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BANK LOAN COMMITMENTS AND INTEREST RATE VOLATILITY

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Bank loan commitments are examined in the context of option pricing models and a valuation formula is obtained. The partial takedown phenomenon, which is both distinctive and vexatious, is considered in detail. Finally, estimates of the value of U.S. bank loan commitments and their sensitivity to interest rate changes are provided.

1. Introduction

Although widely recognized as basic instruments of our credit markets, bank loan commitments remain vaguely understood.[1] These commitments are sources of capital gains and losses in periods of volatile interest rates, yet they are not recorded in bank balance sheets. At best, loan commitments occupy the murky status of off-balance sheet or footnote items. Commitment accounting may well explain a substantial portion of the widely observed insensitivity of bank balance sheets to financial deterioration in periods of economic instability. One apparent reason for the vagueness surrounding loan commitments is that we lack a well-established method for valuing them. This paper clarifies the positive problem of accounting for loan commitments and the normative problem of pricing them.

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[1]As of year-end 1978, unused formal loan commitments at larger commercial banks in the U.S. were approximately \$200 billion, or 15 percent of the banking system's footings. At the same time, loans made under commitments totalled approximately \$115 billion, or 15 percent of gross loans at all commercial banks. See 'Loan Commitments at Selected Large Commercial Banks', Federal Reserve Statistical Release, May 1979. Bank loan commitments are discussed by Crane (1973), Higgins (1972), Summers (1975) and Bartter and Rendleman (1978, 1979).

The following section interprets bank loan commitments as options and develops a pricing formula. Section 3 discusses partial takedowns of loan commitments, a phenomenon that distinguishes commitments from stock options and many reflect banking market imperfections. The fourth section utilizes an option pricing approach to obtain a valuation expression for loan commitments and to assess the sensitivity of commitment values to interest rate changes.

2. The loan commitment

Consider a bank commitment made at time $t=0$ to lend an amount L, at time $t=T$. The loan, if taken, will mature at time τ (the term-to-maturity of the loan is $\tau-T$) and the agreed upon rate of interest on the loan will be $r_{pT}+k$, where r_{pT} is the prime rate of interest at $t=T$ and k is an add-on, expressed in the same units as r_{pT}, reflecting the perceived risk of default and perhaps other customer characteristics as well.

The typical charge for such a commitment will be some fraction, α, of the amount of the loan commitment.[2] In principle, αL is an asset entry on the bank's balance sheet and the difference between αL and the bank's valuation of the commitment liability, U_0, is an addition to net worth.[3]

The marginal gross rate on loans, r_{mt}, is the interest rate the bank would charge on the same loan at time t in the absence of a loan commitment. This interest rate subsumes at least three elements: (i) the bank's cost in making funds available to the borrower, (ii) a premium for sustaining default risk, (iii) a profit margin which will depend on the degree of competition in the loan market. If the loan market is perfectly competitive, this profit margin will be driven to zero, of course. But with inertia in the movement of customers among suppliers, and limited entry into banking, we would expect bank profits to be a random variable with a positive expected value.

Temporal uncertainty in (i), (ii) and (iii) mea~~ns~~ that r_{mt} is a stochastic variable. Whether or not the customer decides to exercise or take down the

[2]It is customary for banks to require a compensating balance in addition to (sometimes in lieu of) an explicit fee. For simplicity, we shall ignore balance requirements.

[3]In a perfectly competitive loan commitment market, one would expect no difference between αL and the bank's valuation of the commitment liability, U_0. However, commitment markets are not perfectly competitive and in pricing their loan commitments banks typically take into account their customers' alternative opportunities in the credit market. Throughout the following discussion we assume that the bank customer sustains non-trivial costs in shifting from one bank to another. This inertia — which may be due to the customer sustaining the cost of the new bank's assessment of the customer's collateral or lending limit — gives rise to the 'bank–customer relationship' which provides the customer's current bank with a measure of monopoly power [see Jaffee and Modigliani (1969), and Stiglitz and Weiss (1981)]. This line of reasoning is similar to the argument presented in Jensen and Meckling (1976) — the part owner-manager of a firm bears the entire cost incurred by the 'outside' shareholders in monitoring his activities.

loan commitment depends on the value of r_{mT} relative to $r_{pT}+k$. If the bank's customer chooses to exercise his option, the bank is required to purchase a claim against the option owner for the agreed upon price of $L>x_T$, where[4]

$$x_T = L\exp\{(r_{pT}+k-r_{mT})(\tau-T)\} \tag{1}$$

is the value of the claim at $t=T$. The cost of the commitment to the bank or the value of the option to the customer at $t=T$ is[5]

$$\begin{array}{lll} 0 & \text{if } x_T \geqq L & \text{and the option is not exercised,} \\ L-x_T & \text{if } x_T < L & \text{and the option is exercised.} \end{array}$$

If $G(x_t|x_0)$ is the cumulative distribution function of x_t (conditional on $x_t=x_0$ at $t=0$), the expected cost of the commitment is given by

$$\int_0^L (L-x_T)\,\mathrm{d}G(x_T|x_0).$$

The value of the option at $t=0$ is the present value of this expected cost. Discounting at some appropriate risk-adjusted rate, r_e, we obtain

$$U_0 = U(x_0, 0) = \exp(-r_e T)\int_0^L (L-x_T)\,\mathrm{d}G(x_T|x_0), \tag{2}$$

where

$$x_0 = L\exp\{(r_{p0}+k-r_{m0})(\tau-T)\}. \tag{3}$$

Eq. (2) can be viewed as both the expected cost to the bank of providing a loan commitment and as an option pricing formula.

Notice that a change in r_{mt}, occurring after the consummation of a loan commitment, will alter the value of U_0. Since αL is invariant, such changes in U_0 are capital losses (gains) to the bank. Note, too, that U_0 is non-negative. Thus, while the potential loss to the bank has an upper bound of L, any gain due to a fall in r_{mt} is limited by the size of the commitment fee, αL. Before

[4]We shall initially assume that the exercise of the loan option means a complete takedown of the loan commitment.

[5]We view the marginal gross rate on loans, r_{mt}, as the marginal opportunity cost of funds to the bank, and hence use it as the appropriate discount rate. Moreover, the marginal cost of debt to the bank customer is also taken to be r_{mt}. While capital market imperfections leading to the creation of financial intermediaries would normally imply a difference between these two rates, in the present context the recognition of still another interest rate complicates the analysis to no apparent advantage.

considering a solution for U_0, we examine variable takedowns, a phenomenon that distinguishes bank loan commitments from stock options and may reflect certain idiosyncrasies of bank credit markets.

3. Commitment takedowns

The assumption that loan commitments are either exercised in full or not at all is superficially plausible, but it does not accord well with practice in commercial banking. Bank customers often exercise only a portion of their 'line', even when the borrowing rate under the commitment is clearly below comparable alternative rates of interest.

While the possibility of partial takedowns need not invalidate the option-pricing approach to valuing loan commitments, it does suggest the need for a more detailed consideration of the institutional arrangements surrounding their creation and exercise. Bankers commonly explain fractional takedowns with the observation that the customer lacks 'need' for all of the loan commitment and/or wishes to foster good relations with its bank by not fully exploiting the windfall of an inexpensive loan.[6] These two explanations are neither mutually exclusive nor are they inconsistent with the option-pricing approach.

The relative persuasiveness of the two explanations depends on the firm's interest elasticity of demand for borrowed funds. The firm may continually substitute one form of borrowing for another based on relative costs with the total demand for debt being determined by the firm's desired capital structure and the availability of profitable investment opportunities. Where the firm's debt-ratio is fixed by capital structure considerations and total assets are invariant to the cost of debt, say because of rigidly limited investment opportunities, the demand for debt will be interest inelastic. On the other hand, if the firm has unlimited investment opportunities and no restriction on financial leverage, the demand for funds would be perfectly elastic.[7] In practice, the firm's demand for funds presumably lies somewhere between these two extremes.

Consider fig. 1 where the firm's demand for loans is depicted by $d(B)$. The supply schedule of funds is described by the interest rate $r(B)$ which consists of only three segments, for simplicity. The lowest (left-most) segment corresponds to the strictly limited funds available to the firm at a cost lower

[6] The loan commitment owner's ongoing relationship with the option-writer distinguishes the stock option from the loan commitment. The owner of the stock option has no knowledge of or concern about the option-writer. Indeed, their relationship may be legally severed in that both may transact with the market maker, as in the case of the Chicago Board of Options Exchange. In this case, the cost of changing trading partners is zero.

[7] If the firm's demand for funds is infinitely elastic, indeterminacy may result. In this case, partial takedowns may still be explained by the subsequently discussed multi-period customer relationship considerations.

than $r_{pT}+k$. The next (center) segment represents funds available under the loan commitment and the final (right-most) segment reflects the cost of alternative funds, such as bank loans unrelated to a commitment. If r_{mT} exceeds $(r_{pT}+k)$, B^* will be demanded and the loan takedown will be (B^*-B_1). The loan commitment option is exercised in full only if $(B^*-B_1)\geqq L^*$, where L^* is the magnitude of the loan commitment.

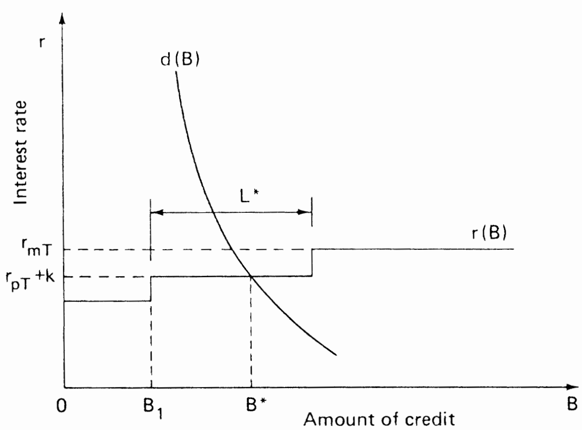

Fig. 1. The firm's demand for and supply of credit.

Alternatively, partial takedowns may be explained in terms of the bank–customer relationship, whereby the bank enjoys a degree of monopoly power as a result of the customer's perceived cost of establishing new sources of (bank) credit [see Wood (1975), and Hodgman (1961)]. The degree to which a customer exercises his loan commitment can be expected to influence the future pricing (availability) of bank services since any gain the customer realizes is an equivalent loss to the bank. In establishing α and k, the lender presumably considers expected borrower behavior under alternate states of the world. Should the borrower surprise the lender by borrowing more than expected, it would seem reasonable to expect the lender to revise his expectations and adjust upward α and/or k applicable to future commitment transactions. Consider two sequential loan commitments. Let α_0 and k_0 represent the commitment fee rate and add-on, respectively, for the first commitment (determined at $t=0$), and let α_1 and k_1 correspond to the second commitment. The add-on for the second loan commitment (exercisable at $t=2$), k_1, is determined at $t=1$, when the first commitment is exercised. Let $k_1=k_0+g$, where g is an increment to the lending rate determined by the firm's borrowing behavior at time 1. Thus k_1 is imbedded in the second loan commitment in light of the customer's use of its earlier commitment. In general, g can be considered a function of the takedown at $t=1$, L/L^*, and $g', g''>0$. Convexity of g means that k_1 increases at an increasing rate with takedowns in period 1. Recall that commitments are only exercised when they mean losses for the bank.[8]

At time 1, the firm minimizes the expected cost of the next loan plus the opportunity loss from not fully taking down the current loan. The total cost is given by[9]

$$C=(1-L/L^*)(L-x_1)+E(L')\cdot \exp[E(r_{p2})+k_0+g-E(r_{m2})], \qquad (4)$$

where x_1 is the same as x_T [defined in eq. (1)] with $T=1$; L^* is the bank's commitment made at $t=0$; L is the takedown at $t=1$ on the commitment at $t=0$; $E(L')$ is the expected loan in period 2 (considered fixed and therefore not a decision variable); and $E(r_{p2})$ and $E(r_{m2})$ are the expected prime rate and gross marginal rate on loans at $t=2$. Both terms on the right-hand side of eq. (4) will vary with (L/L^*) as illustrated in fig. 2. The firm's optimal takedown is determined at the point where C is minimized.

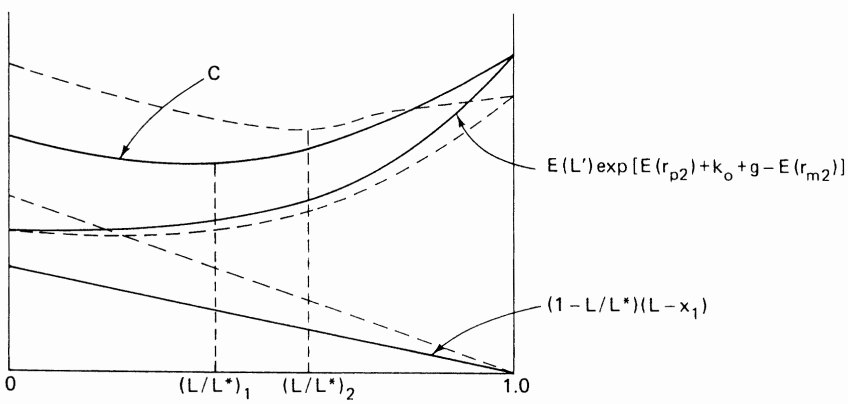

Fig. 2. Optimal takedowns.

[8]More generally $g=g(r_{m1}, r_{p1}, k, L/L^*)$, where r_{m1} and r_{p1} are the gross marginal rate on loans and the prime rate at $t=1$, when the first loan commitment is exercised. In the present analysis, we ignore the effect of all variables except L/L^* on g, i.e., $dg/d(L/L^*)\simeq \partial g/\partial(L/L^*)$. Also, the convexity of g presupposes that (1) the bank expects larger takedowns in period 2 with increasing takedowns in period 1, i.e., expectations are revised on the basis of observed customer behavior, and (2) the bank–customer relationship deteriorates rapidly as the customer takes increasing advantage of an inexpensive loan.

[9]The following discussion is consistent with the assumption that the customer is risk neutral. This assumption is made merely for convenience at this stage and is required neither here nor in the development of the valuation formula in the next section.

Notice that either explanation of partial takedowns will result in takedowns rising with $E(r_{m1})$. In the case of limited demand for debt, the amount of funds available at less than the commitment rates presumably declines with rising $E(r_{m1})$, i.e., the line segment to the left of B_1 in fig. 1 is diminished. This follows from the 'drying up' of alternate sources of credit with rising interest rates. In the case of the customer relationship, x_1 decreases with $E(r_{m1})$ and hence the first term on the right-hand side of eq. (4) increases. This can be seen as a clockwise rotation of $(1-L/L^*)(L-x_1)$ in fig. 2 (broken graphs). Furthermore, the increase in $E(r_{m1})$ may imply an increase in $E(r_{m2})$ relative to $E(r_{p2})$. Hence, the $E(L')\cdot\exp[E(r_{p2})+k_0+g-E(r_{m2})]$ schedule becomes less steep at each value of (L/L^*). Both effects increase the optimal takedown, as illustrated by the move from $(L/L^*)_1$ to $(L/L^*)_2$ in fig. 2.

In periods of increasing interest rates, it seems reasonable to expect that loan commitment owners will exercise increasing proportions of their outstanding commitments. Thus, recognition of fractional takedowns introduces an additional source of capital loss (gain). Not only do losses per dollar of loans made under commitments increase with $E(r_{m1})$, but the amount of loans, L, rises as well, i.e., $\mathrm{d}E(L)/\mathrm{d}E(r_{m1})>0$. When the elasticity of demand for loans $d(B)$ is both positive and finite both proffered explanations for partial takedowns may play a role. The first explanation, based on limited demand for debt, is likely to increase in relative importance as the elasticity of demand declines. The alternative explanation, based on the bank–customer relationship, gains force when investment opportunities are abundantly available and the firm's demand schedule for funds is highly elastic.

4. Estimating loan commitment values

To estimate loan commitment values, we need to incorporate fractional takedowns into eq. (2) and find an analytical solution to the resulting expression. The expected takedown $E(L)$ may be estimated from experience. To solve for U_0, we need to specify $G(x_T|x_0)$ and the discount rate r_e, and evaluate the integral.

Recent work done in contingent claims valuation suggests a solution based on equilibrium in a competitive capital market.[10] A loan commitment permits the purchaser to sell a risky security (the customer's indebtedness) to the option writer (bank) at a specified future date and price.[11] The

[10]An alternative approach was outlined in Greenbaum (1975).

[11]This corresponds to a European put option which may be exercised only at the maturity date. An American put option may be exercised any time before maturity and is worth more than the equivalent European put. In practice, bank loan commitments are often exercised *in parts* over a specified time interval, rather than wholly at a single point in time. This feature implies an extremely complicated type of option which we ignore in the subsequent analysis.

commitment may be viewed as a put option with a striking price equal to the face value of the loan commitment, L^*. The option is sold by the bank at $t=0$ for αL^*, and matures at $t=T$. The underlying asset or state variable x_t is a debt contract from the borrower. The risk of the commitment seller arises primarily from the stochastic nature of r_{pt} and r_{mt}. In the face of rising interest rates, the commitment seller could sustain losses either because of a sticky prime rate or because the appropriate value for k varies positively with the level of interest rates whereas k is fixed under terms of the commitment. If $(r_{pt}+k)\equiv r_{mt}$, then $E(x_T)\equiv x_T$ and the value of the option, U_0, is always zero. In this case, the bank has no risk exposure and banks in competition would presumably bid α to zero. Similarly, borrowers would have no incentive to purchase loan commitments if they knew that the rate at which the bank issues a commitment at $t=0$ is identically equal to the rate at which they would be able to obtain funds at $t=T$ in the absence of a commitment.

In solving eq. (2), one is tempted to follow the Black and Scholes (1972) and Merton (1973a) approach by constructing a hedge portfolio including the loan commitment in question. However, a prerequisite of this approach is that the relevant variables should be traded assets. Since there is no active secondary market for bank loan commitments, this requirement is not satisfied. Fortunately, the difficulty can be overcome by using the general valuation principle developed in Ross (1976), Garman (1977), and Dothan and Williams (1978a), or by appealing to the intertemporal CAPM [see Merton (1973b)]. We shall follow the approach suggested by Constantinedes (1978) in applying the CAPM. To do this, we first need to specify the price dynamics of the state variable, x. Assume that changes in x in the time interval $(t,t+\mathrm{d}t)$ are described by[12]

$$\mathrm{d}x=\mu\,\mathrm{d}t+\sigma\,\mathrm{d}\xi,$$

where $\mu=\bar{\mu}x$, $\sigma=\bar{\sigma}x$, and $\mathrm{d}\xi$ is the increment of a Wiener process. We assume $\bar{\mu}$ and $\bar{\sigma}$ are constants, which means $\ln x$ is normally distributed with mean $\bar{\mu}-\bar{\sigma}^2/2$ and variance $\bar{\sigma}^2$, per unit time.

Assuming that the value of the option $U(x,t)$ is twice continuously differentiable in x and once continuously differentiable in t, we can appeal to

[12]Note that we are assuming that the distribution of x at $t=T$ is lognormal, which means that x takes values in the set $(0,\infty)$. In the discussion that follows, the riskless rate of interest, r, is assumed to be constant and finite. However, no upper bound is placed on either r_{pt} or r_{mt}. Thus, $r_{pt}, r_{mt}\in(r,\infty)$. Moreover,

$$x_T\in(L\exp[(r+k-r_{mT})(\tau-T)],\ L\exp[(r_{pT}+k-r)(\tau-T)]),$$

and $\inf\{x_T\}=0$, $\sup\{x_T\}=\infty$.

Ito's lemma and write

$$\mathrm{d}U(x,t) = (U_t + \mu U_x + (\sigma^2/2)U_{xx})\,\mathrm{d}t + \sigma U_x\,\mathrm{d}\xi,$$

where

$$U_t \equiv \frac{\partial U(x,t)}{\partial t}, \quad U_x \equiv \frac{\partial U(x,t)}{\partial x} \quad \text{and} \quad U_{xx} \equiv \frac{\partial^2 U(x,t)}{\partial x^2}.$$

Therefore

$$\frac{\mathrm{d}U(x,t)}{U(x,t)} = \frac{1}{U(x,t)}\left[U_t + \mu U_x + \frac{\sigma^2}{2}U_{xx}\right]\mathrm{d}t + \frac{\sigma U_x}{U(x,t)}\mathrm{d}\xi.$$

In equilibrium, the loan commitment will satisfy the intertemporal CAPM, if the necessary assumptions hold.[13] If η_p is the expected rate of return on the loan commitment per unit time [i.e., $E(\mathrm{d}U/U) = \eta_p\,\mathrm{d}t$], we have

$$\eta_p = \frac{1}{U}\left(U_t + \mu U_x + \frac{\sigma^2}{2}U_{xx}\right), \tag{5}$$

$$\sigma_{pm} = \rho\sigma_m\sigma U_x/U, \tag{6}$$

$$\eta_p - r = \lambda\sigma_{pm}/\sigma_m, \tag{7}$$

and

$$\lambda = \eta_m - r/\sigma_m, \tag{8}$$

where r is the instantaneous risk-free rate of interest,[14] σ_{pm} is the covariance of the loan commitment with the market per unit time, σ_m^2 is the variance per

[13]See Dothan and Williams (1978b) for a discussion of the assumptions underlying the intemporal CAPM. The essential conditions are that the relevant means, variances, and covariances exist and that the market clears for all assets included in the market index. These conditions are surprisingly general [e.g., see Merton (1973b)] and it is no longer necessary that both the contingent asset and the underlying security be costlessly and continuously traded in idealized, frictionless markets.

[14]In the absence of a riskless asset, r can be interpreted as the instantaneous expected rate of return on the zero-beta portfolio, and all of the subsequent results are sustained. Hence, for concreteness we shall assume the availability of a riskless asset and we shall refer to r as the riskless rate of interest. Further, note that we are assuming no intertemporal uncertainty in r. While stochastic variations in r would impart greater authenticity to the model, it would greatly complicate the analysis and probably destroy any hope of obtaining an analytical solution. Since the loan commitment derives its value from variations in r_{mt} and r_{pt}, it seems reasonable to focus on these two rates of interest.

unit time, η_m is the expected return per unit time of the market portfolio, and ρ is the correlation coefficient between the return on the loan commitment and the return on the market.

Substituting eq. (5) in eq. (7) we get

$$\frac{1}{U}\left(U_t+\mu U_x+\frac{\sigma^2}{2}U_{xx}\right)-r=\frac{\lambda\sigma_{pm}}{\sigma_m}.$$

Substituting eq. (6) above we get

$$\frac{1}{U}\left(U_t+\mu U_x+\frac{\sigma^2}{2}U_{xx}\right)-r=\frac{\lambda\rho\sigma_m\sigma U_x}{U\sigma_m},$$

which implies that

$$U_t+(\mu-\lambda\rho\sigma)U_x+\frac{\sigma^2}{2}U_{xx}-rU=0. \tag{9}$$

To obtain an expression for $U(x,t)$, we have to solve eq. (9) subject to the boundary condition

$$U(x,T)=\max(L-x_T,0). \tag{10}$$

Recalling that $\mu=\bar{\mu}x$ and $\sigma=\bar{\sigma}x$, we can rewrite eq. (9) as

$$U_t+(\bar{\mu}x-\lambda\rho\bar{\sigma}x)U_x+\frac{\bar{\sigma}^2x^2}{2}U_{xx}-rU=0. \tag{11}$$

The solution to eq. (11), subject to the boundary condition (10), is given by[15]

$$\begin{aligned}U(x_0,0)=&-x_0\left\{1-\exp\left[(\bar{\mu}-\lambda\rho\bar{\sigma}-r)T\right]N\right.\\&\left.\times\left[\frac{\ln(x_0/L^*)+(\bar{\mu}-\lambda\rho\bar{\sigma}+\bar{\sigma}^2/2)T}{\bar{\sigma}\sqrt{T}}\right]\right\}\\&+L^*\exp(-rT)\left\{1-N\right.\\&\left.\times\left[\frac{\ln(x_0/L^*)+(\bar{\mu}-\lambda\rho\bar{\sigma}-\bar{\sigma}^2/2)T}{\bar{\sigma}\sqrt{T}}\right]\right\},\end{aligned} \tag{12}$$

[15]The solution to eq. (11), subject to a call option boundary condition instead of (10), appears in Constantinedes (1978).

where

$$x_0 = L^*[\exp\{(r_{p0} + k - r_{m0})(\tau - T)\}], \tag{13}$$

and $N(\cdot)$ is the standard error function.[16]

If there was an active secondary market for loan commitments, and if the state variable were traded, we should obtain $\bar{\mu} - \lambda\rho\bar{\sigma} = r$ and eq. (12) would reduce to the familiar Black–Scholes formula for European put options:

$$U(x_0, 0) = -x_0\left\{1 - N\left[\frac{\ln(x_0/L^*) + (r + \bar{\sigma}^2/2)T}{\bar{\sigma}\sqrt{T}}\right]\right\}$$
$$+ L^* \mathrm{e}^{-rT}\left\{1 - N\left[\frac{\ln(x_0/L^*) + (r - \bar{\sigma}^2/2)T}{\bar{\sigma}\sqrt{T}}\right]\right\}. \tag{14}$$

From option pricing theory or by inspection of eq. (12), we know that $\partial U_0/\partial x_0 < 0$ and $\partial U_0/\partial\bar{\sigma} > 0$. If the expected opportunity cost of capital r_{m0}, increases more than the expected prime rate, x_0 declines. The value of the loan commitment increases, reflecting a decline in the value of the customer's indebtedness. If the increase in the marginal opportunity cost of funds is accompanied by greater volatility in $(r_{mt} - r_{pt})$, as is often the case in periods of rapidly rising interest rates, U_0 will increase further.

For illustrative purposes, consider a one dollar loan commitment drawn up in October 1975 and exercisable in April 1976. The loan, if taken down, has a maturity of one year. The marginal gross rate on loans is taken as the sum of the six-month CD rate in October 1975 plus add-ons for default risk and normal profit (see table 1). The 0.6 percent add-on for default risk was obtained by dividing the difference between loan losses charged to reserves and recoveries by average total loans for 1975, for all insured commercial banks. The 0.85 percent profit margin was obtained by dividing 1975 net income for all insured banks by average total footings. We fix k at 0.5 percent and r at 7.25 percent [the bond equivalent six-month U.S. Treasury bill rate in October 1975, Salomon Brothers (1976)]. Converting all rates to monthly bases and letting $\bar{\mu} = 0.01$, $\rho = 0.8$, $\eta_m = 0.008$, and $\sigma_m = 0.002$, we can use the data in table 1 and eqs. (12) and (13) to obtain $x_0 = 0.9897$,

[16]Note that the loan commitment has the functional form specified in eq. (12) due to our assumption that x_T is lognormal or equivalently that $(r_{pT} - r_{mT})$ is normal. The reasonableness of this assumption is an empirical issue, and alternative distributional specifications will yield different valuation formulae. However, because a bank loan commitment has an isomorphic correspondence with a put option, our observation about the relationship between interest rate volatility and commitment values will be valid regardless of the distributional assumptions made. [See Merton (1973a).]

$\bar{\sigma}=0.00458$, and $U_0=0.009$. Thus, assuming complete takedown, the value of the loan commitment is 0.9 percent.[17]

As of 1976, formal loan commitments at larger U.S. commercial banks totalled \$150 billion and the average takedown was about 45 percent. Since informal commitments probably exceed formal commitments and the above data cover only a fraction of the banking system, \$150 billion may not be an unreasonable estimate of takedowns under all loan commitments. Thus, approximately \$1.35 billion of bank liabilities associated with loan commitments failed to appear on bank balance sheets in 1975.

Table 1

The cost of capital and the prime rate.[a]

1975	r_m (percent) 6-month CD rate	Profit and default rate	r_p (percent) prime rate	$r_m - r_p$	x_0
January	8.70	1.45	10.05	0.10	1.004
February	6.65	1.45	8.96	−0.86	1.014
March	6.40	1.45	7.93	−0.08	1.006
April	6.40	1.45	7.50	0.35	1.002
May	6.50	1.45	7.40	0.55	0.9995
June	6.15	1.45	7.07	0.53	0.9997
July	6.70	1.45	7.15	1.00	0.9952
August	7.45	1.45	7.66	1.24	0.9927
September	7.70	1.45	7.88	1.27	0.9924
October	8.05	1.45	7.96	1.54	0.9897
November	7.00	1.45	7.53	0.92	0.9958
December	6.90	1.45	7.26	1.09	0.9941

[a] *Source:* The prime rate and CD rates are from Salomon Brothers (1976). Default rate (0.6 percent) was obtained from loan loss data appearing in FDIC (1976). The profit rate (0.85 percent) was estimated from net income on assets data in FDIC (1976).

To illustrate the effect of a change in the marginal gross rate on loans r_{mT}, fig. 3 shows U_0 for a \$1 commitment with changing r_{mT}, given that r and $(r_{pT}+k)$ are fixed. Notice that volatility has an increasing influence on U_0 as the spread between r_{mT} and $(r_{pT}+k)$ narrows.

5. Conclusion

Although loan commitments are integral to commercial banking, they tend to be ignored in bank balance sheets. This accounting omission desensitizes

[17] Note that $\bar{\sigma}$ was obtained in the usual fashion as the square root of the variance of the logarithm of the rate of return on the state variable x.

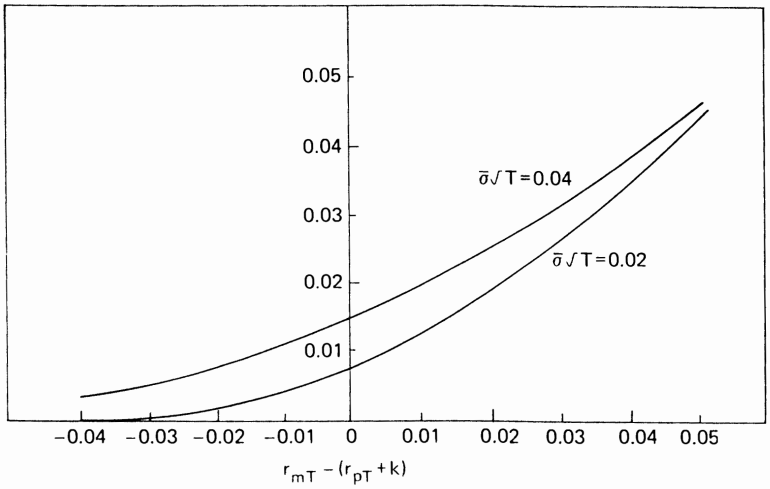

Figure 3. The option value per unit of loan commitment for fixed r and $r_{pT}+k$.

banks' net worth accounts to interest rate changes and insulates balance sheets against deterioration in times of rapidly rising interest rates.

One reason for the accounting omission is the lack of agreement as to how to value commitments. In turn, the valuation problem has led to managerial confusion in pricing loan commitments. This paper has sought to dispel some of the confusion surrounding loan commitments by viewing them as options. It also provides coarse estimates of loan commitment values and their sensitivity to interest rate changes. Properly valued, bank loan commitments probably would not bulk large in the banking system's balance sheet. However, variations in the value of loan commitments arising from interest rate variability could impart significant volatility to the net worth account of larger banks.

This paper constitutes a first step in analyzing a complex problem. We have not integrated the partial takedown phenomenon into the option-pricing approach to loan commitments. Providing an alternative valuation model that incorporates partial takedowns would appear to be the logical next step.

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