

# OPTIMAL CAPITAL STRUCTURE AND PROJECT FINANCING

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## Optimal Capital Structure and Project Financing\*

SALMAN SHAH

*Faculty of Management Studies, University of Toronto,  
246 Bloor St. West, Toronto, Ontario M5S 1V4, Canada*

AND

ANJAN V. THAKOR

*School of Business, Indiana University,  
Bloomington, Indiana 47405*

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We examine the financing and incorporation modes for new projects. There are two objectives. The first is to provide a theory of optimal capital structure that links risk, leverage, and value and is particularly applicable to large firms. Counter to conventional wisdom, we show riskier firms acquire more debt, pay higher interest rates, and have higher values in equilibrium. Second, we provide an economic rationale for project financing which entails organizing a new project legally distinct from the firm's other assets. We explain why project financing involves higher leverage than conventional financing and why highly risky assets are project-financed. *Journal of Economic Literature* Classification Numbers: 022, 521.

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### I. INTRODUCTION

This paper is concerned with how an investment should be financed and whether it should be incorporated separately or within the firm. Our specific objectives are twofold. The first is to develop a theory of optimal capital structure, and the second is to show how the theory can explain a rapidly growing institutional practice, namely project financing.

Notwithstanding the seminal Modigliani and Miller [23, 24] theorem, a firm's capital structure appears to be a key financial decision that is not yet completely understood. One strand of the literature, exemplified by Bulow

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and Shoven [3], Kim [15], and Scott [33], suggests that an interior optimal capital structure results from the tradeoff between the tax advantages and exogenous bankruptcy costs of leverage. More recent papers within this group have demonstrated optimal capital structure without relying on exogenous leverage-related costs by suggesting that the presence of firm-specific tax shields unrelated to leverage (DeAngelo and Masulis [8] and Kim [16]) increases the measure of the set of states characterized by debt-tax shield redundancy. They show that this induces a concavity with respect to leverage in the firm's after-tax expected profit and thus results in an interior optimal capital structure.<sup>1</sup> Another strand, pioneered by Ross [30], and Leland and Pyle [18], and subsequently developed by Heinkel [12], and Lee, Thakor and Vora [17], assumes asymmetric information between managers and investors and shows that a firm's capital structure may be important because it acts as a signal of an a priori unknown parameter of the probability distribution of its future cash flows. In related papers, Jensen and Meckling [14], Hellwig [13], and Chaney and Thakor [4] propose capital structure relevance under moral hazard. Recently, Darrough and Stoughton [5] have integrated moral hazard and adverse selection considerations in a capital structure model.

While these papers and others highlight the multifaceted role of leverage, they do not explain why there are significant leverage differences between conventionally financed ventures (those incorporated as integral components of the sponsoring firms) and project-financed ventures (those incorporated as legally segregated entities), and why project financing has grown so rapidly in the corporate taxation era. Thus, to build a theory of project financing we need an alternative model of optimal capital structure. In this paper we develop such a model, based on corporate taxes and asymmetric information.<sup>2</sup> We demonstrate that, in raising the required investment outlay for a given project, each firm will use a unique (interior) mix of debt and equity—despite corporate taxation and the absence of bankruptcy costs—and show that there are some projects whose values are maximized with separate incorporation; moreover, our model predicts systematic leverage differences between such projects and those which are conventionally financed. In contrast to many existing papers, our theory does not rely on risk aversion, exogenously given signaling cost structures, bankruptcy costs (or any other exogenous costs associated with debt), or

<sup>1</sup> Actually, this induced concavity in DeAngelo and Masulis [8] can be obtained even without nondebt related tax shields. See Talmor, Haugen, and Barnea [35].

<sup>2</sup> A commonly stated reason for developing capital structure models without taxes is that the tax deductibility of corporate interest payments is a post-1913 development, while "interior" capital structures were observed even prior to 1913. However, we are not concerned with this justification for excluding taxes because project financing is a relatively recent development and taxes, we think, are an important factor in its growing popularity.

## OPTIMAL CAPITAL STRUCTURE AND PROJECT FINANCING

nonpecuniary moral hazard in loan contracts. (Specific distinctions between our research and the existing literature are outlined in Sect. III.) The reliance is on asymmetric information, which is in the spirit of Arrow's [1] conjecture about the importance of informational imperfections in understanding financing decisions in a securities market without Arrow-Debreu-type contingent dealings.

Our principal finding on capital structure differs from traditional wisdom. We show that in equilibrium riskier firms choose higher debt levels and pay higher interest rates. Moreover, riskier firms also have higher equilibrium values. (The intuition is provided in the discussion following Theorem 3 in Sect. III.) These results, which play a pivotal role in our examination of project financing, are of interest in their own right because they provide a possible explanation for documented differences in leverage ratios across industries. As Miller [22] notes, these differences have yet to be well understood,

On the other hand, many questions clearly still remain to be answered. What about cross-sectional variations in debt ratios, for example—a subject on which surprisingly little work has yet been done?

Our study of project financing is motivated by the scant attention paid to it in the literature.<sup>3</sup> Yet, over the past decade, this has become an increasingly important financing tool for marshalling capital resources for a wide array of natural resource projects both in the United States and abroad. Project financing is currently a multibillion dollar activity involving the construction of docks for supertankers, supermarkets, massive iron ore pellet processing facilities, drilling rigs for deep sea oil and gas exploration, power generation facilities, etc.

Project financing is defined as an arrangement whereby a sponsor or group of sponsors incorporates a project as a legally separate entity, with project cash flows kept segregated for financing purposes from its sponsors, thereby permitting an appraisal independent of any direct support from the participants themselves. Project financing usually involves the sponsors providing equity and management for the project and issuing debt that is nonrecourse to the sponsors. That is, creditors must rely exclusively on the ability of the project for repayment of project-related debt obligations. Thus, from the creditors' perspective, a key difference between conventional financing and project financing is as follows. If conventional financing is used and the project is organized as a component of the firm, the project cash flows are mingled with the firm's other cash flows. The lenders' evaluation of the project is then based on the "post-project-adoption"

<sup>3</sup> The sole exception is an excellent practitioner-oriented paper by Mao [20] which conjectures that the ultimate economic rationale for project financing may lie in informational imperfections.

## OPTIMAL CAPITAL STRUCTURE AND PROJECT FINANCING

creditworthiness of the total firm—and, of course, the seniority of the project lenders' claims in case of default—rather than the cash flow distribution of the project per se. On the other hand, if project financing is resorted to, the creditors' evaluation of the project is linked *solely* to the project's own cash flow generation capability.

Two project financing-related issues appear to deserve attention. First, we would like an economic rationale for project financing. And second, we would like the theory to explain two stylized facts:<sup>4</sup> (i) project financing involves higher leverage ratios than conventional financing, and (ii) many investments utilizing project financing appear to be highly risky.

Our theory sheds light on the above stylized facts. We show that, for a variety of projects, *the method of incorporation of the project affects both its leverage and economic value*; project financing enhances the values of some of these projects by permitting higher optimal leverage than with conventional financing. The conditions that guarantee this constitute sufficiency conditions for the strict Pareto dominance of project financing and supply the first economic rationale for the practice.<sup>5</sup>

Our research is related to the work of Myerson [26] and others in the theory of resource allocation mechanisms. As in that work, we begin with a description of the economic environment at a primitive level by specifying agents' preferences, endowments, exchange technology, and the initial information structure. Then, assuming that economic agents are limited to debt and equity contracts, but abstaining from any fixed resource allocation scheme, we examine a broad class of schemes to discover the constraints on resource allocation implied by asymmetric information. Properties of the Pareto optimal outcomes are subsequently derived by solving the usual programming problem. An interior optimal capital structure as well as an economic rationale for project financing emerge endogenously as Pareto optimal resource allocation outcomes. Our results can be seen as buttressing Townsend's [36] idea that financial structures—viewed there as *all* payment devices—are communication systems.

The paper is organized as follows. Section II contains a description of the economy and a development of the symmetric information equilibrium. In

<sup>4</sup> See McGuinty [21] and Nevitt [27] for discussions of these stylized facts.

<sup>5</sup> Casual explanations for project financing can be found in the popular literature (see, e.g., the Wall Street Journal). These usually revolve around the conjecture that firms like project financing because it allows them to obtain "off-balance sheet debt" and thus be relatively highly leveraged while maintaining apparently conservative financial statements and financial ratios that do not run afoul of existing loan covenants. This is, however, an unsatisfactory explanation—even in a world with informational limitations—because it requires lenders to systematically ignore highly visible "off-balance sheet" investments by their borrowers. This criticism is further underscored by the fact that most loan covenants these days impose explicit restrictions even on activities that do not completely appear on borrowers' balance sheets, such as project financing.

Section III it is assumed that project sponsors have information about their projects' payoff distributions that outsiders do not have. Adopting Riley's [29] reactive equilibrium concept, we characterize the asymmetric information capital structure equilibrium. The model is extended in Section IV to provide an economic justification for project financing. Section V contains all the formal proofs.

II. DESCRIPTION OF THE ECONOMY

The economy consists of two types of economic agents (project sponsors and creditors), two points in time, and a single commodity. All agents are risk neutral. A firm is defined as a collection of project sponsors. There are many firms in the economy and each has the option to access a production process that is an indivisible point-input, point-output project requiring an investment of  $I$  units of the commodity at the beginning of the period (the first point-in-time) and yielding a random gross return of  $R$  at the end of the period (the second point-in-time). Thus, a project sponsor can either consume its endowment at the beginning of the period or invest it for later consumption. Similarly, a creditor can consume its endowment now or lend it to a firm to obtain a claim to a risky, end-of-period payoff. The marginal rate of substitution (the number of end-of-period units of the commodity that must be offered in exchange for one unit of the commodity now) is  $\rho$ . We assume that all agents have the option to forego a unit of the commodity at the beginning of the period in exchange for a riskless claim to  $\rho$  units of the commodity at the end of the period.

Let  $\mathbb{R}_+ \equiv [0, \infty]$ . We assume  $R$  takes its values in  $\mathbb{R}_+$ . The randomness in  $R$  is described by a probability measure,  $F(R, \theta)$ , defined on a sigma-algebra of the subsets of  $\mathbb{R}_+$ , where  $\theta$  is a parameter that indexes the riskiness of  $R$ . The value of  $\theta$  varies across firms and we normalize it to take values in a compact subset,  $[\theta_1, \theta_2]$ , of  $[0, 1]$ . We also assume that there exists a density function,  $f(R, \theta)$ , associated with  $F(R, \theta)$ . Thus, the project to which a firm has access varies from one firm to the next, and this cross-sectional variation is described by the continuum  $[\theta_1, \theta_2]$ . Increasing values of  $\theta$  imply increasing risk in the Rothschild and Stiglitz [32] sense. That is,

$$\int_{\mathbb{R}_+} Rf(R, \tilde{\theta}) dR = \int_{\mathbb{R}_+} Rf(R, \theta) dR = \bar{R} \tag{1}$$

and for  $\theta < \tilde{\theta}$  and  $\chi \geq 0$ ,

$$\int_0^\chi F(R, \tilde{\theta}) dR \geq \int_0^\chi F(R, \theta) dR \tag{2}$$

with strict inequality for  $\chi > 0$  and  $\theta$  and  $\bar{\theta}$  in the interior of  $[\theta_1, \theta_2]$ . Thus, firms have projects with the same mean return,  $\bar{R}$ , but different risks. Henceforth, for any two density functions  $f(R, \theta)$  and  $f(R, \bar{\theta})$  satisfying (1) and (2), we shall use the notation  $f(R, \bar{\theta}) \textcircled{R} f(R, \theta)$  to indicate that  $f(R, \bar{\theta})$  is riskier than (but has the same mean as)  $f(R, \theta)$  in the above sense. For now we assume  $\theta$  is common knowledge.

In this framework, project sponsors are shareholders and creditors are bondholders. (We temporarily assume that the firm has no risky endowment other than the option to invest in the risky project.) Let  $\alpha I$  be the amount of initial investment a firm finances with debt, and  $[1 - \alpha] I$  the amount it finances with the shareholders' equity. In exchange for every unit of the commodity the creditors lend at the beginning of the period, they require a payment of  $K$  units at the end of the period. We shall assume that each unit of the commodity is worth \$1, so that the single commodity can be called (consumable) cash. A central taxing authority levies a wealth tax at the rate  $\tau$  on the end-of-period cash flow accruing to the shareholders, and creditors operate in a competitive environment.

Denote by  $V^0$  the total expected end-of-period after-tax cash flow generated by the project, by  $B^0$  the expected end-of-period cash flow accruing to the creditors, and by  $\hat{E}^0$  the expected end-of-period cash flow accruing to the shareholders. Discounted back to the present, these variables would represent the total value of the project, the value of debt, and the value of equity, respectively. It is notationally convenient to partition  $\mathbb{R}_+$  as follows,

$$\Gamma \equiv \{R \mid R \leq \alpha K, R \in \mathbb{R}_+\} \quad \text{and} \quad \Gamma^c \equiv \{R \mid R > \alpha K, R \in \mathbb{R}_+\}.$$

Thus,

$$\begin{aligned} V^0 = I \int_{\Gamma} R f(R, \theta) dR + \int_{\Gamma^c} I[R - \alpha K][1 - \tau] f(R, \theta) dR \\ + \int_{\Gamma} I \alpha K f(R, \theta) dR, \end{aligned} \quad (3)$$

$$B^0 = I \bar{R} - I \int_{\Gamma^c} [R - \alpha K] f(R, \theta) dR, \quad (4)$$

$$\hat{E}^0 = \int_{\Gamma^c} I[R - \alpha K][1 - \tau] f(R, \theta) dR. \quad (5)$$

Note

$$V^0 = B^0 + \hat{E}^0. \quad (6)$$

The *net* expected end-of-period cash flow accruing to the shareholders, therefore, is

$$\begin{aligned} E^0 &= \hat{E}^0 - [1 - \alpha] I\rho \\ &= \int_{r^*} I[R - \alpha K][1 - \tau] f(R, \theta) dR - [1 - \alpha] I\rho. \end{aligned} \quad (7)$$

The project sponsors choose  $\alpha$  to maximize (7) and then  $K$  is determined competitively to yield the creditors an expected return of  $\rho$ . That is,

$$B^0 = \alpha I\rho. \quad (8)$$

Combining (4) and (8) gives us

$$\int_{r^*} I[R - \alpha K] f(R, \theta) dR = I\bar{R} - \alpha I\rho \quad (9)$$

and substituting (9) in (7) produces

$$E^0 = I\{[1 - \tau] \bar{R} - \rho\} + \alpha I\rho\tau, \quad (10)$$

which is maximized at  $\alpha = 1$ . Thus, not surprisingly, our perfect information equilibrium replicates the Modigliani and Miller [24] extreme leverage proposition in the presence of corporate taxes.

### III. CAPITAL STRUCTURE EQUILIBRIUM UNDER ASYMMETRIC INFORMATION

Suppose now that each firm knows its own  $\theta$ , but creditors know only the cross-sectional dispersion of probability distributions with the same expected return  $\bar{R}$ . That is, the project sponsors within the firm know the  $\bar{R}$  and  $\theta$  for their project while creditors (and other firms) know  $\bar{R}$  but are a priori uninformed about  $\theta$ . Let  $g(\theta)$  denote the cross-sectional probability density function over  $\theta$ . We assume  $g(\theta)$  is common knowledge and that its support is  $[\theta_1, \theta_2]$ .

To simplify the analysis, suppose  $f(R, \theta)$  is decomposable as follows

$$f(R, \theta) = f^0(R) + \theta[f^1(R) - f^0(R)], \quad (11)$$

where  $f^1(R) \otimes f^0(R)$  and  $F^1(R)$  and  $F^0(R)$  are the associated cumulative distribution functions. We now establish that this decomposition preserves the property that increasing  $\theta$  implies increasing risk.

**THEOREM 1.** *Let  $f(R, \theta)$  be described by (11) with  $\theta \in [\theta_1, \theta_2] \subseteq [0, 1]$ . Then, for  $\hat{\theta} > \theta$  with  $\hat{\theta}, \theta \in [\theta_1, \theta_2]$ , we have  $f^1(R) \otimes f(R, \hat{\theta}) \otimes f(R, \theta) \otimes f^0(R)$ .*

From now on,  $f(R, \theta)$  will be assumed to be of the form specified in (11).

It is easy to see that, if there were no leverage, firms would be indifferent to their  $\theta$ 's because project sponsors are risk neutral. This, of course, is no longer true when debt is introduced, and creditors themselves "care" about the  $\theta$  of the project they help to finance. This observation is formalized in the lemma below.

**LEMMA 1.** *For any pair  $(\alpha, K) \in [0, 1] \times \mathbb{R}_+$ , the expected cash flow accruing to the creditor declines monotonically as  $\theta$  increases.*

Thus, market participants have an incentive to develop information transmission channels to prevent market failure. One solution is costly information production. For the present, however, we assume it is prohibitively costly to do so and look for other alternatives.

The subsequent analysis of optimal capital structure is tailored to enable examination of project financing. It is for this reason that we constrain the firm to raise capital no greater than the required investment. The major concern in project financing is the structuring of a financial package designed to procure the funds necessary to get the project on line; the issue is *not* to decide whether to sell the entire project to "outsiders." In fact, retention of some interest in the project by the initial sponsors is usually essential for access to the project. Thus, our focus is on analyzing how the *required investment* is composed of debt and equity. We now present some notation needed to define equilibrium.

We consider an atomless economy in which lenders compete on the basis of contracts. A *capital structure contract*,  $\omega$ , is defined as a pair  $(\alpha, K) \equiv \omega \in [0, 1] \times \mathbb{R}_+$ . When a firm selects such a pair, it binds itself to financing a fraction  $\alpha$  of its initial investment with debt and paying an interest factor of  $K$ . The contract a firm chooses is costlessly observable, which precludes "secret" leverage. Let  $\omega_i(\theta)$  be the capital structure contract designed by creditor  $i$  for a firm reporting  $\theta$ . The set

$$\Omega_i = \{\omega_i(\theta) | \theta \in [\theta_1, \theta_2]\}$$

therefore describes the *strategic policy* of creditor  $i$ . Let  $N = \{1, \dots, n\}$  denote the set of all possible (competing) creditors (the counting measure of  $N$  could be infinity) and let  $\xi_i(\Omega_1, \dots, \Omega_n)$  be the net expected profit of creditor  $i$  when the vector of strategic policies being offered is  $(\Omega_1, \dots, \Omega_n)$ . A creditor's net expected profit is the expected cash flow accruing to it from its lending less the opportunity cost of not being able to invest the same amount at the riskless rate. We can now formally define equilibrium.

DEFINITION. A reactive capital structure equilibrium (RCSE) is a set of feasible strategic policies,  $\Omega^* = (\Omega_1^*, \dots, \Omega_n^*)$ , for the  $n$  creditors if for any  $i \in N$  and any feasible strategic policy  $\Omega_i$  such that

$$\xi_i(\Omega_1^*, \dots, \Omega_i, \dots, \Omega_n^*) > \xi_i(\Omega^*)$$

$\exists$  another creditor  $j \in N$  and another feasible strategic policy  $\Omega_j$  such that

- (i)  $\xi_j(\Omega_1^*, \dots, \Omega_j, \dots, \Omega_n^*) \leq \xi_j(\Omega^*)$ ,
- (ii)  $\xi_j(\Omega_1^*, \dots, \Omega_i, \dots, \Omega_j, \dots, \Omega_n^*) > \xi_j(\Omega_1^*, \dots, \Omega_i, \dots, \Omega_j^*, \dots, \Omega_n^*)$ ,
- (iii)  $\xi_i(\Omega_1^*, \dots, \Omega_i, \dots, \Omega_j, \dots, \Omega_n^*) < \xi_i(\Omega^*)$ ,
- (iv)  $\forall m \in N, m \neq i, j$ , and all feasible  $\Omega_m$   
 $\xi_j(\Omega_1^*, \dots, \Omega_i, \dots, \Omega_j, \dots, \Omega_m, \dots, \Omega_n^*)$   
 $\geq \xi_j(\Omega_1^*, \dots, \Omega_i, \dots, \Omega_j^*, \dots, \Omega_m^*, \dots, \Omega_n^*)$ .

This definition of the RCSE is due to Riley [29]. Riley's motivation for developing it came from the (by now well-known) problem with the existence of pure strategy Nash equilibria with a continuum of types.<sup>6</sup>

To characterize the properties of the RCSE, we now examine the design of capital structure contracts. We can view each creditor as publicly announcing its strategic policy in the form of a complete menu of capital structure contracts, with firms allowed to select their own contracts. Alternatively, following the revelation principle developed in the literature on resource allocation mechanisms (see, e.g., Myerson [26]), we can think of every creditor asking each firm that approaches it to directly report its true  $\theta$  to the creditor, and then awarding the firm an  $\omega(\theta)$  contingent on that report. Thus, a capital structure contract is a mapping

$$\omega: [\theta_1, \theta_2] \rightarrow [0, 1] \times \mathbb{R}_+.$$

For a firm that reports  $\hat{\theta}$  when its true attribute is  $\theta$ , let  $E(\hat{\theta}|\theta)$  be the difference between the net expected end-of-period cash flow accruing to the shareholders if there is leverage and the net expected end-of-period cash flow accruing to the shareholders if there is no leverage. Similarly, define  $B(\hat{\theta}|\theta)$  as the expected end-of-period cash flow accruing to the creditors when a firm with a true attribute  $\theta$  reports  $\hat{\theta}$ . When there is truthful reporting, we set  $E(\theta|\theta) \equiv E(\theta)$  and  $B(\theta|\theta) \equiv B(\theta)$ . In what follows, we characterize the properties of feasible capital structure allocations before proving the existence of the RCSE. The next result forms the basis of our analysis.

<sup>6</sup> Dasgupta and Maskin [6, 7] have recently shown that the existence of mixed strategy equilibria can usually be proved in market games in which pure strategy Nash equilibria fail to exist.

## OPTIMAL CAPITAL STRUCTURE AND PROJECT FINANCING

LEMMA 2. *Assume the RCSE exists. Then, the RCSE contracts are a solution to the following constrained maximization program*

$$\text{maximize} \quad \int_{\theta_1}^{\theta_2} E(\theta) g(\theta) d\theta \quad (12)$$

subject to

$$\begin{aligned} E(\hat{\theta}|\theta) = & \int_{r(\hat{\theta})} I[R - \alpha(\hat{\theta}) K(\hat{\theta})][1 - \tau] f(R, \theta) dR \\ & - [1 - \alpha(\hat{\theta})] I\rho - \{I\bar{R}[1 - \tau] - I\rho\}, \end{aligned} \quad (13)$$

$$B(\hat{\theta}|\theta) = \int_{r(\hat{\theta})} IRf(R, \theta) dR + \int_{r(\hat{\theta})} I\alpha(\hat{\theta}) K(\hat{\theta}) f(R, \theta) dR, \quad (14)$$

$$B(\theta) = \alpha(\theta) I\rho, \quad \forall \theta \in [\theta_1, \theta_2], \quad (15)$$

$$E(\theta) \geq 0, \quad \forall \theta \in [\theta_1, \theta_2], \quad (16)$$

$$0 \leq \alpha(\theta) \leq 1, \quad \forall \theta \in [\theta_1, \theta_2], \quad (17)$$

$$E(\theta|\theta) \geq E(\hat{\theta}|\theta) \quad \forall \theta, \hat{\theta} \in [\theta_1, \theta_2]. \quad (18)$$

Thus, this lemma says that creditors compete on the basis of the capital structure contracts they offer, and competition induces each creditor to design its offer set to maximize the cross-sectionally weighted expected utilities of its borrowers.<sup>7</sup> The constraints are straightforward; (13) and (14) are definitional constraints, (15) is a competitive consistency condition which says that the creditor must earn zero net expected profit on every borrowing firm in equilibrium, (16) is an individual rationality constraint (no firm will accept a capital structure contract that yields it lower welfare than using all equity), (17) says that the leverage assumed by a firm cannot be negative and it cannot exceed the initial financing required, and finally, (18) is an incentive compatibility constraint which guarantees that all feasible capital structure contracts induce truth-telling. The next lemma shows that this maximization problem can be further simplified.

LEMMA 3. *Assuming the RCSE exists, the RCSE contracts are a solution to the following constrained maximization program*

$$\text{maximize} \quad \int_{\theta_1}^{\theta_2} E(\theta) g(\theta) d\theta, \quad (12)$$

<sup>7</sup> As is always the case with reactive equilibria, the RCSE contracts here are independent of  $g(\theta)$ . That is, despite the fact that the reactive equilibrium is the Pareto dominating member of the set of informationally consistent equilibria—as Lemma 2 claims—the allocations under the reactive equilibrium do not depend on the welfare weights used.

subject to

$$E(\theta) = I\alpha(\theta) \rho\tau, \quad (19)$$

$$\int_{r(\theta)} I[R - L(\theta)] f(R, \theta) dR + IL(\theta) = \alpha(\theta) I\rho, \quad (20)$$

$$L(\theta) \equiv \alpha(\theta) K(\theta), \quad (21)$$

$$E(\theta) \geq 0, \quad (22)$$

$$0 \leq \alpha(\theta) \leq 1, \quad (23)$$

$$E'(\theta) = [1 - \tau] \int_{r(\theta)} I[R - L(\theta)] \{f^0(R) - f^1(R)\} dR,$$

$$\text{for almost every (a.e.) } \theta \in [\theta_1, \theta_2], \quad (24)$$

$$E'(\theta) \geq 0, E''(\theta) \geq 0, \quad \text{wherever } E'(\theta) \text{ exists.} \quad (24')$$

A sequence of lemmata are next presented to examine the nature of the RCSE.

LEMMA 4. *For a fixed  $\alpha$ , the value of  $K$  that satisfies (20) increases monotonically as  $\theta$  increases.*

LEMMA 5. *For a fixed  $\theta$ , the value of  $K$  that satisfies (20) increases monotonically as  $\alpha$  increases.*

Our next result shows that, for any feasible allocation, almost every type of firm gets a level of debt financing that falls short of its first best level. A feasible allocation is defined as one that satisfies the informational consistency requirements (24) and (24') as well as the other constraints in Lemma 3.

THEOREM 2. *For any feasible capital structure schedule, all firms with  $\theta \in [\theta_1, \theta_2)$  that obtain nonzero leverage under that schedule have interior capital structures.*

Thus, if one thinks of a firm's capital structure as a signal of its true risk attribute, the signaling equilibrium—if an equilibrium exists—is dissipative.<sup>8</sup> There is a loss in tax shield, the value of which does not accrue to the creditors. This occurs for all firms, except possibly the riskiest. Whether the riskiest firms obtain their first best level of financing depends on some parametric restrictions, as we show later. The principal message of this theorem, however, is that almost all firm types that are able to borrow

<sup>8</sup> The existence of an equilibrium is proved in Theorem 5. Clearly, the equilibrium is dissipative only relative to the private sector.

have interior optimal capital structures.<sup>9</sup> This contrasts sharply with the conventional notion that firms should be all-debt financed when there is corporate taxation, personal taxes are inconsequential to the corporate capital structure decision, and leverage entails neither exogenously stipulated deadweight losses such as bankruptcy costs nor any endogenous tradeoff of the DeAngelo and Masulis [8] type.<sup>10</sup>

Given next is a description of the dependence of a firm's leverage, debt interest rate, equity value, and total asset value on its risk attribute.

**THEOREM 3.** *For any feasible capital structure schedule, the amount of debt financing available and the interest rate paid are nondecreasing functions of  $\theta$ . Moreover, the firm's debt repayment obligation (face value of debt), the net (per share) value of its equity and the value of its total assets are also nondecreasing functions of  $\theta$ .*

This is a strong and empirically testable prediction about the capital structure policies of firms and about the admissible relationships between leverage, value and risk. As in Ross [30], a cross-sectional regression of leverage and firm values will, according to this theory, have a positive slope. But the causality here is reversed. Unlike Ross the feasible allocation does not involve the higher valued firms using more leverage. Rather, firms that use higher leverages have higher values! Moreover, Ross's model is nondissipative whereas ours is dissipative, and the underlying resource allocation scheme that generates the informational equilibrium there is exogenous—Ross takes the linearity restriction on managerial incentive contracts as given—whereas it is endogenous in our model.<sup>11</sup>

Our modeling and results also differ from Leland and Pyle's [18]. In their model, riskier firms choose lower optimal debt levels (even without bankruptcy costs) and the project sponsors (entrepreneurs) in these firms enjoy lower expected utilities.<sup>12</sup> The Leland and Pyle model, in contrast to

<sup>9</sup> As we show later, incentive compatibility may dictate that a subset of  $[\theta_1, \theta_2]$  should be denied credit. Hence, the interior capital structure finding applies only to these firms in  $[\theta_1, \theta_2]$  that obtain credit. We do prove later, though, that the Lebesgue measure of firm types that obtain credit in equilibrium is strictly positive.

<sup>10</sup> Such a tradeoff is obviously absent in our model or else we would have encountered an interior optimum with perfect information.

<sup>11</sup> There is also a "side payments" issue in Ross [30] that is absent here. Because truthful reporting in that model is predicated upon a managerial incentive contract, managers—whose wealths could generally be expected to be small relative to the values of the firms they manage—could potentially be offered "side payments" by the shareholders to induce them to emit false signals.

<sup>12</sup> Actually, Proposition III in Leland and Pyle [18] states that an increase in risk *increases* the entrepreneur's expected utility. However, it can be shown (see Diamond [9]) that this proposition is erroneous. Fortunately, it does not affect the crux of the Leland and Pyle analysis.

ours, depends critically on the assumption that project sponsors are risk averse and thus requires knowledge of sponsors' preferences in designing an informationally consistent equilibrium valuation schedule.<sup>13</sup> Further, the debt in that model is *riskless* and the parameter that is asymmetrically known is the mean project return. In our model, debt is risky and the mean return is common knowledge. Because of its assumptions, the Leland and Pyle model seems suited for relatively small, closely held (manager controlled) firms with low leverage ratios. By contrast, our model is particularly applicable to large firms in which entrepreneurial risk preferences are relatively unimportant. Under the maintained assumption of universal risk neutrality, our findings apply to firms of any size.

The Leland and Pyle conclusion that optimal leverage should be a decreasing function of risk can be found elsewhere too, particularly in the literature that assigns a central role to bankruptcy costs to explain capital structure (see Scott [33] and Kim [15] for the effects of bankruptcy costs on the capital structure decision). The essence of the argument is that, since *expected* bankruptcy costs increase with the firm's riskiness and its marginal tax rate is invariant to risk, an optimal tradeoff between the bankruptcy costs and tax advantages of debt would imply an inverse association between leverage and risk. For example, Lintner [19] writes

For all such companies, the optimal debt level will vary strongly and inversely with assessments of the company's unlevered bankruptcy risk—one measure of "business" as distinct from "financial" risk assessed at *any* amount of debt commitment).

The inverse relationship between leverage and risk is also implicit in Stiglitz's [34] analysis. In Stiglitz's model, the optimal amount of debt to issue is a level such that any further increment would make the debt risky, i.e., every firm issues *only* riskless debt. Since a higher level of business risk in the firm's assets implies a lower maximum level of riskless debt, Stiglitz's prediction is similar to Lintner's [19]. In view of this, Theorem 3 seems surprising. Note also that our finding does not depend on arbitrage-impeding transactions costs as, for example, in Baumol and Malkiel [2] or moral hazard as in Grossman and Hart [10] and Hellwig [13]. It does, of course, depend on the absence of bankruptcy costs, but then their importance in capital structure theory has been challenged on both theoretical and empirical grounds.<sup>14</sup>

<sup>13</sup> One may argue that it is more likely that there will be an informational asymmetry about project sponsors' preferences—if sponsors are indeed risk averse—than about the attributes of the projects they seek funding for.

<sup>14</sup> See Haugen and Senbet [11] and Warner [37]. The only other paper we know of that also finds that riskier firms use more debt and have higher values is Heinkel [12]. Unlike the endogeneity of our result, however, Heinkel's result stems from an exogenous restriction on the cross-sectional correlation between risk and value. Also related to our work is the paper

A paper related to ours, although it assumes symmetric information, is DeAngelo and Masulis [8]. It also relies only on the tax code—without appealing to exogenous debt-related costs—to show an optimal capital structure.<sup>15</sup> Unlike our work, however, that paper does not predict any direct relationship between leverage and risk. Rather, it predicts that firms with larger nondebt related tax shields will employ lesser leverage.

The intuition behind our main result is as follows. High risk firms have an incentive to understate their  $\theta$ 's because doing so results in lower interest rates. To offset this misrepresentation proclivity, they are offered higher levels of debt financing as a “reward” for reporting higher  $\theta$ 's. Low risk firms, on the other hand, are induced not to perversely mimic the high risk firms by being offered low interest rates as a “reward” for forgoing high leverage levels. The riskier firm, with a greater likelihood of extreme realizations, has a larger set of states in which its income is very high and thus a relatively high tax liability *ceteris paribus*. Consequently, the high leverage-high interest rate combination is appealing for such a firm because it permits greater sheltering of its relatively more likely large income. On the other hand, the less risky firm attaches a smaller probability to the occurrence of very high income states and thus prefers a lower interest rate to a larger tax shelter. These disparate preferences sustain a perfectly separating equilibrium in which a distinct capital structure contract is optimally chosen for each  $\theta$ .

We have thus far discussed feasible capital structure allocations without having proved the existence of equilibrium. Doing so is the next main task. En route, however, we shall establish a number of intermediate results for which additional notation is necessary.

Define  $\bar{L} \in \mathbb{R}_+$  as the first crossover point of the cumulative distribution functions associated with  $f^0(R)$  and  $f^1(R)$ . That is,  $\bar{L}$  satisfies

$$\int_0^{\bar{L}} f^1(R) dR = \int_0^{\bar{L}} f^0(R) dR$$

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by Myers and Majluf [25] which touches briefly upon how a firm may wish to issue debt and equity simultaneously when there is asymmetric information. However, their analysis is aimed at establishing a hierarchial ordering for financing methods and most of it seems to indicate a strict dominance for debt. Recently, Ross [31] has developed a symmetric information model with taxes and uncertainty to explore the relationship between capital structure and risk. He finds that, in a cross section of firms with *identical* total variance of earnings, those with higher (positive) cash flow betas will have lower debt levels.

<sup>15</sup> In a recent paper, Talmor, Haugen, and Barnea [35] show that the DeAngelo and Masulis result is an outcome of their assumption that debt principal is tax deductible. Assuming only interest is tax deductible, they show that the firm's after-tax profit function is usually *convex* with respect to leverage, ruling out interior optima.

and

$$\int_0^y f^1(R) dR > \int_0^y f^0(R) dR, \quad \forall y \in (0, \bar{L}).$$

Clearly,  $(0, \bar{L})$  has strictly positive Lebesgue measure on  $\mathbb{R}_+$ . If this were not true, there would exist an  $\varepsilon > 0$  small enough such that (2) would be violated with  $\varepsilon = \chi$ .

By Theorem 3 we know that the riskiest project gets the maximum leverage. Suppose  $\alpha(\theta_2) = 1$ . Then define  $L_m$  as the value of  $L(\theta_2)$  that satisfies (20) with  $\alpha(\theta_2) = 1$ . That is,

$$\int_0^{L_m} I[R - L_m] f(R, \theta_2) dR + IL_m = I\rho. \quad (25)$$

Let  $K_m(\theta_2)$  satisfy (25), so that  $L_m = 1 \cdot K_m(\theta_2)$ . Next define  $\bar{\alpha}$  as the value of  $\alpha$  satisfying

$$\int_0^{\bar{L}} I[R - \bar{L}] f(R, \theta_2) dR + I\bar{L} = I\bar{\alpha}\rho. \quad (26)$$

It is clear that  $\bar{\alpha} > 0$ .

We can now state a simple but useful fact.

LEMMA 6. *For any feasible capital structure schedule,*

$$L(\theta) \leq \bar{L}, \quad \forall \theta \in [\theta_1, \theta_2].$$

Our next result is concerned with the maximum admissible debt face value and an identification of the condition under which the riskiest project is granted its first best leverage. Notationally,  $\wedge$  designates the min operator.

THEOREM 4. *If the set of informationally consistent schedules is non-empty, then the Pareto dominant member of that set satisfies (stars denote Pareto dominance)  $L^*(\theta_2) = L_m \wedge \bar{L}$ . If  $L_m \leq \bar{L}$ , then  $\alpha^*(\theta_2) = 1$ .*

We now establish the main existence result. The basic idea in the proof is to show that the set of informationally consistent schedules is nonempty.

THEOREM 5. *The RCSE exists and is unique.*

In the context of the Spencian signaling model, a necessary requirement for informational consistency—assuming smoothness—is that  $\partial E(\hat{\theta}|\theta)/\partial \hat{\theta}|_{\hat{\theta}=\theta} = 0$ , and the sufficiency condition is that  $\partial^2 E(\hat{\theta}|\theta)/\partial \hat{\theta}^2|_{\hat{\theta}=\theta} \leq 0$ . If

these two conditions are met, then a firm's equity value is maximized when it signals truthfully. It is easy to verify that analogous conditions hold in our model. We now summarize the properties of the RCSE. Stars are used to designate RCSE values, and  $\mu(\cdot)$  denotes the Lebesgue measure on  $\mathbb{R}_+$ . For some  $\theta^e < \theta_2$ , define  $\Theta^* \equiv [\theta^e, \theta_2] \cap [\theta_1, \theta_2]$ .

THEOREM 6. *In the RCSE,*

(i)  $\alpha^{*\prime}(\theta) \geq 0$ ,  $\alpha^{*\prime\prime}(\theta) \geq 0$ ,  $\forall \theta \in [\theta_1, \theta_2]$ , and  $\exists \Theta^* \subseteq [\theta_1, \theta_2]$  such that

$$\alpha^*(\theta) \begin{cases} = 0, & \forall \theta \notin \Theta^* \\ \in (0, \alpha^*(\theta_2)), & \forall \theta \in \Theta^* \cap [\theta_1, \theta_2), \end{cases}$$

$$\alpha^*(\theta_2) = [1 \wedge \bar{\alpha}] \in (0, 1],$$

$$\mu(\Theta^*) > 0;$$

(ii)  $L^{*\prime}(\theta) \geq 0$ ,  $\forall \theta \in [\theta_1, \theta_2]$ ;

(iii)  $K^{*\prime}(\theta) \geq 0$ ,  $\forall \theta \in [\theta_1, \theta_2]$ .

Thus, whenever incentive compatibility permits it and the exclusionary risk parameter,  $\theta^e$ , equals  $\theta_1$ , firms of all types are granted positive leverage. It is possible for some firms with  $\theta \in [\theta_1, \theta_2)$  to be denied leverage, but the theorem asserts that not *all* firm types are precluded from borrowing. The theorem also says that the riskiest firm obtains *strictly* higher leverage than any other firm a lower risk parameter. This is a stronger statement than Theorem 3 which only claims that  $\alpha(\theta)$  is nondecreasing.<sup>16</sup>

We conclude this section with comments on four points. First, we have defined a capital structure contract as a combination of interest rate and leverage. We could have introduced a third instrument, namely the probability that debt would be made available to the firm. However, we have verified that the equilibrium value of this probability is always either zero or one. Thus, there is little to gain by formally modeling this probability, since a zero probability is equivalent to  $\alpha(\theta) = 0$ .

Second, we have modeled a firm as a collection of shareholders. It is important to note, therefore, that shareholder unanimity obtains. The reason is that in equilibrium each firm's net equity value is maximized subject to informational constraints and this is synonymous with the maximization of each shareholder's personal welfare. Intuitively, unanimity is the consequence of each firm's initial shareholders being homogeneous with respect to their preferences and information structures.

<sup>16</sup> Although we show in the proof of Theorem 2 that  $\alpha^*(\theta) < \alpha^*(\theta_2)$ ,  $\forall \theta < \theta_2$ , that finding is based on the *assumption* that at least some firm types are granted positive leverage.

Third, although we have restricted the firm to raising no more external capital than the required investment outlay, lifting that restriction would be easy. As long as the sponsors retain some—no matter how small—residual interest,<sup>17</sup> the capital structure equilibrium will be qualitatively unaltered even if external capital in excess of the required outlay can be raised.

Finally, we have not formally dealt with personal taxes. The implicit assumption has been that personal income from stocks is taxed at the same rate as personal income from bonds, so that personal taxes are allocationally neutral. Thus, we differ from Miller [22] who assumes income from stocks is tax-exempt whereas income from corporate bonds is taxed.<sup>18</sup>

### IV. A RATIONALE FOR PROJECT FINANCING

In the previous section we assumed that creditors could not profitably produce costly information about the return distributions of firms, or equivalently that designing a revelation game was the Pareto dominant alternative. Suppose now that creditors can feasibly produce information about a project at a cost  $Q^0(\beta)$ , which is a positive, real valued function of a vector,  $\beta$ , of various publicly observable factors related to the project, such as its size, its technical complexity, etc. We assume that information

<sup>17</sup> This condition rules out an (absurd) equilibrium in which the debt face value is set at infinity. That is, the entire project is sold to the bondholders and there is *no* residual equity claim. Because there is now only one class of claims, the risk parameter  $\theta$  is irrelevant given risk neutrality, and all projects will be priced contingent on the commonly known mean return. However, this equilibrium is practically absurd because, if the IRS were to allow such things, every firm would be all-equity financed and would simply relabel its equity as “debt.” No taxes would ever be paid and there would be no bankruptcies. From a theoretical standpoint too, this is unacceptable since it makes little sense to refer to a class of claims as debt unless there is at least a small residual equity claim.

<sup>18</sup> Another important difference is that the debt in Miller's [22] model is riskless whereas it is risky in ours. Risky debt affects Miller's results because the bonds of companies in default will not, in general, yield the issuing stockholders their full tax shield. But even if one ignores this effect and assumes that personal income from stocks is taxed at a lower rate than personal income from bonds, our capital structure equilibrium under asymmetric information will not generally be the same as Miller's. This is because the extreme leverage propensity on the part of firms in Miller's model makes it plausible to argue that eventually a sufficient number of low tax bracket investors will be exhausted by an ever increasing corporate debt supply to move  $\rho[1 - \tau] < \rho_e$  to an equality, where  $\rho_e$  is the riskless tax-exempt yield and  $\rho$  the riskless taxable yield. In our imperfect information model, however, every firm (except those with  $\theta = \theta_2$ ) has an *interior* optimal capital structure. This implies an aggregate leverage constraint in our model that is absent in Miller's. Consequently, the aggregate supply of taxable corporate bonds may be small enough to ensure  $\rho[1 - \tau] < \rho_e$ , and as long as that is true, our findings in this section will be qualitatively unaffected.

production permits perfect ex ante identification of the unknown risk parameter. Consequently, first-best leverage follows information production. Although it is creditors who produce information, in equilibrium the cost of producing information will be borne by the firm itself because the price of credit will impound the information production cost. Thus, the firm now has a choice of playing a revelation game or having creditors produce information at a cost to itself. So far we have also assumed that the firm starts out as an empty shell in the absence of the new project. We now amend that assumption by assuming that the firm currently has  $J$  distinct divisions, i.e., it is a portfolio of  $J$  different projects. It has available a new project calling for an investment of  $I$ . If this project is conventionally financed and incorporated as an additional  $(J + 1)$ th division within the firm, creditors who provide part of the financing must produce information about *all* the divisions in the firm. The reason is that the firm can comingle the cash flows of its various divisions, thereby rendering dubious the usefulness of credit terms based solely on the cash flow distribution of the new project.

From the lenders' perspective, another "problem" with multidivisional firms is that even significant alterations in the composition of the firm's assets may be difficult to detect and prevent. Jensen and Meckling [14] argued that, despite bond covenants, firms could engage in asset substitution and change the riskiness of their underlying assets to the bondholders' detriment *subsequent* to the issuance of the bonds. To capture this notion in a simple fashion, we assume that the firm has private information about a change in its return distribution that will occur after borrowing has taken place. This change will require no additional financing.) That is, the firm knows that some signal,  $\delta \in \{\delta_1, \delta_2, \dots, \delta_T\} \equiv \mathcal{A}$ , will arrive after it has financed the new project, and this signal will affect the *firm's* return distribution. This signal is irrelevant to the project itself only if it is separately incorporated, because we assume it does not affect the project's  $\theta$  per se. The best way to think about  $\mathcal{A}$  is to view it as the firm's feasible investment opportunity set in addition to the project in question. The firm knows precisely which element from this set will become available to it to possibly substitute for some of its existing assets, whereas creditors can only assign a subjective prior density function over the set  $\mathcal{A}$ . Alternatively, creditors can find out  $\delta$  by producing information about every  $\delta_i \in \mathcal{A}$ , at a total cost of  $\sum_{\delta_i \in \mathcal{A}} Q^1(\delta_i)$ .<sup>19</sup>

In a perfect information world these problems would clearly be absent. Subsequent discussion shows that our theory predicts no advantage for project financing in this case. Project financing is advantageous whenever it

<sup>19</sup> It is not essential to our theory that  $Q^1(\cdot)$  be additive. Subadditivity would suffice as long as  $Q^1(\cdot)$  is increasing in the counting measure of  $\mathcal{A}$ .

increases the project's net equity value; under perfect information, every project receives complete debt financing—and thus enjoys its maximum potential value—even when conventionally incorporated because perfect ex post monitoring by creditors can be used to ensure compliance with bond covenants that specify precisely how project cash flows should be utilized. The ensuing analysis is aimed at establishing conditions under which project financing is beneficial when there is imperfect information. The intuition behind the formal analysis is as follows. Whenever the net gains from information production are sufficiently high, project financing is beneficial because creditors incur lower screening costs in evaluating a separately incorporated project. On the other hand, when a revelation game is preferred to information production, project financing is value enhancing if it can attract higher leverage. From the previous section we know that this occurs when the post-project-adoption risk parameter of the firm—considered relevant by the creditors with conventional financing—is lower than the project's own risk parameter (the relevant parameter for project financing).

Suppose the firm's total investment (in the  $J$  divisions) currently is  $I_J$  and its total end-of-period payoff is  $I_J R_J$ , a random variable. The firm's total existing debt obligation (principal plus interest) is for an amount  $D_J$ , payable at the end of the period. Define  $P_J \equiv I_J R_J - D_J$  and let  $h_J(P_J, \theta_J(\delta))$  be the density function of  $P_J$ .  $\theta_J$  is expressed as a function of  $\delta$  to indicate that the firm's risk parameter depends also on the signal it knows for sure it is going to receive. Also, the density function  $h_J(\cdot, \cdot)$  is a mixture of a low risk density function and a high risk density function as in (11) with  $\theta_J$  as the risk parameter. Without the new project, the end-of-period expected cash flow accruing to the firm's shareholders is

$$E_J(\theta_J) = \int_{A_J^c} P_J [1 - \tau] h_J(P_J, \theta_J(\delta)) dP_J, \quad (27)$$

where

$$A_J^c \equiv \{P_J | P_J \geq 0, I_J R_J \in \mathbb{R}_+\}.$$

Now suppose the new project is conventionally financed and incorporated within the firm. If the firm avoids information production and decides to engage in a revelation game as in Section III, it will be asked to report  $\theta_{J+1}(\delta)$ , the (post-signal) risk parameter for the density function of the random variable  $P_{J+1} \equiv I_J R_J - D_J + IR$ , where  $I$  is still the investment required in the new project and  $R$  is the return on the new project. In this case, the incremental expected net cash flow from the new project accruing

## OPTIMAL CAPITAL STRUCTURE AND PROJECT FINANCING

to the shareholders of a firm which has an overall (post-project-adoption) risk parameter  $\theta_{J+1}(\delta)$  and a project risk parameter  $\theta$  is

$$\begin{aligned} \tilde{E}(\theta_{J+1}(\delta), \theta) = & \int_{A_{J+1}^c} [P_{J+1} - L(\theta_{J+1})] h_{J+1}(P_{J+1}, \theta_{J+1}(\delta)) dP_{J+1} \\ & - E_J(\theta_J) - I[1 - \alpha^*(\theta_{J+1})] \rho, \end{aligned} \quad (28)$$

where

$$A_{J+1}^c \equiv \{P_{J+1} \mid P_{J+1} \geq L^*(\theta_{J+1}), I_J R_J + IR \in \mathbb{R}_+\}.$$

Note that  $(\alpha^*(\theta_{J+1}(\delta)), K^*(\theta_{J+1}(\delta)))$  is the capital structure contract awarded the firm in the RCSE when it reports  $\theta_{J+1}(\delta)$ . We assume, for simplicity, that the creditors who lend funds for the project have the lowest seniority among all of the firm's creditors. We also assume that the density functions  $h_{J+1}(\cdot, \cdot)$  can be cross-sectionally ordered by the  $\otimes$  relationship stipulated in (11).

Now suppose the firm opts for conventional financing but decides to have costly information produced. Creditors will produce information about the new project as well as about all the  $J$  divisions and the  $\delta_i$ 's at a total cost of  $Q^0(\beta_{J+1}) + \sum_{\delta_i \in \mathcal{A}} Q^1(\delta_i)$ . Since information production reveals the necessary information to creditors *ex ante*, the firm will be given its first best leverage subsequent to information production. Now using the fact that, in a competitive equilibrium, information production costs must be borne by the firm itself, the incremental net expected end-of-period cash flow accruing to the shareholders from adopting the new project *and* having information produced is

$$\hat{E}(\theta_{J+1}(\delta), \theta) = I\rho\tau + \psi(\theta_{J+1}(\delta)) - Q^0(\beta_{J+1}) - \sum_{\delta_i \in \mathcal{A}} Q^1(\delta_i), \quad (29)$$

where  $\psi(\cdot)$  is the *incremental* benefit to the firm as a whole from having information produced. Because information is produced about all the assets of the firm, its capital structure can now be rearranged to attain the first best leverage for the *whole* firm.  $\psi(\cdot)$  then measures the cash flow increment resulting from an increase in the firm's leverage—for *existing* assets—from its previous level to the new first best level. We shall assume that  $J$  and the counting measure of  $\mathcal{A}$  are sufficiently large so that

$$\tilde{E}(\theta_{J+1}(\delta), \theta) > \hat{E}(\theta_{J+1}(\delta), \theta), \quad \forall \theta, \delta. \quad (30)$$

To us this seems a sensible assumption for large firms that have a rich variety of investment options available. For such firms we should expect that inducing revelation of a single relevant parameter through a self-selec-

## OPTIMAL CAPITAL STRUCTURE AND PROJECT FINANCING

tion mechanism would be less costly than having creditors produce information not only about the firm's existing assets but also about possible post-contract events that could affect the return distribution. Of course, this also presumes that the gain to the firm from an increase in leverage for its other assets is not large relative to information production costs, and implies that information production is never optimal with conventional financing.

Now suppose the firm utilizes project financing and sets up the new project as a legally segregated entity. Creditors are no longer concerned about the attributes of the firm's other assets. Only the project's risk parameter matters. If the firm participates in a revelation game in this case, the net end-of-period expected cash flow accruing to the shareholders from the new project will be

$$\tilde{E}_S(\theta) = I\alpha^*(\theta) \rho\tau. \quad (31)$$

If the firm decides in favor of information production, creditors need produce information *only* about the new project. The cost of doing so is  $Q^0(\beta)$ . Because  $Q^0(\cdot)$  is assumed to be an increasing function of size as well as the number of divisions information is being produced about,  $Q^0(\beta) < Q^0(\beta_{j+1})$ . The net end-of-period expected project cash flow accruing to the shareholders in this case is

$$\hat{E}_S(\theta) = I\rho\tau - Q^0(\beta). \quad (32)$$

Figure 1 is a graph of  $\tilde{E}_S(\theta)$  and  $\hat{E}_S(\theta)$  as functions of  $\theta$ . Both functions are sketched as broken lines. The full curve is the upper envelope of the two functions,  $E_S^p(\theta) = \tilde{E}_S(\theta) \vee \hat{E}_S(\theta)$ , where  $\vee$  is the max operator. *Conditional* on project financing being the chosen mode of incorporation,  $E_S^p(\theta)$  is the value of the net end-of-period expected cash flow accruing to the

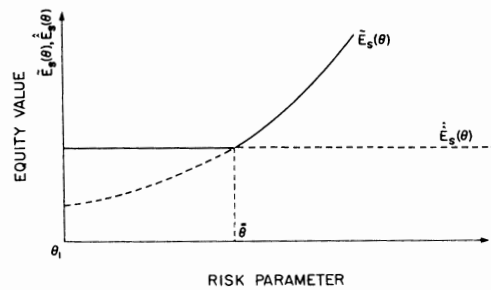


FIG. 1. Equity values as functions of the risk parameter for all possible risk parameter values.

# OPTIMAL CAPITAL STRUCTURE AND PROJECT FINANCING

shareholders, and this value is a function of  $\theta$ . To exclude the uninteresting case in which  $E_S^p(\theta)$  is constant for all  $\theta$ , we assume

$$\tilde{E}_S(\theta_2) = I\alpha^*(\theta_2)\rho\tau > I\rho\tau - Q^0(\beta).$$

This assumption is obviously dispensible when  $\alpha^*(\theta_2) = 1$ . Now, a necessary and sufficient condition for project financing to be preferred to conventional financing by a given firm is that

$$E_S^p(\theta) > \tilde{E}(\theta_{J+1}(\delta), \theta) \tag{33}$$

for that firm. Define

$$\Theta \equiv \{\theta \in [\theta_1, \theta_2] \mid E_S^p(\theta) > \tilde{E}(\theta_{J+1}(\delta), \theta), \theta_{J+1}(\delta) \in \Theta(\Delta)\},$$

where  $\Theta(\Delta)$  is the set of all possible values of  $\theta_{J+1}(\delta)$ . Figure 2 is a graph of  $\tilde{E}_S(\theta)$  and  $\hat{E}_S(\theta)$  with  $\theta$  restricted to take values in  $\Theta$ . Let

$$\Theta^- \equiv \{\theta \in \Theta \mid E_S^p(\theta) = \hat{E}_S(\theta)\}$$

and

$$\Theta^+ \equiv \{\theta \in \Theta \mid E_S^p(\theta) = \tilde{E}_S(\theta)\}.$$

It is apparent that  $\Theta^-$  is a connected set and so is  $\Theta^+$ , and that  $\tilde{E}_S(\theta)$  is convex and nondecreasing over  $\Theta$ .

For a project with a risk parameter  $\theta$ , let  $\theta^0$  be its *pseudo risk*, defined through the relationship  $\tilde{E}(\theta_{J+1}(\delta), \theta) = \tilde{E}_S(\theta^0)$ . We can now state the following result.

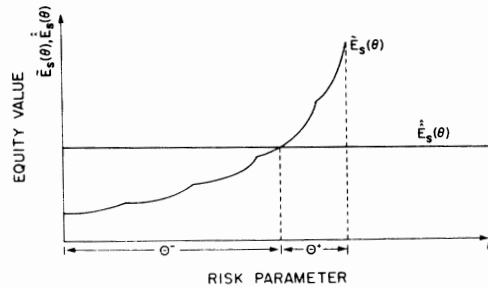


FIG. 2. Equity values as functions of the risk parameter for those risk parameter values for which project financing is preferred.

THEOREM 7. *Whenever project financing is adopted for a project, the following observations hold.*

- (i) *The project has a higher value than it would with conventional financing.*
- (ii) *The project has a higher leverage than it would with conventional financing.*
- (iii) *A project with a relatively low  $\theta \in \Theta$  is more likely to involve information production, whereas a product with a relatively high  $\theta \in \Theta$  is more likely to involve a revelation game.*
- (iv) *The project gets higher leverage if information production is optimal than it would if information production is not optimal.*

*Moreover, as long as  $\mu(\{\theta \in \Theta^* \mid \tilde{E}_S(\theta) < \hat{E}_S(\theta)\}) > 0$ ,  $\exists$  a unique  $\bar{\theta} \in \Theta^*$  such that for all projects with pseudo risks  $\theta^0 \in [\bar{\theta}, \theta_2]$ , conventional financing is Pareto dominant if  $\theta < \theta^0$  and project financing is Pareto dominant if  $\theta \geq \theta^0$ . Regardless of their pseudo risks, however, the riskiest projects (those with  $\theta = \theta_2$ ) always utilize project financing. Finally,  $\exists$  a number  $r \in [0, 1]$  such that, if  $\mu(\Theta^+) \leq r$ , a cross-sectional test will find that the mean leverage in investments utilizing project financing is higher than the mean leverage in conventionally financed projects.*

This theorem has the following empirical connotations. In project financing ventures that employ very high leverage, one should find creditors quite involved in numerous phases of the project since our theory predicts information production by creditors will accompany very high leverage.<sup>20</sup> Moreover, such projects should also be less risky than those project-financed ventures that use somewhat lower leverage and appear to have lesser creditor involvement. The two strongest implications of the theorem are that the riskiest investments will always use project financing and that, at least for projects that have relatively high pseudo risks, a project whose risk exceeds its pseudo risk will be project financed.<sup>21</sup> In combination these two implications suggest the optimality of project financing for highly risky investments and provide a prediction that seems consistent with observed firm behavior.

<sup>20</sup> The contemporary theory of financial intermediation holds the view that costly information production is an essential feature of financial intermediaries; see, for example, Ramakrishnan and Thakor [28].

<sup>21</sup> The theorem says nothing about projects with pseudo risks  $\theta^0 \in [\theta_1, \bar{\theta}]$ . For such projects, project financing is always optimal, sometimes with information production and sometimes with a revelation game. The important point, however, is that for at least a subset of projects one can assert that project financing is Pareto dominant for a project that is riskier—in a sense—when separately incorporated than when it is incorporated as a part of the firm.

If information production is disallowed, matters become much simpler. The following result is immediate from the preceding analysis and is thus presented without proof.

*COROLLARY. Suppose information production is prohibitively costly. Then, any project that is riskier if separately incorporated than it is if adopted within the firm will be organized as a project financing venture.*

This corollary is a more direct prediction. It suggests that if one makes an ex post comparison of project financed ventures with conventionally financed ventures, one should expect the former to be riskier and more highly levered on average.

In closing then, our theory suggests that the economic motivation for project financing stems from a rational attempt to minimize the allocational distortions of asymmetric information rather than the popularly assumed inability on the part of the creditors to detect the off-balance sheet debt associated with project financing.<sup>22</sup>

V. PROOFS

*Proof of Theorem 1.* We know that

$$\begin{aligned} \int_{\mathbb{R}_+} Rf(R, \theta) dR &= \int_{\mathbb{R}_+} Rf^0(R) dR + \theta \int_{\mathbb{R}_+} Rf^1(R) dR - \theta \int_{\mathbb{R}_+} Rf^0(R) dR \\ &= \int_{\mathbb{R}_+} Rf^0(R) dR \quad \text{using } f^1(R) \text{ } \textcircled{R} \text{ } f^0(R) \text{ and (1)} \\ &= \bar{R}. \end{aligned} \tag{34}$$

Similarly, replacing  $\theta$  by  $\hat{\theta}$  gives

$$\int_{\mathbb{R}_+} Rf(R, \hat{\theta}) dR = \bar{R}. \tag{35}$$

Combining (34) and (35) we have, for  $\hat{\theta} > \theta$ ,

$$\bar{R} = \int_{\mathbb{R}_+} Rf^1(R) dR = \int_{\mathbb{R}_+} Rf(R, \hat{\theta}) dR = \int_{\mathbb{R}_+} Rf(R, \theta) dR = \int_{\mathbb{R}_+} Rf^0(R) dR. \tag{36}$$

<sup>22</sup> For an example of the popular viewpoint, see the Wall Street Journal of December 13, 1983.

Now integrating (11) for any  $\theta \in [\theta_1, \theta_2]$ ,

$$\int_0^y F(R, \theta) dR = \int_0^y F^0(R) dR + \theta \left\{ \int_0^y F^1(R) dR - \int_0^y F^0(R) dR \right\}. \quad (37)$$

Since  $\int_0^y F^1(R) dR \geq \int_0^y F^0(R) dR$ , (37) implies that

$$\int_0^y F(R, \theta) dR \geq \int_0^y F^0(R) dR. \quad (38)$$

Thus, (36) and (38) imply that, for  $\theta \in [\theta_1, \theta_2]$ ,

$$f(R, \theta) \otimes f^0(R). \quad (39)$$

Next, add and subtract  $\int_0^y F^1(R) dR$  from the right-hand side (r.h.s.) of (37) to obtain

$$\int_0^y F(R, \theta) dR = \int_0^y F^1(R) dR - [1 - \theta] \left\{ \int_0^y F^1(R) dR - \int_0^y F^0(R) dR \right\}. \quad (40)$$

Since  $\theta \leq 1$  and  $\int_0^y F^1(R) dR \geq \int_0^y F^0(R) dR$ , (36) and (40) jointly imply that

$$f^1(R) \otimes f(R, \theta). \quad (41)$$

Since (41) holds for any  $\theta \in [\theta_1, \theta_2]$ , it must be true that

$$f^1(R) \otimes f(R, \hat{\theta}). \quad (42)$$

Further, from (40) we see that for  $\theta < \hat{\theta}$ ,

$$\int_0^y F(R, \theta) dR \leq \int_0^y F(R, \hat{\theta}) dR. \quad (43)$$

Combining (36) and (43) means that for  $\theta < \hat{\theta}$ ,

$$f(R, \hat{\theta}) \otimes f(R, \theta). \quad (44)$$

Joining (39), (41), (42), and (44) yields the desired result. Q.E.D.

*Proof of Lemma 1.* The expected end-of-period cash flow to the creditors is

$$B(\alpha K, \theta) = \int_r I[R - \alpha K] f(R, \theta) dR + I\alpha K. \quad (45)$$

And using (11) we can write

$$B(\alpha K, \theta) = \int_r I[R - \alpha K][f^0(R) + \theta\{f^1(R) - f^0(R)\}] dR + \alpha KI. \quad (46)$$

Differentiating (46) with respect to  $\theta$ , holding  $\alpha$  and  $K$  fixed, produces

$$\partial B(\alpha K, \theta)/\partial \theta = \int_r I[R - \alpha K][f^1(R) - f^0(R)] dR. \quad (47)$$

Rearranging (47) we obtain,

$$\begin{aligned} & \int_r I[R - \alpha K][f^1(R) - f^0(R)] dR \\ &= \int_r IRf^1(R) dR - \int_r IRf^0(R) dR - \alpha K \int_r f^1(R) dR + \alpha K \int_r f^0(R) dR. \end{aligned}$$

Integrating the above equation by parts, we get

$$\int_r I[R - \alpha K][f^1(R) - f^0(R)] dR = \int_r F^0(R) dR - \int_r F^1(R) dR < 0. \quad \text{Q.E.D.}$$

*Proof of Lemma 2.* From Riley [29, Theorem 7], it follows immediately that the reactive equilibrium is the Pareto dominant member of the set of informationally consistent contracts. Riley has also proved that the reactive equilibrium is always fully separating, which justifies constraint (15). Note that (18) ensures informational consistency. Q.E.D.

*Proof of Lemma 3.* We need to show that (19) is equivalent to (13) with  $\theta = \hat{\theta}$ , (20) and (21) are equivalent to (14) and (15), and (24) and (24') are jointly equivalent to (18).

To see that (19) is equivalent to (13) with  $\hat{\theta} = \theta$ , simplify (13) to get

$$\begin{aligned} E(\theta) &= I\bar{R}[1 - \tau] - \int_{r(\theta)} IR[1 - \tau]f(R, \theta) dR \\ &\quad - \int_{r(\theta)} IL(\theta)[1 - \tau]f(R, \theta) dR \\ &\quad - [1 - \alpha(\theta)]I\rho - I\bar{R}[1 - \tau] + I\rho. \end{aligned} \quad (48)$$

Combining (14) and (15) we see that

$$\alpha(\theta)I\rho = \int_{r(\theta)} IRf(R, \theta) dR + \int_{r(\theta)} IL(\theta)f(R, \theta) dR. \quad (49)$$

Thus,

$$\begin{aligned} & - \int_{r(\theta)} [1 - \tau] IL(\theta) f(R, \theta) dR \\ & = \int_{r(\theta)} [1 - \tau] IRf(R, \theta) dR - [1 - \tau] \alpha(\theta) I\rho. \end{aligned} \quad (50)$$

Substituting (50) into (48) shows the desired equivalence. To see that (14) and (15) are equivalent to (20) and (21), note that (49) can be written as

$$\alpha(\theta) I\rho = \int_{r(\theta)} IRf(R, \theta) dR + IL(\theta) - \int_{r(\theta)} IL(\theta) f(R, \theta) dR.$$

Finally, we will prove that (18) is equivalent to (24) and (24'). First, we show that (18) implies (24) and (24'). Using (13) and (18) we obtain

$$\begin{aligned} E(\theta) & \geq E(\hat{\theta} | \theta) \\ & = E(\hat{\theta}) + [1 - \tau] \int_{r(\hat{\theta})} I[R - L(\hat{\theta})] f(R, \hat{\theta}) dR \\ & \quad - [1 - \tau] \int_{r(\hat{\theta})} I[R - L(\hat{\theta})] f(R, \theta) dR. \end{aligned}$$

Thus, using (11) and rearranging we get

$$E(\theta) - E(\hat{\theta}) \geq I[1 - \tau] \int_{r(\hat{\theta})} [R - L(\hat{\theta})][\theta - \hat{\theta}][f^0(R) - f^1(R)] dR. \quad (51)$$

Reversing the roles of  $\theta$  and  $\hat{\theta}$  produces

$$E(\theta) - E(\hat{\theta}) \leq I[1 - \tau] \int_{r(\theta)} [R - L(\theta)][\theta - \hat{\theta}][f^0(R) - f^1(R)] dR. \quad (52)$$

Taking  $\hat{\theta} < \theta$  and combining (51) and (52) gives us

$$\eta(\hat{\theta}) \leq [E(\theta) - E(\hat{\theta})][\theta - \hat{\theta}]^{-1} \leq \eta(\theta), \quad (53)$$

where

$$\eta(\theta) \equiv I[1 - \tau] \int_{r(\theta)} [R - \alpha(\theta)][f^0(R) - f^1(R)] dR > 0.$$

Since  $\hat{\theta} < \theta$ , (53) implies that the function  $\eta(\theta)$  is nondecreasing in  $\theta$ . Thus, it is Riemann integrable, continuous a.e., and differentiable a.e. in  $\theta$ . Taking limits as  $\hat{\theta} \rightarrow \theta$  we get  $E'(\theta) = \eta(\theta)$  for a.e.  $\theta \in [\theta_1, \theta_2]$ . Moreover, because  $\eta(\theta)$  is nondecreasing, we have  $E''(\theta) \geq 0$  wherever  $E'(\theta)$  exists.

We will now prove that (24) and (24') together imply (18). Note that

$$\begin{aligned}
 E(\theta) - E(\hat{\theta}|\theta) &= E(\theta) - E(\hat{\theta}) - [1 - \tau] \int_{r(\hat{\theta})} [R - L(\hat{\theta})] f(R, \hat{\theta}) dR \\
 &\quad + [1 - \tau] \int_{r(\hat{\theta})} [R - L(\hat{\theta})] f(R, \theta) dR \\
 &= E(\theta) - E(\hat{\theta}) - [1 - \tau] \int_{r(\hat{\theta})} I[R - L(\hat{\theta})][\theta - \hat{\theta}][f^0(R) - f^1(R)] dR \\
 &= E(\theta) - E(\hat{\theta}) - [\theta - \hat{\theta}] E'(\hat{\theta}) \quad \text{for a.e. } \theta \in [\theta_1, \theta_2] \quad (\text{using (24)}) \\
 &= \int_{\hat{\theta}}^{\theta} E'(z) dz - [\theta - \hat{\theta}] E'(\hat{\theta}) \quad \text{for a.e. } \theta \in [\theta_1, \theta_2]. \quad (54)
 \end{aligned}$$

By (24') we know that

$$\int_{\hat{\theta}}^{\theta} E'(z) dz \geq [\theta - \hat{\theta}] E'(\hat{\theta}) \quad \text{for } \theta > \hat{\theta} \quad \text{and a.e. } \theta, \hat{\theta} \in [\theta_1, \theta_2].$$

Thus, the r.h.s. of (54) is nonnegative, implying that (18) holds for a.e.  $\theta$ . Of course, over the set of Lebesgue measure zero that (24) does not hold, (18) must hold. Q.E.D.

*Proof of Lemma 4.* Differentiating (20) with respect to  $\theta$ , holding  $\alpha$  fixed, we obtain

$$\begin{aligned}
 -\alpha I[\partial K/\partial \theta] \int_{r(\theta)} [f^0(R) + \theta\{f^1(R) - f^0(R)\}] dR + I\alpha[\partial K/\partial \theta] \\
 + \int_{r(\theta)} I[R - \alpha K][f^1(R) - f^0(R)] dR = 0.
 \end{aligned}$$

Thus,

$$\begin{aligned}
 I\alpha[\partial K/\partial \theta] \left[ 1 - \int_{r(\theta)} f(R, \theta) dR \right] &= \int_{r(\theta)} I[R - \alpha K][f^0(R) - f^1(R)] dR \\
 &= \int_{r(\theta)} F^1(R) dR - \int_{r(\theta)} F^0(R) dR,
 \end{aligned}$$

which implies

$$\partial K/\partial \theta = \frac{\int_{r(\theta)} F^1(R) dR - \int_{r(\theta)} F^0(R) dR}{I\alpha[1 - \int_{r(\theta)} f(R, \theta) dR]} > 0. \quad \text{Q.E.D.}$$

*Proof of Lemma 5.* Differentiating (20) with respect to  $\alpha$ , holding  $\theta$  fixed, and rearranging gives

$$\partial K/\partial \alpha = \left[ \alpha \left\{ 1 - \int_{\Gamma(\theta)} f(R, \theta) dR \right\} \right]^{-1} \left[ \rho - K \left\{ 1 - \int_{\Gamma(\theta)} f(R, \theta) dR \right\} \right]. \quad (55)$$

From (20) we also obtain

$$\rho = K + \alpha^{-1} \int_{\Gamma(\theta)} Rf(R, \theta) dR - K \int_{\Gamma(\theta)} f(R, \theta) dR. \quad (56)$$

Substituting (56) in (55) yields

$$\partial K/\partial \alpha = \left[ \alpha^2 \left\{ 1 - \int_{\Gamma(\theta)} f(R, \theta) dR \right\} \right]^{-1} \left[ \int_{\Gamma(\theta)} Rf(R, \theta) dR \right] > 0. \quad \text{Q.E.D.}$$

*Proof of Theorem 2.* Any feasible capital structure schedule satisfies (19), (20), and (24) simultaneously. Combining (19) and (24) we obtain

$$I\alpha(\theta) \rho \tau = I\rho\alpha(\theta_2) \tau - [1 - \tau] \int_{\theta}^{\theta_2} \int_{\Gamma(z)} I[R - L(z)][f^0(R) - f^1(R)] dR dz. \quad (58)$$

Assuming that a solution to (57) and (20) exists, denote the solution by

$$L(\theta) = L^*(\theta), \quad (59)$$

$$\alpha(\theta) = \alpha^*(\theta), \quad (60)$$

$$K(\theta) = K^*(\theta) = L^*(\theta)/\alpha^*(\theta). \quad (61)$$

Substituting (59), (60), and (61) into (58) yields

$$\begin{aligned} I\alpha^*(\theta) \rho \tau &= I\rho\alpha^*(\theta_2) \tau - [1 - \tau] \int_{\theta}^{\theta_2} \int_{\Gamma^*(z)} I[R - L^*(z)] \\ &\quad \times [f^0(R) - f^1(R)] dR dz. \end{aligned} \quad (62)$$

Now for any  $\theta \in [\theta_1, \theta_2)$ , the second term on the r.h.s. of (62) is strictly positive since  $\Gamma^*(z)$  has positive measure because of our assumption that the firms under consideration have nonzero leverage. To see this, note that since  $\bar{L} > 0$  and  $L_m > 0$ , we know that  $L^*(\theta_2) > 0$ . Thus,  $\mu(\Gamma^*(\theta_2)) > 0$ . It is also obvious then that we can find a  $\theta'' = \theta_2 - \varepsilon$ , where  $\varepsilon > 0$  is arbitrarily small, such that  $\mu(\Gamma^*(\theta'')) > 0$ . Thus,

$$I\alpha^*(\theta) \rho \tau < I\rho\alpha^*(\theta_2) \tau,$$

$$\text{which means } \alpha^*(\theta) < \alpha^*(\theta_2) \leq 1 \forall \theta \in [\theta_1, \theta_2). \quad \text{Q.E.D.}$$

*Proof of Theorem 3.* Differentiating (19) with respect to  $\theta$  gives

$$E'(\theta) = I\alpha'(\theta) \rho\tau. \quad (63)$$

Joining (24) and (63) yields

$$I\alpha'(\theta) \rho\tau = [1 - \tau] \int_{r(\theta)} I[R - L(\theta)][f^0(R) - f^1(R)] dR \geq 0.$$

Thus,  $\alpha'(\theta) \geq 0$ . Now we have shown in Lemma 4 that for any  $\alpha$ , a higher  $\theta$  implies a higher  $K$ , and we have shown in Lemma 5 that for any  $\theta$ , a higher  $\alpha$  implies a higher  $K$ . Therefore, for  $\theta_i > \theta_j$ , we have

$$\alpha(\theta_i) \geq \alpha(\theta_j),$$

which means

$$K(\theta_i) \geq K(\theta_j).$$

Hence,  $L'(\theta) \geq 0, \forall \theta \in [\theta_1, \theta_2]$ .

To prove that the net equity value is nondecreasing in  $\theta$ , we partially integrate the r.h.s. of (24) to obtain

$$E'(\theta) = I[1 - \tau] \left[ \int_{r(\theta)} F^1(R) dR - \int_{r(\theta)} F^0(R) dR \right] \geq 0. \quad \text{Q.E.D.}$$

*Proof of Lemma 6.* Differentiating (24) with respect to  $\theta$  gives us

$$E''(\theta) = [1 - \tau] L'(\theta) I \int_{r(\theta)} \{f^1(R) - f^0(R)\} dR.$$

From (24') and Theorem 3, we know that

$E''(\theta) \geq 0$  and  $L'(\theta) \geq 0$ . Thus,

$$\int_{r(\theta)} \{f^1(R) - f^0(R)\} dR \geq 0, \quad (64)$$

which implies  $L(\theta) \leq \bar{L}$ .

Q.E.D.

*Proof of Theorem 4.* Integrating (24) produces

$$E(\theta) = E(\theta_2) - [1 - \tau] \int_{\theta}^{\theta_2} \int_{r(z)} I[R - L(z)][f^0(R) - f^1(R)] dR dz. \quad (65)$$

Substituting (65) into (12) yields the following modified objective function

$$\int_{\theta_1}^{\theta_2} E(\theta_2) g(\theta) d\theta - [1 - \tau] \int_{\theta}^{\theta_2} \int_{r(z)} I[R - L(z)][f^0(R) - f^1(R)] dR g(z) dz. \quad (66)$$

Clearly, (66) is maximized when  $E(\theta_2)$  is maximized. From (19),

$$E(\theta_2) = I\alpha(\theta_2) \rho\tau.$$

Therefore, in the Pareto dominant informationally consistent schedule,  $\alpha(\theta_2)$  takes on the maximum value that it can while still satisfying (20) and (64). Combining this with the assertion of Lemma 5 that a higher  $K$  accompanies a higher  $\alpha$ , we have

$$\alpha^*(\theta_2) = 1 \wedge \bar{\alpha} \quad \text{and} \quad L^*(\theta_2) = L_m \wedge \bar{L}.$$

It is transparent that if  $L_m \leq \bar{L}$ , then  $\alpha^*(\theta_2) = 1$ . Q.E.D.

*Proof of Theorem 5.* Riley [29] has shown in his Theorem 7 that, if the set of informationally consistent schedules is nonempty, then there exists a unique reactive equilibrium which is the Pareto dominant member of that set. Thus, we only need to prove that the set of informationally consistent capital structure schedules in our model is nonempty, and that the Pareto dominant member of that set is sustainable as an equilibrium in the sense that it cannot be upset by a pooling allocation.

We know from the proof of Theorem 2 that an informationally consistent schedule must satisfy (58). We also know from Lemma 6 that the Pareto dominant informationally consistent schedule must satisfy

$$\alpha^*(\theta_2) = 1 \wedge \bar{\alpha}.$$

Further, the global incentive compatibility requirement (18) calls for continuity of  $E(\theta)$  everywhere on its domain, and hence continuity of  $\alpha(\theta)$  through (19). It is also easy to verify that  $L(\theta)$  is continuous. The Pareto dominant informationally consistent schedule must, therefore, satisfy

$$\rho\alpha^*(\theta) I\tau + q(\theta) = I\alpha^*(\theta_2) \rho\tau, \quad (67)$$

where

$$q(\theta) \equiv [1 - \tau] \int_{\theta}^{\theta_2} \int_{r(z)} I[R - L(z)]\{f^0(R) - f^1(R)\} dR dz$$

is a nonincreasing function of  $\theta$  that is continuous because  $E(\theta)$  is continuous. Assuming for now that  $\mu(\Gamma(\theta_2)) > 0$  (we shall verify shortly that this is indeed true), we see that, because of the continuity of  $L(\theta)$ ,

$$q'(\theta) = -[1 - \tau] \int_{r(\theta)} I[R - L(\theta)]\{f^0(R) - f^1(R)\} dR$$

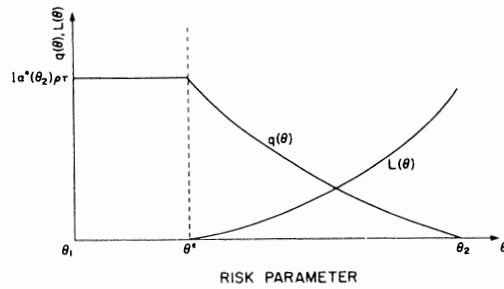


FIG. 3. Equilibrium when some firm types are not granted leverage.

is strictly negative for all  $\theta \leq \theta_2$  at least in a neighborhood of  $\theta_2$ . In fact,  $q'(\theta) < 0 \forall \theta$  such that  $L(\theta) > 0$  and  $q'(\theta) = 0, \forall \theta$  such that  $L(\theta) = 0$ . Thus,  $q(\theta_2) = 0$  and  $q(\theta)$  increases as  $\theta$  decreases. This implies  $\exists \theta^e$  satisfying

$$q(\theta^e) = I\alpha^*(\theta_2) \rho \tau.$$

From (67) then,  $\alpha^*(\theta^e) = 0$ . There are two possibilities. One is that  $\theta^e$  lies in the interior of  $[\theta_1, \theta_2]$ . In this case, (67) implies that  $\alpha(\theta) < 0, \forall \theta < \theta^e$ . However, since negative values of  $\alpha(\theta)$  are infeasible, we set  $\alpha^*(\theta) = 0 \forall \theta < \theta^e$ . This situation is depicted in Fig. 3. The second possibility, sketched in Fig. 4, is that  $\theta^e$  is strictly less than  $\theta_1$ , in which case  $\alpha^*(\theta) > 0, \forall \theta \in [\theta_1, \theta_2]$ . It is easy to see now that

$$q(\theta) < I\alpha^*(\theta_2) \rho \tau, \quad \forall \theta \in [\theta_1, \theta_2] \cap (\theta^e, \theta_2).$$

Hence,  $\exists \alpha^*(\theta) \in (0, \alpha^*(\theta_2))$  for a set of  $\theta$ 's with positive Lebesgue measure such that (67) holds for that set.

Substituting this value of  $\alpha^*(\theta)$  in (20) produces

$$\int_{\Gamma^*(\theta)} I[R - L^*(\theta)] f(R, \theta) dR + IL^*(\theta) = I\alpha^*(\theta) \rho, \quad (68)$$

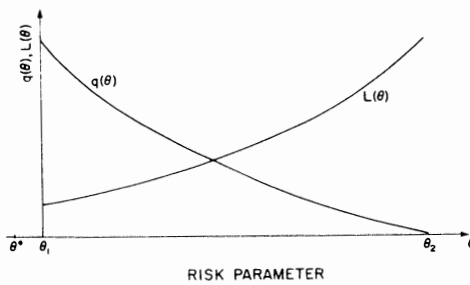


FIG. 4. Equilibrium when all firm types are granted positive leverage.

which yields the value of  $L^*(\theta)$ . Finally,

$$K^*(\theta) = L^*(\theta)/\alpha^*(\theta).$$

Denote  $\Theta^* \equiv [\theta_1, \theta_2] \cap (\theta^c, \theta_2]$  as the set of firm types granted positive leverage in equilibrium.

To complete the proof, we need to verify that  $\mu(\Theta^*)$ —and hence  $\mu(F(\theta_2))$ —is indeed positive and that the second order condition for global incentive compatibility,  $E''(\theta) \geq 0$ , is met. Now we know from the proof of Lemma 6 that this second order condition is met if

$$\int_{F^*(\theta)} \{f^1(R) - f^0(R)\} dR \geq 0, \quad \forall \theta \in \Theta^*. \quad (69)$$

We have proved that  $L'(\theta) \geq 0$  and  $L^*(\theta_2) = L_m \wedge \bar{L}$ . Thus,  $L^*(\theta) \leq L^*(\theta_2) \leq \bar{L}$ ,  $\forall \theta \in \Theta^*$ . Further,

$$\int_0^{\bar{L}} \{f^1(R) - f^0(R)\} dR = 0, \quad (70)$$

and

$$\int_0^y f^1(R) dR > \int_0^y f^0(R) dR, \quad \forall y \in (0, \bar{L}). \quad (71)$$

Combining (70) and (71) yields (69). Thus,  $E''(\theta) \geq 0, \forall \theta \in \Theta^*$ . Note also that, because  $(0, \bar{L})$  has positive Lebesgue measure on  $\mathbb{R}_+$ , (60) can be satisfied for a range  $\theta$ 's with positive measure. That is,  $\mu(\Theta^*) > 0$ . To verify that the candidate reactive equilibrium schedule cannot be upset by a pooling allocation, note that in  $\alpha - K$  space, indifference curves of firms with different  $\theta$ 's have the appropriate "stacking" property with respect to  $\theta$  required in Riley's [29] condition (A-5). Thus, no lender can profitably deviate from the candidate equilibrium with a pooling contract, given the anticipated reactions of other lenders, and we have a reactive equilibrium. Q.E.D.

*Proof of Theorem 6.* The only item that needs to be proved is that  $\alpha^*(\theta) \in (0, \alpha^*(\theta_2)) \forall \theta \in \Theta^*$ . In Theorem 2 we proved this fact under the assumption that  $\mu(\Theta^*) > 0$ . This assumption has now been proven in Theorem 5. Q.E.D.

*Proof of Theorem 7.* Claim (i) follows from (33). To show (ii) note that since project financing is used for the project, the definition of  $\theta^0$  shows that  $E_S^g(\theta) > \tilde{E}_S(\theta^0)$ . Now if  $E_S^g(\theta) = \hat{E}_S(\theta)$ , the project financed venture has complete debt financing. And if  $E_S^g(\theta) = \tilde{E}_S(\theta)$ , then  $\theta > \theta^0$  and again the project financed venture has higher leverage. This establishes claim (ii).

Claim (iii) follows from the fact that  $\tilde{E}_S(\theta)$  in (31) is a nondecreasing, convex function of  $\theta$  whereas  $\hat{E}_S(\theta)$  in (32) is constant for all  $\theta$ . Thus, there exists a  $\theta'$  such that  $\theta \in \Theta^-, \forall \theta \leq \theta'$  and  $\theta \in \Theta^+, \forall \theta > \theta'$ . This proves (iii). Claim (iv) follows from the fact that information production is always accompanied by complete debt financing.

Next, consider a project with a risk parameter  $\theta$  that produces a post-project-adoption risk parameter of  $\theta_{j+1}(\delta)$  for the whole firm. Because  $\tilde{E}_S(\theta_2) > I\rho\tau - Q^0(\beta) = \hat{E}_S(\theta_2)$  by assumption, we know that as long as  $\tilde{E}_S(\theta) > \hat{E}_S(\theta)$  over a set of nonzero Lebesgue measure, the monotonicity and convexity of  $\tilde{E}_S(\theta)$  ensures that  $\exists$  a unique  $\bar{\theta}$  in the interior of  $\Theta^*$  such that  $\tilde{E}_S(\theta) < \hat{E}_S(\theta), \forall \theta < \bar{\theta}$  and  $\tilde{E}_S(\theta) \geq \hat{E}_S(\theta), \forall \theta \geq \bar{\theta}$ . Now suppose the project's  $\theta^0 \in [\bar{\theta}, \theta_2]$ . Looking at Fig. 1 we see that if  $\theta < \theta^0$ , then  $E_S^p(\theta) = \hat{E}_S(\theta) = I\rho\tau - Q^0(\beta) < \tilde{E}_S(\theta^0)$  if  $\theta \leq \bar{\theta}$  and  $E_S^p(\theta) = \tilde{E}_S(\theta) < \tilde{E}_S(\theta^0)$  if  $\theta > \bar{\theta}$ . Thus, conventional financing is better if  $\theta < \theta^0$ . By similar logic, when  $\theta \geq \theta^0 \geq \bar{\theta}$ , we have  $E_S^p(\theta) = \tilde{E}_S(\theta) > \tilde{E}_S(\theta^0)$ . Thus, project financing is better if  $\theta \geq \theta^0$ .

The riskiest projects have  $E_S^p(\theta_2) = I\rho\tau[1 \wedge \bar{\alpha}]$ . Since conventional financing cannot produce an equity value exceeding  $I\rho\tau[1 \wedge \bar{\alpha}]$ , but could produce a lower equity value for a project with  $\theta = \theta_2$ , it follows that the values of the riskiest projects will be maximized with project financing. Finally, if  $\mu(\Theta^+) \leq r$  for  $r$  small enough, most of the ventures involving project financing will have complete debt financing, leading to the cross-sectional claim. Q.E.D.

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## OPTIMAL CAPITAL STRUCTURE AND PROJECT FINANCING

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