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Anjan V. Thakor and Richard Callaway*

I. Introduction

In this paper, we explore the nature of equilibria in an asymmetrically informed bank credit market in which credit applicants know their own (intrinsic) default risks, but potential lenders can discover these default risks only by expending resources to produce information. The resolution of informational asymmetries in the capital market is, in the contemporary view, considered a very important function served by financial intermediaries like commercial banks and, in the opinion of some, even the primary justification for their existence [17]. We, therefore, focus on how the presence of asymmetric information—in particular, the response of (expected) profit-maximizing banks to it—affects the equilibrium prices and quantities of credit offered in the banking system.[1]

We consider an economy containing a "lemons" credit market [1]. Our analytical approach resembles the Stiglitz [25] and Viscusi [29] screening paradigm (which is an alternative to the signaling model developed by Spence [23], [24] and Ross [20]). However, the analogy is not exact since, unlike their model, the information production decision rests with the potential lender rather than the credit applicant. Our formal analysis deals with the commercial credit market, but extension to consumer credit is easy. There are J industries and N_j' firms in industry j. At time zero, nature reveals its state and the (random) cash needs of firms become known. Firms then decide whether they need credit, and if they do, how many banks they should approach. Approaching a greater number of banks may be beneficial because it could increase the likelihood of getting a "better" price.[2] However, the search and applications process also consumes real firm

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[1] For convenience, we refer to all primary lenders as "banks," but our usage subsumes other types of depository financial institutions as well. In the next section, we will introduce a secondary credit source—the bond market—which is more expensive for firms to access.

[2] If a bank rations an applicant, it is equivalent to the bank's offering an infinite interest rate to that applicant. Thus, a firm's search for the best price may also include its attempt to escape rationing, although we prove later that when banks do not have perfect information about each other's

resources, and, thus, the firm must balance this cost against the expected price benefit in choosing the number of banks to approach. After receiving all applications, each bank knows its own portfolio of potential borrowers and this determines, for each applicant, the bank's cost of producing information about that applicant. This process of determining the cost function of producing information is formally portrayed in Section II.

Knowledge of the portfolio of applications received by each bank—and thus its cost functions for information production—may be disseminated in the economy either with perfect accuracy or with (perhaps considerable) noise. We consider both cases and find that the resulting equilibria possess surprising properties, even when the usual competitivity condition of unrestricted entry into the banking system is presumed to hold.

In Section III, we address the case in which perfect information about the applications portfolio of every bank is freely available to all. We prove that a Nash equilibrium exists and has the following characteristics:

(1) As long as a firm applies to one or more banks, only one bank will offer it credit, regardless of the number of banks approached.

(2) Every bank will act as a perfectly discriminating monopolist in pricing its credit, and this (optimal) pricing policy will be unaffected by the number of banks in the system. Thus, banks could earn long run "excess" profits even in the absence of barriers to entry.

(3) Although a firm applying for credit will be denied credit by all but one bank, there is no net rationing in the sense that every credit applicant's needs are accommodated within the banking system as a whole.

In the interest of expositional continuity, proofs of all theorems in this and the following sections are presented in the Appendix.

The more realistic case in which banks can assign only (nondegenerate) subjective probability distributions over the applications portfolios of other banks is analyzed in Section IV, and the equilibrium is shown to display the following properties:

(1) A firm applying to more than one bank may be offered credit by more than one bank.

(2) If an applicant is offered credit by more than one bank, each bank offering credit earns a negative expected profit with respect to that applicant.

(3) If a firm applies to only one bank for credit, the bank's pricing policy is that of a perfectly discriminating monopolist, as in the previous case.

(4) It is possible that an applicant will be denied credit by every bank it has approached, and, thus, there can be net rationing within the banking system as a whole. In fact, the probability of an applicant's being rationed out of the system is an increasing function of the number of banks from whom the applicant requests credit.

applications portfolios, an applicant's strategy of approaching a larger number of banks to increase the probability of being granted credit is actually counterproductive.

Two significant points emerge from this analysis. First, we learn that the economist's intuition that unfettered entry will inevitably drive monopoly rents to zero through an appropriate price effect may not be valid in a world of imperfect information. Secondly, as we discuss in Section V, our model yields a novel insight into the credit rationing phenomenon. The type of net rationing that arises in the case of noisy information dissemination is radically different from the type of rationing discussed in the extant literature [2]. In our model, banks may ration credit without even attempting to process an application, in spite of the fact that all relevant information about the applicant can be acquired. Thus, on a practical note our analysis provides a clear economic rationale for the commonly encountered—but often vaguely understood—lament by bankers that they simply do not have the time to process all loan applications the way they should, and, therefore, occasionally deny credit to potentially worthwhile applicants. Our concluding remarks, with suggestions for further extensions, are presented in Section VI.

II. Asymmetric Information Production Costs among Lenders

In this section, we develop a model that explains why different banks may have different information production costs with respect to the same loan applicant.

Suppose there are M banks in the economy, so that bank m is a member of the vector, $\underline{M} \equiv \{1, \ldots, M\}$. Let there be J industries, so that industry j is an element of the vector, $\underline{J} \equiv \{1, \ldots, J\}$, and let N_j' be the number of firms (potential credit applicants) in industry j.[3] Initially, all banks start with no assets or liabilities on their balance sheets, and the cash needs of every firm are unknown.

The economy opens (at time zero) with the realization of some random state of nature, and upon observing this state each firm knows whether it needs credit or not. Without loss of generality, assume that every firm that needs credit applies for a \$1 loan with a one-period maturity. Thus, if N_{jm} is the number of firms from industry j that apply to bank m for credit, the maximum possible size of that bank's loan portfolio is $\$\sum_j N_{jm}$. Define $\underline{N}_m \equiv (N_{1m}, \ldots, N_{jm}, \ldots, N_{Jm})$. Since a firm may apply to more than one bank, if N_j $(\leqslant N_j')$ is the total number of credit applicants,

$$\sum_m N_{jm} \geqslant N_j \quad \forall j \in \underline{J} \tag{1}$$

$$N_{jm} \leqslant N_j \quad \forall j \in \underline{J}. \tag{2}$$

Our analysis hinges, in part, on the assumptions listed below.

Assumption 1. The industry to which a credit applicant belongs is costlessly known to all.

[3] We implicitly assume $J >> M$, although the assumption can be easily relaxed.

Assumption 2. The default probabilities of credit applicants in industry j are uniformly distributed over the interval, $[\delta_j^-, \delta_j^+] \subseteq (0, 1)$, and this information, for each $j \in \underline{J}$, is freely available to all. Further, $\delta_j^+ = 1 - \epsilon_j$, where ϵ_j is positive and arbitrarily close to zero for every j.

Assumption 3. The specific default probability of any particular applicant is unknown *a priori* to all potential lenders. However, the correct default probability of firm k in industry j can be identified by bank m by investing an amount, $\phi_j^k(N_{jm})$, in information production. Further, $\partial \phi_j^k(N_{jm})/\partial N_{jm} < 0$.

Assumption 4. After all the applications are received, every bank knows the identity of the other banks that each of its applicants has approached for credit. Noiseless information about the exact composition of every other bank's applications portfolio may or may not be available to each bank, however.

Assumption 5. The bank credit market effectively opens only once. That is, once every bank announces the list of applicants it will lend to, no bank can add to its list, and no bank can engage in additional information production. However, if more than one bank agrees to lend to a particular applicant, the banks can compete—on the basis of price—for that applicant's business.

Assumption 6. When banks bid against each other, the prices bid by the other banks are unknown to participants in the bidding process. Each bank can submit at most one bid, and the bidding involves a fixed, known cost of C for each bank. Each bank, however, has the option of submitting its price bid after the customer has informed it which other banks have agreed to lend to it.

Assumption 7. Every applicant has access to the bond market for the necessary credit. However, the total cost of flotation and information production incurred (and borne by the borrower in equilibrium) for any applicant, $\phi_j^k(b)$, exceeds the maximum cost of information production for that applicant in the bank credit market.[4]

Assumption 8. All borrowers and lenders are risk neutral.

The first two assumptions are similar to Akerlof's [1] rational expectations assumption—the average quality is known. Allowing δ_j^+ to be arbitrarily close to one means that we admit the possibility of there being "real lemons" in each industry. This stipulation is a key assumption in our analysis, because, as later discussions will clarify, it tends to induce banks to produce information. Assumption 3 ensures that the equilibrium need not be degenerate, as Akerlof [1] suggested it might. The possibility of costly information production has been explored elsewhere too—for instance, by Stiglitz [25] and Viscusi [29] in the context of labor market equilibria, and by Campbell and Kracaw [6], [7] in studying capital market equilibria. The assumption that the information production cost is declining in N_{jm} is motivated by economies of scale in the production of information—the larger the number of applications a bank receives from an industry, the lower is each applicant's (per unit) share of the "fixed" cost of information generation about that industry. Note that the precise definition of "industry" is irrelevant here; all that we need is a classification of firms into different

[4] These costs are expressed in percentage terms, as a fraction of the size of the debt issue. For simplicity, it is assumed that, unlike the bank credit market, these costs are independent of the pool of borrowers being currently evaluated in the bond market. Moreover, $\phi_j^k(b)$ is known costlessly and without error to all.

risk classes, so that the default probabilities of the firms in a given risk class are systematically related by some common underlying factors.[5]

The fourth, fifth, sixth, and seventh assumptions have an important bearing on the prices and quantities of credit offered by banks, as subsequent sections will highlight. Assumption 6, in particular, needs some explanation. In our model, it is crucial to keep the bidding process secret, because otherwise, banks that do not produce information may be able to learn an applicant's default probability by simply observing the price quoted by a bank that has produced the necessary information. Incentives for costly information generation would then be destroyed. For a similar observation, in the context of informationally efficient securities market, see [10] and [11]. The rationale for permitting each bank to submit only one bid is also related to this possibility. If an initially uninformed bank is allowed to bid a possibly unlimited number of times against a bank that has acquired costly information, the former could deduce the latter's information set, even if bidding is done in secrecy, as long as the applicant reveals the identity of the bank quoting the lowest price at the end of every bid. Assumption 6 precludes such a phenomenon. However, the assumption also gives each bank the flexibility of deciding on the price it wants to quote after observing how many banks are interested in pursuing the applicant's business. Thus, a bank that has not produced information about an applicant but has included the applicant in its initial list could "save" itself the cost, C, of participating in the bidding process for that applicant if it perceives the probability of securing the applicant's business as "negligible." It can do this by simply refusing to extend credit, because Assumption 5 rules out only the banks' adding to their initial lists, not deleting from them. The assumption that bidding involves a fixed cost owes its justification to the fact that banks invariably must incur expenses on paperwork and labor in keeping a loan application on the books and in negotiating terms with the applicant, in addition to whatever expenses may have been incurred in initially producing information about the applicant. This assumption of a nonzero bidding cost also plays an important role in our analysis. Finally, the risk neutrality assumption is made for analytical convenience and can be relaxed, at the expense of increased complexity, without substantially affecting the results.

How does a firm that needs credit decide how many banks it should approach? In the usual analyses of perfectly competitive equilibria, such a question is of no relevance because each firm is guaranteed a unique fair price. However, in our model the number of banks the firm applies to may affect every bank's perception of the firm's demand function and, thus, could affect the prices offered to the firm. Moreover, even ignoring possibly discriminating monopolistic-type behavior by banks, there may be another reason for a firm to approach numerous banks. Since the matrix of information production cost functions for a particular bank is *a priori* unknown, no firm (or bank) knows *ex ante* which bank will be able to produce the relevant information about its risk characteristics at the lowest cost. If banks engage in some variant of cost-plus pricing, a firm can

[5] This type of approach is reminiscent of the "risk classes" assumption of Modigliani and Miller [18].

increase the probability of receiving a low price by applying to a larger number of banks.[6]

An obvious constraint on a firm's propensity to apply for credit to the entire universe of banks in the economy is imposed by the real resources consumed in submitting applications. These resources would, at the very least, include the man-hours expended in preparing applications per standard bank requirements, and may also include the costs of paperwork, contact hours, etc. Thus, the optimal subset of banks, $^*\underline{M}_j^k$ ($\subset \underline{M}$), approached by firm k in industry j, will satisfy

$$(3) \qquad ^*\underline{M}_j^k \in \underset{\underline{M}_j^k \subset \underline{M}}{\operatorname{argmin}} \left\{\bar{S}_j^k + \zeta\left(\underline{M}_j^k\right)\right\}$$

subject to

$$(4) \qquad \bar{S}_j^k = \int \min\left\{\hat{S}_j^k,\, S_j^k(b)\right\} dQ_j^k\left(\hat{S}_j^k \mid \underline{M}_j^k,\, \delta_j^k\right)$$

where

δ_j^k $\equiv$ default probability of firm k in industry j

ζ : $\mathbb{N}_+ \to (0,1)$, is a strictly increasing convex function representing the real resource cost of applying for credit; $\mathbb{N}_+ \equiv \{1, 2, \ldots\}$ is the set of positive integers

$S_j^k(b)$ $\equiv$ interest rate paid by the firm in the bond market

$\hat{S}_j^k$ $\equiv$ the minimum price offered the firm in the bank credit market[7]

$Q_j^k(.|.,.)$ $\equiv$ subjective conditional cumulative distribution function assigned by the firm to the minimum price in the bank credit market.

In specifying a default probability δ_j^k, we assume that the firm uses the credit it acquires at time zero to invest in a risky project that results in an end-of-period (time one) wealth of Z_j^k with probability $(1 - \delta_j^k)$ and zero wealth with probability δ_j^k for the firm. In the subsequent analysis, it is assumed that

$$Z_j^k > \left(1 + S_j^k(b)\right) \quad \forall\, k,\, j.$$

Thus, for bank m

$$(5) \qquad N_{jm} = \sum_{k=1}^{N_j} I_{\left\{^*\underline{M}_j^k\right\}}(m)$$

[6] For other analyses of markets in which search behavior on the part of buyers is optimal, see [21], [15], and [19].

[7] The terms "firms" and "credit applicants" are used somewhat interchangeably, but it should be noted that while all credit applicants are firms, the actual pool of applicants is generally only a subset of the universe of firms in the economy.

where

$$I_{\{{}^*\underline{M}_j^k\}}(m) = \begin{cases} 1 & \text{if } m \in {}^*\underline{M}_j^k \\ 0 & \text{otherwise} . \end{cases} \tag{6}$$

Banks are not assumed to know *a priori* either the random cash needs of firms or their subjective distribution functions, $Q_j^k(.|.,.)$. This means that N_{jm} is a random variable for each bank. Consequently, the information production cost functions for any given applicant generally will not be the same for all banks, and these cost functions will be unknown *ex ante*. In the subsequent sections, this structure forms the basis of our analysis of bank credit market equilibria.

III. Credit Market Equilibrium with Perfect Information about Applications Portfolios

In this section, we assume that after all the credit applications are received by banks, every bank knows not only its own applications portfolio, but also the applications portfolio of every other bank in the economy. In addition to the eight permanent assumptions listed in the previous section, we need the following assumptions.

Assumption 9. All banks face a perfectly elastic supply schedule for deposits. That is, any bank can purchase any quantity of deposits instantaneously at a market determined riskless interest rate, r.[8]

Assumption 10. The interest rate posted by a bank may act as a screening device but not as an incentive device.

The assumption that the deposit supply function is perfectly elastic is standard in models of financial institutions, particularly those dealing with credit rationing. Of course, in light of the recent emergence of liability management as an issue of substantial concern for depository financial institutions, this assumption is apparently unpalatable. However, the introduction of a rising marginal cost of funds for the bank could result in a trivial "quantity rationing" equilibrium in which the bank refuses to extend credit simply because its cost of funds, beyond some point, exceeds the maximum possible price the credit applicant can pay. Assumption 10 implies that the default probability of any specific credit applicant is not altered by the price it has to pay for credit. This is in contrast to Stiglitz and Weiss's [26] assumption that a bank's posted price could influence the characteristics of the investment financed with the funds borrowed from the bank. Also note that our model's specification differs from the model appearing in the recent paper by Stiglitz and Weiss [27] in which the borrower has an effort-supply decision that can affect returns in the "good" state, and a project-selection decision that can, in addition, influence the probability of occurrence of the good state.

[8] This assumption implies that deposit insurance is complete and the probability of any default on the part of the FDIC is zero.

Our exploration of the properties of the equilibrium in this setting proceeds in two steps. First, we show that it is generally not optimal for any (expected) profit-maximizing bank to randomly assign default probabilities to its credit applicants and price its credit without investing in information production. In those cases in which randomization is preferrable to costly information production, banks prefer to ration credit rather than to lend without acquiring information. Then, we prove that in a Nash equilibrium one obtains in this economy, each bank generally grants credit to only a subset of its credit applicants and acts as a perfectly discriminating monopolist in pricing its credit. Although the actual price charged by a bank for credit may differ from the stated interest rate on the proposed loan, we will use the terms "price" and "interest rate" interchangeably henceforth.[9] First, we define equilibrium.

Definition of Equilibrium. An equilibrium satisfied the following conditions:

(i) Each firm chooses the number of banks it should approach in a manner that minimizes its expected cost of borrowing.

(ii) Each bank maximizes its expected profit in choosing whether or not to produce information about an applicant. This choice is made after the bank has noted the number of banks the applicant has approached, and the applications portfolio of each of those banks, if it is observable.

(iii) If a bank's strategy of maximizing expected profit entails not producing information about an applicant, the bank will decide whether or not to bid for the applicant's business after observing how many of the banks approached by the applicants are interested in pursuing its business.

(iv) At the end of the bidding process, each bank will quote a vector of prices for its applicants so as to maximize its expected profit on each application.

Thus, the economy is characterized by four discrete points in time prior to the actual disbursement of credit. At the first point, firms submit credit applications to banks; at the second point, each bank decides which of its applicants it should produce information about; this is followed by the information production phase preceding the third point at which each bank announces a list of applicants to whom it will lend, but does not declare prices; finally, after a "negotiation" or bidding phase, each bank submits to each applicant, at the fourth point, the price of its credit and applicants in turn determine the banks from whom they will borrow. Without loss of generality, it will be assumed that the single period describing the maturity of each loan is substantially longer than the cumulative time that elapses from the point at which applications are submitted to the point of actual credit extension. For discounting purposes then, the model is single pe-

[9] That is, we assume that the interest rates we deal with are real prices rather than nominal rates. For bank loans, a divergence between the loan interest rate and the effective price of the loan is often caused either by items unrelated to the loan being subsumed in the interest rate, or by a part of the price being reflected in parameters other than the interest rate. For instance, on the one hand, the interest rate may include some portion of the fee charged by the bank for making forward commitments to the borrower, and, on the other hand, the bank could reduce the interest rate by requiring the borrower to keep a compensating balance.

riod, with time zero denoting the date on which the applicant borrows and time one denoting the contractually agreed upon repayment date.

Recall that Assumption 7 implies that $\phi_j^k(b)$ exceeds the minimum cost of information production in the bank credit market. Let us now make that assumption more precise by specifying

$$\phi_j^k(b) = \left(1 - \delta_j^-\right)\left(\zeta\left(1_j^k\right) + L_j^0\right) + \phi_j^k\left(1_{jm}\right) + L_j^1 \tag{7}$$

where L_j^0 and L_j^1 are positive constants for any fixed j, $\zeta(1_j^k)$ is the search and applications cost incurred by the applicant in applying to one bank, and $\phi_j^k(1_{jm})$ is the cost of information production for the applicant, namely firm k in industry j.

With these preliminaries, we can explore the incentives of banks to produce information about applicants, when information about the applications portfolios of other banks is costlessly available.

Theorem 1. When perfect information about applications portfolios is freely available,

(i) a bank will prefer to produce information about an applicant rather than "randomize," if it is the only bank the applicant has approached or if it believes that no other bank will agree to extend the applicant credit; and

(ii) a bank will prefer to ration credit rather than produce costly information or "randomize" and offer credit to an applicant if it believes (with probability one) that another bank will extend the applicant credit.

Essentially, the theorem says that a bank, if it wishes to extend credit, will have a preference for generating costly information rather than announcing a uniform nondiscriminating price for all applicants from a given industry. As in Akerlof's model [1], the posting of an imperfect information-based common price induces an adverse selection mechanism that forces out applicants of superior quality (low default probability). Recognizing this, the bank offering such a price responds by making the price high enough to provide it with a risk-adjusted return that is adequate even if only borrowers with the maximum possible default probability accept credit. (See equation (A-10) in the Appendix in which $*D_j^k$ is very large because δ_j^+ is close to 1. Crossmultiplying, it can be seen that $*D_j^k(1 - \delta_j^+)$ is the bank's expected payoff on a borrower with default probability δ_j^+). This means that in spite of the sorting effect of prices, a bank following a "randomization" strategy can earn a positive expected profit as long as cross-sectionally the maximum default probability is below one and no other bank pursues the applicant's business. But, in this case, the bank can earn a higher expected profit by investing in information acquisition. A pivotal assumption leading to this result is that δ_j^+ is arbitrarily close to one. The reasoning behind this is transparent—the lower the maximum default probability in the population of credit applicants, the better is the quality of those who accept credit at the nondiscriminating price, and, consequently, the higher is the bank's expected profit from "randomizing" relative to the expected profit from costly information production. In the case in which there is at least one other bank interested in extending the applicant credit, the gains from randomization diminish as the cost of bidding, C, rises. Again, with sufficiently high δ_j^+ and C, the randomization alternative is rendered unprofitable.

To examine the properties of credit market equilibrium in this setting, we need a few more preliminaries.

For bank m, define

$$\underline{W}_m \equiv \{j \mid N_{jm} > 0, j \in \underline{J}\} \subseteq \underline{J}$$

and

$$\underline{V}_m^0 \equiv \{j \mid j \in J; N_{jm} > N_{jh} \quad \forall\, h \in \underline{M}, h \neq m\} \subseteq \underline{W}_m .$$

Recall that $N_{jm} > N_{jh}$ implies $\phi_j^k(N_{jm}) < \phi_j^k(N_{jh})\ \forall\, k$. Let $\underline{V}_m$ $(\subseteq \underline{W}_m)$ be the set of industries about which bank m produces information, and let $\underline{T}_j^m \equiv \{k \mid m \in {}^*\underline{M}_j^k\}$ be the set of firms within industry j from which bank m receives applications. Thus, if ξ is the counting measure, then $\xi(\underline{T}_j^m) = N_{jm}$. Define $\underline{U}_j^m$ $(\subseteq \underline{T}_j^m)$ as the set of firms within industry j about which bank m produces information. Suppose we allow bank m to offer credit only to firms within industry j that are members of $\underline{U}_j^m$ and to concern itself with only those industries that belong to $\underline{V}_m$.[10] Define $\underline{S}(m) \equiv \{S_j^k(m) \mid k \in \underline{U}_j^m; j \in \underline{V}_m\}$ as the set of interest rates posted by bank m for firm k from industry j, and $\omega_m \equiv (\underline{U}_j^m, \underline{V}_m, \underline{S}(m))$ as the information production and pricing strategies for bank m. The space $\Omega \equiv (\omega_1, \ldots, \omega_m, \ldots, \omega_M)$ then describes the strategic behavior of the entire banking system. (Throughout, stars (*) will be used to denote optimal (profit-maximizing) values.) Assume $\underline{W}_m$ and $\underline{V}_m$ are nonempty for at least some $m \in \underline{M}$. Further, for every $m \in \underline{M}$, assume that the set $\{j \mid j \in \underline{J}; N_{jm} = N_{jh}, h \in \underline{M}, h \neq m\}$ is empty.[11]

With $(\Omega^* \backslash \omega_m) \equiv (\omega_1^*, \ldots, \omega_{m-1}^*, \omega_{m+1}^*, \ldots, \omega_M^*)$, the notation $[\Pi_j^k(m) \mid (\Omega^* \backslash \omega_m)]$ will be used to denote bank m's expected profit conditional upon the rest of the banks in the credit market adopting their optimal strategies. Thus, bank m solves the following maximization program

$$\text{(8)} \qquad \underset{\substack{\underline{V}_m \subseteq \underline{W}_m \\ \underline{U}_j^m \subseteq \underline{T}_j^m \\ \underline{S}(m) \in \mathbb{R}_+^{\theta(m)}}}{\text{Maximize}} \sum_{j \in \underline{V}_m} \sum_{k \in \underline{U}_j^m} \left[\Pi_j^k(m) \mid (\Omega^* \backslash \omega_m)\right]$$

[10] If bank m is the only bank that will ultimately offer these applicants credit, and if for all other applicants there will be at least one other bank in the picture, then Theorem 1 validates this supposition. This line of reasoning will be formalized in Theorem 2.

[11] This assumption is not crucial to the analysis, but its relaxation makes the analytics unnecessarily complex. In the subsequent discussion—notably in the proof of Theorem 2—we ignore the possibility of two banks having exactly the same number of applicants from one industry. Conceptually, dealing with this possibility is not difficult, but it considerably lengthens the formal exposition and calls for some rather *ad hoc* assumptions regarding "tie breaking" rules.

where

$$(9)\quad \overline{\Pi}_j^k(m) \equiv \mathrm{E}\left\{\left(1-\delta_j^k\right)\left(1+S_j^k(m) \mid \tilde{\delta}_j^k\right)\right\} - \left(R+\phi_j^k(m)\right)$$

$$\begin{aligned}
\mathrm{E}\{\,.\,\} &\equiv \text{expectation operator} \\
\mathbb{R}_+ &\equiv [0, \infty), \text{ the feasible set of reals in which the interest rate offered to any firm can take values} \\
\theta(m) &\equiv \xi(\underline{S}(m)) \\
\mathbb{R}_+^{\theta(m)} &\equiv \mathop{\times}_{y \in \{1, \ldots, \theta(m)\}} \mathbb{R}_+(y), \quad \mathbb{R}_+(y) = \mathbb{R}_+ \quad \forall\, y
\end{aligned}$$

and $\phi_j^k(m)$ is as it was defined in the proof of Theorem 1.

The solution to the above maximization program and the nature of the resulting equilibrium are the subjects of the next theorem.

Theorem 2. When perfect information about all applications portfolios is costlessly available to all, a Nash equilibrium exists in which

- (i) every applicant receives an offer of credit from only one bank, irrespective of how many banks it applies to
- (ii) some banks earn positive expected profits and some banks may earn zero profits
- (iii) firm k in industry j pays a rate of interest $1 + S_j^k(b) - \zeta(1_j^k) - e_j$, where $e_j(\in(0, L_j^0))$ is an arbitrarily small positive scalar.

To understand the economic intuition behind this theorem, note that a Nash equilibrium is being discussed. Thus, the question asked is whether it is optimal for a bank to follow some prescribed strategy, if all other banks adopt the same strategy. Now every bank knows that when two or more banks pursue an applicant's business, all the competing banks suffer a loss, regardless of which bank actually wins the bid. Consequently, every bank confronted with direct competition for an applicant's business will want to avoid that applicant. But when this happens, the applicant is left with no creditors, and the first bank to reenter the competition can earn positive profits. It is natural to think of this entrant as the bank with the lowest cost for the applicant, because it has the least to lose (the loss being the sum of the information production cost and the bidding cost) in case some other bank also decides to follow suit. However, once the least cost bank is committed to offering the applicant credit, it is suboptimal for any other bank to reenter the picture, since doing so would yield a negative expected profit. Of course, *ex ante* rationality on the part of banks obviates the need for such a sequence of events; all banks will anticipate these events and the equilibrium sketched in the theorem will result.

Corollary. When perfect information about applications portfolios is freely available *ex post,* every firm applies to only one bank.

Proof. Since the rate offered by a bank does not depend on that bank's information production cost function, which specific bank offers the firm credit is irrelevant from the firm's point of view. Since $\zeta(.)$ is increasing in its argument, the only reason for the firm to approach more than one bank would be to minimize the probability of being rationed. But, by Theorem 2 the firm can avoid being rationed even if it applies to only one bank.

The mechanism described in the proof of Theorem 2 does look somewhat complex, but is fairly intuitive, as the following simple illustration shows.

Illustration. Suppose there are three banks I, II, and III, and two industries, A and B. Let there be five credit applicants, $a_1, \ldots, a_5$, from industry A and six applicants, $b_1, \ldots, b_6$, from industry B. Let the applications portfolios of the three banks be given by the following table.

	Industry	
Bank	A	B
I	a_1	b_1, b_2, b_5
II	a_2, a_3, a_5	b_2, b_6
III	a_4, a_5	b_1, b_3, b_4, b_5

In this case, the equilibrium loan portfolios of banks I, II, and III will be (a_1), $(a_2, a_3, a_5; b_2, b_6)$, and $(a_4; b_1, b_3, b_4, b_5)$, respectively.

Discussion of Equilibrium. An interesting property of the equilibrium is that it maximizes the expected profits of the banking system as a whole. Thus, although banks are assumed not to collude, as long as they have perfect information about each other's applications portfolios they behave as if they were colluding, and, in equilibrium, information is produced only by those banks with the greatest comparative advantage. Consequently, the equilibrium is Pareto optimal in the (restricted) sense that the total resources expended in resolving the informational asymmetry problem are minimized.

A second property of some note is that the entire pool of credit applicants is ''covered'' by the banking system. That is, no applicant is compelled to access the bond market. Thus, while there could be rationing in the ''small''—the denial of credit to the applicant by some but not all of the banks approached—there is no rationing in the ''large.'' But, given that no firm will apply to more than one bank, in equilibrium there is rationing neither in the ''small'' nor in the ''large.'' This highlights another efficiency property of the equilibrium, namely that the total resources consumed by firms in the search and applications process are also minimized.

Note that the equilibrium characterized in Theorem 2 implies that if some entrepreneurs were to observe the excess profits earned by some banks and enter the system to share in those gains, they would not benefit from disturbing the implicitly enforced ''allocation mechanism'' that determines which banks should produce information about and lend to which applicants. Thus, all new entrants will find it optimal to ''conform,'' and free entry will fail to drive expected profits of the banking industry as a whole to zero. To sharpen the intuition behind this argument, recall that the model implies that each firm will choose on a purely random basis the bank it should apply to and will ignore any spatial distribution considerations. A new entering bank will, therefore, be unable to ascertain precisely *ex ante* whether it will earn a zero profit ($\xi(*\underline{U}_j^m) = 0 \; \forall j \in \underline{V}_m^*$) or a positive expected profit ($\xi(*\underline{U}_j^m) > 0$ for some $j \in \underline{V}_m^*$) in equilibrium. The ratio of the number of banks that earn positive expected profits to the number of banks that earn zero profits will, of course, depend on the size of the banking system relative to the size of the pool of potential credit applicants. As a result, new banks will keep entering the system and the size of the banking sector will keep growing until it is so large relative to the expected credit demands of borrowers that the cost of establishing a new bank is exactly equal to the present value of the expected marginal expected profit for a new entrant.[12] But, even after the banking system attains its maximum potential size, the total industry expected profits will be positive in equilibrium. Clearly, each individual bank's slice of the pie will generally diminish as the banking system is augmented in size, simply because there will be a larger number of banks earning zero profits. However, in equilibrium every bank that does offer credit earns a positive expected profit regardless of how many banks there are in the system.

It is easy to see that in the absence of information asymmetries we would get the classical result that a long run competitive equilibrium involves zero expected profits for all banks. Thus, the real culprit responsible for sustaining the feasibility of noncompetitive pricing in this model is the fact that the lender's optimal response to the informational asymmetry is to generate the required information at a cost. As mentioned before, in such a setting, if more than one bank produces information about an applicant, no bank bidding for the applicant's business is able to recoup its cost of information production. Consequently, it is optimal for banks to devise an implicit allocation rule to avoid this problem. It turns out that an allocation rule that works is the one described in Theorem 2—only one bank produces information about and offers credit to a particular applicant. And, when that happens, the bank ends up in a monopoly situation with respect to that applicant. This discussion also points out that the ''excess profits'' equilibrium described here is unlikely to occur in the usual ''goods' industries such as grocery stores or hardware shops. Sellers in these industries do not care who their customers are, and, thus, producing costly information about them is ruled out. What is unique about a bank is that the identity of the borrower is all-important. This induces banks to expend resources in discovering these identities, and has the effect of creating natural incentives for restricting competition (to keep a ''reasonable'' bound on information generation costs).

These observations should be viewed as complementing recent developments in the economics of information, particularly the papers included in [28]. The basic theme of these papers is that the conventional (full information equilibrium) notion of markets clearing at a single price with all individuals and firms viewing themselves as price takers is questionable if the imperfections of information are seriously addressd. More specifically, however, it is perhaps more instructive to compare our model with those of Stiglitz [25] and Viscusi [29]. In proposing a theory of screening, Stiglitz [25] considers groups of individuals indexed by different levels of ability and demonstrates the existence of multiple equilibria, as well as the possibility of nonexistence of equilibria when the individuals themselves have the choice of revealing their *a priori* unknown abilities to prospective employers at a cost. In Viscusi's [29] model, it is the firm that has

[12] Note that this argument holds even if the cost of establishing a new bank is zero.

something to reveal to its prospective employees, namely the hazard involved in the job. Properties of the market unraveling process are explored under the assumption that the revelation (certification) is costly for the firm. In both models, therefore, the option of costly information production rests with the entity whose characteristic is unknown to the rest of the market. This is where our approach is different. In our model, it is the bank, rather than the credit applicant, that decides whether information about the applicant's default probability should be generated at a cost. The fact that our equilibrium has some unusual properties is largely predicated upon this crucial distinction. For instance, if credit applicants could have the necessary information produced on their own, perhaps through an independent accreditation agency, many of the problems we allude to could probably be avoided. However, a serious problem with outside accreditation is the credibility of the information-producing agency itself, particularly in view of the transfer payments that could take place between the agency and the credit applicants.[13]

Thus far, the cost of negotiations for a bank during the bidding phase has played an important role in the analysis. The implications of letting this cost be zero are explored in the following theorem.

Theorem 3. If $C = 0$, multiple (Nash) equilibria are feasible. One of these equilibria is the one described in Theorem 2. The other is one in which every bank

(i) produces costly information about and extends credit to those applicants for which it represents the only bank approached for credit, and

(ii) extends credit without producing information to all of its applicants who have approached at least one other bank.

This theorem highlights the significance of assuming a positive C. With $C = 0$, if every firm applies to more than one bank, no bank in the economy may produce costly information. Consequently, a single price, based on a knowledge of only the cross-sectional distribution of default probabilities, will be announced by every bank, and by adverse selection "superior quality" firms will exit the banking sector and access the bond market.

An obviously restrictive (and somewhat unrealistic) assumption employed in this section is that perfect information about applications portfolios is freely available to all banks. We, therefore, relax this assumption in the next section.

IV. Credit Market Equilibrium with Noisy Information about Applications Portfolios

Assume that after all applications are received, although each bank knows its own portfolio of applications as well as the identity of other banks each of its applicants has approached, it has only noisy information about the portfolios of those banks.

[13] The fact that banks do invest substantial resources in credit evaluation should be viewed as an optimal response to their inability to obtain all relevant information from centralized information production agencies.

Suppose bank m is faced with an application from firm k in industry j. Assume that this firm has also applied to numerous other banks, and bank m knows the identities of these banks but does not precisely know the composition of their applications portfolios. Let $*\underline{M}_j^k$ be the set of banks this applicant has approached, and define

$$\Delta_j^k \equiv \left[*\underline{M}_j^k \setminus m\right] \equiv \left\{h \,|\, h \in *\underline{M}_j^k,\, h \neq m\right\}.$$

In this setting, bank m can assign only (subjective) probabilities over the events that the other banks from whom the applicant has requested credit will indeed extend credit. Let η_h represent bank m's assessment of the probability that bank h will offer this applicant credit, and define

$$\hat{\eta}_h \equiv 1 - \eta_h .$$

Clearly, the equilibrium sketched in Theorem 2 will not be attainable here. Our final theorem analyzes the properties of equilibrium under noisy information about applications portfolios.

Theorem 4. In the absence of noiseless dissemination of information about the applications portfolios of banks,

(i) a credit applicant increases the probability of being rationed out of the entire banking system by applying to a larger number of banks,

(ii) the smaller the number of banks to whom a firm applies, the higher is the probability that at least one of those banks will produce information about it and offer credit, and

(iii) *ex post,* some banks may earn positive expected profits, some negative expected profits, and some zero profits.

Discussion of Equilibrium. Since equilibrium in our model is defined as a situation in which each bank has chosen its strategies so as to maximize its expected profit, it may appear paradoxical that some banks could earn negative expected profits. For reconciliation, note that equilibrium is defined in an *ex ante* sense with every bank taking expectations with respect to two random outcomes.

The first random outcome is the number of banks that eventually elect to bid for the applicant's business, and the second is the amount of repayment actually made by the borrower when the loan matures, conditional on loan disbursement occurring in the first place. The above theorem says that *ex post,* the realization of the first random event for some bank may be such that the expenses incurred by it in information production and bidding exceed the expected profits on lending at the contractually agreed upon prices that emerge at the end of the bidding process.

A significant implication of the theorem is that uncertainty about applications portfolios has a potentially substantive impact on the nature of equilibrium. No longer do we have the efficiency result that information will be produced only by those banks that have the greatest comparative advantage. This departure results because, obviously, no bank knows with certainty how its information production cost functions compare with those of other banks.

Another noteworthy point is that now there can be rationing in the "large" in the sense that an applicant may be denied credit by every bank it approaches. The reason that a firm may apply to numerous banks in spite of the fact that applying increases the probability of getting rationed as well as the cost of applying, is that applying also improves the firm's chances of getting a "bargain" price. For instance, by Theorem 4, if the applicant is fortuitously offered credit by more than one information-producing bank, the applicant must pay only a rate of interest equal to $R(1 - \delta_j^k)^{-1}$, and, therefore, need not bear even the cost of information production, $\phi_j^k(.)$. This result means that search behavior on the part of firms truly may be optimal.[14] Consequently, the total resources consumed by firms in the search and applications process are not necessarily minimized.

It is difficult to say anything definitive about the total expected profits of the banking system in this model. All that can be asserted is that in equilibrium not every bank generally can be guaranteed a zero expected profit, even if the banking system were to grow large in the usual competitive sense. Of course, as mentioned before, some banks may even earn negative expected profits. With no initial capital, this will lead to bankruptcy for the bank, and in our model the FDIC will settle all claims against the bank. This makes it clear that the FDIC's liability is greater in this regime than in the previous one. In other words, if the deposit insurance premium is unaffected, the FDIC bears a part of the cost created by the externality of noisy applications portfolio information. Note, however, that credit applicants could be better off with noisy information, and, thus, there could be conflicts of interest between the FDIC and the credit applicants. In fact, if the FDIC's share of the cost is ultimately passed on to the taxpayers, one effect of the noisy portfolio information is the creation of a transfer payment from the general public to the credit applicants.[15]

Given the lost efficiency induced by a lack of perfect information about applications portfolios, an important question is whether banks have an incentive to eliminate this loss by voluntarily exchanging information about each other's portfolios, contingent perhaps on some unanimously accepted side-payments structure that allows the least-cost bank to lend and share its profits with the others. The answer is no, except under rather special circumstances. The reason is that for such an arrangement to be feasible for any applicant, every bank approached by the applicant should agree to participate. This will be difficult to achieve, because a bank that has a positive profit expectation without sharing information will refuse to participate, unless it can be guaranteed a higher compensation with information sharing. A substantial reward for information sharing will be possible if the customer has approached just a few banks (so that there are fewer banks dividing the "excess" profits). In that case, however, each bank's expectation of profit without sharing information is also high (see equation (A-23) in the proof of Theorem 4 in the Appendix). Thus, in the absence of *ad hoc*

[14] How many banks a firm actually applies to will presumably depend, in part, on the spread between $R(1 - \delta_j^k)^{-1}$ and $1 + S_j^k(b) - \zeta(1_j^k) - e_j$.

[15] A similar argument has been made for the loan guarantees provided by the federal government to the Chrysler Corporation. Since at best Chrysler bears only a portion of the cost of these stock-option type guarantees (the cost borne by Chrysler being the restrictions imposed on it in exchange for the guarantees), there is a transfer payment from the taxpayers to Chrysler.

restrictions, ensuring the existence of private incentives for portfolio information sharing in the general case does not seem within reach.

In the next section, we relate the preceding discussion to the existing literature on credit rationing.

V. Applications to Credit Rationing

A rich menu of papers deals with credit rationing by financial institutions. In the earlier literature, it was argued that non-price rationing is completely inconsistent with profit-maximizing bank behavior [22]. This result led to the emergence of models designed to demonstrate the rationality—from a profit-maximizing standpoint—of credit rationing by banks. Examples of such efforts are the papers by Fried and Howitt [9], Jaffee [12], Jaffee and Modigliani [13], Jaffee and Russell [14], Koskela [16], and most recently, Stiglitz and Weiss [26].

Although these papers invoke different scenarios to justify the existence of credit rationing as a profit-maximizing strategy on the part of lenders, all the papers deal with rationing equilibria in which lenders first assess credit applicants on the basis of all relevant and feasible criteria and then decide to refuse credit to some, irrespective of the price they offer to pay for the credit. For instance, Jaffee and Modigliani make the "complete information" assumption that the lender knows the probability distribution of the wealth to be generated by the credit applicant's investment. They then show that if there are two groups of credit applicants with access to identical investments, credit may be denied to the applicants with the lower demand elasticity, unless the lender can act as a perfectly discriminating monopolist. However, since the absence of monopolisic pricing behavior must be exogenously imposed as an assumption on the model to justify rationing, rather than being derived endogenously within a broader framework, the insights provided by their analysis are somewhat limited. At the other extreme, in two excellent papers, Jaffee and Russell [14] and Stiglitz and Weiss [26] consider an imperfect information world in which lenders know that credit applicants with heterogeneous default characteristics exist, but are unable to identify or control a specific applicant's characteristics. Thus, imperfections in information, and particularly the inability of lenders to rectify these imperfections, play an important role in engendering rationing.

The informational structure we employ is intermediate between the above two extremes. Like Jaffee and Russell [14] and Stiglitz and Weiss [26], we assume that information about the characteristics of credit applicants is imperfect, but we also permit lenders to have the option of removing this imperfection at a cost. In turn, this altered specification gives rise to the possibility of a type of rationing that is routinely practiced by numerous lenders, but has been excluded in the existing discussions of credit rationing. Lenders—particularly depository financial institutions—often do not thoroughly investigate all the credit applications they receive. Rather, they adopt a seemingly *ad hoc* screening procedure to sort out *a priori* unacceptable applications and then proceed to evaluate the remaining applications. Thus, two types of rationing may occur. A subset of the pool of applicants may be denied credit without ever being seriously considered and, subsequently, sòme of the remaining applicants—who are throroughly investigated if possible—may be denied credit because the lender determines that the extension of credit to them is unprofitable at any price.[16] The extant credit rationing literature has focused on the latter type of rationing, whereas we have analyzed the rationale for the former type of rationing.

VI. Concluding Remarks

The importance of imperfections in information in explaining the existence of financial intermediaries has received growing support over the last few years. Some examples are the papers by Bhattacharya [5], Diamond [8], and Leland and Pyle [17]. To the best of our knowledge, however, the model developed in this paper represents the first attempt at exploring the implications of informational asymmetries for the nature of equilibria in the credit markets, although the Stiglitz-Viscusi screening paradigm has been applied in a capital market context by Campbell and Kracaw [6], [7].

The reader may question the somewhat stylized setting we have employed. While some assumptions may be eliminated without risking analytical intractability, most of the economic structure of the model is necessitated by the need to simplify a highly complex problem without losing its essence. The issues we have attempted to deal with are not easily amenable to simple modeling, particularly because of the difficult game-theoretic considerations involved. The majority of our seemingly abstract assumptions are, therefore, designed to ensure that the essential features of the problem are not obscured in a maze of tangential details.

A promising extension of our analysis would be to make it multiperiod. As a conjecture, at least some of the serious problems we have alluded to could be avoided if a bank and a firm established an implicit contract with a built-in intertemporal penalty/reward structure that assured the bank that it would be able to recoup its investments in information production, and assured the firm a competitive price. Such a model could provide an endogenous justification for the simultaneous existence of spot and forward bank credit markets, and could also facilitate an understanding of the pricing of loan commitments under imperfect (or costly) information.

[16] Of course, in our model only the first type of rationing can occur. A more complete analysis would allow both types of rationing to exist. More valuable insights may perhaps be gleaned from such an attempt.

References

[1] Akerlof, G. A. ''The Market for 'Lemons': Qualitative Uncertainty and the Market Mechanism.'' *Quarterly Journal of Economics,* Vol. 84 (August 1970), pp. 488-500.

[2] Baltensperger, E. ''Credit Rationing: Issues and Questions.'' *The Journal of Money, Credit and Banking,* Vol. 10 (May 1978), pp. 170-183.

[3] Bhattacharya, S. ''Imperfect Information, Dividend Policy, and the 'Bird in the Hand' Fallacy.'' *The Bell Journal of Economics,* Vol. 10 (Spring 1979), pp. 259-270.

[4] ________________. ''Nondissipative Signalling Structures and Dividend Policy.'' *Quarterly Journal of Economics,* Vol. 95 (August 1980), pp. 1-24.

[5] ________________. ''Aspects of Monetary and Banking Theory and Moral Hazard.'' *Journal of Finance,* Vol. 37 (May 1982), pp. 371-384.

[6] Campbell, T., and W. Kracaw. ''Information Production, Market Signalling, and the Theory of Financial Intermediation.'' *Journal of Finance,* Vol. 35 (September 1980), pp. 863-882.

[7] ________________. ''Sorting Equilibria in Financial Markets: 'The Incentive Problem.''' *Journal of Financial and Quantitative Analysis,* Vol. 16 (November 1981), pp. 477-492.

[8] Diamond, D. ''Intermediation and Delegated Monitoring.'' CRSP working paper, University of Chicago (1979).

[9] Fried, J., and P. Howitt. ''Credit Rationing and Implicit Contract Theory.'' *The Journal of Money, Credit and Banking,* Vol. 12 (August 1980), pp. 471-487.

[10] Grossman, S. ''On the Efficiency of Competitive Stock Markets Where Traders Have Diverse Information.'' *Journal of Finance,* Vol. 31 (May 1976), pp. 573-585.

[11] Grossman, S., and J. E. Stiglitz. ''Information and Competitive Price Systems.'' *The American Economic Review,* Vol. 66 (May 1976), pp. 246-253.

[12] Jaffee, D. M. ''A Theory and Test of Credit Rationing: Further Notes.'' *The American Economic Review,* Vol. 62 (June 1972), pp. 484-488.

[13] Jaffee, D. M., and F. Modigliani. ''A Theory and Test of Credit Rationing.'' *The American Economic Review,* Vol. 59 (December 1969), pp. 850-872.

[14] Jaffee, D. M., and T. Russell. ''Imperfect Information, Uncertainty, and Credit Rationing.'' *Quarterly Journal of Economics,* Vol. 90 (November 1976), pp. 651-666.

[15] Kohn, M. G., and S. Shavell. ''Optimal Adaptive Search.'' *Journal of Economic Theory,* Vol. 9 (October 1974), pp. 93-123.

[16] Koskela, E. *A Study of Bank Behavior and Credit Rationing.* Helsinki: Adademia Scientarum Fennica (1976).

[17] Leland, H., and D. Pyle. ''Informational Asymmetries, Financial Structure, and Financial Intermediation.'' *Journal of Finance,* Vol. 32 (May 1977), pp. 371-387.

[18] Modigliani, M., and M. H. Miller. ''The Cost of Capital, Corporation Finance, and the Theory of Investment.'' *The American Economic Review,* Vol. 48 (June 1958), pp. 261-297.

[19] Nelson, P. ''Information and Consumer Behavior.'' *Journal of Political Economy,* Vol. 78 (March/April 1970), pp. 311-329.

[20] Ross, S. A. ''The Determination of Financial Structure: The Incentive Signalling Approach.'' *The Bell Journal of Economics,* Vol. 8 (Spring 1977), pp. 23-40.

[21] Rothschild, M.''Searching for the Lowest Price When the Distribution of Prices Is Unknown.'' *Journal of Political Economy,* Vol. 82 (July/August 1974), pp. 689-711.

[22] Samuelson, P. A. In ''Monetary Policy and Management of the Public Debt: Hearings before the Subcommittee on General-Audit Control and Debt Management.'' Joint Committee on the Economic Report, 2nd Congress, and Session (1952), p. 697.

[23] Spence, M. A. ''Job Market Signalling.'' *Quarterly Journal of Economics,* Vol. 87 (August 1973), pp. 355-374.

[24] ________________. "Competitive and Optimal Responses to Signals: Analysis of Efficiency and Distribution." *Journal of Economic Theory,* Vol. 7 (March 1974), pp. 296-332.

[25] Stiglitz, J. E. "A Theory of 'Screening', Education, and the Distribution of Income." *The American Economic Review,* Vol. 65 (June 1975), pp. 283-300.

[26] Stiglitz, J. E., and A. Weiss. "Credit Rationing in Markets with Imperfect Information." *The American Economic Review,* Vol. 71 (June 1981), pp. 393-410.

[27] ________________. "Cutting Off Credit: An Application of Constraints as Incentive Devices." Berkeley/Stanford Finance Colloquim (February 1982).

[28] Symposium on Economics of Information, *Review of Economic Studies,* Vol. 44, (October 1977), pp. 561-572.

[29] Viscusi, K. "A Note on 'Lemons' Markets with Quality Certification." *The Bell Journal of Economics,* Vol. 9 (Spring 1978), pp. 277-280.

Appendix

Proof of Theorem 1. Define $\Pi_j^k(m)$ as the expected profit earned by bank m by lending to firm k from industry j. For notational convenience let $\phi_j^k(m)$ be the shorthand way of writing $\phi_j^k(N_{jm})$, and let $S_j^k(m)$ be the rate of interest charged by bank m to firm k from industry j.

Consider a bank D. We consider three possible cases:

(1) Bank D assumes that at least one other bank produces the necessary information to discover the probability of default δ_j^k and agrees to lend to this applicant.

(2) Bank D assumes that no other bank will offer this applicant credit.

(3) Bank D assumes that if some other bank offers this applicant credit, it will do so without producing information about δ_j^k.

Case 1. Suppose bank D decides not to produce information and knows that bank A will produce the necessary information and then offer the applicant credit. Let $S_j^k(D)$ be the rate of interest chosen by bank D and $S_j^k(A)$ the rate of interest offered by bank A. Bank D's expected profit can then be expressed as

$$\text{(A-1)} \quad \overline{\Pi}_j^k(D) = \left\{ \begin{array}{l} \text{P(applicant borrows from bank } D) \; [1-\lambda]\left[1 + S_j^k(D)\right] \\ \quad - (1+r)\,\text{P(applicant borrows from bank } D) - C \end{array} \right\}$$

where

$\text{P} \equiv$ probability measure

and

$\lambda \equiv$ applicant's expected default probability, conditional on the applicant borrowing from bank D.

Note that since there is more than one bank bidding for the applicant's business, each bank must bear the cost of participating in the bidding process, C. Rewriting (A-1) we get

$$\text{(A-2)} \quad \overline{\Pi}_j^k(D) = \left\{ \begin{array}{l} \left[\text{P}\left(1 + S_j^k(D) < 1 + S_j^k(A)\right)\right] [1-\lambda]\left[1 + S_j^k(D)\right] \\ \quad - (1+r)\,\text{P}\left(1 + S_j^k(D) < 1 + S_j^k(A)\right) - C \end{array} \right\}.$$

Now, once bank A observes another bank bidding for the applicant's business, it will treat its own information production cost, $\phi_j^k(A)$, as a sunk cost, and it will not be reflected in its rate of interest. Thus,

$$\text{(A-3)} \qquad 1 + S_j^k(A) = (1+r)\left(1 - \tilde{\delta}_j^k\right)^{-1}$$

where the tilde indicates a random variable. Define $R \equiv 1 + r$ and $1 + S_j^k(D) \equiv D_j^k$. It can then be verified that

$$\text{(A-4)} \quad \text{P}\left(\tilde{\delta}_j^k > 1 - R\left(D_j^k\right)^{-1}\right) = \left[R - D_j^k\left(1 - \delta_j^+\right)\right]\left[D_j^k\left(\delta_j^+ - \delta_j^-\right)\right]^{-1}$$

and

$$\text{(A-5)} \qquad 1-\lambda = \left[D_j^k\left(1-\delta_j^+\right) + R\right]\left[2D_j^k\right]^{-1}.$$

Thus, substituting (A-4) and (A-5) in (A-2) we get

$$\text{(A-6)} \qquad \overline{\Pi}_j^k(D) = \left\{\begin{array}{l}\left[R^2-\left(D_j^k\left(1-\delta_j^+\right)\right)^2\right]\left[2D_j^k\left(\delta_j^+-\delta_j^-\right)\right]^{-1} \\ \quad -(1+r)\left[R-D_j^k\left(1-\delta_j^+\right)\right]\left[D_j^k\left(\delta_j^+-\delta_j^-\right)\right]^{-1}-C\end{array}\right\}.$$

From bank D's perspective the optimal rate of interest, $^*D_j^k$, is obtained by setting $\partial\overline{\Pi}_j^k(D)/\partial D_j^k = 0$. It is

$$\text{(A-7)} \qquad ^*D_j^k = R\left(1-\delta_j^+\right)^{-1}.$$

Substituting (A-7) in (A-6) gives us bank D's maximum expected profit as $-C$.

If bank D decides to produce information, by (A-3) its optimal interest rate will be $(1+r)(1-\bar{\delta}_j^k)^{-1}$, and its resulting expected profit will be $-\phi_j^k(m)-C$, regardless of whether it secures the applicants business or not.

Case 2. If bank D is the only bank offering credit to the applicant, and decides to randomize, its expected profit is given by

$$\text{(A-8)} \qquad \overline{\Pi}_j^k(D) = \left\{\begin{array}{l}\left[\mathrm{P}\left(D_j^k < 1 + S_j^k(b)\right)\right][1-\lambda]\, D_j^k \\ \quad -(1+r)\,\mathrm{P}\left(D_j^k < 1 + S_j^k(b)\right)\end{array}\right\}$$

where

$$\text{(A-9)} \qquad 1 + S_j^k(b) = \left[R + \phi_j^k(b)\right]\left[1-\bar{\delta}_j^k\right]^{-1}.$$

Repeating the analysis done previously, we get the optimal interest rate

$$\text{(A-10)} \qquad ^*D_j^k = \left[R^2-\phi_j^k(b)^2\right]^{1/2}\left[1-\delta_j^+\right]^{-1}.$$

With this rate, bank D's expected profit is

$$\text{(A-11)} \qquad \left(1-\delta_j^+\right)\left(R - \left[R^2-\phi_j^k(b)^2\right]^{1/2}\right)\left(\delta_j^+-\delta_j^-\right)^{-1} > 0.$$

Let us now compute bank D's expected profit if it produces information. Since there is no other bank in the picture, bank D can charge a rate as high as

$$\text{(A-12)} \qquad \begin{aligned} 1 + S_j^k(D) \equiv D_j^k &= 1 + S_j^k(b) - \zeta\left(1_j^k\right) - e_j \\ &= \left(R + \phi_j^k(b)\right)\left(1-\bar{\delta}_j^k\right)^{-1} - \zeta\left(1_j^k\right) - e_j\,. \end{aligned}$$

Note that to be assured of getting the applicant's business, bank D could charge a rate as high as $1+S_j^k(b)-e_j$. However, if e_j is extremely small, the applicant would be better off directly accessing the bond market without expending any resources in searching and applying in the banking system. Thus, in the long run firms will opt to issue bonds rather than apply for bank loans, unless the rate charged within the banking system is such that each firm finds it cheaper to take a bank loan under the assumption that it applies to only one bank. Note that the applications received by bank m from industry j constitute a random sample from the sample of industry j firms. Thus, the actual average default probability of the bank's industry j applicants will be a random variable, δ_j^o, with $E(\delta_j^o) = \bar{\delta}_j = (\delta_j^+ + \delta_j^-)/2$. Using this fact, the minimum profit bank D can expect to make, prior to producing information, is

$$\text{(A-13)} \qquad \bar{\Pi}_j^k(D_{\min}) = \phi_j^k(b) - \phi_j^k(1_j^k) - \left(\zeta(1_j^k) + e_j\right)\left(1 - \bar{\delta}_j\right).$$

Comparing (A-11) and (A-13), we see that bank D will prefer to produce information rather than "randomize" in this case if

$$\text{(A-14)} \qquad \begin{aligned} &\left(1 - \delta_j^-\right)\left(\zeta(1_j^k) + L_j^0\right) + L_j^1 - \left(\zeta(1_j^k) + e_j\right)\left(1 - \bar{\delta}_j\right) \\ &\quad > \left(1 - \delta_j^+\right)\left(R - \left[R^2 - \phi_j^k(b)^2\right]^{1/2}\right)\left(\delta_j^+ - \delta_j^-\right)^{-1}. \end{aligned}$$

This inequality will hold because δ_j^+ is arbitrarily close to one.

Case 3. When there is another bank offering the applicant credit without producing information, and bank D itself does not produce information, suppose bank D perceives the probability of securing the applicant's business as q.

Then, its expected profit is

$$\text{(A-15)} \qquad \begin{aligned} \bar{\Pi}_j^k(D) &= q\left\{Z^2 - D_j^{k^2}\left(1 - \delta_j^+\right)^2 - 2R\left[Z - D_j^k\left(1 - \delta_j^+\right)\right]\right\} \\ &\quad \times \left\{2D_j^k\left(\delta_j^+ - \delta_j^-\right)\right\}^{-1} - C \end{aligned}$$

where $Z \equiv R + \phi_j^k(b)$. From the first-order condition $\partial\bar{\Pi}_j^k(D)/\partial D_j^k = 0$, we get the optimal interest rate

$$\text{(A-16)} \qquad D_j^{k*} = \left[R^2 - \phi_j^k(b)^2\right]^{1/2}\left(1 - \delta_j^+\right)^{-1},$$

and with this the bank's expected profit is

$$\text{(A-17)} \qquad q\left(1 - \delta_j^+\right)\left(R - \left[R^2 - \phi_j^k(b)^2\right]^{1/2}\right)\left(\delta_j^+ - \delta_j^-\right)^{-1} - C.$$

Again, with $1 - \delta_j^+$ confined to an arbitrarily small (positive) neighborhood of zero, this expression will be negative, indicating that bank D would prefer not to bid for the applicant's business.

Now, if bank D decides to produce information, its expected profit will be (with the expectation taken prior to the information production decision)

$$\begin{aligned} \bar{\Pi}_j^k(D) &= P\left[R\left(1 - \tilde{\delta}_j^k\right)^{-1} < R\left(1 - \delta_j^+\right)^{-1}\right]\left(1 - \tilde{\delta}_j^k\right)\left[R\left(1 - \tilde{\delta}_j^k\right)^{-1}\right] \\ &\quad - R\,P\left[R\left(1 - \tilde{\delta}_j^k\right)^{-1} < R\left(1 - \delta_j^+\right)^{-1}\right] - C - \phi_j^k(D), \end{aligned}$$

recalling that $R(1 - \delta_j^+)^{-1}$ is the optimal rate of interest for a bank that randomizes when it knows there is another information-producing bank extending credit to the applicant, and $R(1 - \tilde{\delta}_j^k)^{-1}$ is the optimal rate for an information-producing bank when it faces competition. Thus,

$$\overline{\Pi}_j^k(D) = -\phi_j^k(D) - C .$$

Proof of Theorem 2. To examine the properties of the solution to (8), we first specify $(\Omega^* \backslash \omega_m)$. Consider the following structure for this strategy space.

Pick an arbitrary bank, $m+1$. Identify the sets $\underline{W}_{m+1}$ and $\underline{V}_{m+1}^0$. Assume that the bank will produce information about and offer credit to all its applicants from industries that belong to $\underline{V}_{m+1}^0$, assuming for the moment that $\xi(\underline{V}_{m+1}^0) > 0$. Define $\underline{V}_{m+1}^1 \equiv \underline{W}_{m+1} \backslash \underline{V}_{m+1}^0 = \{j \mid j \in \underline{W}_{m+1}, j \notin \underline{V}_{m+1}^0\}$, and fix some $j \in \underline{V}_{m+1}^1$. We are assuming that $\xi(\underline{V}_{m+1}^1) > 0$; otherwise, $\underline{V}_{m+1}^* = \underline{W}_{m+1}$ and $^*\underline{U}_j^m = \underline{T}_j^m$. For this industry j, tabulate the sets $\underline{T}_j^h\ \forall\, h \in \underline{M}$, and calulate $\xi(\underline{T}_j^h)\ \forall\, h \in \underline{M}$. Choose the bank h_0 such that $\xi(\underline{T}_j^{h_0}) > \xi(\underline{T}_j^h)\ \forall\, h \in \underline{M},\ h \neq h_0$. Then, for each remaining bank (other than h_0) compute

$$\hat{\underline{T}}_j^h = \left\{k \mid h \in {}^*\underline{M}_j^k;\ k \notin \underline{T}_j^{h_0}\right\},$$

and find a bank h_1 such that $\xi(\hat{\underline{T}}_j^{h_1}) > \xi(\hat{\underline{T}}_j^h)\ \forall\, h \in \underline{M},\ h \neq h_0, h \neq h_1$. Continue in this manner until a set $^*\underline{T}_j^{m+1} = \{k \mid h \in {}^*\underline{M}_j^k;\ k \notin \underline{T}_j^{h_0},\ k \notin \underline{T}_j^{h_1}, \ldots\}$ is found such that

$$\xi\left({}^*\underline{T}_j^{m+1}\right) > \xi\left({}^*\underline{T}_j^h\right) \quad \forall\, h \in \underline{M},\ h \neq h_0,\ h \neq h_1, \ldots, h \neq m+1.$$

The last inequality should be viewed as an equality with zero if $^*T_j^{m+1}$ is the null set. Finally, let $\underline{V}_{m+1}^2 \equiv \{j \mid \xi({}^*\underline{T}_j^{m+1}) > 0\ \forall\, j \in \underline{V}_{m+1}^1\}$ and assume that bank $m+1$ will produce information about every industry $j \in \underline{V}_{m+1}^0 \cup \underline{V}_{m+1}^2$, and offer credit to those firms in industry j which belong to $^*\underline{U}_j^{m+1}$, where

$$\text{(A-18)} \quad {}^*\underline{U}_j^{m+1} = \begin{cases} \underline{T}_j^{m+1} & \forall\, j \in \underline{V}_{m+1}^0; \quad \xi\left(\underline{T}_j^{m+1}\right) = N_{jm} \\ {}^*\underline{T}_j^{m+1} & \forall\, j \in \underline{V}_{m+1}^2; \quad \xi\left({}^*\underline{T}_j^{m+1}\right) \leq N_{jm} . \end{cases}$$

So we assume a strategy space $(\Omega^* \backslash \omega_m)$, where element $m+1$ of the space is $\omega_{m+1}^* \equiv (\underline{V}_{m+1}^*, {}^*\underline{U}_j^{m+1}, \underline{S}^*(m+1))$, with $\underline{V}_{m+1}^* = \underline{V}_{m+1}^0 \cup \underline{V}_{m+1}^2$, $^*\underline{U}_j^{m+1}$ given by (A-18) and $\underline{S}^*(m+1) = \{{}^*\underline{S}_j^k(m+1) \mid k \in \underline{T}_j^{m+1}, j \in \underline{W}_m\}$ where

$$^*S_j^k(m+1) = \begin{cases} 1 + S_j^k(b) - \zeta\left(1_j^k\right) - e_j & \text{if } k \in {}^*\underline{U}_j^{m+1}, j \in \underline{V}_{m+1}^*, \\ \infty & \text{otherwise} . \end{cases}$$

Let ω_m^* be bank m's best reply strategy, given $(\Omega^* \backslash \omega_m)$. Note that bank m will produce information about and offer credit to all of its applicants who, given $(\Omega^* \backslash \omega_m)$, will be denied credit by every other bank in the economy if bank m's expected return from lending to these applicants exceeds the cost. It is obvious that if the sets $\underline{V}_m^*$ $(= \underline{V}_m^0 \cup \underline{V}_m^2)$ and $^*\underline{U}_j^m$ (defined implicitly in (A-18) by replacing $m + 1$ by m) have positive counting measure, those credit applicants who are members of $^*\underline{U}_j^m$, $j \in \underline{V}_m^*$, will be denied credit by every bank except m. Thus, bank m faces no competitive threat from within the banking system with respect to these applicants. If it charges an applicant from this group a rate of interest of $1 + S_j^k(b) - \zeta(1_j^k) - e_j$, its expected profit (for that applicant) will be

$$\begin{aligned}
\Pi_j^k(m) &= \left(1 + S_j^k(b) - \zeta\left(1_j^k\right) - e_j\right)\left(1-\delta_j^k\right) - \left(R + \phi_j^k(m)\right). \\
&= \left\{ \begin{aligned} &\left[\phi_j^k\left(1_{jm}\right) - \phi_j^k(m)\right] + L_j^1 + \zeta\left(1_j^k\right)\left[\delta_j^k - \delta_j^-\right] \\ &\quad + L_j^0\left(1-\delta_j^-\right) - e_j\left(1-\delta_j^k\right) \end{aligned} \right\} \\
&> 0 .
\end{aligned}$$

It remains to be shown that bank m will not want to offer credit to any of its applicants who are not members of $^*\underline{U}_j^m$, $j \in \underline{V}_m^*$. This conclusion follows from Theorem 1 which tells us that if bank m produces information or randomizes and then enters into a bidding process with some other bank (which, by our assumptions regarding $(\Omega^* \backslash \omega_m)$, will offer credit to the applicant), expected profit for bank m will be negative.

Finally, note that the procedure described above ensures that every credit applicant, as long as it applies to at least one bank, will be offered credit within the banking system. Moreover, the only banks that earn zero profits are those for which $\xi(^*\underline{U}_j^m) = 0 \; \forall j \in \underline{V}_m^*$.

Proof of Theorem 3. The fact that the equilibrium characterized in Theorem 2 is feasible even with $C = 0$ is self-evident from an examination of the proof. In Theorem 1 it was proved that if an applicant requests credit from only one bank, that bank's welfare is maximized by producing information about that applicant and extending it credit. This establishes claim (*i*). To see why claim (*ii*) holds, recall from Theorem 1 that when other banks extend an applicant credit without producing information, a given bank's expected profit from randomizing (with $C = 0$) and offering the applicant credit is

$$q\left(1-\delta_j^+\right)\left(R - \left[R^2 - \phi_j^k(b)^2\right]^{1/2}\right)\left(\delta_j^+ - \delta_j^-\right) > 0,$$

where q is the bank's perceived (subjective) probability of securing the applicant's business. The bank's expected profit from producing information in this case is

$$-\,\phi_j^k(.) < 0.$$

Thus, if a bank assumes that the other banks from which the applicant has requested credit will agree to lend without producing information, the bank's own best reply strategy is to lend without producing information rather than ration credit or acquire costly information and offer credit.

Proof of Theorem 4. We must consider three cases:

(1) Bank m produces information about firm k and offers it credit.

(2) Bank m randomizes and offers credit to firm k. If other banks offer credit to the applicant, they produce information and do so.

(3) Bank m randomizes and offers credit to firm k. Other banks that participate in bidding also randomize.

Case 1. If bank m agrees to produce information about firm k and lend to it, the bank's expected profit will be

$$\text{(A-19)} \quad \overline{\Pi}_j^k(m) = \left\{ \begin{array}{l} \left[\mathrm{E}\left\{\left(1-\tilde{\delta}_j^k\right)\left(1+S_j^k\left(m_{\min}\right) \mid \tilde{\delta}_j^k\right)\right\} - \left(R + \phi_j^k(m)\right) - C \right] P_0 \\ + \left[\mathrm{E}\left\{\left(1-\tilde{\delta}_j^k\right)\left(1+S_j^k\left(m_{\max}\right) \mid \tilde{\delta}_j^k\right)\right\} - \left(R + \phi_j^k(m)\right) \right] P_1 \end{array} \right\}$$

where

$S_j^k(m_{\min})$ $\equiv$ minimum possible rate bank m can offer the applicant,

$S_j^k(m_{\max})$ $\equiv$ maximum feasible rate bank m can offer the applicant,

P_0 $\equiv$ probability that at least one bank, other than m,will extend credit to the applicant,

P_1 $\equiv 1 - P_0$.

As we argued previously, when there is another bank bidding for the applicant's business, bank m will be willing to offer a rate as low as

$$\text{(A-20)} \quad 1 + S_j^k\left(m_{\min}\right) = R\left(1-\tilde{\delta}_j^k\right)^{-1}$$

and when there is no other bank in the picture, bank m will charge a rate as high as

$$\text{(A-21)} \quad \begin{aligned} 1 + S_j^k\left(m_{\max}\right) &= 1 + S_j^k(b) - \zeta\left(1_j^k\right) - e_j \\ &= \left(R + \phi_j^k(b)\right)\left(1-\tilde{\delta}_j^k\right)^{-1} - \zeta\left(1_j^k\right) - e_j\,. \end{aligned}$$

Substituting (A-20) and (A-21) in (A-19) gives us

$$\text{(A-22)} \quad \overline{\Pi}_j^k(m) = P_1\left\{\phi_j^k(b) - \left(1-\overline{\delta}_j\right)\left(\zeta\left(1_j^k\right) + e_j\right)\right\} - \phi_j^k(m) - P_0 C\,.$$

Making the appropriate substitution for P_1 we can now get the following necessary condition for bank m to produce the relevant information and extend credit to this applicant

$$\text{(A-23)} \quad \begin{aligned} \phi_j^k(m) < {} & \left[\prod_{h \in \Delta_j^k} \hat{\eta}_h\right] \left\{\phi_j^k(b) - \left(1-\overline{\delta}_j\right)\left(\zeta\left(1_j^k\right) + e_j\right)\right\} \\ & - \left[1 - \prod_{h \in \Delta_j^k} \hat{\eta}_h\right] C\,. \end{aligned}$$

It is obvious that the quantity inside the curly brackets in the first term is positive. The above inequality tells us that as the credit applicant approaches a larger number of banks (and $\xi(\Delta_j^k)$ increases), it becomes progressively more difficult to satisfy (A-23) and, therefore, increasingly unlikely that bank m will extend credit to the applicant. Clearly, this reasoning is valid for every other bank this applicant has approached. Consequently, the probability of the applicant being rationed by every bank rises with the number of banks it applies to. (Note that beyond some point, (A-23) may be violated for every $m^* \in \underline{M}_j^k$. The credit applicant, however, does not know each bank's $\hat{\eta}_h$, and, therefore, in determining the set $^*M_j^k$ it will have to assign its own subjective probabilities as proxies for the $\hat{\eta}_h$'s.)

Case 2. If bank m randomizes and other banks produce information, bank m's expected profit will be

$$\text{(A-24)} \quad \overline{\Pi}_j^k(m) = P_1\left(1-\delta_j^+\right)\left(R - \left\{R^2-\phi_j^k(b)^2\right\}^{1/2}\right)\left(\delta_j^+ - \delta_j^-\right)^{-1} - P_0 C \, .$$

A necessary condition for the bank to desire participation in the bidding process in this case is that the above expression be positive. That is,

$$\text{(A-25)} \quad P_1\left(1-\delta_j^+\right)\left(R - \left\{R^2-\phi_j^k(b)^2\right\}^{1/2}\right)\left(\delta_j^+ - \delta_j^-\right)^{-1} > P_0 C \, .$$

With δ_j^+ close to one, this inequality will be violated. Note, however, that the greater the number of banks the firm applies to, the smaller will be P_1 and the higher will be P_0, and the easier it will be to violate (A-25). Thus, we reach the same conclusion as in Case 1—a firm increases the probability of getting rationed by requesting credit from a larger number of banks.

Case 3. If all banks randomize, bank m's expected profit will be

$$\text{(A-26)} \quad \overline{\Pi}_j^k(m) = P_0\left[q\left(1-\delta_j^+\right)\left(R - \left\{R^2-\phi_j^k(b)^2\right\}^{1/2}\right)\left(\delta_j^+ - \delta_j^-\right)^{-1} - C\right]$$

$$\text{(A-27)} \quad + P_1\left[\left(1-\delta_j^+\right)\left(R - \left\{R^2-\phi_j^k(b)^2\right\}^{1/2}\right)\left(\delta_j^+ - \delta_j^-\right)^{-1}\right] .$$

A necessary condition for bank m to bid for the applicant's business is

$$\left[P_0 q + P_1\right]\left(1-\delta_j^+\right)\left(R - \left\{R^2-\phi_j^k(b)^2\right\}^{1/2}\right)\left(\delta_j^+ - \delta_j^-\right)^{-1} > P_0 C \, .$$

Again, as in the previous cases, a violation of this inequality will occur with δ_j^+ close to one, if the firm applies to numerous banks for credit. The reason is that in this case, a high P_0 will make the right-hand side large, whereas a high δ_j^+ will depress the left-hand side.

To examine bank m's incentive to produce information in preference to randomizing and offering credit, we compare (A-23) with (A-24) and (A-26). The former comparison yields the condition

$$\text{(A-28)} \quad P_1\Big[\phi_j^k(b) - \left(1-\bar{\delta}_j\right)\left(\zeta\left(1_j^k\right) + e_j\right)$$
$$- \left(1-\delta_j^+\right)\left(R - \left\{R^2-\phi_j^k(b)^2\right\}^{1/2}\right)\left(\delta_j^+ - \delta_j^-\right)^{-1}\Big] > \phi_j^k(m) \, ,$$

and the latter comparison gives us

$$\text{(A-29)}\qquad P_1\left[\phi_j^k(b) - \left(1-\bar{\delta}_j\right)\left(\zeta\left(1_j^k\right) + e_j\right)\right]$$

$$- \left[P_0 q + P_1\right]\left[\left(1-\delta_j^+\right)\left(R - \left\{R^2 - \phi_j^k(b)^2\right\}^{1/2}\right)\left(\delta_j^+ - \delta_j^-\right)^{-1}\right] > \phi_j^k(m)\,.$$

Thus, a high P_1 (fewer banks applied to) coupled with a high δ_j^+ ensures that banks prefer to generate costly information. If P_1 is low, banks will prefer to randomize. However, our earlier analysis shows that in this case banks choose rationing credit over bidding without information acquisition. Therefore, as the applicant approaches a smaller number of banks, each bank has a stronger incentive to produce information and extend credit to it.

The above arguments establish claims (*i*) and (*ii*). As for claim (*iii*), note that *ex post* it is possible that every applicant to which a bank extends credit may also be offered credit by some other bank. In this case, the bank's expected profit will obviously be negative. By the same token, expected profits for some banks may be positive (if a sufficiently large proportion of the applicants to whom they offer credit are denied credit by all other banks), and for some zero.