then calculate the average price of the security for 120 periods after the arrival of the private signal. This is plotted in Figure 3.

Before presenting the full simulation results (where $\tilde{\theta}$ and \tilde{s}_1 are drawn from distributions) it is useful to illustrate some of the dynamics of the model. Figure 3 shows the average price path following a private-signal of $\tilde{s}_1 = 1$ when $\tilde{\theta} = 0$, that is when an investor receives a signal that the security is more valuable than it actually is. The price initially jumps from 0 up to 0.5, which is the best estimate of θ given $s_1 = 1$ and $\sigma_{\epsilon}^2 = \sigma_{\theta}^2 (= 1)$. On average, the price *continues* moving up, reaching a maximum of 0.7366 in period 16. At this point the average price starts to decline, and eventually asymptotes to zero. Thus, we see that there is an initial overreaction phase in which the price moves away from the true value as the investor’s attribution bias causes him to place more weight, on average, on his private information. Of course, eventually the public information become precise enough that the investor has to revise down his beliefs about the value of the security. This is the correction phase. Figure 4 shows the investor’s self-perceived precision as a function of time. This plot reveals the reason for the gradual increase in the security price after the arrival of the private information: the increasing average estimated precision. We also see

Figure 5: Average Price-Change Autocorrelations



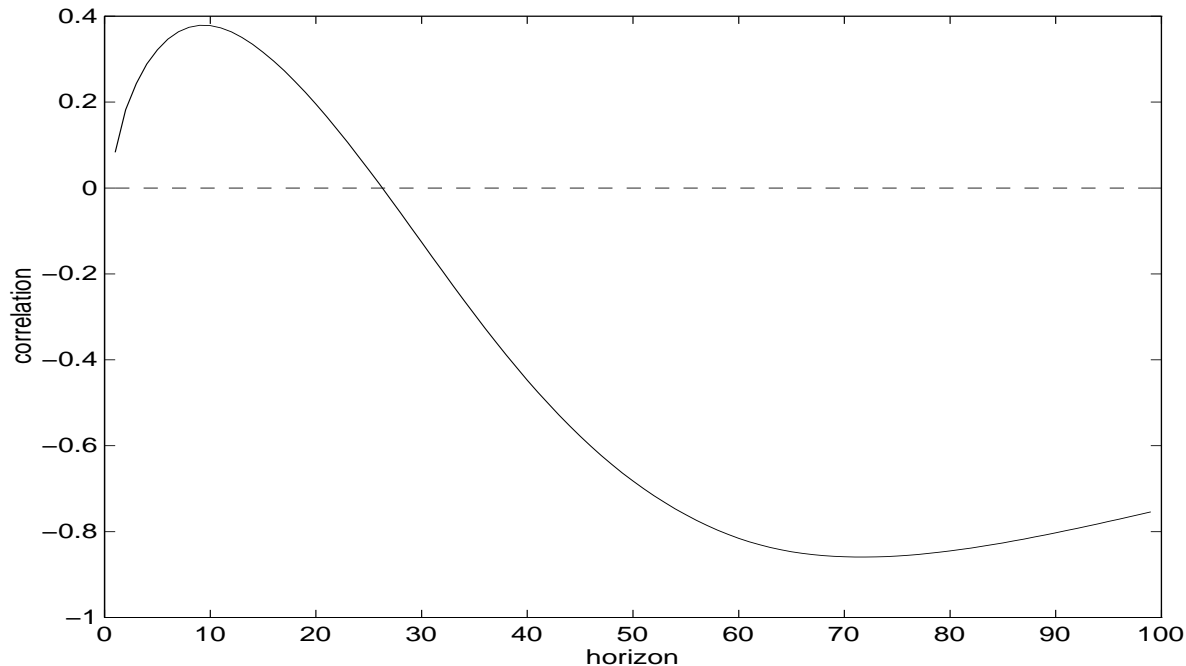
that, after a little over 20 periods, the average self-perceived precision begins to decline as the cumulative public signal becomes more precise. However, this is only part of the reason that the price begins declining toward $\theta = 0$: The price begins to decline, for this example, when the precision of the public signal starts to dominate the precision of the private signal. The investor's estimated precision may still be growing, but if the estimated precision of the public signal grows still faster the price will decline.⁴³

These results suggest that security prices should exhibit momentum in the short run and long run reversal. However, these are all conditional on $s_1 = 1$ and $\theta = 0$. Figure 5 presents the unconditional average autocorrelations of the price changes in the simulation, where now θ and s_1 are resampled for each Monte-Carlo iteration in the simulation. We generated this figure by calculating the price-change autocorrelogram (the set of price-change autocorrelations at lags between 1 period and 119 periods) for each Monte-Carlo iteration. We then find the autocorrelation at each lag by averaging over the 50,000 Monte-Carlo draws.⁴⁴ The resulting plot of average autocorrelations in Figure 5 confirms the intuition derived from Figure 3 that short-lag price change autocorrelations are positive

⁴³This is the reason that the peaks in Figures 3 and 4 do not line up.

⁴⁴The autocorrelation is the unconditional expectation, $\text{cov}(r_t, r_{t+\tau})/\sigma^2(r)$. Given the normality assumption here, the properties of the Monte-Carlo are such that, as the number of Monte-Carlo iterations approaches infinity, the average approaches the true autocorrelation for the system.

Figure 6: Long-Horizon Regression Coefficient for Price Changes on Price Changes



and long-lag autocorrelations are negative.

Since much of the finance literature has concentrated on long-horizon regressions (see, e.g., Fama and French (1988)) rather than multi-lag autocorrelation, we also present the average long horizon regression coefficient from the model:⁴⁵ This is the average (across the Monte-Carlo draws) of the regression coefficient b_τ that is obtained in the regressions:

$$\tilde{R}(t, t + \tau) = a_\tau + b_\tau \tilde{R}(t - \tau, t) + \tilde{\epsilon}(t, t + \tau),$$

where, given the normality of the disturbance terms, we use the price change in place of returns ($R(t, t + \tau) = P(t + \tau) - P(t)$).

To examine long-horizon regression coefficients, Figure 6 presents the corresponding average long-horizon regression coefficients, calculated as a function of the horizon. Again the short horizon coefficients are positive and long-horizon coefficients are negative, consistent with the empirical literature on momentum and reversals. For each lag, long-horizon correlation coefficients will have the same sign as the long-horizon regression coefficients presented in Figure 6.

This implication was based on relatively general assumptions about information arrival process. The main requirement, in addition to the self-attribution bias, is that public

⁴⁵It is straightforward to show that there is a one-to-one mapping between price change autocorrelations and more standard test statistics such as variance ratios or long-horizon regression coefficients.

information arrive gradually. Then, owing to attribution bias, investors will tend to interpret public information as evidence that their private signals are of higher precision than they initially believed, resulting in a *gradual* over-reaction to their private information. Of course, eventually enough public information will arrive so that the overreaction will be corrected.⁴⁶

The conclusions of this initial simulation is summarized as follows.

Result 1 *In the biased self-attribution setting of Section 4.3, if the true share value $\theta = 0$ and the initial private signal $s_1 = 1$, then with sufficient attribution bias the average price at first rises and then gradually declines. This contrasts with a steadily declining price path if here is no attribution bias. In the biased self-attribution setting, average self-perceived precision also initially rises and then declines.*

Result 2 *In the biased self-attribution setting of Section 4.3, short-lag autocorrelations (correlating single-period price changes with single-period price changes) are positive and long-lag autocorrelations are negative.*

Result 3 *In the biased self-attribution setting of Section 4.3, short-term autocorrelations are positive and long-horizon autocorrelations are negative.*

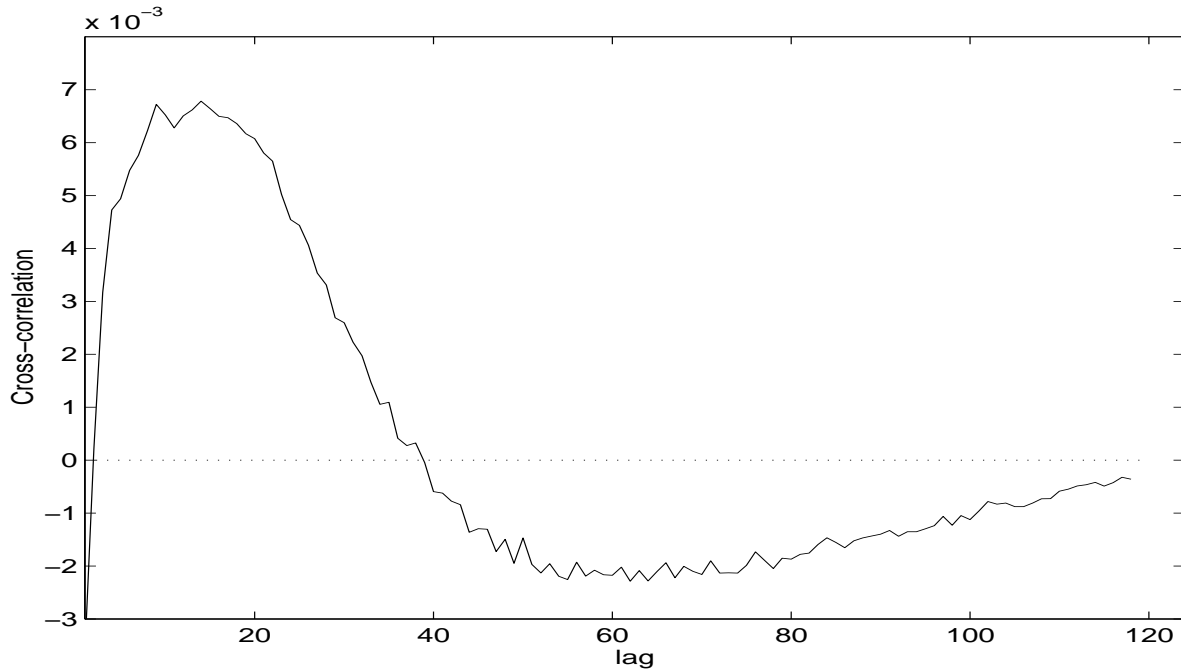
4.3.4 Correlation of Past Accounting Performance with Future Price Changes

Finally, we consider the implications of this model for the correlation between accounting performance and future price changes. Accounting information (sales, earnings, etc.) can be thought of as noisy public signals about θ , so in this subsection we interpret the ϕ 's as changes in accounting performance measures. The results of 4.3.3 for the price change autocorrelations showed that, in response to a positive private signal, the price on average moves up for several periods and then reverses. Consider now the first public signal (at $t = 2$). If the first public signal is positive, it is likely that the first private signal was also positive. Based on the momentum results in this section, this suggests that prices will continue to increase after the arrival date of the public signal. This is consistent with the empirical evidence of Bernard and Thomas (1989, 1990) on post-earnings announcement drift. Of course, eventually prices will move down again as the cumulative public signal becomes more precise and the informed investor begins to put less weight on his own signal. Thus, the analysis of this section suggests that post-earnings announcement drift, like stock-price momentum, may be a phenomenon of continuing overreaction.⁴⁷ In the long-run, of course, the security price will return to its full-information value, implying long-run negative

⁴⁶Similar results apply even if the investor's signal has true precision of 0 (noise trading). All that is required is that he *believe* his precision is positive.

⁴⁷However, the discussion of event-study implications in Subsection 3.3.3 described conditions under which post-earnings announcement drift could be an underreaction effect.

Figure 7: Correlation between Information Changes and Future Price Changes



correlations between accounting performance and future price changes. This is consistent with the results of Lakonishok, Shleifer, and Vishny (1994) who find that the covariance between past 3-5 year financial performance measures such as sales growth and subsequent one-year returns is negative.⁴⁸

To confirm that such a pattern can arise with biased self-attribution, we again calculate the average correlations using our simulation as follows. For each $\tilde{\phi}_t$ (for $t = 2, 120$) we calculate the “earnings change”, defined as:⁴⁹

$$\Delta e_t = \tilde{\phi}_t - \Phi_t = \tilde{\phi}_t - E[\tilde{\phi}_t | \phi_s, s = 2, \dots, t-1]$$

Δe_t is the deviation of ϕ_t from its expected value based on all past public signals.⁵⁰

⁴⁸Several other papers, e.g., DeChow and Sloan (1997), DeBondt and Thaler (1985), and DeBondt and Thaler (1987) also find negative long horizon regression coefficients relating past returns or financial performance measures to subsequent returns. This implies that one or more short horizon, long-lag regression coefficients must be negative (see Appendix D). In contrast, Chan, Jegadeesh, and Lakonishok (1996) do not reject the null of no such a negative relation, possibly owing to a lack of power in detecting long-run reversal.

⁴⁹ Φ_1 is defined to be $\bar{\theta}$, which for the simulation is set to zero.

⁵⁰Note, however, that the expectation ignores information about the private signal (and hence about future public signals which could be extracted from the price. This is consistent with Lakonishok, Shleifer, and Vishny (1994), who use past changes in financial performance measures such as sales and earnings growth, rather than earnings surprises, to forecast future returns.

Then, we calculate the set of sample correlations between the Δe_t and price changes τ periods in the future $\Delta P_{t+\tau} = P_{t+\tau} - P_{t+\tau-1}$. These correlations are then averaged over the 50,000 Monte-Carlo draws. The average correlations are plotted in Figure 7. We see that, as the intuition above suggested, past earnings changes are positively correlated with future price changes in the short-run, but are negatively correlated with long-run future price changes. This pattern of correlations is again consistent with both the evidence of Bernard and Thomas (1989, 1990) and Lakonishok, Shleifer, and Vishny (1994).

Result 3 *In the biased self-attribution setting of Section 4.3, short-lag correlations between single-period stock price changes and past earnings are positive, and long lag correlations are negative.*

4.4 Implications for Underreaction versus Overreaction

In the static model, investors overreact to private information and underreact to public information. We have seen in the dynamic model that public information arrival can be associated with *continuing* overreaction to a private signal. This is *not* a case of short-term underreaction to the private signal, nor of a simple overreaction to public signals. If overconfidence is high, prices still will underreact to public information. The important thing here is that prices underreact less when the signal is consistent with the investor's private signal. Thus, the public signals on average stimulate additional overreaction to the private signal. The resulting post-public-event average trend in price changes could easily be misinterpreted as an underreaction.

Thus, the conclusion from the basic model that investors overreact to private signals holds in the dynamic model. However, while investors underreact on average to public signals, this underreaction is less (and can even be an overreaction) for information that confirms the quality of an investor's signal.

5 Managerial and Related Empirical Implications

The analysis suggests several implications for managerial policy. Suppose that managers serve the interests of current shareholders, and that, in the spirit of Myers and Majluf (1984), current shareholders do not participate in new issues. Then the firm should raise capital when the firm's stock is overvalued, as will tend to be the case after their firm's stock price has recently increased.⁵¹ Similarly, if there is overconfident trading based on information

⁵¹This follows from the result of negative unconditional long-term autocorrelations in the basic model. The short-term momentum that arises in the dynamic model (as in Section 4) could create an incentive to wait until the stock increase peaks, which also supports this effect. Evidence that issuance tends to follow

about the market as a whole that induces mispricing of individual firms, firms should issue after rises in the stock market as a whole.⁵²

The theory also suggests that after stock price run-downs, the firm should tend to be biased towards rights instead of public issues. (This assumes that management maximizes the wealth of current shareholders, and that there are transactions costs associated with disposals of rights and shares, so that shareholders cannot costlessly undo the firm's actions). Similarly, after run-downs, the firm should tilt toward debt rather than equity issues to avoid diluting current shareholders. Thus, the model suggests a solution to the fact (puzzling from the perspective of optimal capital structure theory) that after a rise in market prices, firms tend to issue more equity rather than more debt.

On the reverse side, if we assume that the firm acts in the interest of remaining shareholders, after the firm's stock price has been rising, the firm should be biased in favor of repurchase instead of dividend (since stock is more attractive to buy when it is undervalued).

Korajczyk, Lucas, and McDonald (1991) have emphasized the incentive of managers to disclose as much as possible prior to a new issue and to defer raising capital until after information events such as audited earnings reports. Under full rationality, there are strong pressures to do these things in order to allay investor skepticism associated with the manager's informational advantage. If investors are overconfident, they may not adequately discount for the public revelation of bad news implicit in the decision to issue. In such a setting, managers may hope to exploit investors by not disclosing, or by issuing before credible revelation.

However, in the outcome-dependent confidence model, there is an offsetting effect. Suppose that firms are issuing when the firm is overvalued. In our model, these are periods when investors have overreacted to favorable private signals. Section 4 shows that sometimes an additional public disclosure can on average intensify the overreaction to prior favorable information. This would benefit the issuer.

Since in an outcome-dependent setting confidence during some periods on average increases after an earnings announcement, a less negative stock price reaction to issuance is predicted. This is because, owing to biased self-attribution, public news on average reinforces prior overconfidence. So if managers try to issue when the market overvalues the firm, it may be best to wait until after a noisy public disclosure. Korajczyk, Lucas, and McDonald (1991) provide evidence supporting this prediction. Our theory has the distinct

runups is provided by Lucas and McDonald (1990).

⁵²Corresponding to predictions about issuance after runups are implications about issuance when book/market ratios are low; see Daniel, Hirshleifer, and Subrahmanyam (1997). These implications are also supported by existing evidence.

untested implication that the reduction in the absolute magnitude of the price drop should be greater after a good news disclosure than after a bad news disclosure. This is because good news on average reinforces prior overconfidence, so that investors place less weight on the bad news inherent in the equity issue. This effect is not an implication of the Korajczyk, Lucas, and McDonald (1991) approach, since even a bad news disclosure reduces information asymmetry. In contrast, their model suggests variables relating to reduction in information asymmetry (such as changes in bid-ask spreads or dispersion of analysts' forecasts, etc.).

The event-study interpretation of the model in Section 3.3.3 was based on the premise that managers take actions in response to perceived mispricing by the market. We have in mind a setting where the 'informed' traders of the model are a broad set of outside market participants who are sufficiently numerous to have a major effect on prices. This does not preclude the possibility that managers are irrationally overconfident as well. If their information differs from that of outsiders, insiders will be able to identify outsider's errors and take actions in response to market pricing errors.

Insider trading does not correspond exactly with our model, since it involves two sets of informed traders (corporate insiders, and informed outsiders) privately trading. For highly liquid stocks, if undisclosed insider trading has only a modest impact on price, the event study interpretation of the model should apply to the event of public disclosure of the insider trade. For such firms, the model predicts favorable abnormal performance following insider purchases, and unfavorable performance after sales.

However, if insiders are overconfident and affect price, then there may be overreaction to inside information signals. Although insiders will still on average make money on their trades, the price moves associated with insider trades may partly reverse in the long run. The model suggests that such an effect would be more likely to be found in a thinly traded stock where inside trades can substantially impact price.⁵³

6 Relation to Existing Theories

As mentioned in the introduction, smart individuals can profit by trading against the mispricing. This attenuates the effects of quasi-rational traders. However, so long as overconfi-

⁵³It is not clear that overconfidence is as important for insiders as for outsiders. Overconfidence cannot be important for a decision maker whose true precision is very high. In the limit, an individual whose noise variance is truly zero cannot be overconfident. The empirical evidence discussed earlier of price underreaction to the disclosure of insider trades is consistent with insiders being only modestly overconfident or rational, and therefore not trading sufficiently to cause price overreaction. It is also possible that insider trading regulations, wealth constraints or risk aversion limits the price impact of overconfident insiders.

dent traders are risk averse (in contrast with our simplifying assumption of risk neutrality) and quasi-rational traders are a substantial part of market, prices will be influenced substantially by their errors.

The more significant objection to models with imperfectly rational traders is that wealth may shift from foolish to rational traders until price-setting is dominated by rational traders. In general, this need not be the case. DeLong, Shleifer, Summers and Waldman (1990a, 1991) point out that if traders are risk averse, a trader who underestimates risk will allocate more wealth to risky assets, and may be rewarded for bearing higher risk with higher expected returns.⁵⁴ If, as in our model, traders are overconfident about genuine information signals, the tendency for overconfidence to increase profits may be even stronger.

Indeed, if most individuals have biased self-attribution, then even if overconfident individuals earn lower profits, overconfidence could be important in the long run. This would occur because those who do well end up both controlling more wealth and becoming more overconfident.

A distinct reason why overconfident traders may outperform rational traders is that being overconfident is like a commitment to trade aggressively. When there are only a few informed traders playing strategically, those known to be overconfident may earn higher returns (see Kyle and Wang (1996)).

Several other previous papers have examined overconfidence and securities markets. In Wang (1995), the uninformed are willing to trade with overconfident informed, so information-based trading occurs without noise traders. Odean (1995) examines modifications of standard microstructure models that allow for overconfidence about a private signal, to examine issues of volatility, volume, and market depth. He finds that overconfidence and underweighting of priors implies increased volatility and negative return autocorrelation. We examine a model that includes a public as well as a private signals. We show that in addition to overreaction to private signals, there is underreaction to public signals, and, with time-varying confidence, gradual overreaction to private signals. Thus, in contrast with Odean, we find forces toward positive as well as negative autocorrelation; and we argue that overconfidence can decrease volatility around public news events.

Hirshleifer, Subrahmanyam, and Titman (1994) examine the effect of overconfidence by analyst/traders who over-assess the probability of that they receive information before others. This increases their tendency to herd in selecting stocks to study.

Our approach assumes the maximum amount of rationality consistent with overconfi-

⁵⁴The former paper is based on noise traders who misperceive expected returns. The latter paper focuses on the survival of irrational traders, but does not examine information signals or equilibrium price determination.

dence. In contrast, it could be assumed that the I 's don't even know that the U 's have a different opinion about the private signal noise; or that the U 's don't know that the I 's know that U 's thinks that I are overconfident; and so on in a potentially unlimited set of possible misperceptions. Caballé and Sákovics (1996) analyze this possibility. Benos (1996) examines a microstructure model in which traders are overconfident about their knowledge of the signals of others, and mistakenly believe they they know value perfectly.

There are previous and contemporaneous models implying positive or negative returns autocorrelations. Wang (1993) provides a model where stock returns are negatively autocorrelated as a consequence of mechanistic noise trades. Our theory differs in showing that overconfidence and biased self-attribution can cause imperfect adjustment to new information. It is therefore consistent with either positive or negative autocorrelations at different lags. De Long, Shleifer, Summers, and Waldmann (1990b) point out that a number of empirical observations about security return autocorrelations and overreactions can be explained by the assumption that some traders mechanistically follow positive feedback investment strategies. Our approach differs in explicitly modeling the quasi-rational behavior of (overconfident) agents, and in focusing on the differing effects of public versus private information arrival. The model here provides a possible psychological microfoundation for a stochastic tendency for positive feedback trading as a consequence of changes in confidence. The positive feedback behavior derived here comes from traders who took positions previously, and whose confidence increases asymmetrically more when subsequent public information confirms their actions.

In a contemporaneous paper, Barberis, Shleifer, and Vishny (1996) provide a model consistent with the short and long term return autocorrelations and financial performance/return correlations empirically observed. In their model, earnings follow a random walk, and individuals believe that either earnings follow a steady growth trend, or else that earnings are mean-reverting. They derive implications for returns autocorrelations and earnings-return correlations based on investors revising their beliefs as to which process a firm's earnings are obeying. They do not explore issues related to public-information releases and event-studies.

The "noise trader" approach to securities anomalies is based on the recognition that there is variability in prices arising from unpredictable and possibly irrational trading that seems unrelated to valid information. However, we also think that an important class of mistakes by investors involves the misinterpretation of *genuine* new private information. Thus, irrational errors may be correlated with fundamentals, and informed traders may not be free of bias.

Noise trading can be viewed as a limiting case of overconfident trading in which, holding constant the individual's perceived precision, the signal becomes very noisy. This leads

to non-negligible trades resulting from very little informational stimulus. Thus, the noise trader approach can be viewed as a special kind of overconfidence model. This paper derives some implications of both overconfident noise trading, and of overconfidence when traders have non-negligible private information.

Our model endogenizes noise trading quasi-rationally, in the sense that individuals maximize utility rationally, except for their mistaken beliefs about signal precision. Many securities markets models include very different kinds of individuals, rational versus mechanically noisy. Quasi-rationality of noise trading places restrictions on the time pattern of trades, and makes optimal noise positions dependent on how price moves. These time-patterns and rational price-dependence play a crucial role in several of the implications of our theory.⁵⁵

7 Conclusion

Empirical securities markets research in the last three decades has presented a body of evidence that is not easy to explain using plausible, purely rational asset pricing models. Some studies conclude that the market underreacts to information, while others find evidence of overreaction. We have lacked a theory to integrate this evidence. Although behavioral hypotheses of either underreaction or overreaction have been proposed, an integrated theory is needed to make predictions about when over- or underreaction will occur.

This paper offers a theory of security returns anomalies based on investor overconfidence and on changes in confidence resulting from biased self-attribution of the implications of news arrival. These assumptions are based on extensive psychological evidence. The theory implies that investors will overreact to private information signals and underreact to public information signals. We show that positive autocorrelations and post-public-announcement ‘drifts’ are not necessarily a result of underreaction. The theory also provides a framework that clarifies what such evidence *does* imply about investor over- or underreaction to information.

We apply the theory to derive an integrated explanation for a variety of classes of

⁵⁵The behavior of discounts and premiums in closed-end mutual funds has been adduced as evidence that noise trading or ‘market sentiment’ are important in equity markets (see De Long, Shleifer, Summers, and Waldmann (1990a), Lee, Shleifer, and Thaler (1991)). Under the market segmentation assumptions made by these authors, the relations found in several empirical studies between small firm returns and closed-end fund discounts (which are essentially like market/book ratios) will generally hold in an overconfidence settings as well. Since, in an overconfidence setting, mispricing arises from overreaction to *genuine* information, changes in fund discounts should also be able to predict future future accounting performance, not just stock returns. Swaminathan (1996) finds such predictive power. For further discussion, see Daniel, Hirshleifer, and Subrahmanyam (1997).

anomalous security price patterns documented in numerous empirical studies. The theory also has implications for corporate financing and payout patterns, and provides a variety of new empirical implications.

Appendix A: Covariance and Variance Calculations for the Basic Model

Covariances and Variances of Subsection 3.3

(All signs are under the overconfidence assumption that $\sigma_\epsilon^2 > \sigma_C^2$.)

From (2), the covariance between the date 3 and the date 2 price changes is

$$\text{cov}(P_3 - P_2, P_2 - P_1) = \frac{\sigma_\theta^6 \sigma_C^2 \sigma_p^2 (\sigma_\epsilon^2 - \sigma_C^2)}{(\sigma_\theta^2 + \sigma_C^2) [\sigma_\theta^2 (\sigma_C^2 + \sigma_p^2) + \sigma_C^2 \sigma_p^2]^2}.$$

This is positive since $\sigma_\epsilon^2 > \sigma_C^2$.

The covariance between the date 1 price change and the date 2 price change is

$$\text{cov}(P_2 - P_1, P_1 - P_0) = -\frac{\sigma_\theta^6 \sigma_C^2 (\sigma_\epsilon^2 - \sigma_C^2)}{(\sigma_\theta^2 + \sigma_C^2)^2 [\sigma_\theta^2 (\sigma_C^2 + \sigma_p^2) + \sigma_C^2 \sigma_p^2]},$$

which, with overconfidence, is negative.

It is also easy to show that

$$\begin{aligned} \text{cov}(P_3 - P_1, P_1 - \bar{\theta}) &= -\frac{\sigma_\theta^4 (\sigma_\epsilon^2 - \sigma_C^2)}{(\sigma_\theta^2 + \sigma_C^2)^2} < 0 \\ \text{cov}(P_3 - P_2, P_1 - \bar{\theta}) &= -\frac{\sigma_\theta^4 \sigma_p^2 (\sigma_\epsilon^2 - \sigma_C^2)}{(\sigma_\theta^2 + \sigma_C^2) [\sigma_\theta^2 (\sigma_C^2 + \sigma_p^2) + \sigma_\theta^2 \sigma_p^2]} < 0. \end{aligned} \quad (5)$$

Since $P_3 = \theta$, and using the expression for P_2 in (2),

$$\text{cov}(P_3 - P_2, \epsilon^*) = \frac{\sigma_\epsilon^2 \sigma_\eta^2 \sigma_\theta^2 (\sigma_\theta^2 + \sigma_p^2) (\sigma_C^2 - \sigma_\epsilon^2)}{[\sigma_\epsilon^2 (\sigma_\theta^2 + \sigma_p^2) + \sigma_\theta^2 \sigma_p^2] [\sigma_C^2 (\sigma_\theta^2 + \sigma_p^2) + \sigma_\theta^2 \sigma_p^2]},$$

which is positive so long as $\sigma_C^2 < \sigma_\epsilon^2$.

The variance of the date 2 price change is

$$\text{var}(P_2 - P_1) = \frac{\sigma_C^4 [\sigma_C^4 \sigma_\theta^6 + \sigma_\theta^8 \sigma_\epsilon^2 + \sigma_\theta^4 \sigma_p^2 (\sigma_C^2 + \sigma_\theta^2)^2]}{[\sigma_C^2 (\sigma_p^2 + \sigma_\theta^2) + \sigma_\theta^2 \sigma_p^2]^2 (\sigma_\theta^2 + \sigma_C^2)^2},$$

which can either increase or decrease in σ_C^2 .

The date 1 price volatility:

$$\text{var}(P_1 - P_0) = \frac{\sigma_\theta^4 (\sigma_\theta^2 + \sigma_\epsilon^2)}{(\sigma_\theta^2 + \sigma_C^2)^2}$$

decreases with σ_C^2 ,

The unconditional volatility is just the average of $\text{var}(P_3 - P_2)$, $\text{var}(P_2 - P_1)$, and $\text{var}(P_1 - P_0)$, which is

$$\frac{1}{3} \left[\frac{\sigma_\epsilon^2 \sigma_p^4 \sigma_\theta^4 + \sigma_C^4 \sigma_\eta^2 \sigma_\theta^2 (\sigma_\eta^2 + \sigma_\theta^2)}{[\sigma_C^2 (\sigma_p^2 + \sigma_\theta^2) + \sigma_\theta^2 \sigma_p^2]^2} + \frac{\sigma_C^4 \{ \sigma_C^4 \sigma_\theta^6 + \sigma_\theta^8 \sigma_\epsilon^2 + \sigma_\theta^4 \sigma_p^2 (\sigma_C^2 + \sigma_\theta^2)^2 \}}{(\sigma_C^2 + \sigma_\theta^2)^2 [\sigma_C^2 (\sigma_p^2 + \sigma_\theta^2) + \sigma_\theta^2 \sigma_p^2]^2} + \frac{\sigma_\theta^4 (\sigma_\theta^2 + \sigma_\epsilon^2)}{(\sigma_\theta^2 + \sigma_C^2)^2} \right].$$

When there is no overconfidence, $\sigma_C^2 = \sigma_\epsilon^2$, this reduces to $\sigma_\theta^2/3$. The excess volatility is the difference between the above expression and $\sigma_\theta^2/3$,

$$\frac{2\sigma_\theta^4 (\sigma_\epsilon^2 - \sigma_C^2) [\sigma_C^4 (\sigma_p^4 + \sigma_\theta^2 \sigma_p^2 + \sigma_\theta^4) + \sigma_C^2 \sigma_p^2 \sigma_\theta^2 (2\sigma_p^2 + \sigma_\theta^2) + \sigma_p^4 \sigma_\theta^4]}{3(\sigma_C^2 + \sigma_\theta^2)^2 [\sigma_C^2 (\sigma_\theta^2 + \sigma_p^2) + \sigma_p^2 + \sigma_\theta^2]^2}.$$

This is positive so long as there is overconfidence, $\sigma_C^2 < \sigma_\epsilon^2$.

Proofs of Some Claims in Subsection 3.3.3

Proposition 5: Denote the date 2 mispricing as MP_2 . Suppressing arguments on $P_2^R(s_2)$ and $P_2^C(s_2)$ for expositional convenience, we have that $MP_2 = P_2^R - P_2^C = -E(\theta - P_2^C(s_2)|s_1, s_2)$. By the properties of normal random variables, this implies that the variable $x = \theta - P_2^C + MP_2$, which is the residual from the regression of $\theta - P_2^C$ on s_1 and s_2 , is orthogonal to s_1 and s_2 . Suppose we pick a variable $y = f(s_1, s_2)$ which is orthogonal to MP_2 . Such a variable will be orthogonal to x , so that we have $\text{cov}(\theta - P_2^C + MP_2, y) = 0$. Since $\text{cov}(MP_2, y) = 0$ by construction, it follows from the linearity of the covariance operator that $\text{cov}(\theta - P_2^C, y) = 0$. A converse argument shows that if we pick a variable $y' = g(s_1, s_2)$ which is orthogonal to the post event return $\theta - P_2^C$ then $\text{cov}(MP_2, y') = 0$. Thus, all functions of s_1 and s_2 are orthogonal to MP_2 if and only if they are orthogonal to the post event return $\theta - P_2^C$.

For the specific case when the event depends linearly on s_2 , we have from (2) that

$$P_3 - P_2 = \frac{\sigma_C^2 \sigma_p^2 \theta - \sigma_p^2 \sigma_\theta^2 \epsilon - \sigma_C^2 \sigma_\theta^2 \eta}{\sigma_\theta^2 (\sigma_C^2 + \sigma_p^2) + \sigma_C^2 \sigma_p^2}$$

Since s_2 is defined as $\theta + \eta$, from the above expression, it immediately follows that

$$\text{cov}(P_3 - P_2, s_2) = \frac{\sigma_C^2 \sigma_p^2 \sigma_\theta^2 - \sigma_C^2 \sigma_\theta^2 \sigma_p^2}{\sigma_\theta^2 (\sigma_C^2 + \sigma_p^2) + \sigma_C^2 \sigma_p^2}$$

which equals zero, thus showing that all events that depend only on s_2 are non-selective events.

Proposition 6: Using standard normal distribution properties,

$$\epsilon^* = E[\epsilon | P_1, \theta + \eta] = \frac{\sigma_\epsilon^2 (\sigma_\theta^2 + \sigma_p^2) (\theta + \epsilon) - \sigma_\theta^2 \sigma_\epsilon^2 (\theta + \eta)}{\sigma_\epsilon^2 (\sigma_\theta^2 + \sigma_p^2) + \sigma_\theta^2 \sigma_p^2}. \quad (6)$$

It is straightforward to show that the ratio of the date 2 mispricing to ϵ^* is given by

$$\frac{\sigma_\epsilon^2 [\sigma_C^2 (\sigma_\theta^2 + \sigma_p^2) + \sigma_\theta^2 \sigma_p^2]}{\sigma_p^2 \sigma_\theta^2 (\sigma_C^2 - \sigma_\theta^2)},$$

which is a constant (for a given level of confidence). Thus, selective events can alternatively be viewed as events that are linearly related to ϵ^* .

High values of ϵ^* signify overpricing and low values underpricing. The proposition follows by observing that

$$\text{cov}(P_3 - P_2, \epsilon^*) = \frac{\sigma_\epsilon^2 \sigma_\eta^2 \sigma_\theta^2 (\sigma_\theta^2 + \sigma_p^2) (\sigma_C^2 - \sigma_\epsilon^2)}{[\sigma_\epsilon^2 (\sigma_\theta^2 + \sigma_p^2) + \sigma_\theta^2 \sigma_p^2] [\sigma_C^2 (\sigma_\theta^2 + \sigma_p^2) + \sigma_\theta^2 \sigma_p^2]} < 0$$

and

$$\text{cov}(P_2 - P_1, \epsilon^*) = -\frac{\sigma_C^2 \sigma_\epsilon^2 \sigma_\theta^4 [\sigma_\epsilon^2 (\sigma_p^2 + \sigma_\theta^2) + \sigma_p^2 \sigma_\theta^2]}{(\sigma_C^2 + \sigma_\theta^2) [\sigma_C^2 (\sigma_\theta^2 + \sigma_p^2) + \sigma_p^2 \sigma_\theta^2]^2} < 0$$

□

Proposition 7:

For part (2), the analyst observes the private signal $\theta + \epsilon$ and rationally assesses its error variance. His forecast is therefore

$$E[\theta + \eta|\theta + \epsilon] = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\epsilon^2}(\theta + \epsilon).$$

It follows that

$$\text{cov}(P_3 - P_2, z) = \frac{\sigma_p^2 \sigma_\theta^4 (\sigma_\epsilon^2 - \sigma_C^2)}{[\sigma_C^2 (\sigma_p^2 + \sigma_\theta^2) + \sigma_p^2 \sigma_\theta^2] (\sigma_\epsilon^2 + \sigma_\theta^2)},$$

which is positive under overconfidence ($\sigma_C^2 < \sigma_\epsilon^2$).

Proposition 8: We interpret the ‘fundamental/price’ ratio as $\bar{\theta} - P_1$. For part (1),

$$\text{cov}(P_1, \epsilon^*) = \frac{\sigma_\epsilon^2 [\sigma_\epsilon^2 (\sigma_p^2 + \sigma_\theta^2) + \sigma_p^2 \sigma_\theta^2]}{\sigma_C^2 (\sigma_p^2 + \sigma_\theta^2) + \sigma_p^2 \sigma_\theta^2} > 0$$

Also,

$$\epsilon^* = k_1 s_1 + k_2 s_2,$$

where

$$k_1 = \frac{\sigma_\epsilon^2 (\sigma_\theta^2 + \sigma_p^2)}{\sigma_C^2 (\sigma_p^2 + \sigma_\theta^2) + \sigma_p^2 \sigma_\theta^2} \quad (7)$$

$$k_2 = -\frac{\sigma_\epsilon^2 \sigma_\theta^2}{\sigma_C^2 (\sigma_p^2 + \sigma_\theta^2) + \sigma_p^2 \sigma_\theta^2}. \quad (8)$$

This implies that the distribution of ϵ^* conditional on $\theta + \epsilon$ is normal with mean

$$\frac{(k_1 + k_2)\sigma_\theta^2 + k_1\sigma_\epsilon^2}{\sigma_\theta^2 + \sigma_\epsilon^2}(\theta + \epsilon)$$

and variance

$$\frac{[(k_1 + k_2)\sigma_\theta^2 + k_1\sigma_\epsilon^2]^2}{\sigma_\theta^2 + \sigma_\epsilon^2}(\theta + \epsilon).$$

The standardized cumulative normal distribution function of a normal random variable with nonzero mean and variance is increasing in its mean. Since $E[\epsilon^*|\theta + \epsilon]$ is proportional to $\theta + \epsilon$, the probability conditional on P_1 that ϵ^* exceeds a given threshold value (indicating occurrence of the positive event) is increasing in $\theta + \epsilon$. The reverse holds for a negative event, proving part (2).

Appendix B: Discrete Model of Outcome-Dependent Overconfidence

At time 0, θ has a value of +1 or -1 and an expected value of zero. At time 1, the player receives a signal s_1 , and, at time 2, a signal s_2 . s_1 may be either H or L while s_2 may be either U or D . After each signal, the player updates his prior expected value of θ .

$$\begin{aligned} Pr(s_1 = H|\theta = +1) &= p = Pr(s_1 = L|\theta = -1) \\ Pr(s_2 = U|\theta = +1) &= q = Pr(s_2 = D|\theta = -1). \end{aligned}$$

The probabilities that $\theta = +1$, given s_1 and s_2 are

$$\begin{aligned} Pr(\theta = +1|s_1 = H) &= \frac{Pr(s_1 = H|\theta = +1)Pr(\theta = +1)}{Pr(s_1 = H)} \\ &= \frac{p/2}{p/2 + (1-p)/2} = p. \end{aligned}$$

When s_2 confirms s_1 (either $s_1 = H, s_2 = U$ or $s_1 = L, s_2 = D$), the player becomes overconfident and acts as if his precision were p_C instead of p , so

$$\begin{aligned} Pr(\theta = +1|s_1 = H, s_2 = U) &= \frac{Pr(s_1 = H, s_2 = U|\theta = +1)Pr(\theta = +1)}{Pr(s_1 = H, s_2 = U)} \\ &= \frac{p_C q}{p_C(2q-1) + (1-q)}. \end{aligned}$$

When s_2 is informative ($q > 1/2$), this probability exceeds p_C . When s_2 does not confirm s_1 , the player does not become overconfident, so

$$\begin{aligned} Pr(\theta = +1|s_1 = H, s_2 = D) &= \frac{Pr(s_1 = H, s_2 = D|\theta = +1)Pr(\theta = +1)}{Pr(s_1 = H, s_2 = D)} \\ &= \frac{p(1-q)}{p(1-q) + q(1-p)}. \end{aligned}$$

When evaluated with an informative signal s_2 ($q > 1/2$), this probability is less than p . With a risk neutral player, the price of the asset with value θ can be calculated linearly using the above probabilities. The price at time 0 (P_0) is, by definition, equal to 0. As θ can take on a value of $+1$ or -1 , the price is $(\rho)(+1) + (1-\rho)(-1)$ or, $2\rho - 1$, where ρ is the probability that θ is $+1$.

$$\begin{aligned} P_1|_{s_1=H} &= 2Pr(\theta = +1|s_1 = H) - 1 = 2p - 1 \\ P_2|_{s_1=H, s_2=U} &= 2Pr(\theta = +1|s_1 = H, s_2 = U) - 1 = \frac{p_C + q - 1}{p_C(2q-1) + (1-q)} \\ P_2|_{s_1=H, s_2=D} &= 2Pr(\theta = +1|s_1 = H, s_2 = D) - 1 = \frac{p-q}{p+q-2pq}. \end{aligned}$$

The remaining prices are just the negative of their respective symmetrical price above.

$$\begin{aligned} P_1|_{s_1=L} &= 1 - 2p \\ P_2|_{s_1=L, s_2=D} &= \frac{1 - p_C - q}{p_C(2q-1) + (1-q)} \\ P_2|_{s_1=L, s_2=U} &= \frac{q-p}{p+q-2pq}. \end{aligned}$$

The price changes are $\Delta P_1 = P_1 - P_0 = P_1$ and $\Delta P_2 = P_2 - P_1$. $E[P_1] = 0$, so $\text{cov}(\Delta P_1, \Delta P_2) = E[\Delta P_1 \Delta P_2]$. The probabilities of the eight possible outcomes are:

$$\begin{aligned} Pr(\theta = +1, s_1 = H, s_2 = U) &= Pr(\theta = -1, s_1 = L, s_2 = D) = pq/2 \\ Pr(\theta = -1, s_1 = H, s_2 = U) &= Pr(\theta = +1, s_1 = L, s_2 = D) = (1-p)(1-q)/2 \\ Pr(\theta = +1, s_1 = H, s_2 = D) &= Pr(\theta = -1, s_1 = L, s_2 = U) = p(1-q)/2 \\ Pr(\theta = -1, s_1 = H, s_2 = D) &= Pr(\theta = +1, s_1 = L, s_2 = U) = (1-p)q/2. \end{aligned}$$

The product $\Delta P_1 \Delta P_2$ can only take on two values, based upon the various signal combinations:

$$\begin{aligned}
[\Delta P_1 \Delta P_2]_{s_1=H, s_2=U} &= [\Delta P_1 \Delta P_2]_{s_1=L, s_2=D} \\
&= (2p-1) \left(\frac{p_C + q - 1}{p_C(2q-1) + (1-q)} - (2p-1) \right) \\
&\equiv X. \\
[\Delta P_1 \Delta P_2]_{s_1=H, s_2=D} &= [\Delta P_1 \Delta P_2]_{s_1=L, s_2=U} \\
&= (2p-1) \left(\frac{p-q}{p+q-2p} - (2p-1) \right) \\
&\equiv Y.
\end{aligned}$$

Combining these two values and the probabilities above, $E[\Delta P_1 \Delta P_2]$ can be written as $(1-a)X + aY$ where $a = p + q - 2pq$. After some calculation, the two components of this expression are as follows:

$$\begin{aligned}
(1-a)X &= \frac{2(2p-1)(2pq-p-q+1)(p_C p + p_C q + pq - 2p_C p q - p)}{p_C(2q-1) + (1-q)} \\
aY &= 2p(2q-1)(2p-1)(p-1).
\end{aligned}$$

Combining these two terms and a great deal of factoring produces the final result,

$$E[\Delta P_1 \Delta P_2] = \frac{2q(2p-1)(p_C - p)(1-q)}{p_C(2q-1) + (1-q)} > 0. \quad (9)$$

When there is no overconfidence (and $p_C = p$) this expression is zero and the price changes have zero correlation.

A Second Noisy Public Signal

The model so far shows that overreaction can be exaggerated by a possible rise in confidence triggered by a noisy public signal. We now add a second noisy public signal to the model, so that we can consider gradual correction of mispricing. Signal $s_{3'}$ follows s_2 and can take on one of two value: G or B . The precision of this signal is as follows:

$$Pr(s_{3'} = G | \theta = +1) = r = Pr(s_{3'} = B | \theta = -1).$$

This signal does not affect confidence. If the player becomes overconfident (and replaces p with p_C) after s_2 , then the player will continue to use p_C as his measure of the precision of s_1 , regardless of whether $s_{3'}$ confirms s_1 . As there are two possible prices after the first signal and four possible prices after the second, there are eight possible prices after observation of the third signal. As above, by symmetry, only half of these prices need to be calculated. Using the conditional probabilities, the period three prices are:

$$\begin{aligned}
P_{3'} |_{s_1=H, s_2=U, s_{3'}=G} &= \frac{p_C q r - (1-p_C)(1-q)(1-r)}{p_C q r + (1-p_C)(1-q)(1-r)} \\
P_{3'} |_{s_1=H, s_2=U, s_{3'}=B} &= \frac{p_C q(1-r) - (1-p_C)(1-q)r}{p_C q(1-r) + (1-p_C)(1-q)r} \\
P_{3'} |_{s_1=H, s_2=D, s_{3'}=G} &= \frac{p(1-q)r - (1-p)q(1-r)}{p(1-q)r + (1-p)q(1-r)} \\
P_{3'} |_{s_1=H, s_2=D, s_{3'}=B} &= \frac{p(1-q)(1-r) - (1-p)qr}{p(1-q)(1-r) + (1-p)qr}.
\end{aligned}$$

With two possible values for θ , there are now sixteen possible sets of $\{\theta, s_1, s_2, s_3\}$ realizations. Only $\{s_1, s_2, s_3\}$ are observed by the player, resulting in eight sets of possible signal realizations. When calculating the covariances of price changes, only half of these realizations can result in unique products of price changes, so we define

$$\begin{aligned} A_{ij} &\equiv \Delta P_i \Delta P_j |_{H,U,G} = \Delta P_i \Delta P_j |_{L,D,B} \\ B_{ij} &\equiv \Delta P_i \Delta P_j |_{H,U,B} = \Delta P_i \Delta P_j |_{L,D,G} \\ C_{ij} &\equiv \Delta P_i \Delta P_j |_{H,D,G} = \Delta P_i \Delta P_j |_{L,U,B} \\ D_{ij} &\equiv \Delta P_i \Delta P_j |_{H,D,B} = \Delta P_i \Delta P_j |_{L,U,G}. \end{aligned}$$

Each of these four possible products must then be weighted by their probability of occurrence to calculate the expected value of the products of the price changes (the expected value of each price is zero). The weights for the A_{ij} component of covariance are:

$$\begin{aligned} Pr(H, U, G | \theta = +1) + Pr(H, U, G | \theta = -1) &= pqr/2 + (1-p)(1-q)(1-r)/2 \\ Pr(L, D, B | \theta = -1) + Pr(L, D, B | \theta = -1) &= pqr/2 + (1-p)(1-q)(1-r)/2. \end{aligned}$$

Proceeding in this manner, the covariances are:

$$\begin{aligned} E[\Delta P_i \Delta P_j] &= [pqr + (1-p)(1-q)(1-r)]A_{ij} + \\ &\quad [pq(1-r) + (1-p)(1-q)r]B_{ij} + \\ &\quad [p(1-q)r + (1-p)q(1-r)]C_{ij} + \\ &\quad [p(1-q)(1-r) + (1-p)qr]D_{ij}. \end{aligned}$$

When calculating $E[\Delta P_1 \Delta P_2]$ earlier, A_{12} and B_{12} were equal to X and C_{12} and D_{12} were equal to Y , with the r and $1-r$ factors from s_3' summing to one. To simplify the algebra, temporarily let all signals have a precision of p , with p_C replacing p as the precision of the first signal if overconfidence occurs (i.e., $q = r = p$). The covariances in this case are:

$$\begin{aligned} E[\Delta P_1 \Delta P_2]_{r=q=p} &= \frac{2p(1-p)(2p-1)(p_C-p)}{p_C(2p-1)+1-p} > 0 \\ E[\Delta P_1 \Delta P_3]_{r=q=p} &= \frac{2pp_C(p-1)(p_C-p)(1-p_C)(2p-1)^3}{[p_C(2p-1)+1-p][p_C(2p-1)+(1-p)^2]} < 0 \\ E[\Delta P_2 \Delta P_3]_{r=q=p} &= \frac{4p^2 p_C(p-1)^2(p_C-p)(p_C-1)(2p-1)^2(2p_C-1)}{[p_C(2p-1)+1-p]^2[p_C(2p-1)+(1-p)^2]} < 0. \end{aligned}$$

The size of $E[\Delta P_1 \Delta P_2]_{r=q=p}$ and $E[\Delta P_2 \Delta P_3]_{r=q=p}$ can be compared directly. Observe the following:

$$E[\Delta P_2 \Delta P_3]_{r=q=p} = -\frac{2p(1-p)p_C(1-p_C)(2p-1)(2p_C-1)}{[p_C(2p-1)+(1-p)][p_C(2p-1)+(1-p)^2]} E[\Delta P_1 \Delta P_2]_{r=q=p}.$$

That is, the covariance between the price changes in periods two and three is the negative of the covariance of price changes in periods one and two multiplied by a factor. Begin by examining the numerator of this factor. The first three components, $2p(1-p)$, are maximized when $p = 1/2$ while the next two components, $p_C(1-p_C)$, are maximized when $p_C = 1/2$. The product of these first five components must then be upwardly bounded by $1/8$. As the final two components,

$(2p - 1)(2p_C - 1)$, are each upwardly bounded by one, the entire numerator is upwardly bounded at $1/8$. As for the denominator, the expression $p_C(2p - 1) + (1 - p)$ is minimized when $p_C = p$, resulting in an expression that is minimized at $1/2$ with a value of $p = 1/2$. The second component of the denominator is similarly minimized when $p_C = p$, resulting in an expression that is minimized when $p = 1/2$, which produces a lower bound of $1/4$. The denominator, then, is bounded below by $1/8$, which combined with a numerator bounded above by the same value produces a fraction bounded above by one. Therefore, the negative covariance between period two and period three price changes must be, in absolute value, equal to or smaller than the positive covariance between period one and period two price changes, resulting in an overall one-period covariance that is positive.

When $q = r$ differs from p , the covariances are:

$$\begin{aligned} E[\Delta P_1 \Delta P_2]_{r=q} &= \frac{2q(1-q)(2p-1)(p_C-p)}{p_C(2q-1)+1-q} > 0 \\ E[\Delta P_1 \Delta P_{3'}]_{r=q} &= \frac{2qp_C(q-1)(p_C-p)(1-p_C)(2q-1)^2(2p-1)}{[p_C(2q-1)+1-q][p_C(2q-1)+(1-q)^2]} < 0 \\ E[\Delta P_2 \Delta P_{3'}]_{r=q} &= \frac{4qp_C(q-1)(p_C-p)(1-p_C)(2q-1)^2(pp_C+pq+p_Cq-2pp_Cq-p)}{[p_C(2q-1)+1-q]^2[p_C(2q-1)+(1-q)^2]} < 0. \end{aligned}$$

Now let the signal $s_{3'}$ have a precision of r that differs from both precisions of p and q . Proceeding as above, the covariances are as follows.

$$\begin{aligned} E[\Delta P_1 \Delta P_2] &= \frac{2q(1-q)(2p-1)(p_C-p)}{p_C(2q-1)+1-q} > 0 \\ E[\Delta P_1 \Delta P_{3'}] &= \alpha \frac{2qp_C(q-1)(p_C-p)(1-p_C)(2r-1)^2(2p-1)}{[p_C(2q-1)+1-q][p_C(q+r-1)+(1-q)(1-r)]} < 0 \\ E[\Delta P_2 \Delta P_{3'}] &= \alpha \frac{4qp_C(q-1)(p_C-p)(1-p_C)(2r-1)^2(pp_C+pq+p_Cq-2pp_Cq-p)}{[p_C(2q-1)+1-q]^2[p_C(q+r-1)+(1-q)(1-r)]} < 0 \\ E[\Delta P_{3'} \Delta P_3] &= \alpha^2 \frac{4rp_C(p_C-p)(1-p_C)(1-r)(2r-1)^2(p_C+q-1)}{[p_C(q+r-1)+(1-q)(1-r)]^2} > 0 \\ E[\Delta P_1 \Delta P_3] &= \alpha \frac{2r(p_C-p)(2p-1)(r-1)(p_C(2q-1)+(1-q))}{p_C(q+r-1)+(1-q)(1-r)} < 0, \end{aligned}$$

where

$$\alpha = \frac{q(q-1)}{p_C(r-q)+r(q-1)}.$$

Comparing to the previous case (where $s_{3'}$ had the same precision as that of s_2), the α multiplier is one when r equals q and positive otherwise⁵⁶, while the remainder of the expressions for $E[\Delta P_1 \Delta P_{3'}]$ and $E[\Delta P_2 \Delta P_{3'}]$ contain a few replacements of q with r as compared to the previous case where r is held at the value of q .

The magnitude of $E[\Delta P_2 \Delta P_{3'}]$ varies non-monotonically with q . The impact of a change in the precision of $s_{3'}$ is clearer.

$$\frac{\partial E[\Delta P_2 \Delta P_{3'}]}{\partial r} = \frac{4q^2 p_C(q-1)^2(p_C-p)(p_C-1)(2r-1)(pp_C+pq+p_Cq-2pp_Cq-p)}{[p_C(r-q)+r(q-1)]^2[p_C(q+r-1)+(1-q)(1-r)]^2} < 0.$$

⁵⁶The numerator is negative. The denominator is maximized with respect to r when r is at its maximum value of one, which yields a negative denominator and a positive fraction, α .

As r rises (the precision of $s_{3'}$ is increased), $E[\Delta P_2 \Delta P_{3'}]$ becomes more negative (increases in absolute value). As $r \rightarrow .5$, this covariance approaches zero. Thus, when the second noisy public signal is not very informative, this negative single-lag covariance becomes arbitrarily small in absolute value.

Confidence increases when s_2 confirms s_1 , but its effects are mitigated as s_2 becomes more informative. Thus, an increase in the precision of s_2 has an ambiguous effect on $E[\Delta P_2 \Delta P_{3'}]$. This increase results in a greater likelihood of overconfidence occurring, yet also places greater, rational, confidence in s_2 itself, yielding less leverage to the effects of overconfidence.⁵⁷ Based on simulation, it appears that the greater information resulting from higher values of q tends to over-shadow the increased likelihood of overconfidence, resulting in generally lower absolute values for $E[\Delta P_2 \Delta P_{3'}]$.

Larger values of r , the precision of $s_{3'}$, result in more negative values of $E[\Delta P_2 \Delta P_{3'}]$. In this case, a more informative second noisy public signal can only place less weight on previous signals and result in a stronger correction of the previous overreaction. Thus, the final one-period covariance is more negative as the precision of $s_{3'}$ rises.

Appendix C: Covariance Calculations for the Dynamic Model (Subsection 4.2.1)

Since the probability p is an exogenous constant, and the probability that the date 1 price move was positive is $1/2$, by the law of iterated expectations

$$\begin{aligned} \text{cov}(P_2 - P_1, P_1 - P_0) &= E_{s_2}[E[\{(P_2 - P_1)(P_1 - P_0)\} | s_2]] \\ &= \frac{k\sigma_\theta^4(\sigma_\theta^2 + \sigma_\epsilon^2)}{2(\sigma_\theta^2 + \sigma_C^2)^2(\sigma_\theta^2 + \sigma_C^2 - k)} > 0. \end{aligned}$$

Since $P_3 = \theta$,

$$\begin{aligned} \text{cov}(P_3 - P_2, P_2 - P_1) &= E_{s_2}[E[\{(P_3 - P_2)(P_2 - P_1)\} | s_2]] \\ &= -\frac{\sigma_\theta^2[k^2\sigma_\theta^2 + k(\sigma_\epsilon^2 - \sigma_C^2)]}{2(\sigma_\theta^2 + \sigma_C^2)(\sigma_\theta^2 + \sigma_C^2 - k)^2}. \end{aligned} \tag{10}$$

Further,

$$\text{cov}(P_3 - P_1, P_1 - P_0) = E[(\theta - P_1)P_1] = -\frac{\sigma_\theta^4(\sigma_\epsilon^2 - \sigma_C^2)}{(\sigma_\theta^2 + \sigma_C^2)^2}. \tag{11}$$

The covariances in (10) and (11) are both negative.

It is easy to show that

$$\begin{aligned} \text{cov}(P_3 - P_{3'}, P_{3'} - P_2) &= \frac{\sigma_\theta^6}{2} \left[\frac{\sigma_C^2(\sigma_\epsilon^2 - \sigma_C^2)}{(\sigma_\theta^2 + \sigma_C^2)^2[\sigma_\theta^2(\sigma_C^2 + \sigma_p^2) + \sigma_C^2\sigma_p^2]} \right. \\ &\quad \left. + \frac{(\sigma_C^2 - k)(\sigma_\epsilon^2 + k - \sigma_C^2)}{(\sigma_\theta^2 + \sigma_C^2 - k)^2[\sigma_\theta^2(\sigma_C^2 - k + \sigma_p^2) + (\sigma_C^2 - k)\sigma_p^2]} \right] > 0, \end{aligned} \tag{12}$$

⁵⁷At the extreme, a value of q equal to one yields the greatest chances of s_2 confirming s_1 yet results in zero values for all covariances as the perfect information of s_2 entirely determines all subsequent prices.

$$\text{cov}(P_3 - P'_3, P_2 - P_1) = -\frac{k\sigma_p^2\sigma_\theta^4(k + \sigma_\epsilon^2 - \sigma_C^2)}{2(\sigma_C^2 - k + \sigma_\theta^2)[(\sigma_C^2 - k)(\sigma_\theta^2 + \sigma_p^2) + \sigma_p^2\sigma_\theta^2]} < 0 \quad (13)$$

$$\begin{aligned} \text{cov}(P_3 - P'_3, P_1 - P_0) = & -\frac{1}{2} \left[\frac{\sigma_\theta^4\sigma_p^2(\sigma_\epsilon^2 - \sigma_C^2)}{(\sigma_\theta^2 + \sigma_C^2)[\sigma_C^2(\sigma_\theta^2 + \sigma_p^2) + \sigma_\theta^2\sigma_p^2]} \right. \\ & \left. + \frac{\sigma_\theta^4\sigma_p^2(k + \sigma_\epsilon^2 - \sigma_C^2)}{(\sigma_\theta^2 + \sigma_C^2)[(\sigma_C^2 - k)(\sigma_\theta^2 + \sigma_p^2) + \sigma_p^2\sigma_\theta^2]} \right] < 0 \end{aligned} \quad (14)$$

and

$$\begin{aligned} \text{cov}(P'_3 - P_2, P_1 - P_0) = & -\frac{1}{2} \left[\frac{\sigma_\theta^6\sigma_C^2(\sigma_\epsilon^2 - \sigma_C^2)}{(\sigma_\theta^2 + \sigma_p^2)^2[\sigma_C^2(\sigma_\theta^2 + \sigma_p^2) + \sigma_\theta^2\sigma_p^2]} \right. \\ & \left. + \frac{\sigma_\theta^6(\sigma_C^2 - k)(k + \sigma_\epsilon^2 - \sigma_C^2)}{(\sigma_\theta^2 + \sigma_C^2 - k)[(\sigma_C^2 - k)(\sigma_\theta^2 + \sigma_p^2) + \sigma_p^2\sigma_\theta^2]} \right] < 0. \end{aligned} \quad (15)$$

Furthermore, straightforward calculations also indicate that

$$\text{cov}(P_3 - P_2, P_2 - P_1) = -\frac{k\sigma_\theta^6(\sigma_C^2 - k)(\sigma_\epsilon^2 + k - \sigma_C^2)}{2(\sigma_C^2 + \sigma_\theta^2 - k)^2[\sigma_p^2\sigma_\theta^2 + (\sigma_p^2 + \sigma_\theta^2)(\sigma_C^2 - k)]} < 0. \quad (16)$$

The three single-lag contiguous covariances are given by (11), (12), and (16). First, compare the covariances (10) and (16). It is evident upon this comparison that the sum of these two covariances will be negative so long as σ_p^2 is sufficiently large since the latter (negative) covariance varies inversely with σ_p^2 and the former covariance does not depend on σ_p^2 . Since the covariance in (12) is always positive, unconditional momentum (defined as the simple arithmetic average of the three single covariances) will obtain if σ_p^2 is sufficiently small.

Next, compare (12) and (16). As k goes to zero, the covariance in (16) goes to zero, while the covariance in (12) remains strictly positive. Thus, if k is sufficiently small, the sum of these two covariances will be positive. Since (11) is always nonnegative, unconditional momentum will obtain if k is sufficiently small.

Appendix D: The Relation Between Long-Term Overreaction and Reversals

For there to exist long-term overreaction to returns as discussed in DeBondt and Thaler (1987), or to performance measures such as earnings or sales as in Lakonishok, Shleifer, and Vishny (1994), there must be a “reversal” at some horizon, meaning that, for some n , $\text{cov}(r_t, r_{t+n}) < 0$ in the case of return reversals, and $\text{cov}(a_t, r_{t+n})$, where a_t denotes growth in an accounting variable such as earnings or sales. This is a simple mathematical relationship that is a result of the linearity of the covariance operator, as we show here.

Return overreaction means that the covariance between past long-horizon returns and future long horizon returns is negative.⁵⁸

$$\text{cov}(r(t - \tau, t), r(t, t + \tau)) < 0$$

⁵⁸We perform this derivation assuming equal horizons for past and future returns, but the result is valid for any horizon of past and future returns.

where here $r(t - \tau, t)$ denotes the log return between $t - \tau$ and t , ($=\log(P_{t-\tau}/P_t)$). Now, let r_t denote the one-period log return ($=\log(P_{t-1}/P_t)$). Then

$$\begin{aligned}
cov(r(t - \tau, t), r(t, t + \tau)) &= cov\left(\sum_{s=t-\tau+1}^t r_s, \sum_{u=t+1}^{t+\tau} r_u\right) \\
&= \sum_{s=t-\tau+1}^t \sum_{u=t+1}^{t+\tau} cov(r_s, r_u) \\
&= \sum_{n=1}^{2\tau-1} \min(n, 2\tau - n) \cdot cov(r_s, r_{s+n})
\end{aligned}$$

Since $\min(n, 2\tau - n)$ is always negative (for this range of n), it follows that if $cov(r(t - \tau, t), r(t, t + \tau)) < 0$, then $cov(r_s, r_{s+n}) < 0$ for some value(s) of n . In other words, there must be a return reversal.

A similar derivation shows that if there is a negative covariance of past long-term earnings/sales-growth with future long-horizon returns, then there must be some reversal in response to positive accounting growth in a quarter (*i.e.*, $cov(a_t, r_{t+n})$ will be negative for some $n > 0$).

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